Lecture II on
axion theory and model building

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Lecce, Sep. 2023

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Axions from higher-dimensional gauge fields

In the 2nd lecture, we will study a different type of axion models in which 4-dim axions originate from higher-dimensional p-form gauge field, which is most naturally realized in string/M theory.

In such models, PQ symmetry is not introduced by hand, but it appears in 4D effective theory as a consequence of the higher-dimensional gauge symmetry associated with the p-form gauge field.

More concretely, the PQ symmetry corresponds to a locally well-defined, but globally ill-defined higher-dimensional gauge symmetry.

Therefore the PQ symmetry can be broken only by **non-local effects** in extra dimension, which are exponentially suppressed in the limit where the volume of the relevant extra dimension is large compared to the fundamental length scale.

As it is locally a gauge transformation of the p-form gauge field, the PQ symmetry is intrinsically a non-linear symmetry.

Therefore such models do not admit a cosmological phase transition from the linear PQ phase to non-linear phase.

Low energy couplings of the axions from p-form gauge field have a different pattern from the axion couplings in models with a linear PQ symmetry, which might be experimentally testable.

Axion from a 5-dim gauge field

(This is a simple 5-dim theory which exhibits many features of axions in string/M theory. Part of this discussion is based on KC, hep-ph/0308024.)

5-dim QCD compactified on S^1/Z_2 with the minimal structure for 4-dim axion which couple to the 4-dim gluons and quarks:

$$Z_2 : \; y \to -y \quad (y \cong y + 2\pi R)$$

5D U(1) gauge boson

5D gluons

$$\begin{aligned} S_{5\mathrm{D}} = \int d^5x \sqrt{-\tilde{g}} \, \Big[& \frac{1}{2} M_5^3 \mathcal{R}_5(\tilde{g}) - \frac{1}{4g_{5A}^2} A^{MN} A_{MN} - \frac{1}{4g_{5S}^2} G^{aMN} G^a_{MN} \\ & + i \sum_I \bar{Q}_I (\tilde{\gamma}^M D_M + \mu_I A_{MN} \tilde{\gamma}^{MN}) Q_I + D^M \xi^* D_M \xi - M_\xi^2 |\xi|^2 \\ & + \frac{k_{\mathrm{CS}}}{32\pi^2} \frac{\epsilon^{MNPQR}}{\sqrt{-\tilde{g}}} A_M G^a_{NP} G^a_{QR} + \ldots \Big] \end{aligned}$$

U(1)-charged massive scalar

U(1)-neutral quarks with U(1) magnetic moment

U(1)-SU(3)$_c$-SU(3)$_c$ Chern-Simons term

$$(A_{MN} = \partial_M A_N - \partial_N A_M, \quad G^a_{MN} = D_M G^a_N - D_N G^a_M)$$

Z_2 boundary condition on 5D fields

$$\tilde{g}_{MN}(y) = \tilde{g}_{MN}(y+2\pi R) = \epsilon_M \epsilon_N \tilde{g}_{MN}(-y)$$

$$G^a_M(y) = G^a_M(y+2\pi R) = \epsilon_M G^a_M(-y)$$

$$A_M(y) = A_M(y+2\pi R) = -\epsilon_M A_M(-y)$$

$$Q_I(y) = Q_I(y+2\pi R) = \gamma_5 Q_I(-y)$$

$$\xi(y) = \xi(y+2\pi R) = \xi(y)$$

$$\big(\epsilon_M = (\epsilon_\mu, \epsilon_5) = (1, -1)\big)$$

Z_2-even gravitons and gluons

Z_2-odd U(1) gauge boson

Z_2-odd U(1) gauge symmetry

$$U(1)_A: \quad A_M \to A_M + \partial_M \Lambda, \quad \Phi \to e^{iq_\Phi \epsilon(y)\Lambda(x^\mu, y)}\Phi$$

$$\Lambda(x,y) = \Lambda(x, y+2\pi R) = -\Lambda(x,-y)$$

$$\epsilon(y) = \epsilon(y+2\pi R) = \begin{cases} 1 & (0 < y < \pi R) \\ -1 & (-\pi R < y < 0) \end{cases}$$

$$D_M\Phi = \partial_M \Phi - i\big(q_\Phi \epsilon(y) A_M + T_a(\Phi) G^a_M\big)\Phi$$

Z_2 boundary condition eliminates the unwanted partners of the 4-dim graviton, gluons, axion and quarks.

$$A_M(x,y) = A_M(x, y+2\pi R)$$
$$= A^+_{0M}(x) + \sum_{n=1} \left[A^+_{nM}(x)\cos\left(\frac{ny}{R}\right) + A^-_{nM}(x)\sin\left(\frac{ny}{R}\right)\right]$$

$m_n^2 = -\partial_5^2 = \dfrac{n^2}{R^2}$: only zero modes are light 4-dim particles

$$A_M(-y) = \big(A_\mu(-y), A_5(-y)\big) = \big(-A_\mu(y), A_5(y)\big)$$

allows to have an axion zero mode from A_5 without a vector zero mode from A_μ.

$$
\begin{aligned}
\Psi(x,y) &= \Psi(x, y+2\pi R) \\
&= \psi_0(x) + \sum_{n=1} \left[\psi_n^+(x)\cos\left(\frac{ny}{R}\right) + \psi_n^-(x)\sin\left(\frac{ny}{R}\right)\right]
\end{aligned}
$$

$\Psi(-y) = \gamma_5\Psi(y)$ allows to have a 4-dim left-handed quark zero mode without a right-handed zero mode having the same gauge charge.

In string/M theory, such a projection onto the desired form of 4-dim zero modes is achieved by a more complicate geometrical and/or topological structure of the compact internal space.

4-dim angular axion from 5-dim gauge field

$$\frac{a(x)}{f_a} = \oint dy\, A_5(x,y) \quad \left(y \cong y + 2\pi R \cong -y\right)$$

For subsequent discussion, let us consider a locally well-defined, but globally ill-defined 5-dim U(1) gauge transformation:

$$U(1)_\alpha : \quad A_5 \to A_5 + \partial_y \Lambda_\alpha(y), \quad \Phi_{5D} \to e^{iq_\Phi \epsilon(y)\Lambda_\alpha(y)}\Phi_{5D}$$

$$\left(\Lambda_\alpha(y) = \alpha \frac{y}{2\pi R} \text{ for continuous real parameter } \alpha, \quad q_\Phi \in \mathbb{Z}\right)$$

$$\Omega_\alpha(y) \equiv e^{i\alpha\epsilon(y)y/2\pi R} = \Omega_\alpha(-y) = \Omega_\alpha(y+2\pi R) \text{ only for } \alpha = 2\pi\times \text{ integer}$$

$\Rightarrow$ $U(1)_\alpha$ is globally well-defined only for $\alpha = 2\pi\times$ integer

$$U(1)_{\alpha=2\pi} : \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + 2\pi, \quad \Phi_{5D} \to [\Omega_{2\pi}(y)]^{q_\Phi}\Phi_{5D}$$

There is no U(1)-charged 4-dim zero mode, then the $U(1)_{\alpha=2\pi}$ transformation of Φ_{5D} corresponds to an invariant reshuffling of the massive KK modes.

$$\Rightarrow \quad a(x) \cong a(x) + 2\pi f_a \quad \text{in 4-dim effective theory}$$

$$U(1)_\alpha: \quad A_5 \to A_5 + \frac{\alpha}{2\pi R}, \quad \Phi \to e^{iq_\Phi \epsilon(y)\alpha y/2\pi R}\Phi$$

$$(\alpha = \text{continuous real parameter})$$

Because it is locally a good gauge symmetry, $U(1)_\alpha$ can be broken only by non-local effects stretched over the 5$^{\text{th}}$ dimension S^1/Z_2:

$$U(1)_\alpha \text{ breaking} \propto e^{-2\pi MR} \quad \text{in the limit } MR \gg 1$$

In case that there is no U(1)-charged light matter field in 4-dim effective theory, which is the case in our 5-dim theory and also for the axions from p-form gauge field in string/M theory, the low energy consequence of $U(1)_\alpha$ is a nonlinear PQ symmetry in the GKR basis, whose breaking is exponentially suppressed in the limit that the orbifold radius is larger than the fundamental length scale of the 5-dim theory:

$$U(1)_{\mathrm{PQ}}: \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha$$

Dimensional reduction to 4D effective theory

Flat 5D spacetime background:

$$ds^2 = \tilde{g}_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \qquad (y \cong -y \cong y + 2\pi R)$$

