Feynman Integral
Role of Intersection Theory

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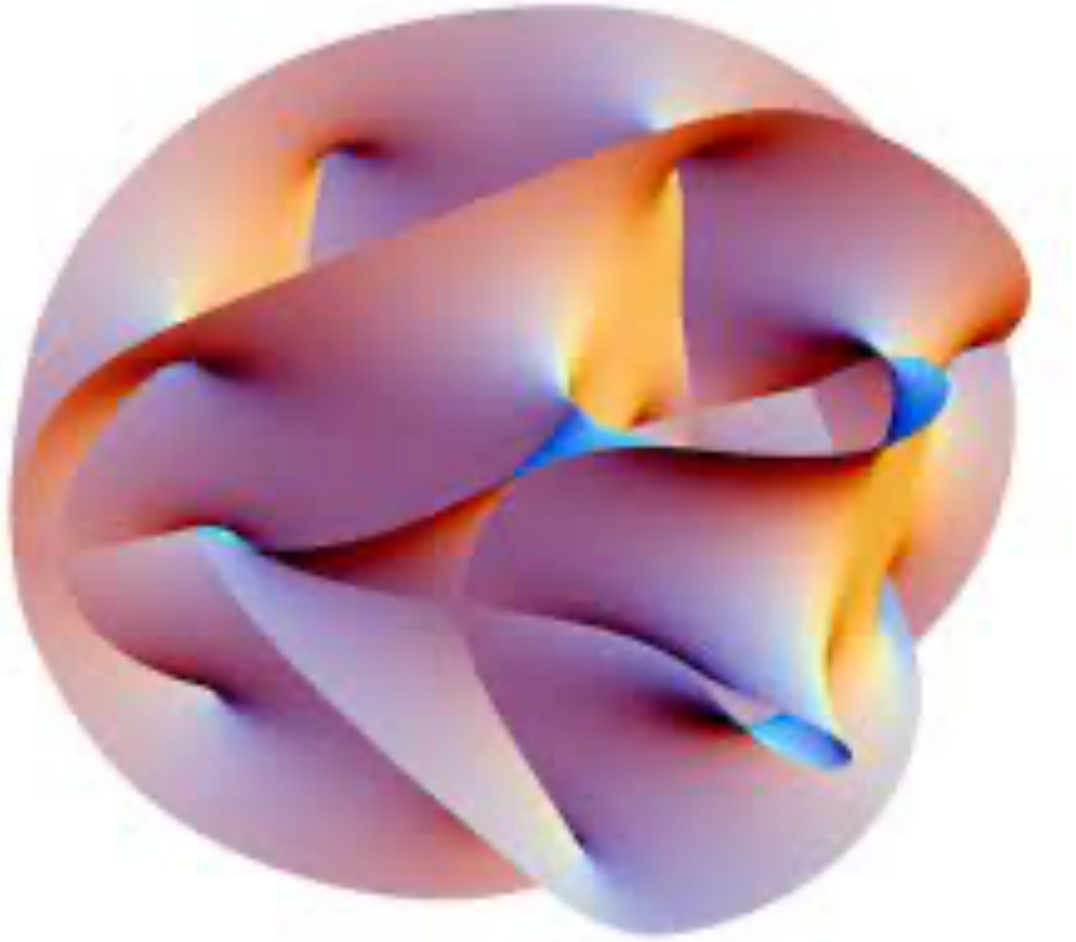
G.A. 754496

INFN
Istituto Nazionale di Fisica Nucleare

Why Scattering Amplitude ?

