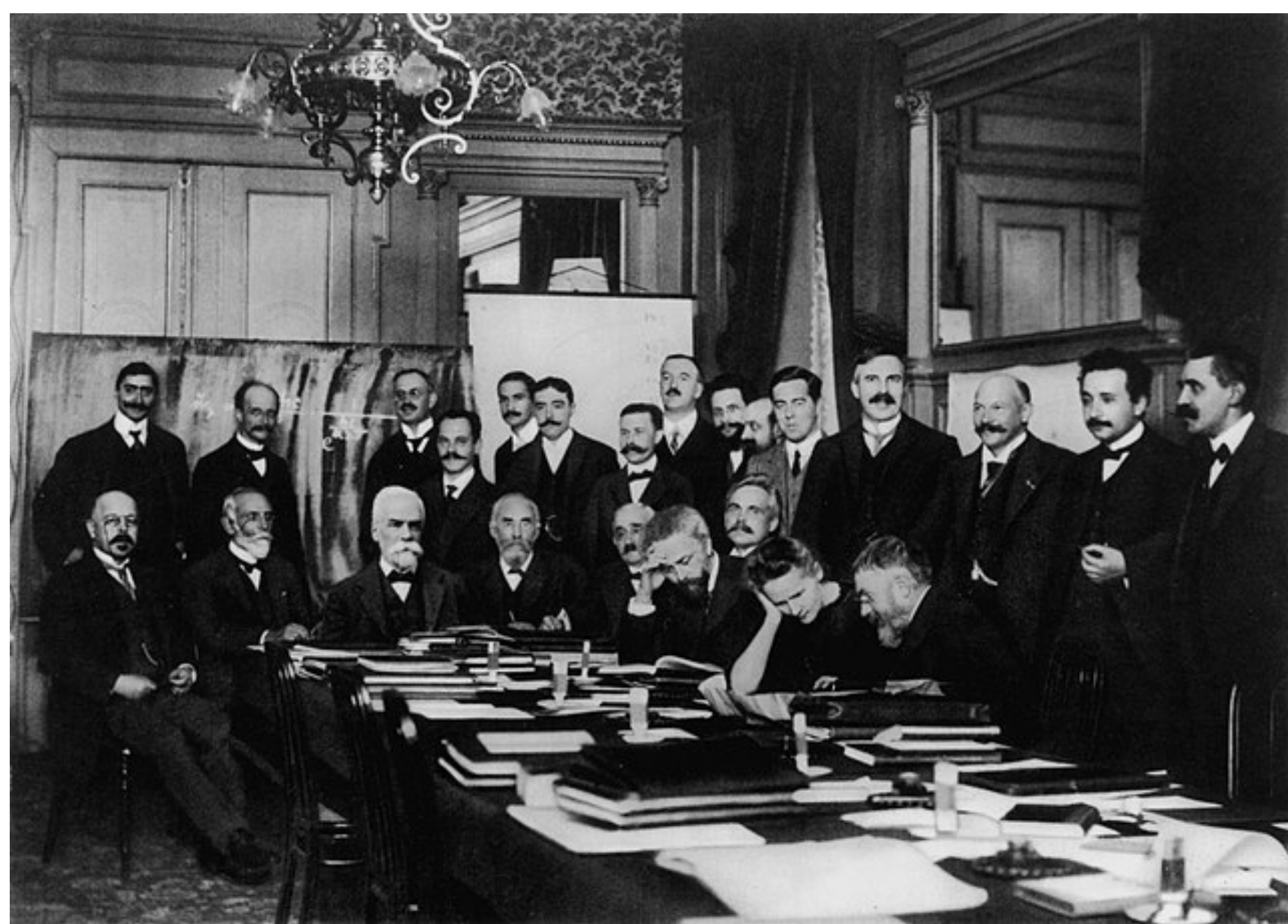


Tests of Quantum Mechanics with Ion Interferometry

Surjeet Rajendran
The Johns Hopkins University

Why?

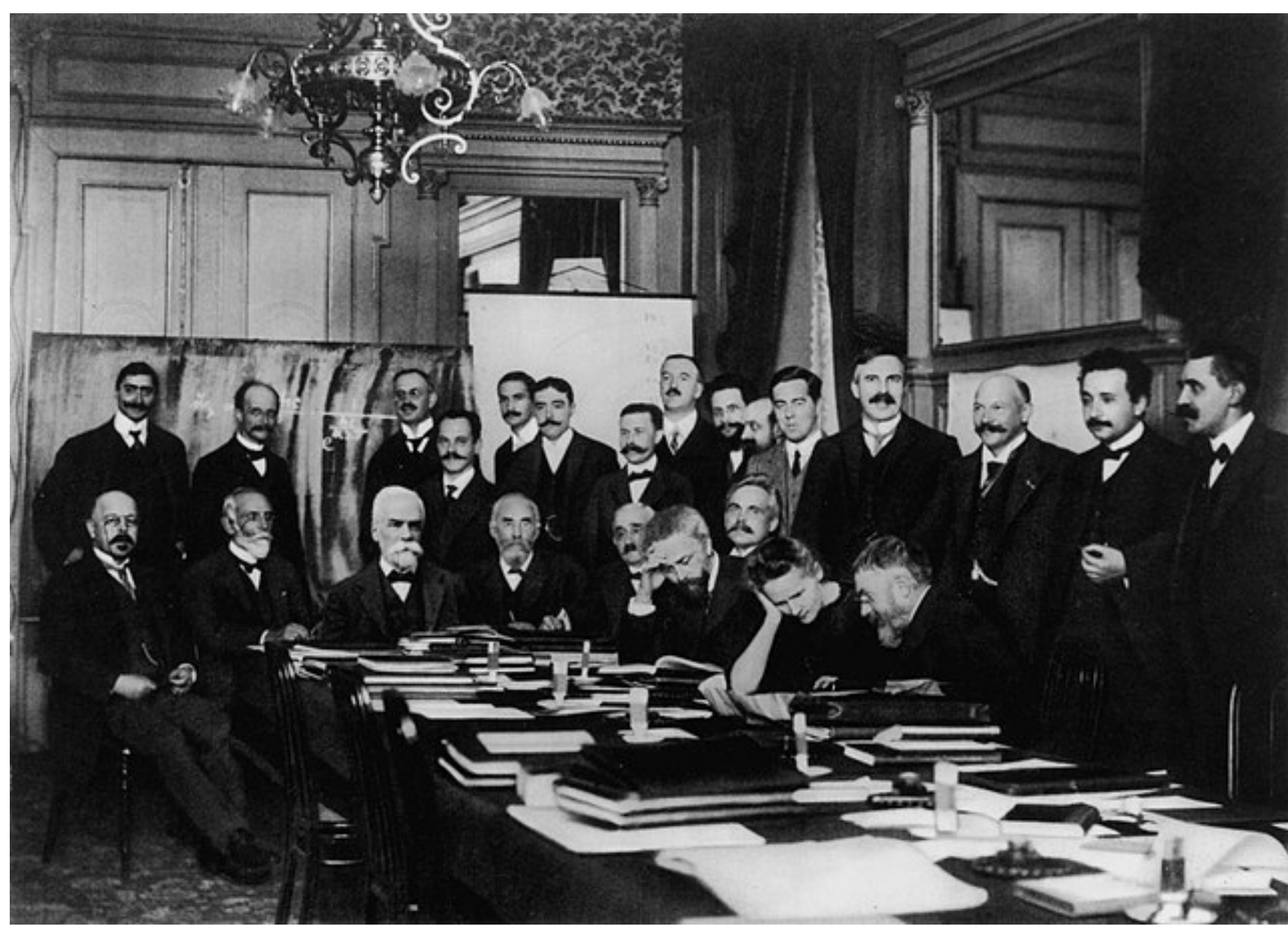
Why?



Quantum Mechanics

**Theory built on observations in the 1900s
Why should it be “the absolute truth”?**

Why?



Quantum Mechanics

Theory built on observations in the 1900s
Why should it be “the absolute truth”?

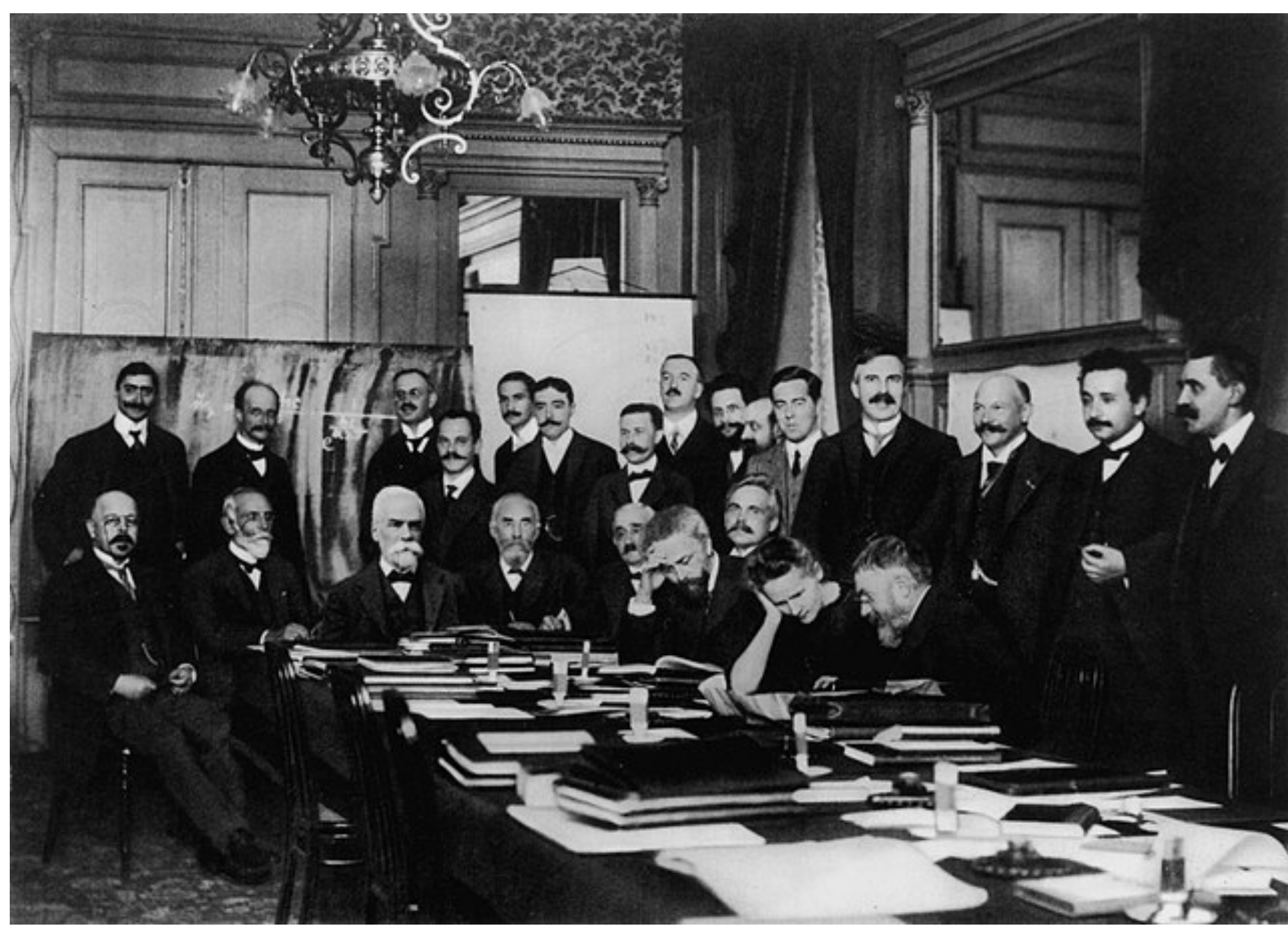
What?

Two Postulates of Quantum Mechanics

Probability

Linearity

Why?



Quantum Mechanics

Theory built on observations in the 1900s
Why should it be “the absolute truth”?

What?

Two Postulates of Quantum Mechanics

Probability

Linearity

Which?

Probability

Probability

Finite system has a finite set of energies

Continuous observables and symmetries

Probability

Finite system has a finite set of energies
Continuous observables and symmetries } **Deterministic
Observables?**

Probability

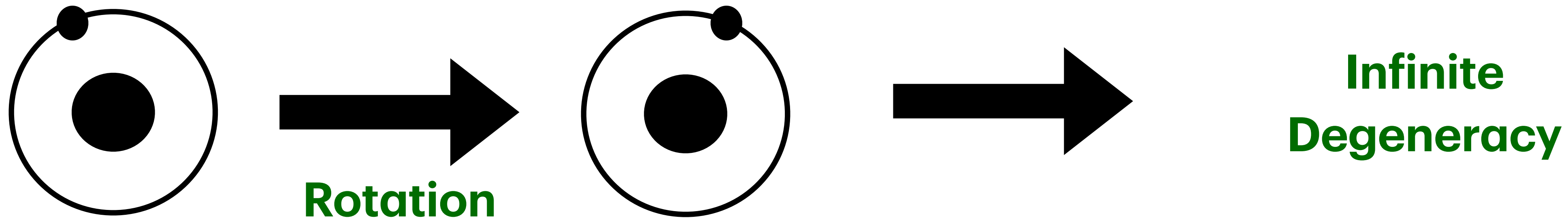
Finite system has a finite set of energies
Continuous observables and symmetries } **Deterministic
Observables?**

Could an electron in an atom have a well defined position?

Probability

Finite system has a finite set of energies
Continuous observables and symmetries } **Deterministic Observables?**

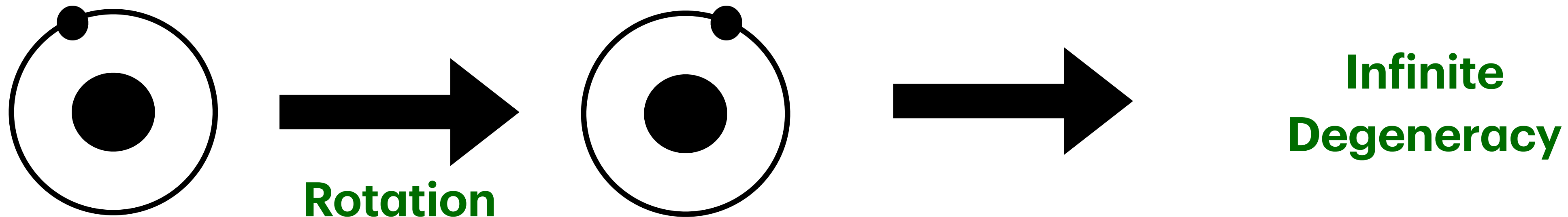
Could an electron in an atom have a well defined position?



Probability

Finite system has a finite set of energies
Continuous observables and symmetries } Deterministic Observables?

Could an electron in an atom have a well defined position?



Quantum Mechanics

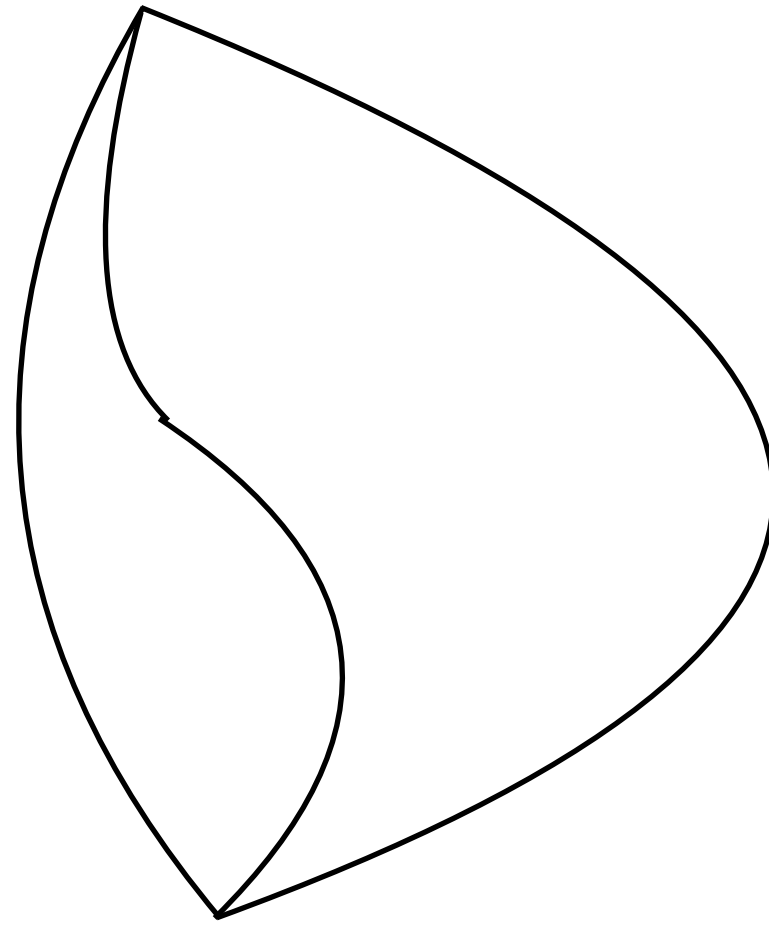
Sacrifice Determinism.

