Lectures on axion theory and model building

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Axions (or axion-like particles) are light pseudo-scalar bosons described by a periodic (angular) field variable. There are many reasons for why we are interested in axions, including the followings:

1) A specific type of axion called the QCD axion solves the strong CP problem which is one of the major naturalness problems of the SM of particle physics.

   Unlike the other naturalness problems, i.e. the weak scale hierarchy problem and the cosmological constant problem, the strong CP problem can not be explained by the anthropic selection in multiverse, thus it strongly requires a physical explanation.

2) Light axions may constitute the dark sector in our Universe, including the dark matter, dark radiation, or even the dark energy.

   They also have many interesting astrophysical, cosmological, and laboratory implications.

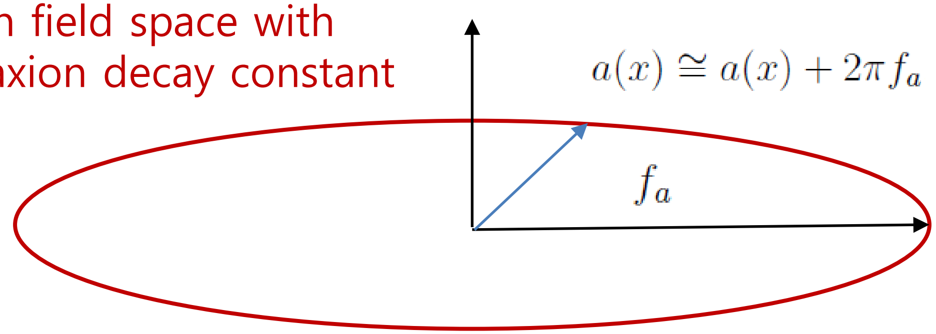
3) Axions generically appear in 4D effective theory of string/M theory which is considered to be the best candidate for a theory incorporating both particle physics and quantum gravity.

Axion is a pseudo-Nambu-Goldstone boson associated with a non-linearly realized (or spontaneously broken) approximate global U(1) symmetry.

Peccei-Quinn (PQ) symmetry:

$$U(1)_{\rm PQ}: \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha, \ \ldots \quad (\alpha = \text{constant})$$

Circular axion field space with
radius f_a = axion decay constant



Generically axions can have a potential of the form

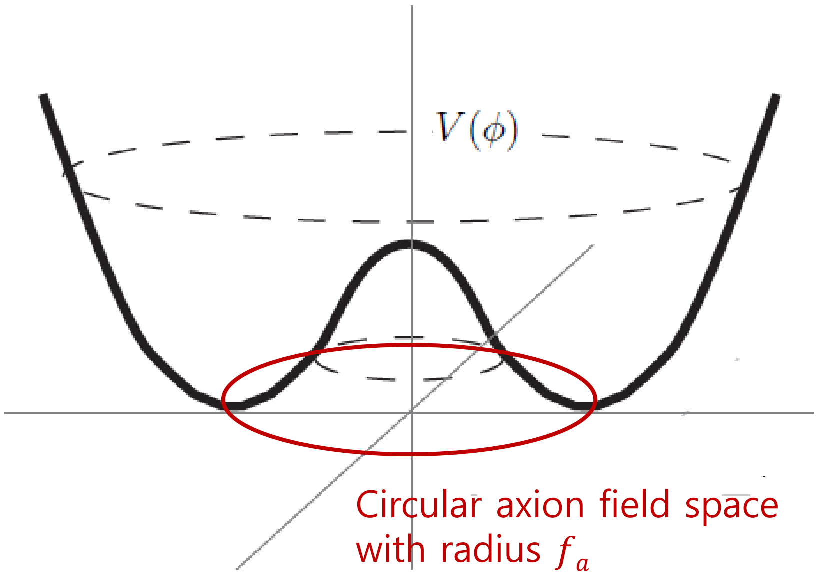
$$V(a) = \sum_n \Lambda_n^4 \cos\left(n\frac{a}{f_a} + \delta_n\right)$$

Any nontrivial axion potential means a change of energy density under the PQ transformation of the axion field, so it is a consequence of the explicit breaking of the PQ symmetry.

(nearly) exact $U(1)_{\rm PQ}$ $\Rightarrow$ (nearly) massless axion

In modern viewpoint, when quantum gravity is included, global symmetry can not be an exact symmetry. It is still a plausible possibility that the PQ symmetry is nearly exact, so the associated axion can be light as much as you wish.

Angular axion may originate from the phase of a (elementary or composite) complex scalar field ϕ developing a nonzero VEV.



$$\phi = \frac{1}{\sqrt{2}}\big(f_a + \rho(x)\big)e^{ia(x)/f_a}$$

$$\big(f_a = \sqrt{2}\langle\phi\rangle\big)$$

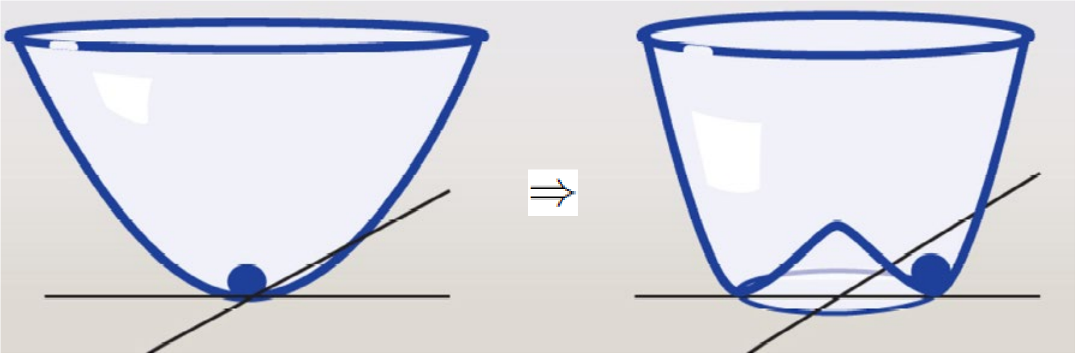
In such models, nonlinear PQ symmetry appears as a low energy consequence of the spontaneous breakdown of a linearly realized PQ symmetry:

$$[U(1)_{\rm PQ}]_{\rm linear}: \;\; \phi \to e^{i\alpha}\phi, \;\; \dots \text{ defined on the entire complex plane of } \phi$$

$$\Rightarrow \quad [U(1)_{\rm PQ}]_{\rm nonlinear}: \;\; \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha, \;\; \dots$$

defined on the circular axion field space with $|\phi| = f_a/\sqrt{2}$

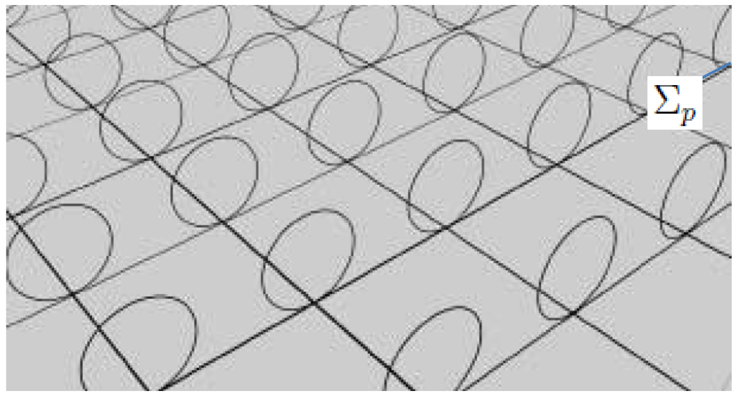
As the underlying theory can describe the entire sub-Planckian region in the complex plane of ϕ , including $\phi = 0$, such models allow a cosmological phase transition from the linear PQ phase with $\langle\phi\rangle = 0$ to a non-linear phase with $\langle\phi\rangle \neq 0$:



From an article by A. Cho, Science 2012.

Alternatively, angular axion may originate from a higher-dimensional (p-form) gauge field, which is most naturally realized in string/M theory:

$$\frac{a(x)}{f_a} = \oint dy\, A_5(x,y) \quad \left(\int_{\Sigma_p} dy^{m_1}..dy^{m_p} A_{[m_1..m_p]} \right)$$



Closed p-dimensional surface in extra dimension

In this case, the nonlinear PQ symmetry corresponds to a locally well-defined, but globally ill-defined higher-dimensional gauge symmetry.

$$U(1)_{\rm PQ}: \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha \quad \cong \quad \begin{array}{l} A_5(x,y) \to A_5(x,y) + \partial_y \Lambda(y) \\ \text{with} \quad \oint_{y \cong y+2\pi R} dy\, \partial_y \Lambda(y) = \Lambda(y+2\pi R) - \Lambda(y) = \alpha \end{array}$$

PQ symmetry is broken only by non-local effects which can be naturally small.

Discrete PQ symmetry for $\alpha = 2n\pi \;\; (n \in \mathbb{Z})$ is a genuine higher-dimensional gauge symmetry, which ensures the angular nature of the axion field:

$$e^{iq\Lambda(y)} = e^{iq\left[\Lambda(y+2\pi R)+2n\pi\right]} \quad (q, n \in \mathbb{Z}) \quad \Rightarrow \quad \frac{a(x)}{f_a} \cong \frac{a(x)}{f_a} + 2n\pi$$

PQ symmetry is intrinsically a nonlinear symmetry, so it does not admit a cosmological phase transition from the linear phase to nonlinear phase.

$f_a \to 0$ corresponds to the limit where the volume of the extra dimension becomes infinity, in which the theory becomes higher-dimensional theory with a p-form gauge field:

$$\frac{a(x)}{f_a} = \oint dy\, A_5(x,y) \quad \left(\int_{\Sigma_p} dy^{m_1}..dy^{m_p} A_{[m_1..m_p]} \right)$$

In this lecture, we will study both type of axions while focusing on the low energy couplings of those axions to the SM gauge and matter fields.