Collider Phenomenology

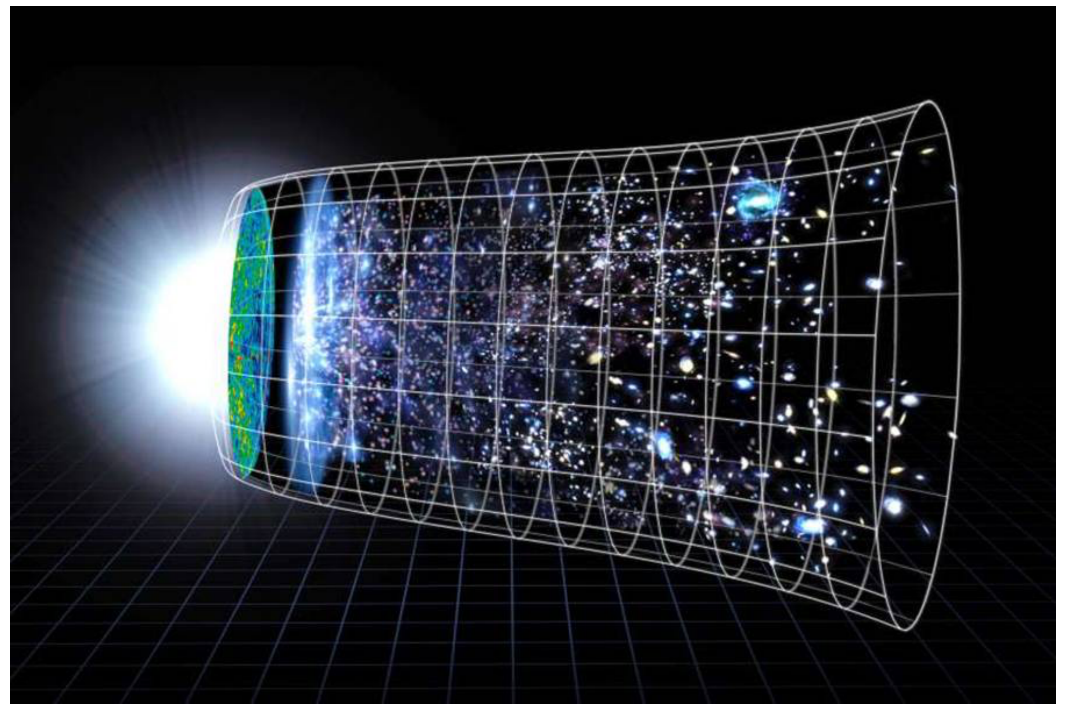
Geometry and QFT



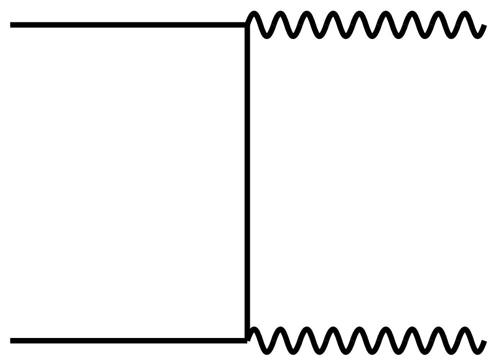
Scattering Amplitude

Gravitational Waves

Cosmology

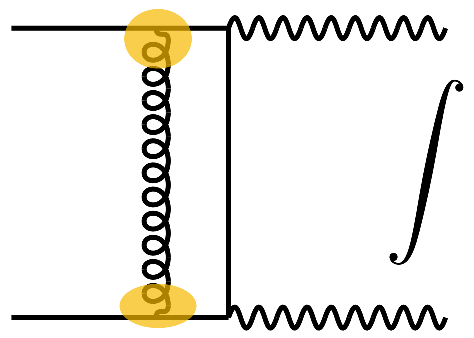


Scattering Amplitude

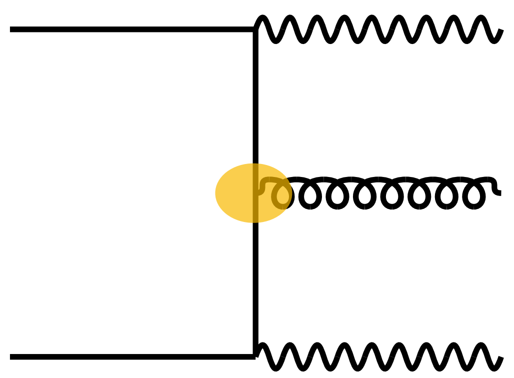


LO

$$\sigma^0 \approx \int |\mathcal{M}_N^{(0)}|^2 d\Phi_N$$



$$\int \left[\frac{V_2}{\epsilon^2} + \frac{V_1}{\epsilon^1} + V_0 \right] d\phi_2$$



$$\int [R_0]\, d\phi_3$$

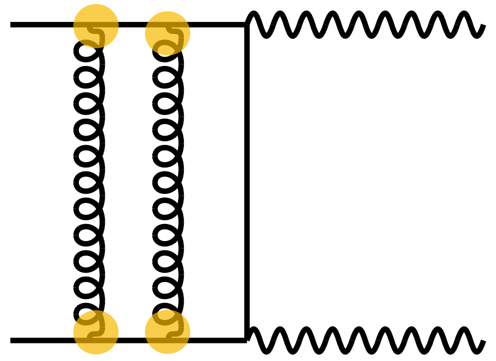
NLO

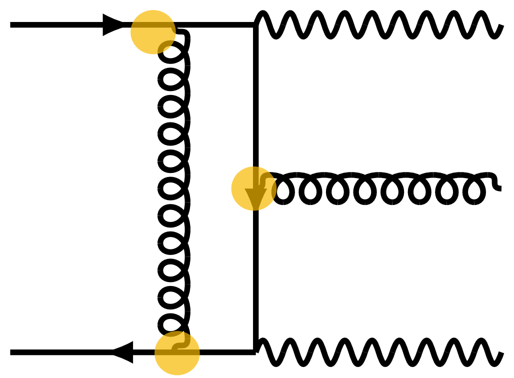
$$\sigma_N^{(1)} \approx \int 2\mathrm{Re}\left(\mathcal{M}_N^{(0)*}\mathcal{M}_N^{(1)}\right) d\Phi_N + \int |\mathcal{M}_{N+1}^{(0)}|^2 d\Phi_{N+1}$$

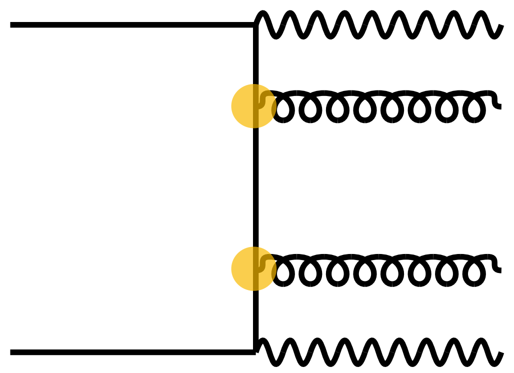
$$\int \left[\frac{VV_4}{\epsilon^4} + \frac{VV_3}{\epsilon^3} + \frac{VV_2}{\epsilon^2} + \frac{VV_1}{\epsilon^1} + VV_0 \right] d\phi_2$$

$$\int \left[\frac{RV_2}{\epsilon^2} + \frac{RV_1}{\epsilon^1} + RV_0 \right] d\phi_3$$