Preserve finite set of energy states, continuous symmetries and observables

Bell Inequalities, Kochen-Specker, SSC Theorems

Linearity

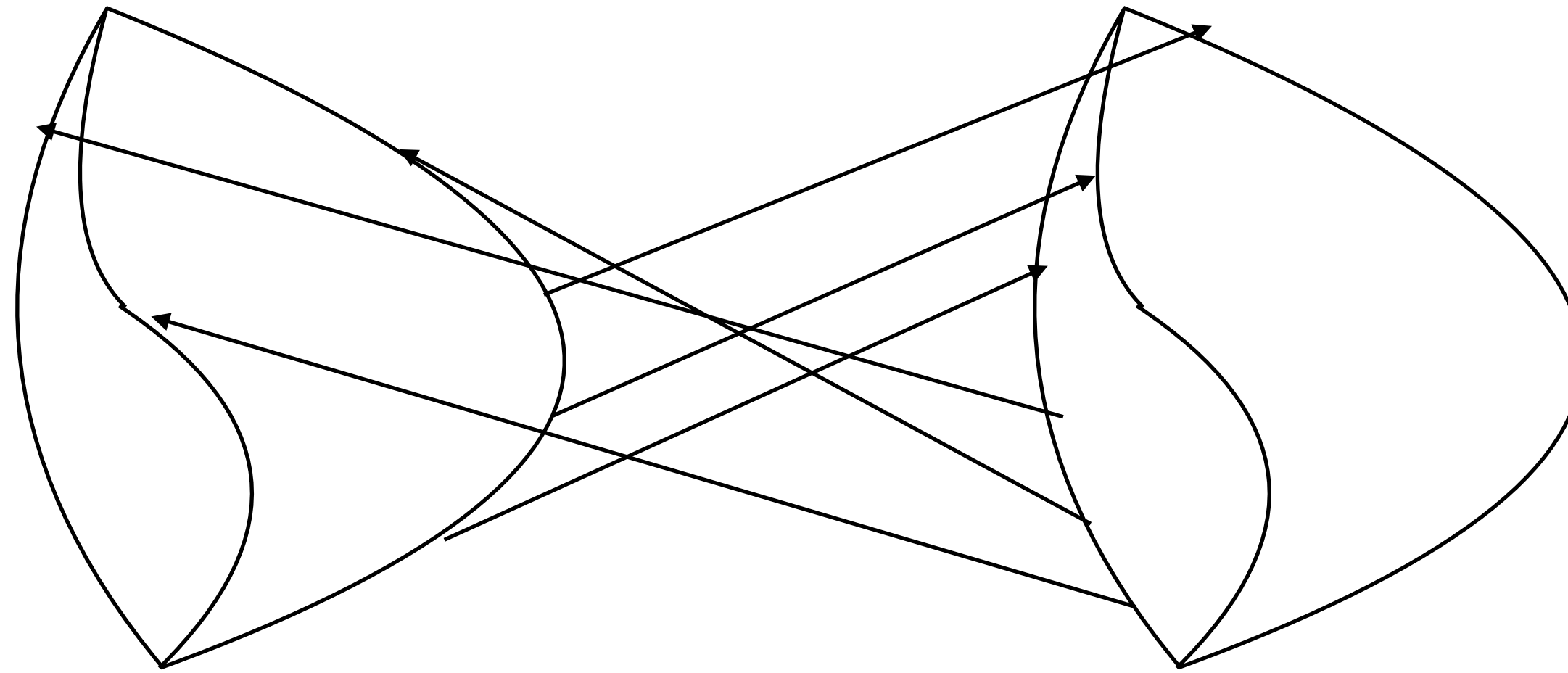
Electron Coupled to Electromagnetism



**Electron paths do not
interact via
electromagnetism**

Linearity

Electron Coupled to Electromagnetism

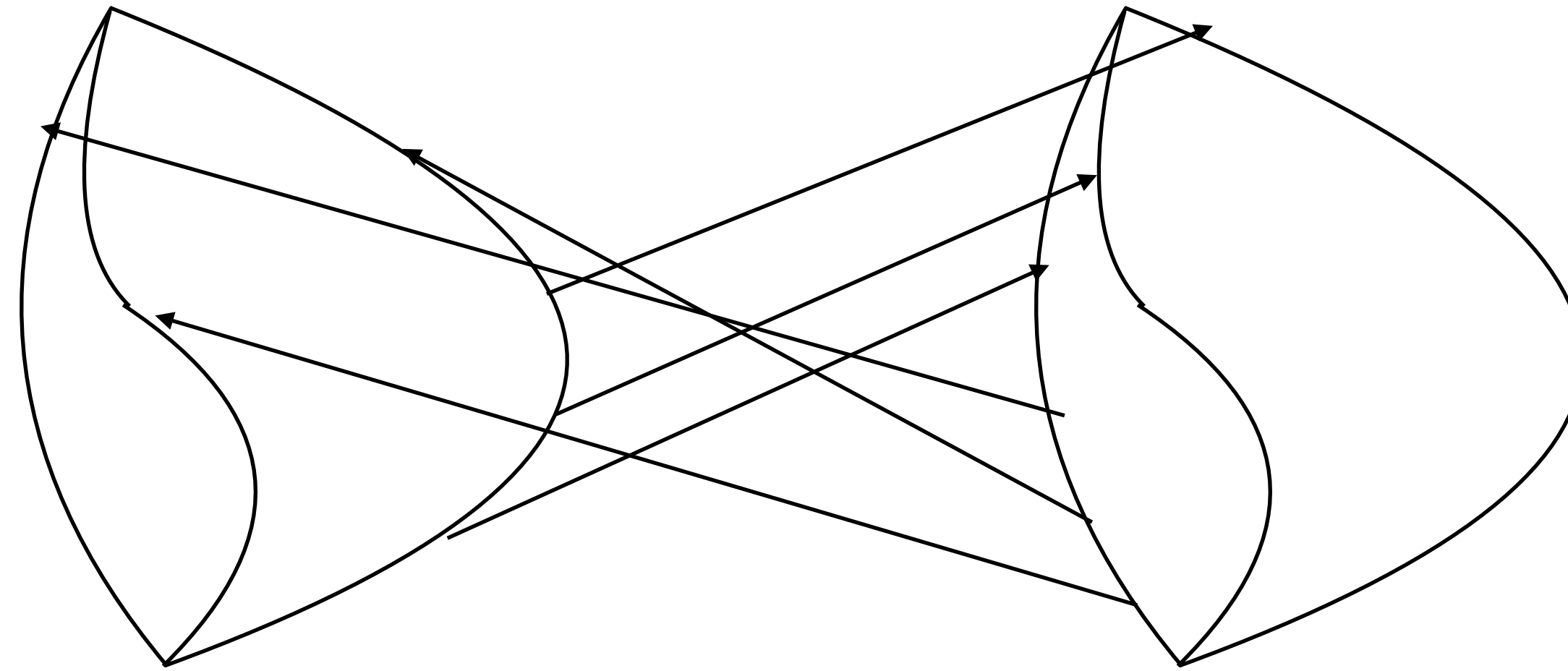


**Electron paths do not
interact via
electromagnetism**

**Paths of two electrons
interact causally (QFT)**

Linearity

Electron Coupled to Electromagnetism



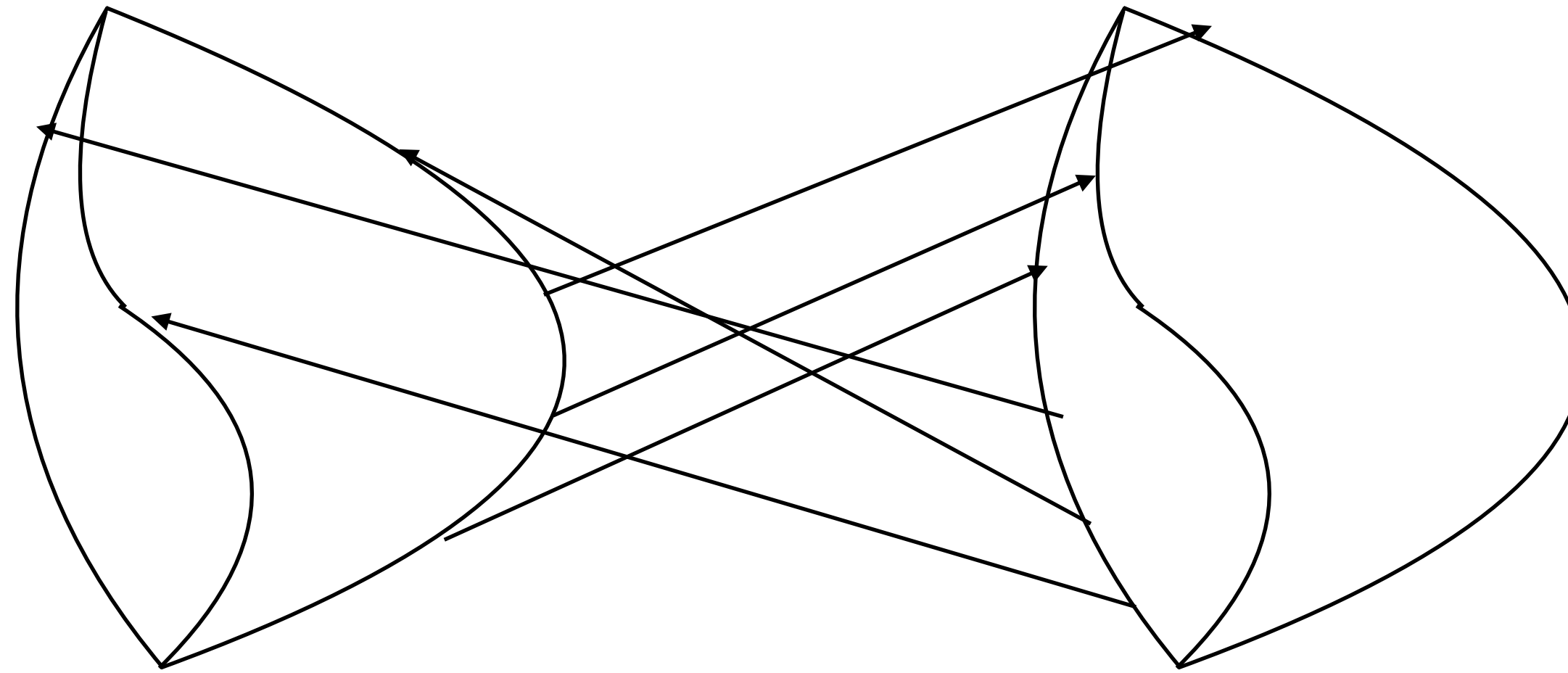
**Electron paths do not
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**Paths of two electrons
interact causally (QFT)**

Why can't path talk to itself?

Linearity

Electron Coupled to Electromagnetism



**Electron paths do not
interact via
electromagnetism**

**Paths of two electrons
interact causally (QFT)**

Why can't path talk to itself?

$$A_\mu \rightarrow A_\mu + \epsilon \langle A_\mu \rangle$$

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \epsilon \langle g_{\mu\nu} \rangle$$

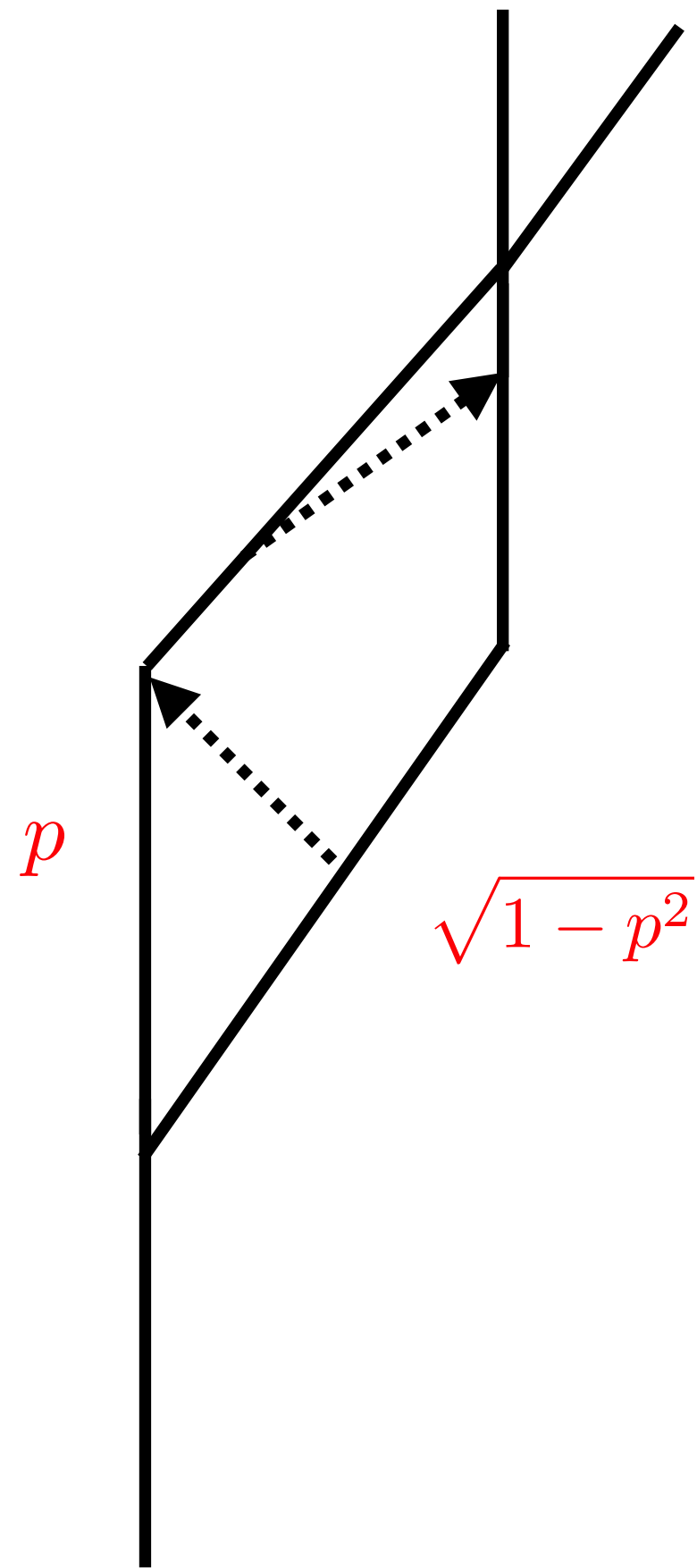
Experimental Tests

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function

Coulomb Field of one path interacts with the other path



Experimental Tests

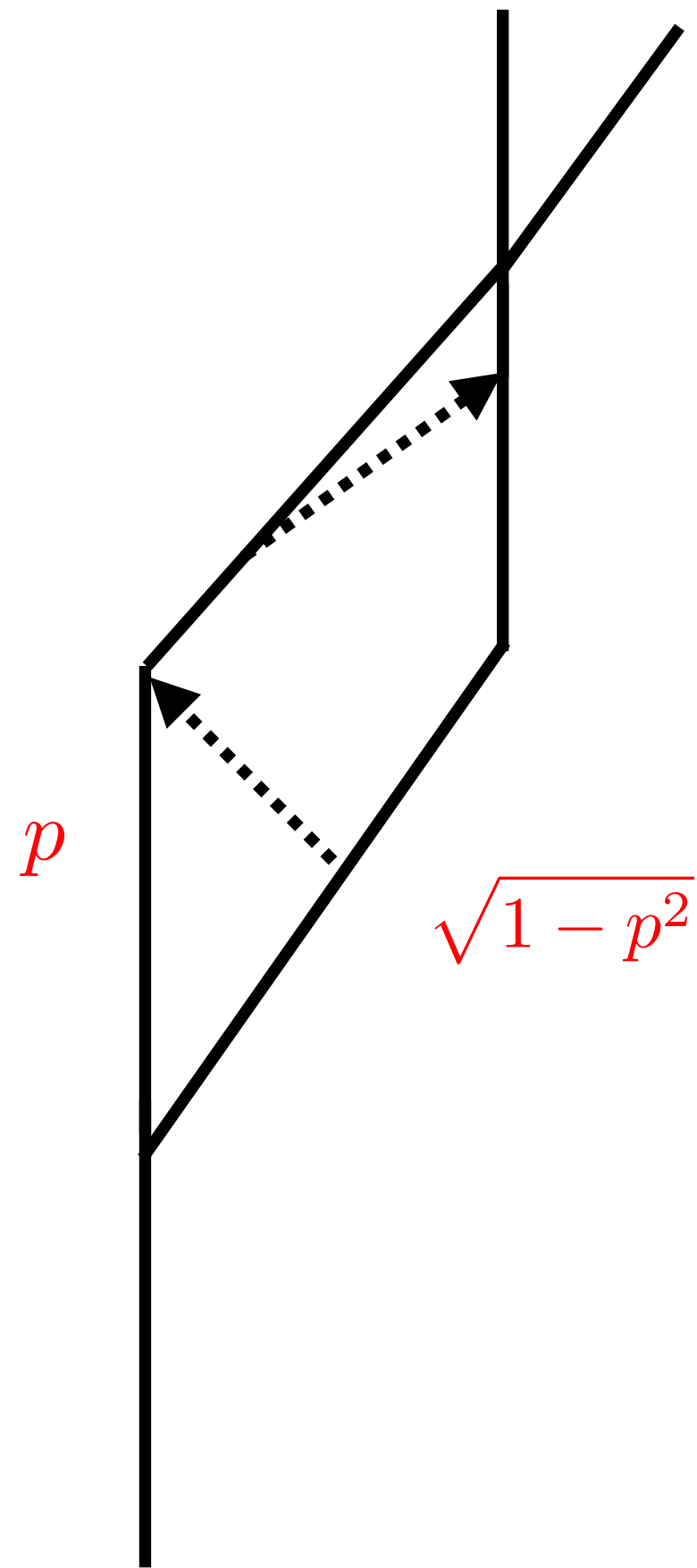
Interferometry - interaction between paths