U(1)-neutral 5D quarks & antiquarks:

$$Q_I = (Q_i, Q_i^c)$$

U(1)-charged massive 5D scalar:

ξ with $M_\xi \gg 1/\pi R$

(Weak gravity conjecture for the 5D U(1): $M_\xi^2 \lesssim q_\xi^2 g_{5A}^2 M_5^3$)

Light 4D fields (zero modes) from the 5D fields:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}(x), \quad G_\mu^a = G_\mu^a(x), \quad A_5 = \frac{\theta(x)}{2\pi R}, \quad (Q_i, Q_i^c) = \frac{(q_i(x), q_i^c(x))}{\sqrt{\pi R}}, \quad \xi = 0$$

$$\left(\theta(x) = \frac{a(x)}{f_a} = \oint dy\, A_5(x, y)\right)$$

$$S_{5\mathrm{D}} = \int d^5x\sqrt{-\tilde{g}}\,\Big[\frac{1}{2}M_5^3\mathcal{R}_5(\tilde{g}) - \frac{1}{4g_{5A}^2}A^{MN}A_{MN} - \frac{1}{4g_{5S}^2}G^{aMN}G^a_{MN}$$
$$+\, i\sum_I \bar{Q}_I(\tilde{\gamma}^M D_M + \mu_I A_{MN}\tilde{\gamma}^{MN})Q_I + D^M\xi^* D_M\xi - M_\xi^2|\xi|^2$$
$$+\, \frac{k_{\mathrm{CS}}}{32\pi^2}\frac{\epsilon^{MNPQR}}{\sqrt{-\tilde{g}}}A_M G^a_{NP}G^a_{QR} + \ldots\Big]$$

$$\mathcal{L}_{4D} = \frac{1}{2}M_P^2 R(g) + \frac{1}{2}f_a^2\partial_\mu\theta\partial^\mu\theta - \frac{1}{4g_s^2}G^{a\mu\nu}G^a_{\mu\nu} + \sum_{\psi=q_i,q_i^c} i\bar{\psi}\gamma^\mu D_\mu\psi$$
$$+\, \frac{c_G}{32\pi^2}\theta G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \sum_{\psi=q_i,q_i^c} C_\psi\partial_\mu\theta\bar{\psi}\gamma^\mu\psi - \delta V(\theta)$$

4D Planck scale, axion scale and the QCD coupling:

$$M_P^2 = \pi R M_5^3, \quad f_a^2 = \frac{1}{4\pi g_{5A}^2 R}, \quad \frac{1}{g_s^2} = \frac{\pi R}{g_{5S}^2}$$

4D axion couplings from the 5D CS term and the 5D U(1) magnetic moments:

$$c_G = k_{\mathrm{CS}}, \quad (C_{q_i}, C_{q_i^c}) = \frac{(\mu_{Q_i}, \mu_{Q_i^c})}{\pi R}$$

Weak gravity conjecture (WGC) applied for our model implies the existence of a U(1)-charged 5-dim particle ξ with a mass bounded as

Arkani-Hamed et al, hep-th/0601001

$$M_\xi^2 \lesssim q_\xi^2 g_{5A}^2 M_5^3$$

Such U(1)-charged 5-dim field have axion-dependent KK masses:

$$m_n^2(\xi) = M_\xi^2 + \frac{(n + q_\xi\theta/2\pi)^2}{R^2} \qquad (n \in \mathbb{Z})$$

generating an axion-dependent 1-loop Casimir energy density.

Summing over the KK modes of 5-dim field can be rearranged to be a sum over the number of windings of the 5D field around S^1/Z_2 .

For more details, see for instance M. Reece, arXiv:2304.08512.

In the limit $M_\xi \gg 1/R$, the rearranged Casimir energy can be interpreted as an axion potential induced by the Euclidean worldline of ξ winding S^1/Z_2, i.e. worldline instanton with the Euclidean action $S_{\rm ins}^{\rm WL} = 2M_\xi \pi R$:

$$\delta V = \mathcal{O}\left(\frac{1}{8\pi^2}\frac{M_\xi^2}{(2\pi R)^2} e^{-2\pi M_\xi R} \cos q_\xi\theta\right)$$

Weak gravity conjecture (WGC) implies the existence of an instanton generating a bare axion potential

$$\delta V \propto e^{-S_{\rm ins}} \Lambda_{\rm UV}^4 \cos(a/f_a),$$

which would require

$$S_{\rm ins} \gtrsim 2 \ln(\Lambda_{\rm UV}^2/m_a f_a).$$

This is likely to be a generic feature of the axions from higher-dimensional gauge field.

PQ-breaking

1) QCD anomaly:

$$\frac{c_G}{32\pi^2}\frac{a}{f_a}G\tilde{G} \quad \Rightarrow \quad V_{\rm QCD}(a) \sim m_\pi^2 f_\pi^2 \propto \Lambda_{\rm QCD}^3 \sim \mu^3 e^{-8\pi^2/bg_s^2(\mu)} \propto e^{-2M_*\pi R}$$

2) PQ breaking by the instanton implied by the WGC:

Worldline instanton $\quad \Rightarrow \quad \delta V_{\rm UV} \propto e^{-2M_\xi \pi R}$

In the limit $M_* R,\, M_\xi R \gg 1$, allowing M_ξ / M_* to vary by just a factor of few, the axion $\theta(x)$ can be identified as any of

QCD axion: $\delta V_{\rm UV} < 10^{-10} V_{\rm QCD} \sim 10^{-10} m_\pi^2 f_\pi^2 \quad (c_G \neq 0)$

Heavy ALP: $\delta V_{\rm UV} \gg m_\pi^2 f_\pi^2$

Ultralight ALP: $\delta V_{\rm UV} \ll m_\pi^2 f_\pi^2 \quad (c_G = 0)$

Axion scale and couplings in the experimentalist's notation

$$\mathcal{L}_{\rm axion} = -\frac{1}{4}F^{A\mu\nu}F^A_{\mu\nu} + i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi + \frac{1}{2}\partial_\mu a\partial^\mu a + g_{a\psi}\partial_\mu a(x)\bar{\psi}\bar{\sigma}^\mu\psi + \frac{1}{4}g_{aA}a(x)F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$g_{aG} = \frac{g_s^2}{8\pi^2}\frac{c_G}{f_a}, \quad g_{a\psi} = \frac{C_\psi}{f_a} \qquad c_G = k_{\rm CS}, \quad (C_{q_i}, C_{q_i^c}) = \frac{(\mu_{Q_i}, \mu_{Q_i^c})}{\pi R}$$

$$M_P^2 = \pi R M_5^3, \quad f_a^2 = \frac{1}{4\pi g_{5A}^2 R}, \quad \frac{1}{g_s^2} = \frac{\pi R}{g_{5S}^2}$$

$$S_{\rm ins}^{\rm QCD} = \frac{8\pi^2}{g_s^2}, \quad S_{\rm ins}^{\rm WL} = 2M_\xi \pi R$$

4D Weak Gravity Conjecture (WGC) on axion from the 5D WGC on U(1) gauge coupling:

$$M_\xi^2 \lesssim q_\xi^2 g_{5A}^2 M_5^3 \quad \Rightarrow \quad \frac{f_a}{M_P} = \frac{1}{\sqrt{g_{5A}^2 M_5^3}}\frac{1}{2\pi R} \lesssim \frac{1}{2\pi M_\xi R} = \frac{1}{S_{\rm ins}^{\rm WL}}$$

Scales and couplings in 5D theory

4D | 5D | String/M theory?