$$\int [RR_0]\, d\phi_4$$



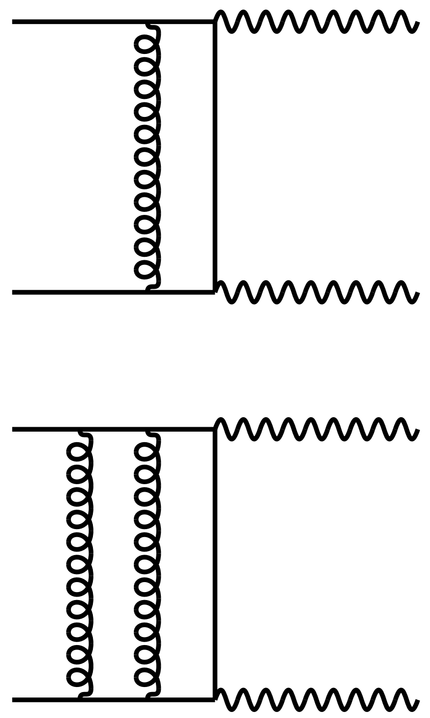




NNLO

$$\sigma_N^{(2)} \approx \int 2\mathrm{Re}\left(\mathcal{M}_N^{(0)*}\mathcal{M}_N^{(2)}\right) d\Phi_N + \int 2\mathrm{Re}\left(\mathcal{M}_{N+1}^{(0)*}\mathcal{M}_{N+1}^{(1)}\right) d\Phi_{N+1} + \int |\mathcal{M}_{N+2}^{(0)}|^2 d\Phi_{N+2}$$

Computation of the Loop Amplitude



Generation of the Diagrams via QGRAF

↓

Dirac algebra, Color sum, Trace in the numerators

↓

Reduction to scalar integrals

$$\mathcal{M} = \sum_i a_i I_i \qquad i = \mathcal{O}(10^5)$$

Loop Amplitude

Reduction of scalar integrals to Master integrals

↓

Compute the Master Integrals

Number of Master Integrals

$$\mathcal{M} = \sum_i c_i J_i \qquad i = \mathcal{O}(10^2)$$

Decomposition of Feynman Integrals using Intersection Theory

Intersection Theory and Feynman Integral

$$I = \sum_{i=1}^{\nu} c_i J_i$$

$$I \cdot J_i$$

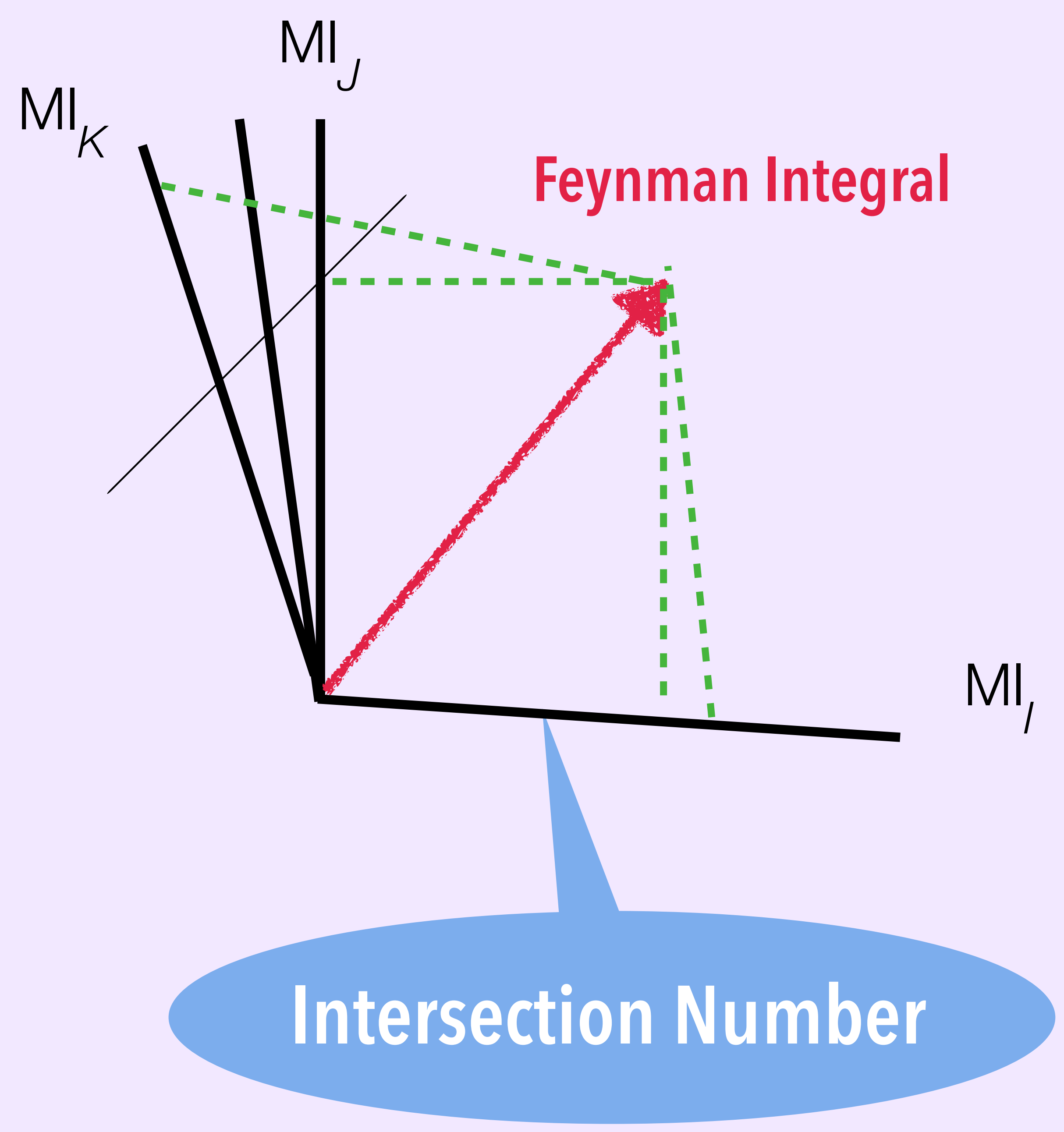
$$J_i \cdot J_j = \delta_{ij}$$

$$I \cdot J_j \left(C^{-1}\right)_{ji}$$

$$J_i \cdot J_j = C_{ij} \neq \delta_{ij}$$

Intersection Theory — **Feynman Integral**

Feynman Integral decomposition



Mastrolia, Mizera (2018)

Frellesvig, Gasparotto, MKM, Mastrolia, Mattiazzi, Mizera (2019)

Frellesvig, Gasparotto, Laporta, MKM, Mastrolia, Mattiazzi, Mizera (2019)

Frellesvig, Gasparotto, MKM, Mastrolia, Mattiazzi, Mizera (2020)

Chestnov, Frellesvig, Gasparotto, MKM, Mastrolia (2022)

What is the Vector Space ?