Take an ion - split its wave-function

Coulomb Field of one path interacts with the other path

Gives rise to phase shift that depends on the intensity p of the split

Use intensity dependence to combat systematics



Implementation

Put ion in an ion trap

**Place in spatial superposition, want a way to read out
relative phase**

Implementation

Put ion in an ion trap

Place in spatial superposition, want a way to read out
relative phase

Harmonic Oscillator States: $|0\rangle$, $|1\rangle$

Ion Electronic States: $|S\rangle$, $|D\rangle$

Implementation

Put ion in an ion trap

Place in spatial superposition, want a way to read out relative phase

Harmonic Oscillator States: $|0\rangle$, $|1\rangle$

Ion Electronic States: $|S\rangle$, $|D\rangle$

Steps

(1) Initial State: $\alpha |0,S\rangle + \beta |0,D\rangle$

Implementation

Put ion in an ion trap

Place in spatial superposition, want a way to read out relative phase

Harmonic Oscillator States: $|0\rangle$, $|1\rangle$

Ion Electronic States: $|S\rangle$, $|D\rangle$

Steps

(1) Initial State: $\alpha |0,S\rangle + \beta |0,D\rangle$

(2) Send $|0,S\rangle \rightarrow |1,D\rangle$

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Spatial Superposition

Implementation

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Free Evolution for time T

$$(\alpha |0\rangle + \beta e^{i\Delta ET} |1\rangle) |D\rangle$$

Implementation

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Free Evolution for time T

$$(\alpha |0\rangle + \beta e^{i\Delta ET} |1\rangle) |D\rangle$$

(3) Send $|1,D\rangle \rightarrow |0,S\rangle$ $(\alpha |0, D\rangle + \beta e^{i\Delta ET} |0, S\rangle)$

Implementation

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Free Evolution for time T

$$(\alpha |0\rangle + \beta e^{i\Delta ET} |1\rangle) |D\rangle$$

(3) Send $|1,D\rangle \rightarrow |0,S\rangle$ $(\alpha |0, D\rangle + \beta e^{i\Delta ET} |0, S\rangle)$

(4) Final Splitter

$$|0, D\rangle \rightarrow \frac{1}{\sqrt{2}} (|0, D\rangle + |0, S\rangle) \quad |0, S\rangle \rightarrow \frac{1}{\sqrt{2}} (|0, D\rangle - |0, S\rangle)$$

Implementation

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Free Evolution for time T

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Final State $\frac{1}{\sqrt{2}} ((\alpha + \beta e^{i\delta ET}) |0, D\rangle + (\alpha - \beta e^{i\delta ET}) |0, S\rangle)$

Implementation

State: $(\alpha |0\rangle + \beta |1\rangle) |D\rangle$

Free Evolution for time T

$$(\alpha |0\rangle + \beta e^{i\Delta ET} |1\rangle) |D\rangle$$

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Final State $\frac{1}{\sqrt{2}} ((\alpha + \beta e^{i\delta ET}) |0, D\rangle + (\alpha - \beta e^{i\delta ET}) |0, S\rangle)$

Count ions in $|0, D\rangle$ vs $|0, S\rangle$

Results

$^{40}\text{Ca}^+$ Ion

**T ~ 10 ms
(Ion decoherence)**

O(1200) measurements total

Trap Localization ~ 10 nm

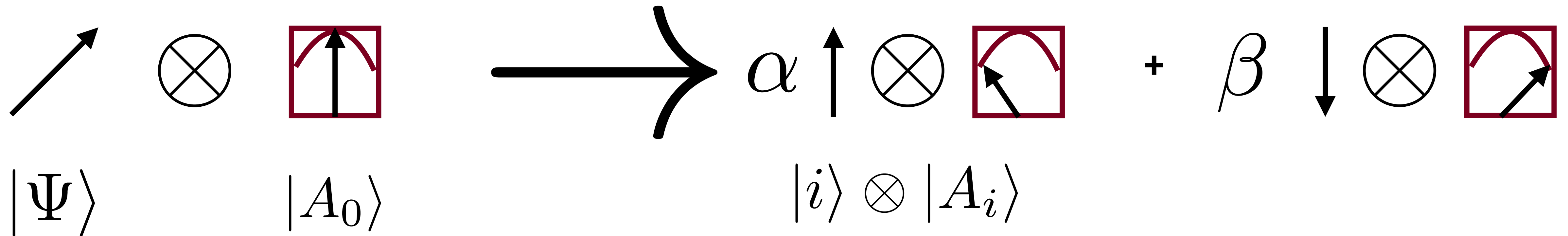
$$\epsilon \lesssim 5 \times 10^{-12}$$

Macroscopic Effects

Measurement in Quantum Mechanics

Not some mysterious process

Interaction between quantum state and measuring device

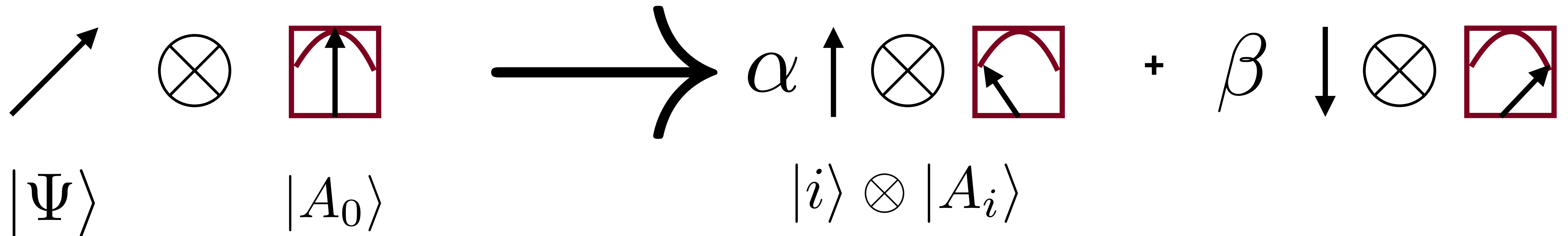


$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle$$

Measurement in Quantum Mechanics

Not some mysterious process

Interaction between quantum state and measuring device

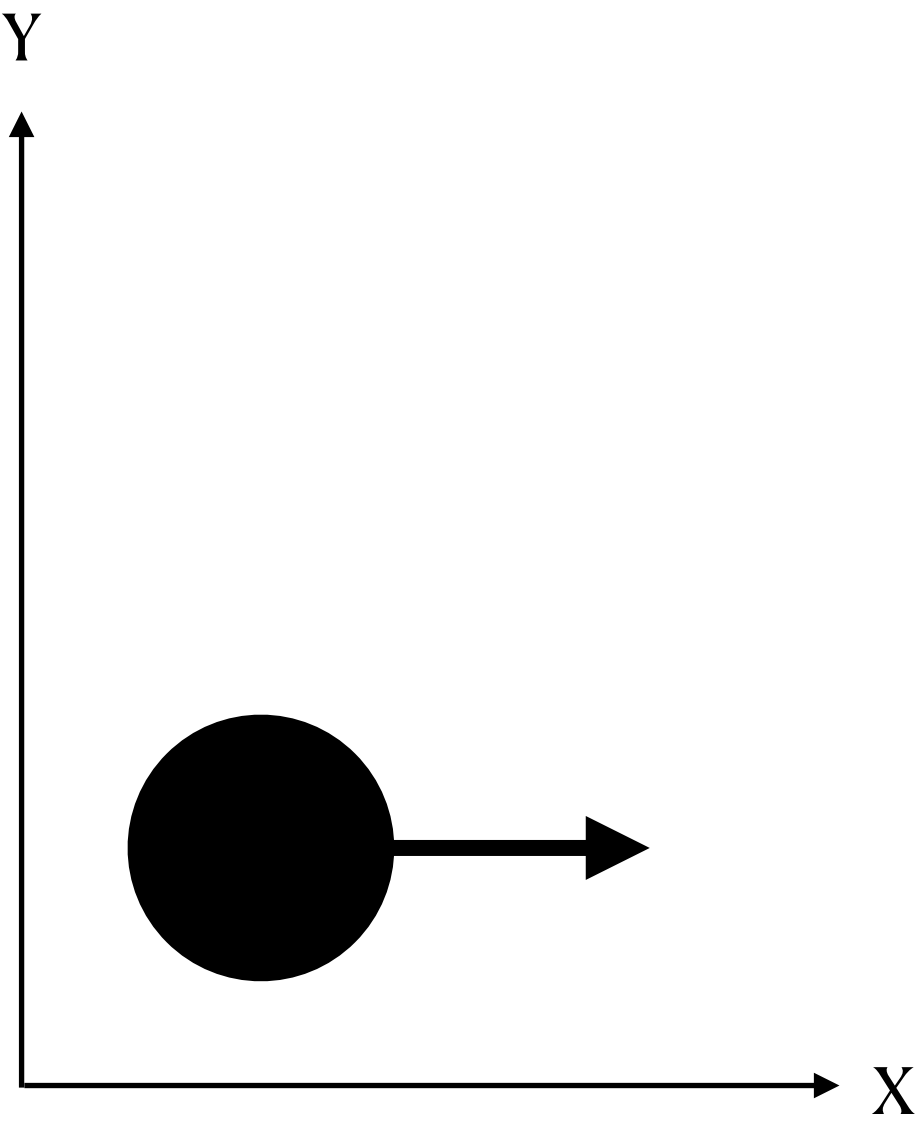


$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle$$

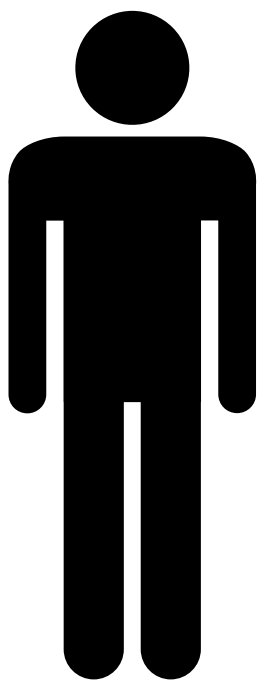
Prediction of Quantum Mechanics ("Many Worlds"), Not an interpretation

Pick: $\langle A_j | A_i \rangle = \delta_{ij} \implies \rho_{|\Psi\rangle} = \sum_i c_i c_i^* |i\rangle \langle i|$ "Interpret" as direct sum of "worlds"

