

**1st General Meeting of COST Action
COSMIC WISPERS (CA21106)**
Bari - September 5-8, 2023

Birefringence in CMB anisotropies due to cosmological pseudoscalar fields



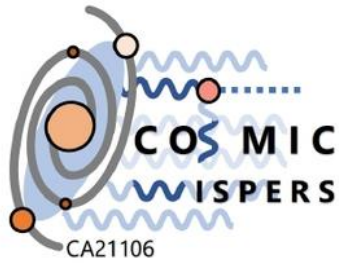
**Vatican
Observatory**



Matteo Galaverni

Based on a work with:

Fabio Finelli and **Daniela Paoletti** (INAF/OAS Bologna & INFN Bologna)
Phys.Rev.D **107** (2023) 8, 083529 (arXiv 2301.07971 [astro-ph.CO])



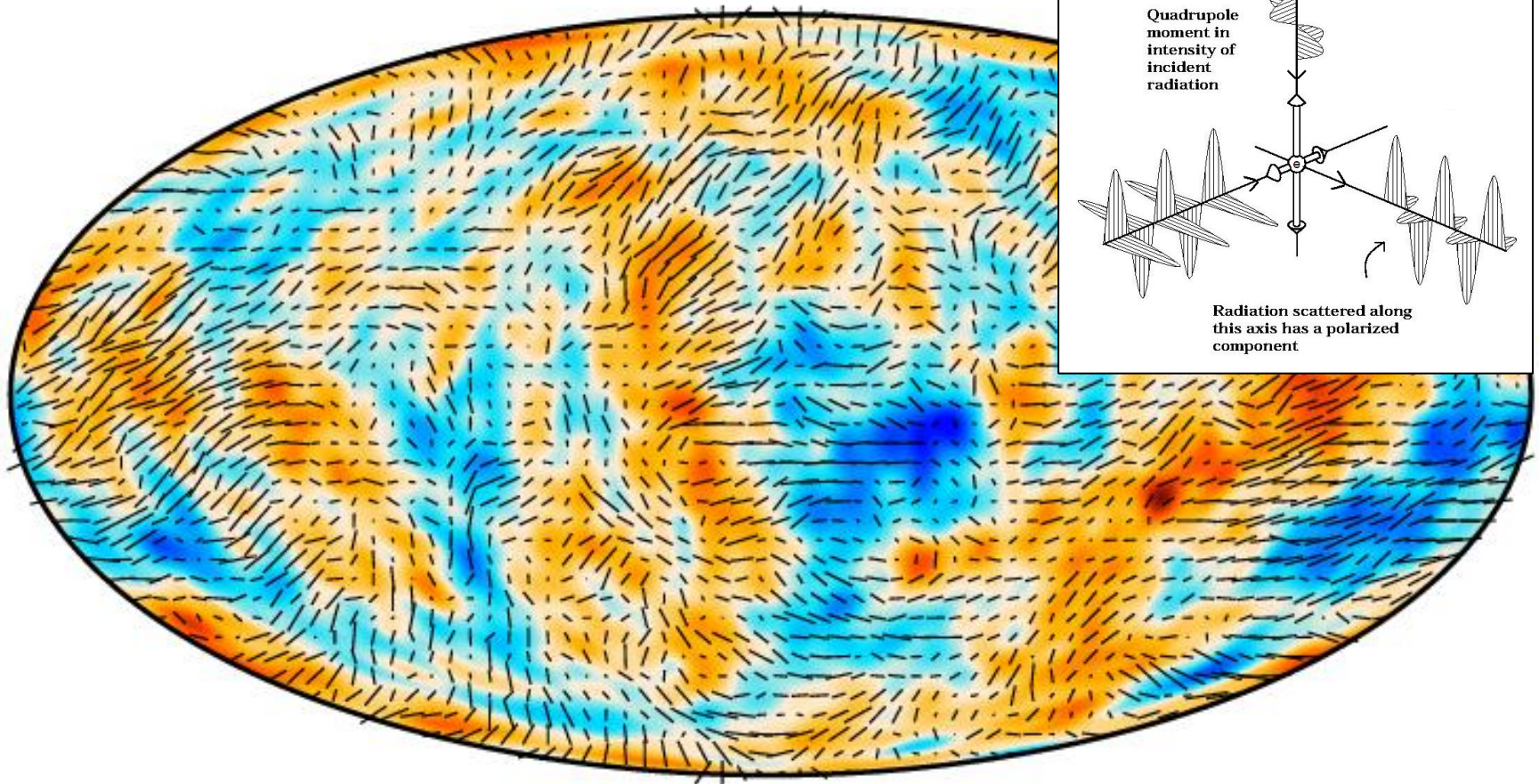
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Plan of the talk

- **CMB polarization anisotropies & cosmic birefringence;**
- **Redshift/time evolution** of cosmic birefringence and CMB;
- **Cosmological pseudoscalar field: Early Dark Energy (EDE), Quintessence (DE) or axion-like Dark Matter (DM);**
- **Conclusions.**

CMB polarization anisotropies



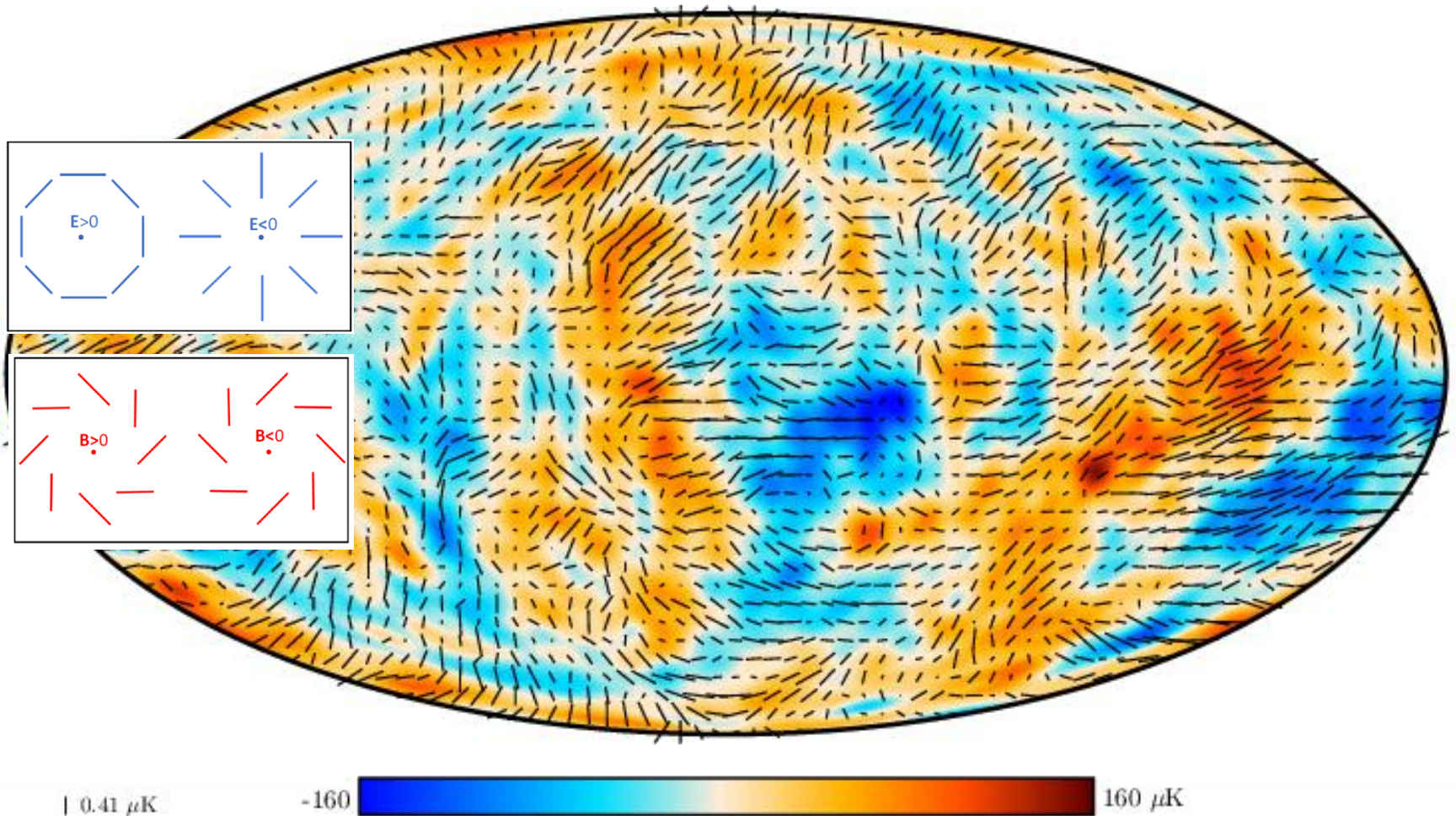
| 0.41 μK

-160

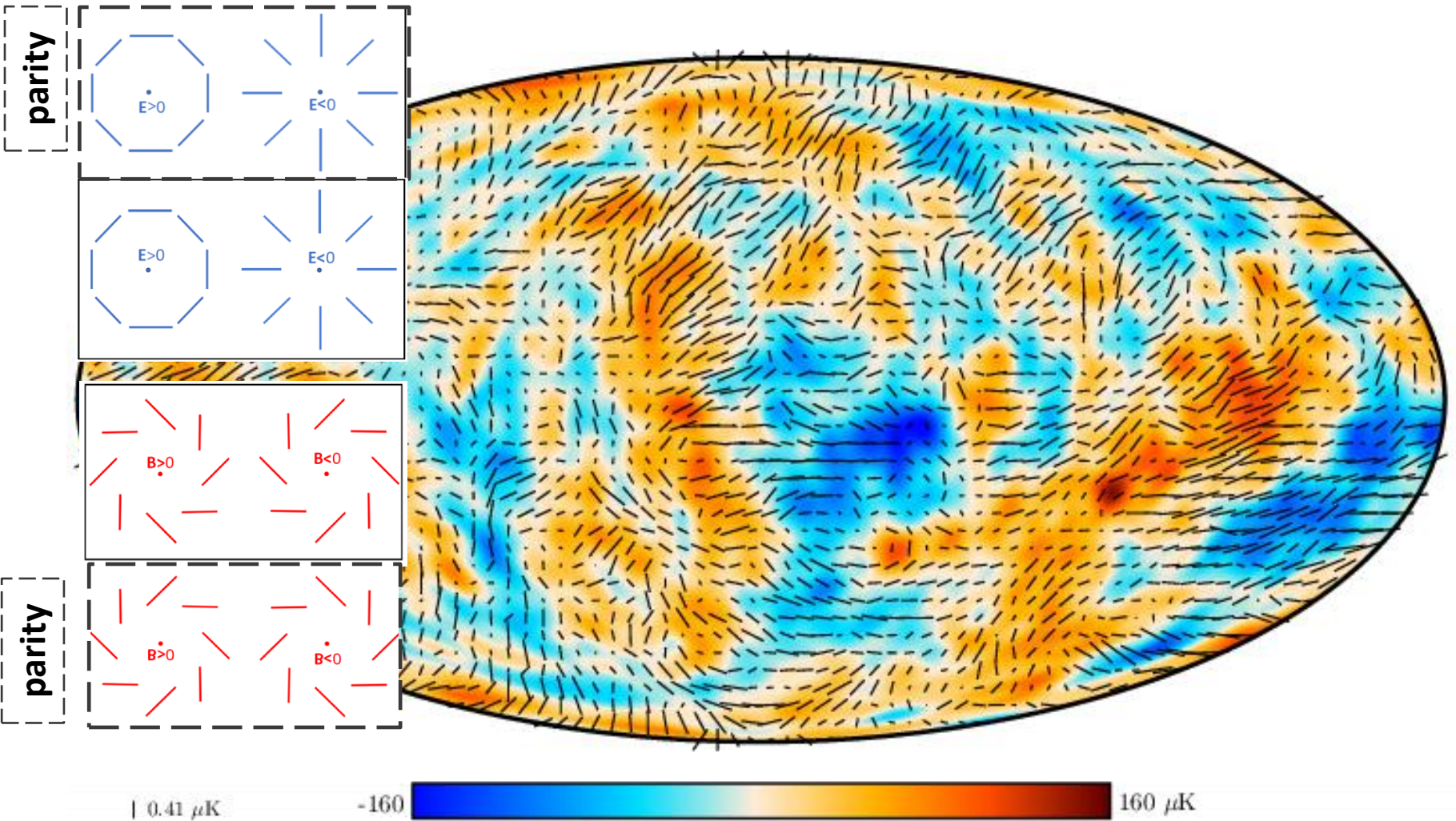
160 μK

[2018 Planck map of the polarized CMB anisotropies]

