Resonant Cavities for Gravitational Waves

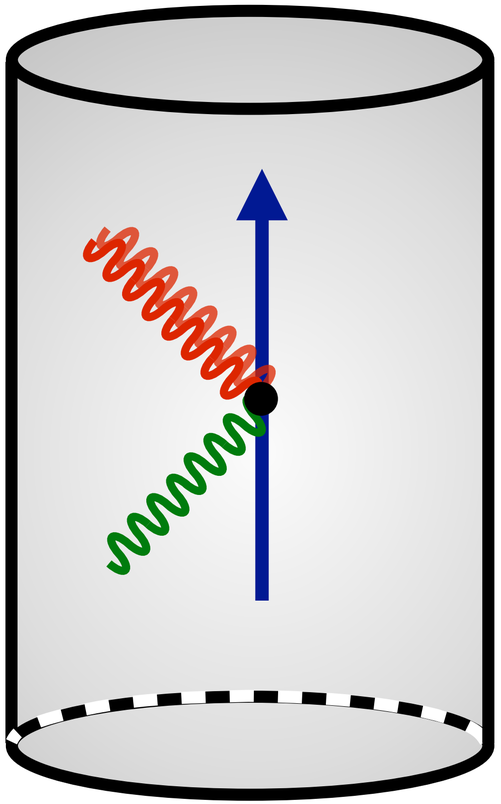
Sebastian A. R. Ellis

University of Geneva

w/ A. Berlin, D. Blas, R. T. D'Agnolo, R. Harnik, Y. Kahn, J. Schütte-Engel *& M. Wentzel*

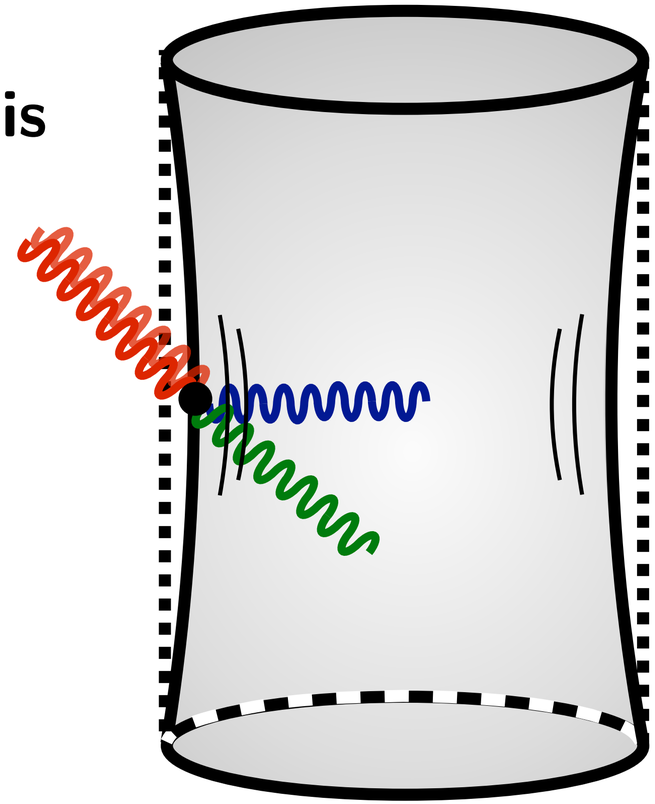
arXiv: 2112.11465 *arXiv:2303.01518*

Resonant Cavities for Gravitational Waves



Sebastian A. R. Ellis

University of Geneva

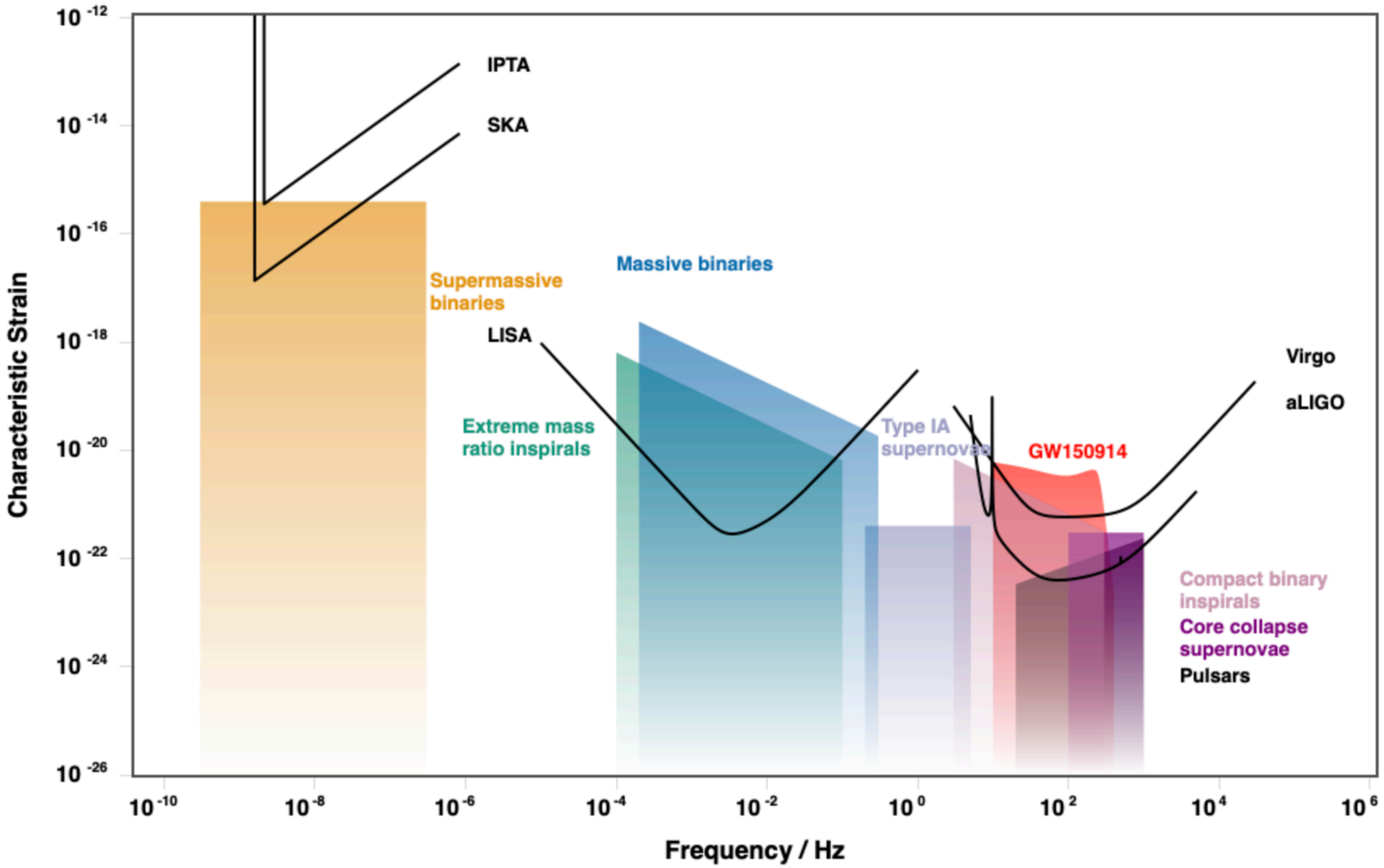


w/ A. Berlin, D. Blas, R. T. D'Agnolo, R. Harnik, Y. Kahn, J. Schütte-Engel
arXiv: 2112.11465

& M. Wentzel
arXiv:2303.01518

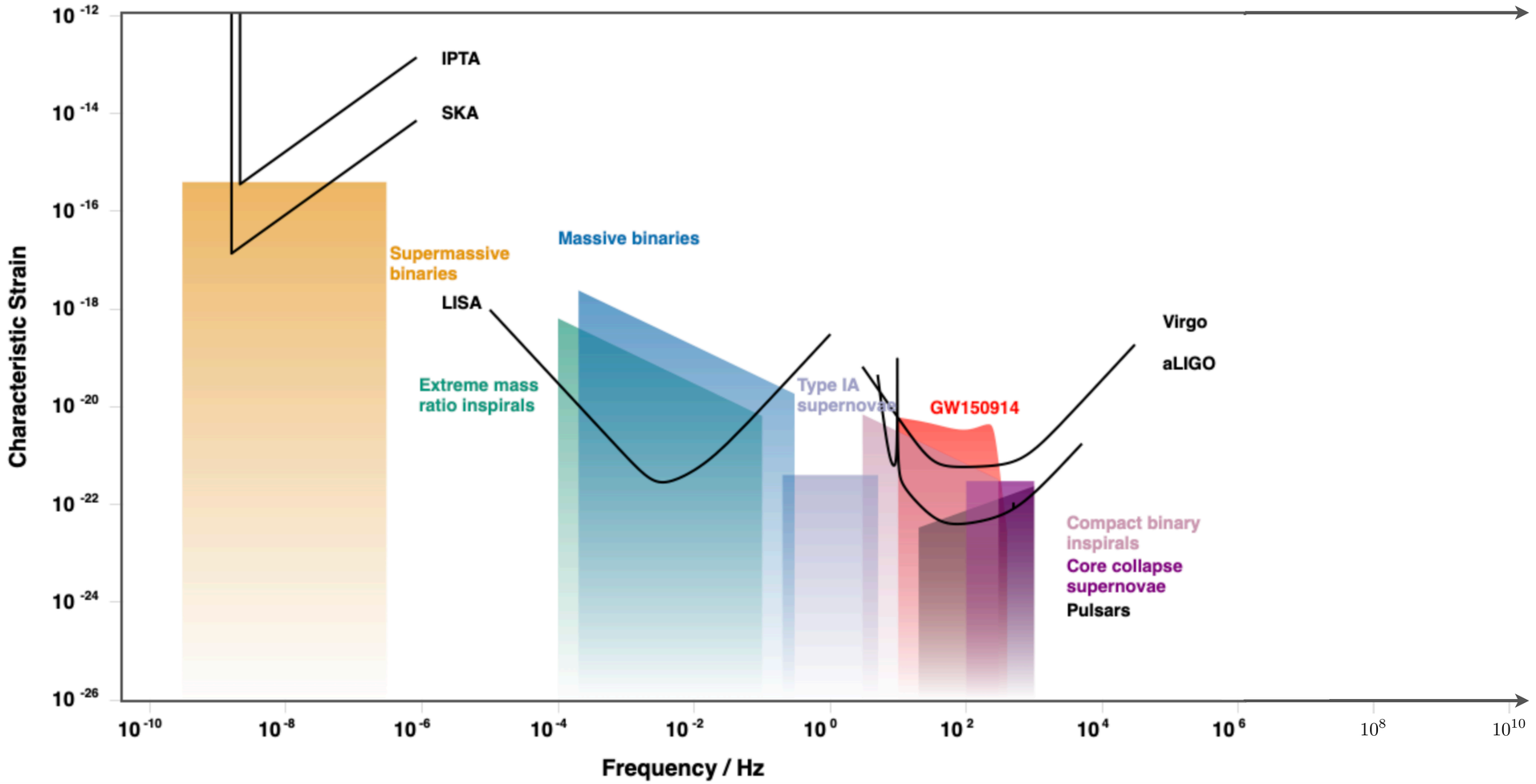
Gravitational Waves

By Christopher Moore, Robert Cole and Christopher Berry, formerly of the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge



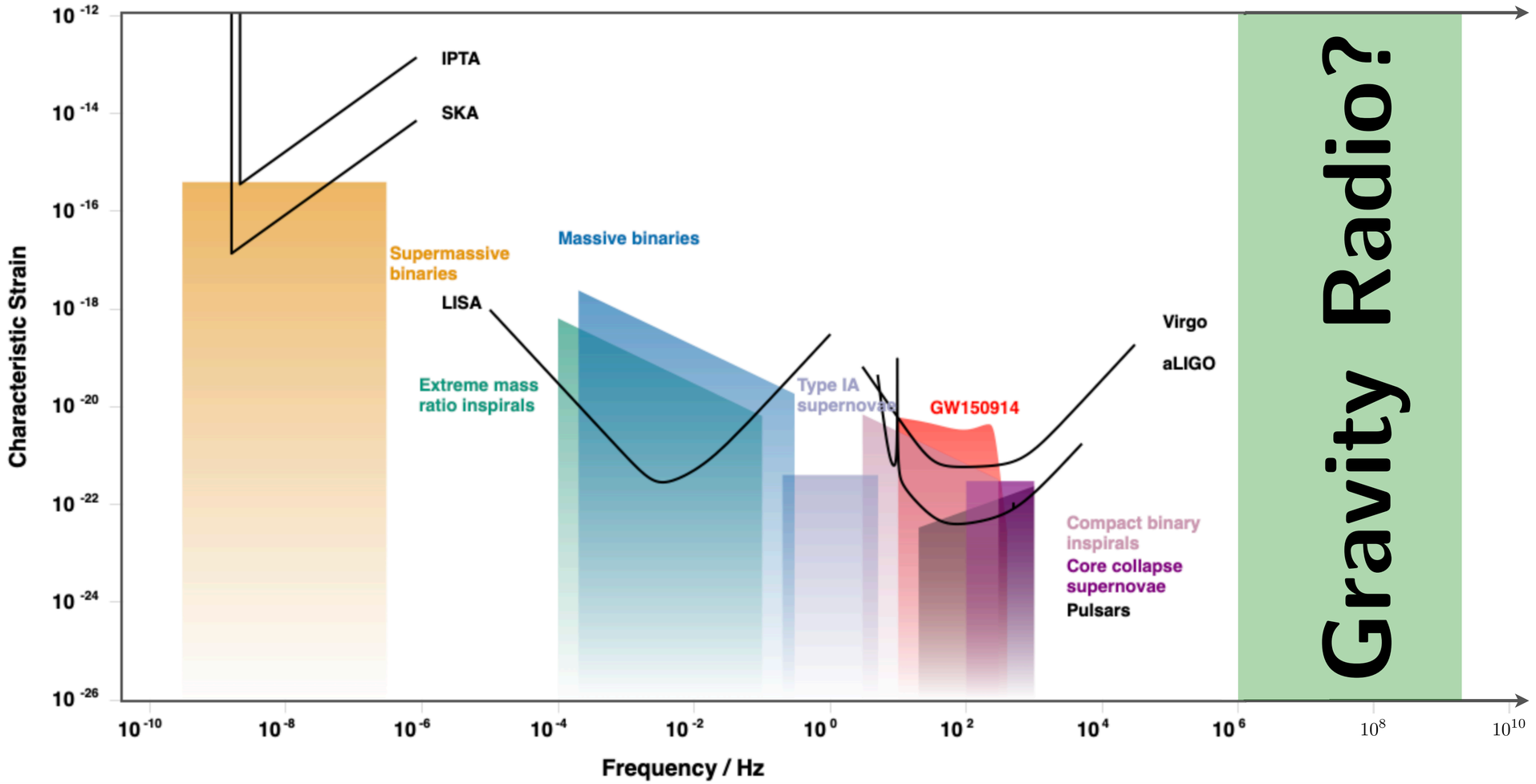
Gravitational Waves

By Christopher Moore, Robert Cole and Christopher Berry, formerly of the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge



Gravitational Waves

By Christopher Moore, Robert Cole and Christopher Berry, formerly of the Gravitational Wave Group at the Institute of Astronomy, University of Cambridge

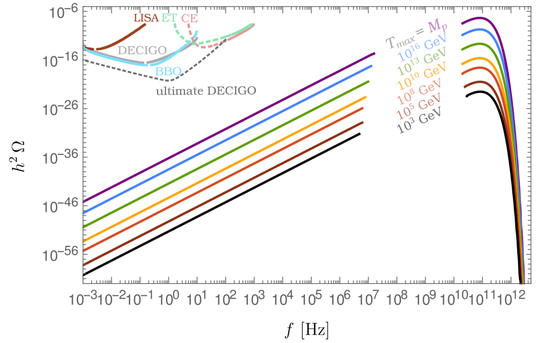


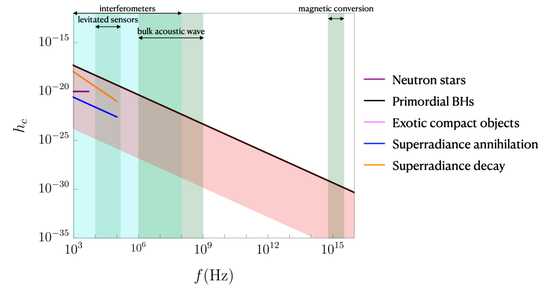
Sources of HFGWs

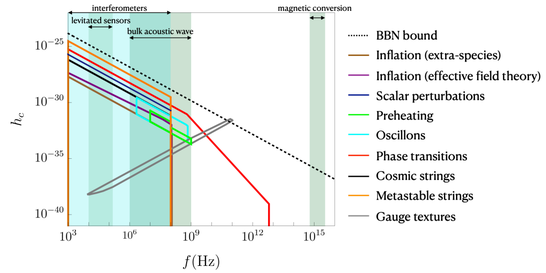
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:		
BSM:		

Ringwald et al, 2011.04731







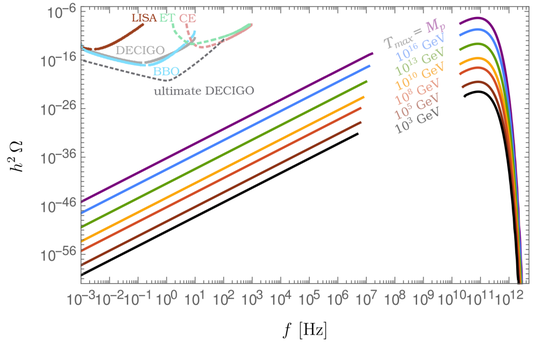
Aggarwal et al, 2011.12414

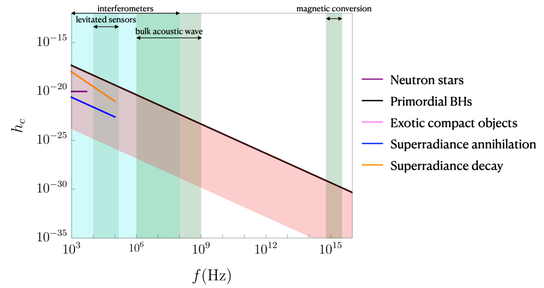
Sources of HFGWs

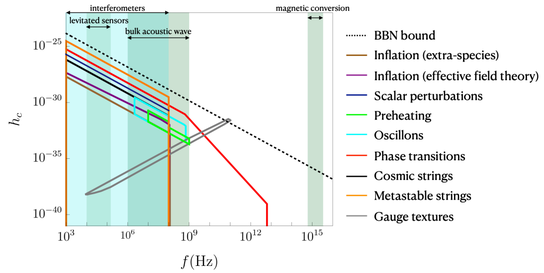
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	
BSM:		