Plan

Lecture 1:

PQ solution of the strong CP problem
Low energy axion couplings
Axion models with a linearly realized PQ symmetry

Lecture 2:

Axions from a 5D gauge field
Axions from p-form gauge field in string/M theory
Axions in string models with anomalous U(1) gauge symmetry

Useful recent reviews or lectures:

A. Hook, arXiv:1812.02669

L. Di Luzio, M. Giannoti, E. Nardi, L. Visinelli, arXiv:2003.01100

KC, S. H. Im, C. S. Shin, arXiv:2012.05029

M. Reece, arXiv: 2304.08512

Strong CP problem

The SM describes the known particle physics up to the energy scale ~ few TeV.

$$\mathcal{L}_{\rm SM} = -\frac{1}{4g_s^2}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4g_2^2}W^{i\mu\nu}W^i_{\mu\nu} - \frac{1}{4g_1^2}B^{\mu\nu}B_{\mu\nu} + D_\mu H^\dagger D^\mu H$$
$$+ \sum_{\psi=q_i,u_i^c,d_i^c,\ell_i,e_i^c} i\bar{\psi}\sigma^\mu D_\mu\psi + \frac{\theta_{\rm QCD}}{32\pi^2}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$$
$$+ \Big((y_u)_{ij}q_i u_j^c H^* + (y_d)_{ij}q_i d_j^c H + (y_e)_{ij}\ell_i e_j^c H + \text{h.c}\Big) - \lambda\Big(H^\dagger H - \frac{1}{2}v^2\Big)^2$$

Two CP-violating angles in the SM:

$\delta_{\rm KM} = \arg\cdot\det([y_u y_u^\dagger, y_d y_d^\dagger])$ (CPV in the weak interactions)

$\bar{\theta} = \theta_{\rm QCD} + \arg\cdot\det(y_u y_d)$ (CPV in the strong interactions)

Similar angles $\theta_{1,2}$ for the EW gauge bosons are irrelevant since θ_2 can be rotated away by $U(1)_B$ or $U(1)_L$ (in any case θ_2-dependence is suppressed by the extremely small $e^{-8\pi^2/g_2^2}$) and the remained θ_1-dependence appears only in unusual situations involving a magnetic monopole or topologically nontrivial spacetime, e.t.c.

Observed CPV in weak interactions & the absence of CPV in strong interactions imply that these two angles are so different, causing a fine tuning problem:

$$\delta_{\rm KM} \sim 1 \qquad |\bar{\theta}| < 10^{-10} \qquad \Big(d_n \sim \frac{e}{m_n}\frac{m_u}{m_n}\bar{\theta} < 1.8\times10^{-26}\, e\cdot{\rm cm}\Big)$$

Since $\bar{\theta}$ is essentially the coefficient of $G\tilde{G} = \partial_\mu K^\mu$ which is a total derivative, $\bar{\theta}$-dependence of physical amplitudes appears only through the gluon field configurations having a nonzero surface integral of K^μ at infinity, e.g. instantons, whose effects $\propto e^{-8\pi^2/g_s^2(\mu)}$ in the weak QCD coupling limit.

As a consequence, $\bar{\theta}$ is important mostly at low energy scales where

$$g_s^2(\mu)/8\pi^2 \sim 1 \quad (\mu \lesssim 1\,\mathrm{GeV})$$

Therefore, we can limit the discussion to the low energy QCD around 1 GeV, which is defined as the low energy effective theory of the SM:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g_s^2} G^{a\mu\nu} G^a_{\mu\nu} - i\bar{q}\gamma^\mu D_\mu q - \frac{\bar{\theta}}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} - \left(\bar{q}_L M q_R + \mathrm{h.c}\right)$$

$$q_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Real and positive quark masses

Peccei-Quinn solution to the strong CP problem

Introduce a global U(1) symmetry called the PQ symmetry which is

(1) non-linearly realized (or spontaneously broken) at least in low energy limit, with the associated Nambu-Goldstone boson called "the QCD axion",

(2) explicitly broken dominantly by the QCD anomaly   Peccei, Quinn '77

For non-linear PQ symmetry, one can always choose a field basis for which only the axion transforms under $U(1)_{PQ}$, while all other fields are invariant:

$$U(1)_{\rm PQ}: \quad \frac{a(x)}{f_a} \rightarrow \frac{a(x)}{f_a} + \alpha, \quad \Phi \rightarrow \Phi \quad (\alpha = \text{constant})$$

Georgi-Kaplan-Randall (GKR) basis '86

If any matter field transforms under $U(1)_{PQ}$, one can make an appropriate axion-dependent field redefinition to make the redefined fields are all PQ-invariant.

Generic effective axion lagrangian in the GKR basis at scales above the EW scale, but of course below f_a, with the couplings defined for the angular field $\theta(x) = a(x)/f_a$:

PQ-conserving

PQ-breaking by the SM gauge anomalies

$$\mathcal{L}_{\rm axion} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\partial_\mu a}{f_a}\left(C_\psi J^\mu_\psi + C_\phi J^\mu_\phi\right) + \frac{c_A}{32\pi^2}\frac{a}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + \delta\mathcal{L}_{\rm axion}$$

additional axion interactions such as a bare axion potential and higher-order interactions

$$J^\mu_\psi = \bar{\psi}\sigma^\mu\psi, \quad J^\mu_\phi = i(\phi^* D_\mu \phi - {\rm h.c.}), \quad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}$$

$$c_A = (c_G, c_W, c_B) \quad \text{for} \quad F^A_{\mu\nu} = (G^a_{\mu\nu}, W^i_{\mu\nu}, B_{\mu\nu})$$

Integer or rational number to be compatible with $a(x) \cong a(x) + 2\pi f_a$

$$\delta\mathcal{L}_{\rm axion} = -\delta V(a) + \ldots$$

Bare axion potential induced by PQ-breaking other than those by the SM gauge anomalies

$$U(1)_{\rm PQ}: \quad \frac{a(x)}{f_a} \to \frac{a(x)}{f_a} + \alpha \quad (\alpha = \text{constant})$$

$$J^\mu_{\rm PQ} = \sum_\Phi \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\frac{\delta\Phi}{\delta\alpha} = f_a\partial^\mu a + C_\psi J^\mu_\psi + C_\phi J^\mu_\phi + f_a\frac{\partial\delta\mathcal{L}_{\rm axion}}{\partial(\partial_\mu a)}$$

$$\partial_\mu J^\mu_{\rm PQ} = \sum_\Phi \frac{\partial\mathcal{L}}{\partial\Phi}\frac{\delta\Phi}{\delta\alpha} = \sum_{A=G,W,B}\frac{c_A}{32\pi^2}F^{A\mu\nu}F^A_{\mu\nu} + f_a\frac{\partial\delta\mathcal{L}_{\rm axion}}{\partial a}$$

Different axions depending on the dominant PQ breaking

(PQ breaking by $c_{W,B}$ (EW anomaly) can be safely ignored.)

QCD axion:

PQ-breaking by $\frac{c_G}{32\pi^2}\frac{a}{f_a}G\tilde{G}$ (QCD anomaly) highly dominates over other breakings:

Axion potential induced by the QCD anomaly

$$V(a) = V_{\rm QCD}(a) + \delta V(a) \quad \text{with} \quad \delta V(a) < 10^{-10}\, V_{\rm QCD}(a) \sim 10^{-10} m_\pi^2 f_\pi^2$$

$$\Rightarrow \quad |\bar{\theta}_{\rm eff}| < 10^{-10} \quad \left(\bar{\theta}_{\rm eff} = \bar{\theta} + c_G \frac{\langle a \rangle}{f_a}\right), \quad m_a \sim \frac{m_\pi f_\pi}{f_a}$$

Heavy axion-like particle (ALP):