How to define the scalar product ?

Computation of Intersection Number

Fibration Method

- Matsumoto (1998)
- Mizera (2019)
- Frellesvig, Gasparotto, Laporta, MKM, Mastrolia, Mattiazzi, Mizera (2019)
- Frellesvig, Gasparotto, MKM, Mastrolia, Mattiazzi, Mizera (2020)
- Wienzierl (2020)
- Caron-Huot, Pokraka (2021)

Secondary Equation Matsubara-Heo (2019)

Multivariate Differential Equation Matsumoto (1998)

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Macaulay matrix for Feynman integrals: linear relations and intersection numbers

Vsevolod Chestnov,[a,b] Federico Gasparotto,[a,b] Manoj K. Mandal,[b] Pierpaolo Mastrolia,[a,b] Saiei J. Matsubara-Heo,[c,d] Henrik J. Munch[a,b] and Nobuki Takayama[c]

Intersection Numbers from Higher-order Partial Differential Equations

2022

Vsevolod Chestnov,[a,b] Hjalte Frellesvig,[c] Federico Gasparotto,[a,b] Manoj K. Mandal,[b] Pierpaolo Mastrolia[a,b]

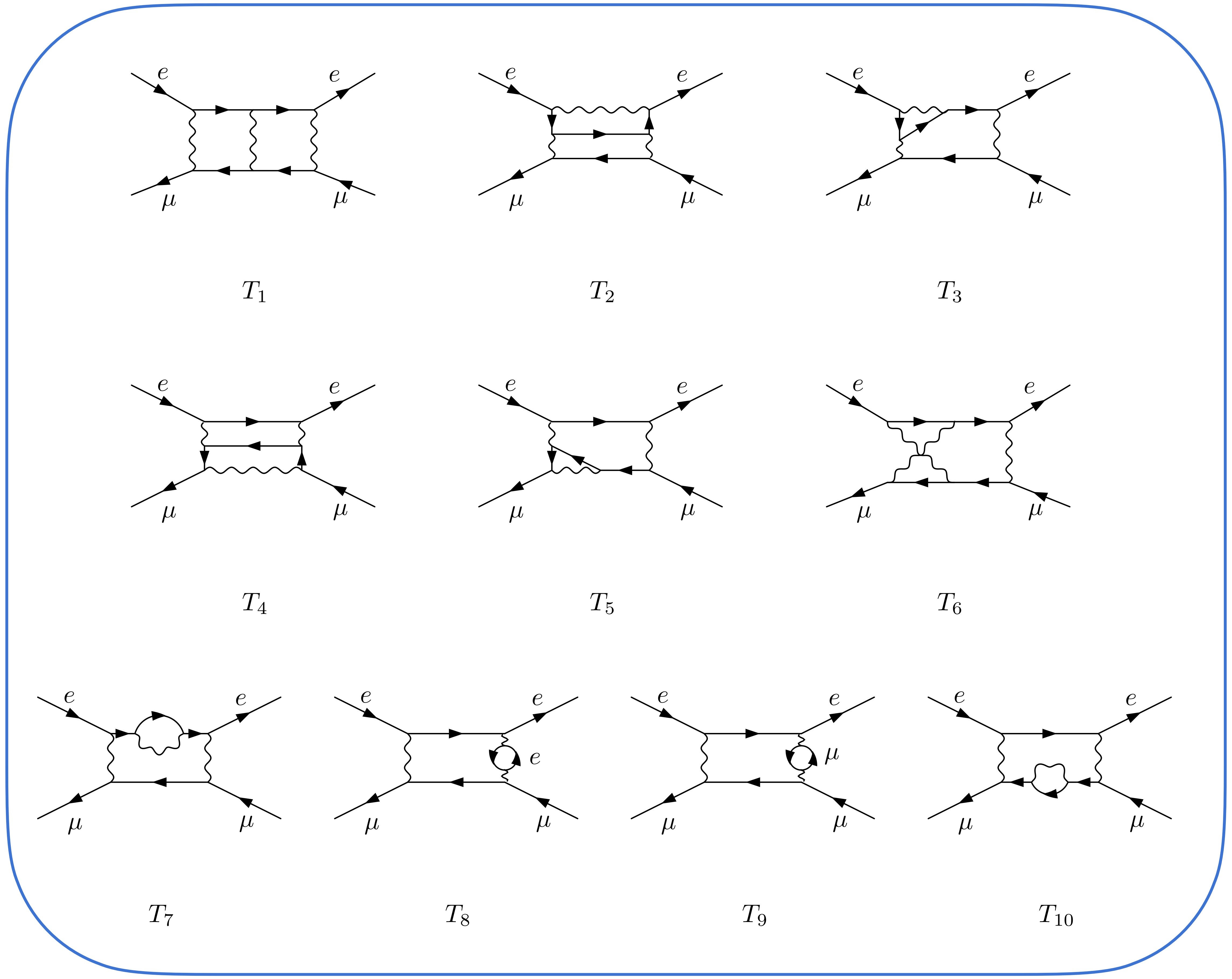
Outlook

Feynman Integral
Quantum Field Theory
Graph Polynomials
Quadratic Relations
Theoretical Physics
Master Integrals
Differential Equations
Vector Space Structure
IBP Identities
Intersection Theory
For
Twisted de-Rham Co-homology
Aomoto-Gel'fand Integral
Differential and Algebraic Geometry
Multivalued Functions
Twisted Reimann Period Relations
Mathematics
Independent Forms And Contours
Differential Equations
Cohomology Group
Contiguity Relation

