Quantum measurement of weakly coupled fields

Giacomo Marocco

with J. Beckey & D. Carney

Introduction

Q: How can we detect weakly coupled physics?

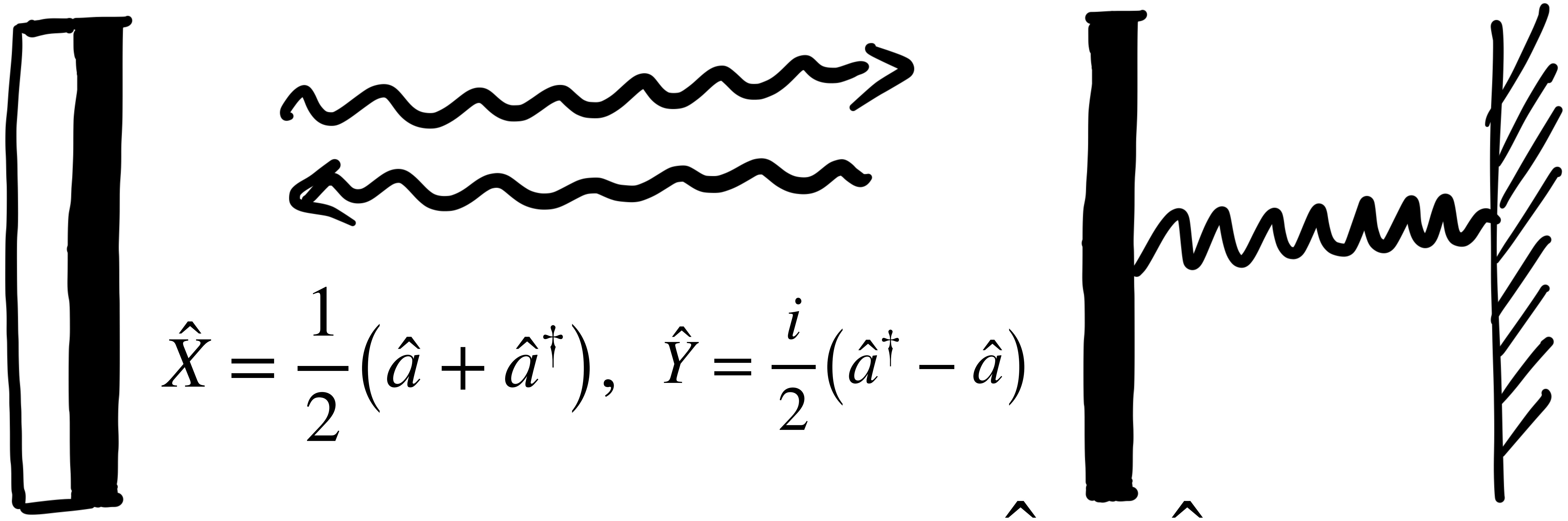
- Gravitational waves (black hole mergers, stochastic background, …)
- Axions (dark matter, solar, terrestrially sourced, produced in situ, …)

An A: Through weak, continuous measurement of quantum sensors

- PVM ($\{\mathscr{P}_i\} : \sum_i \mathscr{P}_i = 1$ and $\mathscr{P}_i \mathscr{P}_j = \delta_{ij}$) vs POVM ($\{\mathscr{O}_i\} : \sum_i \mathscr{O}_i = 1$)

There's a simple theoretical framework in which to formulate these systems.

Fabry-Pérot interferometer



$$\hat{X} = \frac{1}{2}\left(\hat{a} + \hat{a}^{\dagger}\right), \quad \hat{Y} = \frac{i}{2}\left(\hat{a}^{\dagger} - \hat{a}\right)$$

$\hat{x}_m \,, \hat{p}_m$

$$H_{\text{light}} = \omega_c(\hat{x}_m)\left(\hat{X}^2 + \hat{Y}^2\right) \qquad H_{\text{mass}} = \frac{1}{2}M\omega_M^2\,\hat{x}_M^2 + \frac{1}{2M}\hat{p}_M^2$$

How do physics systems couple to this?