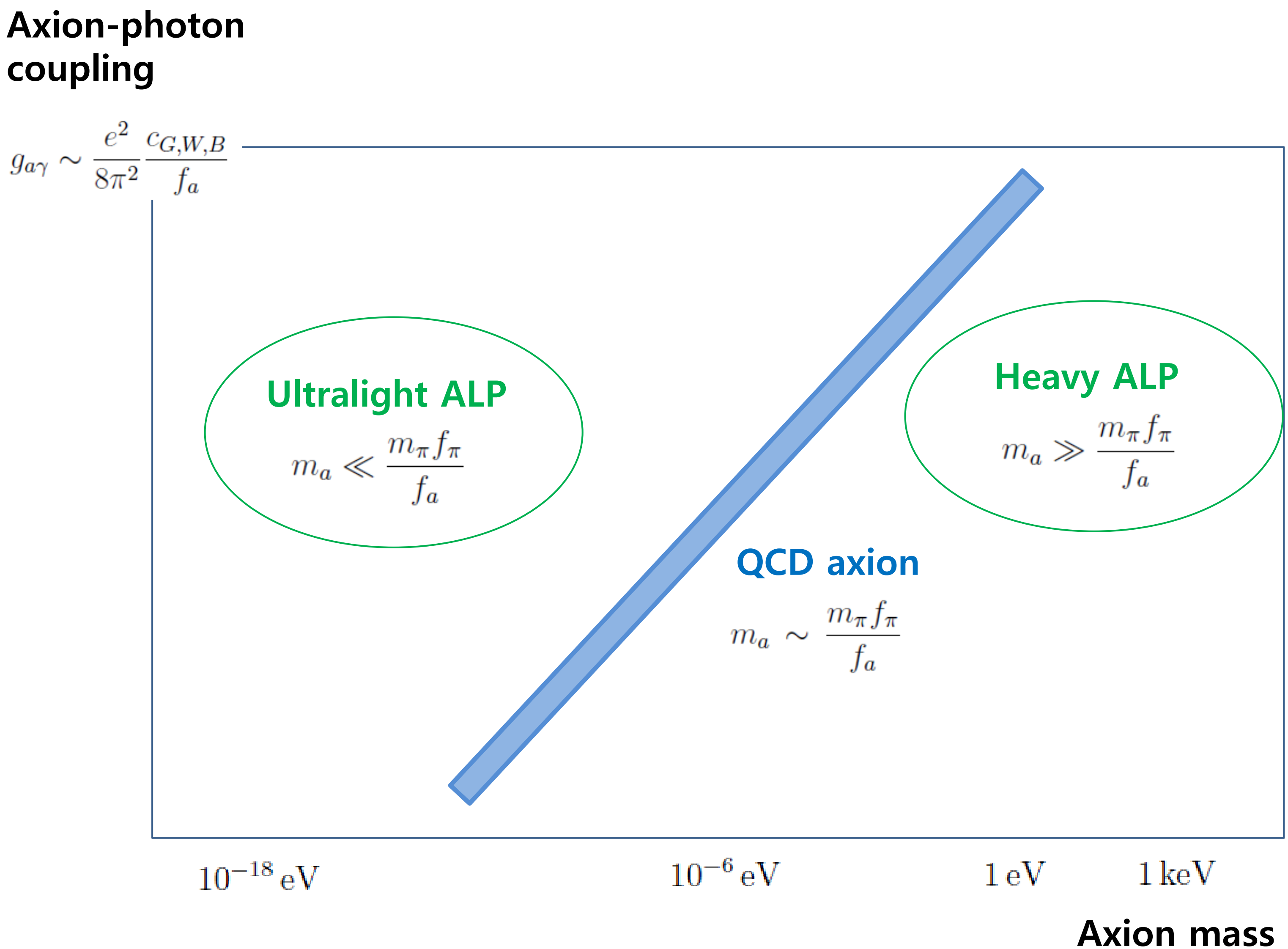
$$\delta V(a) \gg m_\pi^2 f_\pi^2 \quad \Rightarrow \quad m_a \gg \frac{m_\pi f_\pi}{f_a}$$

Ultralight ALP:

$c_G = 0$, so there is no $V_{\rm QCD}(a)$, and $\delta V(a) \ll m_\pi^2 f_\pi^2 \quad \Rightarrow \quad m_a \ll \frac{m_\pi f_\pi}{f_a}$

If any of $c_{G,W,B}$ is nonzero, these 3-type of axions all have the coupling to the photon given by

$$g_{a\gamma} \sim \frac{e^2}{8\pi^2}\frac{c_{G,W,B}}{f_a}$$



Axion-photon coupling
$g_{a\gamma} \sim \frac{e^2}{8\pi^2}\frac{c_{G,W,B}}{f_a}$
Ultralight ALP
$m_a \ll \frac{m_\pi f_\pi}{f_a}$
Heavy ALP
$m_a \gg \frac{m_\pi f_\pi}{f_a}$
QCD axion
$m_a \sim \frac{m_\pi f_\pi}{f_a}$
$10^{-18}\,\mathrm{eV}$
$10^{-6}\,\mathrm{eV}$
$1\,\mathrm{eV}$
$1\,\mathrm{keV}$
Axion mass

Some features of the GKR basis

"PQ-conserving" and "PQ-breaking" are manifestly distinguished from each other, i.e. derivative and non-derivative axion couplings.

Simple correspondence between "the axion-couplings" and "the PQ current and its divergence". One consequence is that the axion-gauge boson couplings are quantized.

Usually the GKR basis is more convenient for studying axion physics at lower energy scales.

It is straightforward to incorporate the RG running of axion couplings and the modification of couplings coming from integrating out heavy fields.

Most suitable for describing the axions originating from p-form gauge field in higher-dimensional theory such as string/M theory

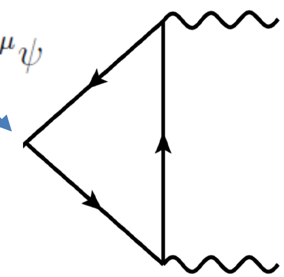
Symmetry breaking by anomalies

$$U(1)_\psi : \quad \psi \to \psi' = e^{i\alpha}\psi \quad (\psi = \text{gauge-charged chiral fermion})$$

Matrix element of the Noether current in background gauge field:

$$\partial_\mu J^\mu_\psi = \frac{1}{16\pi^2}\sum_A \mathrm{tr}(T_A^2(\psi))F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$J^\mu_\psi = \bar{\psi}\sigma^\mu\psi$



Variation of the (Euclidean) path integral measure in back ground gauge field:

$$\int [\mathcal{D}\psi'][\mathcal{D}\bar{\psi}']_{A_\mu} = e^{i\int d^4x_E\,\frac{\alpha}{16\pi^2}\mathrm{tr}(T_A^2(\psi))F^{A\mu\nu}\tilde{F}^A_{\mu\nu}} \int [\mathcal{D}\psi][\mathcal{D}\bar{\psi}]_{A_\mu}$$

$$\frac{\delta\mathcal{L}_{\mathrm{eff}}}{\delta\alpha} = \partial_\mu J^\mu_\psi = \frac{1}{16\pi^2}\sum_A \mathrm{tr}(T_A^2(\psi))F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

PQ-breaking by the fermion anomaly can be converted to the PQ breaking by the axion coupling $aF^A\tilde{F}^A$ through an axion-dependent change of the fermion field variables.

$$\mathcal{L}(\psi_n, \bar{\psi}_n) = i\bar{\psi}_n\sigma^\mu D_\mu\psi_n - \left(\frac{1}{2}M_{nm}e^{-i(q_n+q_m)a(x)/f_a}\psi_n\psi_m + \mathrm{h.c}\right)$$

$$U(1)_{\mathrm{PQ}}: \quad \frac{a}{f_a} \to \frac{a}{f_a} + \alpha, \quad \psi_n \to e^{iq_n\alpha}\psi_n$$

In this description, PQ-breaking is entirely from the anomalous variation of the fermion path integral measure.

Consider the same theory, but defined in terms of $\psi'_n = e^{-ik_n a(x)/f_a}\psi_n$.

$$\int \prod_n [\mathcal{D}\psi_n][\mathcal{D}\bar{\psi}_n] e^{-\int d^4x_E\, \mathcal{L}_E(\psi_n, \bar{\psi}_n)}$$
$$= \int \prod_n [\mathcal{D}\psi'_n][\mathcal{D}\bar{\psi}'_n] e^{-\int d^4x_E\, \mathcal{L}'_E(\psi'_n, \bar{\psi}'_n)}$$

$$\mathcal{L}'(\psi'_n, \bar{\psi}'_n) = i\bar{\psi}'_n\sigma^\mu D_\mu\psi'_n - \left(\frac{1}{2}M_{nm}e^{-i(q'_n+q'_m)a(x)/f_a}\psi'_n\psi'_m + \mathrm{h.c}\right) \quad (q'_n = q_n - k_n)$$
$$+ k_n\frac{\partial_\mu a}{f_a}\bar{\psi}'_n\sigma^\mu\psi'_n + \frac{1}{16\pi^2}k_n \mathrm{tr}(T_A^2(\psi_n))\frac{a}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$\mathcal{L}(\psi_n, \bar{\psi}_n) = i\bar{\psi}_n \sigma^\mu D_\mu \psi_n - \left(\frac{1}{2} M_{nm} e^{-i(q_n+q_m)a(x)/f_a} \psi_n \psi_m + \mathrm{h.c}\right)$$

$$U(1)_{\mathrm{PQ}}: \quad \frac{a}{f_a} \to \frac{a}{f_a} + \alpha, \quad \psi_n \to e^{iq_n \alpha} \psi_n$$

$$J^\mu_{\mathrm{PQ}} = f_a \partial^\mu a + q_n \bar{\psi}_n \sigma^\mu \psi_n, \quad \partial_\mu J^\mu_{\mathrm{PQ}} = \frac{1}{16\pi^2} q_n \mathrm{tr}(T_A^2(\psi_n)) F^{A\mu\nu} \tilde{F}^A_{\mu\nu}$$

$$\mathcal{L}'(\psi'_n, \bar{\psi}'_n) = i\bar{\psi}'_n \sigma^\mu D_\mu \psi'_n - \left(\frac{1}{2} M_{nm} e^{-i(q'_n+q'_m)a(x)/f_a} \psi'_n \psi'_m + \mathrm{h.c}\right)$$
$$+ k_n \frac{\partial_\mu a}{f_a} \bar{\psi}'_n \sigma^\mu \psi'_n + \frac{1}{16\pi^2} k_n \mathrm{tr}(T_A^2(\psi_n)) \frac{a}{f_a} F^{A\mu\nu} \tilde{F}^A_{\mu\nu}$$

$$U(1)_{\mathrm{PQ}}: \quad \frac{a}{f_a} \to \frac{a}{f_a} + \alpha, \quad \psi'_n \to e^{iq'_n \alpha} \psi'_n \quad (q'_n = q_n - k_n)$$

$$J^\mu_{\mathrm{PQ}} = f_a \partial^\mu a + (k_n + q'_n) \bar{\psi}'_n \sigma^\mu \psi'_n = f_a \partial^\mu a + q_n \bar{\psi}'_n \sigma^\mu \psi'_n,$$
$$\partial_\mu J^\mu_{\mathrm{PQ}} = \frac{1}{16\pi^2} (k_n + q'_n) \mathrm{tr}(T_A^2(\psi_n)) F^{A\mu\nu} \tilde{F}^A_{\mu\nu} = \frac{1}{16\pi^2} q_n \mathrm{tr}(T_A^2(\psi_n)) F^{A\mu\nu} \tilde{F}^A_{\mu\nu}$$

The form of the lagrangian, i.e. the lagrangian couplings, severely depends on the used field basis, while the Noether current and its anomalous divergence are more directly related to the observables, therefore they are invariant under this change of the field basis.