Ringwald et al, 2011.04731







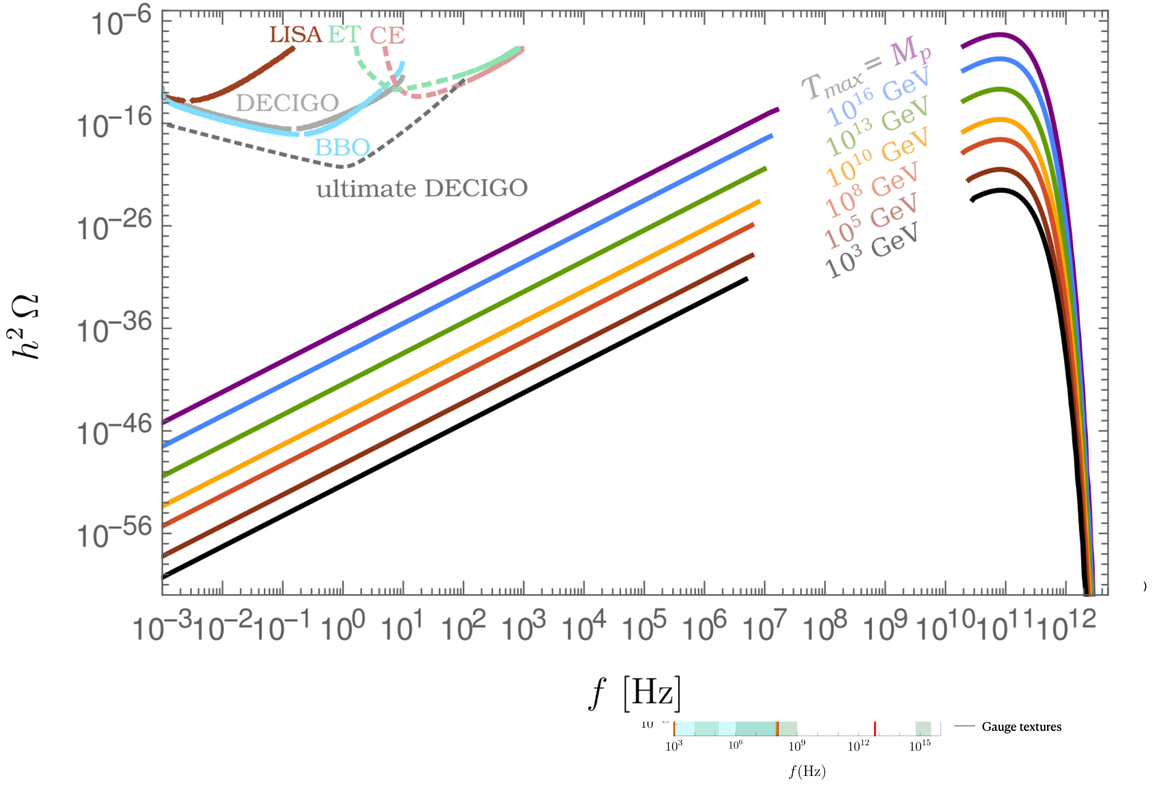
Aggarwal et al, 2011.12414

Sources of HFGWs

see F. Muia's talk for more on sources

Ringwald et al, 2011.04731

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	
BSM:		



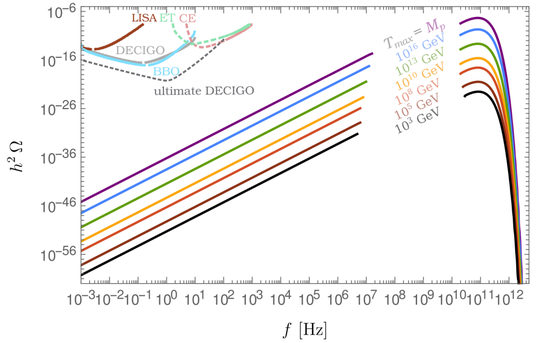
Aggarwal et al, 2011.12414

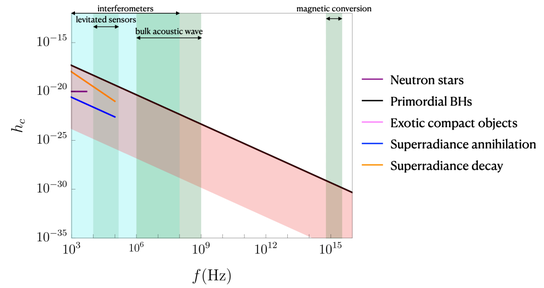
Sources of HFGWs

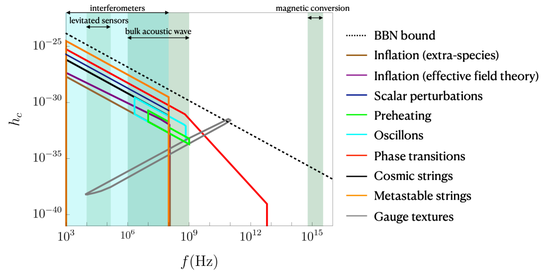
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	
BSM:		

Ringwald et al, 2011.04731







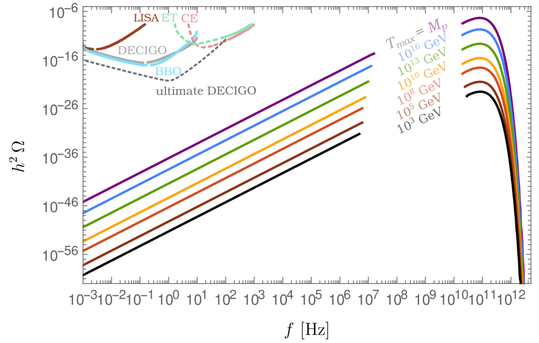
Aggarwal et al, 2011.12414

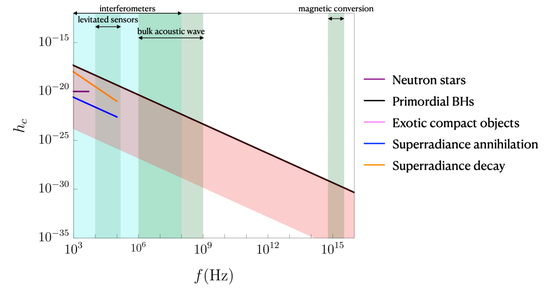
Sources of HFGWs

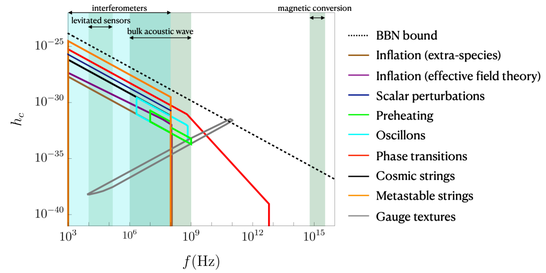
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:		

Ringwald et al, 2011.04731







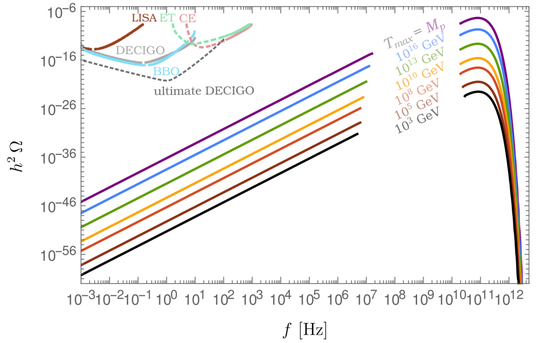
Aggarwal et al, 2011.12414

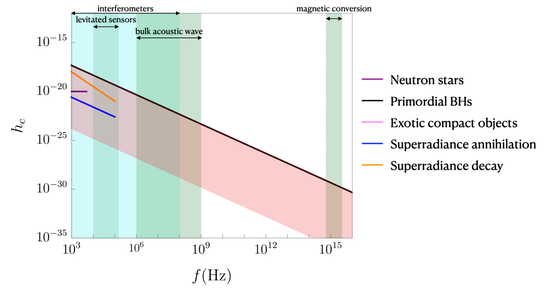
Sources of HFGWs

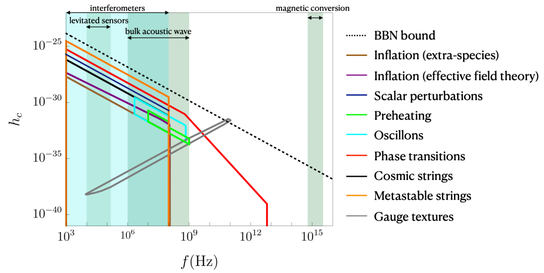
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:		✓ PBH inspirals Superradiance Exotic objects ...

Ringwald et al, 2011.04731







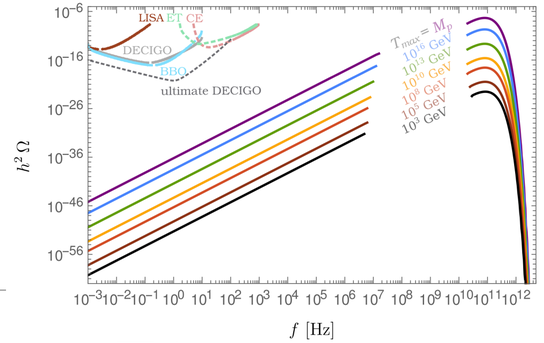
Aggarwal et al, 2011.12414

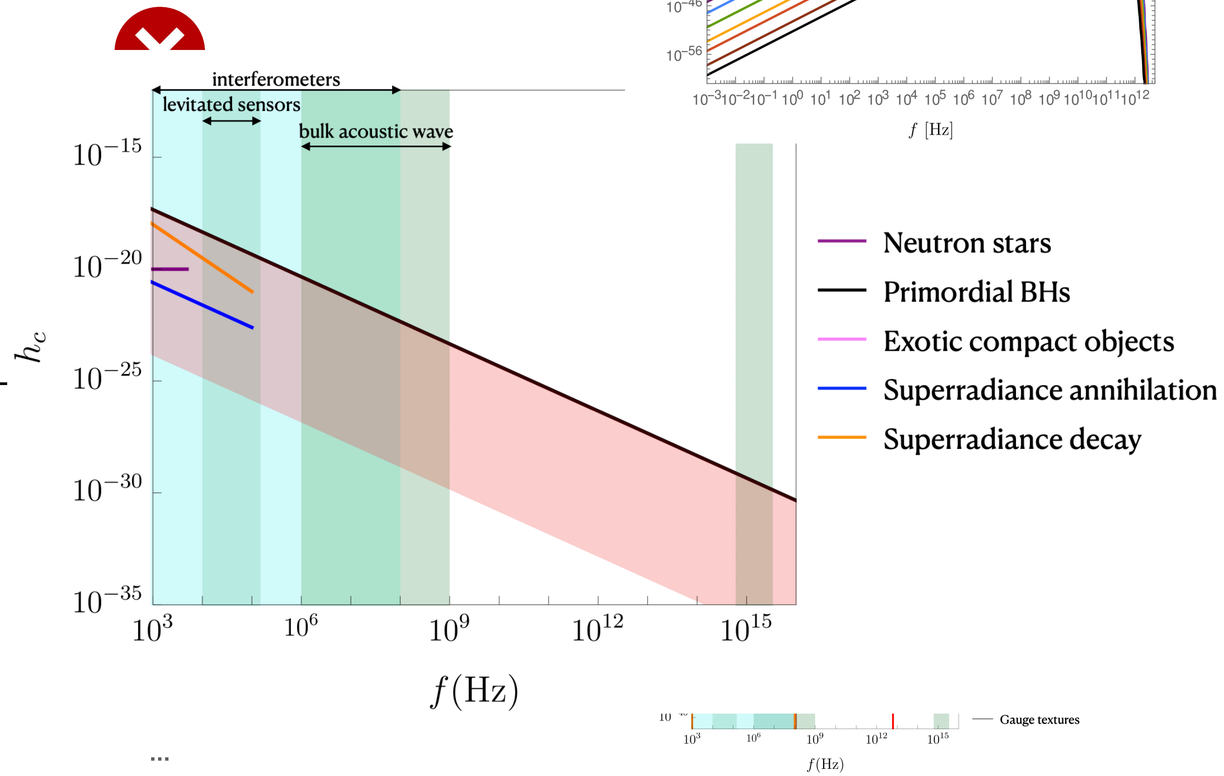
Sources of HFGWs

see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗
BSM:		...

Ringwald et al, 2011.04731





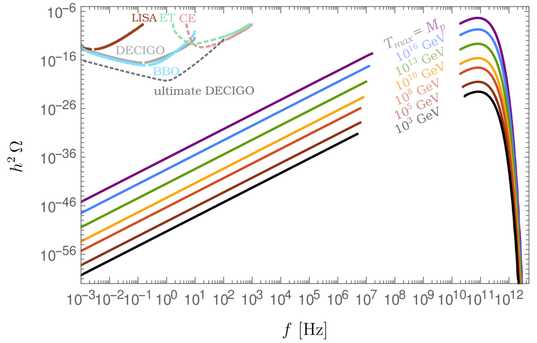
Aggarwal et al, 2011.12414

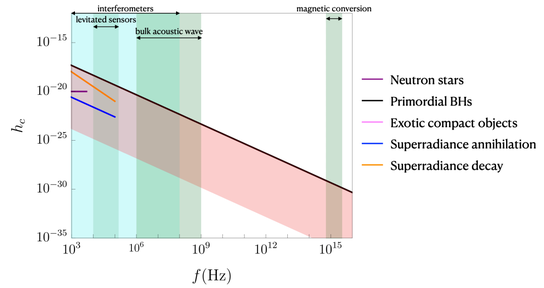
Sources of HFGWs

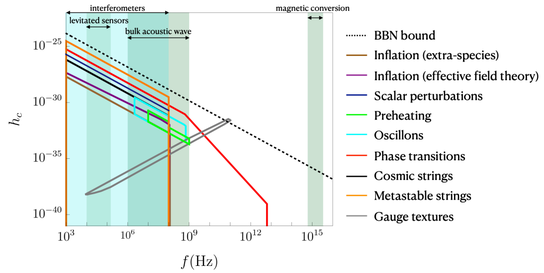
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:		✓ PBH inspirals Superradiance Exotic objects ...

Ringwald et al, 2011.04731







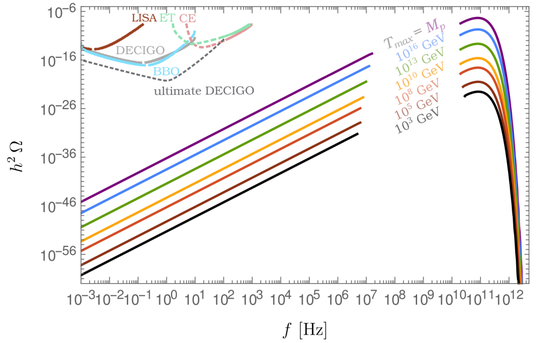
Aggarwal et al, 2011.12414

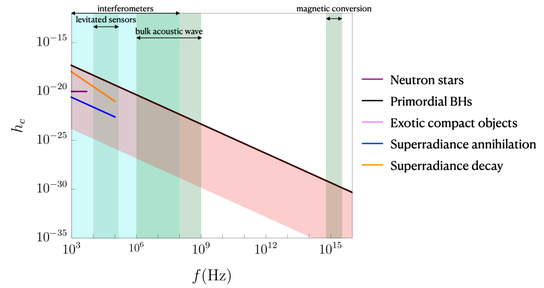
Sources of HFGWs

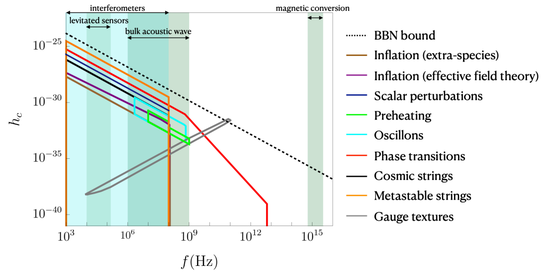
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:	✓ Inflation Phase transitions Cosmic Strings ...	✓ PBH inspirals Superradiance Exotic objects ...