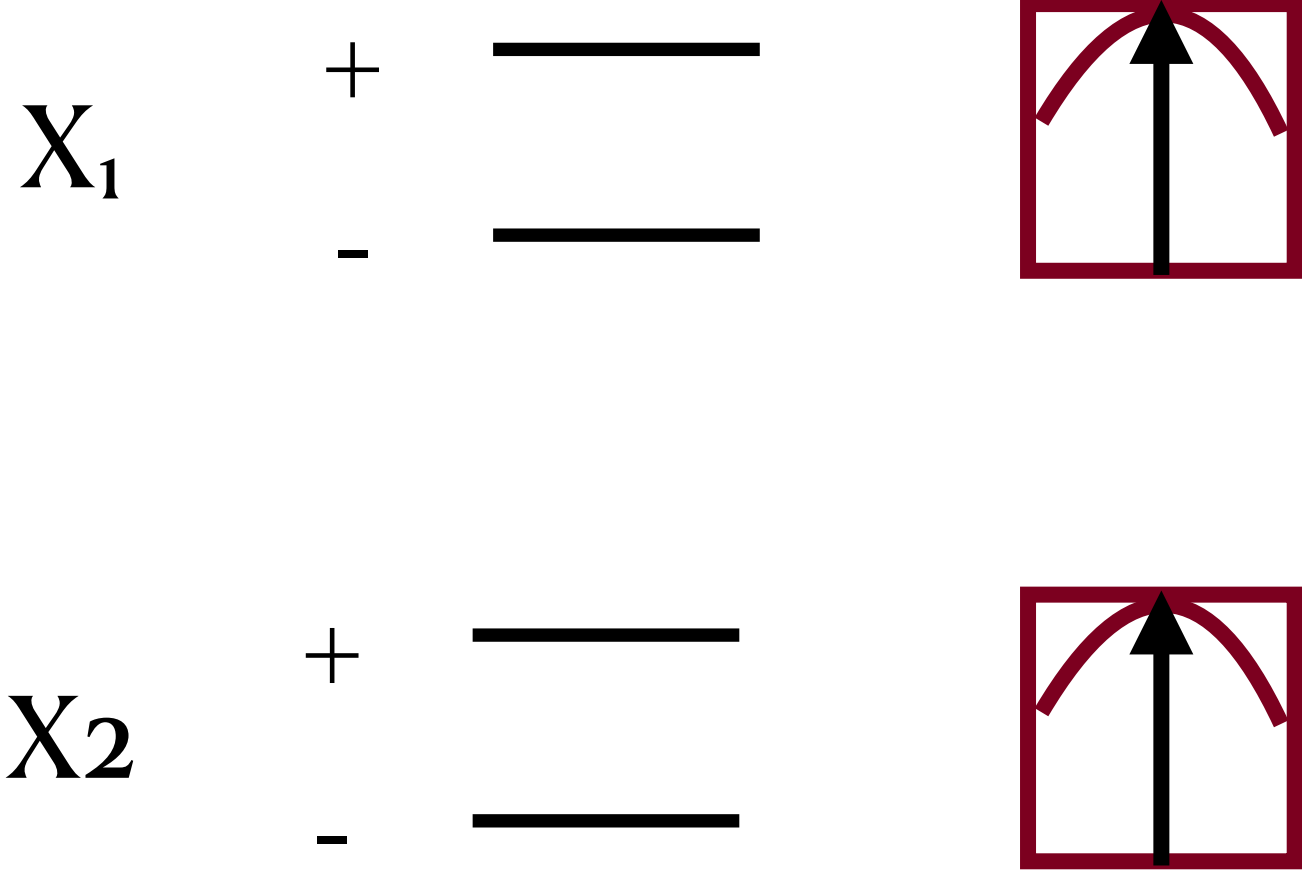
Linear Quantum Mechanics



Spin
Along x



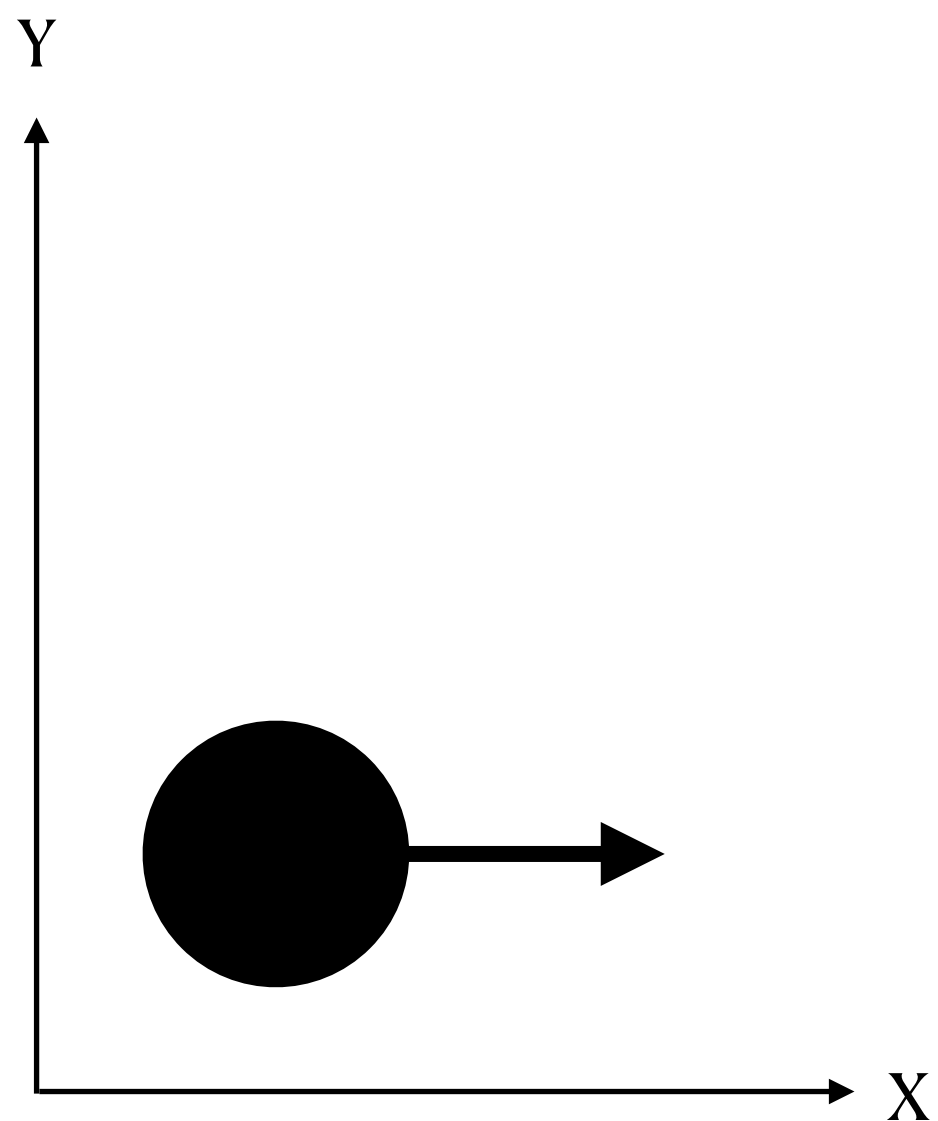
Experimentalist



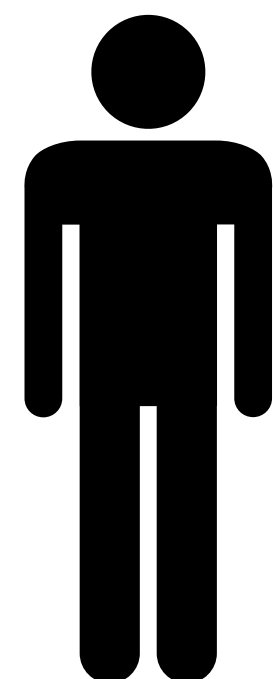
Initial State : $|x(0)\rangle$

Represents Full Quantum State (spin, experimentalist...)

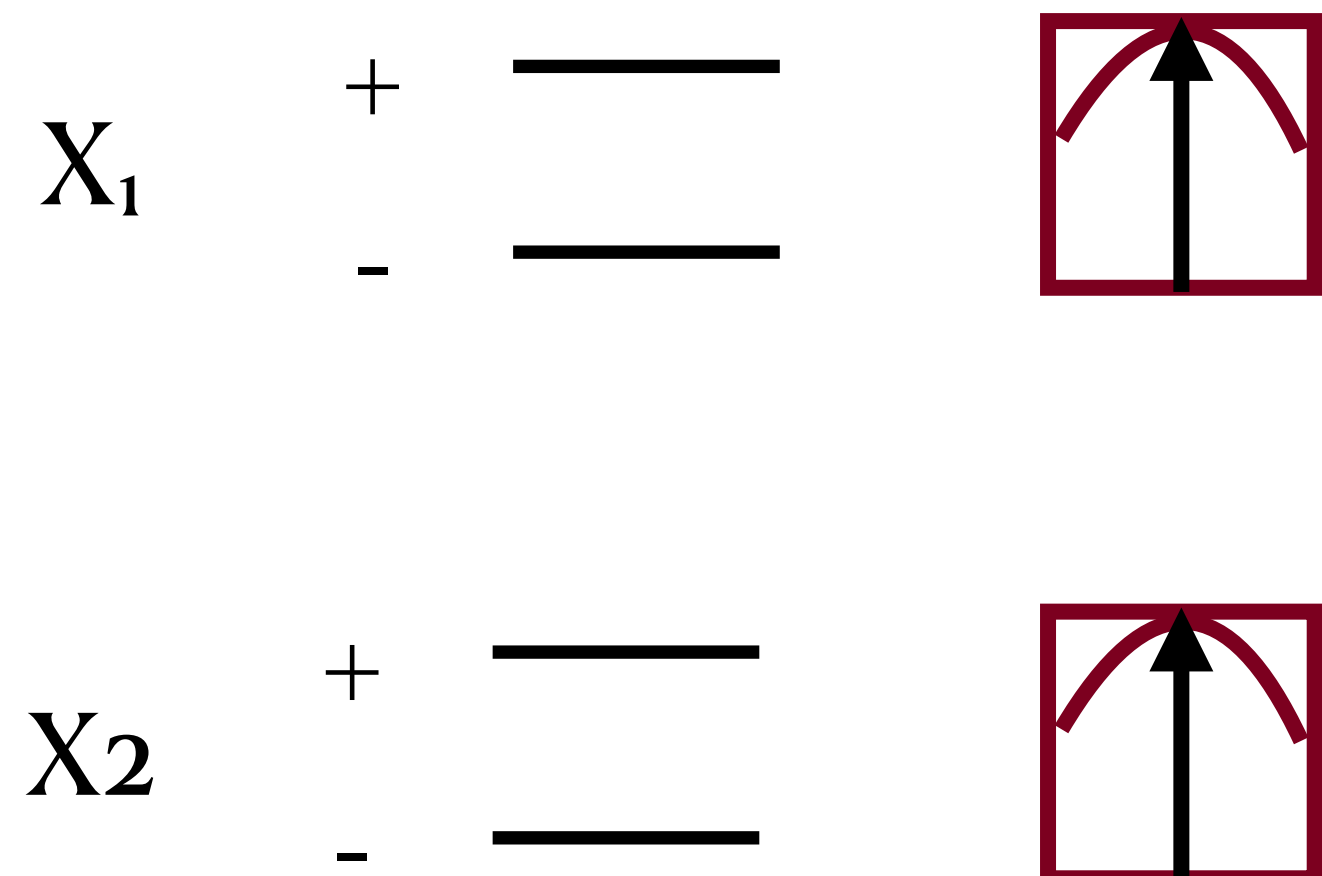
Linear Quantum Mechanics



**Spin
Along x**



Experimentalist



Initial State : $|\chi(0)\rangle$

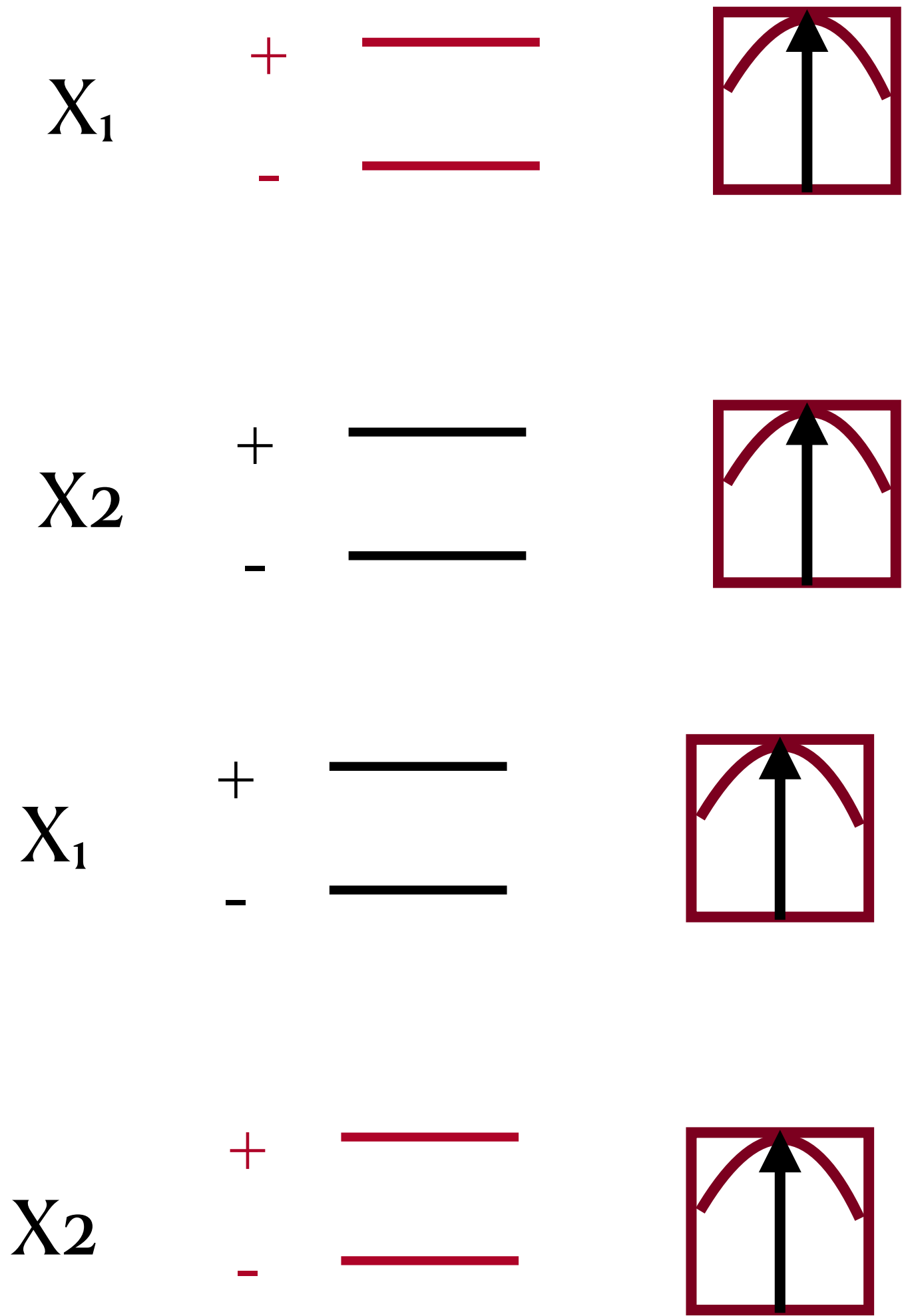
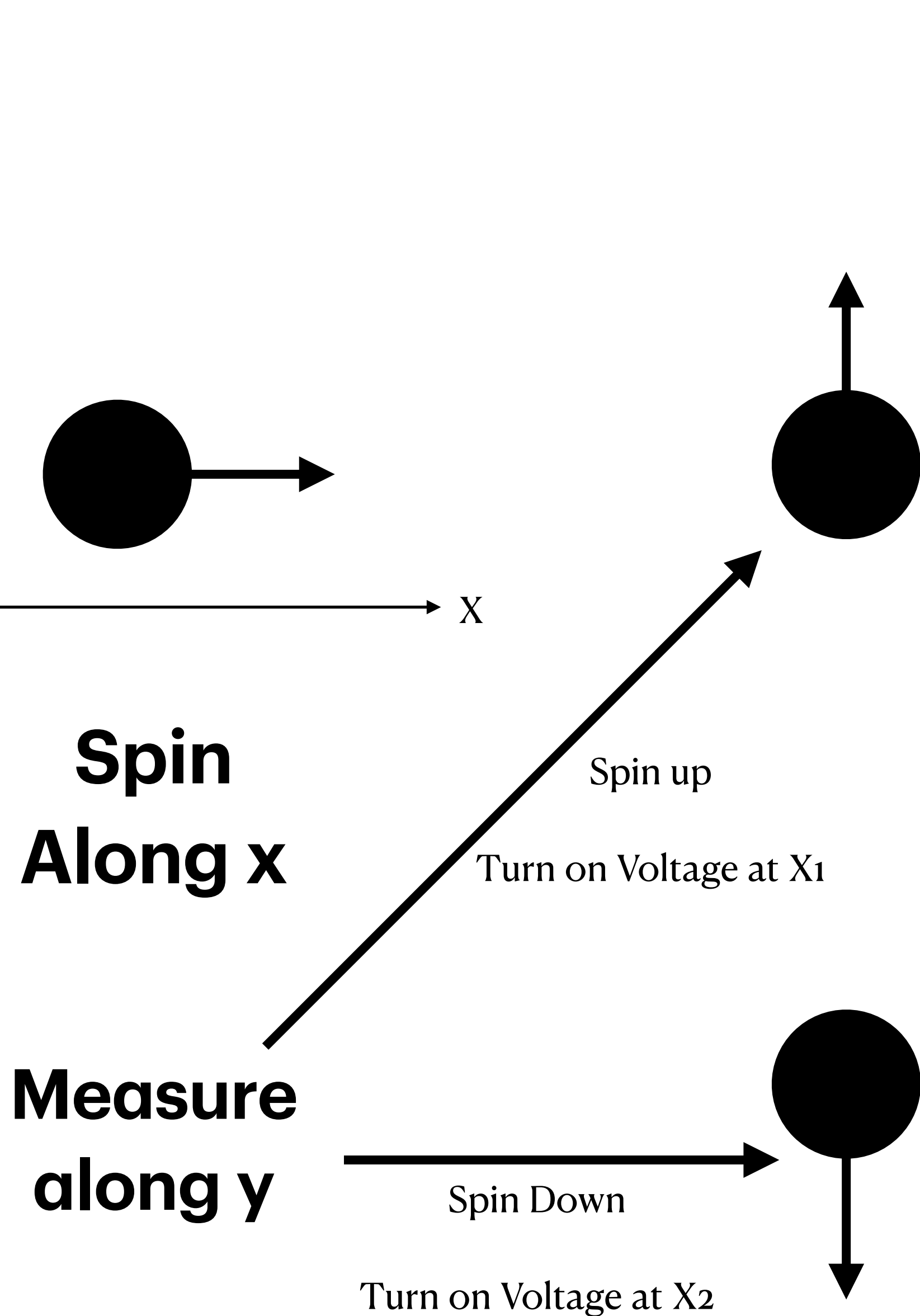
Represents Full Quantum State (spin, experimentalist...)

Measure spin along y.