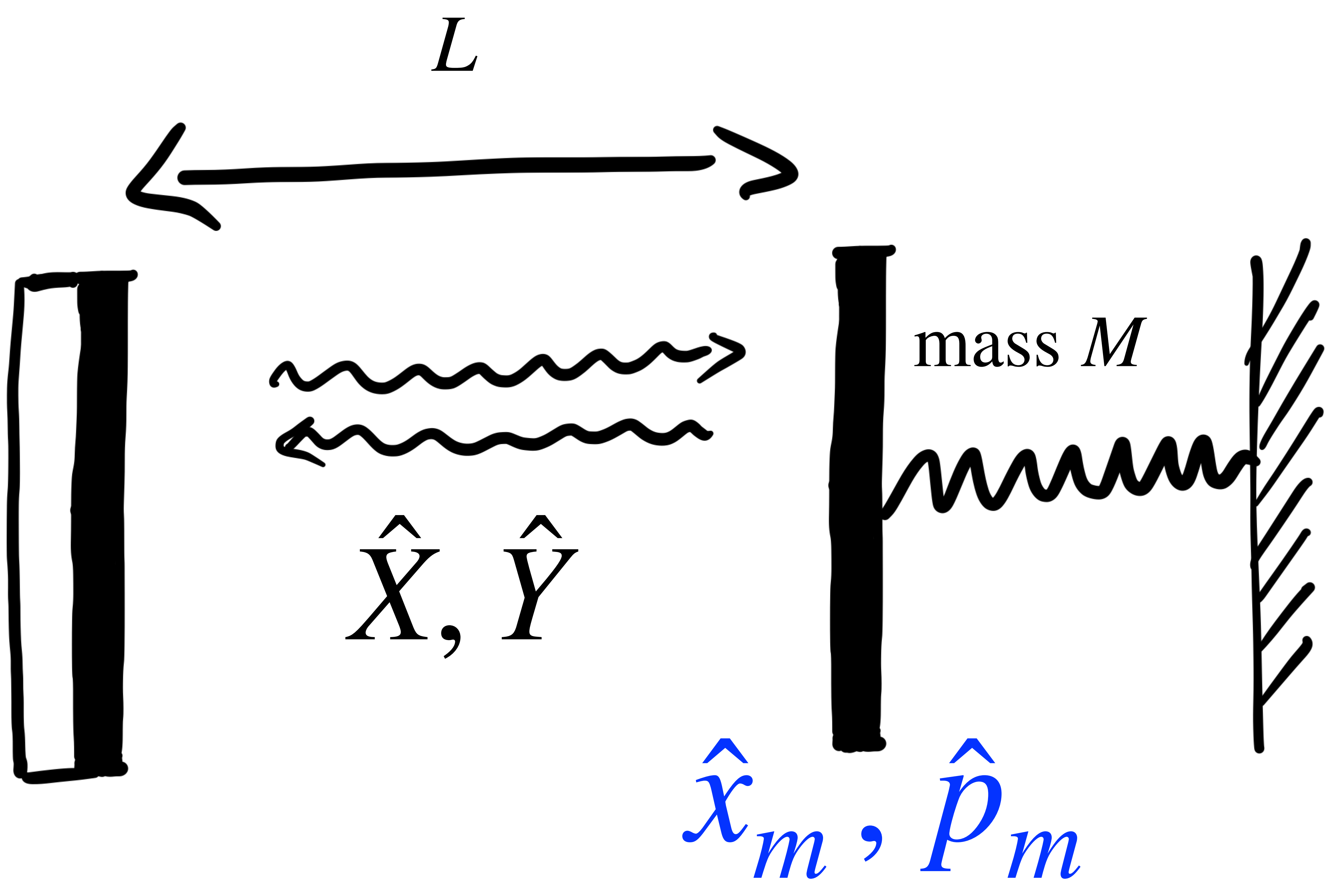
Gravitational wave detection I

$$H_{\text{int}} = -\frac{1}{2}\int d^3\mathbf{x}\, h_{\mu\nu}T^{\mu\nu}$$

$$\xrightarrow{\text{PD Frame}} \frac{1}{2}ML\ddot{h}\hat{x}_m$$

$$\dot{p}_m \supset i[H_{\text{int}}, p_m]$$

$$= -\frac{1}{2}ML\ddot{h}$$

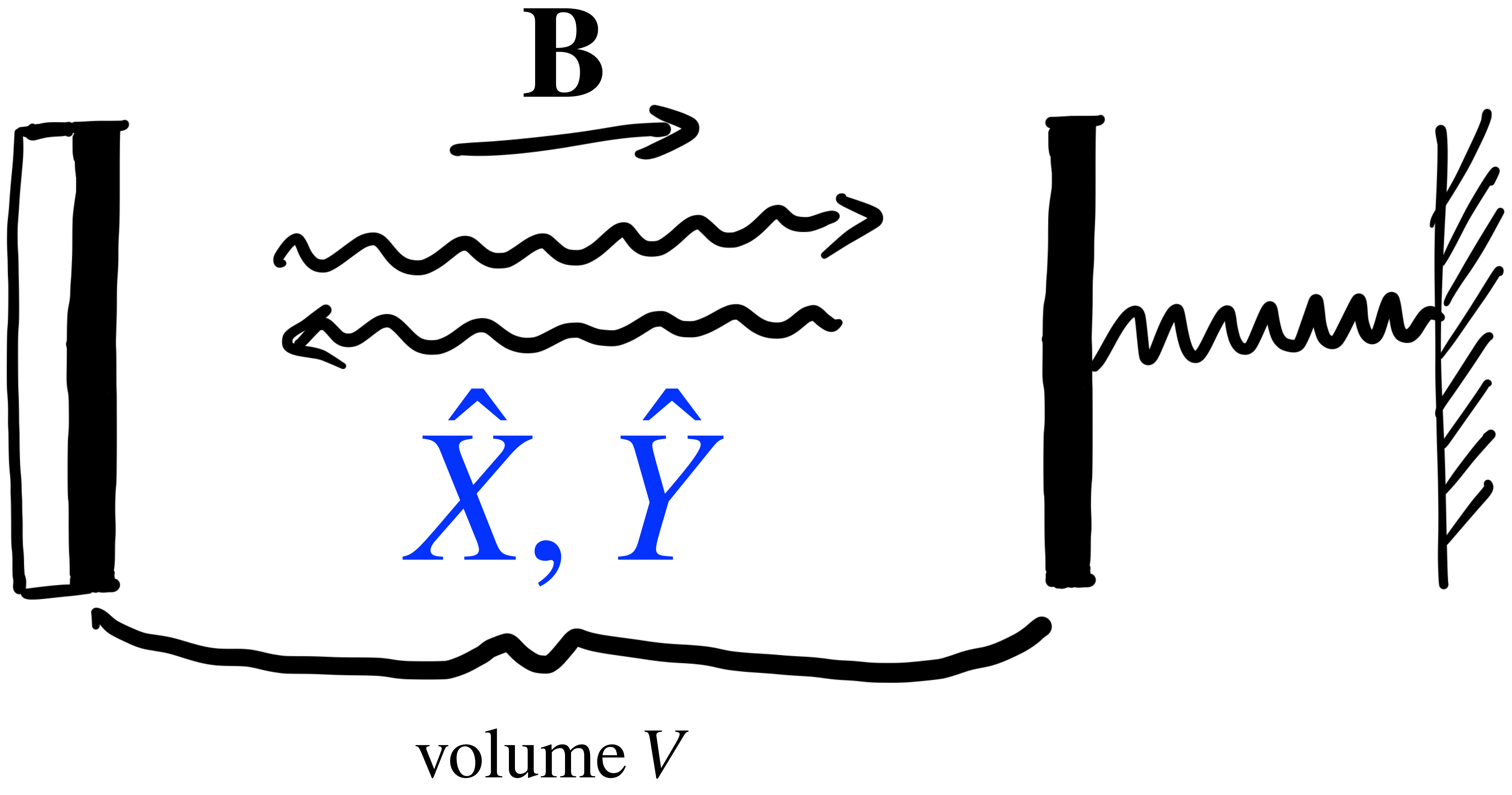


Gravitational wave (GW) imprints as a force on the test mass (LIGO/VIRGO/...)

Gravitational wave detection II

$$H_{\text{int}} = -\frac{1}{2}\int d^3\mathbf{x}\, h_{\mu\nu}T^{\mu\nu}$$

$$\longrightarrow \langle \mathbf{B}\cdot\mathbf{h}\cdot\boldsymbol{\varepsilon}\rangle_V\, V^{1/2}\left(\sin\phi\,\hat{X}+\cos\phi\,\hat{Y}\right)$$