Ringwald et al, 2011.04731





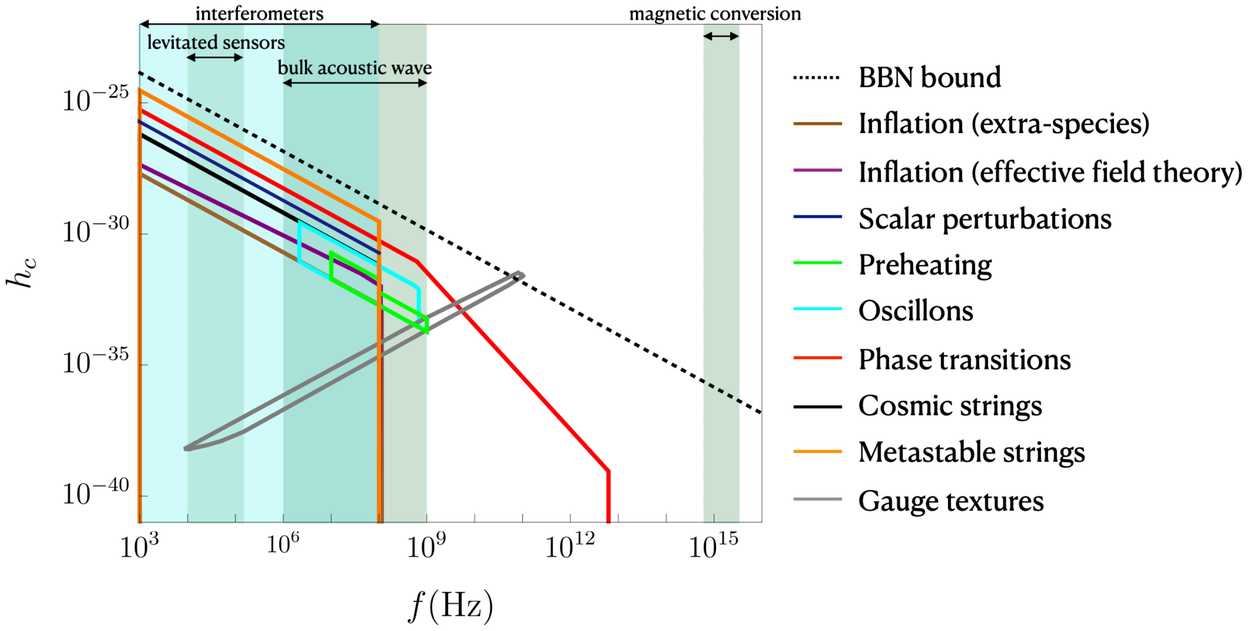


Aggarwal et al, 2011.12414

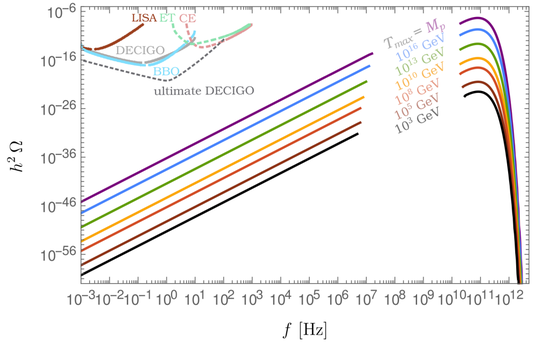
Sources of HFGWs

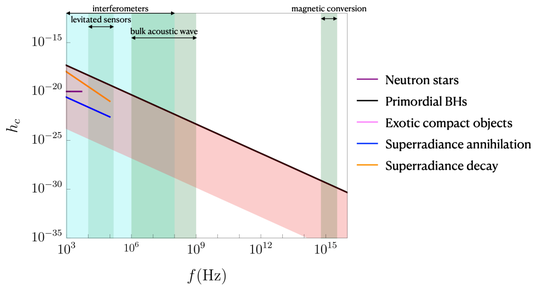
see F. Muia's talk for more on sources

Standard Model



Ringwald et al, 2011.04731





BSM:

Inflation

Phase transitions

Cosmic Strings

...

PBH inspirals

Superradiance

Exotic objects

...

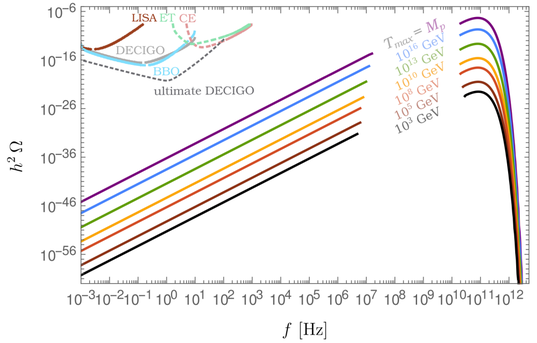
Aggarwal et al, 2011.12414

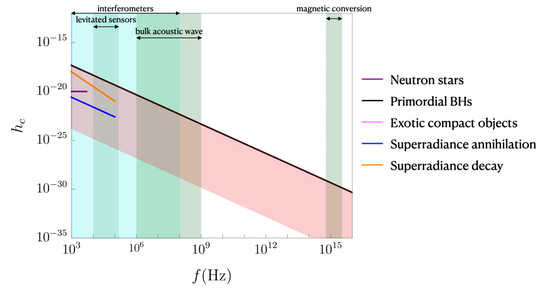
Sources of HFGWs

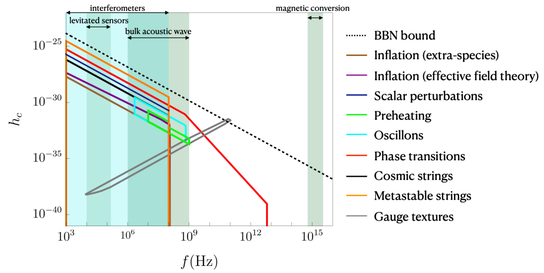
see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓ Thermal plasma fluctuations Ghiglieri & Laine (2015) Ghiglieri et al (2020) Ringwald et al (2020)	✗ ?
BSM:	✓ Inflation Phase transitions Cosmic Strings ...	✓ PBH inspirals Superradiance Exotic objects ...

Ringwald et al, 2011.04731







Aggarwal et al, 2011.12414

Sources of HFGWs

see F. Muia's talk for more on sources

	Stochastic	Coherent
Standard Model:	✓	✗
BSM:	✓	✓

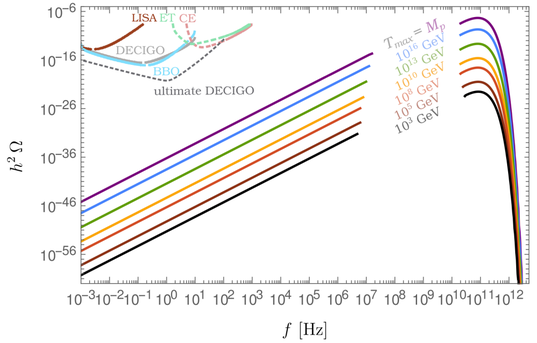
HFGW searches — high risk, high return

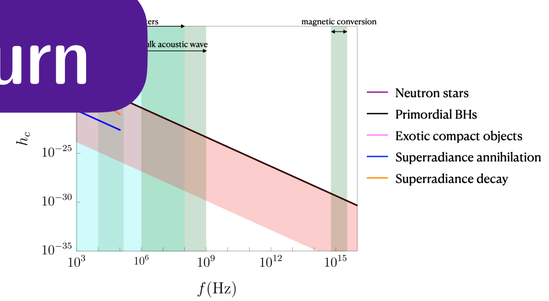
Ringwald et al (2020)

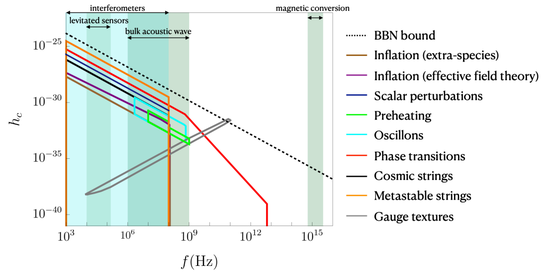
Inflation
Phase transitions
Cosmic Strings
...

PBH inspirals
Superradiance
Exotic objects
...

Ringwald et al, 2011.04731



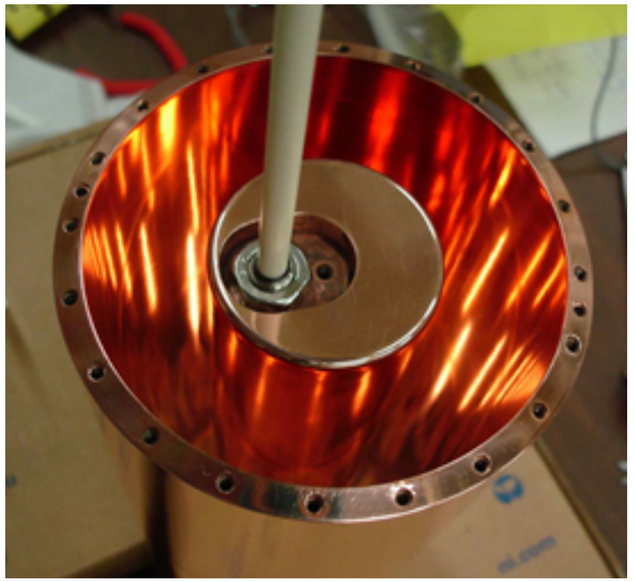




Aggarwal et al, 2011.12414

Resonant Cavities

Why?



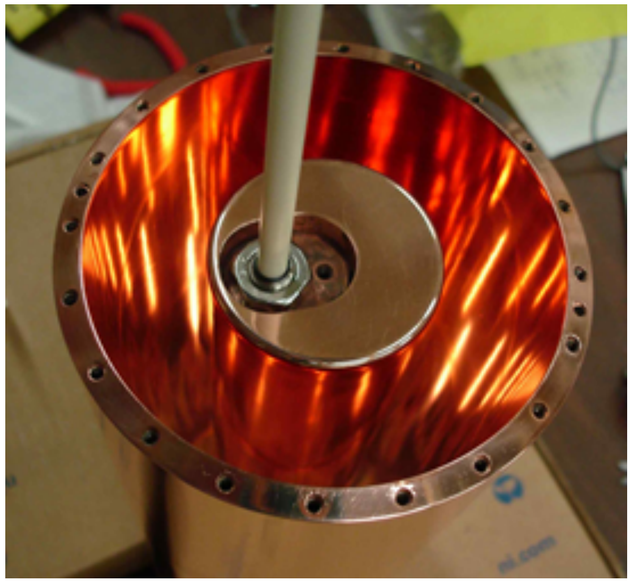
HAYSTAC

Resonant Cavities

Why?

Mature technology & constantly improving

Benefit from decades of development for accelerator use



HAYSTAC

Resonant Cavities

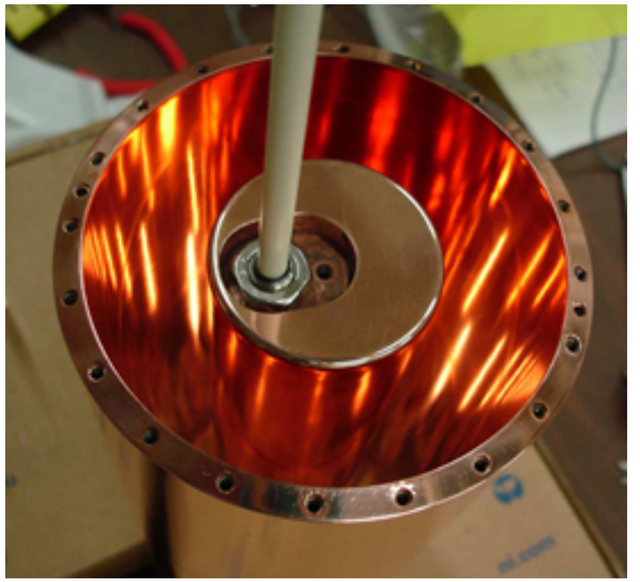
Why?

Mature technology & constantly improving

Benefit from decades of development for accelerator use

Cavities for fundamental physics already in use:

e.g. Axion Dark Matter



HAYSTAC

Resonant Cavities

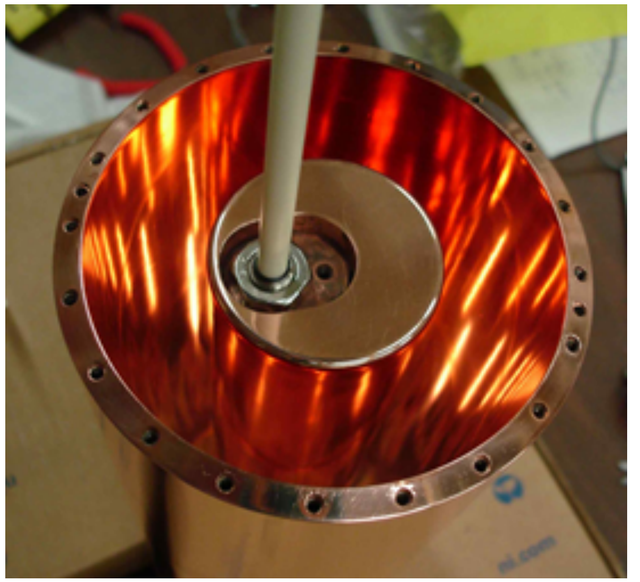
Why?

Mature technology & constantly improving

Benefit from decades of development for accelerator use

Cavities for fundamental physics already in use:

e.g. Axion Dark Matter



HAYSTAC

More on the connection between GWs and axions later...

Outline

Outline

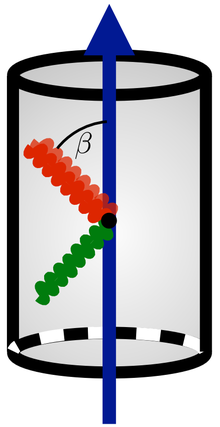
How can we use resonant cavities to search for GWs?

Outline

How can we use resonant cavities to search for GWs?

Electromagnetic Interactions of GWs

- Parallels with Axion searches
- Importance of reference frame

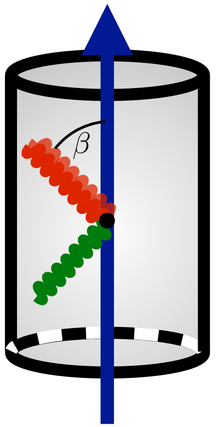


Outline

How can we use resonant cavities to search for GWs?

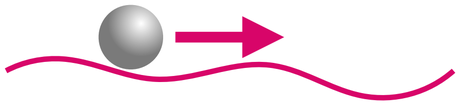
Electromagnetic Interactions of GWs

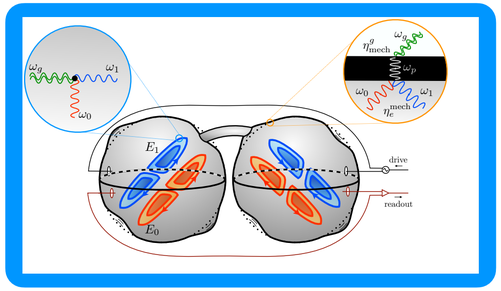
- Parallels with Axion searches
- Importance of reference frame



Tidal deformation by GWs

- Motion of test masses





Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

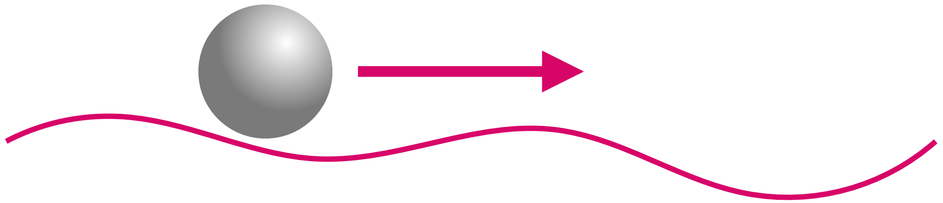
$$\eta_{\mu\nu} = (-, +, +, +)$$

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-, +, +, +)$$

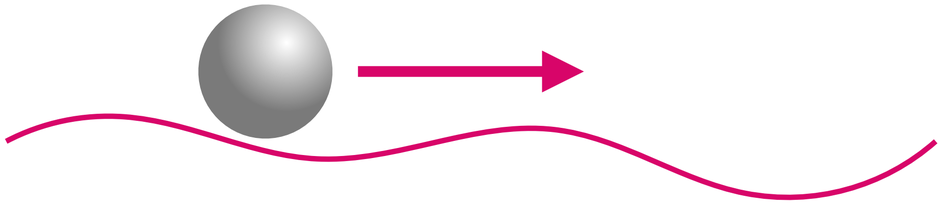


Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



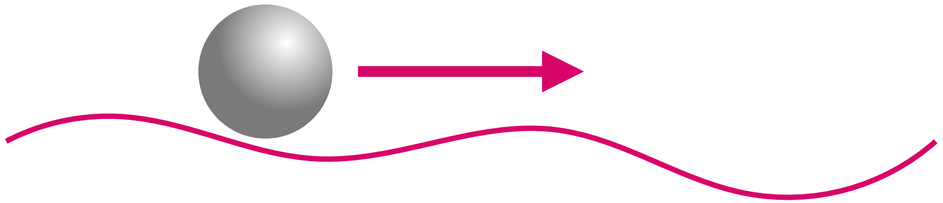
Geodesic equation

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



Geodesic equation

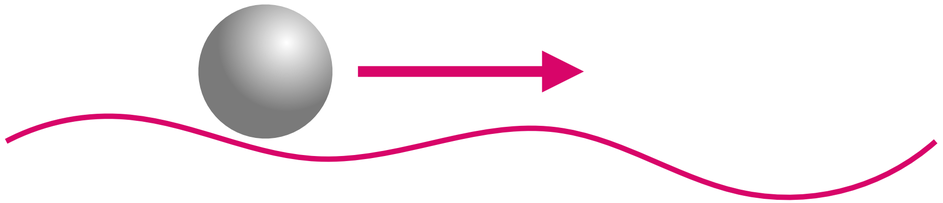
MAGO 2.0
see A. Berlin next

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



Geodesic equation

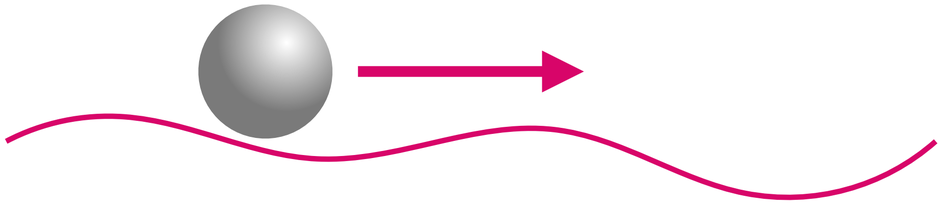
MAGO 2.0
see A. Berlin next

Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

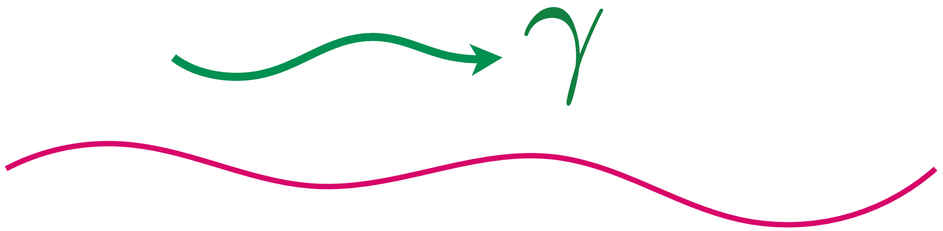
Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



Geodesic equation

MAGO 2.0
see A. Berlin next

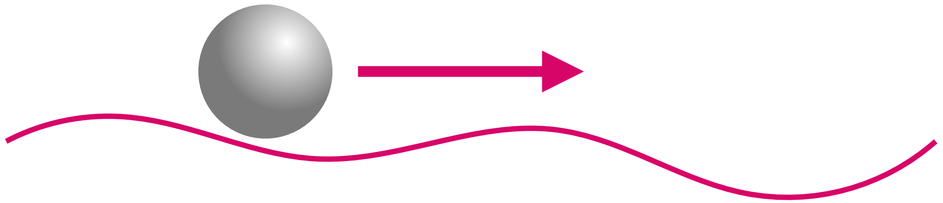


Gravitational Waves $\otimes$ Resonant Cavities

How do GWs interact with cavities?