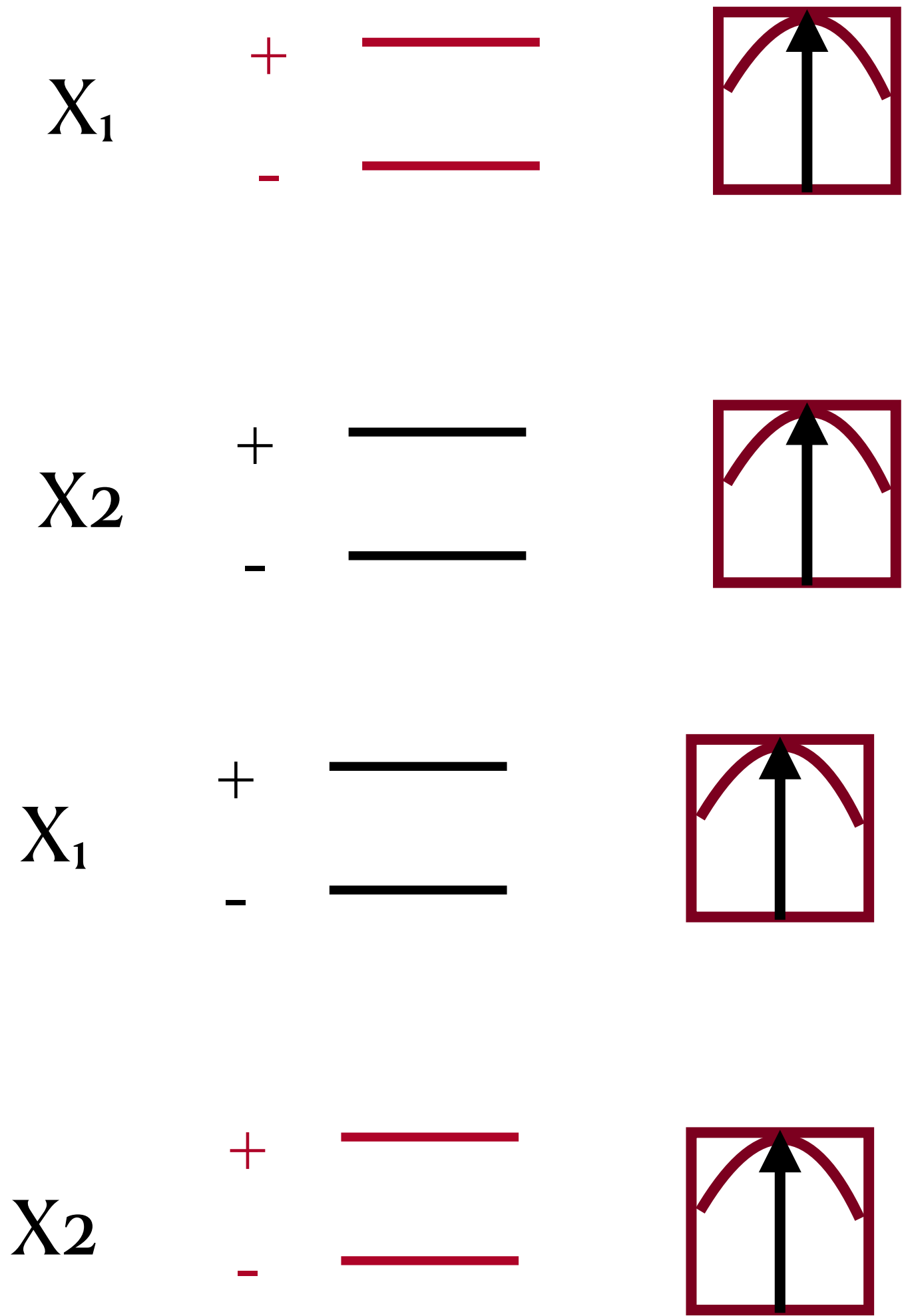
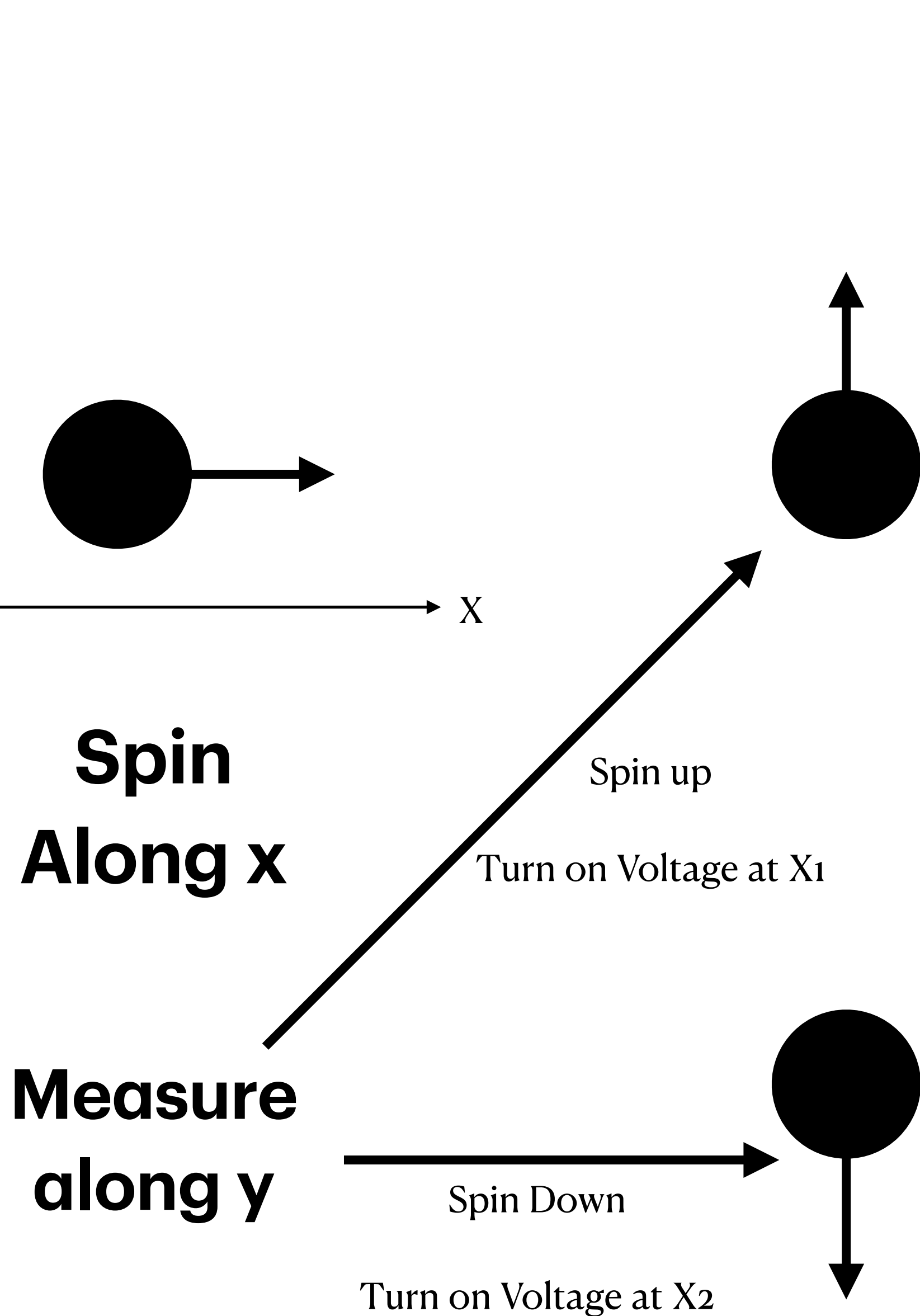
Based on outcome, turn on voltage source at X_1 or X_2 .

What is the quantum state after measurement?

Macroscopic Superposition



Macroscopic Superposition



Final State: $|X\rangle = |U\rangle|V_1\rangle|E_1\rangle + |D\rangle|V_2\rangle|E_2\rangle$

Prediction of QM
(Many Worlds)

Linear Quantum Mechanics

Which Voltage sensors light up?

$$|\chi\rangle = |\mathbf{U}\rangle|\mathbf{V}_1\rangle|\mathbf{E}_1\rangle + |\mathbf{D}\rangle|\mathbf{V}_2\rangle|\mathbf{E}_2\rangle$$

$$\mathcal{L} \supset eA_\mu \bar{\Psi} \gamma^\mu \Psi$$

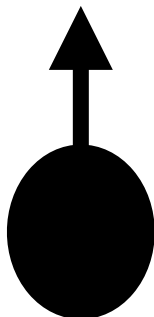
Compute Transition Matrix Elements

$$\langle U|\langle V_1|\langle E_1|eA_\mu(x_1)\bar{\Psi}(x_1)\gamma^\mu\Psi(x_1)|\chi\rangle\neq 0$$

$$\langle U|\langle V_1|\langle E_1|eA_\mu(x_2)\bar{\Psi}(x_2)\gamma^\mu\Psi(x_2)|\chi\rangle=0$$



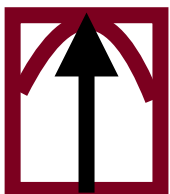
$$\langle V_1|A_\mu(x_2)|V_1\rangle=0$$



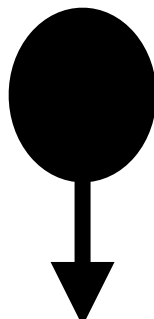
X_1



X_2



X_1



X_2



Linear Quantum Mechanics

Which Voltage sensors light up?

$$|\chi\rangle = |U\rangle|V_1\rangle|E_1\rangle + |D\rangle|V_2\rangle|E_2\rangle$$

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$$\langle U | \langle V_1 | \langle E_1 | eA_\mu(x_1) \bar{\Psi}(x_1) \gamma^\mu \Psi(x_1) | \chi \rangle \neq 0$$

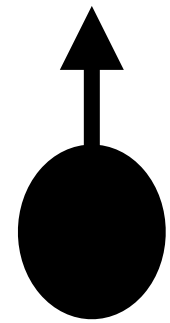
$$\langle U | \langle V_1 | \langle E_1 | eA_\mu(x_2) \bar{\Psi}(x_2) \gamma^\mu \Psi(x_2) | \chi \rangle = 0$$



$$\langle V_1 | A_\mu(x_2) | V_1 \rangle = 0$$

But in both $|V_1\rangle, |V_2\rangle$:

$$\langle \chi | A_\mu(x_1) | \chi \rangle \neq 0, \langle \chi | A_\mu(x_2) | \chi \rangle \neq 0$$



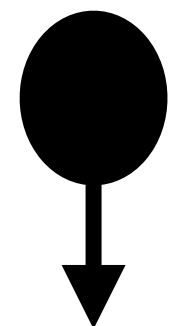
X_1



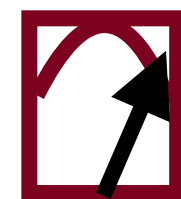
X_2



X_1

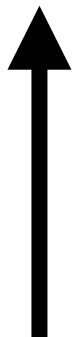


X_2



Non-Linear Quantum Mechanics

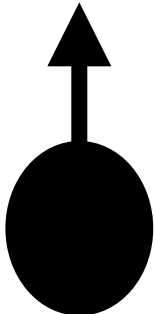
$$\mathcal{L} \supset e A_\mu \bar{\Psi} \gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi} \gamma^\mu \Psi$$



State Dependent Non-linear Term

But in both $|V_1\rangle, |V_2\rangle$:

$$\langle \chi | A_\mu (x_1) | \chi \rangle \neq 0, \langle \chi | A_\mu (x_2) | \chi \rangle \neq 0$$



X_1



X_2



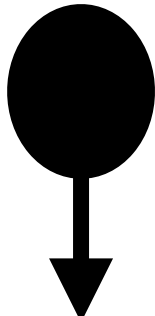
ϵ

X_1



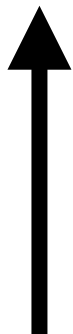
ϵ

X_2



Non-Linear Quantum Mechanics

$$\mathcal{L} \supset e A_\mu \bar{\Psi} \gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi} \gamma^\mu \Psi$$



State Dependent Non-linear Term

But in both $|V_1\rangle, |V_2\rangle$:

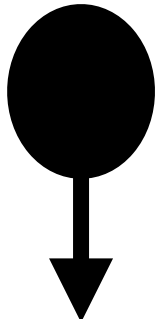
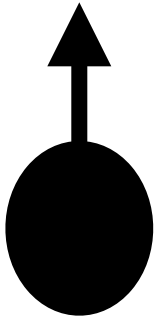
$$\langle \chi | A_\mu (x_1) | \chi \rangle \neq 0, \langle \chi | A_\mu (x_2) | \chi \rangle \neq 0$$

Communication between “worlds”

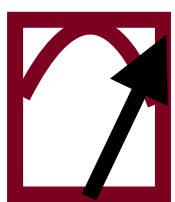
Consequence of Causality - trace over entangled particles

Non-linearity visible despite Environmental De-coherence!

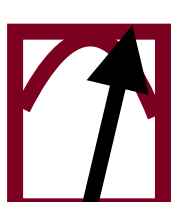
Polchinski: “Everett Phone”



X_1

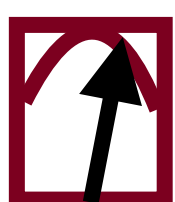


X_2



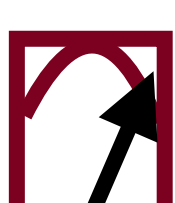
ϵ

X_1



ϵ

X_2



Experimental Tests

Key Point: Create macroscopic superposition

Create Expectation value of EM/Gravity

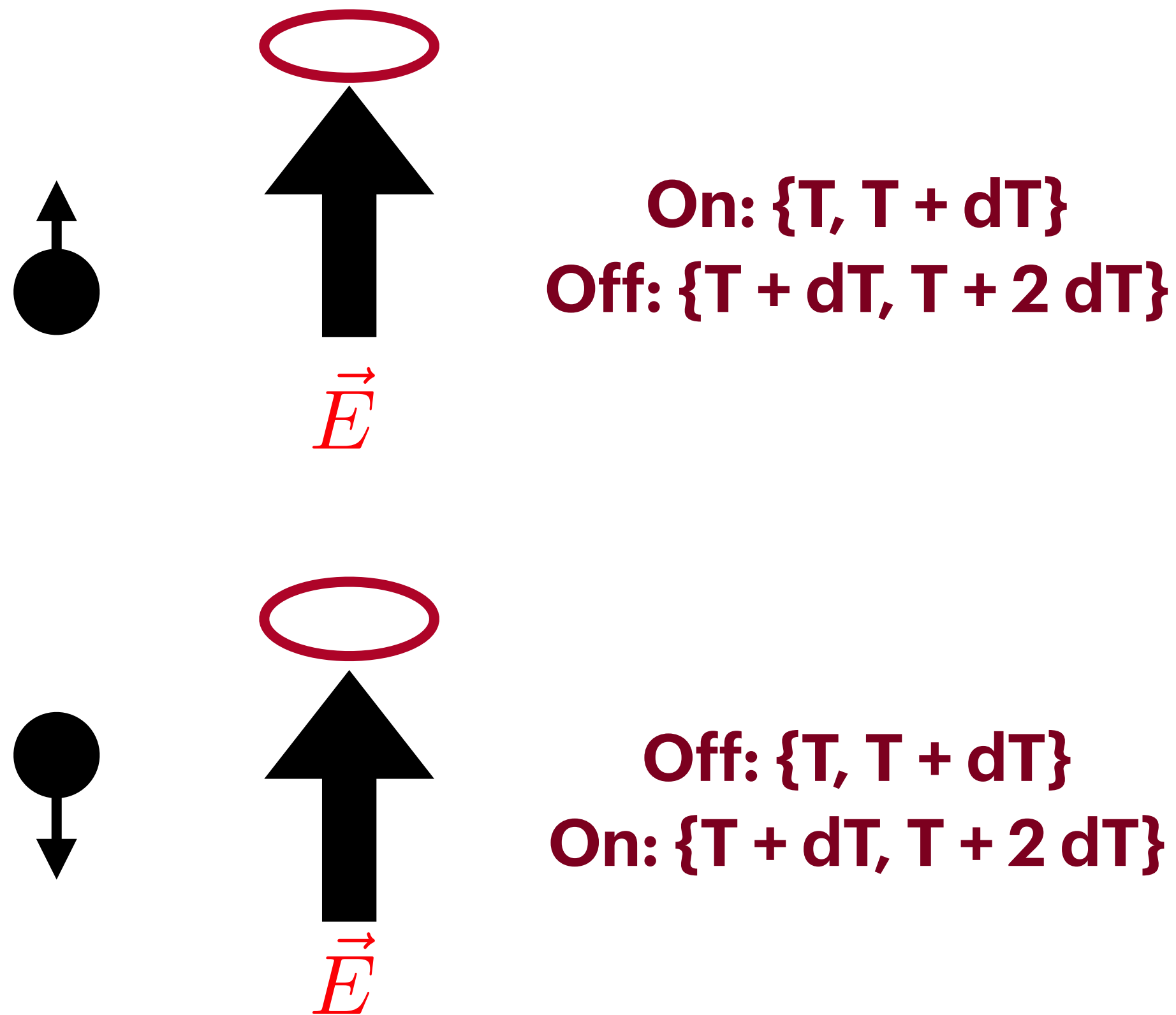
Search for Expectation value

Experimental Tests

Key Point: Create macroscopic superposition

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Search for Expectation value