GKR basis : $k_n = q_n$

$$\mathcal{L}_{\rm GKR} = i\bar{\psi}_n \sigma^\mu D_\mu \psi_n - \Big(\frac{1}{2} M_{nm}\psi_n\psi_m + {\rm h.c}\Big) + q_n \frac{\partial_\mu a}{f_a}\bar{\psi}_I \sigma^\mu \psi_I + \frac{1}{16\pi^2} q_n {\rm tr}(T_A^2(\psi_n))\frac{a}{f_a} F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

In the GKR basis, PQ-breaking is entirely from the axion coupling $aF^A\tilde{F}^A$.

Axion potential $V_{\rm QCD}(a)$ induced by $\frac{c_G}{32\pi^2}\frac{a(x)}{f_a}G\tilde{G}$

Euclidean QCD partition function for a constant axion background:

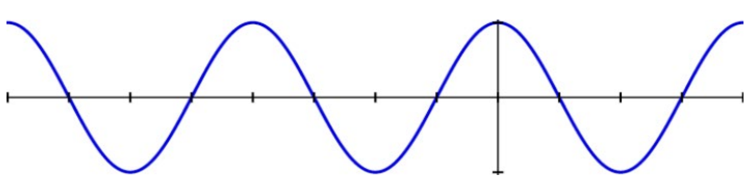
$$\exp\left(-\int d^4x_E\, V_{\rm QCD}(a)\right)$$
$$= \int [\mathcal{D}G][\mathcal{D}q][\mathcal{D}\bar{q}]\ \exp\left[-\int d^4x_E\, \mathcal{L}^{(E)}_{\rm QCD}(G, q;\, M,\, \bar{\theta} + c_G\frac{a}{f_a})\right]$$
$$= \int [\mathcal{D}G]\, {\rm Det}(D_E + M)\, e^{-\int d^4x_E\, \frac{1}{4g_3^2}GG}\, e^{i\frac{1}{32\pi^2}\int d^4x_E\, \left(\bar{\theta}+c_G\frac{a}{f_a}\right)G\tilde{G}}$$

$${\rm Det}(D_E + M) = \prod_{\lambda_n}(M - i\lambda_n) = \left(\prod_{\lambda_n=0} M\right)\left(\prod_{\lambda_n>0}(M^2+\lambda_n^2)\right) > 0$$

Vafa, Witten '84

$\Rightarrow$ $V_{\rm QCD}(a)$ has the global minima at

$$\bar{\theta}_{\rm eff} = \bar{\theta} + c_G\langle a\rangle/f_a = 2n\pi \quad (n \in \mathbb{Z})$$



$$\theta_{\rm eff} = \bar{\theta} + c_G\frac{a}{f_a} \quad (c_G = 3)$$

PQ mechanism and PQ quality problem

$$V_{\rm axion} \;=\; V_{\rm QCD}(a) + \delta V_{\rm CPV} + \delta V_{\rm UV}$$

$V_{\rm QCD}(a)$ = axion potential generated by low energy QCD dynamics through

$$\frac{1}{32\pi^2}\Big(\bar\theta + c_G\frac{a}{f_a}\Big)G\tilde G$$

$\Rightarrow$ global minima at $\bar\theta_{\rm eff} = \bar\theta + c_G\langle a\rangle/f_a = 2n\pi \;\; (n\in\mathbb{Z})$

So, if there is no additional axion potential, QCD becomes CP-conserving regardless of the value of $\bar\theta$. (PQ mechanism '77)

However, even in the absence of additional PQ breaking, when combined with CP-violating SM weak interactions or BSM physics, $\frac{1}{32\pi^2}\Big(\bar\theta + c_G\frac{a}{f_a}\Big)G\tilde G$ generates an additional axion potential

$$\delta V_{\rm CPV} = \big(\epsilon_{\rm SM}+\epsilon_{\rm BSM}\big)m_\pi^2 f_\pi^2 \sin\left(\bar\theta + c_G\frac{a}{f_a}\right)$$

$$\Rightarrow \qquad \bar\theta_{\rm eff} = \bar\theta + c_G\frac{\langle a\rangle}{f_a} \sim \epsilon_{\rm SM}+\epsilon_{\rm BSM} \quad \big(V_{\rm QCD}(a)\sim m_\pi^2 f_\pi^2\big)$$

$$\big(\epsilon_{\rm SM}\sim 10^{-19}\sin\delta_{\rm KM}, \quad \epsilon_{\rm BSM} < 10^{-10} \;\text{ for BSM scale} > \text{multi-TeV}\big)$$

PQ quality problem

In modern viewpoint, additional PQ breaking, in particular those from quantum gravity, appears to be inevitable.

Quantum gravity instanton, axionic wormhole, …

$$\delta V_{\rm UV} \ni -e^{-S_{\rm ins}}\Lambda^4 \cos\left(\frac{a}{f_a}+\delta\right)$$

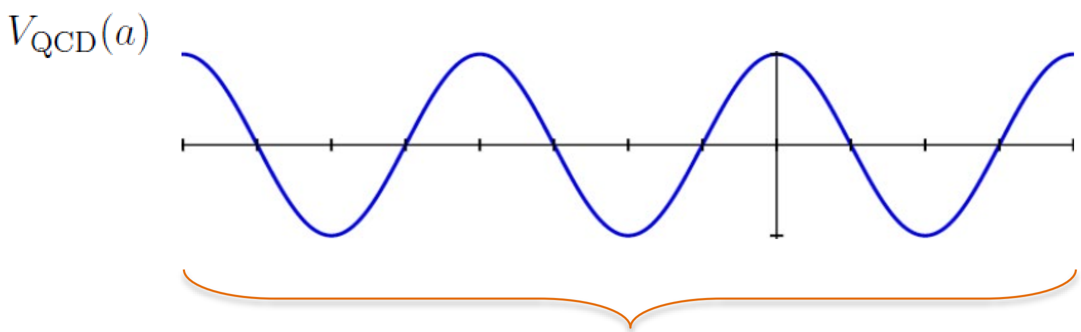
$$\Rightarrow \quad \bar{\theta}_{\rm eff} = \bar{\theta} + c_G\frac{\langle a\rangle}{f_a} \sim \frac{e^{-S_{\rm ins}}\Lambda^4}{m_\pi^2 f_\pi^2}\sin\left(\bar{\theta}-\delta\right) < 10^{-10}$$

$$\Lambda^4 \sim m_{3/2}M_P^3 \quad \Rightarrow \quad e^{-S_{\rm ins}} < 10^{-64}\left(\frac{100\,{\rm TeV}}{m_{3/2}}\right)$$

Additional PQ-breaking, including those from quantum gravity, is required to be highly suppressed (PQ-quality problem), which is an implicit assumption involved in the axion solution of the strong CP problem.

Apparently, this issue can not be addressed within the EFT framework, while string theory can provide a theoretical framework to examine the PQ quality problem more concretely.

Axion solution to the strong CP problem predicts an axion string with a tension $\mu_s \sim f_a^2$ attached by the domain walls with a surface energy density $\mu_w \sim m_\pi^2 f_\pi^2 / m_a$, which have interesting cosmological implications:

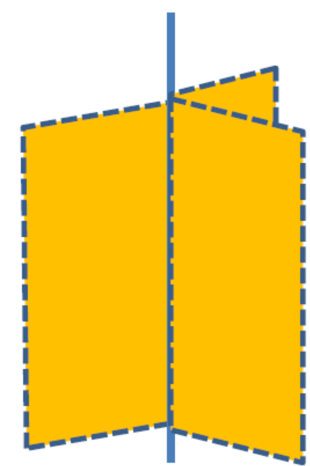


$\theta_{\rm eff} = \bar{\theta} + c_G \frac{a}{f_a}$ $(c_G = 3)$

$2\pi f_a$ = axion field range

$c_G = N_{\rm DW}$ = Number of discrete degenerate vacua

= Number of axion domain walls attached to an axion string around which $a \to a + 2\pi f_a$



Axion potential and axion-photon coupling induced by $\frac{c_G}{32\pi^2}\frac{a(x)}{f_a}G\tilde{G}$ (at leading order in chiral perturbation theory)

Redefine the axion as

$$U(1)_{\rm PQ}: \quad \bar{\theta} + c_G\frac{a}{f_a} \;\rightarrow\; c_G\frac{a}{f_a} \quad \Rightarrow \quad \bar{\theta}_{\rm eff} = c_G\frac{\langle a\rangle}{f_a}$$

Light quark-antiquark condensation in background axion field:

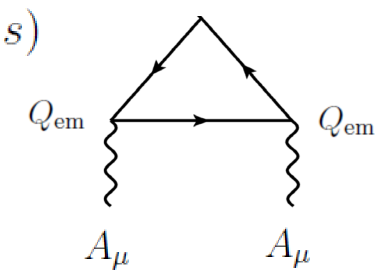
$$\langle \bar{q}_L q_R\rangle_{\rm axion} \simeq \frac{m_\pi^2 f_\pi^2}{(m_u+m_d)}e^{i\phi_q(a)} \quad (q=u,d,s)$$

$$\left(\pi^0 \propto \phi_u - \phi_d, \;\; \eta \propto \phi_u+\phi_d-2\phi_s, \;\; \eta' \propto \phi_u+\phi_d+\phi_s\right)$$

Axial U(1) transformations of the light quarks and axion in QCD with a background axion field:

$$U(1)_q: \quad q_{L,R} \rightarrow e^{\mp i\alpha_q}q_{L,R}, \quad c_G\frac{a}{f_a} \rightarrow c_G\frac{a}{f_a} - 2\alpha_q \quad (q=u,d,s)$$

$$\partial_\mu J^\mu_q = \left(\frac{1}{16\pi^2}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} - \frac{1}{16\pi^2}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}\right) + \frac{N_c}{8\pi^2}Q^2_{\rm em}F^{\mu\nu}\tilde{F}_{\mu\nu} + \ldots$$