$\frac{1}{R}$ KK scale

G^a_M, Q, Q^c 5D QCD with light colored particles

$\Lambda_5 \sim M_5,\ M_\xi,\ \frac{1}{\mu_Q},\ \frac{1}{g^2_{5A}}$ 5D cutoff scale

PQ quality for light axion: $\delta V = \mathcal{O}\left(\frac{1}{8\pi^2}\frac{M_\xi^2}{(2\pi R)^2}e^{-2\pi M_\xi R}\cos q_\xi\theta\right)$

$$\Rightarrow \quad S^{\rm WL}_{\rm ins} = 2\pi M_\xi R \gtrsim 2\ln(f_a/m_a) = \mathcal{O}(100)$$

$$2\pi M_\xi R \lesssim 2\pi\Lambda_5 R = \frac{2}{g_s^2}\Lambda_5 g^2_{5S}$$

Loop/Tree = O(1) for strongly coupled 5D QCD

$$g^2_{5S}\int^{\Lambda_5}\frac{d^5k}{(2\pi)^5}\frac{1}{k^2}\frac{1}{k^2} \lesssim \mathcal{O}(1) \quad \Rightarrow \quad g^2_{5S}\Lambda_5 \lesssim \mathcal{O}(6\pi^3)$$

$$\Rightarrow \quad 2\ln(f_a/m_a) \lesssim S^{\rm WL}_{\rm ins} = 2\pi M_\xi R \lesssim \mathcal{O}\left(\frac{12\pi^3}{g_s^2}\right)_{\mu\sim f_a}$$

$$\Rightarrow \quad S^{\rm WL}_{\rm ins} = \mathcal{O}\left(\frac{8\pi^2}{g_s^2}\right)_{\mu\sim f_a} = \mathcal{O}\left(\frac{8\pi^2}{g^2_{\rm GUT}}\right)$$

Two characteristic features of the axion from 5D gauge field, which are shared by the axions from p-form gauge field in string/M theory

1) Axion decay constant nearly saturates the WGC bound in relatively simple compactification without a big hierarchy among the moduli VEVs:

$$\frac{f_a}{M_P} \sim \frac{1}{S_{\rm ins}} \sim \frac{g^2}{8\pi^2} = \mathcal{O}(10^{-2})$$

WL instanton suggested by the WGC

2) Comparable couplings to the gauge and matter fields at $\mu \sim f_a$:

$$C_\psi = \frac{\mu_\psi}{\pi R} \sim \frac{1}{\pi \Lambda_5 R} \sim \frac{1}{S_{\rm ins}} = \frac{1}{2\pi M_\xi R} \quad \Rightarrow \quad \left(\frac{g_{a\psi}}{g_{aA}}\right)_{\mu\sim f_a} \sim \frac{8\pi^2}{g^2}\frac{1}{S_{\rm ins}} = \mathcal{O}(1)$$

which might be compared to

$$\left(\frac{g_{a\psi}}{g_{aA}}\right)^{\rm KSVZ}_{\mu\sim f_a} \sim \frac{g^2}{8\pi^2}, \quad \left(\frac{g_{a\psi}}{g_{aG}}\right)^{\rm DFSZ}_{\mu\sim f_a} \sim \frac{8\pi^2}{g^2}$$

For QCD axion which couples to the gluons, this parametric difference might be faded away below the QCD scale where $g_s^2(\mu)/8\pi^2 \sim 1 \;\; (\mu \lesssim 1\,{\rm GeV})$, while for an ALP without the coupling to the gluons, this difference survives down to the observable low energy scales.

Hierarchy between the axion and Planck scales from geometry

1) Large extra n-dimensions through which the gravity can propagates, while the axion and the SM fields are confined on 5D spacetime

$$\int d^{5+n}x\sqrt{-\tilde{g}}\,\frac{1}{2}M_{5+n}^{3+n}\mathcal{R}_{5+n} \quad\Rightarrow\quad \int d^4x\sqrt{-g}\,\frac{1}{2}M_P^2\mathcal{R}_4(g) \quad \left(M_P^2 = M_{5+n}^{n+3}\pi R V_n\right)$$

(5+n)-dim Planck mass

Large volume of the extra n-dimension

$$\Rightarrow \quad \frac{f_a}{M_P} \sim \frac{1}{\sqrt{V_n M_{5+n}^n}}\frac{1}{S_{\rm ins}^{\rm WL}}$$

Additional suppression (relative to the Planck scale) of the axion decay constant by the large volume factor

2) Warped 5-th dimension along which the axion is localized near the IR-end, for which the axion scale is red-shifted by an exponentially small warp factor

Randall-Sundrum background

KC, hep-ph/0308024

$$ds^2 = \tilde{g}_{MN} dx^M dx^N = e^{-2k|y|} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (k = \text{AdS curvature})$$

$$\left(y = 0 \ (\text{UV-end}), \quad y = \pi R \ (\text{IR-end}) \right)$$

$$\int d^5x \sqrt{-\tilde{g}}\, \frac{1}{2} M_5^3 \mathcal{R}_5 \quad \Rightarrow \quad \int d^4x \sqrt{-g}\, \frac{1}{2} M_P^2 \mathcal{R}_4(g) \quad \left(M_P^2 = \frac{M_5^3 (1 - e^{-2k\pi R})}{2k} \right)$$

<u>Axion zero mode</u>

To eliminate the mixing between A_5 and A_μ, introduce the gauge-fixing

$$S_{\rm gf} = -\frac{1}{2g_{5A}^2} \int d^5x \sqrt{\tilde{g}} \left(\tilde{g}^{\mu\nu} \partial_\mu A_\nu - \tilde{g}^{55} e^{2k|y|} \partial_y (e^{-2k|y|} A_5) \right)^2$$

$$\Rightarrow \quad \frac{1}{2g_{5A}^2} \int d^5x \left[e^{-2k|y|} (\partial_\mu A_5)^2 - \left(\partial_y (e^{-2k|y|} A_5) \right)^2 \right]$$

$$\Rightarrow \quad A_5 = \frac{k e^{2k|y|}}{e^{2k\pi R} - 1} \theta(x)$$ (Axion zero mode localized near the IR-end $y = \pi R$)

$$\Rightarrow \quad f_a^2 = \frac{1}{g_{5A}^2}\left(\frac{k}{e^{2k\pi R}-1}\right)^2 \int_0^{\pi R} dy\, e^{2ky} = \frac{1}{2g_{5A}^2}\frac{k}{e^{2\pi kR}-1}$$

$$\frac{f_a}{M_P} \simeq \left(\frac{k^2}{M_5^3 g_{5A}^2}\right)^{1/2} e^{-\pi kR} \quad (k\pi R \gg 1)$$

(Axion decay constant exponentially red-shifted relative to the Planck scale)

Gluon and quark zero modes in warped background:

$$G_\mu^a = G_\mu^a(x), \qquad (Q_i, Q_i^c) = \frac{e^{3k|y|/2}}{\sqrt{\pi R}}(q_i(x), q_i^c(x))$$

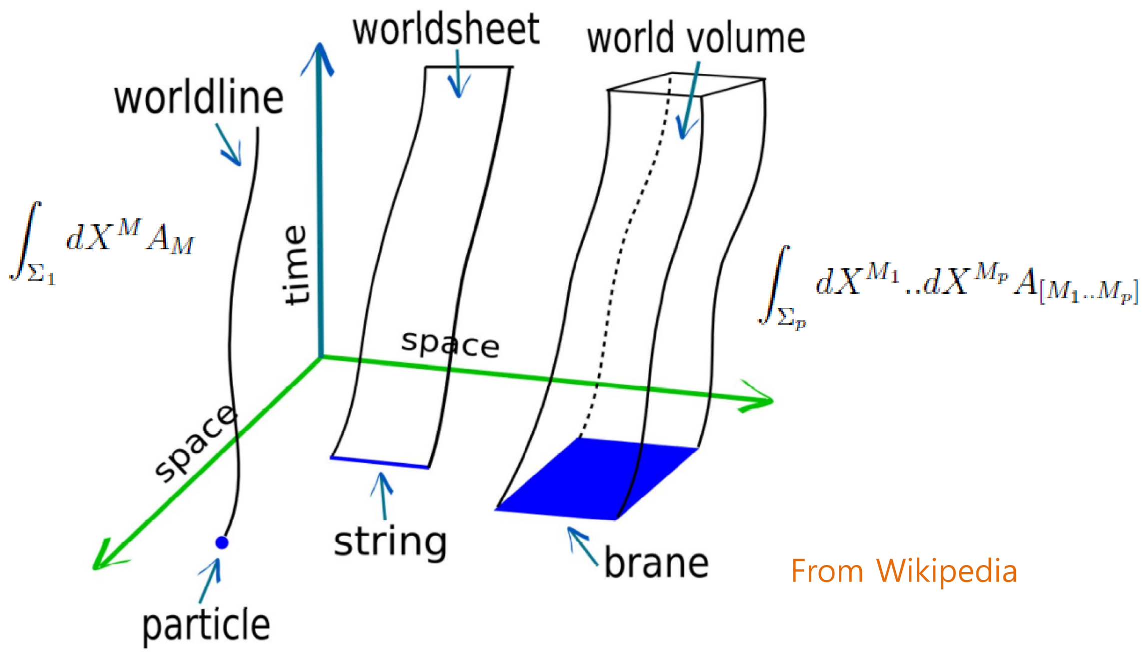
$$\Rightarrow \quad c_G = k_{\rm CS}, \quad (C_{q_i}, C_{q_i^c}) = \frac{(\mu_{Q_i}, \mu_{Q_i^c})}{\pi R}$$

Introducing nonzero masses for the 5D quarks, the shape of
the quark zero modes and their axion couplings can be modified.