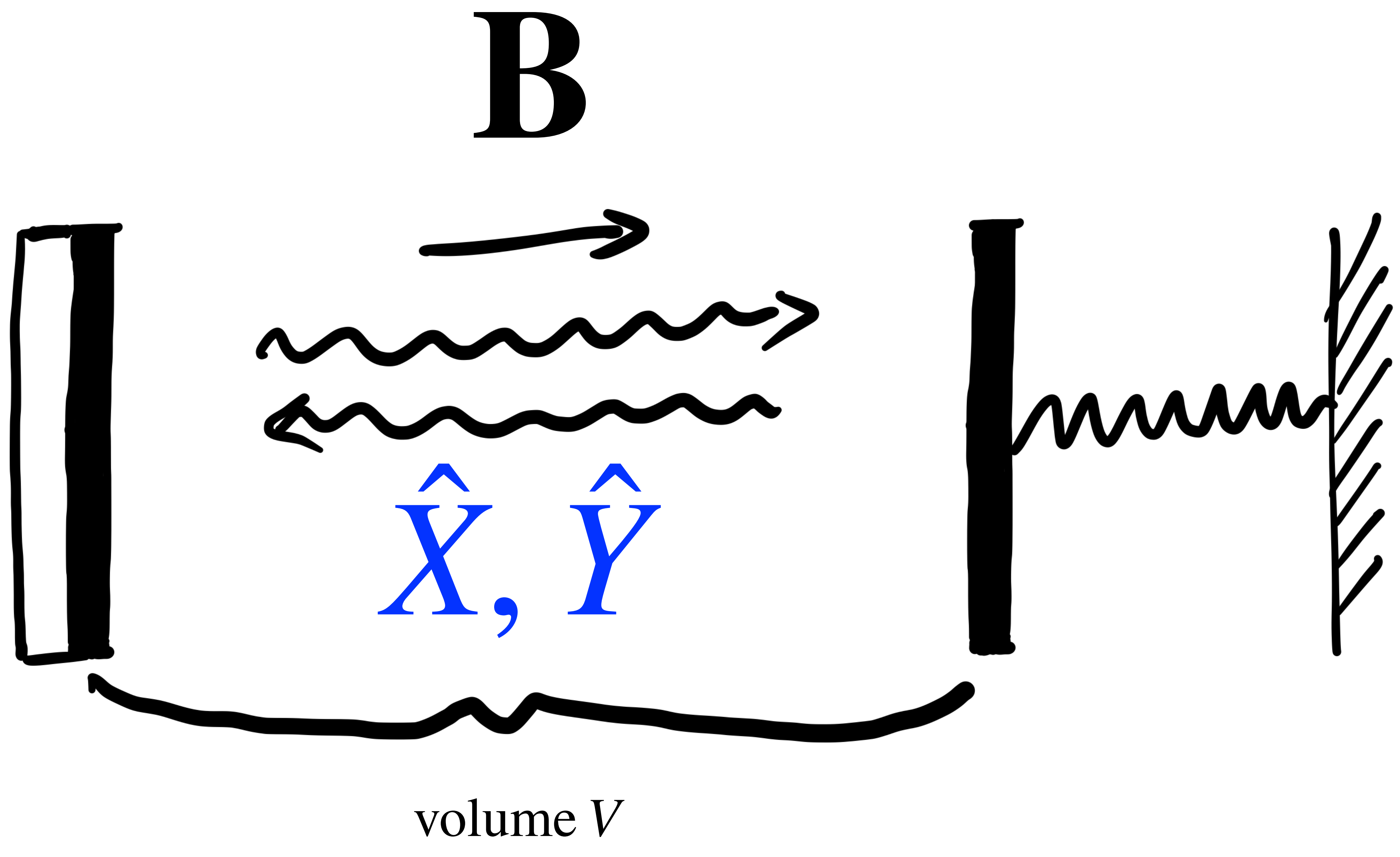


Gravitational wave imprints directly on electromagnetic field (CAST, ...)

An axion dark matter detector

$$H_{\text{int}} = g_{a\gamma\gamma} \int d^3\mathbf{x}\, a\mathbf{E}\cdot\mathbf{B}$$

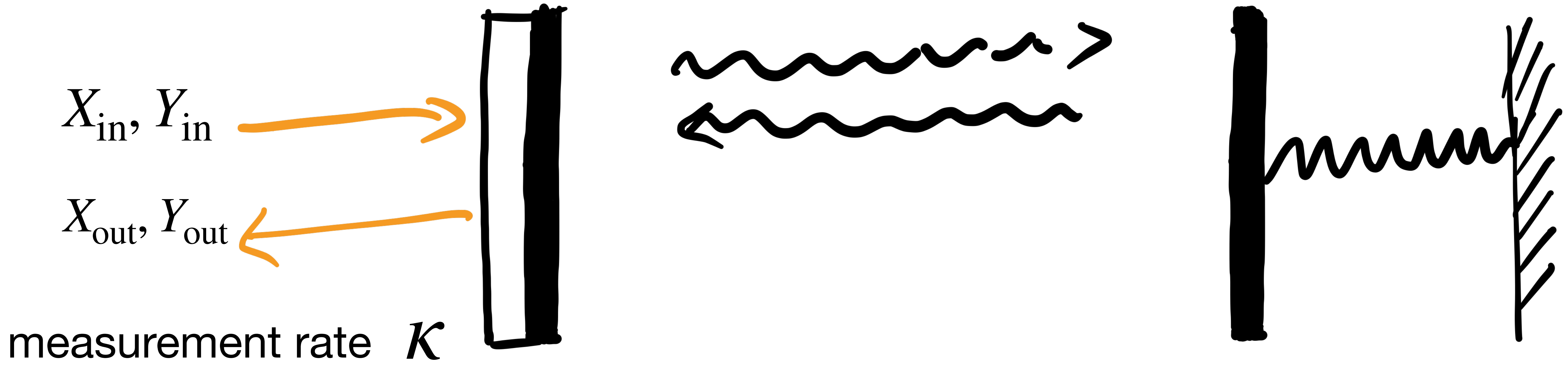
$$\longrightarrow g_{a\gamma\gamma} \langle \mathbf{B}\cdot\boldsymbol{\varepsilon}\rangle_V V^{1/2} a_0 \big(\sin\phi(t)\,\hat{X} + \cos\phi(t)\,\hat{Y}\big)$$



Axion DM imprints on EM field with stochastic phase

Input and output modes

How do we monitor what is happening inside the cavity?