CMB polarization anisotropies

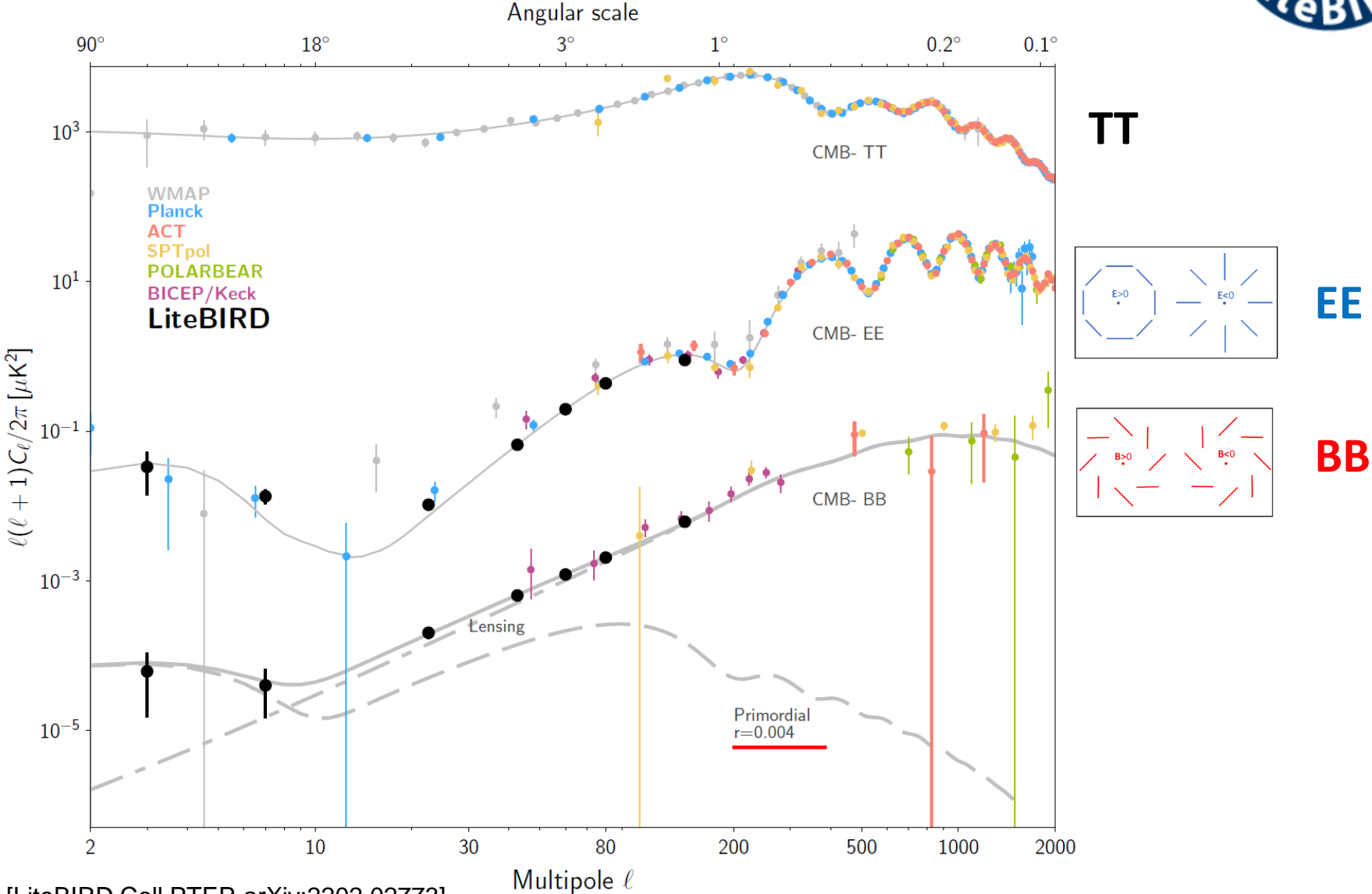


CMB polarization anisotropies



Present measurements + expected sensitivity for LiteBIRD

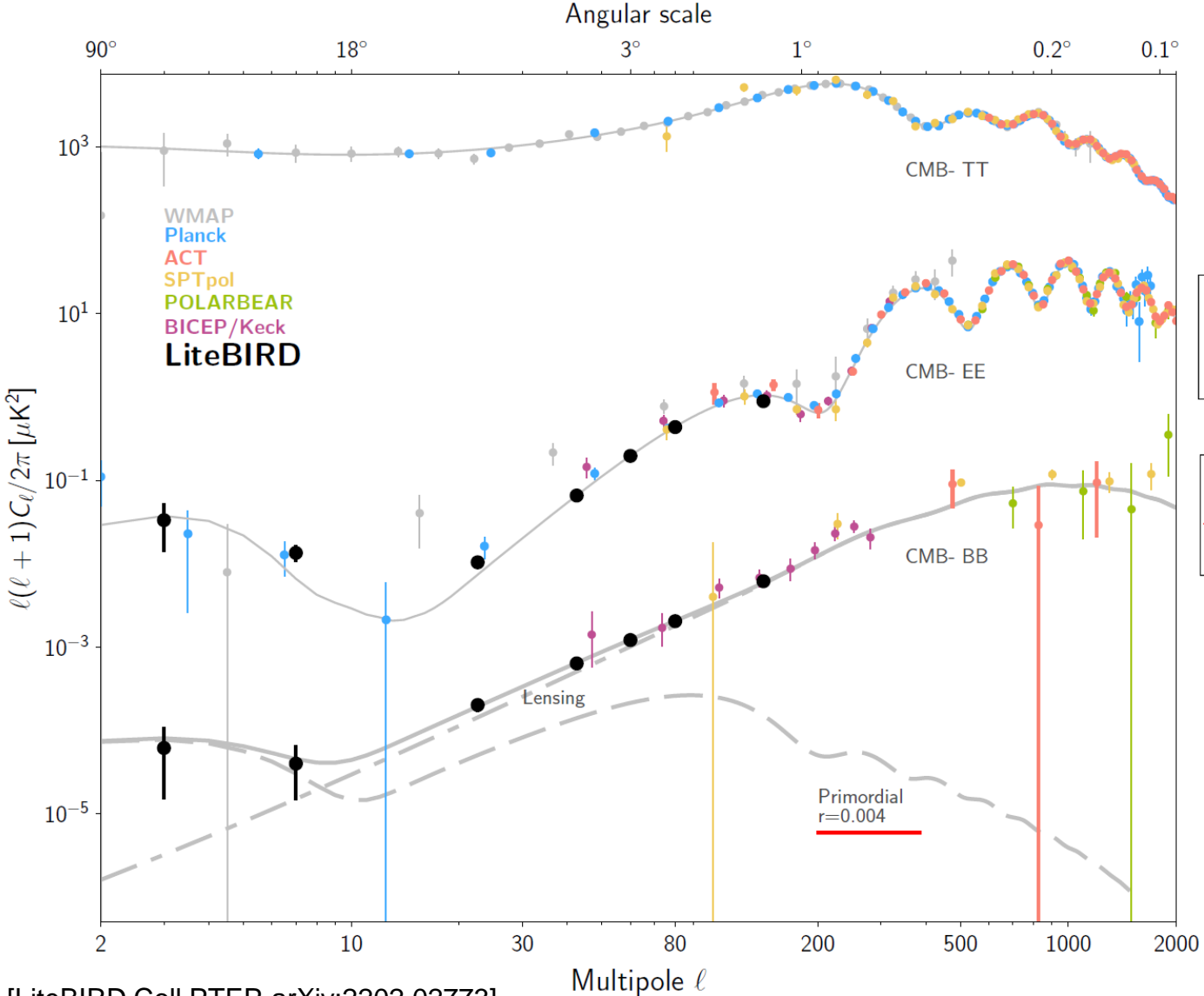
Lite (Light) satellite for the study of B-mode polarization and Inflation from cosmic background Radiation Detection



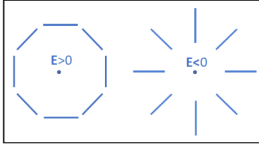
[LiteBIRD Coll PTEP-arXiv:2202.02773]

Present measurements + expected sensitivity for LiteBIRD

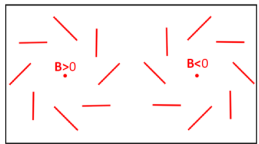
Lite (Light) satellite for the study of B-mode polarization and Inflation from cosmic background Radiation Detection”



TT



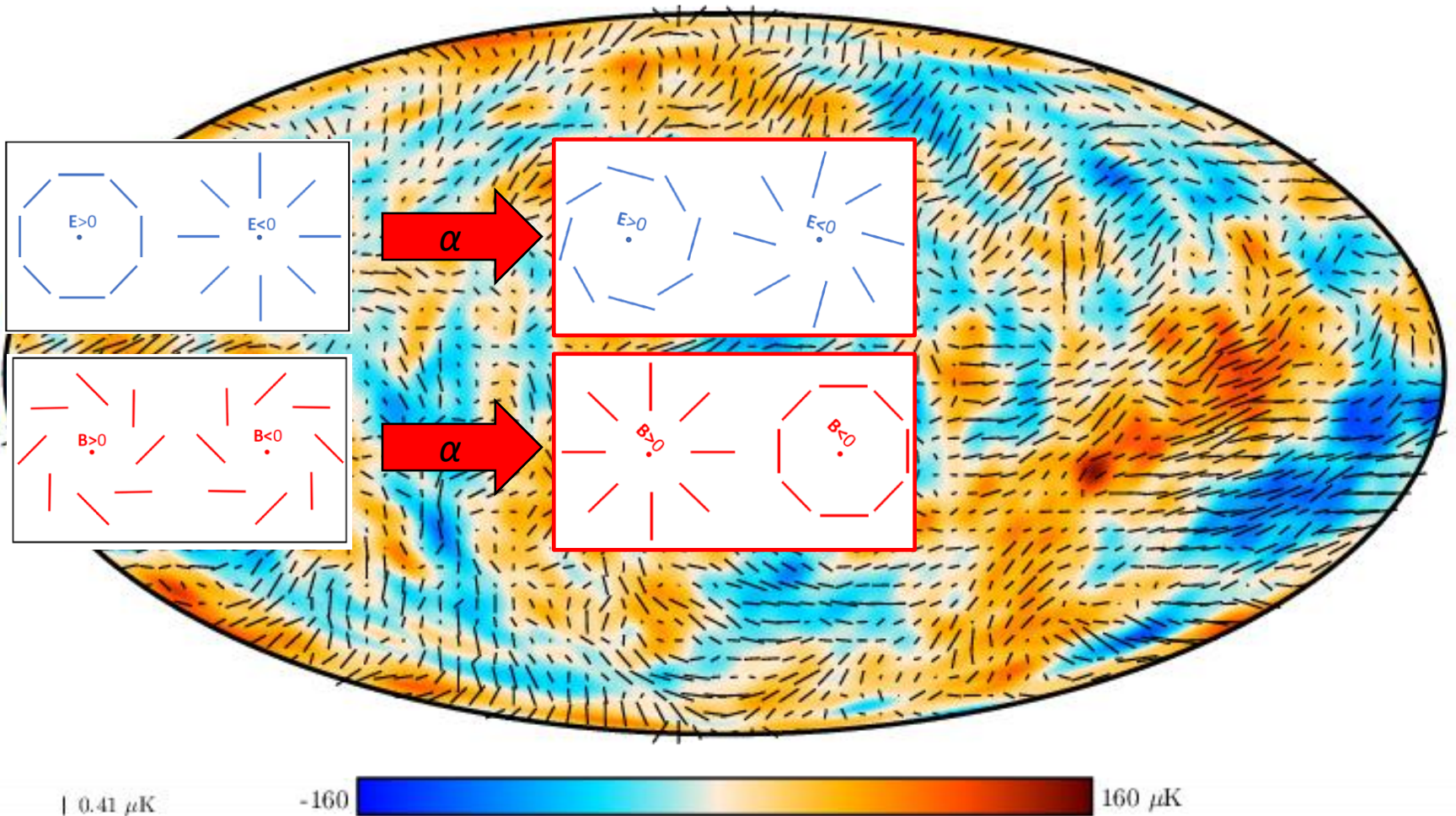
EE



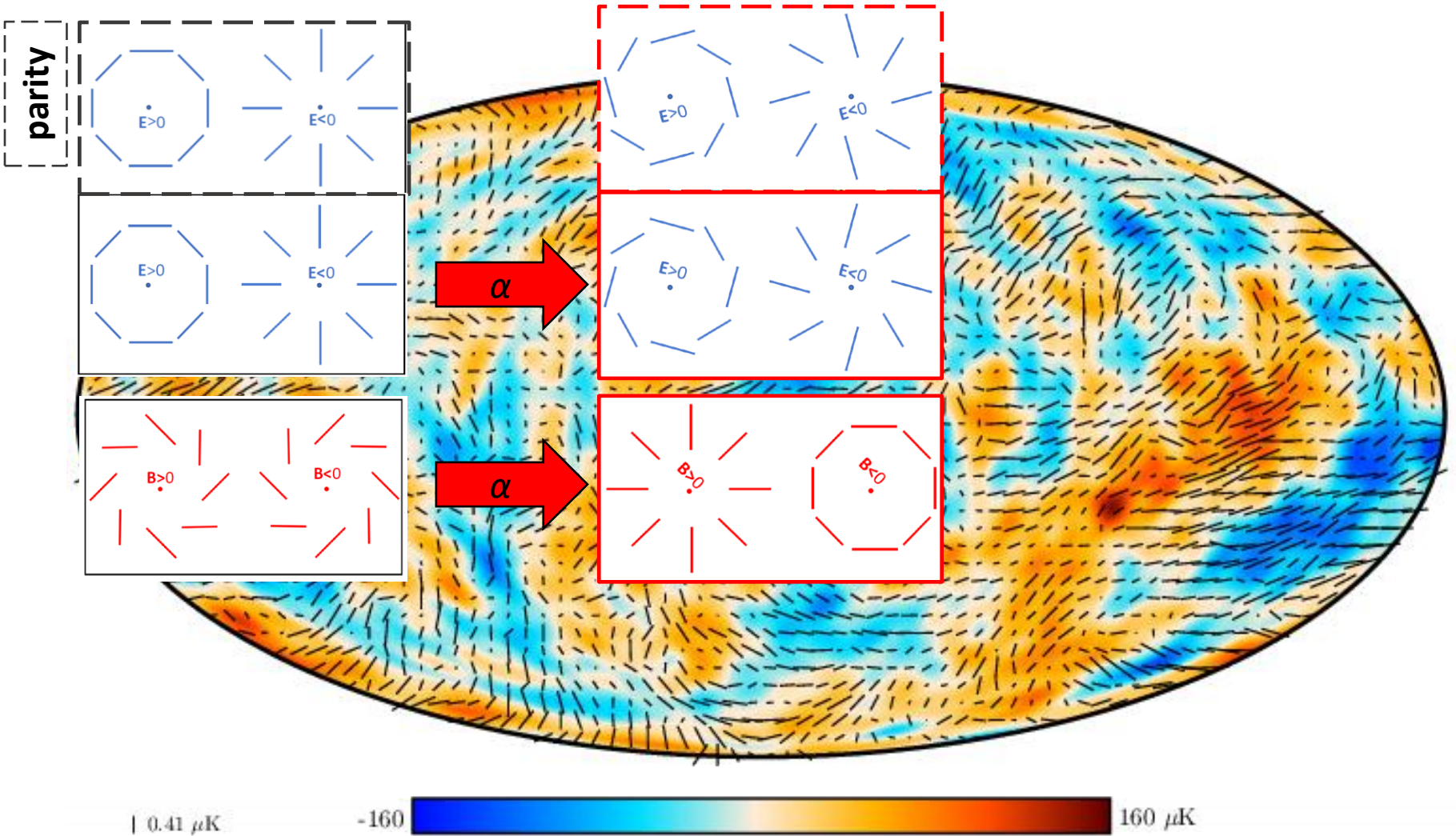
BB

- Potential sources of B mode:**
- Gravitational lensing;
 - Inflationary gravitational w.;
 - Primordial magnetic fields;
 - Cosmic strings and other topological defects;
 - Cosmic birefringence: coupling of CMB photons with a pseudoscalar field; ...