Axions in string/M theory

String/M theory involves a variety of (p-1)-dimensional objects ((p-1)-brane) which couples to a p-form gauge field

$$A_p = \frac{1}{p!} A_{[M_1 M_2 \ldots M_p]} dX^{M_1} \wedge dX^{M_2} \wedge \ldots \wedge dX^{M_p} \quad \left(X^M = (X^0, X^1, .., X^9) \right)$$

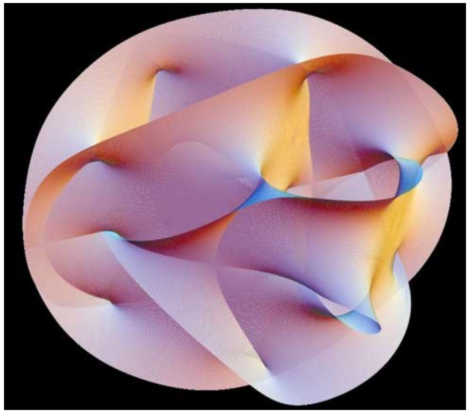


From Wikipedia

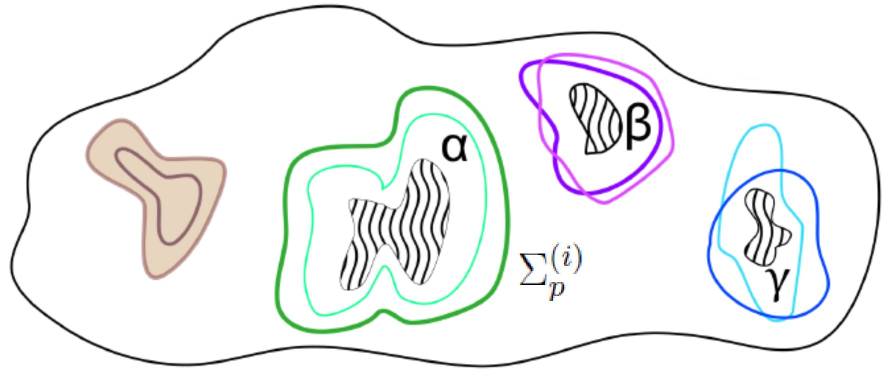
(p-1)-dim brane which couples to p-form gauge potential over the p-dim worldline (p=1), worldsheet (p=2), worldvolume (p>2).

String compactifications also involve p-dimensional cycles $\Sigma_p^{(i)}$ and the associated harmonic p-forms $\omega_p^{(i)}$

From arXiv:2102.00532



6D Calabi-Yau



p-cycle= p-dim closed surfaces in CY, which can not be smoothly deformed to a point or to other cycles

Zero modes of the p-form gauge fields include the 4D axions:

Witten '84

$$A_p(x,y) = \sum_{i=1}^{N_p} \theta^i(x)\omega_p^{(i)}(y) \quad \Leftrightarrow \quad \theta^i(x) = \int_{\Sigma_p^{(i)}} A_p$$

$$\left(A_5 = \frac{\theta(x)}{2\pi R}\right) \quad \left(\int_{\Sigma_p^{(i)}} \omega_p^{(j)} = \delta_i^j\right) \quad \left(\theta(x) = \oint dy\, A_5\right)$$

2-form gauge field gives an additional axion: $\partial_\mu\theta = \epsilon_{\mu\nu\rho\sigma}\partial^\nu A^{\rho\sigma}$

Axion in 4-dim N=1 Supergravity

We are mostly interested in compactifications preserving 4-dim N=1 SUSY. Then the axion physics near the compactification scale can be described by 4-dim N=1 SUGRA lagrangian.

With N=1 SUSY, each axion has its modulus (saxion) partner forming the complex scalar component of a chiral superfield:

$$T^i = \tau^i + i\theta^i \qquad (\theta^i \cong \theta^i + 2\pi,\ \tau^i = S_{\rm ins}^{(i)})$$

Modulus VEV corresponds to the Euclidean action of a (p-1) brane-instanton which couples to the p-form gauge potential while wrapping the p-cycle:

$$\mathcal{A}_{\rm ins} \propto e^{-\left(S_{\rm ins} + i\int_{\Sigma_p} A_p\right)} = e^{-\left(\tau + i\theta\right)} \qquad (S_{\rm ins} \propto \text{brane-tension} \times p\text{-cycle volume})$$

Axion= p-form gauge field over a p-cycle

$\Leftrightarrow$ Brane instanton= Euclidean (p-1)-brane wrapping the p-cycle

Non-linear PQ symmetry in the GKR basis:

$$U(1)_{\rm PQ}:\ \ T \to T + i\alpha \quad (\alpha = \text{real constant})$$

Axion scales and the couplings to the gauge and matter fields are determined by the PQ-invariant Kaehler potential and the PQ-breaking holomorphic gauge kinetic functions:

Moduli Kaehler potential

Matter Kaehler metric

$$K = K_0(T^i + T^{i*}) + Z_\Phi(T^i + T^{i*})\Phi^*\Phi$$

SM matter superfields

Integer or rational number

$$8\pi^2 \mathcal{F}_A = c_{Ai}T^i + ... \quad (\mathrm{Re}(\mathcal{F}_A) = 1/g_A^2)$$

$$\mathcal{L}_{\rm axion} = \frac{1}{2}f_{ij}^2 \partial_\mu\theta^i\partial^\mu\theta^j + \frac{1}{32\pi^2}c_{Ai}\theta^i F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + \partial_\mu\theta^i\left(C_{\phi i}J^\mu_\phi + C_{\psi i}J^\mu_\psi\right) \quad \left(\theta^i = \frac{a_i}{f_i}\right)$$

$$\left(J^\mu_\phi = i(\phi^* D_\mu\phi - \mathrm{h.c}), \quad J^\mu_\psi = \bar{\psi}\bar{\sigma}^\mu\psi \right)$$

$$\frac{1}{2}f_{ij}^2 = M_P^2 \frac{\partial^2 K_0}{\partial T^i \partial T^{j*}}$$

$$C_{\phi i} = \frac{\partial \ln Z_\Phi}{\partial T^i}, \quad C_{\psi i} = \frac{\partial \ln(e^{-K_0/2} Z_\Phi)}{\partial T^i}$$

In most case, the brane instanton generates a PQ-breaking nonperturbative superpotential or Kaehler potential, yielding a bare axion potential as

Dine et al. '86; Blumenhagen et al, arXiv:0902.3251

$$\delta_{\rm ins} W \sim e^{-T} \quad \Rightarrow \quad \delta V_{\rm UV} \sim e^{-\tau} m_{3/2} M_P^3 \cos(a/f_a)$$

$$\delta_{\rm ins} K_0 \sim e^{-T} \quad \Rightarrow \quad \delta V_{\rm UV} \sim e^{-\tau} m_{3/2}^2 M_P^2 \cos(a/f_a)$$

4-dim N=1 SUSY expressions show some generic features of the axions from p-form gauge field, which we already noticed for the axion from 5-dim gauge field.

$$C_{\phi i} = \frac{\partial \ln Z_\Phi}{\partial T^i}, \quad C_{\psi i} = \frac{\partial \ln (e^{-K_0/2} Z_\Phi)}{\partial T^i}$$

$$\Rightarrow \quad C_{\phi,\psi} = \mathcal{O}\left(\frac{1}{\tau}\right) \quad \text{in the limit } \tau \gg 1$$

Couplings to the canonically normalized SM gauge and matter fields:

$$\frac{g_{aA}}{4} a(x) F^{A\mu\nu} \tilde{F}^{A}_{\mu\nu} + g_{a\psi} \partial_\mu a(x) \bar{\psi} \sigma^\mu \psi$$

$$g_{aA} = c_A \frac{g_A^2}{8\pi^2} \frac{1}{f_a} \sim \frac{g_{\rm GUT}^2}{8\pi^2} \frac{1}{f_a}, \qquad g_{a\psi} \sim \frac{1}{\tau} \frac{1}{f_a}$$