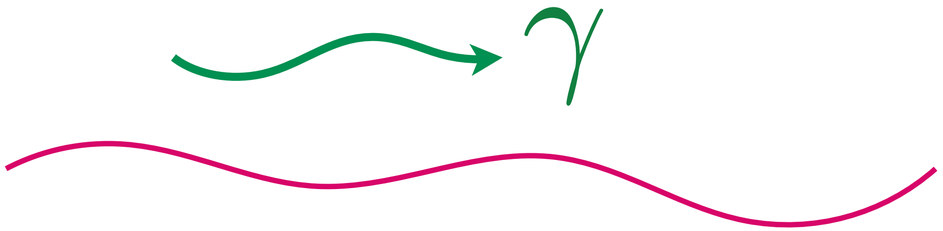
Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



Geodesic equation

MAGO 2.0
see A. Berlin next



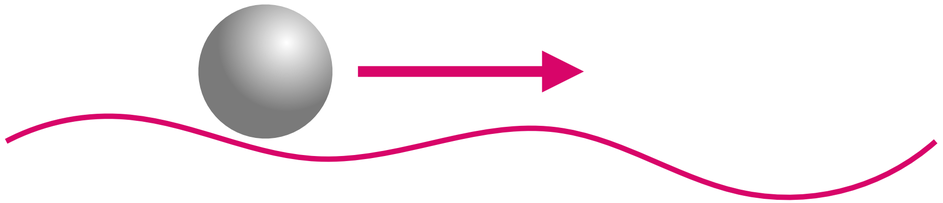
Maxwell equations in curved spacetime

Gravitational Waves ⊗ Resonant Cavities

How do GWs interact with cavities?

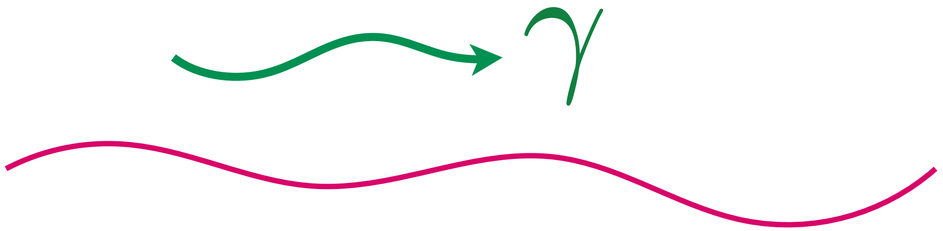
Metric encodes effects of curvature

$$\eta_{\mu\nu} = (-,+,+,+)$$



Geodesic equation

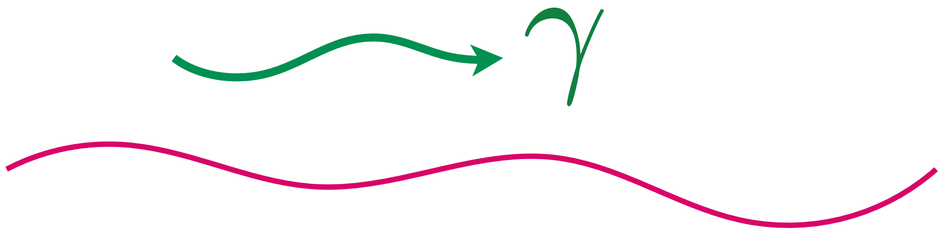
MAGO 2.0
see A. Berlin next



Maxwell equations in curved spacetime

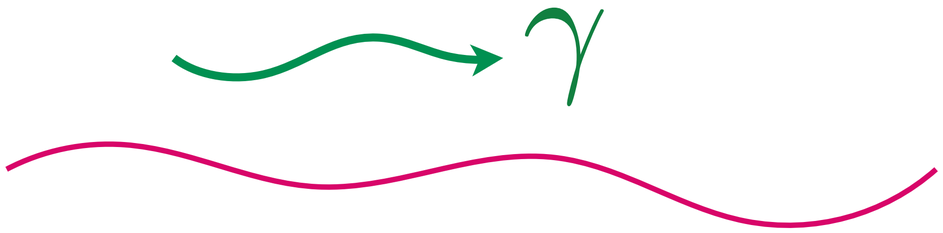
Haloscope + MAGO 2.0
see A. Berlin next

Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

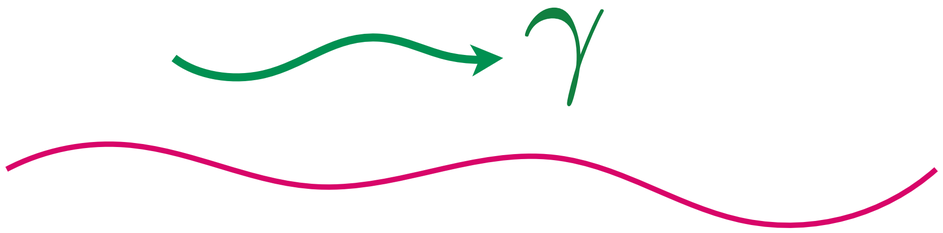
Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \longrightarrow \mathcal{L} \supset \mathcal{O}(hF^2)$$

Interactions of Gravitational Waves *with light*

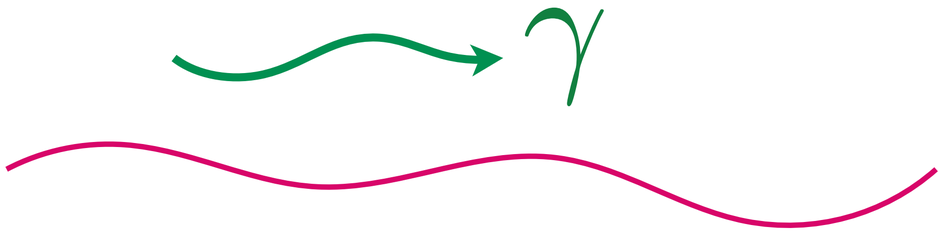


$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \longrightarrow \mathcal{L} \supset \mathcal{O}(hF^2)$$

Equation of motion: $\partial F \sim -\partial\left(h\,F\right)$

Interactions of Gravitational Waves *with light*



$$S_{\rm EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + g^{\mu\nu}J_\mu A_\nu\right)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \longrightarrow \mathcal{L} \supset \mathcal{O}(hF^2)$$

Equation of motion: $\partial F \sim -\partial\left(h\,F\right)$

Effective current from spatial or temporal variations of h or F

$$j^\mu_{\rm eff} \equiv \partial_\nu\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^\nu{}_\alpha\,F^{\alpha\mu} - h^\mu{}_\alpha\,F^{\alpha\nu}\right)$$

Cur Cavis?* Part I: Electromagnetic Signal

Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\text{eff}} \equiv \partial_{\nu} \left(\frac{1}{2} h F^{\mu\nu} + h^{\nu}{}_{\alpha} F^{\alpha\mu} - h^{\mu}{}_{\alpha} F^{\alpha\nu} \right)$$

* "Why Cavities?" in Latin

Cur Cavis?* Part I: Electromagnetic Signal

Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu}\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^{\nu}{}_{\alpha}\,F^{\alpha\mu} - h^{\mu}{}_{\alpha}\,F^{\alpha\nu}\right)$$

Currents can excite cavity modes $\mathbf{E}_{\rm cav}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav}\cdot\mathbf{J}_{\rm eff}$$

* "Why Cavities?" in Latin

Cur Cavis?* Part I: Electromagnetic Signal

Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu} \left(\frac{1}{2} h F^{\mu\nu} + h^{\nu}{}_{\alpha} F^{\alpha\mu} - h^{\mu}{}_{\alpha} F^{\alpha\nu} \right)$$

Currents can excite cavity modes $\mathbf{E}_{\rm cav}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav} \cdot \mathbf{J}_{\rm eff}$$

Should be reminiscent of axion physics...

* "Why Cavities?" in Latin

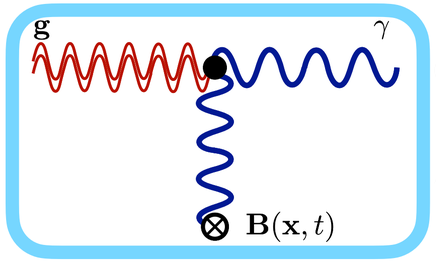
Cur Cavis?* Part I: Electromagnetic Signal

Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu}\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^{\nu}{}_{\alpha}\,F^{\alpha\mu} - h^{\mu}{}_{\alpha}\,F^{\alpha\nu}\right)$$

Currents can excite cavity modes $\mathbf{E}_{\rm cav}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav}\cdot\mathbf{J}_{\rm eff}$$



Gertsenshtein effect (1962)

Also Zeldovich (1973)

$$j_{\rm g} \sim \partial\,(h\,F)$$

Should be reminiscent of axion physics...

* “Why Cavities?” in Latin

Cur Cavis?* Part I: Electromagnetic Signal

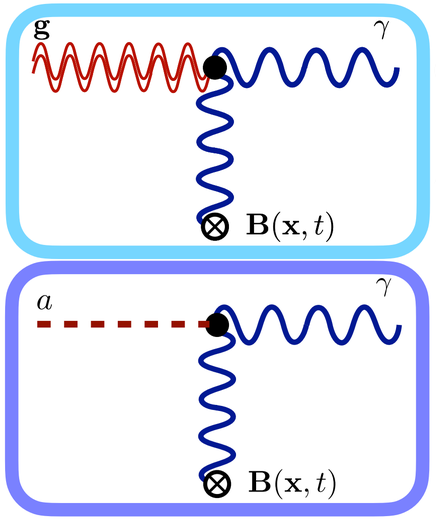
Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu}\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^{\nu}{}_{\alpha}\,F^{\alpha\mu} - h^{\mu}{}_{\alpha}\,F^{\alpha\nu}\right)$$

Currents can excite cavity modes $\mathbf{E}_{\rm cav}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav}\cdot\mathbf{J}_{\rm eff}$$

Should be reminiscent of axion physics...



Gertsenshtein effect (1962)

Also Zeldovich (1973)

$$j_{\rm g} \sim \partial\,(h\,F)$$

$$j_{\rm a} \sim g_{a\gamma\gamma}\partial\,(a\,F)$$

* "Why Cavities?" in Latin

Cur Cavis?* Part I: Electromagnetic Signal

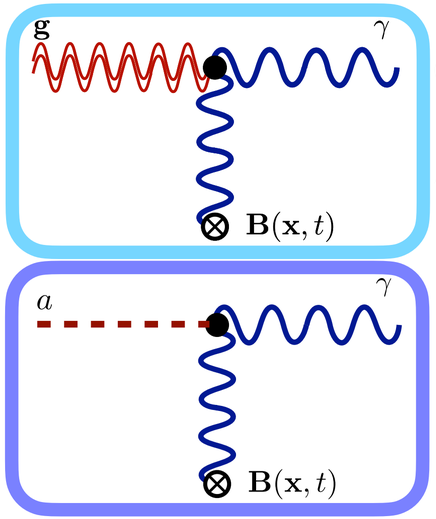
Effective current from spatial or temporal variations of h or F

$$j^{\mu}_{\rm eff} \equiv \partial_{\nu}\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^{\nu}{}_{\alpha}\,F^{\alpha\mu} - h^{\mu}{}_{\alpha}\,F^{\alpha\nu}\right)$$

Currents can excite cavity modes $\mathbf{E}_{\rm cav}$ as long as η non-zero:

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav}\cdot\mathbf{J}_{\rm eff}$$

Should be reminiscent of axion physics...



Gertsenshtein effect (1962)

$$j_{\rm g} \sim \partial\,(h\,F)$$

Also Zeldovich (1973)

$$j_{\rm a} \sim g_{a\gamma\gamma}\partial\,(a\,F)$$

Raffelt & Stodolsky (1988)

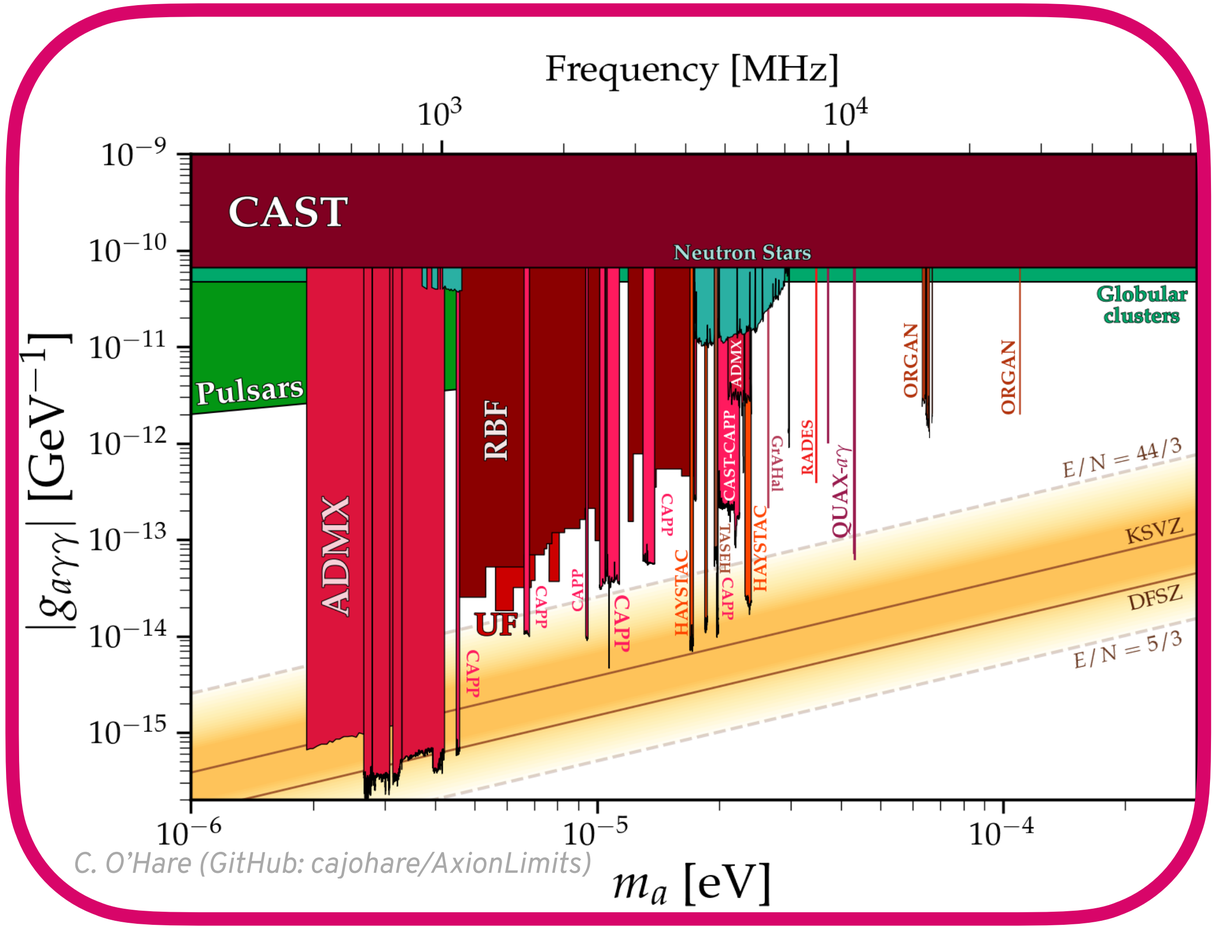
* "Why Cavities?" in Latin

Intuition for EM signal

Intuition for EM signal

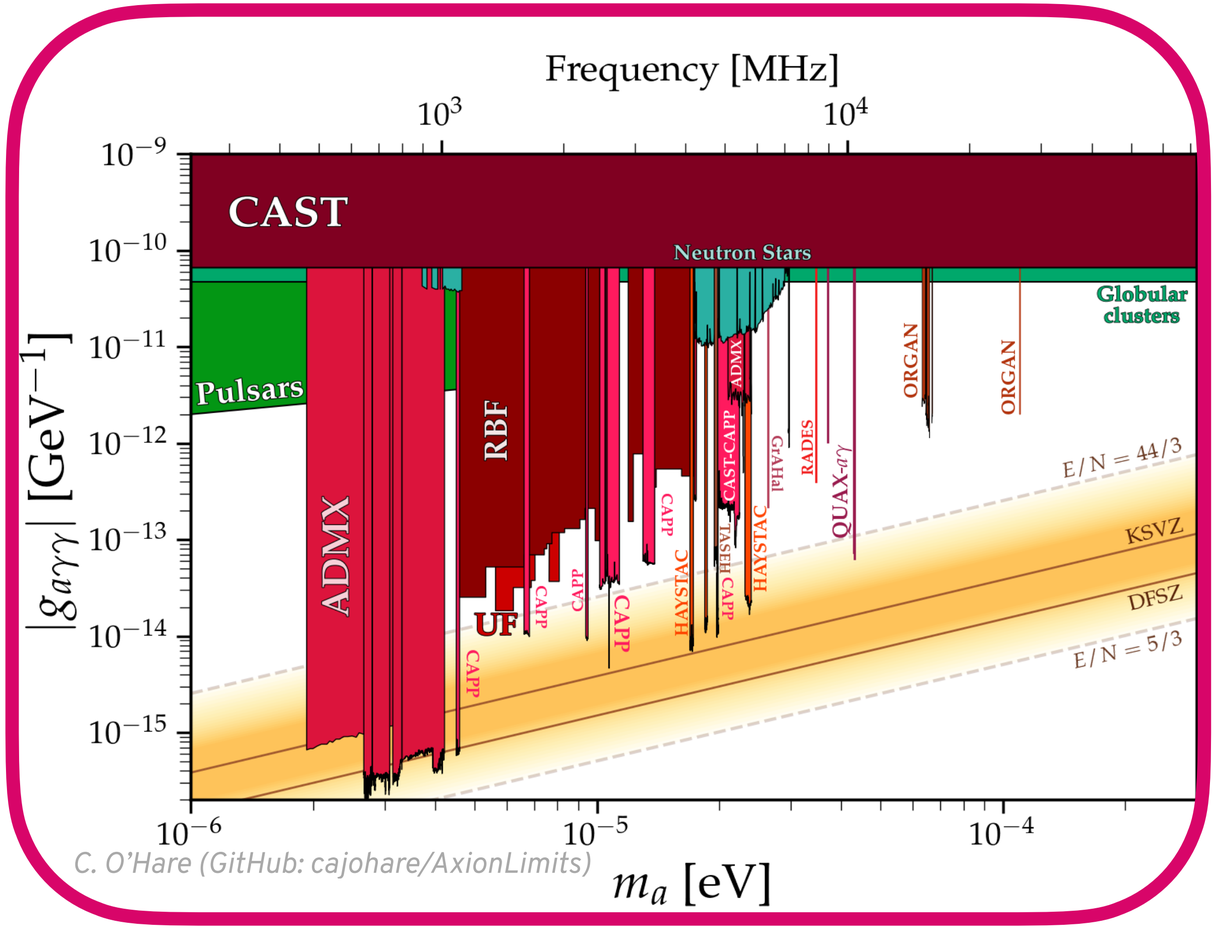
Estimate sensitivity to GWs by comparing sizes of currents