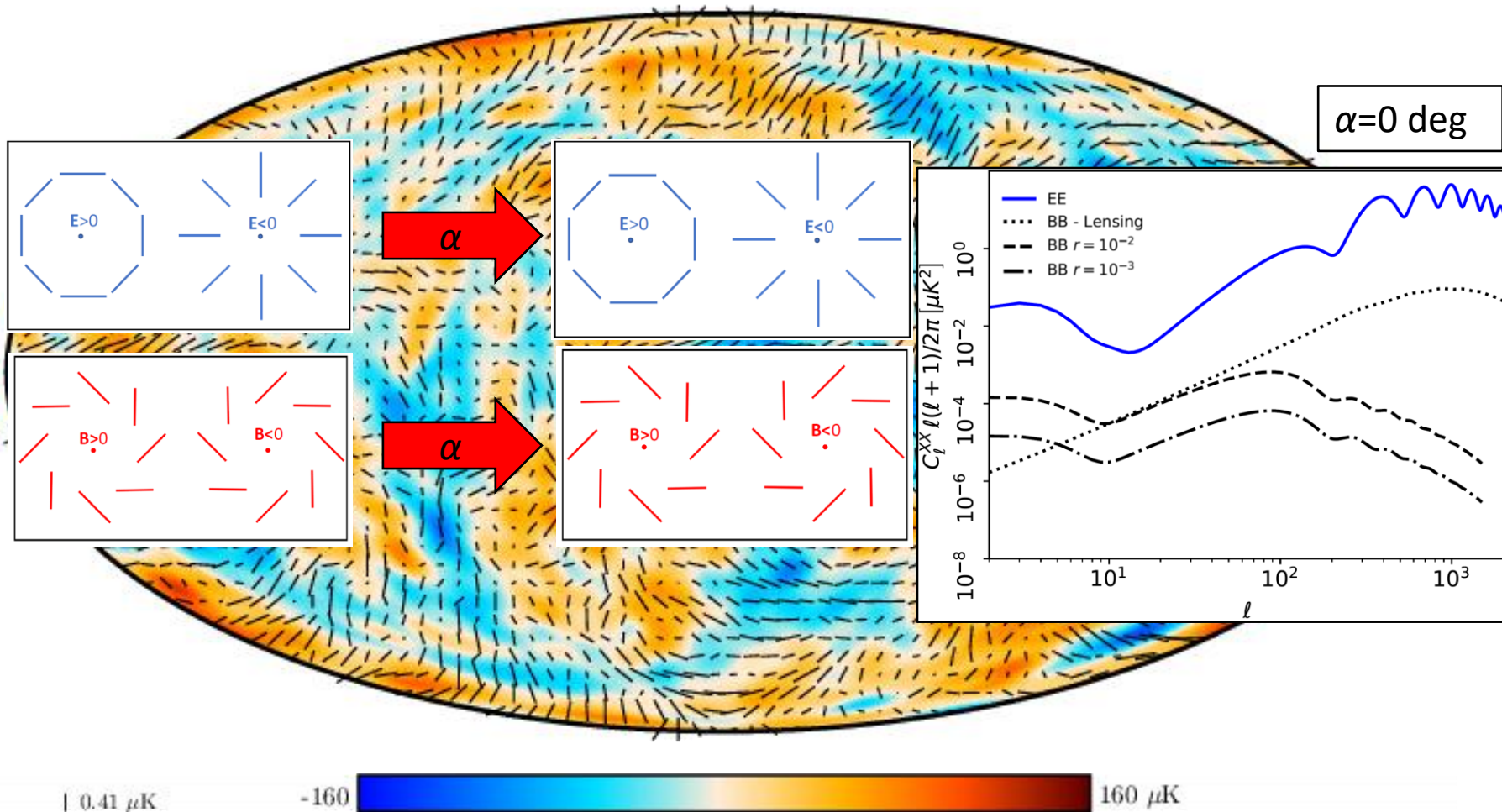
Cosmic Birefringence



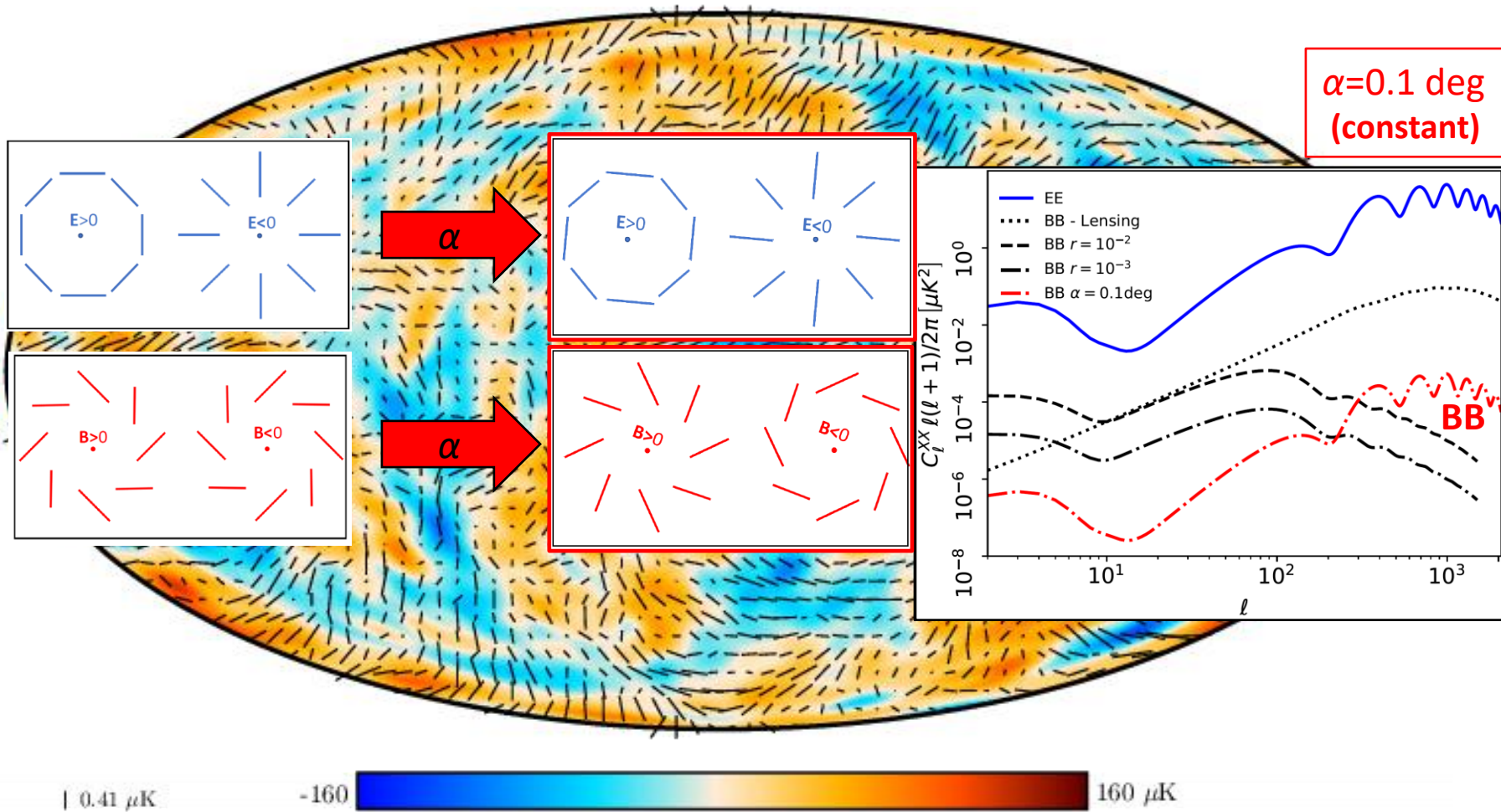
Cosmic Birefringence



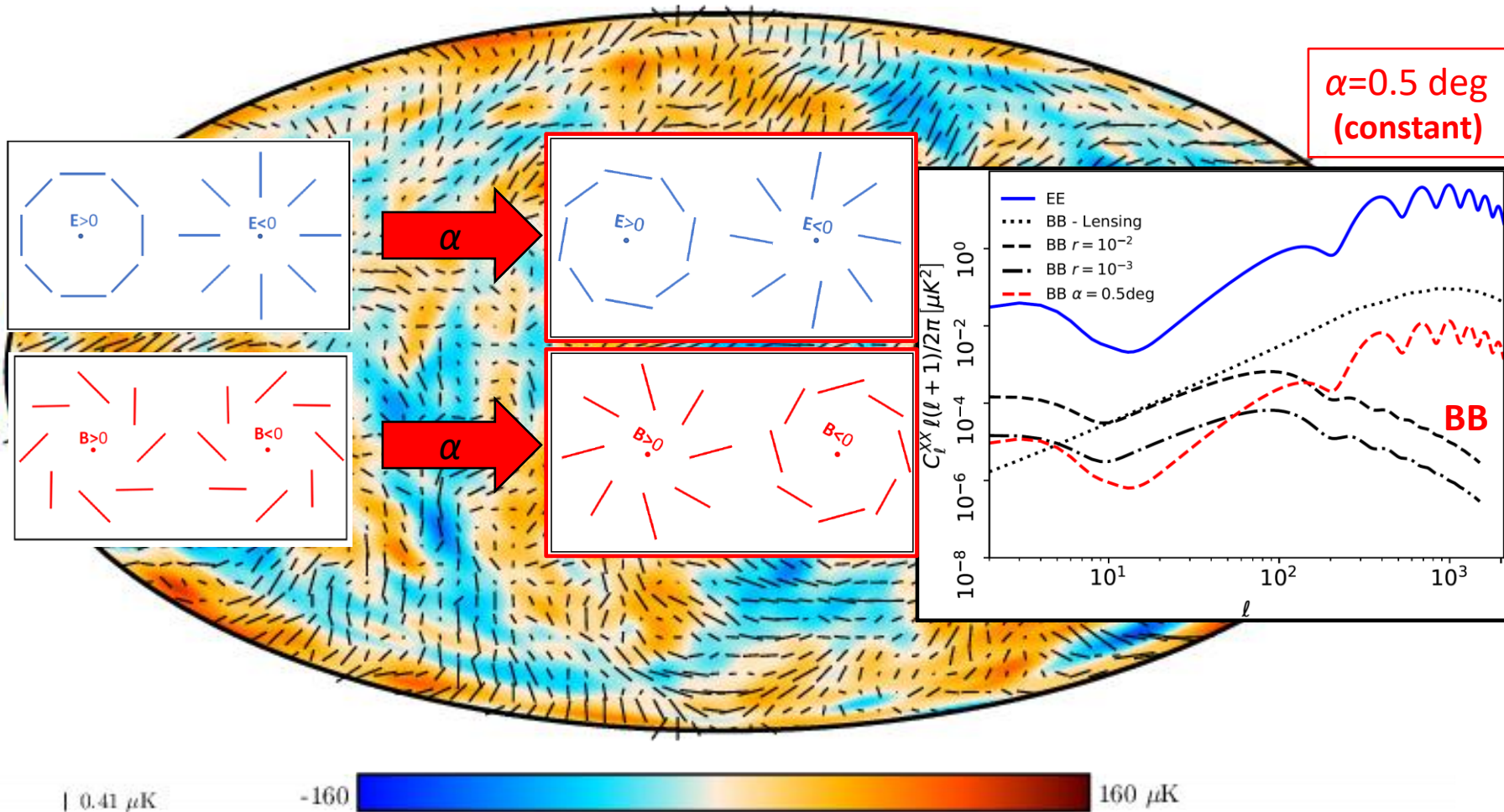
Cosmic Birefringence



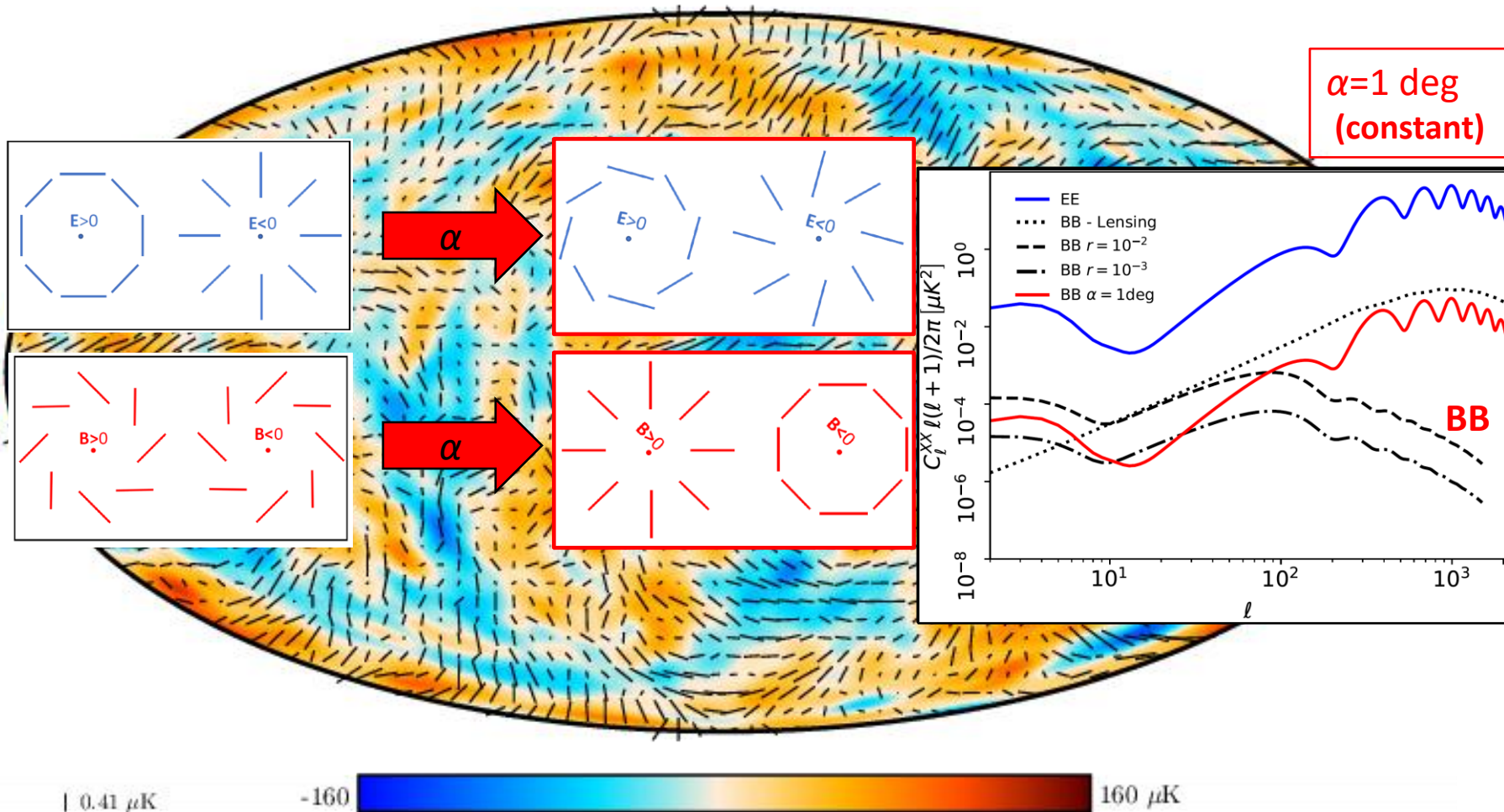
Cosmic Birefringence



Cosmic Birefringence



Cosmic Birefringence



α [deg]: isotropic and constant in redshift

For a cosmological birefringence:

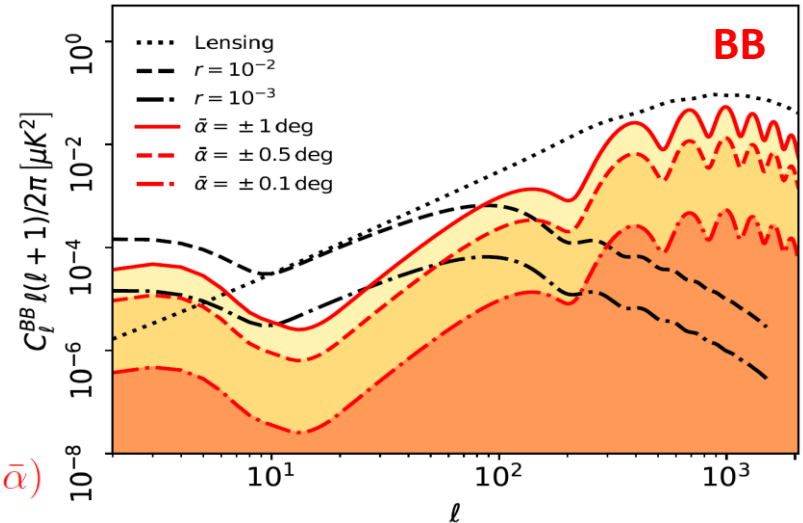
-*isotropic*

-*constant in redshift*

→ only “total rotation” is important:

$$\bar{\alpha} \equiv \alpha(\eta_{\text{rec}}) - \alpha(\eta_0) = \frac{g\phi}{2} [\phi(\eta_{\text{rec}}) - \phi(\eta_0)]$$