Brane-instanton induced axion potential:

$$\delta V_{\rm UV} \sim e^{-\tau} m_{3/2} M_P^3 \cos(a/f_a) \;\; \text{or} \;\; e^{-\tau} m_{3/2}^2 M_P^2 \cos(a/f_a)$$

$$\Rightarrow \quad \tau \gtrsim \ln(m_{3/2} M_P / m_a^2) \;\; \text{or} \;\; \ln(m_{3/2}^2/m_a^2) = \mathcal{O}(60-100)$$

For axions which couple to the SM gauge fields,

$$\frac{c_A}{32\pi^2} \frac{a}{f_a} F^{A\mu\nu} \tilde{F}^{A}_{\mu\nu} \;\; \Rightarrow \;\; \frac{1}{g_A^2} = c_A \frac{\tau}{8\pi^2} + \ldots \;\; \Rightarrow \;\; \tau \lesssim \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) = \mathcal{O}(100)$$

$$\Rightarrow \quad \tau = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) = \mathcal{O}(100)$$

$$\Rightarrow \quad \left(\frac{g_{a\psi}}{g_{aA}}\right)_{\mu\sim f_a} \sim \frac{8\pi^2}{g_{\rm GUT}^2}\,\frac{1}{\tau} = \mathcal{O}(1)$$

which might be compared to

$$\left(\frac{g_{a\psi}}{g_{aA}}\right)^{\rm KSVZ}_{\mu\sim f_a} \sim \frac{g_{\rm GUT}^2}{8\pi^2}, \qquad \left(\frac{g_{a\psi}}{g_{aA}}\right)^{\rm DFSZ}_{\mu\sim f_a} \sim \frac{8\pi^2}{g_{\rm GUT}^2}$$

The implication of this parametric difference for the low energy (below the QCD scale) couplings of the QCD axion needs a more careful analysis, which can be found in arXiv:2106.05816.

Axions from 3-form (2-form) field in heterotic E_8xE_8 M/string theory

Heterotic M-theory (string theory) compactified on $\mathrm{CY}\times S^1/Z_2$ with $\omega_L=\omega_{\mathrm{YM}}$:

(Het-M → Het-string in the limit R_{11} → 0.)

$$C_3=A_{\mu\nu}(x)\,dx^\mu\wedge dx^\nu\wedge dx^{11}+\theta^i(x)\,\omega_2^{(i)}\wedge dx^{11}\qquad (i=1,..,h_{1,1})$$

$$S=t_S+i\theta_S\quad(\partial_\mu\theta_S\propto\epsilon_{\mu\nu\rho\sigma}\partial^\nu A^{\rho\sigma}),\qquad T^i=t^i+i\theta^i$$

$t_S\propto\mathcal{V}_{\mathrm{CY}}$: M5 brane-instanton wrapping CY

$t^i\propto R_{11}\mathcal{V}_{\Sigma_2^{(i)}}$: M2 brane-instanton wrapping $\Sigma_2^{(i)}$ and stretched along the 11th dim

$$\mathcal{V}_{\mathrm{CY}}=\frac{1}{6}d_{ijk}t^it^jt^k\quad\left(d_{ijk}=\int_{\mathrm{CY}}\omega_2^{(i)}\wedge\omega_2^{(j)}\wedge\omega_2^{(k)}\right)$$

$$K \simeq -\ln(S+S^*) - \ln \mathcal{V}_{\rm CY} + \frac{1}{\mathcal{V}_{\rm CY}^{1/3}} \Phi^*_{\overline{27}} \Phi_{\overline{27}} \quad (t_S > t^i \gg 1)$$

$$\mathcal{F}_{E6} = \frac{1}{8\pi^2}(S + \ell_i T^i), \quad \mathcal{F}_{E8} = \frac{1}{8\pi^2}(S - \ell_i T^i)$$

$$\left(\ell_i = \frac{1}{8\pi^2}\int \omega_2^{(i)} \wedge \left[{\rm tr}(F\wedge F) - \frac{1}{2}{\rm tr}(R\wedge R)\right]\right)$$

$$t_S = 4\pi^2\left(\frac{1}{g^2_{E6}} + \frac{1}{g^2_{E8}}\right), \quad \ell_i t^i = 4\pi^2\left(\frac{1}{g^2_{E6}} - \frac{1}{g^2_{E8}}\right)$$

$$\frac{S}{2\pi} \leftrightarrow \frac{2\pi}{S}, \quad \frac{T}{2\pi} \leftrightarrow \frac{2\pi}{T} \qquad \Rightarrow \qquad t_S, t^i \gtrsim \mathcal{O}(2\pi)$$

$$\Rightarrow \qquad \mathcal{O}(2\pi) \lesssim t_S, t^i \lesssim \mathcal{O}\left(\frac{8\pi^2}{g^2_{E_6}}\right)$$

All moduli have similar VEVs of the order of $8\pi^2/g^2_{\rm GUT} \sim 10^2$, yielding

$$f_a \sim 10^{16}\,{\rm GeV}, \quad g_{aA}(\mu \sim f_a) \sim g_{a\psi}(\mu \sim f_a) \sim \frac{1}{10^{18}\,{\rm GeV}}$$

Axions from 4-form field in Type IIB string theory

$$T^i = \tau^i + i\theta^i$$

$$\theta^i = \int_{\Sigma_4^{(i)}} C_4 \qquad \tau^i = \frac{\mathcal{V}_{\rm CY}}{\partial t^i} = \frac{1}{2} d_{ijk} t^j t^k \propto \mathcal{V}_{\Sigma_4^{(i)}}$$

(D3 brane-instanton wrapping $\Sigma_4^{(i)}$)

SM gauge fields on D7 branes wrapping a combination of $\Sigma_4^{(i)}$

$$8\pi^2 \mathcal{F}_A = \ell_i T^i$$

$$K = -2\ln \mathcal{V}_{\rm CY}(T^i + T^*) + Z_\Phi(T^i + T^{i*})\Phi^*\Phi$$

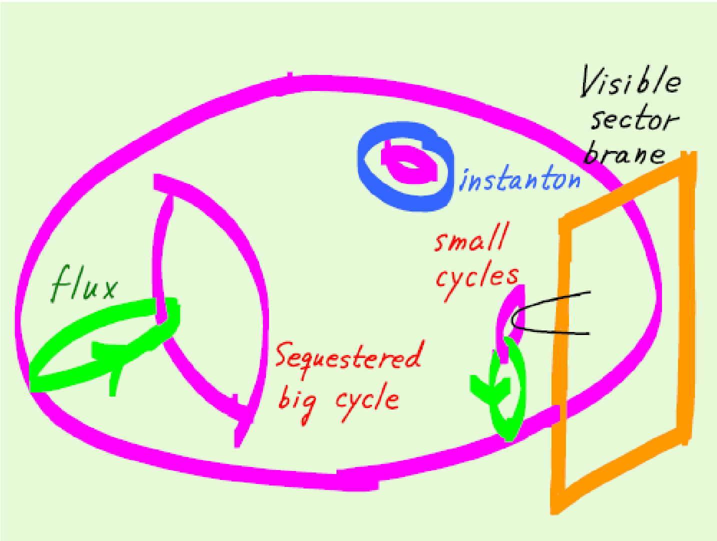
$$\mathcal{V}_{\rm CY}(\tau_i) = \frac{1}{6} d_{ijk} t^i t^j t^k \quad \text{for} \;\; \tau^i = \frac{1}{2} d_{ijk} t^j t^k$$

Moduli-dependence of the matter Kaehler metric $Z_\Phi(\tau^i)$ is more model-dependent, but it reveals the limiting behavior

$$\frac{\partial \ln Z_\Phi}{\partial \tau} \sim \frac{1}{\tau} \quad (\tau \gg 1)$$

Axions in Large Volume Scenario of Type IIB string theory

Balasubramanian et al, hep-th/0502058
Cicoli et al, arXiv:1206.0819



Large CY space involving

1) a big 4-cycle with the volume $\sim \tau_b$
2) smaller 4-cycles with the volume $\sim \tau_s$