Intuition for EM signal



Estimate sensitivity to GWs by comparing sizes of currents

Intuition for EM signal

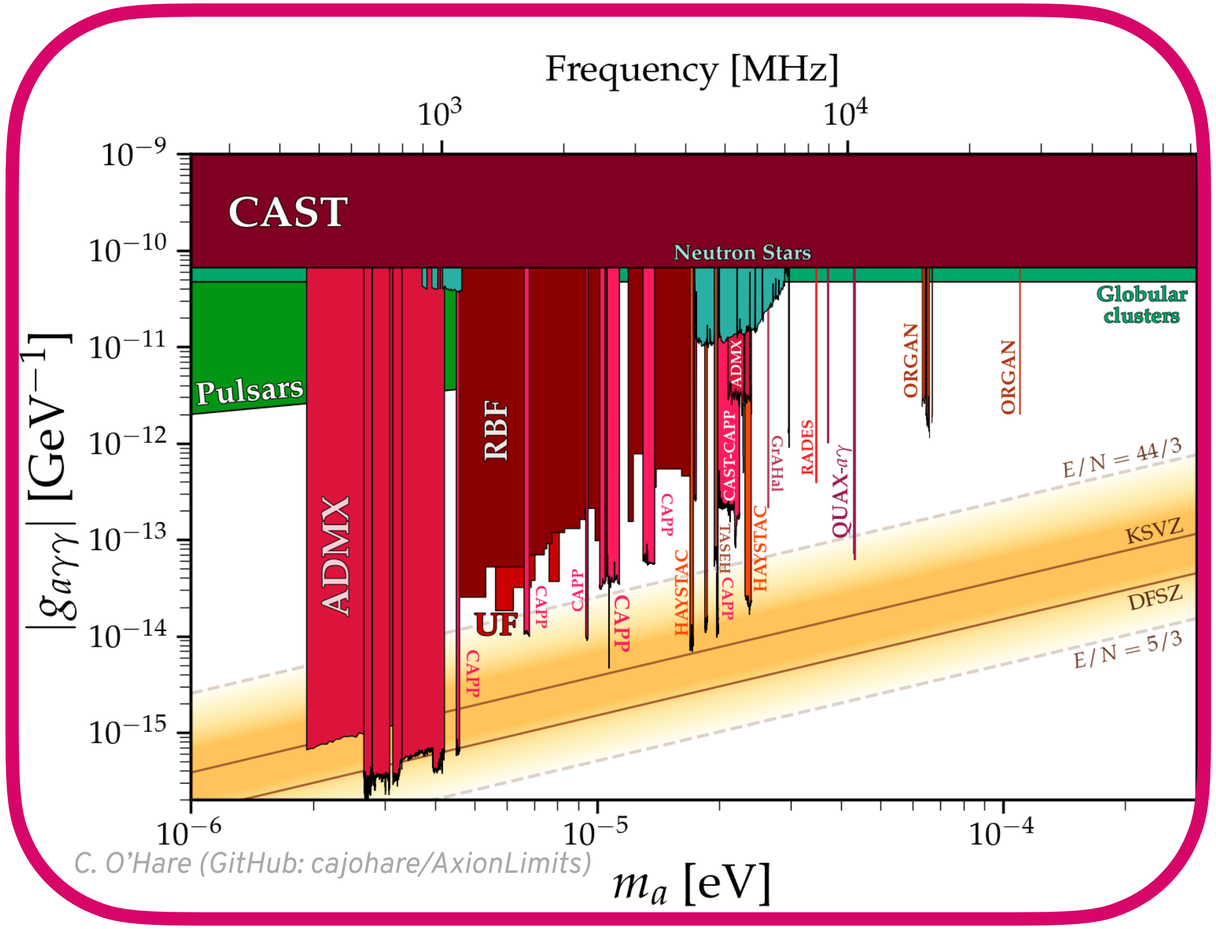


Estimate sensitivity to GWs by comparing sizes of currents

$$j_{\rm eff}^{\rm axion} \sim g_{a\gamma\gamma}\partial_t(a\mathbf{B}) + \mathcal{O}(v)$$

$$j_{\rm eff}^{\rm axion} \lesssim 10^{-19}\,{\rm T/m}$$

Intuition for EM signal



Estimate sensitivity to GWs by comparing sizes of currents

$$j_{\rm eff}^{\rm axion} \sim g_{a\gamma\gamma}\partial_t(a\mathbf{B}) + \mathcal{O}(v)$$

$$j_{\rm eff}^{\rm axion} \lesssim 10^{-19}\,{\rm T/m}$$

$$\Downarrow$$

$$j_{\rm eff}^{\rm GW} \sim \partial_t(h\mathbf{B}) + \ldots$$

$$h \lesssim 10^{-21}$$

Framing the Question

A more detailed estimate requires some GR

Framing the Question

A more detailed estimate requires some GR

GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

Framing the Question

A more detailed estimate requires some GR

GW in TT gauge: $\partial_\mu h^{\mu\nu} = 0, \quad h_\mu{}^\mu = 0, \quad h_{00} = h_{0i} = 0$

Riemann tensor invariant at O(h):

$$R_{0i0j} = -\frac{1}{2}\partial_t^2 h_{ij}^{\mathrm{TT}},$$

$$R_{0ijk} = \frac{1}{2}\partial_t \left(\partial_k h_{ij}^{\mathrm{TT}} - \partial_j h_{ik}^{\mathrm{TT}}\right),$$

$$R_{ikjl} = \frac{1}{2}\left(\partial_k\partial_j h_{il}^{\mathrm{TT}} + \partial_i\partial_l h_{jk}^{\mathrm{TT}} - \partial_i\partial_j h_{kl}^{\mathrm{TT}} - \partial_k\partial_l h_{ij}^{\mathrm{TT}}\right)$$

Framing the Question

Crucial to work in appropriate reference frame!

Framing the Question

Crucial to work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

$$\hat{\boldsymbol{n}} \times \boldsymbol{E} = 0 \qquad \hat{\boldsymbol{n}} \cdot \boldsymbol{B} = 0$$

Maxwell (19th century)

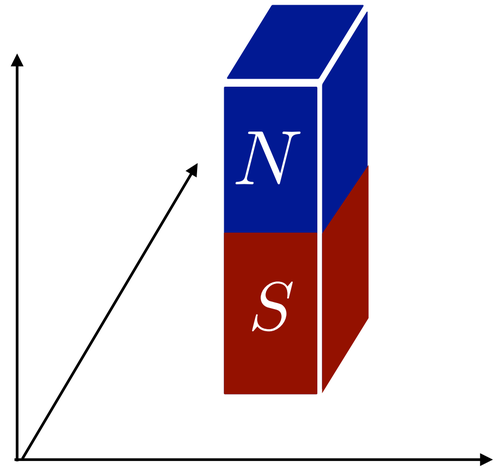
Framing the Question

Crucial to work in appropriate reference frame!

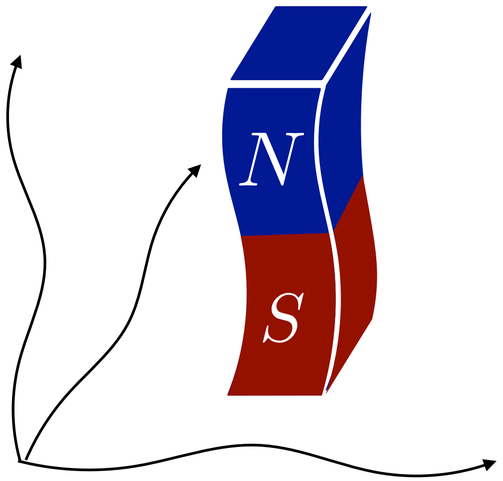
Detector in Local Inertial Frame (LIF)

$$\hat{\boldsymbol{n}} \times \boldsymbol{E} = 0 \qquad \hat{\boldsymbol{n}} \cdot \boldsymbol{B} = 0$$

Maxwell (19th century)



B-field in LIF $\neq$ B-field in TT



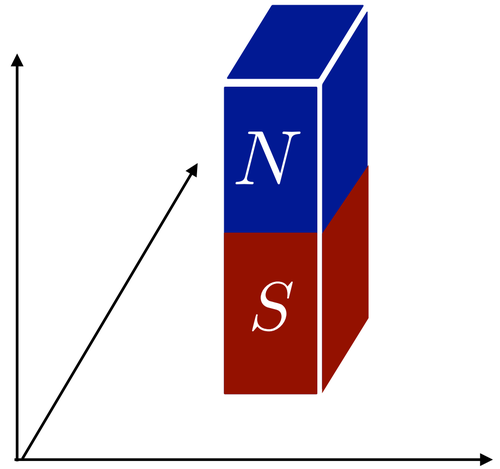
Framing the Question

Crucial to work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

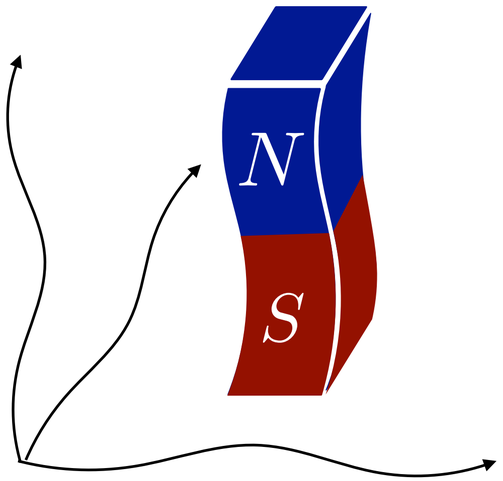
$$\hat{\boldsymbol{n}} \times \boldsymbol{E} = 0 \qquad \hat{\boldsymbol{n}} \cdot \boldsymbol{B} = 0$$

Maxwell (19th century)



B-field in LIF $\neq$ B-field in TT

Which frame is the right one to use?



Framing the Question

Proper Detector Frame — complication

Framing the Question

Proper Detector Frame — complication

Textbooks give long-wavelength approximation $\omega_g \, R_{\rm cav} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$ *e.g. Maggiore (2007)*

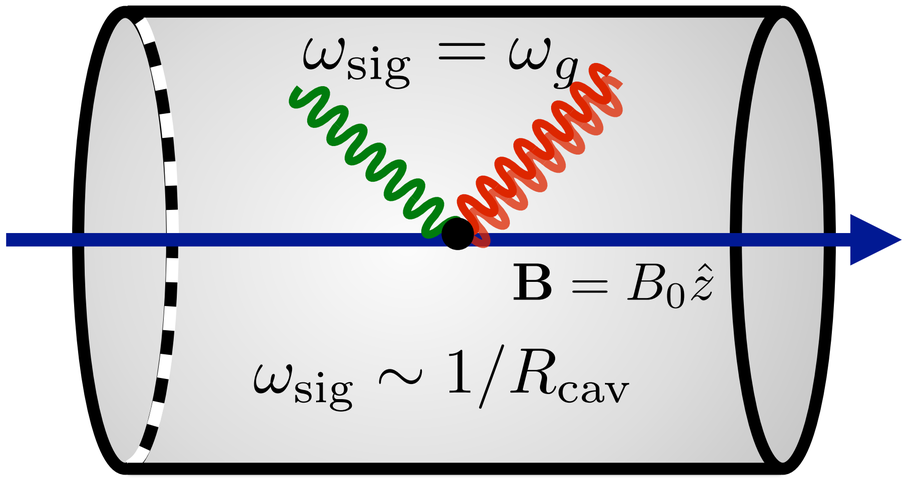
Framing the Question

Proper Detector Frame — complication

Textbooks give long-wavelength approximation $\omega_g \, R_{\rm cav} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$ *e.g. Maggiore (2007)*

Resonant Cavity:



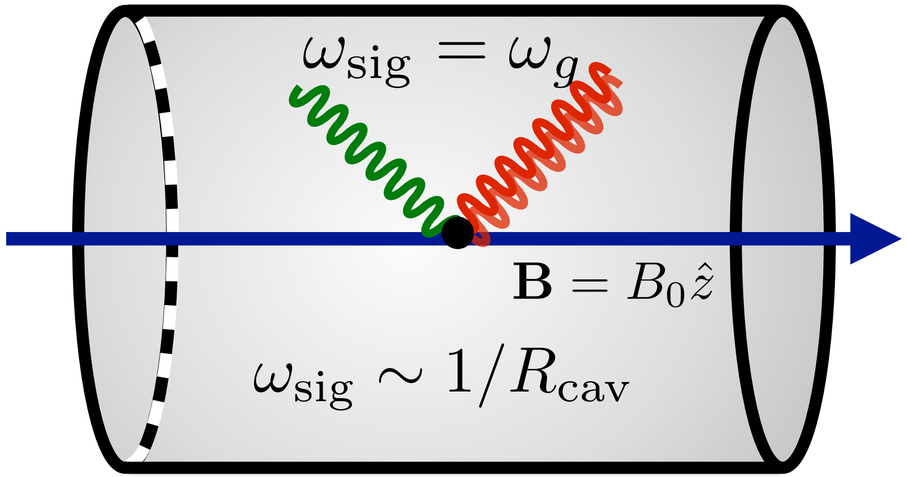
Framing the Question

Proper Detector Frame — complication

Textbooks give long-wavelength approximation $\omega_g \, R_{\rm cav} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$ *e.g. Maggiore (2007)*

Resonant Cavity:



Framing the Question

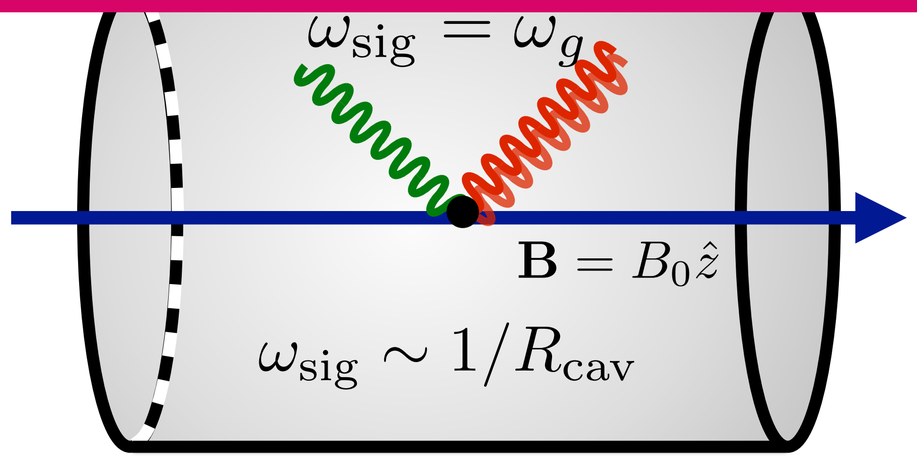
Proper Detector Frame — complication

Textbooks give long-wavelength approximation $\omega_g \, R_{\rm cav} \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$ *e.g. Maggiore (2007)*

Long-wavelength limit invalid!