If the in/outgoing modes are linearly coupled to cavity, we have I-O relations [Gardiner, Collett '84]

$$X_{\text{out}}(t) = X_{\text{in}}(t) + \sqrt{\kappa}X(t)$$

$$Y_{\text{out}}(t) = Y_{\text{in}}(t) + \sqrt{\kappa}Y(t)$$

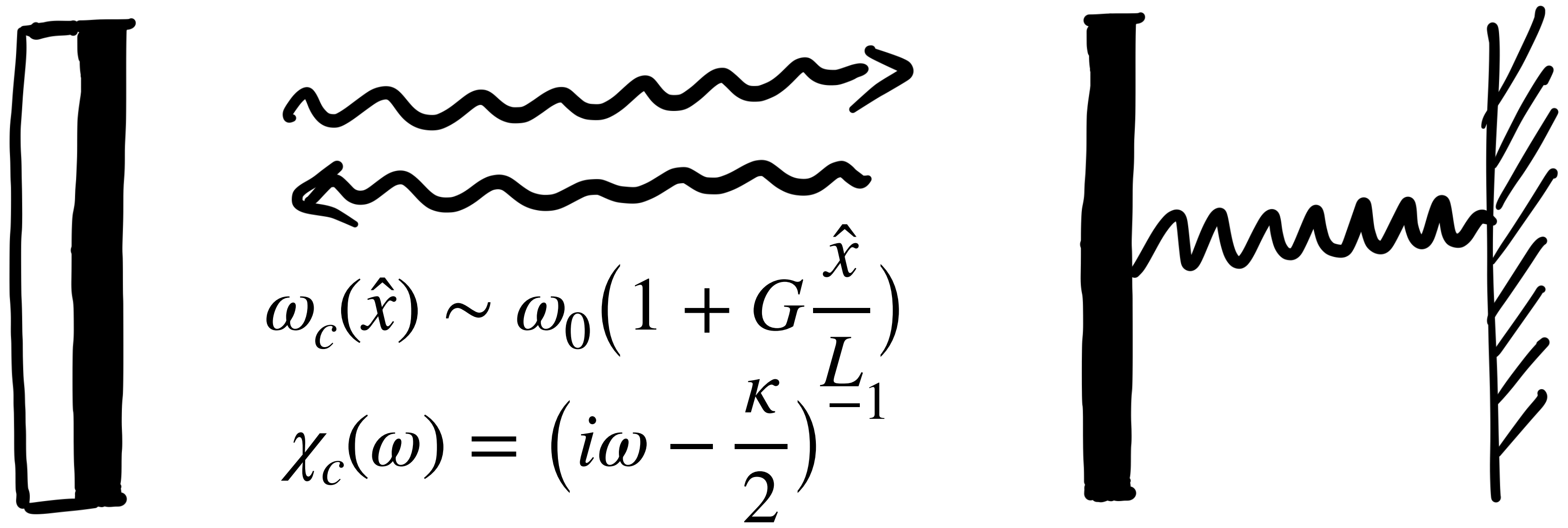
I-O for GW I

Read Y_{out} via homodyne detection:

$$Y_{\text{out}}(\omega) = \overbrace{-2\,\kappa\,G^2\chi_c^2\,\chi_m}^{\chi_{YX}(\omega)}\;X_{\text{in}}(\omega) \quad + \quad \overbrace{\big(1+\kappa\,\chi_c\big)}^{\chi_{YY}(\omega)}\;Y_{\text{in}}(\omega)$$

$$+ \quad \underbrace{\sqrt{\frac{\kappa}{2}}\,G\,M\,L\,\omega^2\chi_c\,\chi_m}_{\chi_{Yh}(\omega)}\;h(\omega)$$

$$\chi_m(\omega) = \left[m\,(\omega_M^2 - \omega^2)\right]^{-1}$$



$$\omega_c(\hat{x}) \sim \omega_0\big(1 + G\frac{\hat{x}}{L}\big)$$

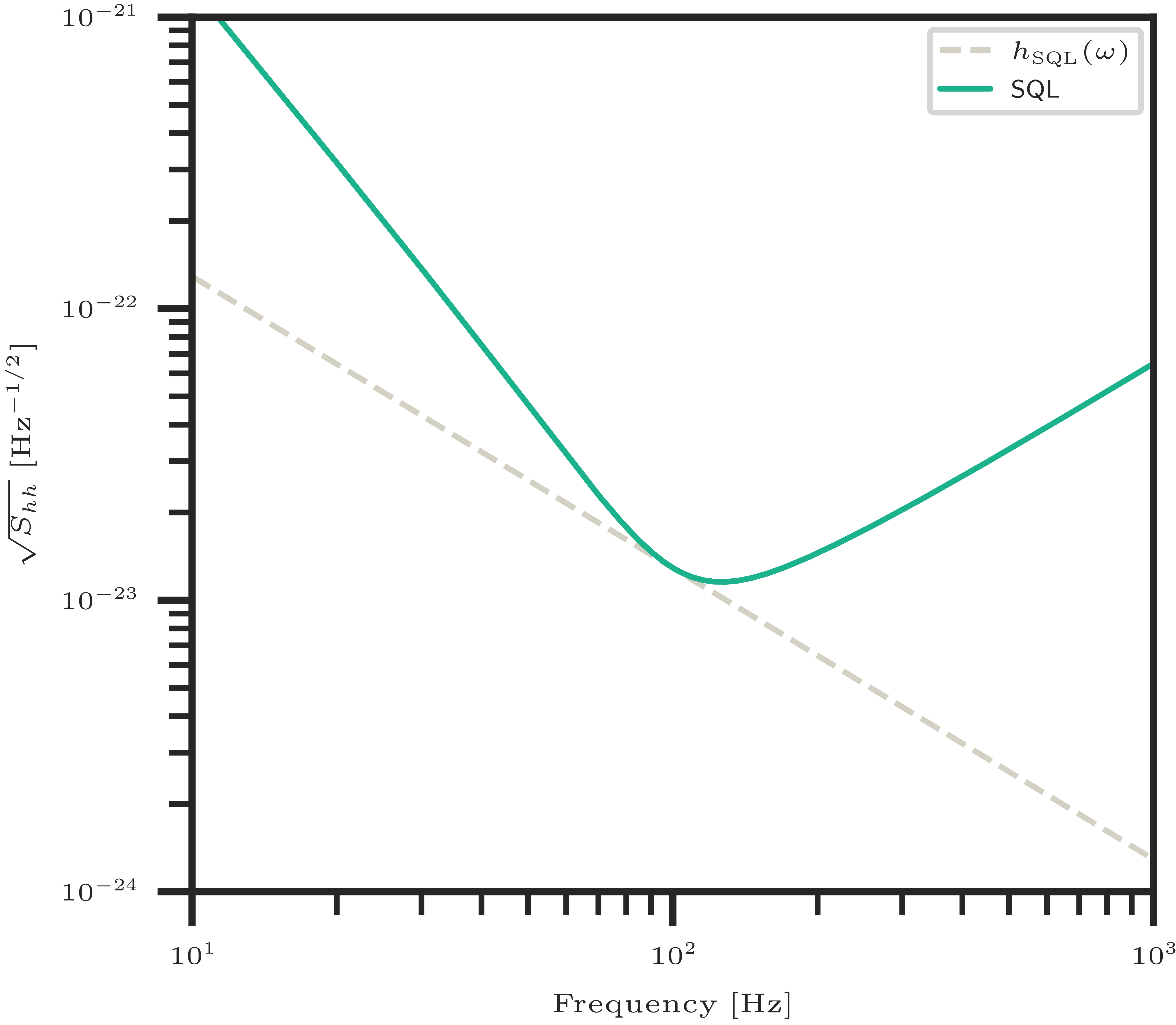
$$\chi_c(\omega) = \big(i\omega - \frac{\kappa}{2}\big)^{-1}$$

