Massive Gravitons in Very Special Relativity: Theory[1] and Observations[2]

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September 12, 2023

[1] J. Alfaro and A. Santoni. "Very special linear gravity: A gauge-invariant graviton mass", Phys. Lett. B 829:137080 (2022)

[2] A. Santoni, J. Alfaro and A. Soto, "Graviton mass bounds in very special relativity from binary pulsar's gravitational waves", Phys. Rev. D 108, 044072

- Original idea from Cohen and Glashow (2006).[3]
- New Spacetime symmetry: **T(4) + Lorentz Subgroups***

*Subgroups of interest $(T_1 \equiv K_x + J_y\,,\; T_2 \equiv K_y - J_x)$

- $T(2) = T_1 + T_2$
- $E(2) = T(2) + J_z$ and $HOM(2) = T(2) + K_z$
- $SIM(2) = T(2) + J_z + K_z$

All these subgroups get enlarged to the full Lorentz group when adding discrete simmetries like P, T or CP, so that:

Small **CP VIOLATION** $\Longleftrightarrow$ Small **VSR EFFECTS** .

[3] Andrew G Cohen and Sheldon L Glashow. "Very special relativity", Physical review letters, 97(2):021601 (2006)

T(2), E(2) - Have an invariant 4−Vector

Possibility to build local terms that violate Lorentz invariance
→ Free Particle's propagation gets affected

HOM(2) - No new Invariants

- Implies Special Relativity's kinematics
- CPT not granted

SIM(2) - No new Invariants

- Implies Special Relativity's kinematics
- CPT granted

However, we have a preferred light-like spacetime direction $n_\mu = (1, 0, 0, 1)$ so that

$$n_\mu \underset{SIM(2)}{\longrightarrow} e^{\phi} n_\mu \, .$$

Then, ratios of scalar products with n^μ are invariants under VSR.

Very Special Linear Gravity (VSLG)

Quadratic VSR Lagrangian of the $h_{\mu\nu}$ spin-2 field ($|h_{\mu\nu}| << \eta_{\mu\nu}$):

$$L = \frac{1}{2} h^{\mu\nu} O_{\mu\nu\alpha\beta} h^{\alpha\beta} \tag{1}$$

Ingredients:

- Metric $\eta_{\mu\nu}$
- Momentum p^μ
- VSR vector $N^\mu = n^\mu/(n \cdot p)$

$$O = 3\,\eta\eta + 9\,pp\eta + 12\,pN\eta + 12\,ppNN\ , \tag{2}$$

Rules:

- Index Symmetries: $\mu \Longleftrightarrow \nu$, $\alpha \Longleftrightarrow \beta$, $\mu\nu \Longleftrightarrow \alpha\beta$
- Gauge Invariance: $O_{\mu\nu\alpha\beta} p^\alpha = 0$

Equations of Motion (E.o.M.): $O_{\mu\nu\alpha\beta} h^{\alpha\beta} = 0$, where[4]

$$\begin{aligned}
O_{\mu\nu\alpha\beta} = \ \chi \Big(& p_\mu p_\nu \eta_{\alpha\beta} - \frac{1}{2} p_\mu p_\alpha \eta_{\nu\beta} - \frac{1}{2} p_\mu p_\beta \eta_{\nu\alpha} + p_\alpha p_\beta \eta_{\mu\nu} - \frac{1}{2} p_\nu p_\beta \eta_{\mu\alpha} - \frac{1}{2} p_\nu p_\alpha \eta_{\mu\beta} \\
& - p^2 \eta_{\mu\nu} \eta_{\alpha\beta} + \frac{1}{2} p^2 \eta_{\mu\alpha} \eta_{\nu\beta} + \frac{1}{2} p^2 \eta_{\mu\beta} \eta_{\nu\alpha} + m_g^2 \eta_{\mu\nu} \eta_{\alpha\beta} - \frac{m_g^2}{2} \eta_{\mu\alpha} \eta_{\nu\beta} - \frac{m_g^2}{2} \eta_{\mu\beta} \eta_{\nu\alpha} \\
& - m_g^2 N_\mu N_\nu p_\alpha p_\beta + \frac{m_g^2}{2} N_\mu N_\alpha p_\nu p_\beta + \frac{m_g^2}{2} N_\mu N_\beta p_\nu p_\alpha + \frac{m_g^2}{2} N_\nu N_\alpha p_\mu p_\beta + \frac{m_g^2}{2} N_\nu N_\beta p_\mu p_\alpha - m_g^2 N_\alpha N_\beta p_\mu p_\nu \\
& + m_g^2 p^2 N_\mu N_\nu g_{\alpha\beta} - \frac{m_g^2}{2} p^2 N_\mu N_\alpha g_{\nu\beta} - \frac{m_g^2}{2} p^2 N_\mu N_\beta g_{\nu\alpha} - \frac{m_g^2}{2} p^2 N_\nu N_\beta \eta_{\mu\alpha} - \frac{m_g^2}{2} p^2 \eta_{\mu\beta} N_\nu N_\alpha + m_g^2 p^2 N_\alpha N_\beta \eta_{\mu\nu} \\
& - m_g^2 \eta_{\mu\nu} N_\alpha p_\beta - m_g^2 \eta_{\mu\nu} p_\alpha N_\beta + \frac{m_g^2}{2} \eta_{\mu\alpha} N_\nu p_\beta + \frac{m_g^2}{2} \eta_{\mu\alpha} p_\nu N_\beta + \frac{m_g^2}{2} \eta_{\mu\beta} N_\nu p_\alpha + \frac{m_g^2}{2} \eta_{\mu\beta} p_\nu N_\alpha \\
& + \frac{m_g^2}{2} \eta_{\nu\alpha} N_\mu p_\beta + \frac{m_g^2}{2} \eta_{\nu\alpha} p_\mu N_\beta + \frac{m_g^2}{2} \eta_{\nu\beta} N_\mu p_\alpha + \frac{m_g^2}{2} \eta_{\nu\beta} p_\mu N_\alpha - m_g^2 \eta_{\alpha\beta} N_\mu p_\nu - m_g^2 \eta_{\alpha\beta} p_\mu N_\nu \Big).
\end{aligned}$$