Resonant Cavity:



Framing the Question

Solution — GW as sum of plane waves

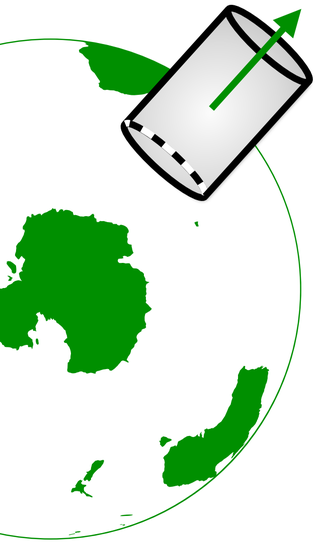
$$h \propto e^{i\omega_g(t-z)} \longrightarrow \partial_i h^{\mathrm{TT}}_{jk} \sim -\delta_{iz}\partial_t h^{\mathrm{TT}}_{jk}$$

$$x^{k_1}\dots x^{k_r} R_{\mu\nu\rho\sigma,k_1\dots k_r} = (-i\omega_g z)^r R_{\mu\nu\rho\sigma}$$

$$h_{00} = -2\sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n,k_1,\cdots k_r} x^m x^n x^{k_1}\cdots x^{k_r}$$

$$h_{0i} = -2\sum_{r=0}^{\infty} \frac{r+2}{(r+3)!} R_{0nin,k_1,\cdots k_r} x^m x^n x^{k_1}\cdots x^{k_r}$$

$$h_{ij} = -2\sum_{r=0}^{\infty} \frac{r+1}{(r+3)!} R_{injn,k_1,\cdots k_r} x^m x^n x^{k_1}\cdots x^{k_r}$$



Märzlin (1994)
Rakhmanov (2014)

Framing the Question

Solution — GW as sum of plane waves

$$h \propto e^{i\omega_g(t-z)} \longrightarrow \partial_i h^{\mathrm{TT}}_{jk} \sim -\delta_{iz}\partial_t h^{\mathrm{TT}}_{jk}$$

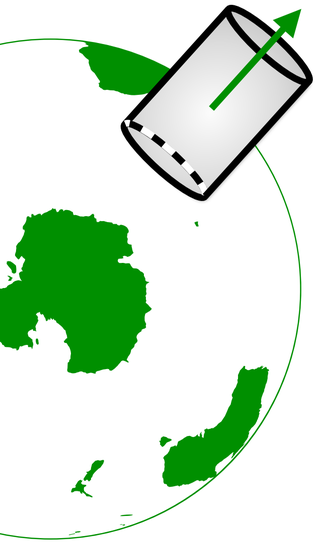
$$x^{k_1}\dots x^{k_r} R_{\mu\nu\rho\sigma,k_1\dots k_r} = (-i\omega_g z)^r R_{\mu\nu\rho\sigma}$$

$$h_{00} = -2R_{0m0n}x^m x^n \left(-\frac{i}{\omega_g z} + \frac{1+e^{-i\omega_g z}}{(\omega_g z)^2}\right)$$

$$h_{0i} = -2R_{0min}x^m x^n \left(-\frac{i}{2\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

$$h_{ij} = -2R_{imjn}x^m x^n \left(-\frac{1+e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\frac{1-e^{-i\omega_g z}}{(\omega_g z)^3}\right)$$

Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)



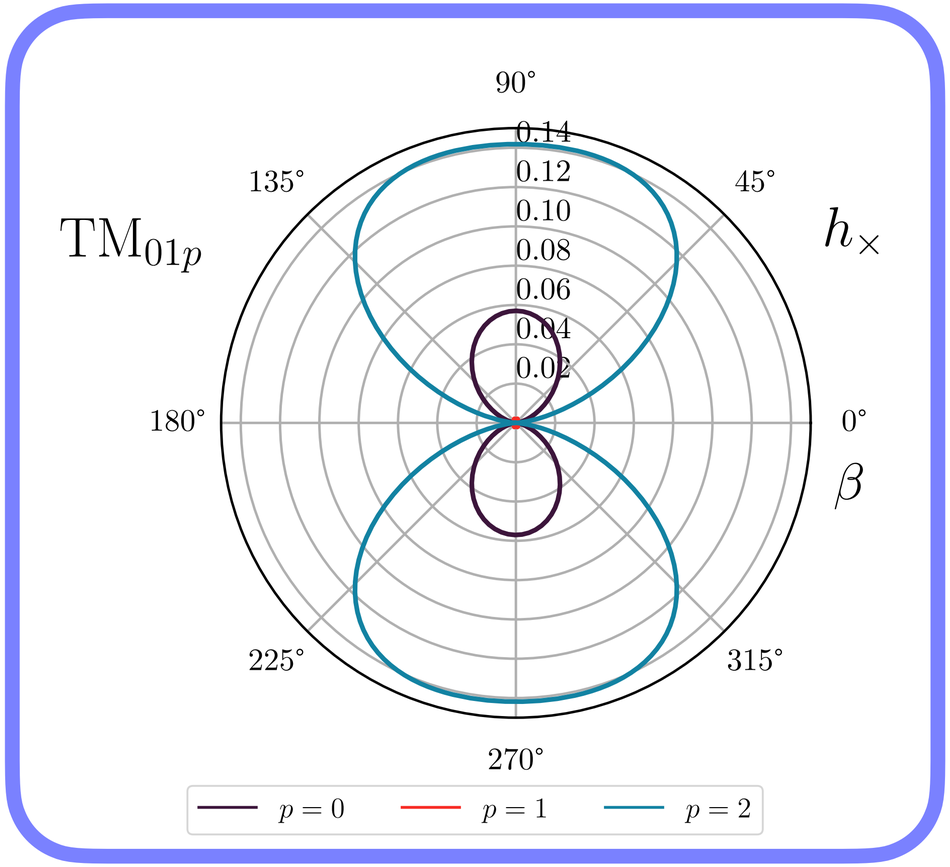
Märzlin (1994)
Rakhmanov (2014)

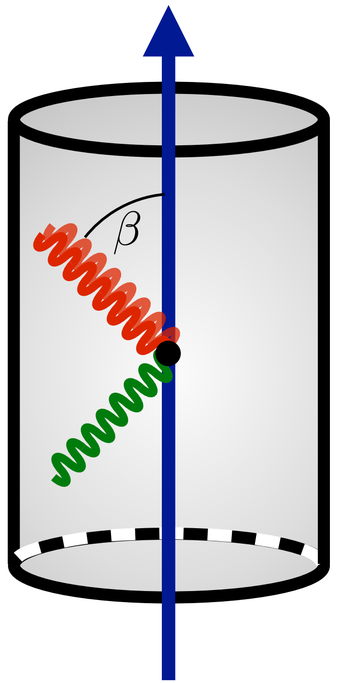
Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\mathrm{cav}} \cdot \mathbf{J}_{\mathrm{eff}}$$

Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav} \cdot \mathbf{J}_{\rm eff}$$

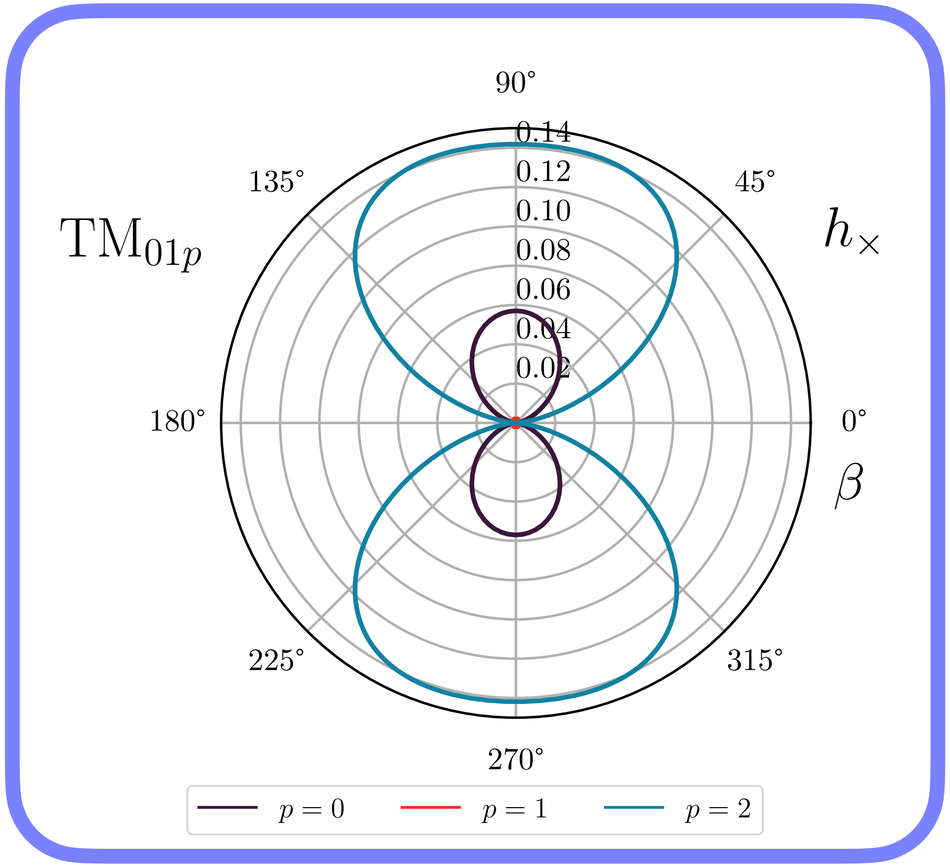




Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

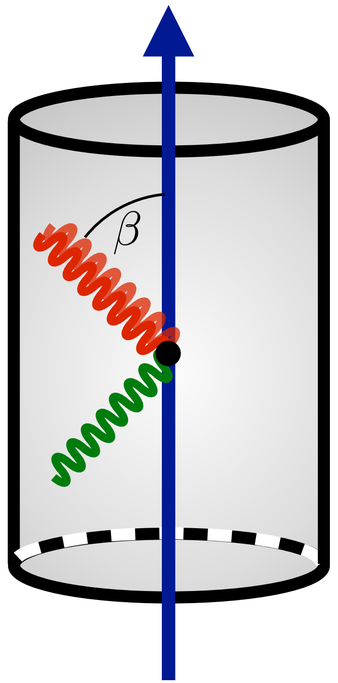
Axion Cavity Modes Couple to GWs

$$\eta \propto \int_V \mathbf{E}^*_{\rm cav} \cdot \mathbf{J}_{\rm eff}$$

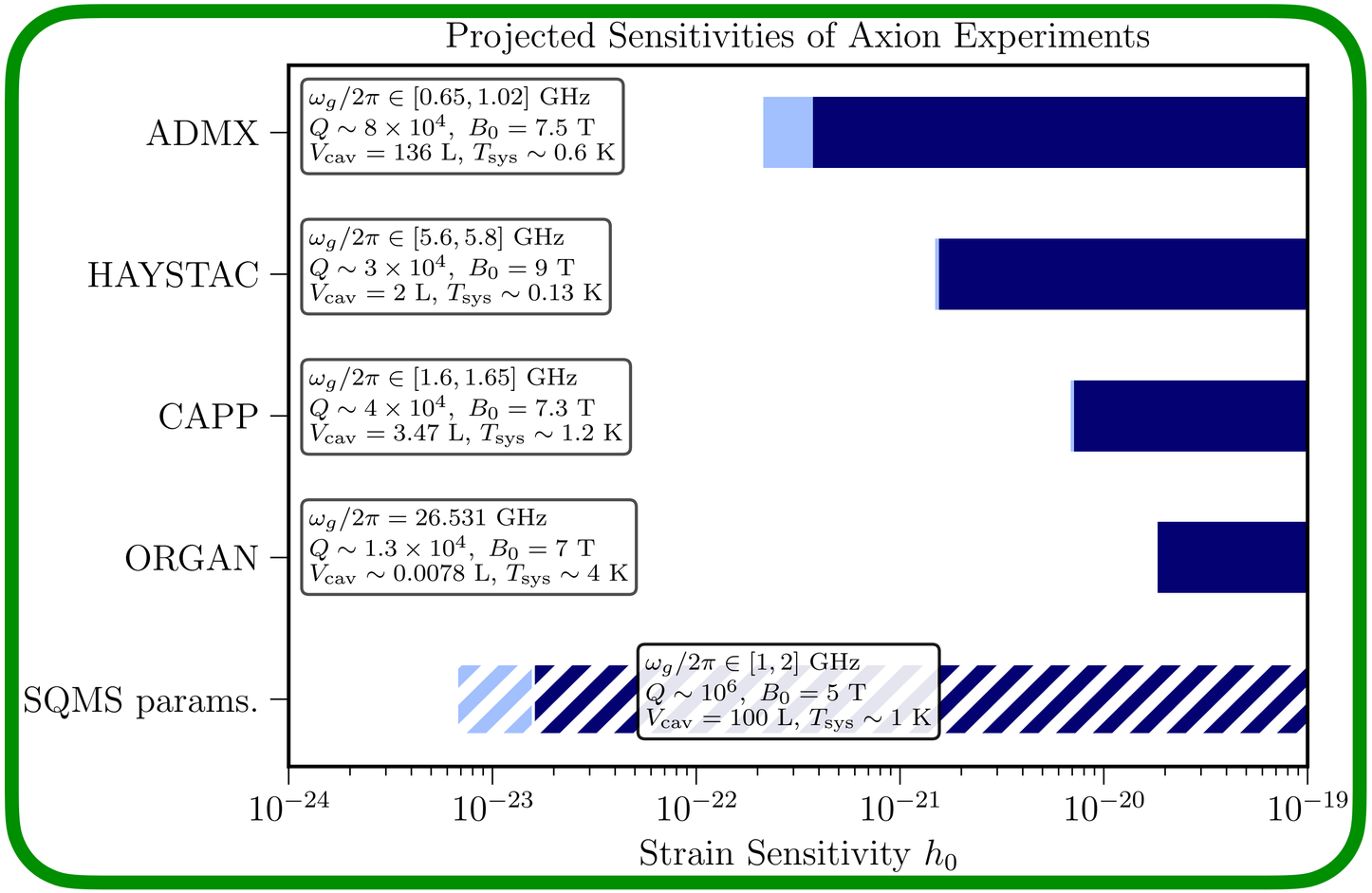


Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

But TM modes not optimal...



Axion Cavity Sensitivity



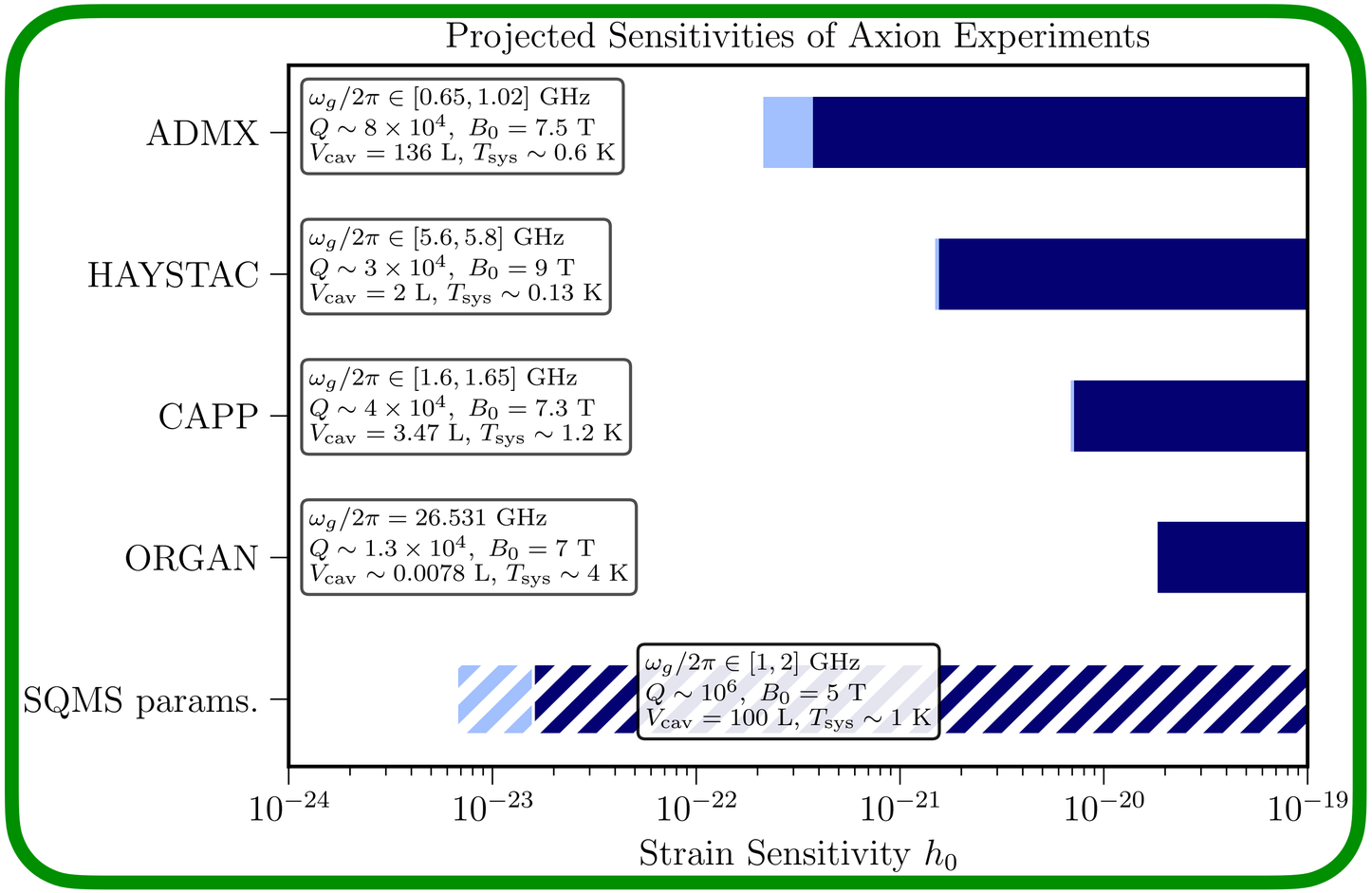
Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{\rm sig} = \frac{1}{2} Q \, \omega_g^3 \, V_{\rm cav}^{5/3} \, (\eta_n \, h_0 \, B_0)^2$$

see F. Muia's talk for more on signals

Axion Cavity Sensitivity



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{\rm sig} = \frac{1}{2} Q \, \omega_g^3 \, V_{\rm cav}^{5/3} \, (\eta_n \, h_0 \, B_0)^2$$