Collider Applications

Two-Loop amplitude with massless and massive particles

- ✦ Computation of 2-loop virtual amplitude for electron-muon scattering, relevant for the MUonE experiment
- ✦ Computation of 2-loop virtual amplitude for top pair production



PHYSICAL REVIEW LETTERS **128,** 022002 (2022)

Two-Loop Four-Fermion Scattering Amplitude in QED

R. Bonciani,[1,*] A. Broggio,[2,†] S. Di Vita,[3,4] A. Ferroglia,[5,6,‡] M. K. Mandal,[7,8,§] P. Mastrolia,[8,7,∥] L. Mattiazzi,[7,8,¶] A. Primo,[9,**] J. Ronca,[10,††] U. Schubert,[11,‡‡] W. J. Torres Bobadilla,[12,§§] and F. Tramontano[10,∥∥]

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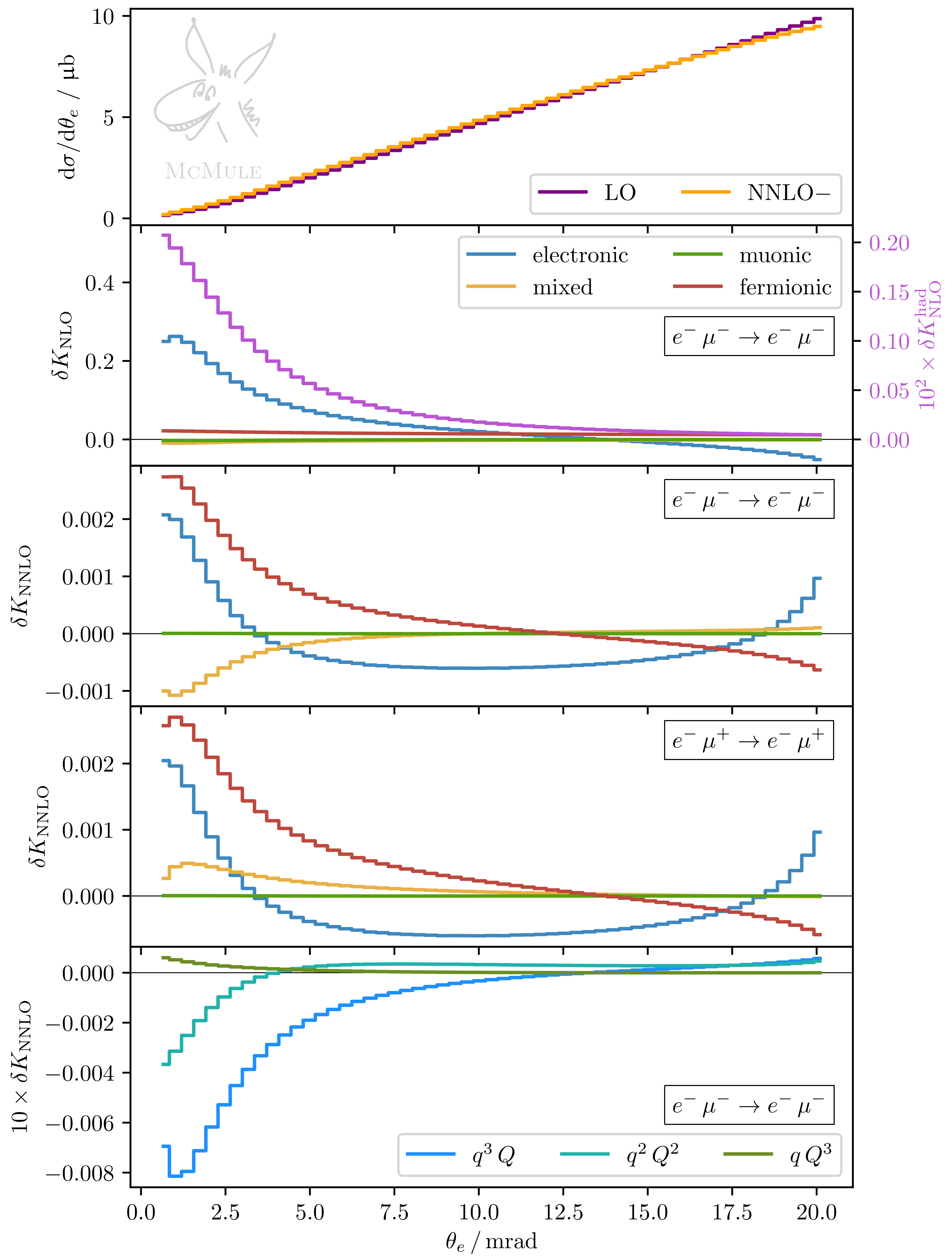
Two-loop scattering amplitude for heavy-quark pair production through light-quark annihilation in QCD

Manoj K. Mandal,[a] Pierpaolo Mastrolia,[a,b] Jonathan Ronca[c] and William J. Torres Bobadilla[d]

NNLO Prediction for Muon-Electron Scattering



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Muon-electron scattering at NNLO

A. Broggio,[a] T. Engel,[b,c,d] A. Ferroglia,[e,f] M.K. Mandal,[g,h] P. Mastrolia,[i,g] M. Rocco,[b] J. Ronca,[j] A. Signer,[b,c] W.J. Torres Bobadilla,[k] Y. Ulrich[l] and M. Zoller[b]