$$C_\ell^{BB,\text{obs}} = C_\ell^{EE,\text{rec}} \sin^2(2\bar{\alpha})$$



α [deg]: isotropic and constant in redshift

For a cosmological birefringence:

-*isotropic*

-*constant in redshift*

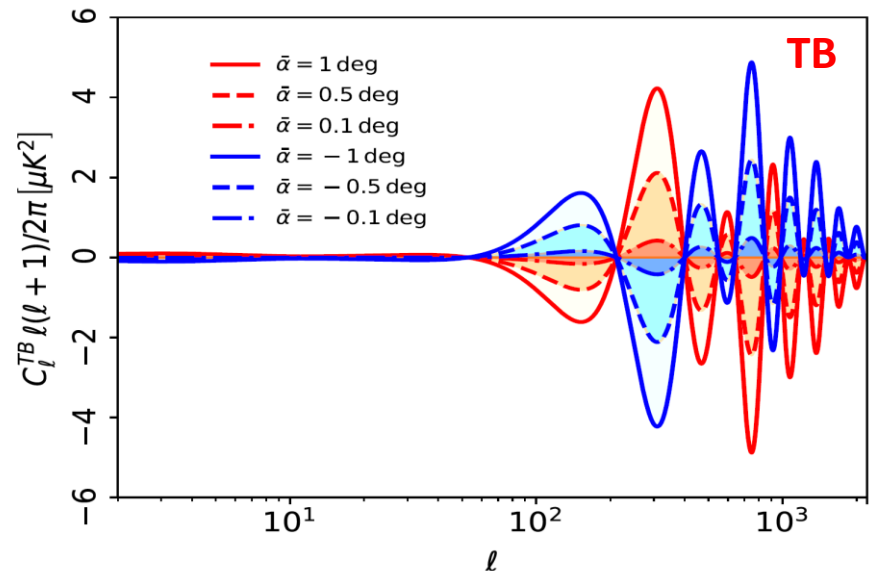
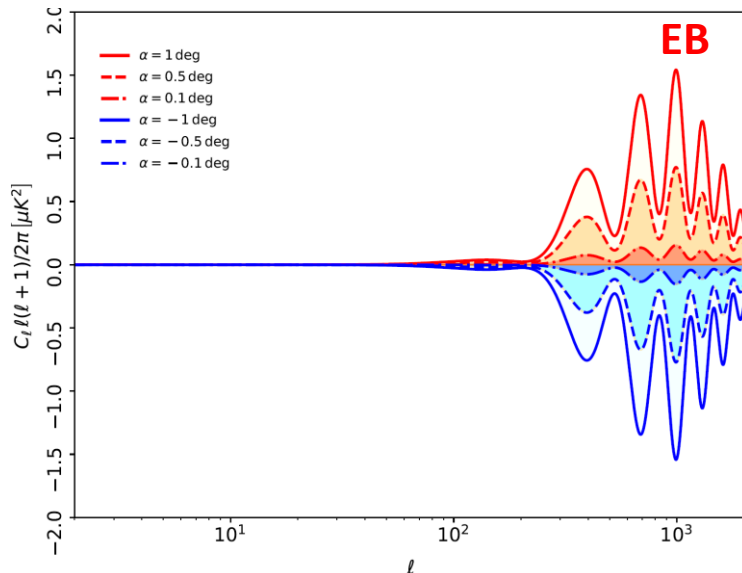
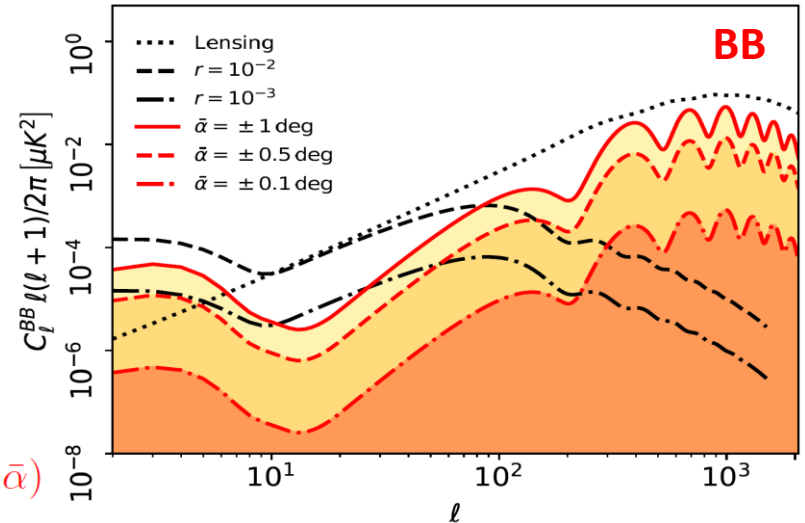
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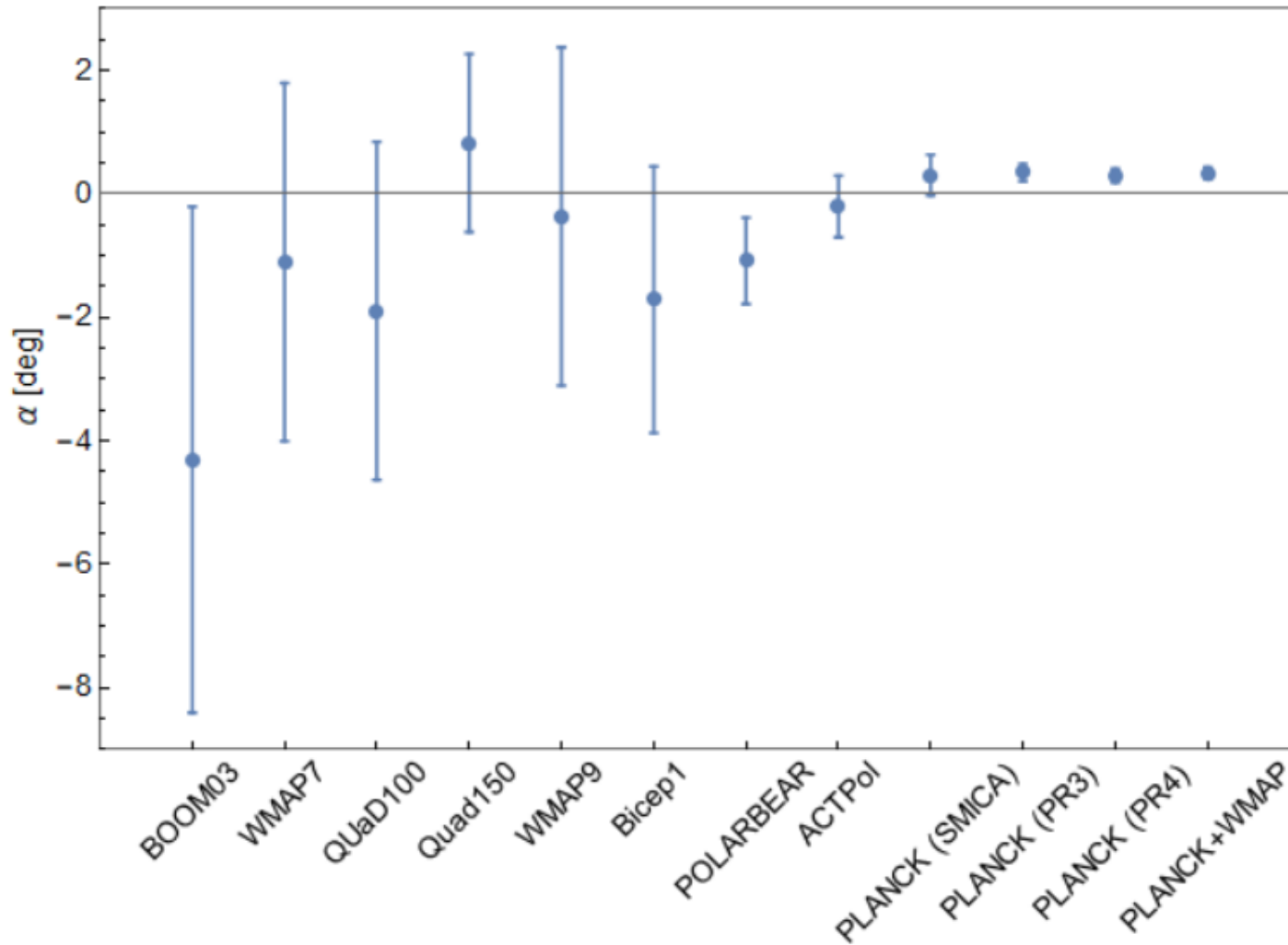
$$C_\ell^{BB,\text{obs}} = C_\ell^{EE,\text{rec}} \sin^2(2\bar{\alpha})$$

$$C_\ell^{EB,\text{obs}} = \frac{1}{2} \left(C_\ell^{EE,\text{rec}} - C_\ell^{BB,\text{rec}} \right) \sin(4\bar{\alpha})$$