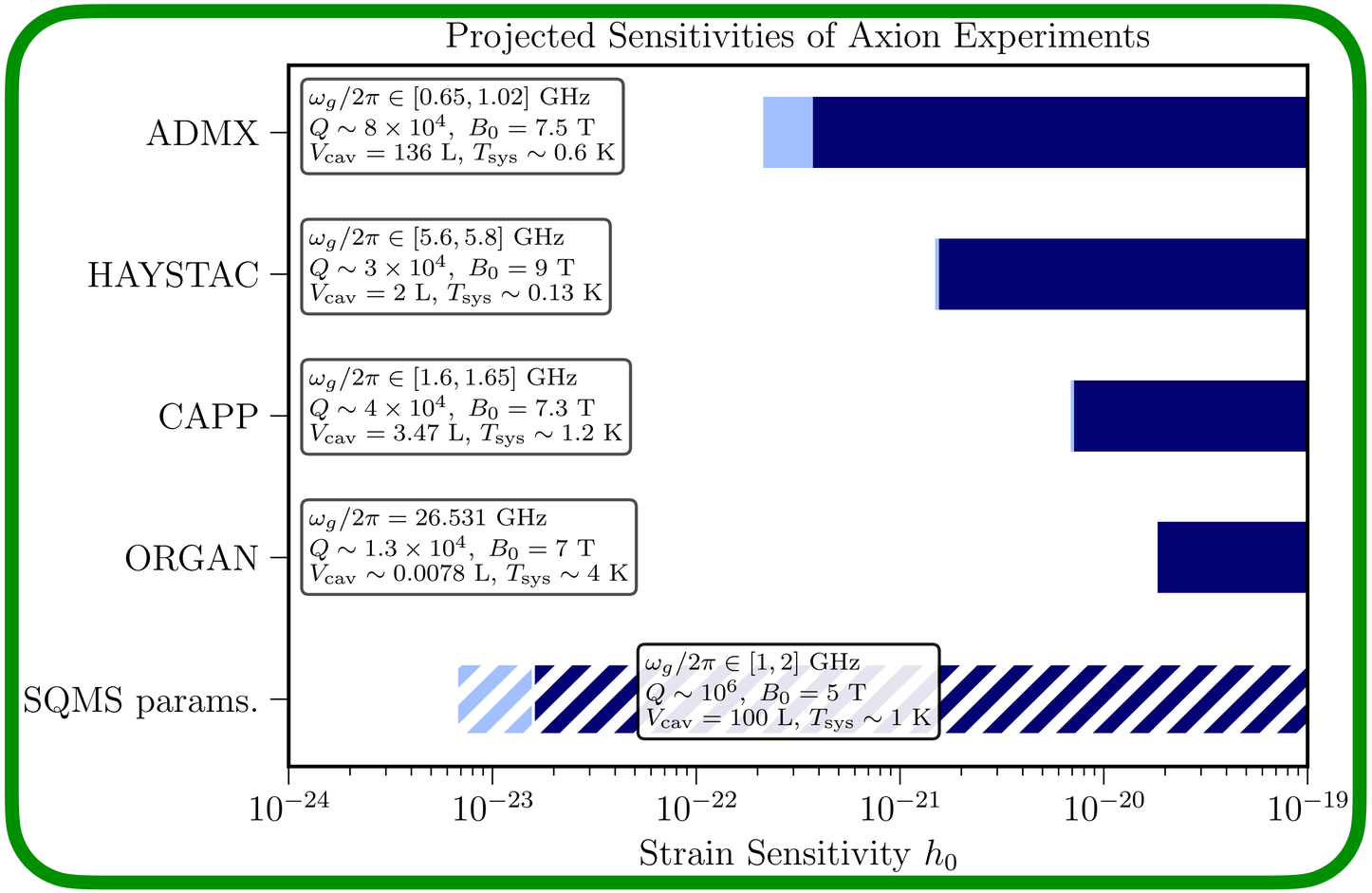
Stochastic GWs

$$\mathrm{SNR} \sim Q \, \omega_g \; \eta_{\rm stoch}^2 \, B_0^2 \, V_{\rm cav} \, S_h(\omega_g)/T_{\rm sys}$$

$$\Omega_g(\omega_g) \sim \omega_g^3 \, S_h(\omega_g)/H_0^2$$

see F. Muia's talk for more on signals

Axion Cavity Sensitivity



Berlin, Blas, D'Agnolo, SARE, Harnik, Kahn, Schutte-Engel (PRD 2022)

Coherent GW

$$P_{\rm sig} = \frac{1}{2} Q \, \omega_g^3 \, V_{\rm cav}^{5/3} \, (\eta_n \, h_0 \, B_0)^2$$

Stochastic GWs

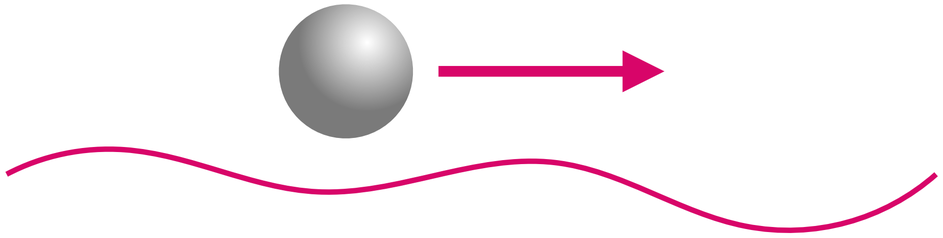
$$\mathrm{SNR} \sim Q \, \omega_g \; \eta_{\rm stoch}^2 \, B_0^2 \, V_{\rm cav} \, S_h(\omega_g)/T_{\rm sys}$$

$$\Omega_g(\omega_g) \sim \omega_g^3 \, S_h(\omega_g)/H_0^2$$

Not beyond BBN bound...

see F. Muia's talk for more on signals

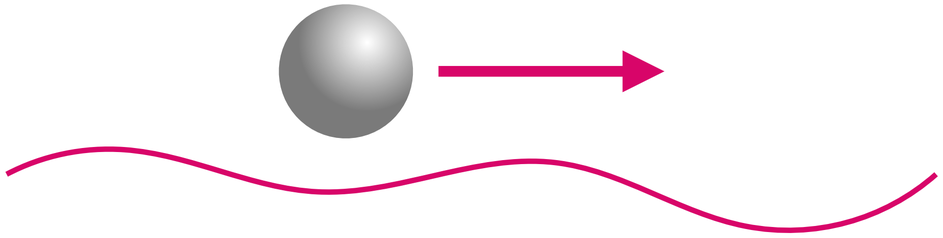
Interactions of Gravitational Waves *with masses*



$$S = -\int dt\; m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

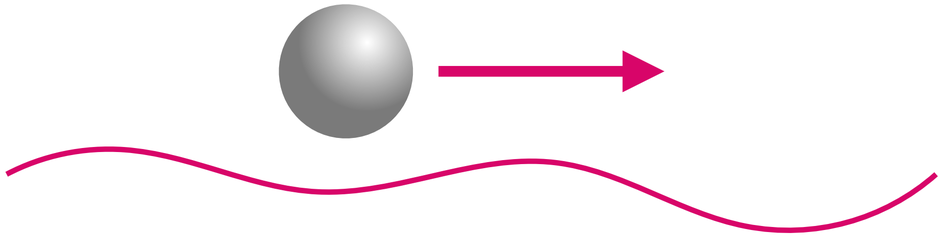
Interactions of Gravitational Waves *with masses*



$$S = -\int dt\; m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

Equation of motion: $\dfrac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x)\dfrac{dx^\nu}{d\tau}\dfrac{dx^\rho}{d\tau} = 0$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$

Interactions of Gravitational Waves *with masses*



$$S = -\int dt\; m\sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}$$

Equation of motion: $\dfrac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}(x)\dfrac{dx^\nu}{d\tau}\dfrac{dx^\rho}{d\tau} = 0$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$

Effect of GW encoded in Christoffel symbol $\qquad \Gamma \propto \partial h$

Encore: Framing the Question

Work in appropriate reference frame!

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\frac{F_i}{m}$$

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\frac{F_i}{m} \qquad \frac{d^2\xi_i}{d\tau^2} \simeq -\partial_i \, \Gamma^j_{00} \, \xi^i$$

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\frac{F_i}{m}$$

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\partial_i \, \Gamma^j_{00} \, \xi^i$$

$$F_i \simeq \frac{m}{2} \ddot{h}^{\mathrm{TT}}_{ij} x^i$$

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\frac{F_i}{m} \qquad \frac{d^2\xi_i}{d\tau^2} \simeq -\partial_i \Gamma^j_{00}\, \xi^i \qquad F_i \simeq \frac{m}{2} \ddot{h}^{\mathrm{TT}}_{ij} x^i$$

Long-wavelength approximation valid because materials have $c_s \ll 1$

e.g. Maggiore (2007)

Encore: Framing the Question

Work in appropriate reference frame!

Detector in Local Inertial Frame (LIF)

Effect of GW in LIF is that of a Newtonian Force

$$\frac{d^2\xi_i}{d\tau^2} \simeq -\frac{F_i}{m}$$

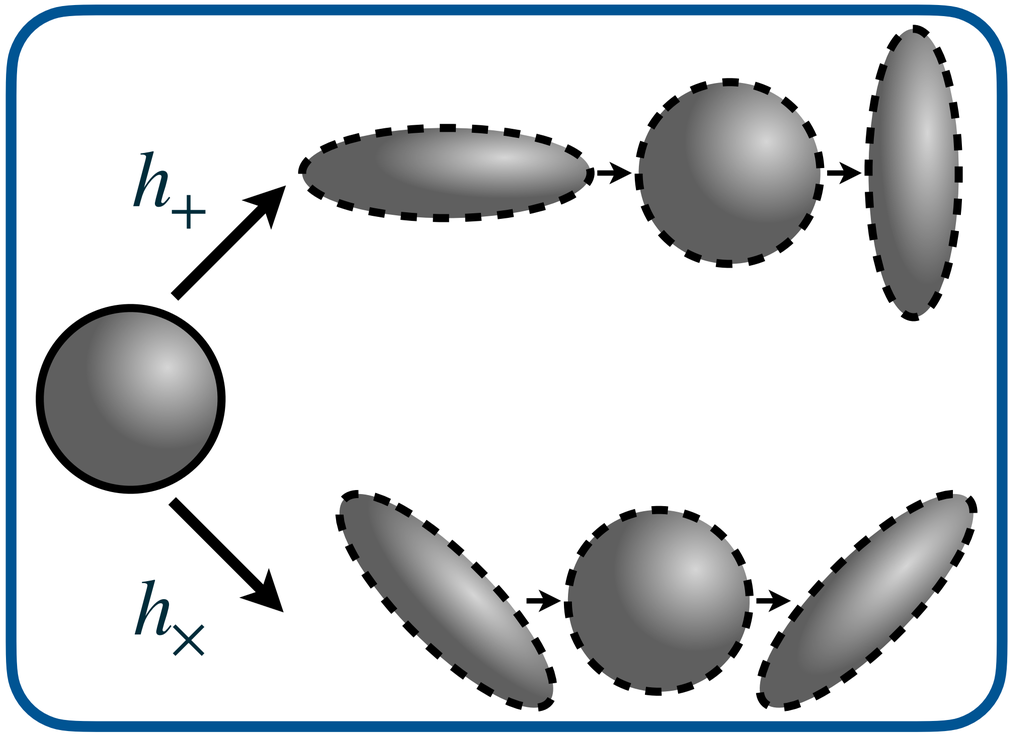
$$\frac{d^2\xi_i}{d\tau^2} \simeq -\partial_i \, \Gamma^j_{00} \, \xi^i$$

$$F_i \simeq \frac{m}{2} \ddot{h}^{\mathrm{TT}}_{ij} x^i$$

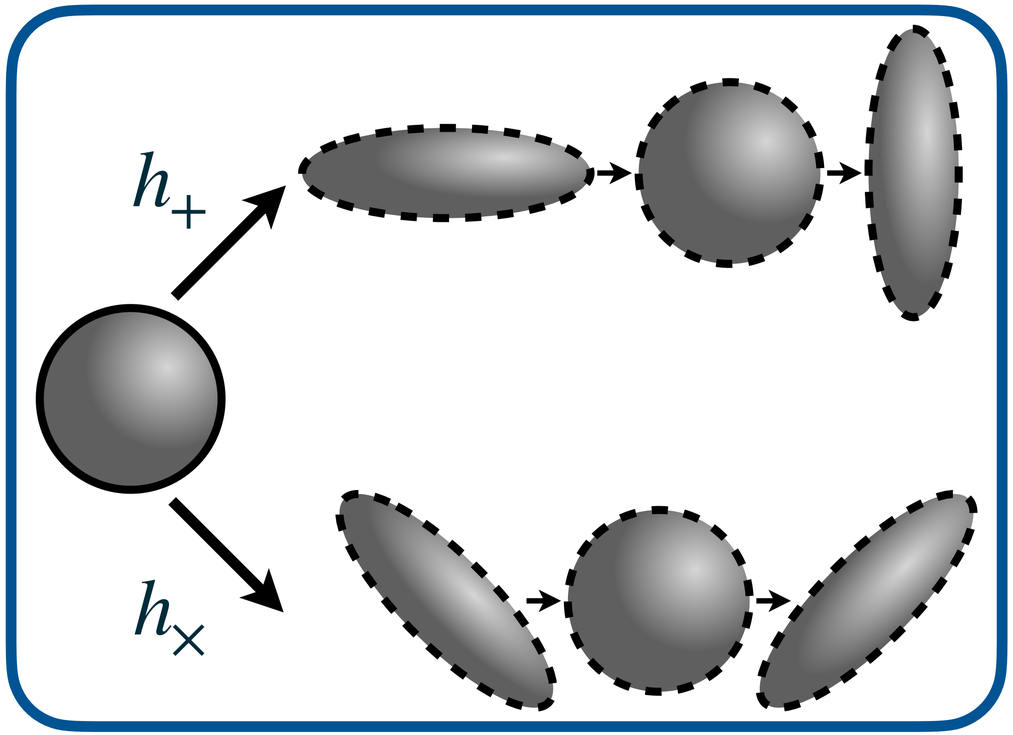
Long-wavelength approximation valid because materials have $c_s \ll 1$

$$ds^2 \simeq -dt^2(1 + R_{0i0j}x^i x^j) - \frac{4}{3}\, dt\, dx^i \left(R_{0ijk}x^j x^k\right) + dx^i\, dx^j \left(\delta_{ij} - \frac{1}{3}R_{ikjl}x^k x^l\right)$$ *e.g. Maggiore (2007)*

Gravitational Wave and a Hollow Sphere



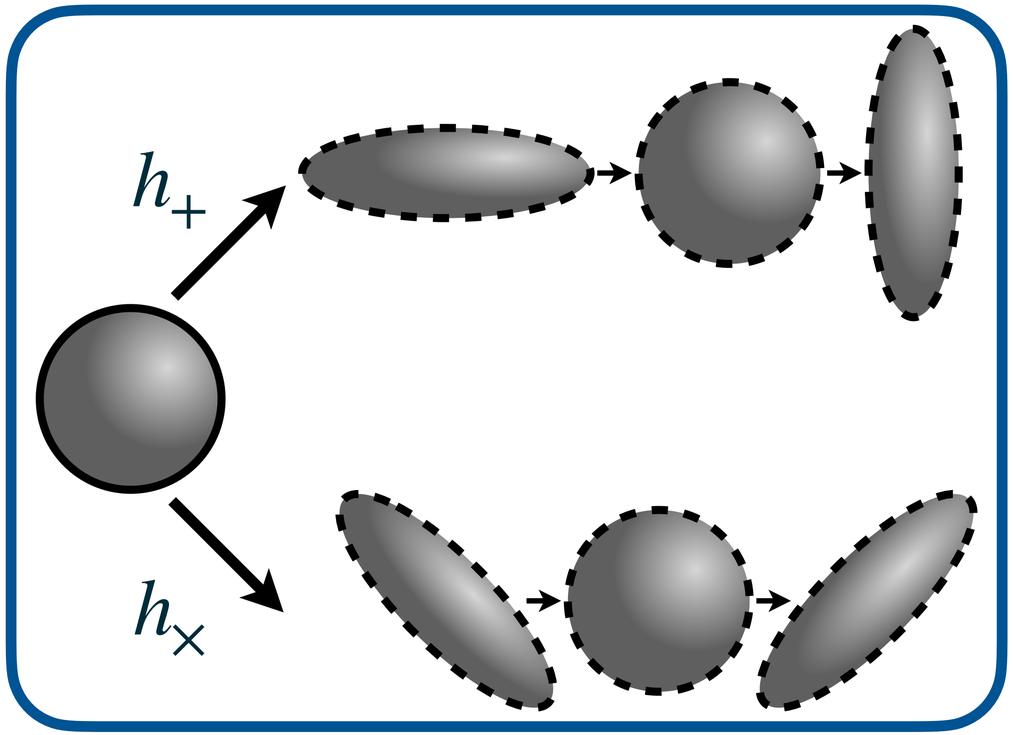
Gravitational Wave and a Hollow Sphere



Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla \times \mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

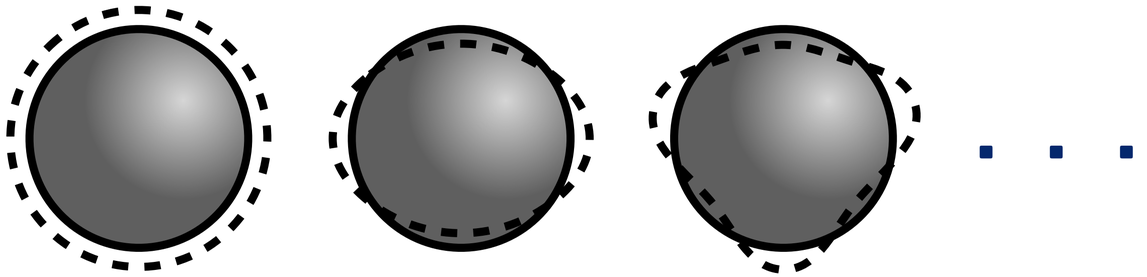
Gravitational Wave and a Hollow Sphere



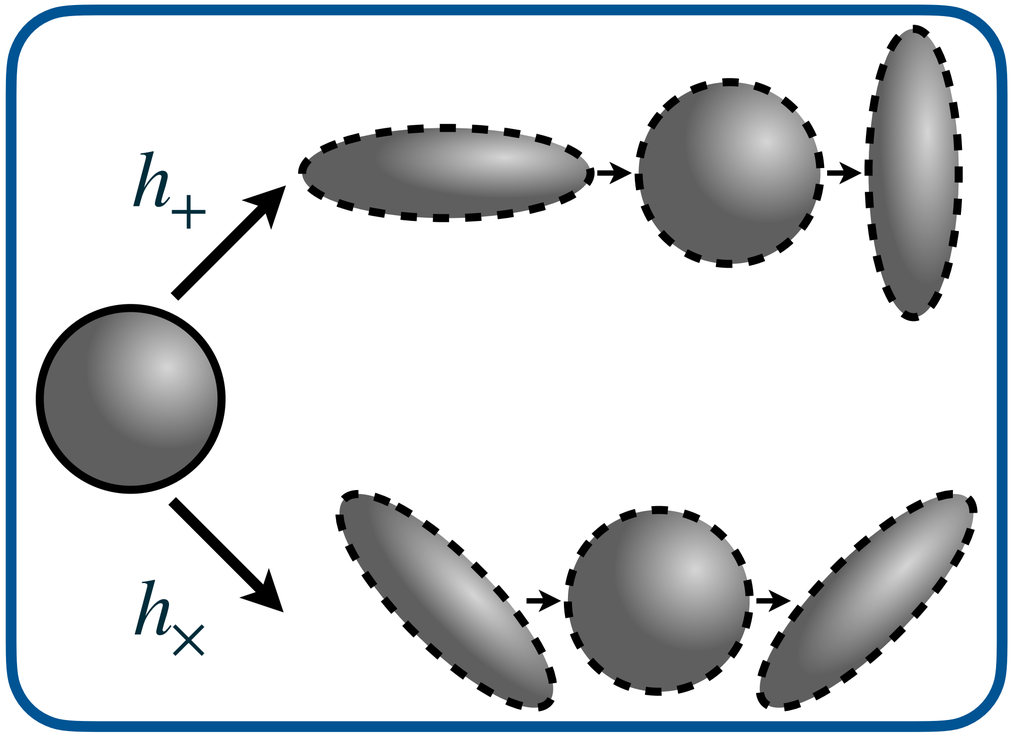
Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