Estimator for gravitational wave strain: $h_{\text{est}}(\omega) = \dfrac{1}{\chi_{Yh}}Y_{\text{out}}(\omega)$

Noise in GW detector

$$S_{hh}(\omega) = \langle h_{\rm est}(\omega) h_{\rm est}(-\omega) \rangle$$
$$= |\chi_{Yh}|^{-2} \Big(\underbrace{|\chi_{YX}|^2 S_{XX}^{\rm in}}_{\text{shot−noise}} + \underbrace{|\chi_{YY}|^2 S_{YY}^{\rm in}}_{\text{back−action}} \Big)$$

Increasing the optomechanical coupling G increases both χ_{Yh} and χ_{YX} — optimise!



A Quantum “Limit”

We measure an integrated signal, whose noise is characterized by $\Delta^2 Y(t) = \int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\, S_{YY}^{\text{out}}(\omega)$;

Heisenberg Uncertainty Principle $S_{XX}^{\text{in}} \cdot S_{YY}^{\text{in}} \geq \frac{1}{4} \left| \langle [X_{\text{in}}, Y_{\text{in}}] \rangle \right|^2$ implies that, if X_{in} and Y_{in} are uncorrelated,

$$\Delta^2 Y(t) \geq \int_{-\infty}^{\infty} d\omega\, e^{i\omega t} \left(|\chi_{YX}|^2 S_{XX}^{\text{in}} + \frac{|\chi_{YY}|^2}{4 S_{XX}^{\text{in}}} \right)$$

which is minimised for $S_{XX}^{\text{in}} = \left| \frac{\chi_{YY}}{2\chi_{YX}} \right|$ [Caves ’81]

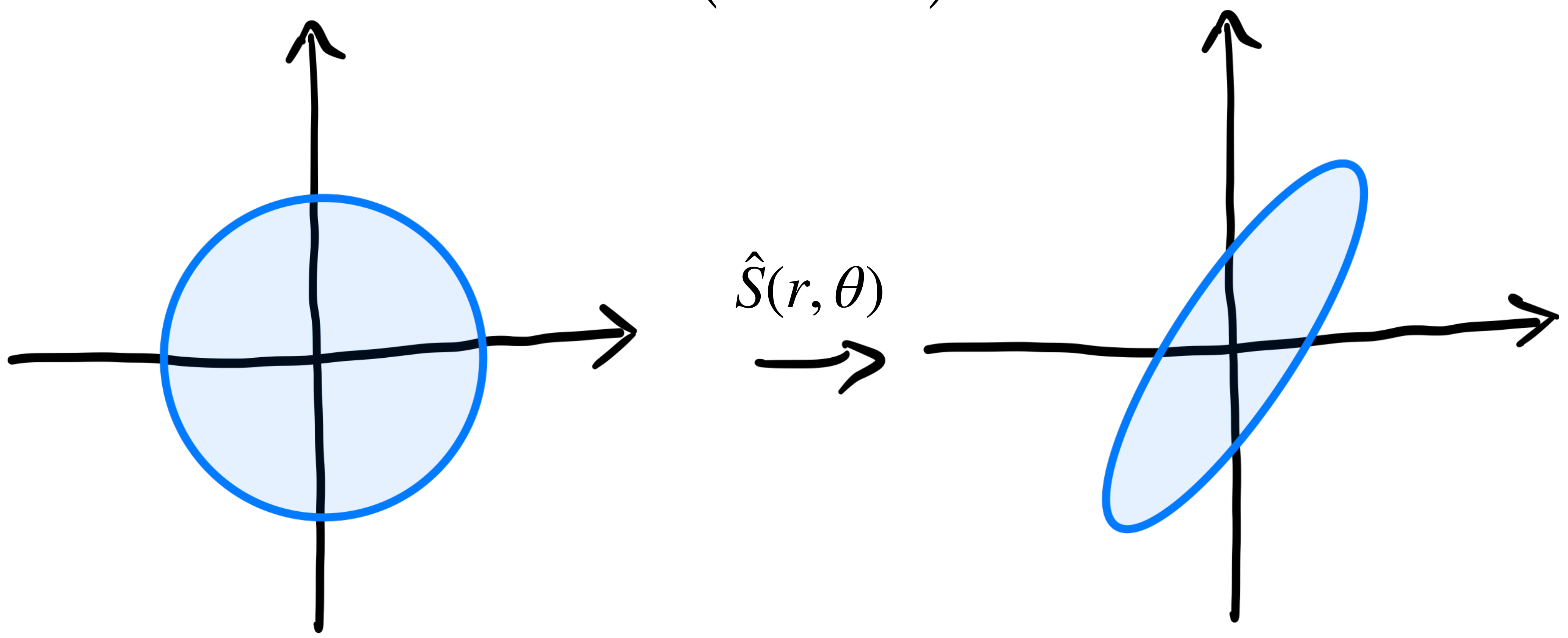
The noise is bounded: $\Delta^2 Y(t) \geq \int_{-\infty}^{\infty} d\omega\, e^{i\omega t}\, |\chi_{YX}\, \chi_{YY}|$

Beyond the Standard Quantum Limit

Two ways to beat the SQL:

1. Squeeze/rotate: $S_{XY}^{\text{in}} \neq 0$

$$S_{YY}^{\text{out}} = |\chi_{YX}|^2 S_{XX}^{\text{in}} + |\chi_{YY}|^2 S_{YY}^{\text{in}}$$
$$+2\,\text{Re}\left(\chi_{YX}\chi_{YY}^{*}\right) S_{XY}^{\text{in}}$$