$$C_\ell^{TB,\text{obs}} = C_\ell^{TE,\text{rec}} \sin^2(2\bar{\alpha})$$

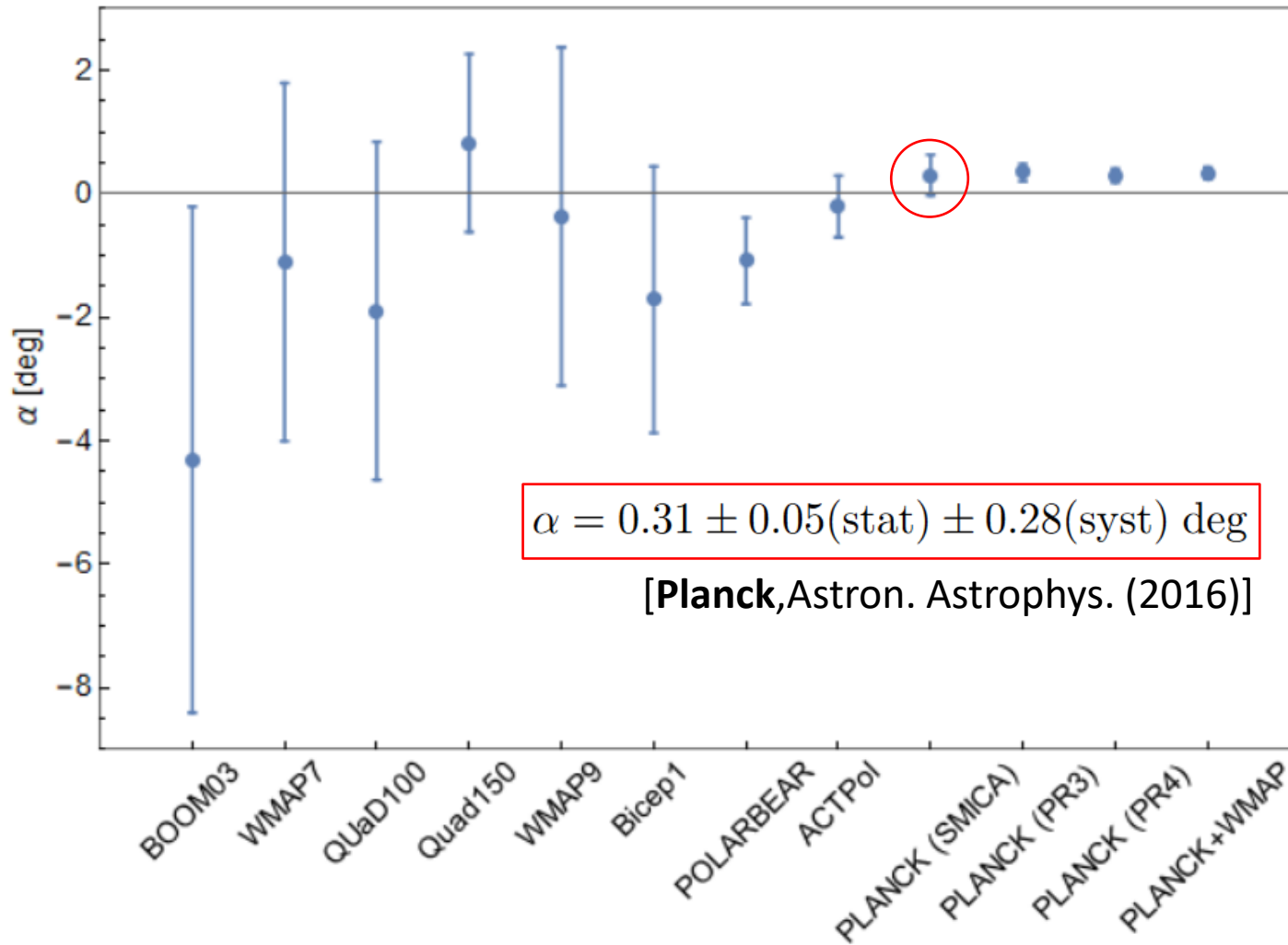


Constraints for α (*isotropic and constant in z*)



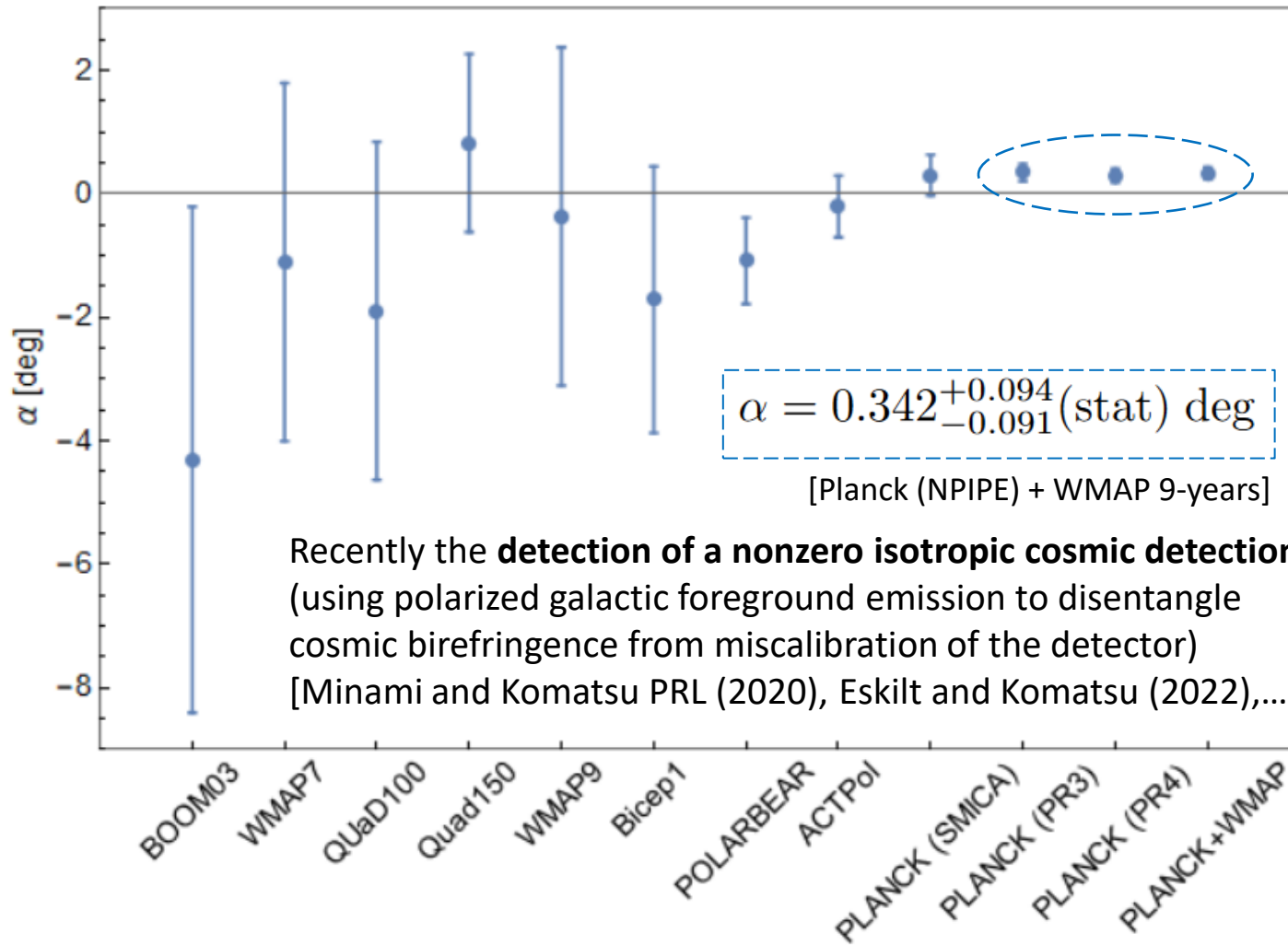
CMB polarization experiments constraints on isotropic cosmological birefringence as reviewed in Kaufman *et al.* (2016), Planck (2016), Greco *et al.* (2023), Williams (2023).

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CMB polarization experiments constraints on isotropic cosmological birefringence as reviewed in Kaufman *et al.* (2016), Planck (2016), Greco *et al.* (2023), Williams (2023).

α [deg]: *isotropic and z dependent*

Boltzmann equation for linear polarization **with cosmic birefringence** (Liu, Lee and Ng [PRL 2006], Finelli and Galaverni [PRD 2009]):

$$\Delta'_{Q\pm iU}(k, \eta) + ik\mu\Delta_{Q\pm iU}(k, \eta) = -n_e\sigma_T a(\eta) \left[\Delta_{Q\pm iU}(k, \eta) + \sum_m \sqrt{\frac{6\pi}{5}} {}_{\pm 2}Y_2^m S_P^{(m)}(k, \eta) \right] \mp i2\alpha'(\eta)\Delta_{Q\pm iU}(k, \eta)$$

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we follow the
line-of-sight

$$C_\ell^{EE/BB/EB} = (4\pi)^2 \frac{9(\ell+2)!}{16(\ell-2)!} \int k^2 dk [\Delta_{E/B/E}(k, \eta_0)\Delta_{E/B/B}(k, \eta_0)]$$

strategy [Seljak and Zaldarriaga (1996)]

where the **source terms for scalar perturbations** are modified:

$$\Delta_E(k, \eta_0) = \int_0^{\eta_0} d\eta g(\eta) S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \cos 2[\alpha(\eta) - \alpha(\eta_0)]$$

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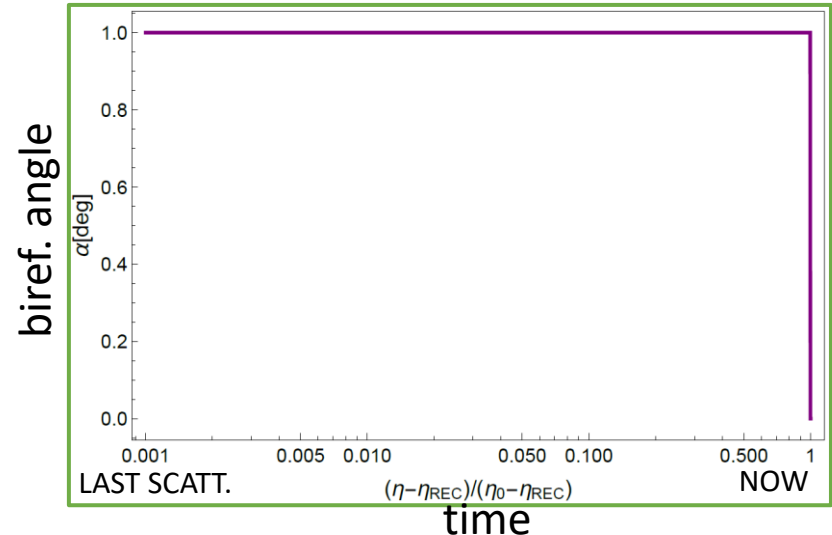
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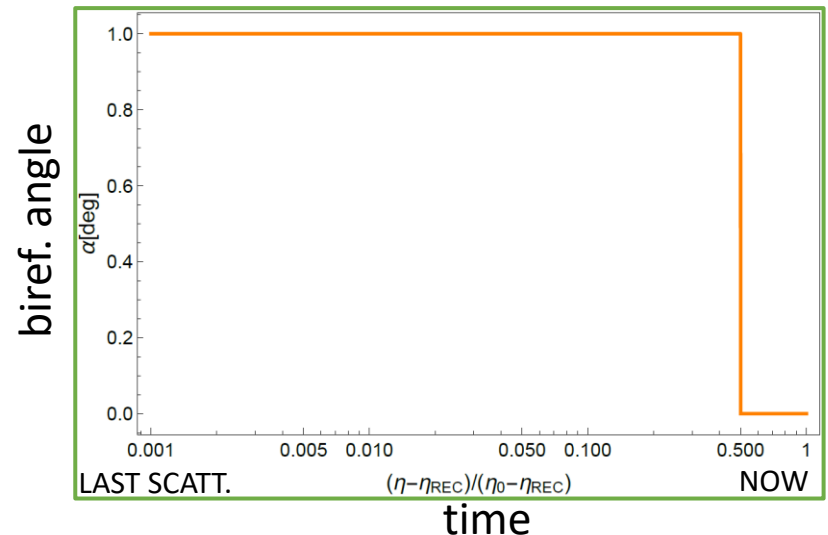
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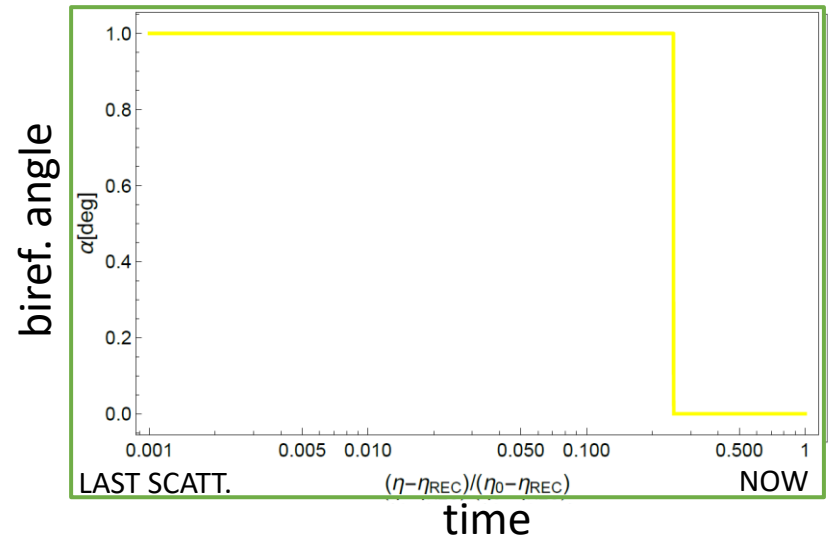
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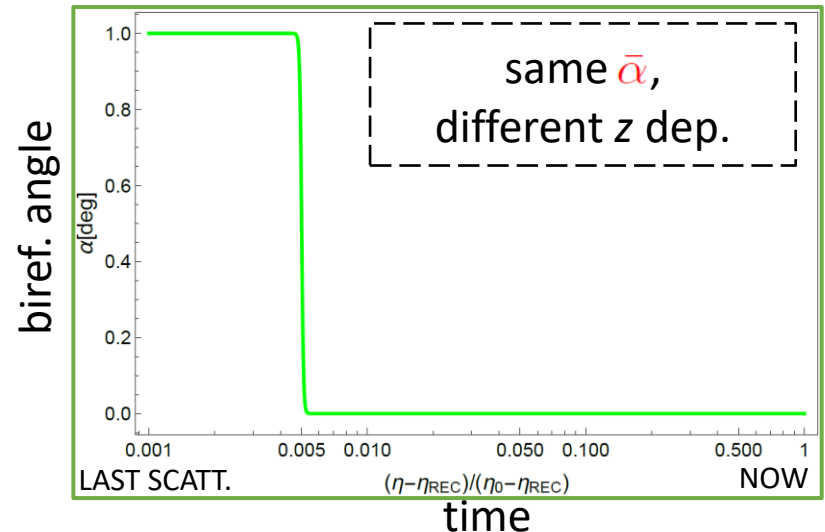
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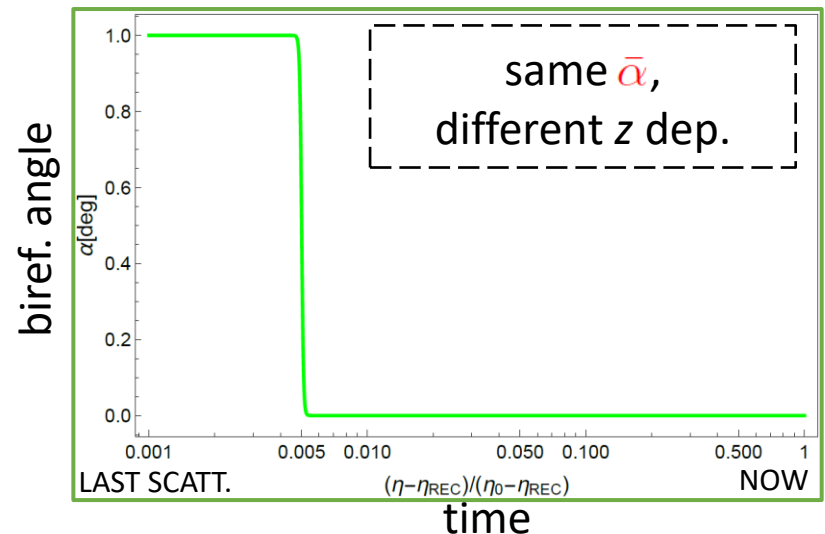
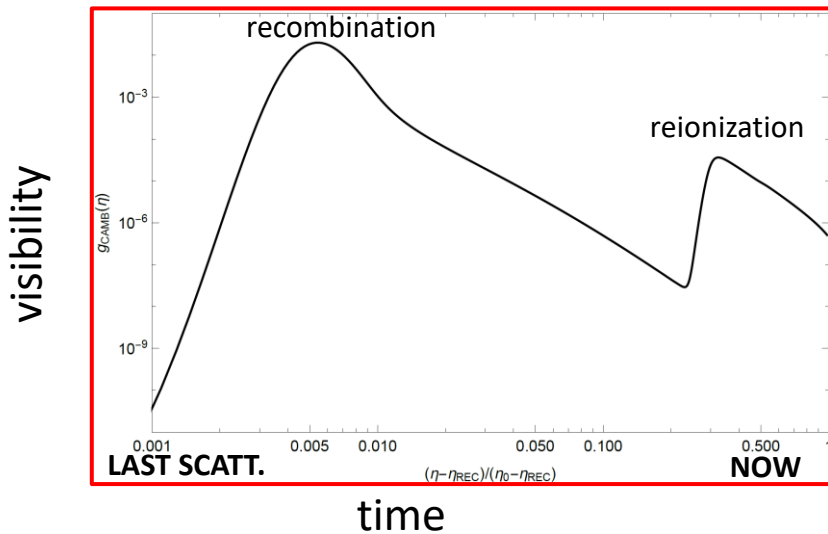
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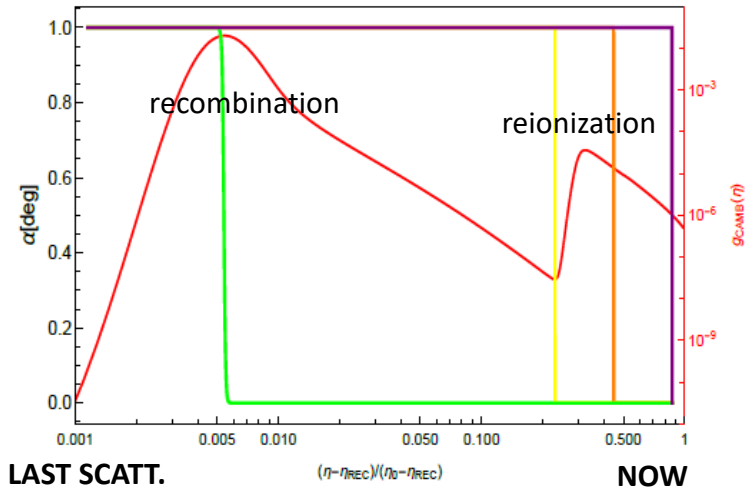
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α [deg]: isotropic and z dependent