Spheroidal



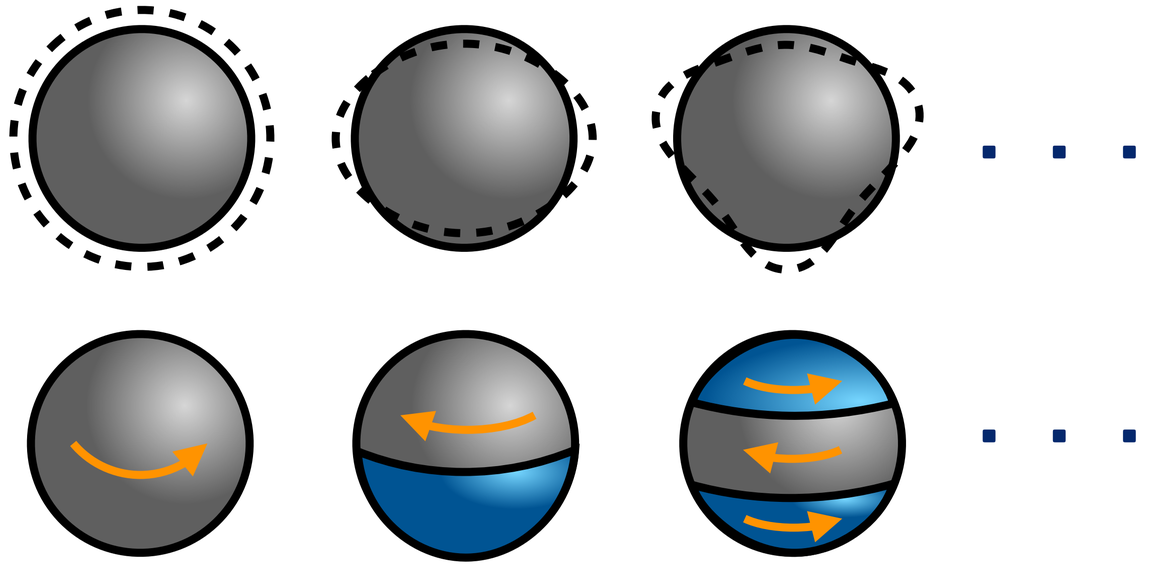
Gravitational Wave and a Hollow Sphere



Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\,.$$

Spheroidal Toroidal



Gravitational Wave and a Hollow Sphere

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla \times \mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x}, t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x},t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{TT}_{ij}}{V_{\rm cav}^{1/3}\,V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}\;U_p^{*i}\,x^j$$

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x},t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{TT}_{ij}}{V_{\rm cav}^{1/3}\,V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}\;U_p^{*i}\,x^j$$

$$\langle\mathbf{U}_p\rangle \sim h_0\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\times\begin{cases}\dfrac{\omega_g^2}{\omega_g^2-\omega_p^2}\ , & |\omega_g-\omega_p|\gg\omega_p/Q_p\\ Q_p\ , & |\omega_g-\omega_p|\ll\omega_p/Q_p\end{cases}$$

Tiny displacement $\ll$ nm

Gravitational Wave and a Hollow Sphere

Mechanical modes of a sphere

$$\mathbf{U}_{lmn} = \nabla\phi_L + i\,\nabla\times\mathbf{L}\phi_{T_1} + i\,\mathbf{L}\phi_{T_2}\ .$$

$$\mathbf{U}(\mathbf{x},t) = u_p(t)\,\mathbf{U}_p(\mathbf{x})$$

Equation of motion

$$\ddot{u}_p + \frac{\omega_p}{Q_p}\,\dot{u}_p + \omega_p^2\,u_p \simeq -\frac{1}{2}\,\omega_g^2\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\,h_0\,e^{i\omega_g t}$$

$$\eta^g_{\rm mech} = \frac{\hat{h}^{TT}_{ij}}{V_{\rm cav}^{1/3}\,V_{\rm shell}}\int_{V_{\rm shell}} d^3\mathbf{x}\;U_p^{*i}\,x^j$$

$$\langle\mathbf{U}_p\rangle \sim h_0\,V_{\rm cav}^{1/3}\,\eta^g_{\rm mech}\times\begin{cases}\dfrac{\omega_g^2}{\omega_g^2-\omega_p^2}\ , & |\omega_g-\omega_p|\gg\omega_p/Q_p\\ Q_p\ , & |\omega_g-\omega_p|\ll\omega_p/Q_p\end{cases}$$

Tiny displacement $\ll$ nm

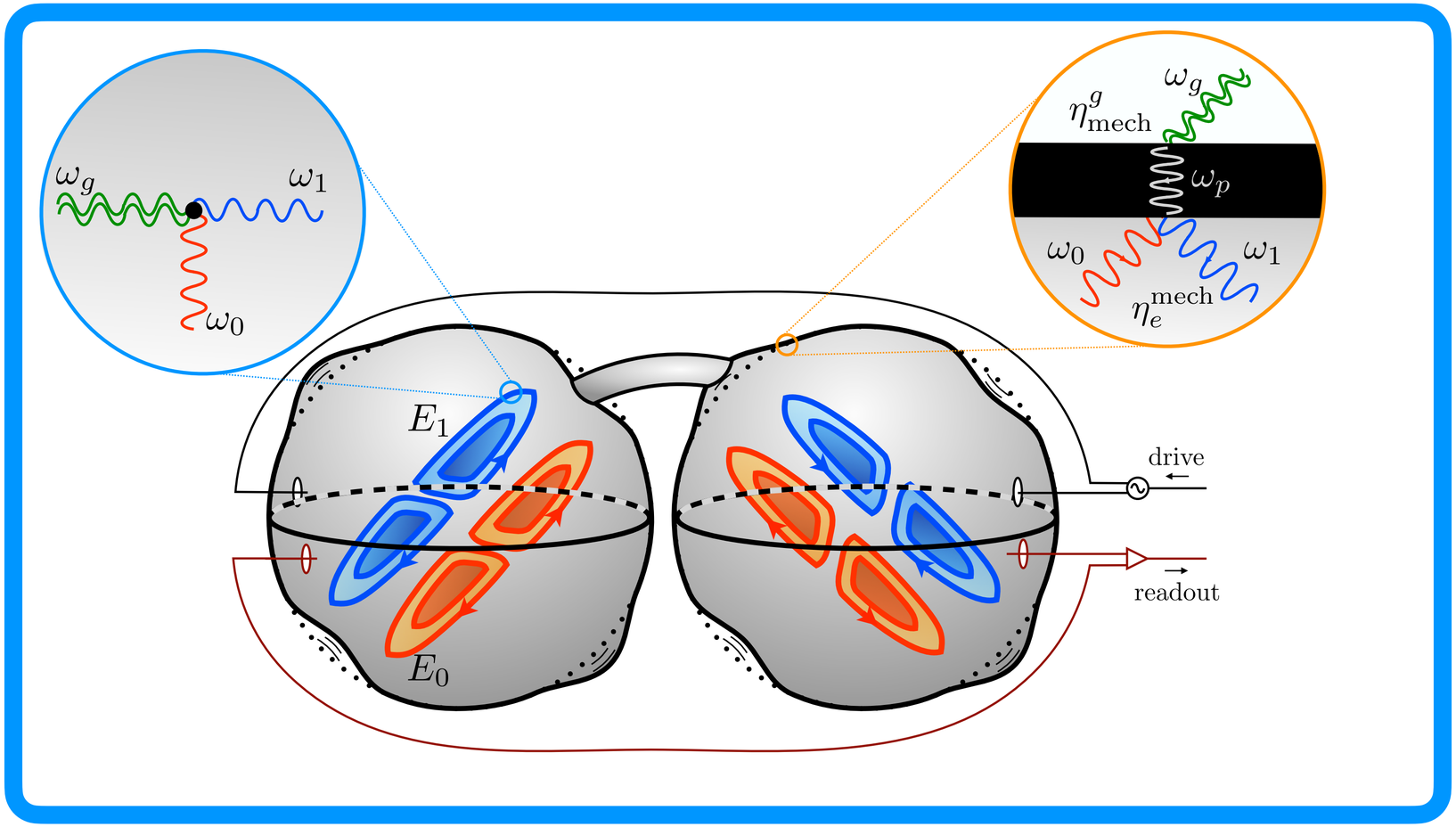
Cur Cavis?* pt. 2

MAGO 2.0

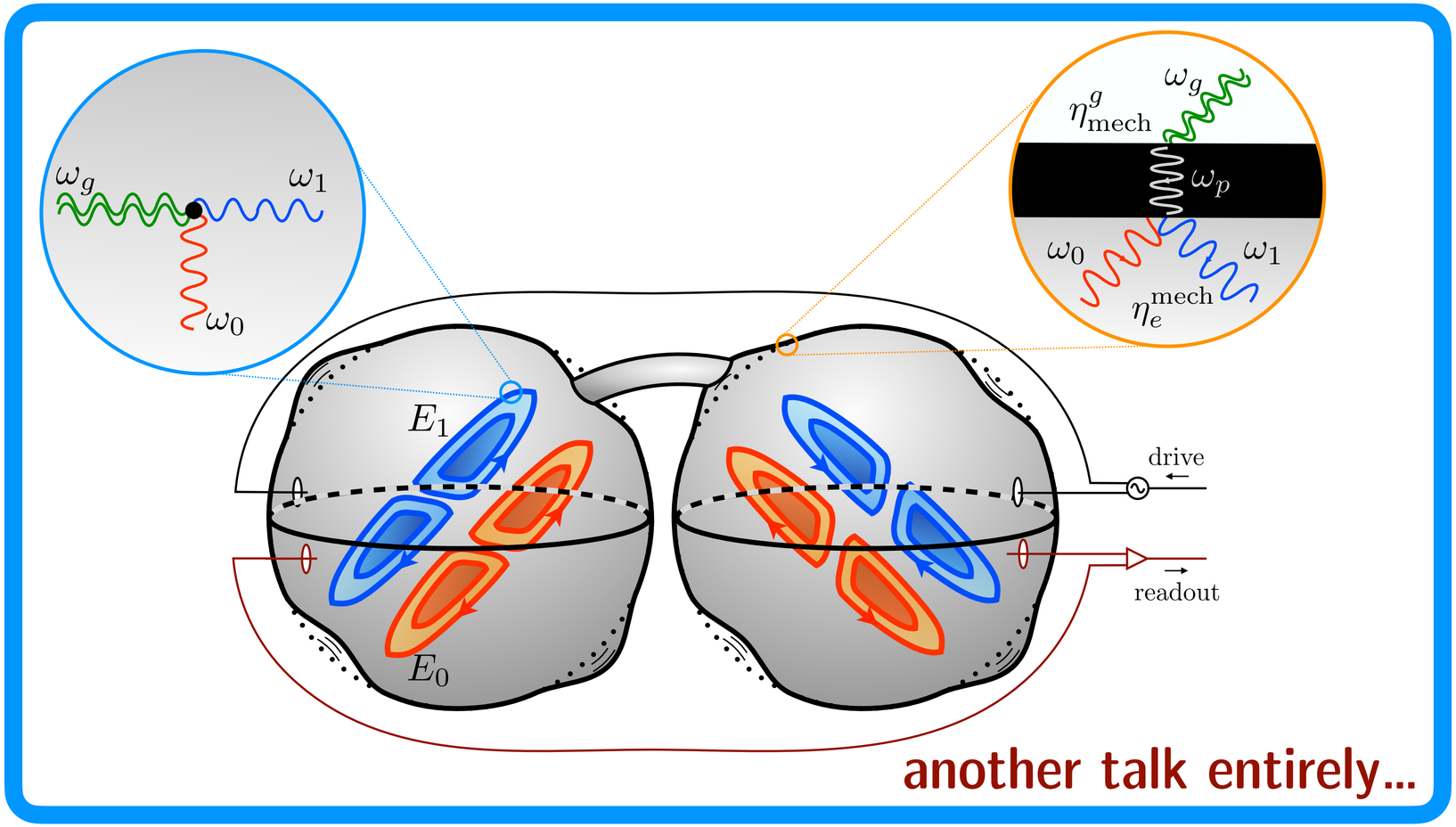
see A. Berlin next

* "Why Cavities?" in Latin

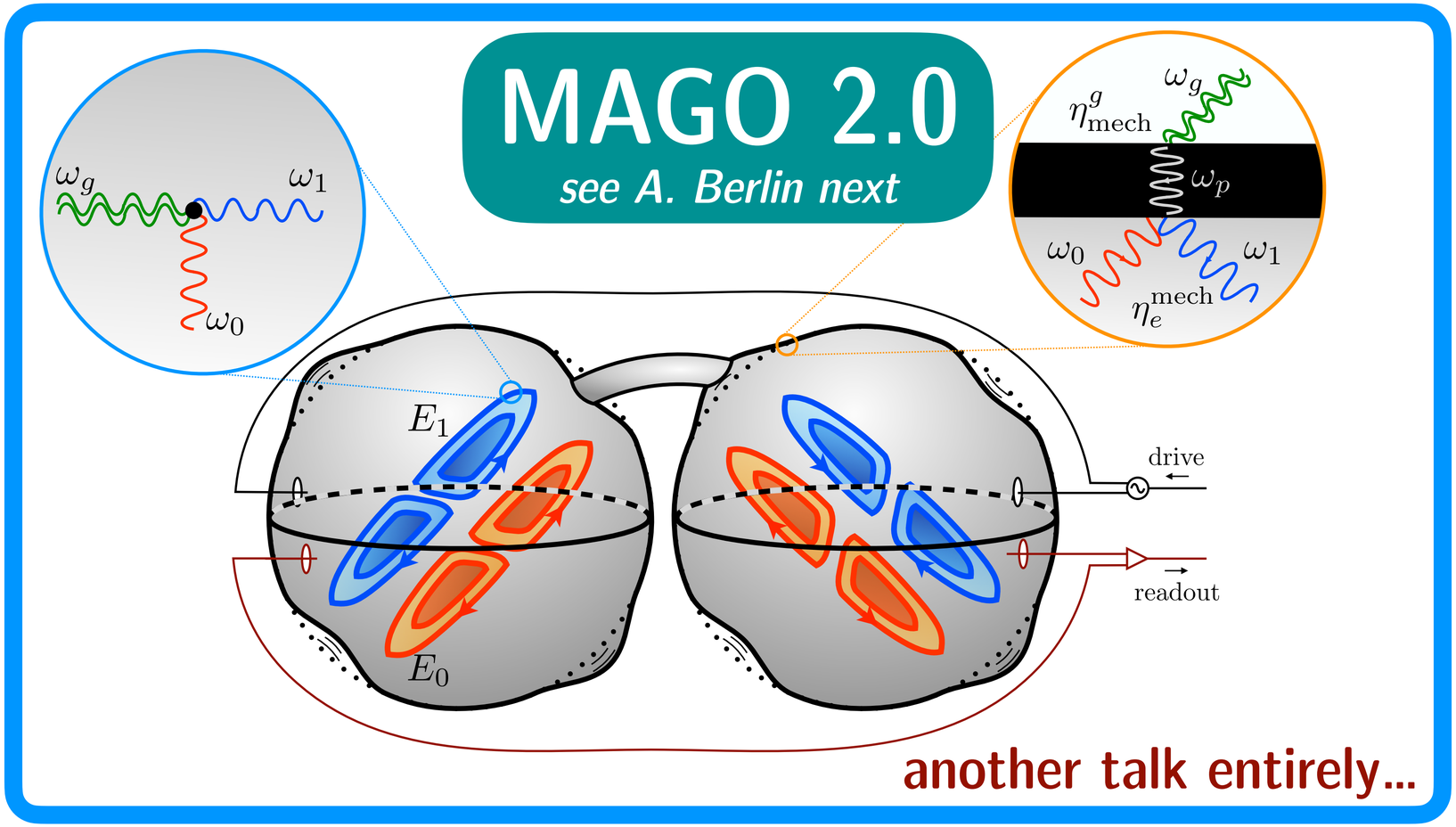
Mechanical and EM Signals Coexist



Mechanical and EM Signals Coexist



Mechanical and EM Signals Coexist



Open questions

Are resonant cavities the best we can do in the radio band?

Open questions

Are resonant cavities the best we can do in the radio band?

What advances in readout can be applied? *synergies w/ Axion searches, QC(?)*

Open questions

Are resonant cavities the best we can do in the radio band?

What advances in readout can be applied? *synergies w/ Axion searches, QC(?)*

Networks of resonant cavities? *see e.g. arXiv:2308.11497 by Schmieden & Schott*

Open questions

Are resonant cavities the best we can do in the radio band?

What advances in readout can be applied? *synergies w/ Axion searches, QC(?)*

Networks of resonant cavities? *see e.g. arXiv:2308.11497 by Schmieden & Schott*

Outside of radio band, what other approaches? *see Asher's talk*