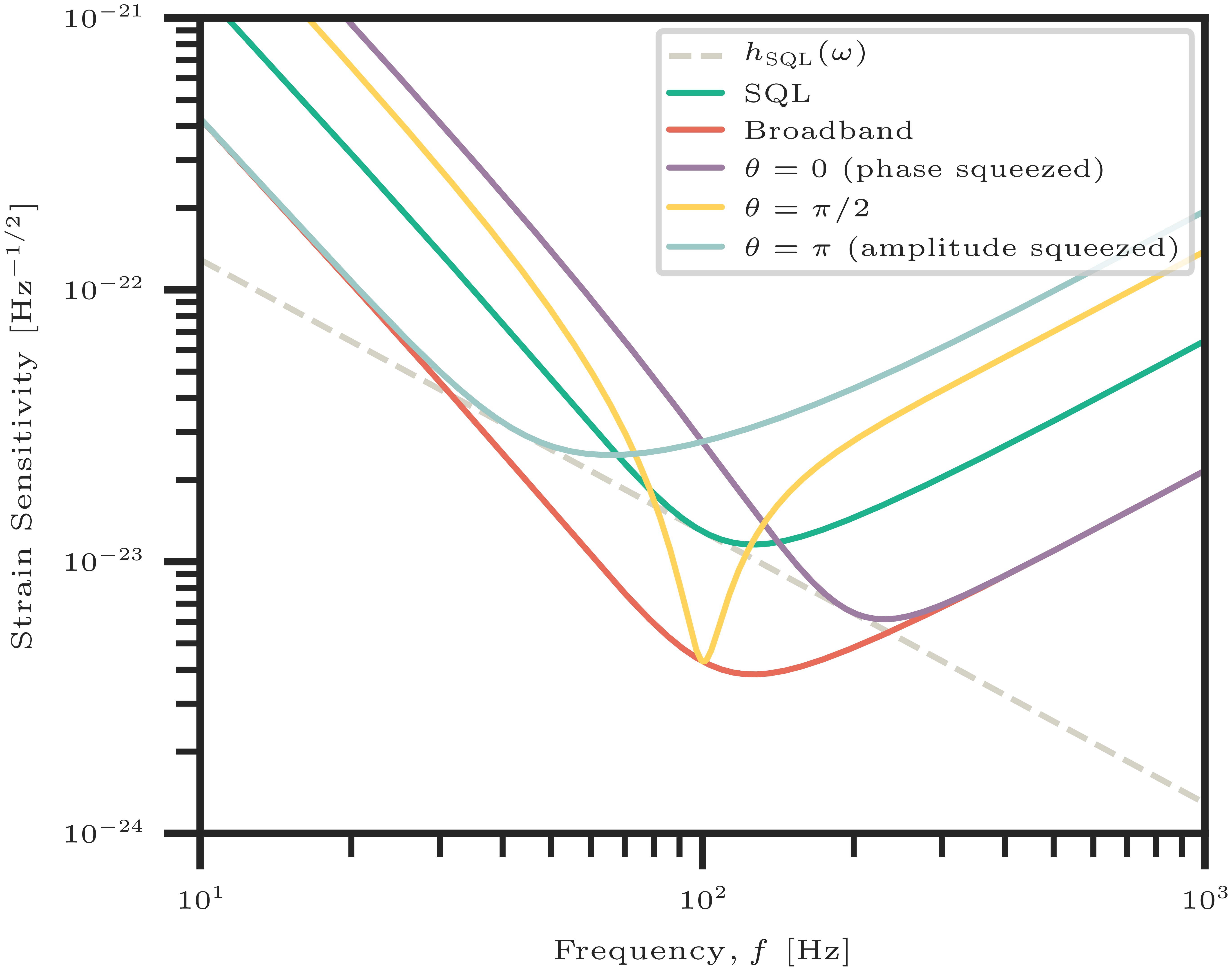


2. Back-action evasion (BAE): $\chi_{YX} \to 0$

$$S_{YY}^{\text{out}} = |\chi_{YY}|^2 S_{YY}^{\text{in}}$$

$S_{YY}^{\text{out}} \to 0$ in a Y-eigenstate

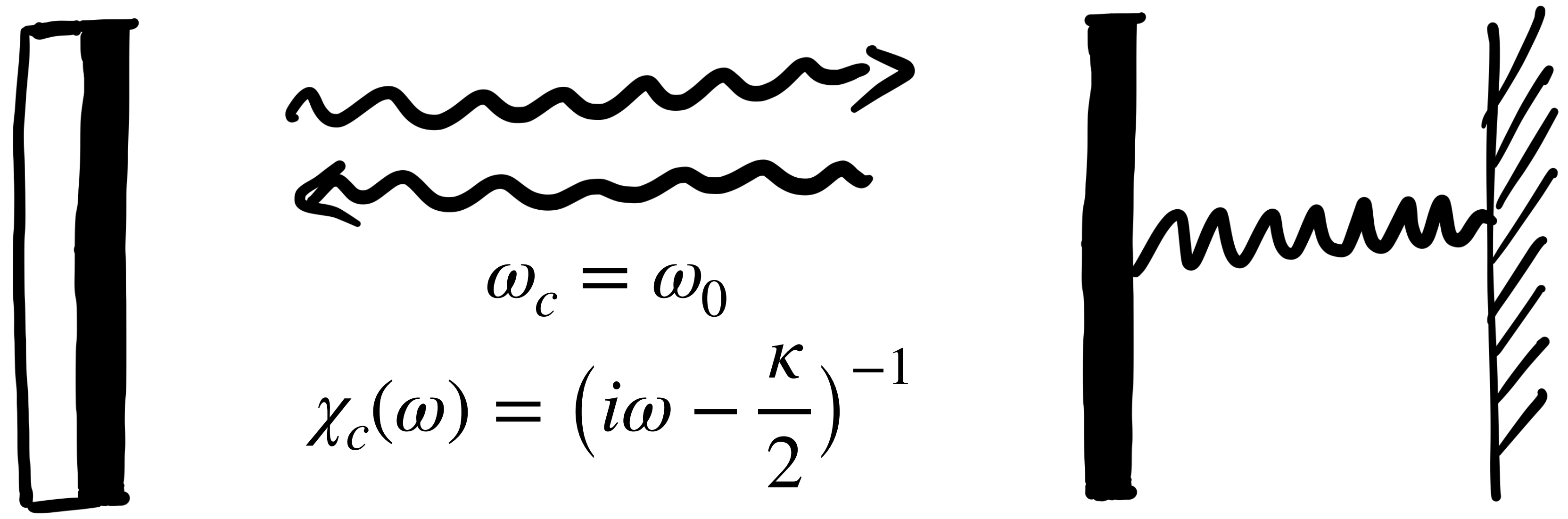
Beyond SQL for GWs



I-O for homodyne axion search

X-Y susceptibility for a fixed mirror: $\chi_{YX} = -2\,\kappa\, G^2\, \chi_c^2\, \chi_m \xrightarrow{\chi_m \to 0} 0$

$$Y_{\text{out}} = \chi_{YY} Y_{\text{in}} + \chi_{Ya}\, a$$



Measurement noise arises only from variance in Y_{in}:

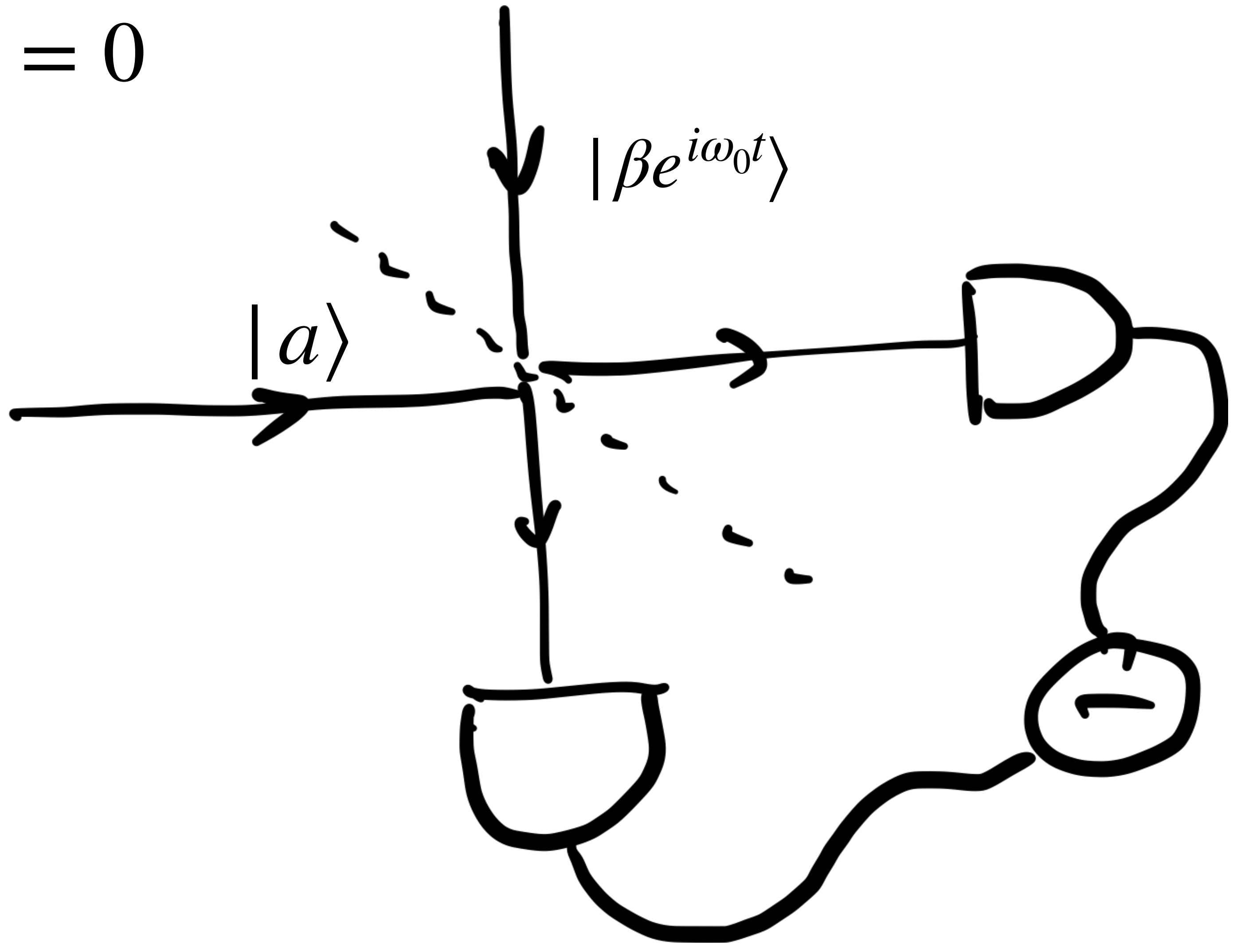
$$S_{aa} = \frac{\chi_{YY}}{\chi_{Ya}} S_{YY}^{\text{in}}$$

Fixed cavities have no back-action.

Homodyne detection as back-action evasion

Homodyne detection lets us measure the Schrödinger picture operator $Y_{\mathrm{S}}(t) = \frac{1}{\sqrt{2}}\left(a^{i\omega_0 t}a_S + e^{-i\omega_0 t}a_S^\dagger\right)$

Since $a_{\mathrm{H}}(t) = e^{-i\omega_0 t}a_{\mathrm{S}}$, the observable is conserved: $\frac{dY_{\mathrm{H}}}{dt} = i[H, Y_H] + \partial_t Y_{\mathrm{S}} = 0$



No back-action when measuring conserved quantities. Despite BAE, still may be better to measure number operator (see previous talks)!

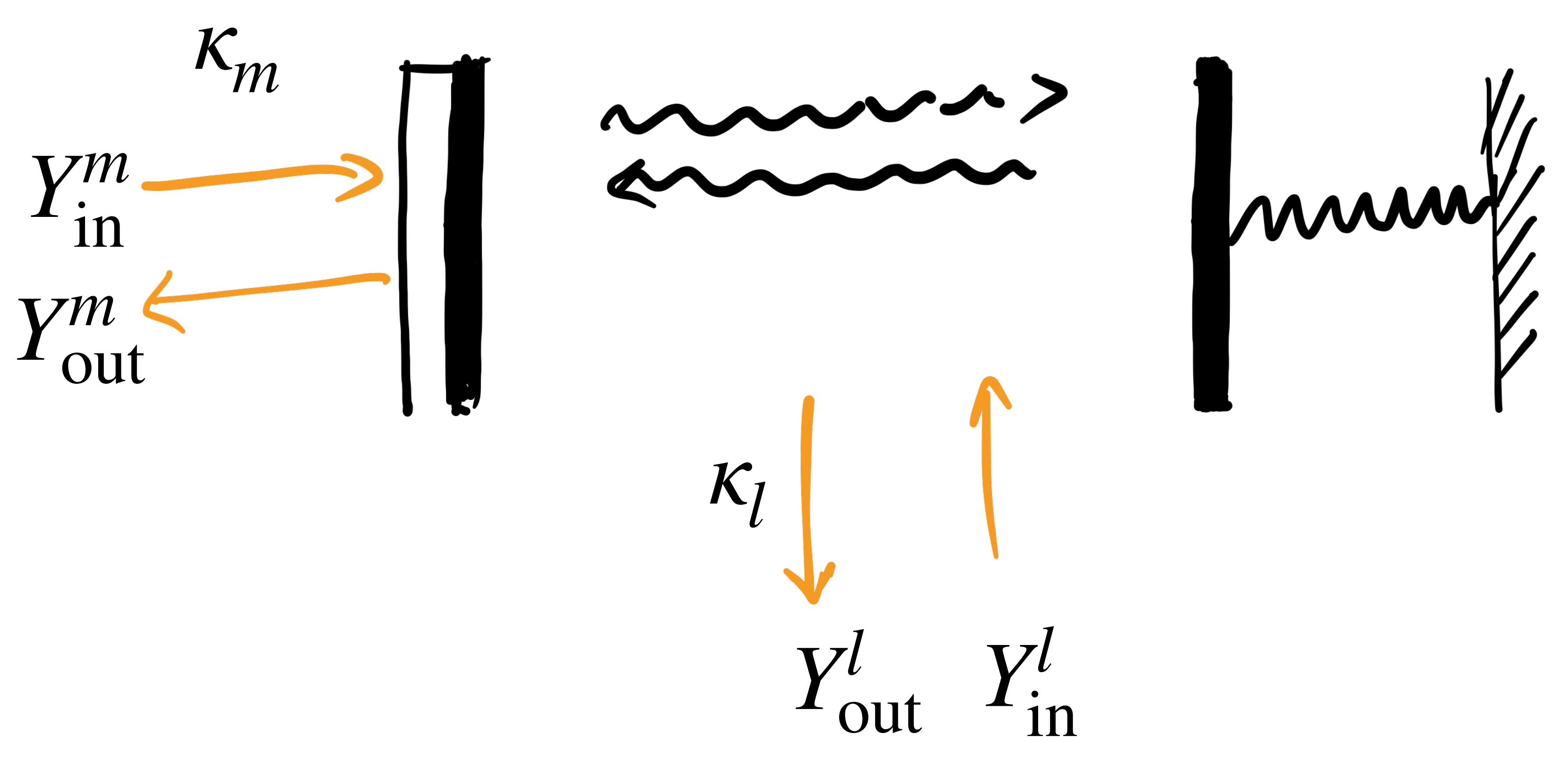
(If mirror is free to move, $a_{\mathrm{H}}(t) = e^{-i\int \omega_c(\hat{x})\,dt}a_{\mathrm{S}} \neq e^{-i\omega_0 t}a_{\mathrm{S}}$, so no BAE in general)