Quite messy!

[4] Jorge Alfaro and Alessandro Santoni. "Very special linear gravity: A gauge-invariant graviton mass", Physics Letters B 829:137080 (2022)

Using the **Gauge Choices** $\partial_\mu h^{\mu\nu} = 0$; $N_\mu h^{\mu\nu} = 0$; $h = h^\mu_\mu = 0$

$\rightarrow$ The new E.o.M. is Klein-Gordon like

$$(p^2 - m_g^2) h_{\mu\nu} = 0 \tag{3}$$

m_g is a graviton mass!

Gauge Invariance $\iff$ only **2 Physical Degrees of Freedom** .

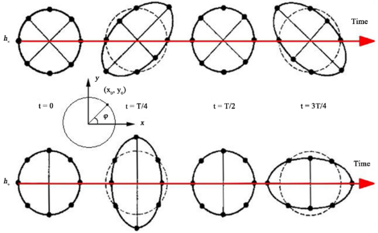
That could be of interest for

- Universe's Accelerated Expansion
- Dark Matter[5]

[5] K. Aoki and S. Mukohyama. "Massive gravitons as darkmatter and gravitational waves", Physical Review D94:024001 (2016)

Geodesic Deviation for Gravitational Waves (GW)

In General Relativity we have only **transverse** motion:



In VSLG we have **motion** also in the **direction of propagation**!

$$\delta\xi^z = \delta\xi_0^z + \frac{1}{2}\frac{m_g^2}{E^2}h_{13}\delta\xi_0^x + \frac{1}{2}\frac{m_g^2}{E^2}h_{23}\delta\xi_0^y + \frac{1}{2}\frac{m_g^4}{E^4}h_{33}\delta\xi_0^z\,. \qquad (4)$$

Using **Graviton mass bound** from time delay in GW170817

$$m_g \lesssim 10^{-22}eV \rightarrow \frac{m_g^2}{E^2} \lesssim 10^{-20} \quad \text{for LIGO/VIRGO}\,[10Hz, 10kHz]$$

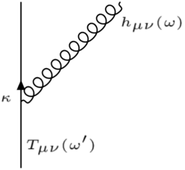
Binary stars are astronomical objects of great interest and,

since the discovery of the Hulse Taylor binary pulsar in 1974, they have demonstrated to be a great experimental tool.

So, why Binaries for VSR?

1. One of the simplest sources of GW to study
2. Experimental precision on gravitational energy loss is increased by very long observation times

"Effective Field theory" calculation with classical source[6]



The vertex should be gauge invariant and linear in $h_{\mu\nu}$

$$\rightarrow \frac{\kappa}{2}\, T_{\mu\nu}\, h^{\mu\nu} \text{ with } \kappa = \sqrt{32\pi G} \tag{5}$$

Implying the **unpolarized squared amplitude**

$$\sum_{\lambda} |A_\lambda|^2 = \frac{\kappa^2}{4}\, \tilde{T}^{*}_{\mu\nu}\, \tilde{T}_{\alpha\beta} \sum_{\lambda} \epsilon^{*\,\mu\nu}_{\lambda}\, \epsilon^{\alpha\beta}_{\lambda} \tag{6}$$

[6]Goldberger, Walter D. "Effective field theories and gravitational radiation." Les Houches. Vol. 86. Elsevier, 351-396 (2007)

Differential emission probability:

$$dP = \frac{d^3k}{2\omega(2\pi)^3} \sum_\lambda |A_\lambda|^2 \quad (7)$$

Then, the total energy loss rate will be simply

$$\dot{E} = \frac{dE}{dt} = \int \frac{\omega}{T} dP = \frac{d^3k}{2T(2\pi)^3} \sum_\lambda |A_\lambda|^2 \quad (8)$$

Defining $S^{\mu\nu\alpha\beta} = \sum_\lambda \epsilon_\lambda^{*\mu\nu} \epsilon_\lambda^{\alpha\beta}$
and replacing the squared amplitude we have

$$\dot{E} = \frac{\kappa^2}{8(2\pi)^3 T} \int \rho(\omega)\, \omega^2\, \tilde{T}^*_{\mu\nu}\, \tilde{T}_{\alpha\beta} S^{\mu\nu\alpha\beta}\, d\omega\, d\Omega_k \quad (9)$$

The energy momentum tensor of a Binary system is

$$T^{\mu\nu}(t,\vec{x}) = \mu\, U^{\mu} U^{\nu} \delta^3(\vec{x} - \vec{r}(t))\,, \tag{10}$$

where $\mu = m_1 m_2 / M$ is the reduced mass, $\vec{r}(t)$ is the reduced mass trajectory and $U^{\mu} = (1, \dot{r}_x, \dot{r}_y, 0)$ its non-relativistic four-velocity.