Axions associated with the 4-cycles:

$$\theta_b = \int_{\Sigma_{4b}} C_4, \quad \theta_s = \int_{\Sigma_{4s}} C_4$$

SM living on D7 branes wrapping the smaller cycle:

$$\Rightarrow \qquad \tau_b \gg \tau_s = \frac{8\pi^2}{g^2_{\rm GUT}} \qquad (\mathcal{V}_{\rm CY} \sim \tau_b^{3/2})$$

$$8\pi^2 \mathcal{F}_A = T_s$$

$$K = -3\ln(T_b + T_b^*) + \frac{\hat{K}_0(T_s + T_s^*)}{(T_b + T_b^*)^{3/2}} + \frac{\hat{Z}_\Phi(T_s + T_s^*)}{(T_b + T_b^*)}\Phi^*\Phi$$

The decay constant of the small-cycle axion is suppressed by a certain power of the big-cycle volume, therefore it can have any value above the astrophysical bound~ 10^8 GeV:

$$f_{a_s} \sim \frac{M_P}{\tau_b^{3/4}\tau_s} \ll 10^{16}\,\mathrm{GeV}$$

The couplings of the small-cycle axion to the SM gauge and matter fields have the anticipated pattern that we have discussed about:

$$g_{a_s A}(\mu \sim f_{a_s}) = \frac{g_{\rm GUT}^2}{8\pi^2}\frac{1}{f_{a_s}}, \quad g_{a_s\psi}(\mu \sim f_{a_s}) \sim \frac{1}{\tau_s f_{a_s}} \sim g_{a_s A}(\mu \sim f_{a_s})$$

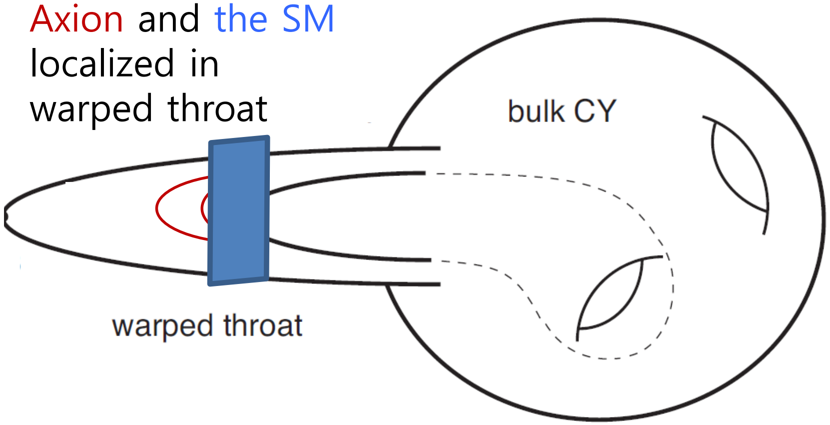
The decay constant of the big-cycle axion is also suppressed by the large volume factor, but by a different amount. The big-cycle axion does not couple to the SM gauge fields, while it has a Planck-scale-suppressed coupling to the matter fields:

$$f_{a_b} \sim \frac{M_P}{\tau_b} \ll 10^{16}\,\mathrm{GeV} \qquad g_{a_b A} = 0, \quad g_{a_b\psi}(\mu \sim f_{a_b}) \sim \frac{1}{\tau_b f_{a_b}} \sim \frac{1}{M_P}$$

Axion and the SM at strongly warped region

We can imagine a situation that both the SM and some axions are localized in a strongly warped region (warped throat) in string compactification.

Axion and the SM localized in warped throat



The axion decay constant is red-shifted by an exponentially small warp factor, while the couplings to the SM fields take a similar form as other cases.

$$K = K_b(X,X^*) + XX^*\hat{K}_0(T+T^*) + \Omega(X,X^*)\hat{Z}_\Phi(T+T^*)\Phi\Phi^*$$

$$\langle |X| \rangle = \text{warp factor} \ll 1$$

$$f_a \sim |X|\frac{M_P}{\tau} \sim |X|\frac{g_{\rm GUT}^2}{8\pi^2}M_P \ll 10^{16}\,\text{GeV}$$

$$g_{aA} = \frac{g_{\rm GUT}^2}{8\pi^2}\frac{1}{f_a}, \quad g_{a\psi} \sim \frac{1}{\tau f_a} \sim g_{aA}$$

Summary on axions from p-form gauge field in string/M theory

For each axion from p-form gauge field, there exist a modulus partner parameterizing the associated brane-instanton suggested by the WGC:

$$T = \tau + i\theta \quad \left(\theta(x) = \frac{a(x)}{f_a} \cong \theta(x) + 2\pi, \;\; \tau = S_{\rm ins}\right)$$

Often the brane-instanton generates a bare axion potential as

$$\delta_{\rm ins} W \sim e^{-T} \quad \Rightarrow \quad \delta V_{\rm UV} \sim e^{-\tau} m_{3/2} M_P^3 \cos(a/f_a)$$

$$\Rightarrow \quad \tau \gtrsim \ln(m_{3/2} M_P / m_a^2) = \mathcal{O}(100)$$

For axions with nonzero coupling to the SM gauge fields,

$$\frac{c_A}{32\pi^2} \frac{a}{f_a} F^{A\mu\nu} \tilde{F}^A_{\mu\nu} \quad \Rightarrow \quad \frac{1}{g_A^2} = c_A \frac{\tau}{8\pi^2} + \ldots \quad \Rightarrow \quad \tau \lesssim \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) = \mathcal{O}(100)$$

$$\Rightarrow \quad \tau = \mathcal{O}\left(\frac{8\pi^2}{g_{\rm GUT}^2}\right) = \mathcal{O}(100)$$

For axions which do not couple to the SM gauge fields, their moduli partner can have a much bigger VEV.

Axion decay constant is generically given by

$$f_a \sim \epsilon \frac{M_P}{\tau} \sim \epsilon \times 10^{16}\,\mathrm{GeV}$$

$$(\epsilon = \text{volume suppression or small warp factor})$$

Couplings to the SM gauge and matter fields:

$$\frac{g_{aA}}{4} a(x) F^{A\mu\nu} \tilde{F}^{A}_{\mu\nu} + g_{a\psi} \partial_\mu a(x) \bar{\psi} \sigma^\mu \psi$$

$$g_{aA} = c_A \frac{g_A^2}{8\pi^2} \frac{1}{f_a} \sim \frac{g_{\rm GUT}^2}{8\pi^2} \frac{1}{f_a}, \qquad g_{a\psi} \sim \frac{1}{\tau} \frac{1}{f_a}$$

$$\Rightarrow \quad \left(\frac{g_{a\psi}}{g_{aA}} \right)_{\mu \sim f_a} \sim \frac{8\pi^2}{g_{\rm GUT}^2} \frac{1}{\tau} = \mathcal{O}(1)$$

which might be compared to

$$\left(\frac{g_{a\psi}}{g_{aA}} \right)^{\rm KSVZ}_{\mu \sim f_a} \sim \frac{g_{\rm GUT}^2}{8\pi^2}, \qquad \left(\frac{g_{a\psi}}{g_{aA}} \right)^{\rm DFSZ}_{\mu \sim f_a} \sim \frac{8\pi^2}{g_{\rm GUT}^2}$$

It should be stressed that all of our discussions are about the axions at high scales ~ f_a , not the light axions at low energy world.

It is quite likely that some or even most of these high scale axions obtain a mass heavier than O(100) GeV by certain UV physics, e.g. by hidden gaugino condensation generating

$$\delta V_{\rm UV} \propto e^{-(\tau+i\theta)/N} \ (N>1)$$

It is also possible that some of high scale axions are eaten by U(1) gauge bosons by the Stuekelberg mechanism, thus do not appear in low energy world.

Therefore, for a complete story about light axions in string/M theory, we should carefully examine what happens to axions at scales below f_a .

In regard to this, a particularly relevant question is how some of those high scale axions remain to be light, while their moduli (saxion) partners get heavy masses, presumably heavier than 10 TeV, to avoid the cosmological moduli problem.

In view of the difficulties of stabilizing moduli, a large part of the story about light axions in string/M theory is still an open question, and it is beyond the scope of this lecture.

Yet, if some of the high scale axions from p-form gauge field survive down to the low energy world, which looks like a plausible possibility, their low energy properties can be determined by what we have found at high scales ~ f_a .

Axions in models with anomalous U(1) gauge symmetry

We have discussed axions from higher-dimensional p-form gauge field in string/M theory, whose PQ symmetries are intrinsically nonlinear.

A mechanism to convert an intrinsically nonlinear PQ symmetry to a linear PQ symmetry has been first discussed by Witten in 84.