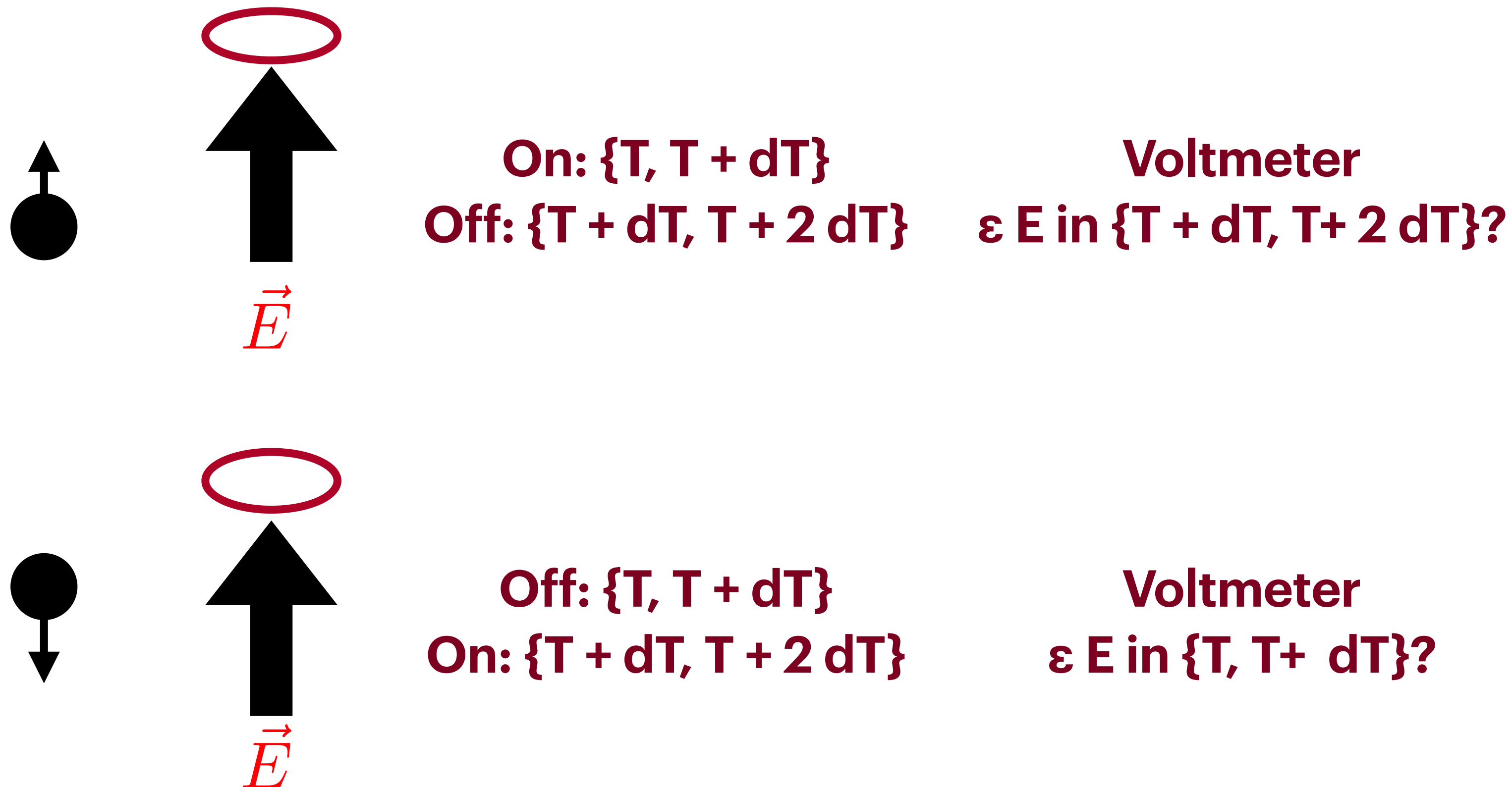


Experimental Tests

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Search for Expectation value

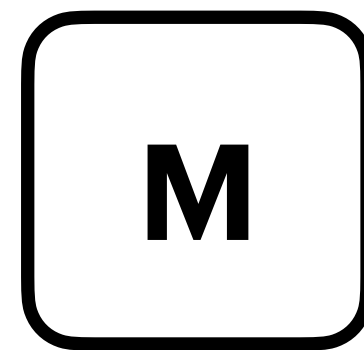
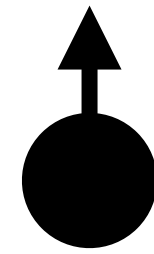


Experimental Tests

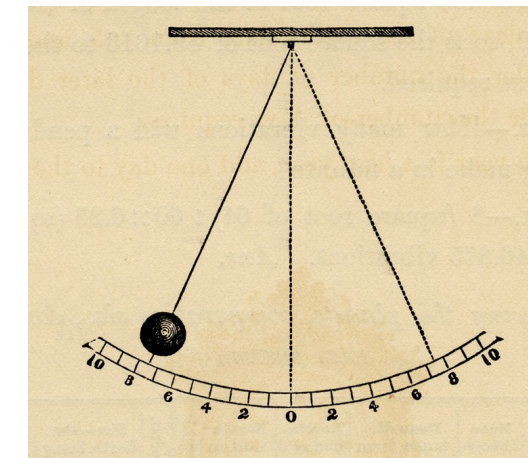
Key Point: Create macroscopic superposition

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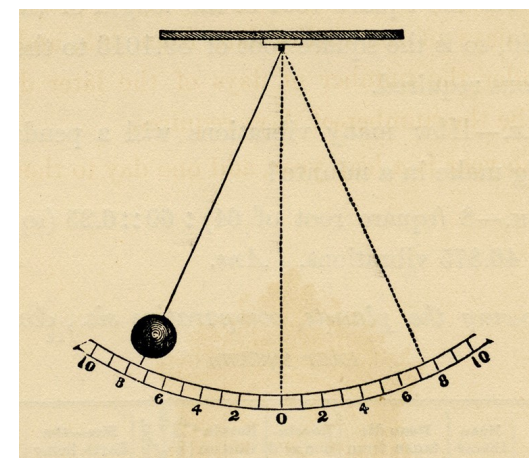
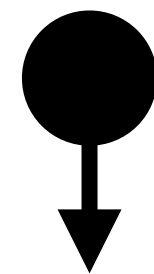
Search for Expectation value



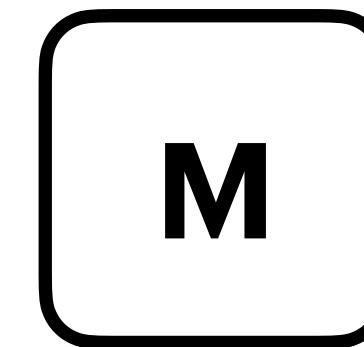
X₁



X₂



X₁



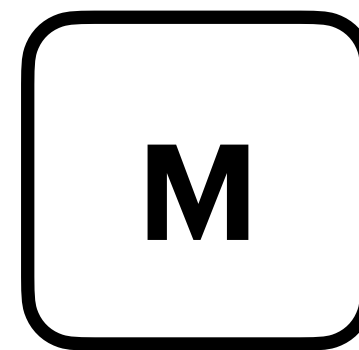
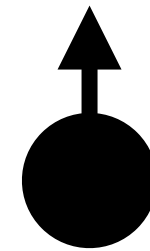
X₂

Experimental Tests

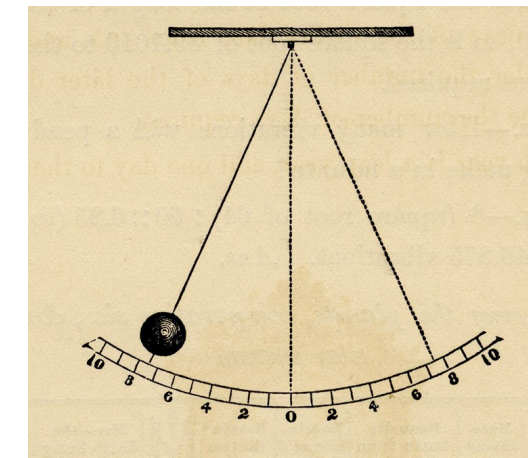
Key Point: Create macroscopic superposition

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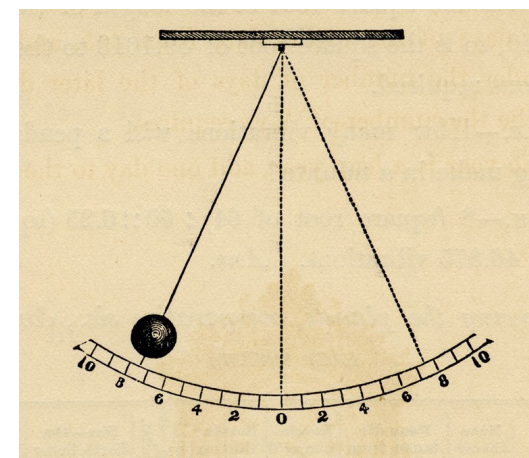
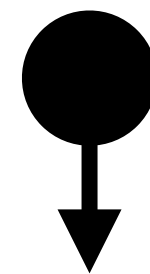
Search for Expectation value



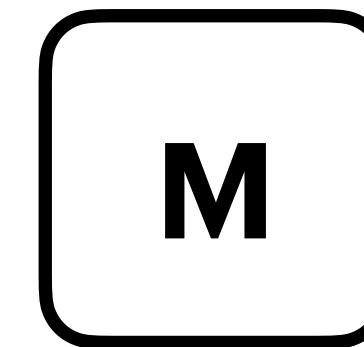
X₁



X₂



X₁



X₂

Even Null Result is Interesting: $G_{\mu\nu} = \langle T_{\mu\nu} \rangle$

Conclusions

- 1. Quantum Field Theory can be generalized to include non-linear, state dependent time evolution**
- 2. Conventional tests of quantum mechanics in atomic and nuclear systems do NOT probe causal non-linear quantum mechanics**
- 3. Straightforward set of experimental tests possible to probe non-linear quantum mechanics**
- 4. Motivation to test other extensions as well - e.g. Lindblad Decoherence**

Backup

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = (y\phi + \epsilon\langle\chi|\phi|\chi\rangle)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = (y\phi + \epsilon\langle\chi|\phi|\chi\rangle)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

At first order, use zeroth order solution - expectation value is simply a background field

Perform standard QFT on this background field to compute first order correction

Perturbation Theory

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Single Particle states? Causality for Multi-particle states?

Single Particle

$$H \supset y \Phi \bar{\Psi} \Psi = y (\phi + \tilde{\epsilon} \langle \chi | \phi | \chi \rangle) \bar{\Psi} \Psi$$

Suppose we have a ψ particle - how does its wave-function evolve?

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Straightforward Computation of Expectation Value

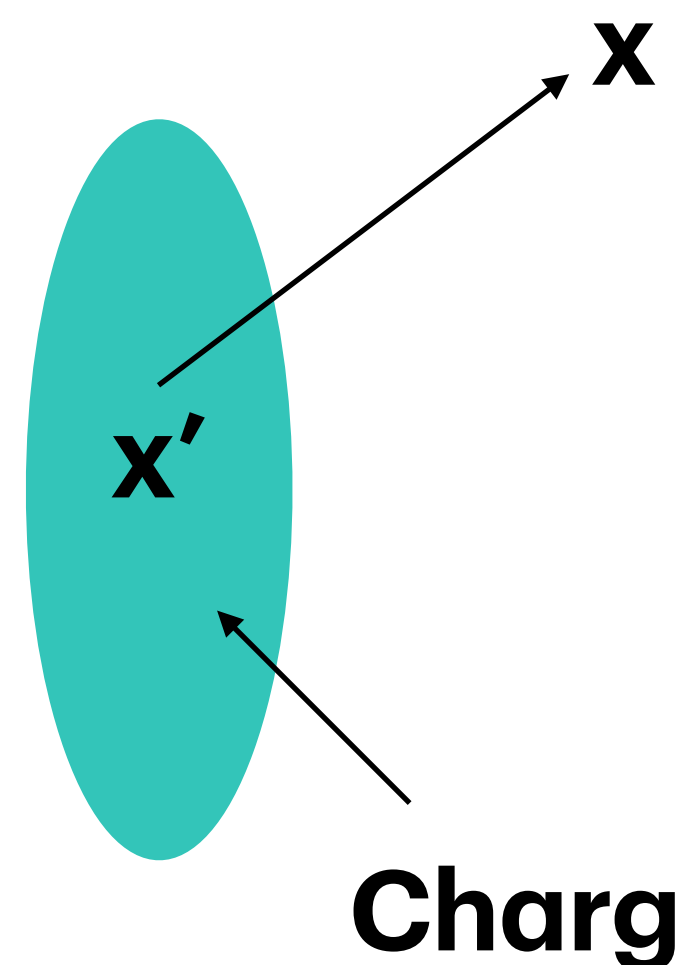
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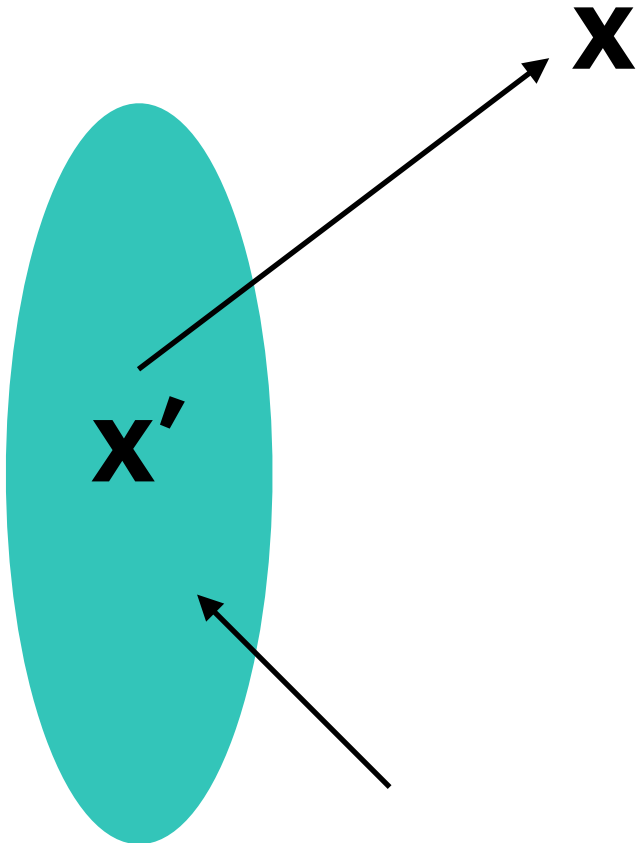
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Charge Density of ψ

Causal Green's Function

Schrodinger Equation

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Single particle equation derived from field theory

Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \tilde{\epsilon}y \int d^4x' \Psi^*(x) \Psi(x') G_R(x;x')\right) \Psi(t,\mathbf{x})$$

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Hermitean Form of Hamiltonian implies conserved norm

Maintain Probabilistic Interpretation

Entangled Systems

$$\Psi (x, y; t) = \sum_{i,j} c_{ij} (t) \alpha_i (x) \beta_j (y)$$

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To change evolution, need to change ϕ

ϕ changes via causal Green's function - naturally comes from field theory!

Gauge Theories and Gravitation

Linear QFT Lagrangian, Shift bosonic field by expectation value

To Path Integral, add:

$$e^{iS_0 + i \int d^4x \left(e \frac{(A_\mu + \epsilon_\gamma \langle \chi | A_\mu | \chi \rangle)}{1 + \epsilon_\gamma} J^\mu + \epsilon \tilde{\gamma} \langle \chi | F_{\mu\nu} | \chi \rangle F^{\mu\nu} \right)}$$

Background Field

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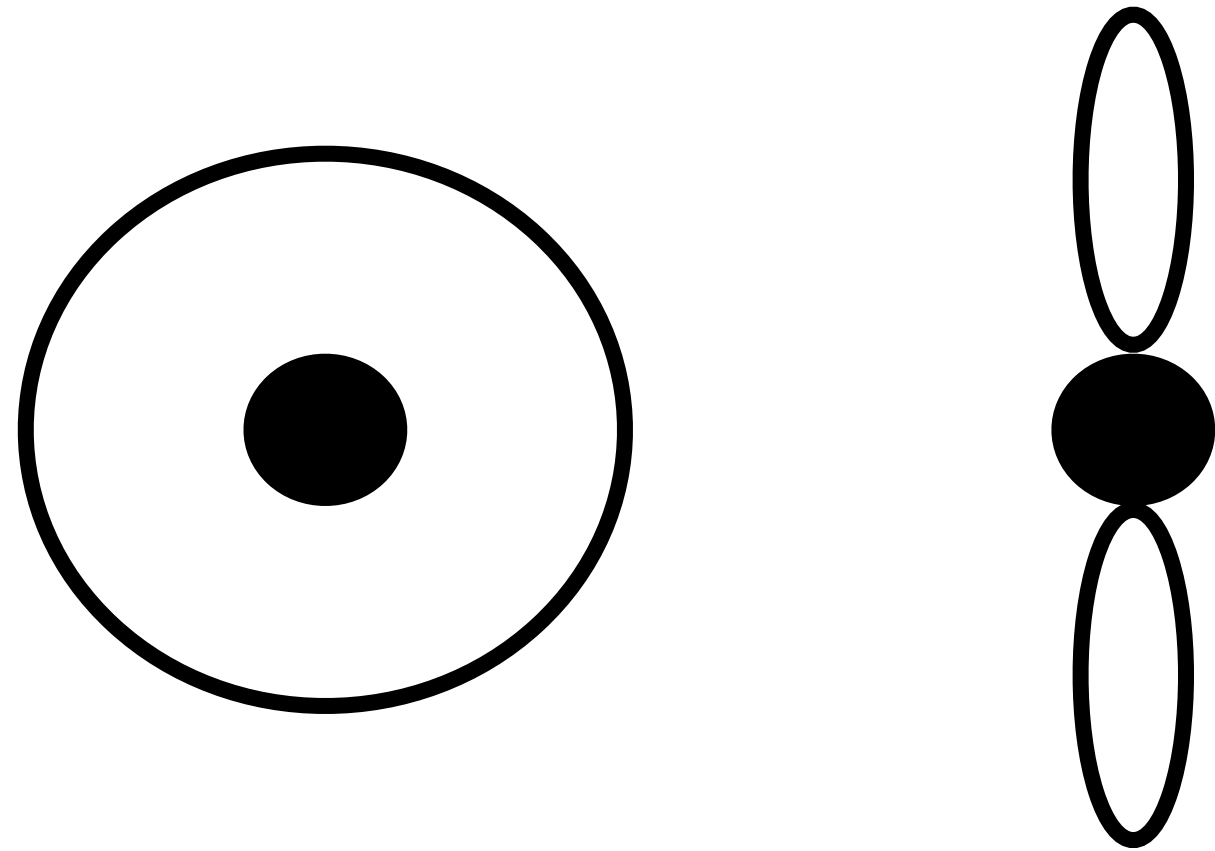
Background Field

Gravitation

$$e^{iS_0 + i \int d^4x (\epsilon_G \langle \chi | g_{\mu\nu} | \chi \rangle \partial^\mu \phi \partial^\nu \phi)}$$

Constraints

What does this do to the Lamb Shift?