In the effective theory of mesons in the same background axion field,

$$U(1)_q: \quad \phi_q \to \phi_q + 2\alpha_q, \quad c_G\frac{a}{f_a} \to c_G\frac{a}{f_a} - 2\alpha_q$$

$$\frac{\delta\mathcal{L}_{\rm eff}(\phi_q)}{\delta\alpha_q} = \frac{N_c}{32\pi^2}Q^2_{\rm em}F^{\mu\nu}\tilde{F}_{\mu\nu} + \ldots$$

Meson potential in the chiral limit M=0, which explains why $m^2_{\eta'} \gg m^2_{\pi,\eta}$

Part of the WZW term which explains $\pi^0, \eta, \eta' \to 2\gamma$

$$\Rightarrow \quad \mathcal{L}_{\rm eff}(\phi_q) \ni -V_0\big(\phi_u + \phi_d + \phi_s + c_G\frac{a}{f_a}\big) + \frac{N_c e^2}{16\pi^2}\Big(\frac{4}{9}\phi_u + \frac{1}{9}\phi_d + \frac{1}{9}\phi_s\Big)F^{\mu\nu}\tilde{F}_{\mu\nu}$$

$$= -\frac{1}{2}m^2_{\eta'}\eta'^2 + \frac{N_c e^2}{16\pi^2}\left[\frac{\partial}{\partial a}\Big(\frac{4}{9}\phi_u + \frac{1}{9}\phi_d + \frac{1}{9}\phi_s\Big)\right]_{a=0} a(x)F^{\mu\nu}\tilde{F}_{\mu\nu} + \ldots$$

Axion-dependence ($\bar{\theta}_{\rm eff}$-dependence) of the meson VEVs is essentially due to $V_0\big(\phi_u + \phi_d + \phi_s + c_G\frac{a}{f_a}\big)$ which solves the QCD $U(1)_A$ problem by providing $m^2_{\eta'} \gg m^2_{\pi,\eta}$.

$$V(\phi_q) = V_0\Big(c_G \frac{a}{f_a} + \sum_q \phi_q\Big) - \frac{2m_\pi^2 f_\pi^2}{(m_u+m_d)} \sum_q m_q \cos\phi_q$$

$$\frac{\partial V(\phi_q)}{\partial \phi_q} = 0 \quad \Rightarrow \quad m_u \sin\phi_u = m_d \sin\phi_d = m_s \sin\phi_s, \quad \phi_u + \phi_d + \phi_s + c_G \frac{a}{f_a} = 0$$

For $m_{u,d} \ll m_s$,

$$\sin\phi_u = -\frac{m_d \sin(c_G a/f_a)}{\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(c_G a/f_a)}}$$

$$\sin\phi_d = -\frac{m_u \sin(c_G a/f_a)}{\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(c_G a/f_a)}}$$

$$\sin\phi_s = -\frac{m_u m_d \sin(c_G a/f_a)}{m_s\sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(c_G a/f_a)}}$$

Di Vecchia, Veneziano '80; Witten '80

$$V_{\rm QCD}(a) = V(\phi_q = \phi_q(a)) \simeq -\frac{f_\pi^2 m_\pi^2}{(m_u+m_d)} \sqrt{m_u^2 + m_d^2 + 2m_u m_d \cos(c_G a/f_a)}$$

Axion mass and axion-photon coupling induced by $\frac{c_G}{32\pi^2}\frac{a(x)}{f_a}G\tilde{G}$

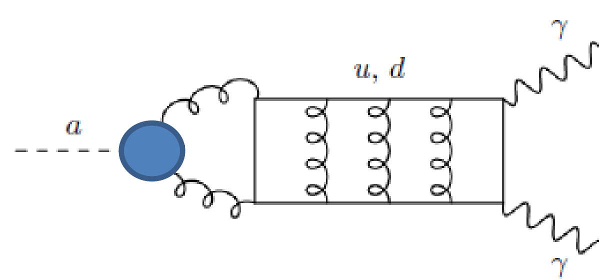
(QCD anomaly)

Axion mass:

$$m_a = f_\pi m_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{c_G}{f_a} \simeq 5.7 \times \left(\frac{10^{12}\,\mathrm{GeV}}{f_a/c_G}\right)\mu\mathrm{eV}$$

Axion-photon coupling:

$$-\frac{e^2}{48\pi^2}\left(\frac{4m_d + m_u}{m_u + m_d}\right)\frac{c_G a(x)}{f_a}F^{\mu\nu}\tilde{F}_{\mu\nu}$$



Axion mass vs axion couplings

$$\mathcal{L}_{\rm axion} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 + \frac{1}{2} g_{a\gamma} a(x) \vec{E}\cdot\vec{B} + g_{aN}\partial_\mu a \bar{N}\gamma^\mu\gamma_5 N + \ldots \quad (N = n, p)$$

<u>QCD axion</u>: Plausible range of the coupling to mass ratio can be identified.

$$m_a = f_\pi m_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{c_G}{f_a} \simeq 5.7 \times \left(\frac{10^{12}\,{\rm GeV}}{f_a/c_G}\right)\mu{\rm eV}$$

From the axion-W/B coupling

$$\frac{g_{a\gamma}}{m_a} = -\frac{e^2}{8\pi^2}\frac{1}{f_\pi m_\pi}\left(\frac{2(m_u+4m_d)}{3\sqrt{m_u m_d}} - \frac{m_u+m_d}{\sqrt{m_u m_d}}\frac{c_W + c_B}{c_G}\right) = \mathcal{O}\left(\frac{e^2}{8\pi^2}\frac{1}{f_\pi m_\pi}\right)$$

$$\frac{g_{aN}}{m_a} = \mathcal{O}\left(\frac{1}{f_\pi m_\pi}\right)$$

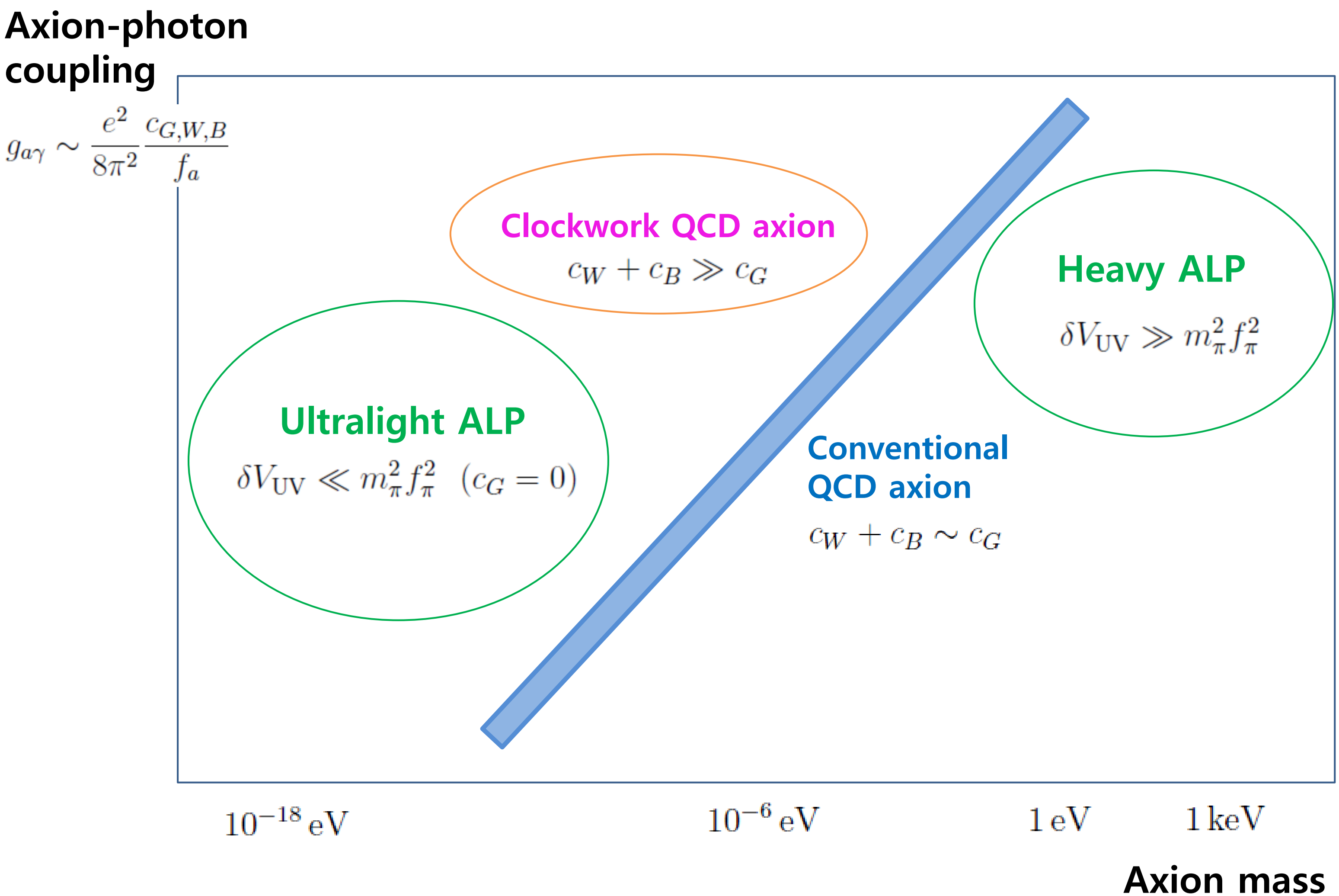
From the axion-gluon coupling

In some models, e.g. the clockwork QCD axion model, $c_{W,B}$ can be exponentially bigger than c_G, which results in an exponentially bigger value of $g_{a\gamma}/m_a$.