- same $\bar{\alpha} = 1$ deg
- but linear polarization rotation happens at different epochs!

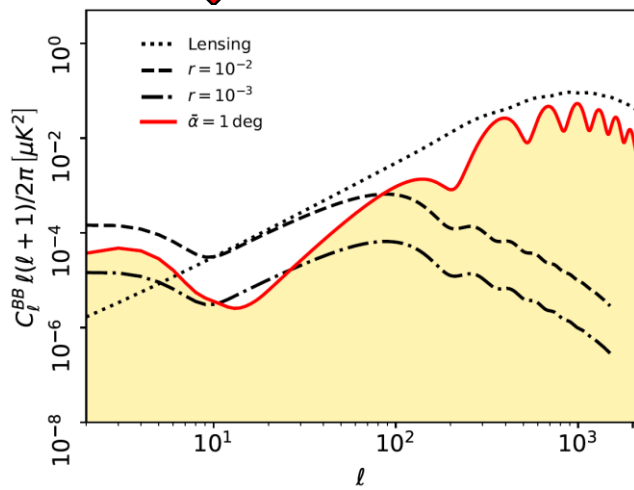


LAST SCATT.

NOW



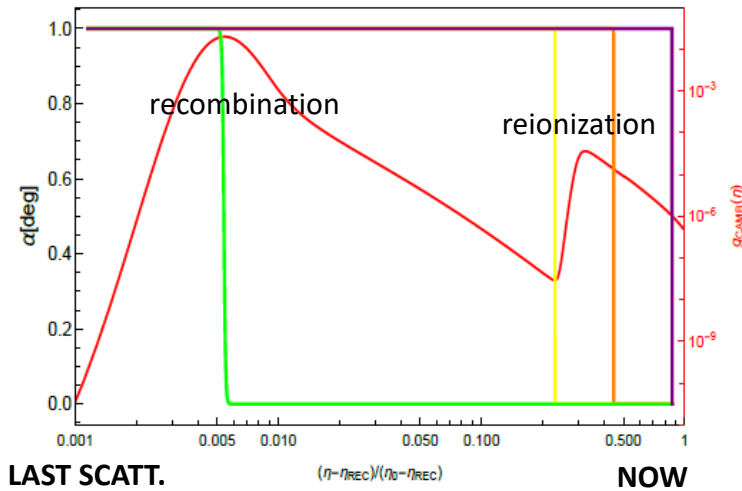
$\bar{\alpha} = 1$ deg



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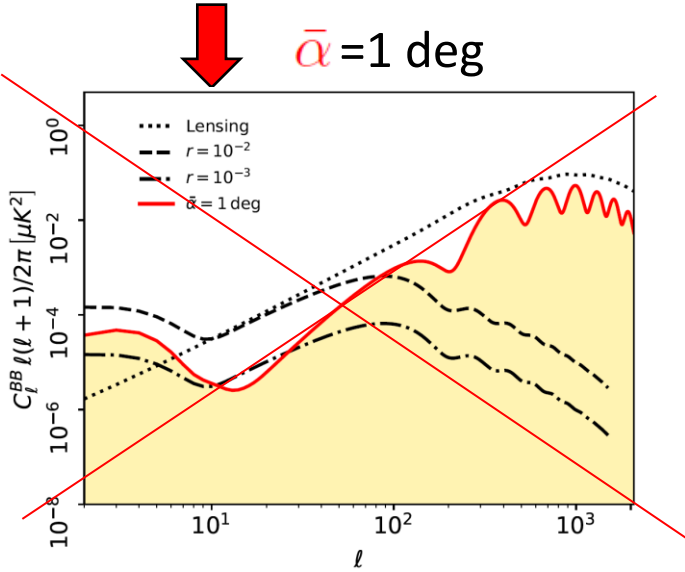
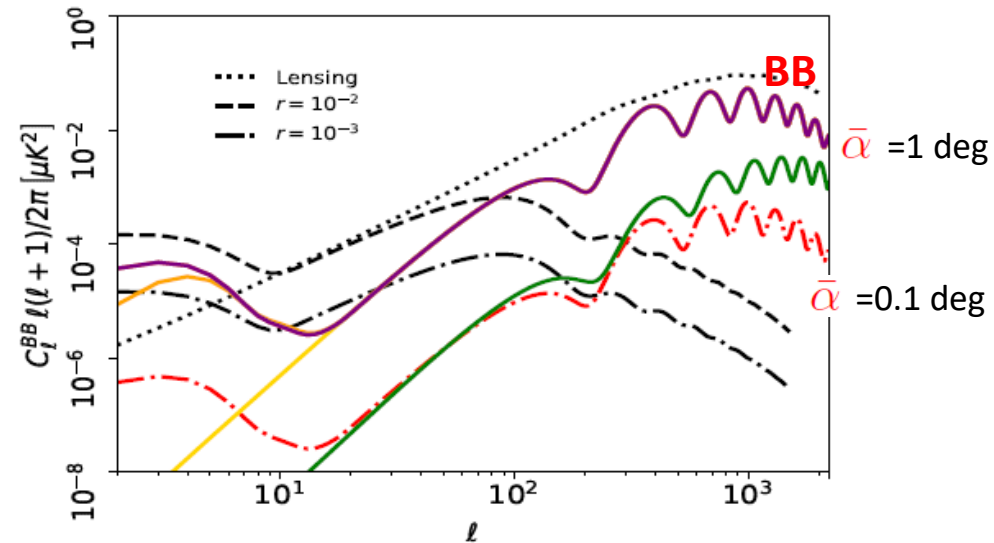


LAST SCATT.

NOW



Integrating the Boltzmann equation along the line-of-sight:



$\bar{\alpha} = 1$ deg

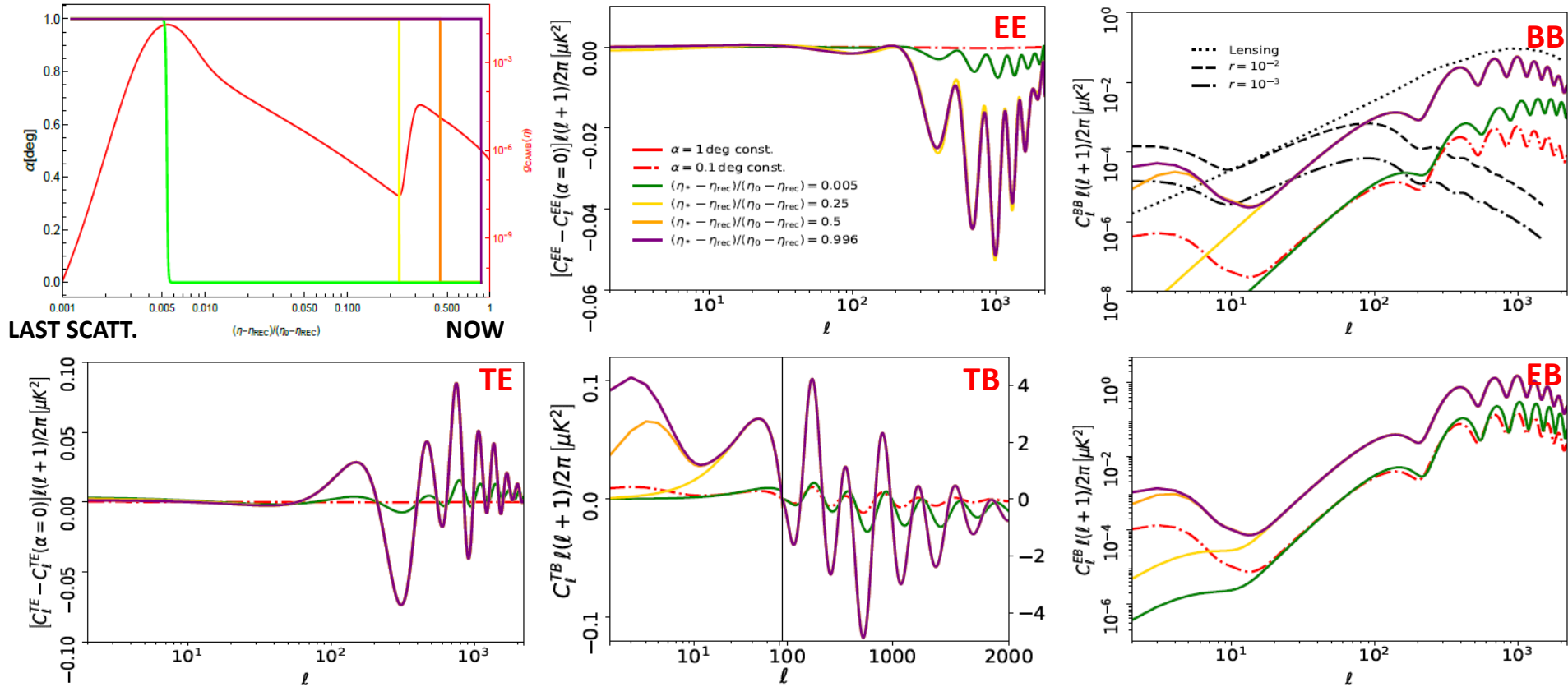
~~$$C_l^{BB, \text{obs}} = C_l^{EE, \text{rec}} \sin^2(2\bar{\alpha})$$~~

(in agreement only if $\bar{\alpha}$ is constant from last scattering to today)

[see also Nakatsuka, Namikawa and Komatsu PRD 2022]

α [deg]: *isotropic* and *z dependent*

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- but linear polarization rotation happens at different epochs!