Proton at Fixed Location

2S and 2P electron have different charge distribution

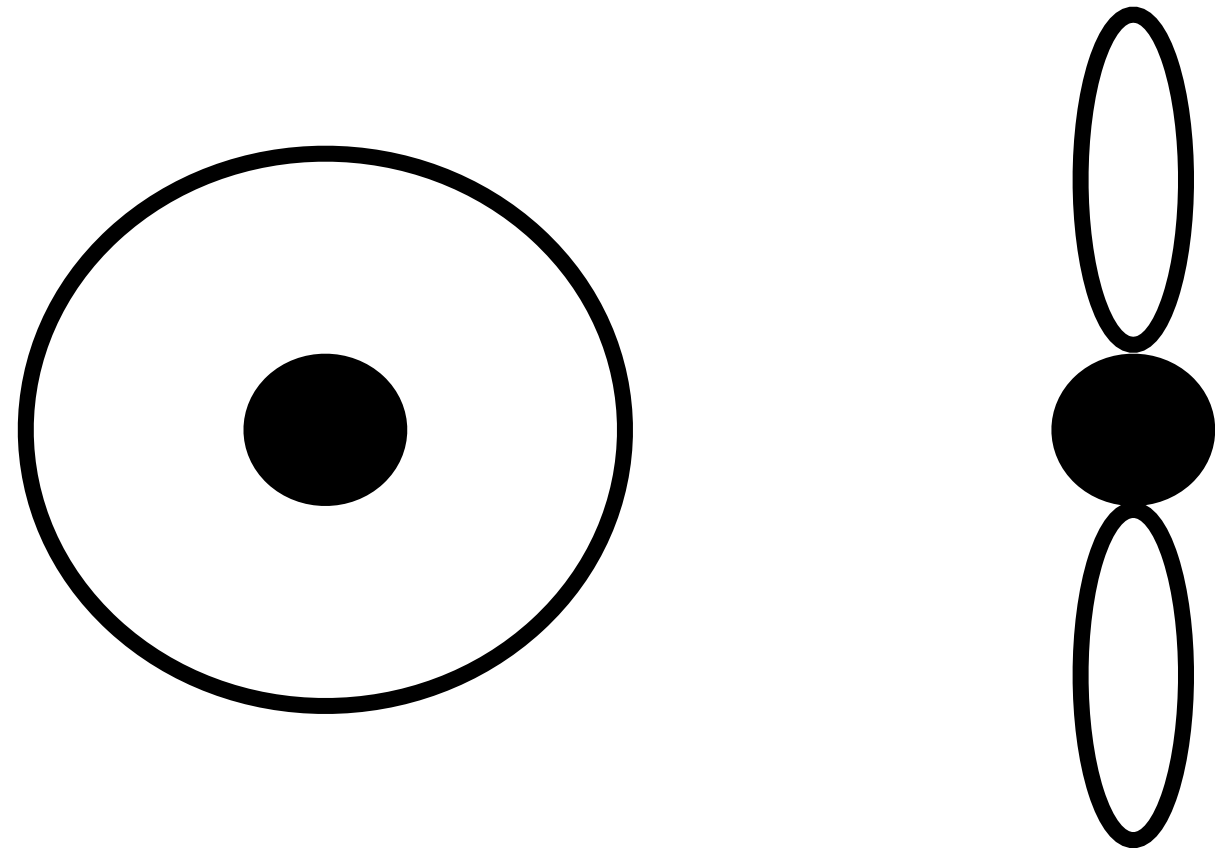
Different expectation value of electromagnetic field

Level Splitting!

$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

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Level Splitting!

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BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)

Expectation value of electromagnetic field diluted

In neutral atom - heavily suppressed, except at edges!

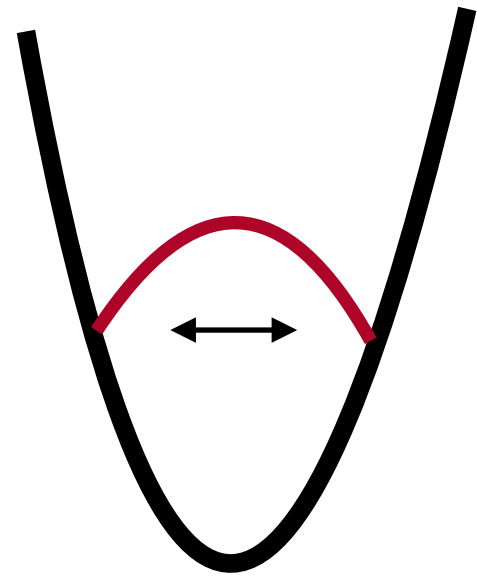
$$\varepsilon < 10^{-2}$$

Similarly, kills possible bounds on QCD and gravity

Constraints

Leading Constraint?

For $\varepsilon > 0$ (repulsive interaction)



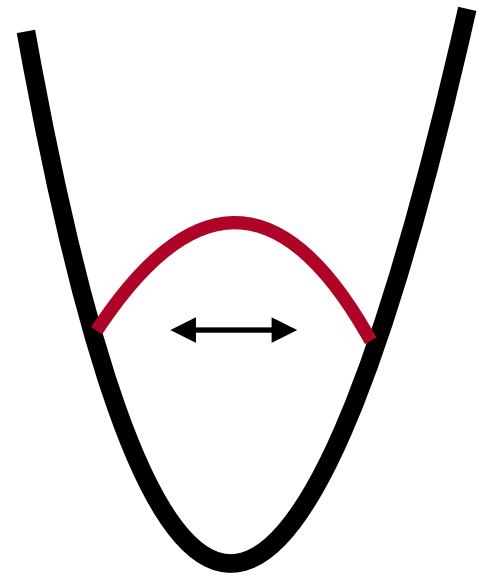
Too large a repulsion, Cant trap ion in trap

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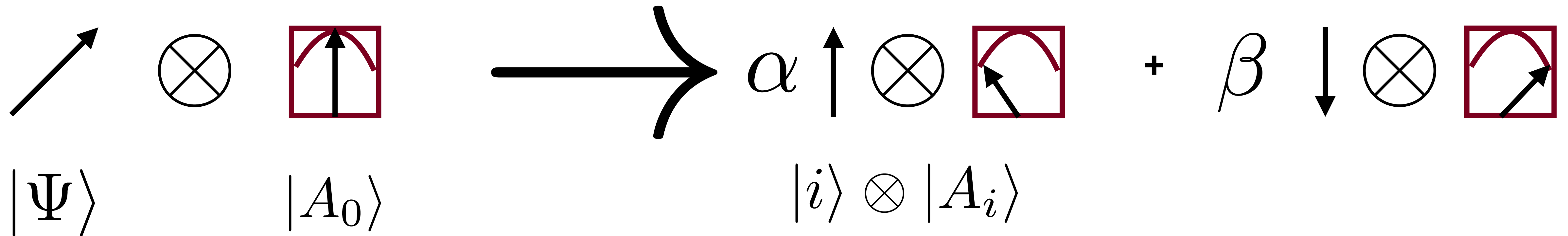
No direct limit on $\varepsilon < 0$ (attractive interaction)

Perhaps from mapping of ion in trap?

Measurement in Quantum Mechanics

Not some mysterious process

Interaction between quantum state and measuring device

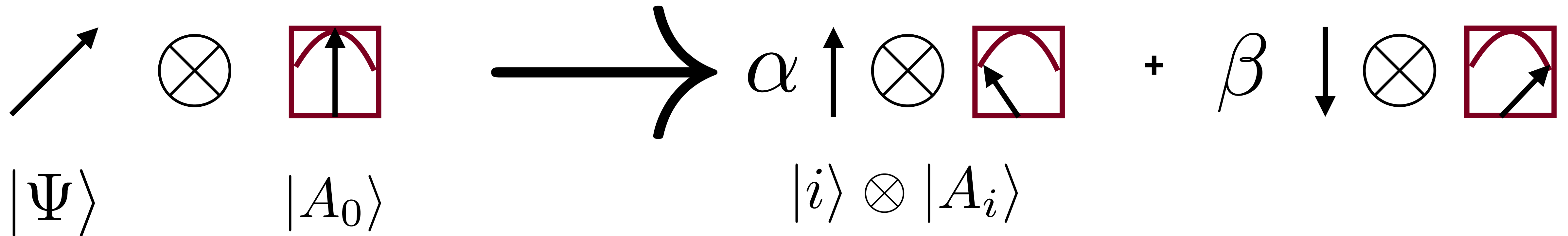


$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle$$

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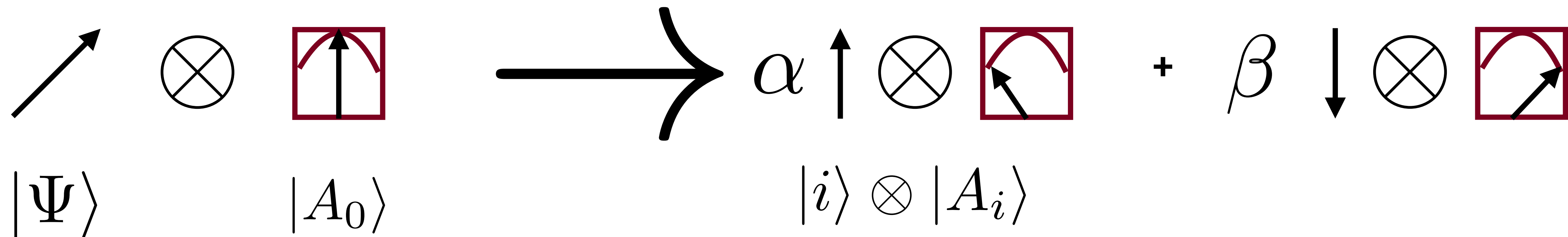
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Prediction of Quantum Mechanics ("Many Worlds"), Not an interpretation

Pick: $\langle A_j | A_i \rangle = \delta_{ij} \implies \rho_{|\Psi\rangle} = \sum_i c_i c_i^* |i\rangle \langle i|$ "Interpret" as direct sum of "worlds"

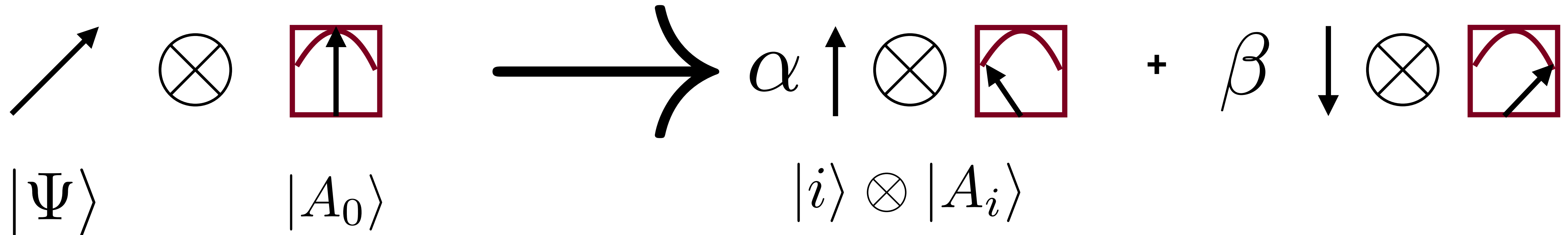
Measurement in Non-Linear Quantum Mechanics

Interaction between quantum state and measuring device



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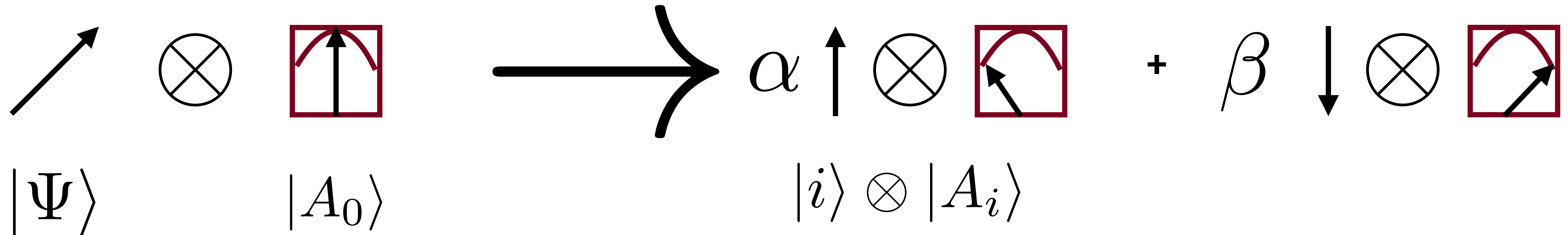
In linear QM, just need to know the basis vectors

Interaction Hamiltonian independent of unknown quantum state

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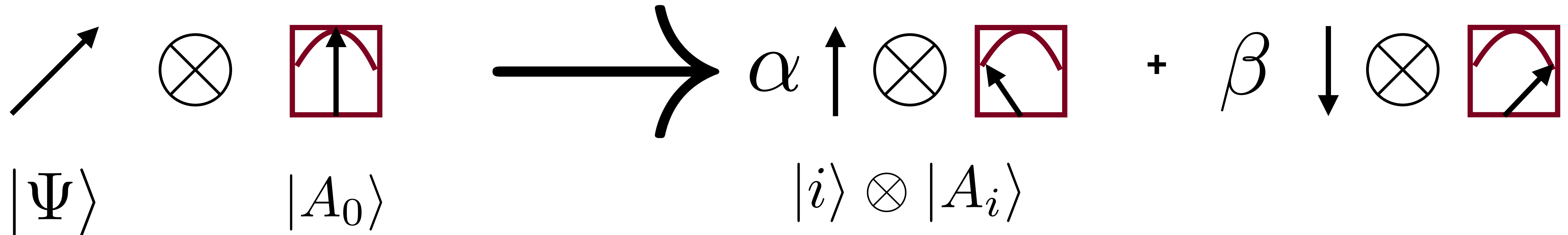
Key Point: Non-linear Hamiltonian depends upon unknown quantum state

$$\text{No Guarantee: } \langle A_j | A_i \rangle = 0$$

$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle + \epsilon \sum_{i,j} d_{i,j} |i\rangle \otimes |A_j\rangle$$