Lossy cavities and axions

Cavity loss implies we have other input/output ports apart from measurement port:

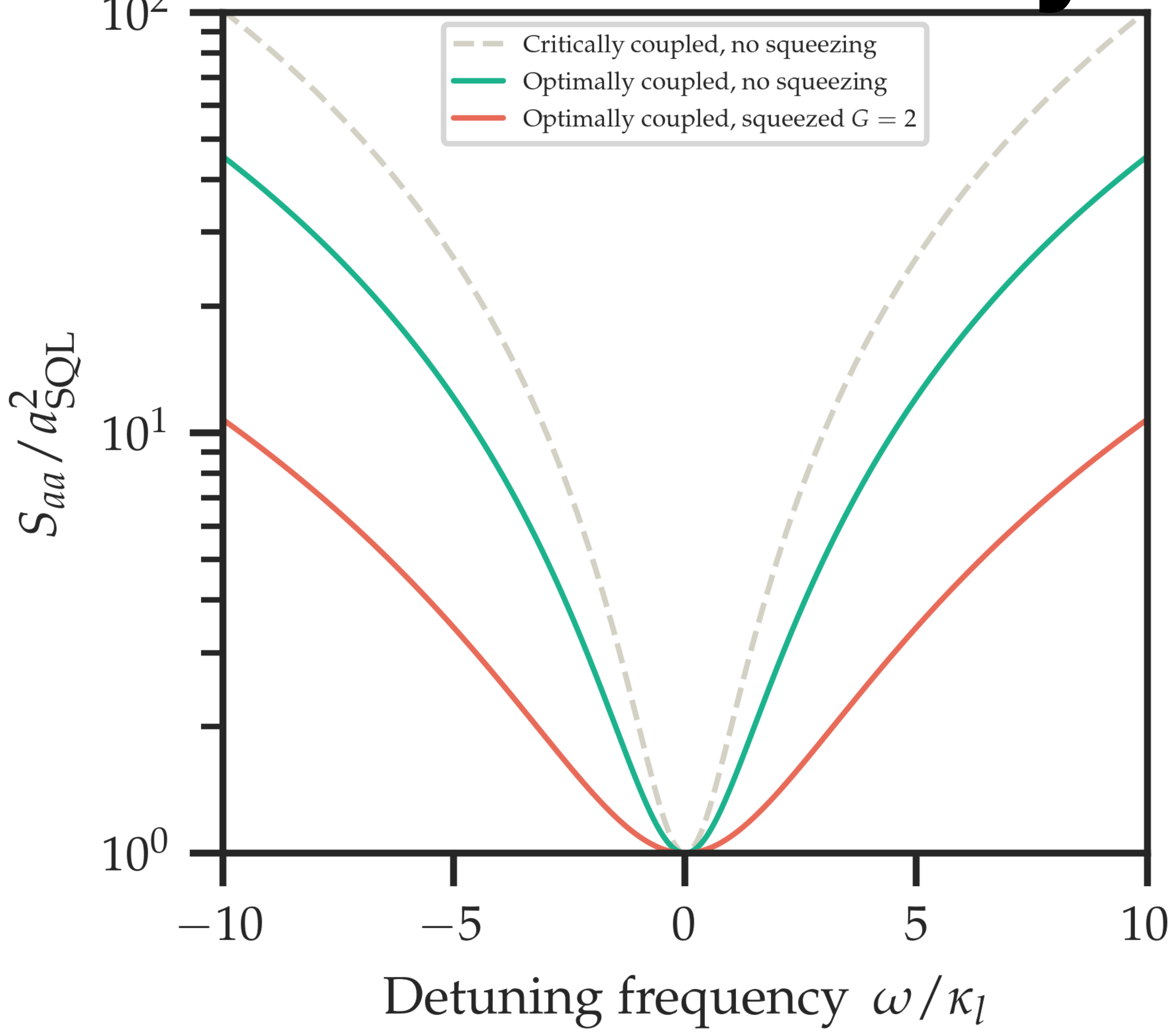
$$Y_{\text{out}}^m = \chi_{mm} Y_{\text{in}}^m + \chi_{ml} Y_{\text{in}}^l + \chi_{ma} a$$

$$S_{aa} = |\chi_{ma}|^{-2} \left(|\chi_{mm}|^2 S_{YY}^m + |\chi_{ll}|^2 S_{YY}^l \right)$$



The axion frequency/mass is unknown;
maximise the Fisher information/scan rate by tuning $\kappa_m = 2e^{-2r}\kappa_l$

Axion sensitivity



Conclusions

- Input-output theory gives a useful, unified framework in which to understand noise (including quantum, thermal, intrinsic losses, etc.) in weak, continuous measurements
- There are simple pipelines to go from physical couplings, to signal and noise, to physics reach of many detectors

Thank you!