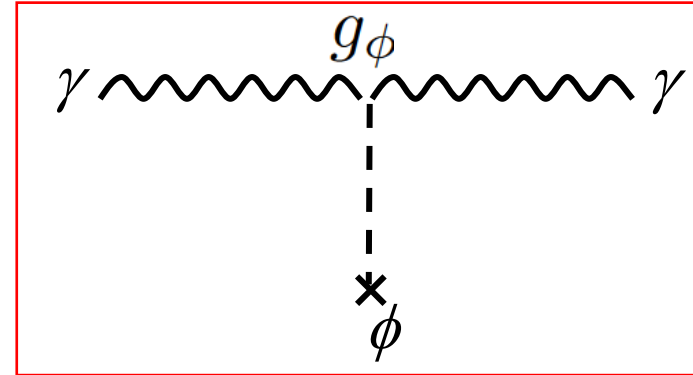
Cosmological pseudoscalar field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\nabla_{\mu}\phi\nabla^{\mu}\phi - V(\phi) - \frac{g_{\phi}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu},$$

Cosmological **pseudoscalar field** ϕ acting as:

- **Dark Matter** (e.g. axion-like particles);
- **Dark Energy** (e.g. ultralight pseudo Nambu-Goldstone bosons);
- **Early Dark Energy.**

$V(\phi)$



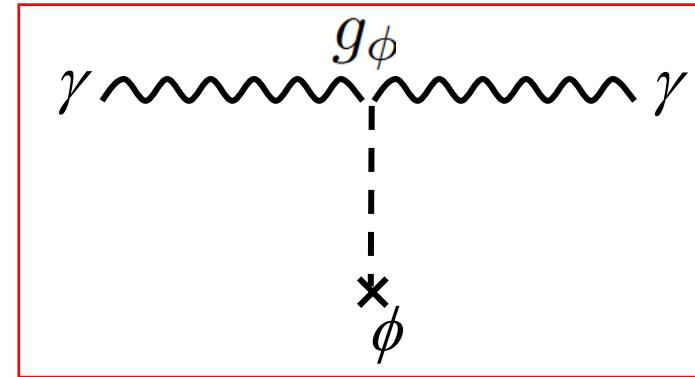
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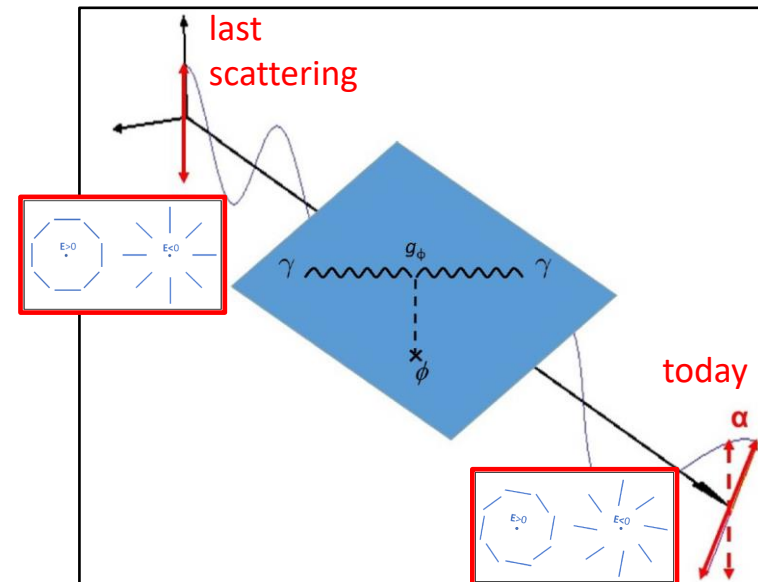
$V(\phi)$



$$\alpha(x) = \frac{g_{\phi}}{2} [\phi(x) - \phi(x_{em})],$$

rotation of the polarization plane
(single photon)

Carrol, Field and Jackiw [PRD 1990],
Harari and Sikivie [Phys. Lett. B 1992],...



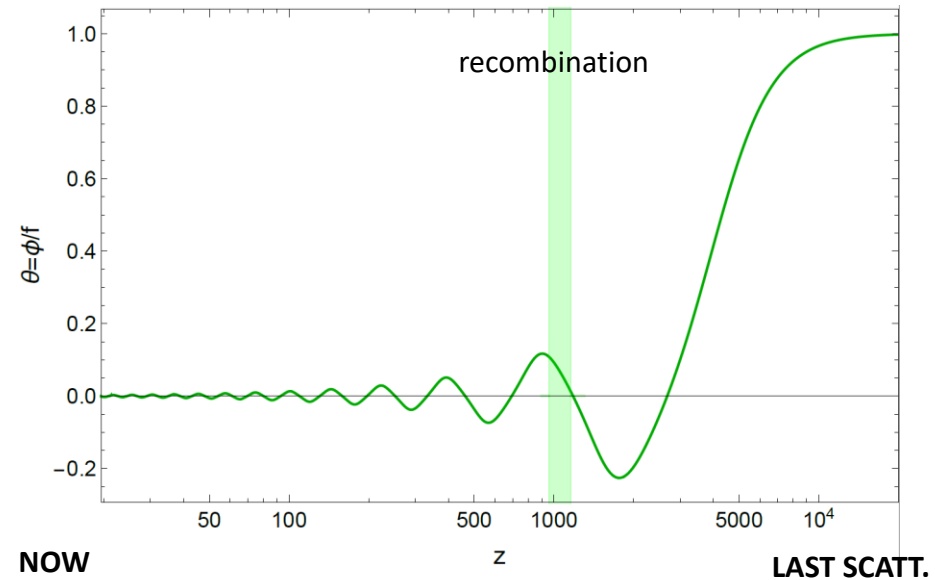
Early Dark Energy

$$V(\phi) = \Lambda^4 \left(1 - \cos \frac{\phi}{f}\right)^n$$

[fixed:

$n=2$, $\Lambda=0.417$ eV, $f=0.05 M_{\text{pl}}$,

$(\phi/f)_{\text{in}}=1$ and $(\dot{\phi}/f)_{\text{in}}=0$]

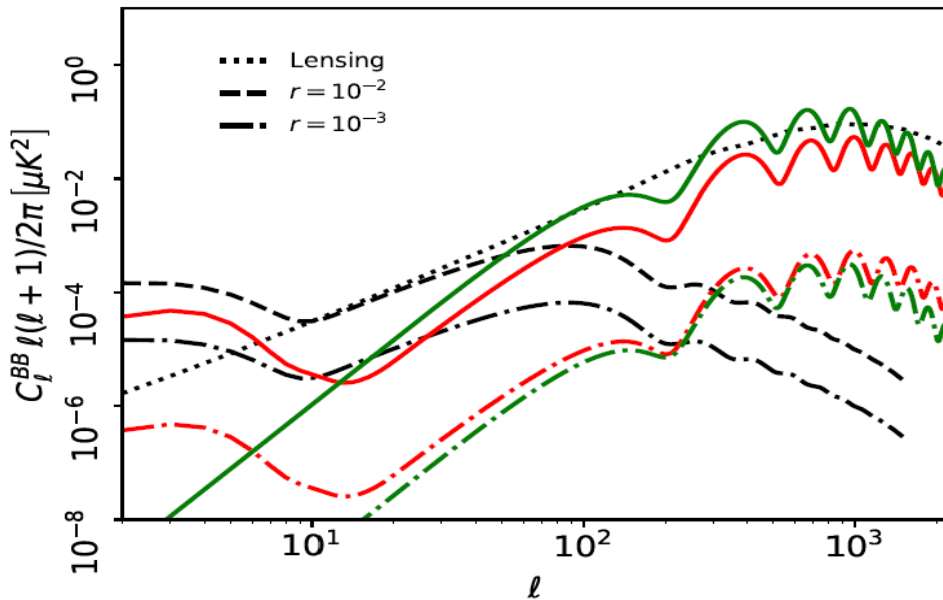
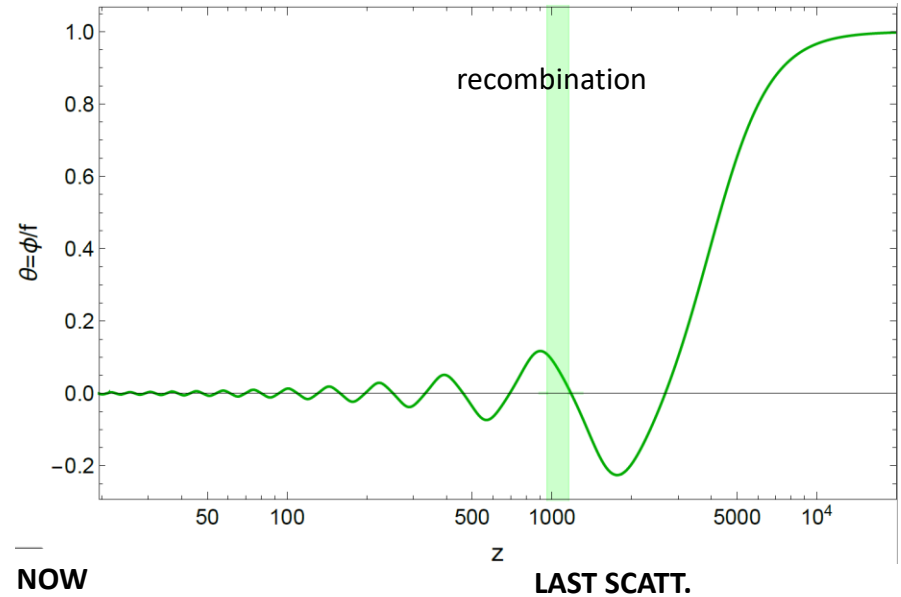


Early Dark Energy

$$V(\phi) = \Lambda^4 \left(1 - \cos \frac{\phi}{f}\right)^n$$

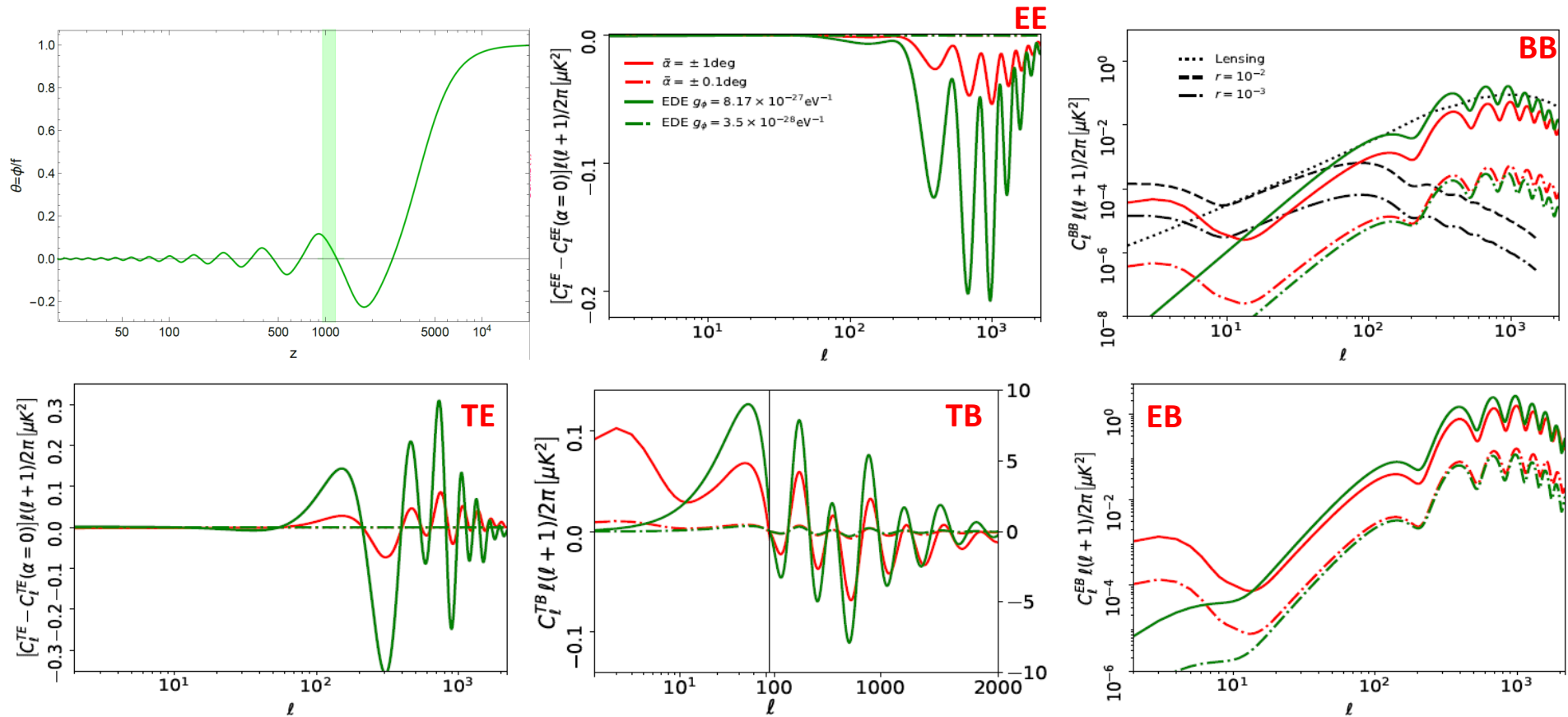
[fixed:

$n=2$, $\Lambda=0.417$ eV, $f=0.05 M_{\text{pl}}$,
 $(\phi/f)_{\text{in}}=1$ and $(\dot{\phi}/f)_{\text{in}}=0$]



- ← BB induced by EDE $g_\phi = 8.17 \times 10^{-18} \text{ GeV}^{-1}$
- ← BB induced by a const. rot $\alpha = 1$ deg
- ← BB induced by a const. rot $\alpha = 0.1$ deg
- ← BB induced by EDE $g_\phi = 3.5 \times 10^{-19} \text{ GeV}^{-1}$

Early Dark Energy



Constraints for a LiteBIRD-like mission for EDE

The parity violating nature of the interaction generates **nonzero parity odd correlators (TB and EB)**, therefore we consider the **full theoretical covariance matrix**:

$$\bar{C}_l = \begin{pmatrix} \bar{C}_l^{TT} & \bar{C}_l^{TE} & \bar{C}_l^{TB} \\ \bar{C}_l^{TE} & \bar{C}_l^{EE} & \bar{C}_l^{EB} \\ \bar{C}_l^{TB} & \bar{C}_l^{EB} & \bar{C}_l^{BB} \end{pmatrix}$$