Volume 149B, number 4,5 PHYSICS LETTERS 20 December 1984

SOME PROPERTIES OF O(32) SUPERSTRINGS

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Received 28 September 1984

Some properties of the anomaly-free O(32) superstring theory recently discovered by Green and Schwarz are discussed. With proper choice of ground state, the theory leads in four dimensions to an SU(5) theory with any desired number of standard generations (and no exotic or mirror fermions). It predicts axions and stable Nielsen–Olesen vortex lines. It can be consistently compactified only if certain topological conditions are imposed.

It involves an anomalous U(1) gauge symmetry under which certain axion from p-form gauge field is eaten by the U(1) gauge boson to form a massive vector boson.

After this massive U(1) gauge boson is integrated out, the U(1) symmetry for the remained light fields becomes a linear global PQ symmetry.

At lower energy scale, this linear PQ symmetry is spontaneously broken, giving an axion that is originated from the phase of U(1)-charged complex scalar field.

In many cases (including the SU(5) models above), the $B \operatorname{Tr} F^4$ coupling [3] leads in $d = 4$ to a coupling $\epsilon^{\alpha\beta\mu\nu} B_{\alpha\beta} Y_{\mu\nu}$ where $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$ is the field strength of a U(1) gauge boson Y. In this case (similarly to our discussion of P), B is "eaten", becoming the longitudinal component of Y_μ, and the Y symmetry becomes at low energies a global Peccei–Quinn symmetry.

Axion from p-form gauge field and the associated nonlinear global PQ symmetry:

$$\theta_1(x) = \int_{\Sigma_p} C_p(x,y) \cong \theta_1(x) + 2\pi$$

$$\tilde{U}(1)_{\rm PQ}: \;\; \theta_1(x) \;\to\; \theta_1(x) + \beta \quad (\beta = \text{real constant})$$

U(1) gauge symmetry under which the axion $\theta_1(x)$ is charged:

$$U(1)_X: \;\; X^\mu \to X^\mu + \partial_\mu \Lambda(x), \quad \theta_1(x) \to \theta_1(x) + q_1 \Lambda(x)$$

$$\phi \to e^{-iq_\phi \Lambda(x)}\phi, \quad \psi \to e^{-iq_\psi \Lambda(x)}\psi$$

$$\mathcal{L} = \frac{1}{2} f^2 D_\mu \theta_1 D^\mu \theta_1 + D_\mu \phi^* D^\mu \phi + \frac{c_{1A}}{32\pi^2}\theta_1 F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + C_{1\phi} D_\mu \theta_1 J^\mu_\phi + C_{1\psi} D_\mu \theta_1 J^\mu_\psi + \dots$$

$$\left(D_\mu \theta_1 = \partial_\mu \theta_1 - q_1 X_\mu, \;\; D_\mu \phi = (\partial_\mu + i q_\phi X_\mu)\phi, \;\; \dots\right)$$

$U(1)_X$-G_A-G_A anomaly cancellation condition:

$$q_1 c_{1A} - 2\sum_\psi q_\psi \,{\rm tr}(T_A^2(\psi)) = 0$$

In the regime where $f^2 \gg |\phi|^2$, X_μ obtains most of its mass by eating θ_1 :

$$\mathcal{L} = \frac{1}{2} f^2 D_\mu \theta_1 D^\mu \theta_1 + D_\mu \phi^* D^\mu \phi + ... \quad \Rightarrow \quad M_X^2 \simeq g_X^2 q_1^2 f^2 \quad (f^2 \gg |\phi|^2)$$

At energy scales below M_X, where X_μ and θ_1 are integrated out, the remained symmetry is the combination of $U(1)_X$ and $\tilde{U}(1)_{\rm PQ}$ under which X_μ and θ_1 do not transform:

$$U(1)_{\rm PQ} = \big(U(1)_X\big)_{\Lambda(x)=-\alpha} + \big(\tilde{U}(1)_{\rm PQ}\big)_{\beta = q_1\alpha} :$$

$$\phi \to e^{iq_\phi \alpha}\phi, \quad \psi \to e^{iq_\psi \alpha}\psi \quad (\alpha = \text{real constant})$$

This is a linear global PQ symmetry that we have assumed for instance in KSVZ and DFSZ models.

If the SM Higgs and fermions are charged under $U(1)_X$, then they are also PQ-charged, so the model corresponds to a DFSZ-type axion model.

If all SM fields are neutral under $U(1)_X$, the model is KSVZ-type.

This PQ symmetry is explicitly broken by the SM gauge anomalies and also by brane-instanton effects which can be naturally small enough:

$$\partial_\mu J^\mu_{\rm PQ} = \frac{2\sum_\psi q_\psi {\rm tr}(T_A^2(\psi))}{32\pi^2} F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + \ldots = \frac{q_1 c_{1A}}{32\pi^2} F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + \ldots$$

$$\left(\delta V \propto e^{-\tau_1}\phi^{|q_1/q_\phi|} \;\; {\rm or} \;\; e^{-\tau_1}\phi^{*|q_1/q_\phi|} \;\; {\rm for} \; q_1/q_\phi \in \mathbb{Z}\right)$$

At lower energy scale, a PQ-charged complex scalar ϕ can develop a nonzero VEV, breaking the PQ symmetry spontaneously as in KSVZ or DFSZ model:

$$\phi = \frac{v}{\sqrt{2}} e^{ia(x)/v} \quad (f_a = v \ll f)$$

One can then make an axion-dependent field redefinition to move to the GKR basis for the resulting non-linear PQ symmetry:

$$\psi \;\to\; e^{iq_\psi a(x)/q_\phi}\psi$$

$$\Rightarrow \quad U(1)_{\rm PQ}: \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + q_\phi \alpha$$

The resulting axion couplings are those of the DFSZ or KSVZ axions, depending on whether the SM fermions are U(1)$_X$-charged or U(1)$_X$-neutral:

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{c_A}{32\pi^2}\frac{a}{f_a} F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + C_\psi \frac{\partial_\mu a}{f_a}\bar{\psi}\sigma^\mu\psi + \ldots \qquad (f_a = v)$$

$$c_A = \frac{q_1 c_{1A}}{q_\phi} = \frac{2\sum_\psi q_\psi {\rm tr}(T_A^2(\psi))}{q_\phi}, \quad C_\psi = \frac{q_\psi}{q_\phi} \quad (\mu \sim f_a)$$

The key condition for converting nonlinear PQ symmetry to a linear PQ symmetry is $v^2 \ll f^2$:

$$\tilde{U}(1)_{\rm PQ}: \ \ \theta_1(x) \ \rightarrow \ \theta_1(x) + \beta \quad (\beta = \text{real constant})$$

$$\underset{v^2 \ll f^2}{\Rightarrow} \qquad U(1)_{\rm PQ} = \big(U(1)_X\big)_{\Lambda(x)=-\alpha} + \big(\tilde{U}(1)_{\rm PQ}\big)_{\beta=q_1\alpha}:$$

$$\phi \rightarrow e^{iq_\phi \alpha}\phi, \quad \psi \rightarrow e^{iq_\psi \alpha}\psi \quad (\alpha = \text{real constant})$$

In the opposite limit $v^2 \gg f^2$, the original nonlinear PQ symmetry $\tilde{U}(1)_{\rm PQ}$ remains to be the PQ symmetry in low energy effective theory and the associated axion originates mostly from the p-form gauge field.

For string compactification preserving 4-dim N=1 SUSY, it is somewhat nontrivial to have $v^2 \ll f^2$.

$$\frac{1}{2}v^2 = \langle|\phi|^2\rangle \ll \frac{1}{2}f^2 = M_P^2 \frac{\partial^2 K_0}{\partial T_1 \partial T_1^*} \quad (T_1 = \tau_1 + i\theta_1)$$

For any gauge symmetry in SUSY model, there exists the auxiliary D-term whose nonzero VEV breaks SUSY spontaneously:

$$U(1)_X : \quad X^\mu \to X^\mu + \partial_\mu \Lambda(x), \quad \theta_1(x) \to \theta_1(x) + q_1 \Lambda(x)$$

$$\phi \to e^{-iq_\phi \Lambda(x)}\phi, \quad \psi \to e^{-iq_\psi \Lambda(x)}\psi$$

$$\Rightarrow \quad D_X = \xi_{\rm FI} - q_\phi|\phi|^2 \quad \left(\xi_{\rm FI} = q_1 M_P^2 \frac{\partial K_0}{\partial T_1}\right)$$

For SUSY preserved at the compactification scale $\sim f$,

$$v^2 \ll f^2 \quad \Leftrightarrow \quad |\xi_{\rm FI}| = \left| q_1 M_P^2 \frac{\partial K_0}{\partial T_1} \right| \ll f^2 = 2M_P^2 \frac{\partial^2 K_0}{\partial T_1 \partial T_1^*}$$

$|\xi_{\rm FI}| \ll f^2$ should be achieved while keeping the moduli VEV large enough to suppress the PQ-breaking brane instanton effect.