The **keplerian orbit** is parametrized as

$$\begin{aligned} \vec{r}(t) &= b\left(\cos\phi - e, \sqrt{1-e^2}\sin\phi, 0\right)\,, \\ \Omega t &= \phi - e\sin\phi \;\text{ with }\; \Omega = \sqrt{\frac{GM}{b^3}} = \frac{2\pi}{P_b}\,. \end{aligned} \tag{11}$$

Where b and e are respectively the semi-major axis and the eccentricity of the orbit.

Main steps of the Calculation

$$\dot{E} = \frac{\kappa^2}{8(2\pi)^3 T} \int \rho(\omega)\,\omega^2\,\tilde{T}^*_{\mu\nu}\,\tilde{T}_{\alpha\beta}\,S^{\mu\nu\alpha\beta}\,d\omega\,d\Omega_k \qquad (12)$$

1. Calculating $S^{\mu\nu\alpha\beta}$ through symmetry arguments
2. Working in the "far zone" or "radiation zone" approximation $b << \lambda << d$ so that

$$\tilde{T}^{ij}(\omega,\vec{k}) \simeq \int d^3x\,\tilde{T}^{ij}(\omega,\vec{x}) \equiv \tilde{T}^{ij}(\omega) \qquad (13)$$

3. Exploiting the (quasi-)periodic motion of binaries, to switch the $\omega-$integral with a sum over the keplerian modes $N \rightarrow \omega_N = N\Omega$
4. Realizing the $\int d\Omega_k$ integral using the tensorial structure of its components

Classically, from $\dot{E}$ we get the period decrease rate

$$\dot{P}_b = \frac{dP_b}{dt} = -6\pi \frac{b^{\frac{5}{2}} G^{-\frac{3}{2}}}{m_1 m_2 \sqrt{m_1 + m_2}} \frac{dE}{dt} \tag{14}$$

which is measured experimentally. Thus, defining $\delta \equiv m_g/\Omega$

$$\dot{P}_{VSR} = -\frac{192\pi\, T_{\odot}^{\frac{5}{3}}}{5} \frac{\tilde{m}_1 \tilde{m}_2}{\tilde{M}^{\frac{1}{3}}} \left(\frac{P_b}{2\pi}\right)^{-\frac{5}{3}} \sum_{N_{min}} f(N, e, \delta, \hat{n}), \tag{15}$$

with $f(N, e, \delta, \hat{n})$ made up of combinations of Bessel functions, and so that we recover the **GR limit**[7]

$$\lim_{\delta \to 0} f(N, e, \delta, \hat{n}) = \frac{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4}{(1 - e^2)^{\frac{7}{2}}} \tag{16}$$

[7] Peters, P. C. and Mathews, J. (1963). Gravitational radiation from point masses in a Keplerian orbit. Physical Review, 131(1), 435.

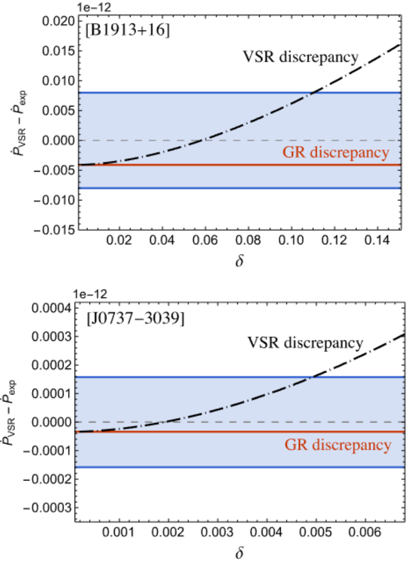
Using data from

- Hulse-Taylor binary PSR B1913+16
- Double Pulsar PSR J0737-3039

We can constrain the VSR origin of graviton mass

$$m_g \lesssim 10^{-21} eV$$

Still worse than the kinematical bound from GW170817 ($\lesssim 10^{-22} eV$)



- VSLG represents a gauge invariant theory of massive gravitons
- Calculation of massive graviton emission from Effective field theory perspective
- Bounds on the VSR graviton mass from binary systems $m_g \lesssim 10^{-21} eV$ still weaker than kinematical bounds

Upcoming Work

Apart from a more complete statistical analysis on binaries, there are many **other applications** to explore for this model

- Primordial Gravitational Waves
- Pulsar Timing Arrays

Thanks for your attention!