<u>Heavy or ultralight ALP</u>

No plausible range of the coupling to mass ratio is available mainly because the ALP mass can exponentially depend on unknown model parameter.

$$m_a^2 \sim e^{-S_{\rm ins}}\frac{\Lambda_{\rm UV}^4}{f_a^2}, \quad g_{a\gamma} \sim \frac{e^2}{8\pi^2}\frac{1}{f_a} \qquad \Rightarrow \qquad \frac{g_{a\gamma}}{m_a} \sim e^{S_{\rm ins}/2}\frac{e^2}{8\pi^2}\frac{1}{\Lambda_{\rm UV}^2}$$



Axion-photon coupling
$g_{a\gamma} \sim \frac{e^2}{8\pi^2}\frac{c_{G,W,B}}{f_a}$
Clockwork QCD axion
$c_W + c_B \gg c_G$
Heavy ALP
$\delta V_{\rm UV} \gg m_\pi^2 f_\pi^2$
Ultralight ALP
$\delta V_{\rm UV} \ll m_\pi^2 f_\pi^2 \quad (c_G = 0)$
Conventional QCD axion
$c_W + c_B \sim c_G$
$10^{-18}\,\mathrm{eV}$
$10^{-6}\,\mathrm{eV}$
$1\,\mathrm{eV}$
$1\,\mathrm{keV}$
Axion mass

Axion couplings at lower energy scales

For more details, see for instance arXiv:2106.05816

(1) Modification of axion couplings coming from integrating out heavy particles

* Multiple scalars involving both heavy and light combinations with

$$i\frac{\partial_\mu a}{f_a}\Big[\sum_n C_{\phi_n}\big(\phi_n^* D_\mu \phi_n - \mathrm{h.c}\big)\Big]$$

Integrating out the heavy scalars while leaving the light combination ϕ_L :

$$\phi_n = \hat{k}_n \phi_L \quad \Rightarrow \quad C_{\phi_L} = \sum_n |\hat{k}_n|^2 C_{\phi_n}$$

* Gauge boson which becomes massive by the VEV of ϕ which couples to the axion:

$$|D_\mu\phi|^2 + iC_\phi\frac{\partial_\mu a}{f_a}\left(\phi^* D_\mu\phi - \mathrm{h.c}\right) + i\bar{\psi}\sigma^\mu D_\mu\psi$$

$$= \frac{M_X^2}{2}X_\mu X^\mu + C_\phi\frac{M_X^2}{q_\phi}\frac{\partial_\mu a}{f_a}X^\mu + q_\psi X^\mu\bar{\psi}\sigma_\mu\psi + \ldots$$

Integrating the massive X_μ :

$$X_\mu = \frac{C_\phi}{q_\phi}\frac{\partial_\mu a}{f_a} \quad \Rightarrow \quad \Delta C_\psi = \frac{q_\psi}{q_\phi}C_\phi$$

Often one can avoid this step by choosing a basis with $C_\phi = 0$ through an axion-dependent gauge transformation which has the same consequence:

$$\phi \to e^{iC_\phi a(x)/f_a}\phi, \quad \psi \to e^{i(q_\psi/q_\phi)C_\phi a(x)/f_a}\psi \qquad \Rightarrow \quad \Delta C_\psi = \frac{q_\psi}{q_\phi}C_\phi$$

(2) Perturbative RG running

(Important when the corresponding tree level axion couplings are small, which is the case for KSVZ axion, string axions, flavor-changing couplings, ..)

$$\mathcal{L}_{\rm axion} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{\partial_\mu a}{f_a}\left(C_\psi J^\mu_\psi + C_\phi J^\mu_\phi\right) + \frac{c_A}{32\pi^2}\frac{a}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu}$$

$$J^\mu_\psi = \bar{\psi}\sigma^\mu\psi, \quad J^\mu_\phi = i(\phi^* D_\mu \phi - \mathrm{h.c.})$$

For the Yukawa coupling of the form $\mathcal{L}_{\rm Yukawa} = y\phi\psi\psi^c$

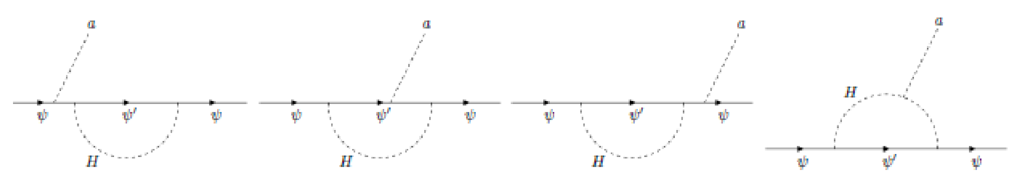
Quantized

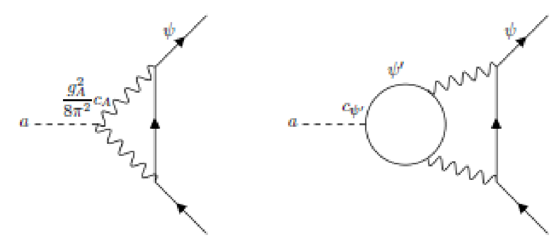
$$\frac{dc_A}{d\ln\mu} = 0$$

$$\frac{dC_\phi}{d\ln\mu} = \frac{|y|^2}{8\pi^2}(C_\phi + C_\psi + C_{\psi^c})$$

$$\frac{dC_{\psi,\psi_c}}{d\ln\mu} = \frac{|y|^2}{16\pi^2}(C_\phi + C_\psi + C_{\psi^c}) - \frac{3}{2}\left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(\psi,\psi^c)\Big(c_A - 2\sum_{\psi'=\psi,\psi^c} C_{\psi'}\mathrm{tr}(T_A^2(\psi'))\Big)$$

Quadratic Casimir





(3) Nonperturbative QCD effects

(chiral perturbation theory, lattice calculations, ...)

After including (1) and (2), we can get the axion couplings at $\mu = \mathcal{O}(1)\ \mathrm{GeV}$, which are given by

$$\frac{1}{32\pi^2}\frac{a(x)}{f_a}\left(c_\gamma e^2 F^{\mu\nu}\tilde{F}_{\mu\nu} + c_G g_s^2 G^{a\mu\nu}\tilde{G}^a_{\mu\nu}\right) + \sum_{\Psi=u,d,s,e,\mu} C_\Psi \frac{\partial_\mu a}{f_a}\bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$c_\gamma = c_W + c_B$$

$$C_u(\mu) = \frac{1}{2}\left(C_{q_1}(v) + C_{u_1^c}(v)\right) + \Delta C_u(v/\mu)$$

$$C_d(\mu) = \frac{1}{2}\left(C_{q_1}(v) + C_{d_1^c}(v)\right) + \Delta C_d(v/\mu)$$

$$C_e(\mu) = \frac{1}{2}\left(C_{\ell_1}(v) + C_{e_1^c}(v)\right) + \Delta C_e(v/\mu)$$

RG running from v to μ

in the basis for which $C_H(v) = 0$ at the EW scale $v = 246\ \mathrm{GeV}$

Axion couplings below the QCD scale

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \frac{1}{2} g_{a\gamma} a(x) \vec{E}\cdot\vec{B} \\ &+ \partial_\mu a(x)\Big[g_{ap}\bar{p}\gamma^\mu\gamma_5 p + g_{an}\bar{n}\gamma^\mu\gamma_5 n + \frac{g_{a\pi N}}{f_\pi}\left(i\pi^+\bar{p}\gamma^\mu n - i\pi^-\bar{n}\gamma^\mu p\right)\Big] \\ &+ \frac{g_{a\pi}}{f_\pi}\partial^\mu a(x)\left(\pi^0\pi^+\partial_\mu\pi^- + \pi^0\pi^-\partial_\mu\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0\right) \\ &+ \sum_{\ell=e,\mu} g_{a\ell}\bar{\ell}\gamma^\mu\gamma_5\ell\end{aligned}$$

$$\begin{aligned} g_{a\gamma} &= \frac{\alpha_{\rm em}}{2\pi}\frac{1}{f_a}\left(c_W + c_B - \frac{2}{3}\frac{(4m_d+m_u)}{(m_u+m_d)}c_G\right) \\ g_{ap} - g_{an} &= \frac{g_A}{2f_a}\left(C_u - C_d + \left(\frac{m_u-m_d}{m_u+m_d}\right)c_G\right) \\ g_{ap} + g_{an} &= \frac{g_0}{2f_a}\left(C_u + C_d - c_G\right) \\ g_{a\pi} &= \frac{2\sqrt{2}}{3} g_{a\pi N} = \frac{2(g_{ap}-g_{an})}{3g_A} \\ g_{a\ell} &= \frac{1}{f_a}C_\ell \quad (\ell = e, \mu)\end{aligned}$$

$$g_A s^\mu = \langle N|\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d|N\rangle$$

$$g_0 s^\mu = \langle N|\bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d|N\rangle$$

$$g_A \simeq 1.27, \quad g_0(\mu = 2\ \text{GeV}) \simeq 0.52$$

$$\begin{aligned} g_{a\gamma} &\simeq \frac{\alpha_{\rm em}}{2\pi}\frac{1}{f_a}\left(c_W + c_B - 1.92 c_G\right) \\ g_{ap} &\simeq \frac{1}{2f_a}\left(0.94c_G + 0.88C_u(2\,\text{GeV}) - 0.39C_d(2\,\text{GeV})\right) \\ g_{an} &\simeq \frac{1}{2f_a}\left(0.04c_G - 0.39C_u(2\,\text{GeV}) + 0.88C_d(2\,\text{GeV})\right)\end{aligned}$$

We now have the full recipe to derive the low energy axion couplings in a given axion model, which are relevant for studying the laboratory, cosmological, or astrophysical implications of axion.