It is usually not possible when only a single axion+modulus participates in the game. Such a case even leads to the opposite limi $|\xi_{\rm FI}| \gg f^2$.

$$K_0 = -3\ln(T+T^*) \quad (T = \tau + i\theta)$$

$$\frac{\xi_{\rm FI}}{M_P^2} = -\frac{3q}{2\tau}, \quad \frac{f^2}{M_P^2} = \frac{3}{2\tau^2} \quad \Rightarrow \quad \frac{|\xi_{\rm FI}|}{f^2} = \frac{\tau}{|q|} \gg 1$$

However, in models involving multiple axions which have **opposite sign of U(1) charges**, it is rather straightforward to achieve $|\xi_{\rm FI}| \ll f^2$.

$$U(1)_X: \quad X_\mu \to X_\mu + \partial_\mu \Lambda(x), \quad \theta^i(x) \to \theta^i(x) + q^i \Lambda(x), \quad \phi \to e^{-iq_\phi \Lambda(x)}\phi, \ \ldots$$

$$(i = 1, 2, .., h_{1,1})$$

$$K_0 = -\ln \mathcal{V}_{\rm CY} \quad \left(\mathcal{V}_{\rm CY} = \frac{1}{6} d_{ijk}\tau^i\tau^j\tau^k, \quad T^i = \tau^i + i\theta^i\right)$$

$$\frac{\xi_{\rm FI}}{M_P^2} = \sum_i q^i \frac{\partial K_0}{\partial T^i} = -\frac{\sum_i q^i \left(\sum_{jk} d_{ijk}\tau^j\tau^k\right)}{4\mathcal{V}_{\rm CY}} = 0$$

$$\text{Eigenvalues of } \frac{f_{ij}^2}{M_P^2} = 2\frac{\partial^2 K_0}{\partial T^i \partial T^{j*}} \sim \frac{1}{\tau^2}$$

For the case that axions have opposite sign of U(1) charges, there exists a ($h_{1,1}$-1)-dim hypersurface in the $h_{1,1}$-dim moduli space over which $\xi_{\rm FI} = 0$, while all axion decay constants are of the order of 10^{16} GeV.

Such a hypersurface in moduli space satisfying

$$\xi_{\rm FI} = |\phi|^2 = D_X = 0,$$

$$\text{Eigenvalues of } \; f_{ij}^2 = 2M_P^2 \frac{\partial^2 K_0}{\partial T^i \partial T^{j*}} \sim \frac{M_P^2}{\tau^2} \sim (10^{16}\,{\rm GeV})^2$$

is a supersymmetric stationary point of the scalar potential:

$$V = \frac{1}{2} g_X^2 D_X^2 + \ldots$$

Turning on SUSY breaking, this SUSY solution is shifted to a nearby local minimum where a nonzero VEV of ϕ is developed as a consequence of SUSY breaking: KC et al, arXiv:1104.3274

$$v = \langle \phi \rangle \sim (10^9 - 10^{12})\,{\rm GeV} \;\ll\; f \sim 10^{16}\,{\rm GeV}$$

Examples

Heterotic string

Bushbinder et al, arXiv:1412.8696

$$T^i = t^i + i\theta^i \quad (i = 1, 2, .., h_{1,1}) \quad \left(\theta^i(x) = \int_{\Sigma_2^{(i)}} B_2(x,y), \quad t^i = \int_{\Sigma_2^{(i)}} J_2\right)$$

$$K_0 = -\ln \mathcal{V}_{\rm CY} \quad \left(\mathcal{V}_{\rm CY} = \int_{\rm CY} J_2 \wedge J_2 \wedge J_2 = \frac{1}{6} d_{ijk} t^i t^j t^k\right)$$

Turn on $U(1)_X$ magnetic flux satisfying $\int_{\rm CY} \langle F_X \rangle \wedge J_2 \wedge J_2 = 0$

This defines a $(h_{1,1}-1)$-dim surface in $h_{1,1}$-dim moduli space, on which

$$\int_{\rm CY} \langle F_X \rangle \wedge J_2 \wedge J_2 = 0 \;\Leftrightarrow\; d_{ijk} q^i t^j t^k = 0 \;\Leftrightarrow\; \xi_{\rm FI} = M_P^2 \sum_i q^i \frac{\partial K_0}{\partial T^i} = 0$$

$$\left(q^i = \int_{\Sigma_2^{(i)}} \langle F_X \rangle\right)$$

$$\Rightarrow \qquad \xi_{\rm FI} = |\phi|^2 = D_X = 0$$

$$\text{Eigenvalues of} \quad f_{ij}^2 = 2M_P^2 \frac{\partial^2 K_0}{\partial T^i \partial T^{j*}} \sim \frac{M_P^2}{t^2} \sim (10^{16}\,{\rm GeV})^2$$

Type IIB string

Similar Type IIA model by
Honecker and Staessens, arXiv:1312.4517

$$T^i = \tau^i + i\theta^i \quad \left(\theta^i(x) = \int_{\Sigma_4^{(i)}} C_4(x,y), \quad \tau^i = \int_{\Sigma_4^{(i)}} J_2 \wedge J_2 = \frac{\partial \mathcal{V}_{\rm CY}}{\partial t^i} = \frac{1}{2} d_{ijk} t^j t^k\right)$$

$$K_0 = -\ln \mathcal{V}_{\rm CY}$$

Turn on $U(1)_X$ magnetic flux satisfying $\int_{\Sigma_4} \langle F_X \rangle \wedge J_2 = 0$

$$\int_{\Sigma_4} \langle F_X \rangle \wedge J_2 = 0 \;\Leftrightarrow\; \sum_i q^i t^i = 0 \;\Leftrightarrow\; \xi_{\rm FI} = M_P^2 \sum_i q^i \frac{\partial K_0}{\partial T^i} = 0$$

$$\left(q^i = \int_{\Sigma_4} \langle F_X \rangle \wedge \omega_2^{(i)}\right)$$

$$\Rightarrow \quad \xi_{\rm FI} = |\phi|^2 = D_X = 0$$

$$\text{Eigenvalues of } f_{ij}^2 = 2M_P^2 \frac{\partial^2 K_0}{\partial T^i \partial T^{j*}} \sim \frac{M_P^2}{\tau^2} \sim (10^{16}\,\text{GeV})^2$$

Summary

String theory generically predict multiple axions at the compactification scale, which originate from higher-dimensional p-form gauge fields.

After the 4-dim physics for SUSY breaking and moduli stabilization is taken into account, some of those axions remain to be light, e.g. have masses light than 100 GeV, while the others get a mass much heavier than 100 GeV.

The remained light axions may include the QCD axion solving the strong CP problem, which is the best motivated axion, as well as a variety of ALPs in wide range of mass, e.g. from multi-GeV to $O(H_0) \sim 10^{-32}$ eV.

Some axions from the higher-dimensional p-form gauge field can be eaten by U(1) gauge bosons at the compactification scale, while leaving as their low energy remnants the same number of linear PQ symmetries.

At lower energy scale, these linear PQ symmetries can be spontaneously broken, giving more conventional axions such as the KSVZ and DFSZ axions that originate from the phase of complex scalar fields.

As usual, the space of possible string compactifications is huge, so it can accommodate most of the possibilities that have been discussed in the context of 4-dim EFT.

Yet string theory provides an attractive rationale for global PQ symmetry well protected by quantum gravity. PQ symmetry corresponds to a locally well-defined but globally ill-defined gauge symmetry, which can be broken only by nonlocal effects in extra dimension.

In relatively simple compactification, light axions from higher-dimensional p-form gauge field have a decay constant $f_a \sim M_P/S_{ins} \sim 10^{16}$ GeV.

On the other hand, the decay constant of the axions from the phase of complex scalar field can be more model-dependent, particularly depend on the SUSY breaking scale.

Light axions that originate from higher-dimensional p-form gauge field have a characteristic pattern of low energy couplings different from those of the conventional KSVZ or DFSZ axions.

This difference might be experimentally testable if we can measure multiple axion couplings including the couplings to the photon, nucleons and electron.