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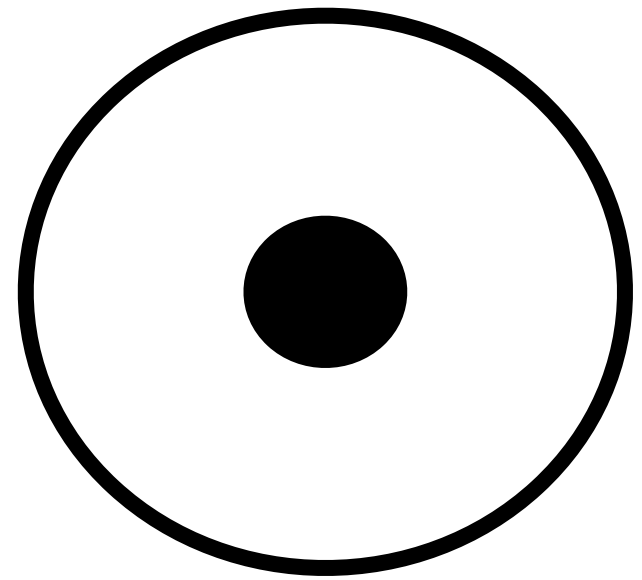
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Measurement Noise

Atom Aging

Interferometry - interaction between paths

Decaying Radioactive nucleus



$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

$$|\chi(0)\rangle = |X\rangle$$

$$|\chi(t)\rangle = \alpha(t) e^{-\frac{\Gamma t}{2}} |X\rangle + \beta(t) |Y\rangle$$

$$\langle \chi | A_\mu | \chi \rangle = \langle X | A_\mu | X \rangle \propto e^{-\Gamma t}$$

**Time dependent self-interaction - time dependent
shift to the energy of atomic states!**

Delicate Non-Linearity

Suppose $|X\rangle = |U\rangle$

Alex performs experiment on July 6 - discovers non-linear quantum mechanics!

$$|\chi\rangle = \frac{1}{\sqrt{2}} (|U\rangle|O_U\rangle + |D\rangle|O_D\rangle)$$

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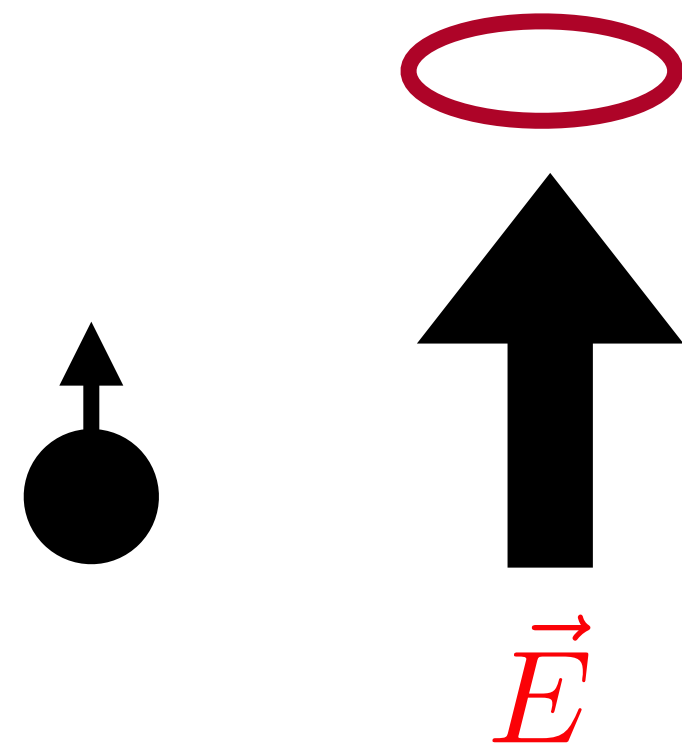
State on 9 AM on July 10

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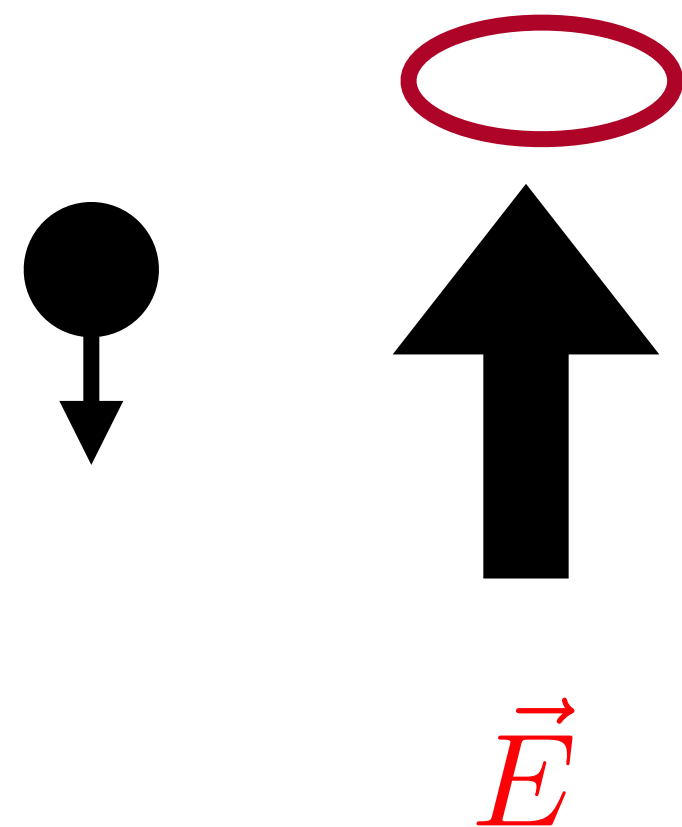
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Particles have been scattering for 13 billion years. Cosmological dilution?

Cosmological Relaxation of Non-Linear QM?

$$\mathcal{L} \supset e A_\mu \bar{\Psi} \gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi} \gamma^\mu \Psi$$

All we need is the expectation value. Non-Linear effects are resistant to decoherence.

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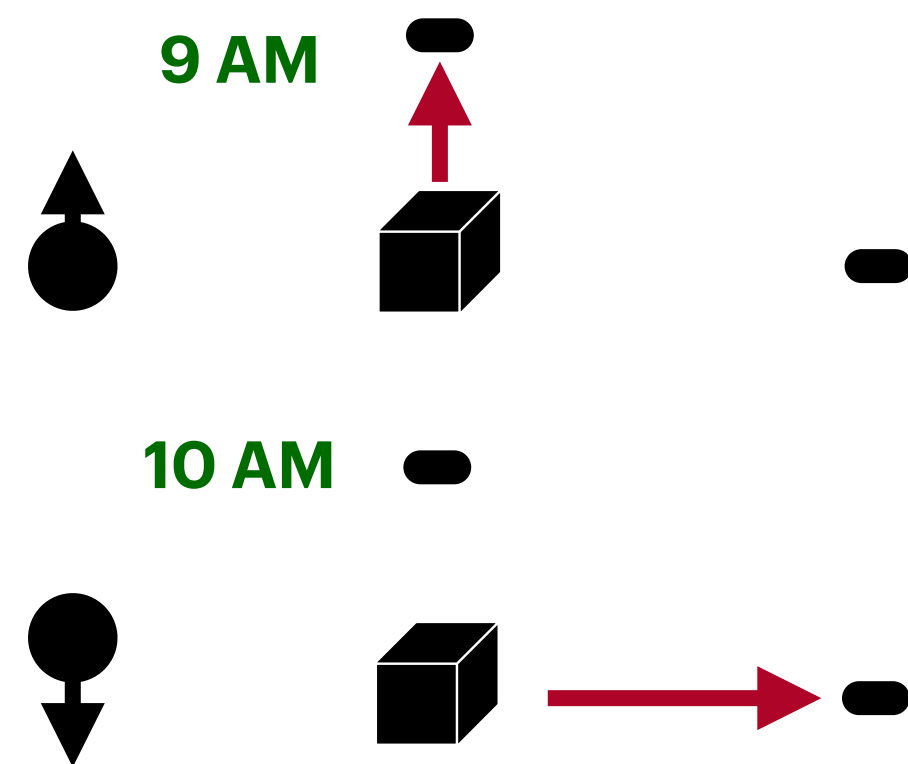
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Superpositions where expectation values of fields are very different



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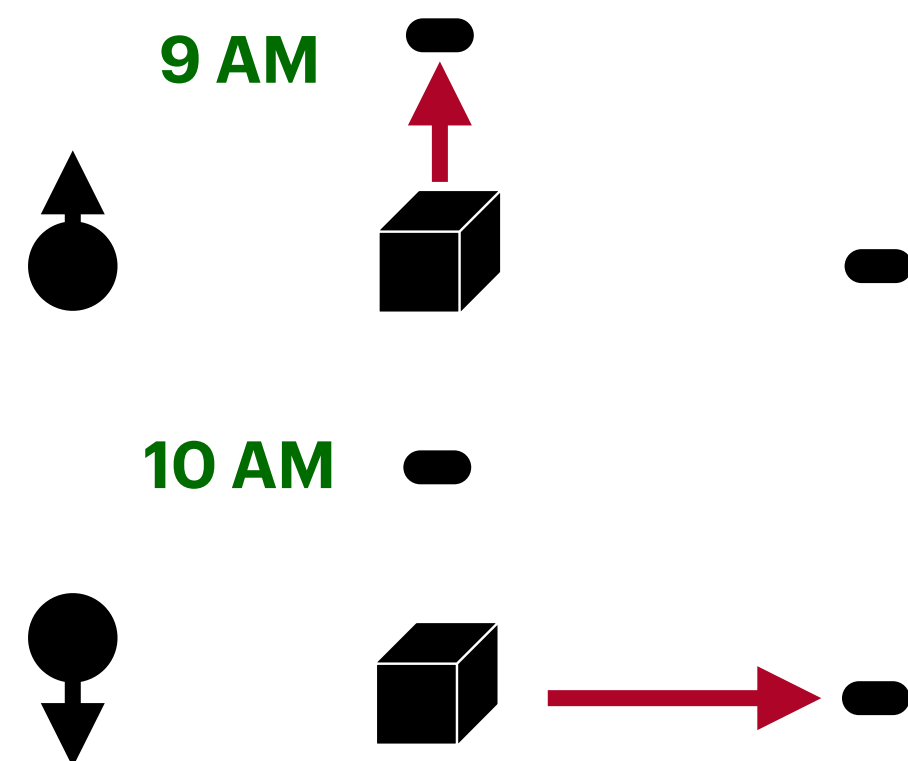
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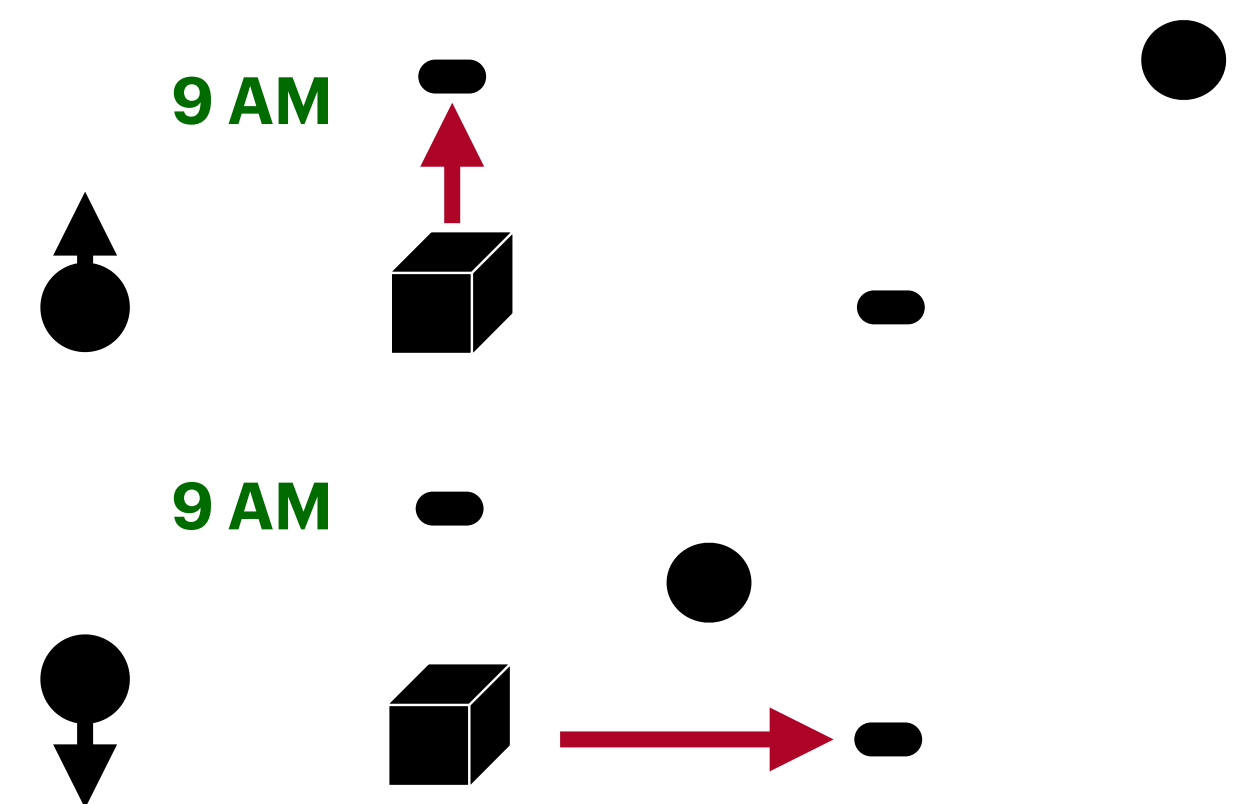
Relevant

Superpositions where expectation values of fields are very different




Irrelevant

Scattering where expectation values are not significantly changed




Classical Universe?

Suppose $|X\rangle = |U\rangle$

$$|U(t=0)\rangle =$$


Classical Universe?

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$|U(t=0)\rangle =$ 

Time Evolve

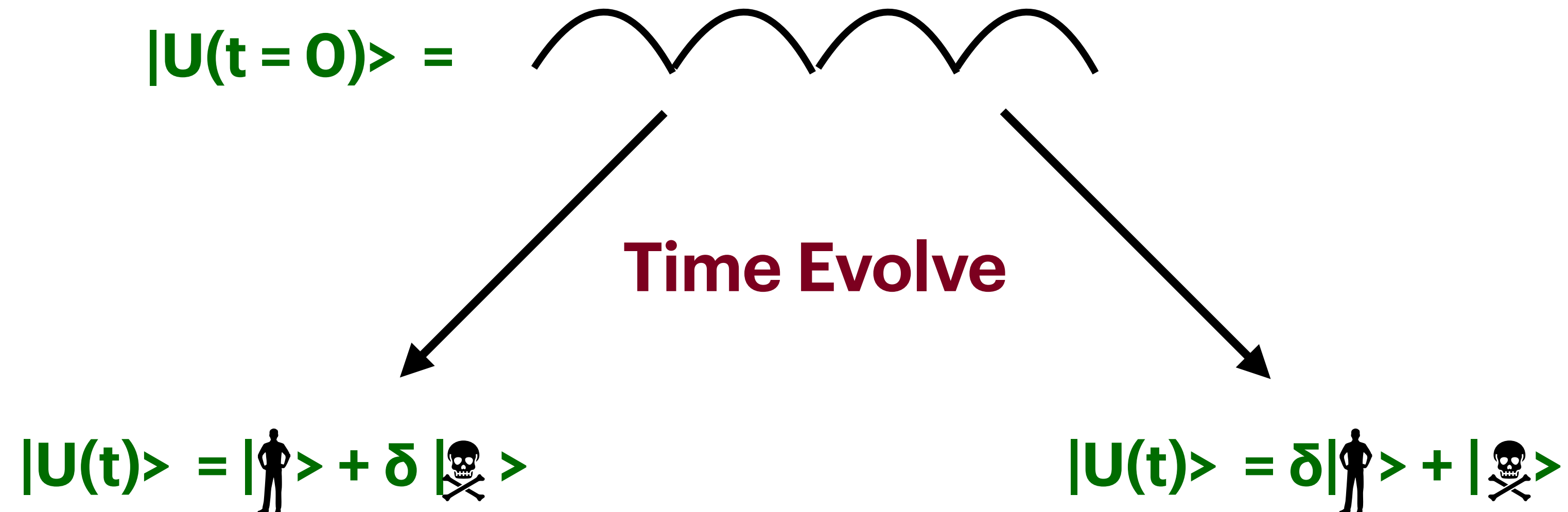
$|U(t)\rangle = |\text{person}\rangle + \delta |\text{skull}\rangle$

$|U(t)\rangle = \delta |\text{person}\rangle + |\text{skull}\rangle$

Can quantum events (scattering, decay etc.) lead to wildly different classical outcomes?

Classical Universe?

Suppose $|X\rangle = |U\rangle$



Can quantum events (scattering, decay etc.) lead to wildly different classical outcomes?

Clearly Possible - e.g. Human choosing to act differently based on quantum event

But, fundamentally - this is because humans can be quantum amplifiers

Are there natural quantum amplifiers, for e.g. in chaotic systems?