Axion models with a linear PQ symmetry

(Axions originating from the phase of complex scalar field)

Recipe for model building

1) Introduce a linear PQ symmetry with quantized PQ charges, which is explicitly broken by the SM gauge anomalies that arise from the loops of PQ and gauge-charged fermion (=anomalous variation of the fermion path integral measure)

$$U(1)_{\rm PQ}: \ \Phi \to e^{iq_\Phi \alpha}\Phi \quad (\Phi = \phi, \psi)$$

$$\partial_\mu J^\mu_{\rm PQ} = \frac{1}{32\pi^2}\sum_A c_A F^A \tilde{F}^A \quad \Big(c_A = 2\sum_\psi q_\psi {\rm tr}(T_A^2(\psi))\Big)$$

2) Introduce a dynamics to generate a VEV of PQ-charged local operator, breaking the PQ symmetry spontaneously:

$$\langle \mathcal{O}(\Phi)\rangle \sim f_a^{n_\mathcal{O}} e^{ia(x)/f_a} \qquad \big(n_\mathcal{O} = {\rm dim}(\mathcal{O}), \ \ a(x) \cong a(x) + 2\pi f_a\big)$$

In such models, much of the physical properties of axion is determined by the PQ charges of the matter fields in the model.

KSVZ model

Kim '79
Shifman, Vainshtein, Zakharov '80

$$\mathcal{L}_{\rm KSVZ} = \partial_\mu \sigma \partial^\mu \sigma^* + i\bar{Q}\bar{\sigma}^\mu D_\mu Q + i\bar{Q}^c \bar{\sigma}^\mu D_\mu Q^c + \mathcal{L}_{\rm SM}$$
$$- \left(y\sigma Q Q^c + {\rm h.c} \right) - \lambda \left(\sigma\sigma^* - \frac{1}{2} f_a^2 \right)^2$$

$$U(1)_{\rm PQ}: \quad \sigma \to e^{i\alpha}\sigma, \quad (Q, Q^c) \to e^{-i\alpha/2}(Q, Q^c)$$

Exotic PQ-charged left-handed quarks and anti-quarks

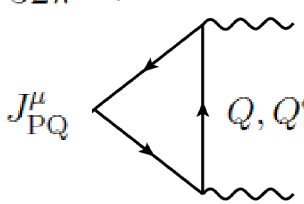
$$Q = (3,2)_{Y_Q}, \quad Q^c = (\bar{3},2)_{-Y_Q}$$

A key feature of the model is that all SM fields are PQ-neutral, leading to a distinctive pattern of axion couplings.

Explicit PQ breaking by anomalies:

If Q, Q^c are $SU(2)_W$ singlet, c_G= N_{DW}=1.

$$\partial_\mu J^\mu_{\rm PQ} = \frac{1}{32\pi^2}\left(2G\tilde{G} + 3W\tilde{W} + 6Y_Q^2 B\tilde{B} \right) \qquad c_A = (c_G, c_W, c_B) = (2, 3, 6Y_Q^2)$$



Spontaneous PQ breaking by $\langle \sigma \rangle = \frac{1}{\sqrt{2}} f_a e^{ia(x)/f_a}$

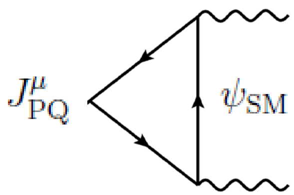
DFSZ model

Zhitnitsky '80
Dine, Fischler, Srednicki '81

$$\mathcal{L}_{\rm DFSZ} = \partial_\mu \sigma \partial^\mu \sigma^* + D_\mu H_u^\dagger D^\mu H_u + D_\mu H_d^\dagger D^\mu H_u$$
$$- \Big(y_u H_u q u^c + y_d H_d q d^c + H_d y_\ell \ell e^c + {\rm h.c} \Big)$$
$$- \lambda \Big(\sigma\sigma^* - \frac{1}{2} f_a^2 \Big)^2 - \Big(\kappa H_u H_d \sigma^n + {\rm h.c} \Big) + \ldots$$

$$U(1)_{\rm PQ}: \quad \sigma \to e^{i\alpha}\sigma, \quad H_{u,d} \to e^{-i\alpha} H_{u,d}, \quad \psi_{\rm SM} \to e^{i\alpha/2}\psi_{\rm SM}$$

$$\partial_\mu J^\mu_{\rm PQ} = \frac{1}{32\pi^2}\Big(6G\tilde{G} + 6W\tilde{W} + 10B\tilde{B} \Big) \quad c_A = (c_G, c_W, c_B) = (6, 6, 10)$$



$$\langle \sigma \rangle = \frac{1}{\sqrt{2}} f_a e^{ia(x)/f_a}$$

SM fields are PQ-charged, yielding different pattern of axion couplings from the KSVZ axion and the axions from p-form gauge field in string/M theory.

Composite axion

In the KSVZ and DFSZ models, PQ symmetry is spontaneously broken by the scalar potential introduced by hand, so the model does not provide an explanation for the origin of the axion scale.

In composite axion models, PQ symmetry is spontaneously broken by a new confining dynamics **(axicolor)** whose confinement scale can be identified as the axion scale.

Composite axion models avoid the potential fine-tuning problem associated with the mass parameters in the scalar potential, and in some models the PQ symmetry is not introduced by hand, but corresponds to an accidental symmetry that appears as a consequence of the gauge symmetries of the model.

In most models of composite axion, the SM fields are PQ-neutral, therefore axion couplings are similar to those of the KSVZ axion.

Model with vector-like axicolor

Kim '85; KC, Kim '85

Gauge group: $G = SU(N_a) \times SU(3)_c$

PQ-charged & axicolored fermions: $\left(\psi_{(N_a,3)}, \psi_{(N_a,1)}\right), \quad \left(\psi^c_{(\bar{N}_a,\bar{3})}, \psi^c_{(\bar{N}_a,1)}\right)$

$$q_{\psi_{(N_a,3)}} = q_{\psi^c_{(\bar{N}_a,\bar{3})}} = \frac{1}{2}, \quad q_{\psi_{(N_a,1)}} = q_{\psi^c_{(\bar{N}_a,1)}} = -\frac{N_c}{2}$$

Spontaneous and explicit breaking of the PQ symmetry:

$$\langle \psi_{(N_a,3)} \psi^c_{(\bar{N}_a,\bar{3})} \rangle = \Lambda_a^3 e^{ia(x)/f_a}, \quad \langle \psi_{(N_a,1)} \psi^c_{(\bar{N}_a,1)} \rangle = \Lambda_a^3 e^{-3ia(x)/f_a} \qquad \partial_\mu J^\mu_{\rm PQ} = \frac{N_a}{32\pi^2} G\tilde{G}$$

Model with chiral axicolor

Gavela et al, arXiv:1812.08174

$$G = SU(5)_a \times SU(3)_c$$

$$10 = \left(\psi_{(10,3)}, \psi_{(10,\bar{3})}\right), \quad \bar{5} = \left(\psi_{(\bar{5},3)}, \psi_{(\bar{5},\bar{3})}\right) \qquad q_{10} = \frac{1}{10}, \quad q_{\bar{5}} = -\frac{3}{10}$$

$$\langle 10 \cdot \bar{5} \cdot \bar{5} \cdot 10 \cdot \bar{5} \cdot \bar{5} \rangle \sim \Lambda_5^9 e^{ia/f_a} \qquad \partial_\mu J^\mu_{\rm PQ} = \frac{2}{32\pi^2} G\tilde{G}$$

Gauge symmetry of this model forbids a PQ-breaking term in the lagrangian up to dim=8 operators, so the PQ symmetry is well protected from quantum gravity.

KSVZ axion couplings

Let us consider a model with $SU(2)_W$ singlet Q, Q^c and also the scale separations:

$$f_a \gg m_Q = y\frac{f_a}{\sqrt{2}} \gg v = 246\,\mathrm{GeV}$$

At the scale $\mu \sim f_a$, $\sigma = \frac{1}{\sqrt{2}}(f+\rho)e^{ia(x)/f}$

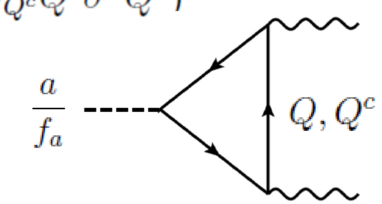
$$\Rightarrow \quad \mathcal{L}_{\rm axion}(\mu \sim f_a) = \frac{1}{2}\partial_\mu a \partial^\mu a - \left(m_Q e^{ia(x)/f} QQ^c + \mathrm{h.c}\right)$$

One may make the field redefinition at this scale

$$Q \to e^{-iq_Q a(x)/f_a} Q, \quad Q^c \to e^{-iq_{Q^c} a(x)/f_a} Q^c \quad (q_Q = q_{Q^c} = \frac{1}{2})$$

to move to the GKR basis:

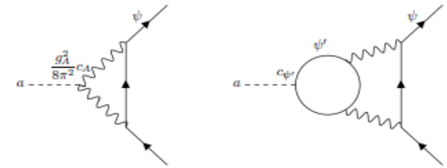
$$\mathcal{L}_{\rm GKR}(\mu \sim f_a) = \frac{1}{2}\partial_\mu a \partial^\mu a - m_Q \bar{Q}Q + \frac{\partial_\mu a(x)}{f}(C_Q \bar{Q}\bar{\sigma}^\mu Q + C_{Q^c}\bar{Q}^c \bar{\sigma}^\mu Q^c)$$
$$+ \frac{1}{32\pi^2}\frac{a(x)}{f}(c_G G\tilde{G} + c_W W\tilde{W} + c_B B\tilde{B})$$



$$c_A = 2\sum_\psi q_\psi \mathrm{tr}(T_A^2(\psi)) = (1, 3, 6Y_Q^2) \quad (A = G, W, B)$$

$$C_{Q,Q^c} = q_{Q,Q^c} = \frac{1}{2}$$

There is no RG running of the axion couplings over the scales $f_a > \mu > m_Q$, which is manifest in the original basis, but it is a consequence of cancellation in the GKR basis:



$$\left(\frac{dC_{Q,Q^c}}{d\ln\mu}\right)_{\rm GKR} = -\frac{3}{2}\sum_A \left(\frac{g_A^2}{8\pi^2}\right)^2 \mathbb{C}_A(Q,Q^c)\Big(c_A - 2\sum_{\psi'=Q,Q^c} C_{\psi'}\mathrm{tr}(T_A^2(\psi')\Big) = 0$$

Consider the scattering amplitude of $gg \to ga$ for $m_Q < \sqrt{s} \sim \sqrt{t} < f_a$.