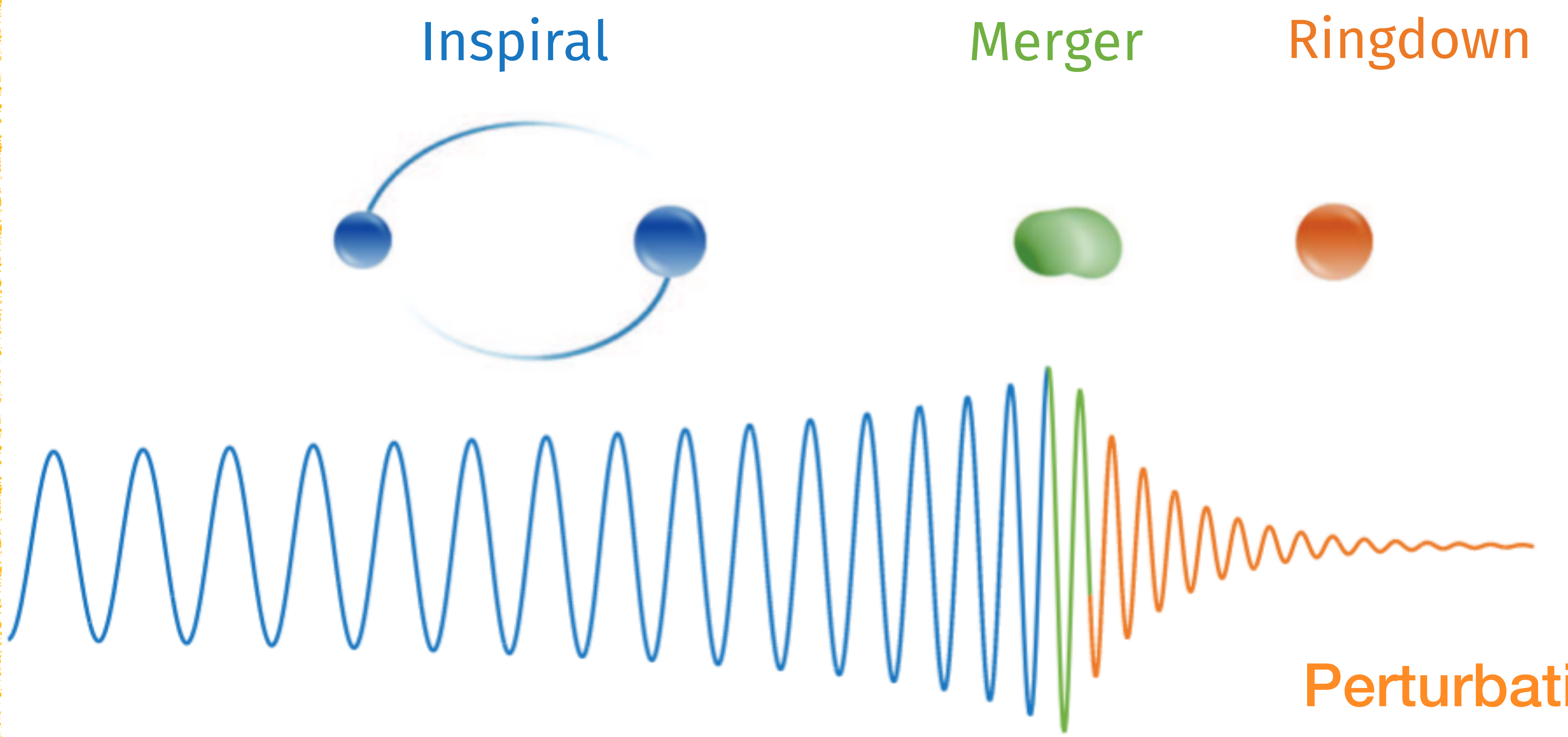
- ☑ First complete and fully differential NNLO calculation of a $2 \to 2$ process with two different non-vanishing masses on the external lines
- ☑ Successful collaboration with the group at PSI

Gravitational Wave Observables

Solving two-body problem in GR

Goldberger, Rothstein
Foffa, Sturani, Sturm, Mastrolia (2016)

Antelis, moreno (2016)