In the original basis, the suppression by m_Q^2 is manifest:

$$\mathcal{A}(gg \to ga) \sim \frac{g_s^3}{16\pi^2}\frac{m_Q^2}{f_a}\frac{1}{\sqrt{s}}$$

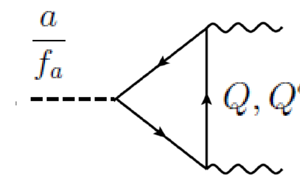
On the other hand, in the GKR basis, it is again the result of cancellation

$$\begin{aligned}\mathcal{A}(gg \to ga) &= \mathcal{A}_{\rm GKR}(gg \to ga) \\ &\sim \frac{g_s^3}{16\pi^2}\frac{\sqrt{s}}{f_a}\Big[c_G - (C_Q + C_{Q^c})(1 + \mathcal{O}(m_Q^2/s)\Big] \sim \frac{g_s^3}{16\pi^2}\frac{m_Q^2}{f_a}\frac{1}{\sqrt{s}}\end{aligned}$$

The GKR basis is inconvenient for the KSVZ axion at $f_a > \mu > m_Q$.

However, below m_Q , we are inevitably in the GKR basis as there is no PQ-charged light field.

$$\mathcal{L}_{\rm axion}(\mu = m_Q) = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{32\pi^2}\frac{a(x)}{f}(G\tilde{G} + 3W\tilde{W} + 6Y_Q^2 B\tilde{B})$$



$C_\Phi(\mu = m_Q) = 0 \quad (\Phi = \psi_{\rm SM}, H)$ because of the vanishing PQ charge of the SM fermions and Higgs

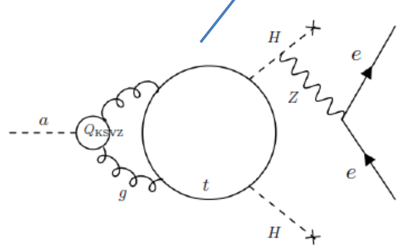
Subsequent RG evolution is the dominant source of the axion couplings to the SM fermions:

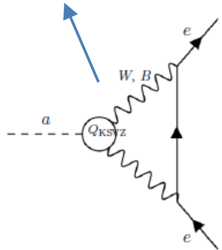
$$C_u(\mu = 2\,{\rm GeV}) \sim C_d(\mu = 2\,{\rm GeV}) \sim 10^{-2}$$

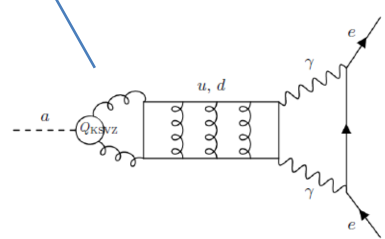
KSVZ axion-electron coupling: KC, Im, Kim, Seong, arXiv:2106.05816

$$C_e(m_e) = \kappa_1 \frac{g_s^4(m_t)}{(8\pi^2)^2}\frac{y_t^2(m_t)}{8\pi^2}c_G + \kappa_2 \frac{e^4}{(8\pi^2)^2}c_{W,B} + \kappa_3 \frac{e^4}{(8\pi^2)^2}\frac{m_u + 4m_d}{m_u + m_d}c_G$$

$$\simeq \Big(0.83 c_G + 0.54 c_W + 0.13 c_B - 0.03 c_G\Big) \times 10^{-3} \quad \text{for } m_Q = 10^{10}\ {\rm GeV}$$







DFSZ axion couplings

Again consider the case with wide scale separations:

$$f_a \gg m_{\tilde{H}} \sim \sqrt{\kappa} f_a \gg v = 246\,\mathrm{GeV}$$

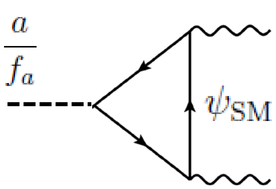
Heavy combination of H_u and H_d

At the scale $\mu \sim f_a$, $\quad \sigma = \frac{1}{\sqrt{2}}(f_a + \rho)e^{ia(x)/f_a}$

$$\Rightarrow \quad \mathcal{L}_{\rm axion}(\mu \sim f_a) = \frac{1}{2}\partial_\mu a \partial^\mu a - \left(\kappa f^2 H_u H_d e^{i2a(x)/f_a} + \mathrm{h.c}\right)$$

Field redefinition to the GKR basis:

$$H_{u,d} \to e^{-iq_{H_{u,d}}a(x)/f_a} H_{u,d}, \quad \psi \to e^{-iq_\psi a(x)/f_a}\psi$$



$$\mathcal{L}_{\rm GKR}(\mu \sim f_a) = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{32\pi^2}\frac{a(x)}{f}\left(c_G G\tilde{G} + c_W W\tilde{W} + c_B B\tilde{B}\right)$$
$$- \frac{1}{2}\frac{\partial_\mu a(x)}{f}\left(\sum_{H_i} iC_{H_i}(H_i^\dagger D^\mu H_i - \mathrm{h.c}) + \sum_\psi C_\psi \bar{\psi}\bar{\sigma}^\mu \psi\right)$$

$$c_A = 2\sum_\psi q_\psi \mathrm{tr}(T_A^2(\psi)) = (6, 6, 10) \quad (A = G, W, B)$$

$$C_{H_{u,d}} = q_{H_{u,d}} = -1, \quad C_\psi = q_\psi = 1/2 \quad (\psi = q_i, u_i^c, d_i^c, \ell_i, e_i^c)$$

Often the GKR basis for DFSZ axion is inconvenient for axion physics at scales above $m_{\tilde{H}}$ or above the heaviest PQ & gauge charged fermion mass m_t .

Nonzero tree level axion couplings to the SM fermions and Higgs, which are dominant in most case.

So let's consider only the tree level couplings to the SM matter.

At $\mu \sim m_{\tilde{H}}$, integrate out the heavy Higgs doublet, yielding

$$H_u = H^* \sin\beta, \quad H_d = H \cos\beta \quad (\tan\beta = \langle H_u \rangle / \langle H_d \rangle)$$

The SM Higgs doublet

$$C_H(\mu = m_{\tilde{H}}) = q_{H_d} \cos^2\beta - q_{H_u} \sin^2\beta = -\cos 2\beta$$

At $\mu \sim v = 246\,\mathrm{GeV}$, make a field redefinition to rotate away C_H :

$$\psi \;\to e^{i(Y_\psi/Y_H)C_H a(x)/f_a}\psi \quad \Rightarrow \quad C_H = 0, \quad \Delta C_\psi = Y_\psi C_H / Y_H$$

$$C_q = \frac{1}{2} - \frac{1}{3}\cos 2\beta, \quad C_{u^c} = \frac{1}{2} + \frac{4}{3}\cos 2\beta, \quad C_{d^c} = \frac{1}{2} - \frac{2}{3}\cos 2\beta$$

$$C_\ell = \frac{1}{2} + \cos 2\beta, \quad C_{e^c} = \frac{1}{2} - \cos 2\beta$$

Couplings to the light quarks and electron at $\mu \sim 1\,\mathrm{GeV}$:

$$C_\Psi \frac{\partial_\mu a}{f_a} \bar{\Psi}\gamma^\mu\gamma_5\Psi \quad (\Psi = u, d, e)$$

$$C_u = \frac{1}{2}\left(C_q + C_{u^c}\right) = \cos^2\beta$$

$$C_d = \frac{1}{2}\left(C_q + C_{d^c}\right) = C_e = \frac{1}{2}\left(C_\ell + C_{e^c}\right) = \sin^2\beta$$

KSVZ and DFSZ axions have a quite different pattern of couplings.

$$C_\Phi \frac{\partial_\mu a(x)}{f_a} J^\mu_\Phi + c_A \frac{g_A^2}{32\pi^2} \frac{a(x)}{f_a} F^{A\mu\nu} \tilde{F}^A_{\mu\nu} \quad (\Phi = \psi, \phi;\ A = G, W, B)$$

$$= g_{a\Phi} \partial_\mu a(x) J^\mu_\Phi + \frac{g_{aA}}{4} a(x) F^{A\mu\nu} \tilde{F}^A_{\mu\nu} \quad \left(g_{a\Phi} = \frac{C_\Phi}{f_a}, \quad g_{aA} = \frac{g_A^2}{8\pi^2} \frac{c_A}{f_a} \right)$$

$$\left(J^\mu_\psi = \bar{\psi} \sigma^\mu \psi, \quad J^\mu_\phi = i(\phi^* D_\mu \phi - \mathrm{h.c.}) \right)$$

KSVZ axion

$$\frac{g_{a\psi_{\rm SM}}(\mu)}{g_{aA}} \sim \frac{g_A^2}{8\pi^2} \ln(m_Q/\mu)$$

DFSZ axion

$$\frac{g_{a\psi_{\rm SM}}(\mu)}{g_{aA}} \sim \frac{8\pi^2}{g_A^2}$$