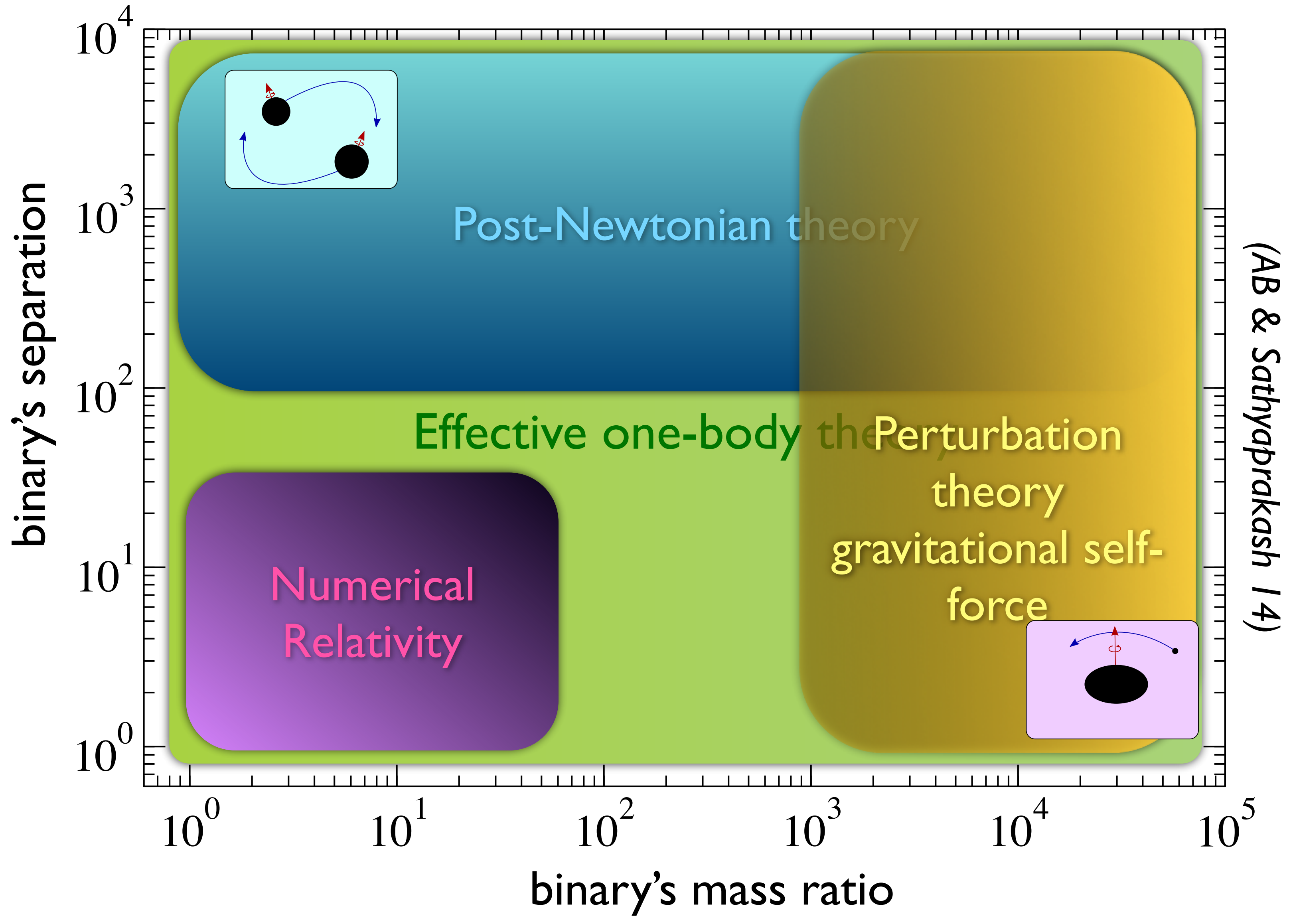


Perturbation Theory

Post-Newtonian (PN)

Numerical Relativity

Post-Minkowskian (PM)



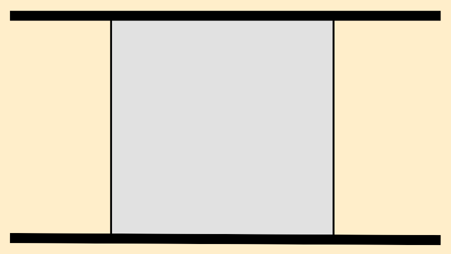
Hierarchy of scales

$$r_\star << r << \lambda_{GW}$$

Tower of EFTs

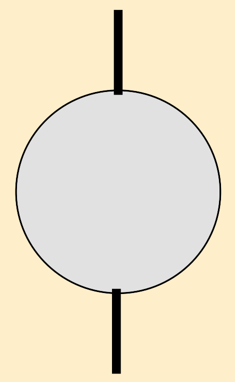
Key Observation

Diagrams in GR



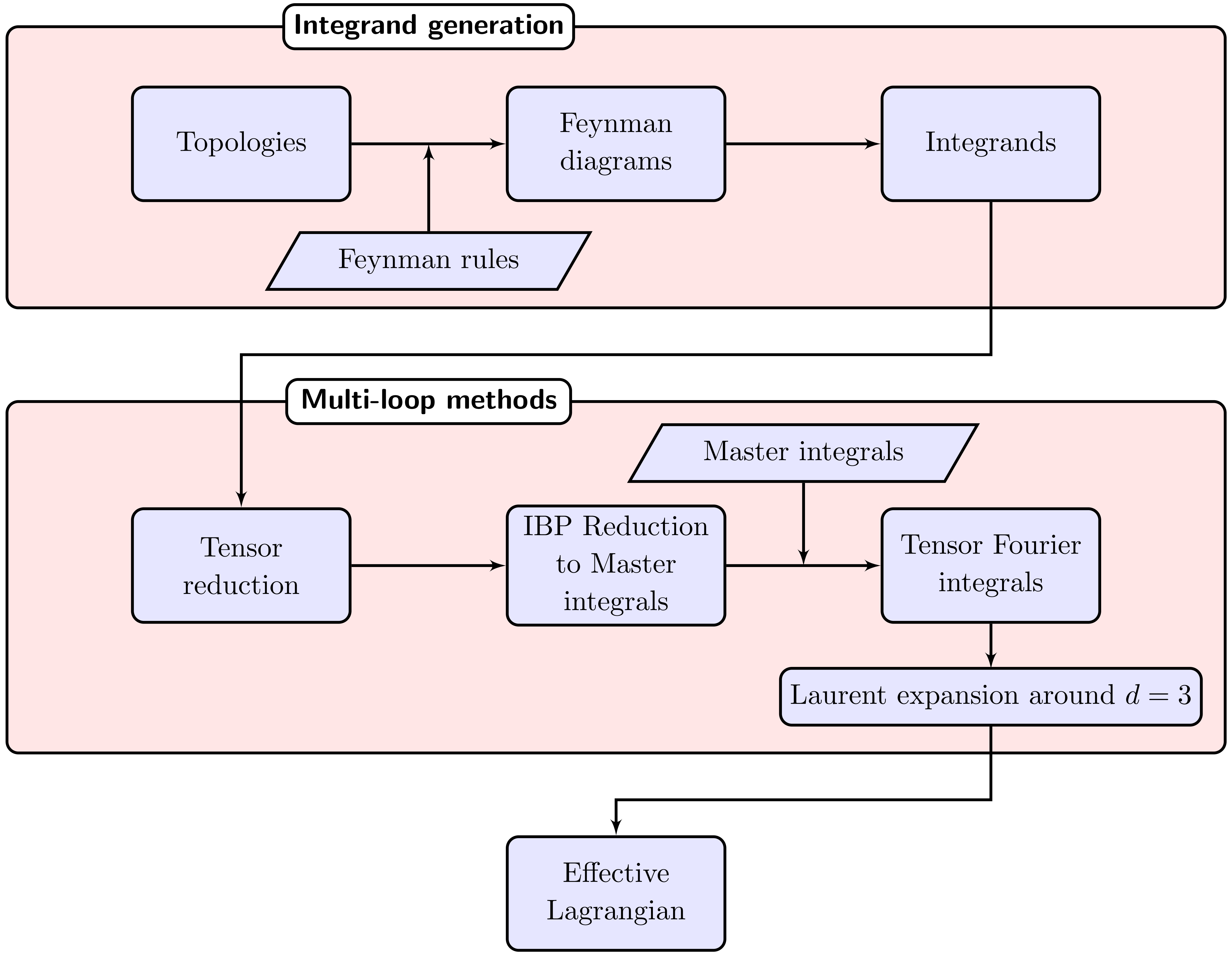
$=$

Diagrams in QFT



$p^2 \equiv s \neq 0$

Computational Algorithm



Gravitational Spin-Orbit Hamiltonian at NNLO in the post-Newtonian framework 2022

Manoj K. Mandal,[a] Pierpaolo Mastrolia,[b,a] Raj Patil,[c,d,e] Jan Steinhoff[c]

Gravitational Quadratic-in-Spin Hamiltonian at NNLO in the post-Newtonian framework 2022

Manoj K. Mandal,[a] Pierpaolo Mastrolia,[b,a] Raj Patil,[c,d,e] Jan Steinhoff[c]

☑ Automated in-house codes

Aim to publish the code in future

Status of Higher Order PN Corrections

PN order	0	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5
	N	1PN		2PN		3PN		4PN		5PN		6PN	
			LO SO		NLO SO		NNLO SO		N3LO SO		N4LO SO		NLO SO
				LO S2		NLO S2		NNLO S2		N3LO S2		N4LO S2	
							LO S3		NLO S3		N2LO S3		N3LO S3
								LO S4		NLO S4		N2LO S4	
											LO S5		NLO S5
												LO S6	

Kim, Levi, Yin (2022)

MKM, Mastrolia, Patil, Steinhoff (2022)

Levi, Yin (2022)

MKM, Mastrolia, Patil, Steinhoff (2022)

Secondment at UCLA

☑ Currently, I am having my secondment at UCLA with Prof. Zvi Bern

Calculating supergravity divergences at high loop order

Color Kinematics Duality

Generalized double copy

Building integrands using maximal cuts

$$(\mathcal{N}=5 \text{ sugra}) \quad : \quad (\mathcal{N}=4 \text{ sYM}) \otimes (\mathcal{N}=1 \text{ sYM})$$

Other Activities

- ✦ Successfully organised the international conference on the EFT methods from bound states to binary systems
- ✦ On the Editorial board of the Proceedings of the MathemAmplitudes Conference, 2019
- ✦ Presented several talks in international conferences
- ✦ Co-supervised Giacomo Brunello for his Master's Thesis

PoS PROCEEDINGS OF SCIENCE

MA2019 --- MathemAmplitudes 2019: Intersection Theory & Feynman Integrals

Member of the Editorial Board

arXiv:2211.01321v1 [hep-th] 16 Oct 2022

UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Fisica e Astronomia "Galileo Galilei"

Master Degree in Physics

Final Dissertation

Effective Field Theory Approach to General Relativity and Feynman Diagrams for Coalescing Binary Systems

Thesis supervisor
Prof. Pierpaolo Mastrolia
Thesis co-supervisor
Dr. Manoj K. Mandal

Candidate
Giacomo Brunello

Academic Year 2021/2022

MathemAmplitudes 2023

May, 2023

Università degli Studi di Padova — INFN — Fellini H2020 MSCA COFUND G.A. 754496

Organizers

M. K. Mandal
P. Mastrolia
R. Groeber
…..

Conclusion

- Novel Algebraic Property Unveiled
  - The algebra of Feynman Integrals is controlled by intersection numbers
  - Intersection Numbers : Scalar Product/Projection between Feynman Integrals
  - Useful for both Physics and Mathematics
- Applications to GW and Collider phenomenology
  - muon-electron scattering at NNLO has been obtained
  - top-pair production from quark annihilation has been computed analytically
  - progress in understanding spin effects in the compact binaries
  - A number of observables e.g binding energy, scattering angle has been computed to high precision
- Future and ongoing works
  - Progress towards computing intersection number using relative twisted co-homology
  - computation of tidal effects to higher PN order in case of compact binaries

Collaboration and Networking

Zvi Bern
Alessandro Broggio
Andrea Ferroglia
Saiei Matsubara-Heo
Adrian Signer
Jan Steinhoff
Nobuki Takayama
Francesco Tramontano

Seva Chestnov
Hjalte Frellesvig
Federico Gasparotto
Luca Mattiazzi
Jonathan Ronca
William J. Torres-Bobadilla

Giacomo Brunello
Giulio Crisanti
Raj Patil

Networking

Sergio Cacciatori
Yoshiaki Goto
Keiji Matsumoto
Tiziano Perraro

Donato Bini
Thibault Damour
Stefano Foffa
Riccardo Sturani

Thank You