Following Xia *et al.* [Astron. Astrophys. 2008] we introduce the effective χ^2

For Early Dark Energy:

\bar{C}_l theoretical (EDE) $+N_\ell$	\hat{C}_l observed $+N_\ell$	χ_{eff}^2
$C_\ell(g_\phi = 1.65 \times 10^{-18} \text{GeV}^{-1})$	$C_\ell(\bar{\alpha} = 0.35 \text{ deg})$	67.3
$C_\ell(g_\phi = 6.0 \times 10^{-20} \text{GeV}^{-1})$	$C_\ell(\alpha = 0 \text{ deg})$	10.5

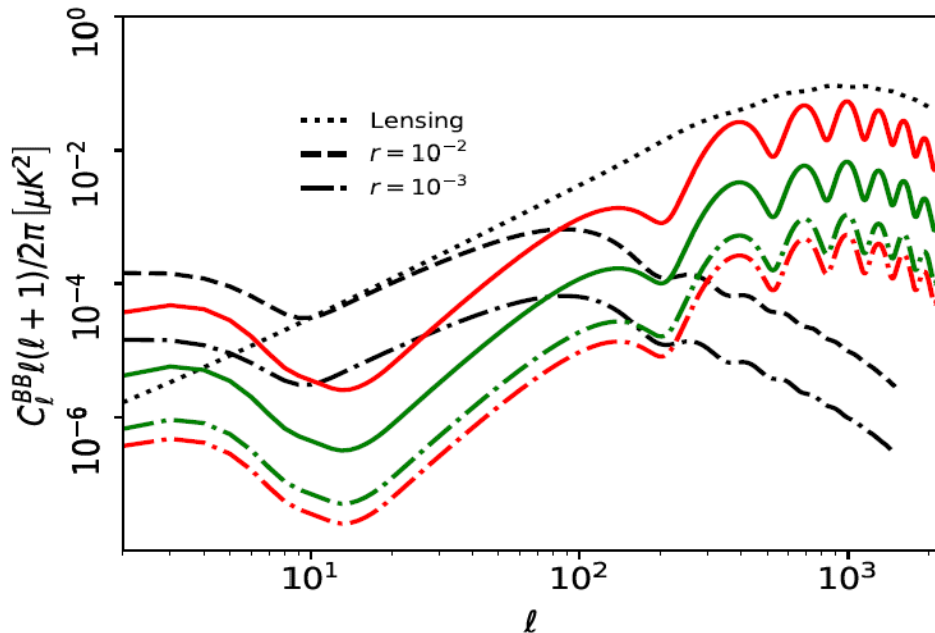
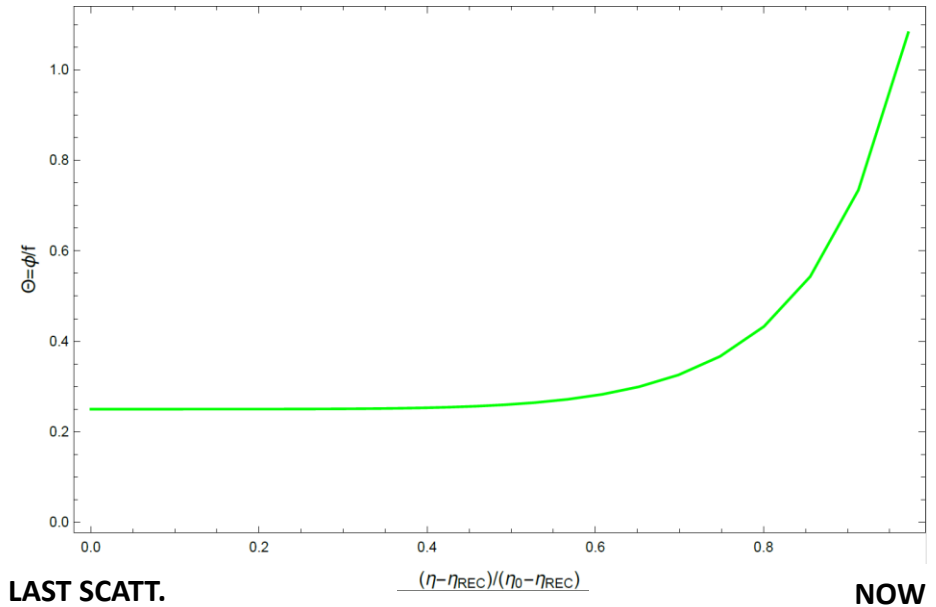
Axion-like Dark Energy

$$V(\phi) = M^4 \left(1 + \cos \frac{\phi}{f} \right)$$

assuming:

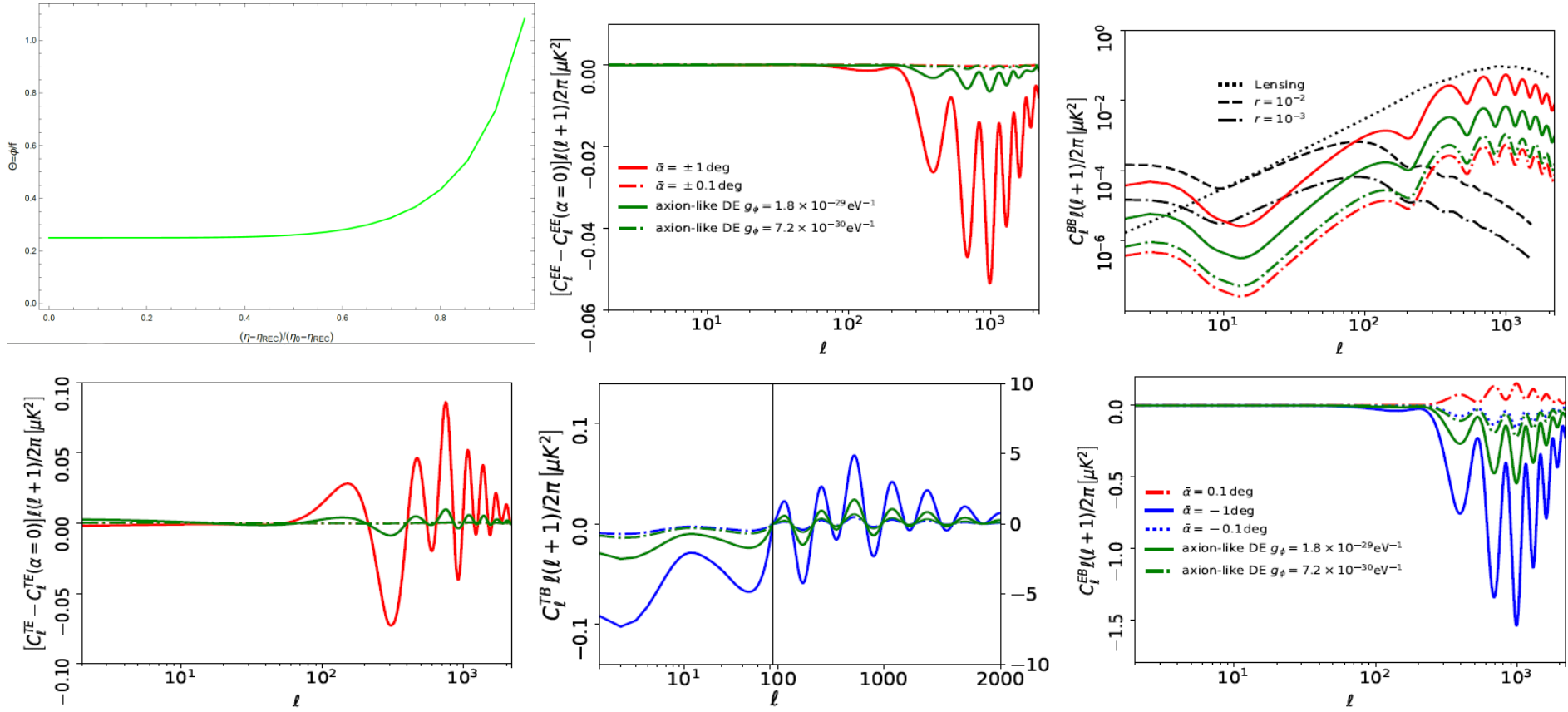
$M = 1.95 \times 10^{-3} \text{ eV}$, $f = 0.25 M_{\text{pl}}$,

$(\phi/f)_{\text{in}} = 0.25$ and $(\dot{\phi}/f)_{\text{in}} = 0$



- ← BB induced by a const. rot $\alpha=1$ deg
- ← BB induced by DE $g_\phi = 1.8 \times 10^{-20} \text{ GeV}^{-1}$
- ← BB induced by DE $g_\phi = 7.2 \times 10^{-21} \text{ GeV}^{-1}$
- ← BB induced by a const. rot $\alpha=0.1$ deg

Constraints for a LiteBIRD-like mission for DE



\bar{C}_l theoretical (DE) $+N_\ell$	\hat{C}_l observed $+N_\ell$	χ_{eff}^2
$C_\ell(g_\phi = 1.8 \times 10^{-20} \text{GeV}^{-1})$	$C_\ell(\bar{\alpha} = -0.35 \text{ deg})$	0.30
$C_\ell(g_\phi = 1.8 \times 10^{-20} \text{GeV}^{-1})$	$C_\ell(\alpha = 0 \text{ deg})$	3.78×10^3
$C_\ell(g_\phi = 9.0 \times 10^{-22} \text{GeV}^{-1})$	$C_\ell(\alpha = 0 \text{ deg})$	9.4

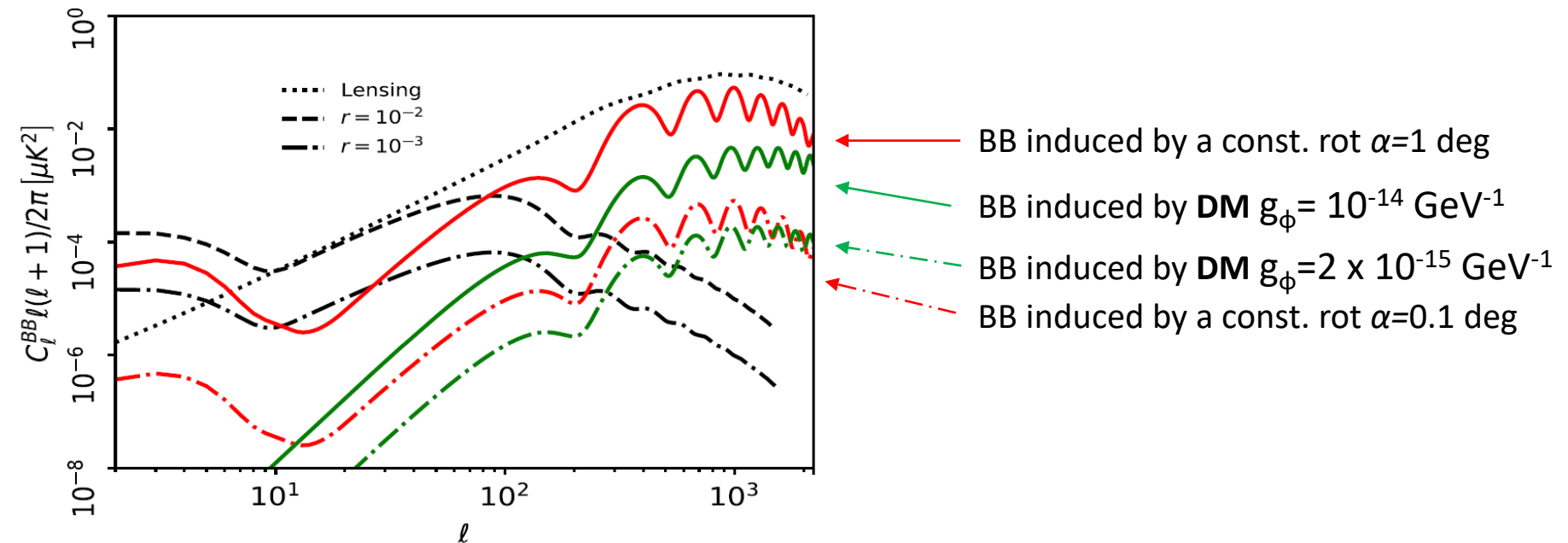
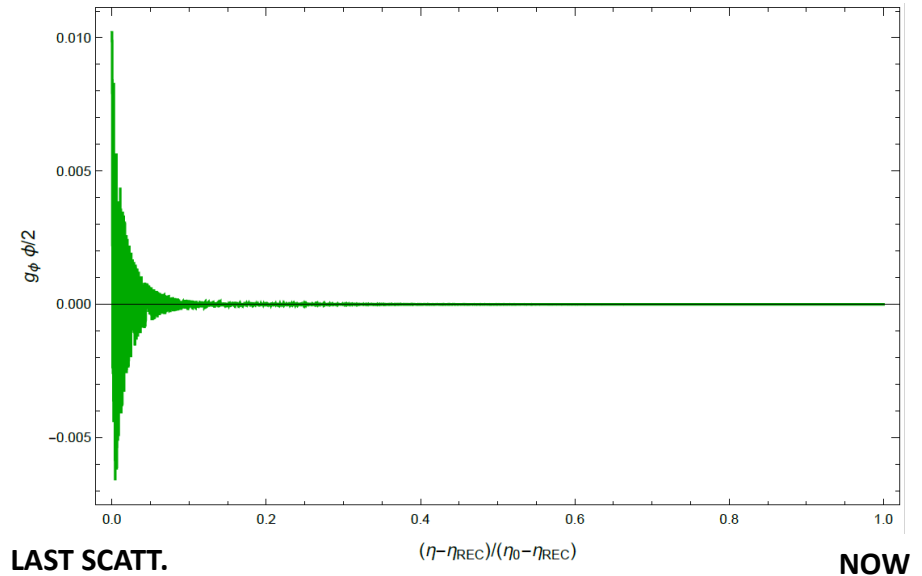
Axion-like Dark Matter

$$V(\phi) = m^2 f^2 \left(1 - \cos \frac{\phi}{f} \right),$$

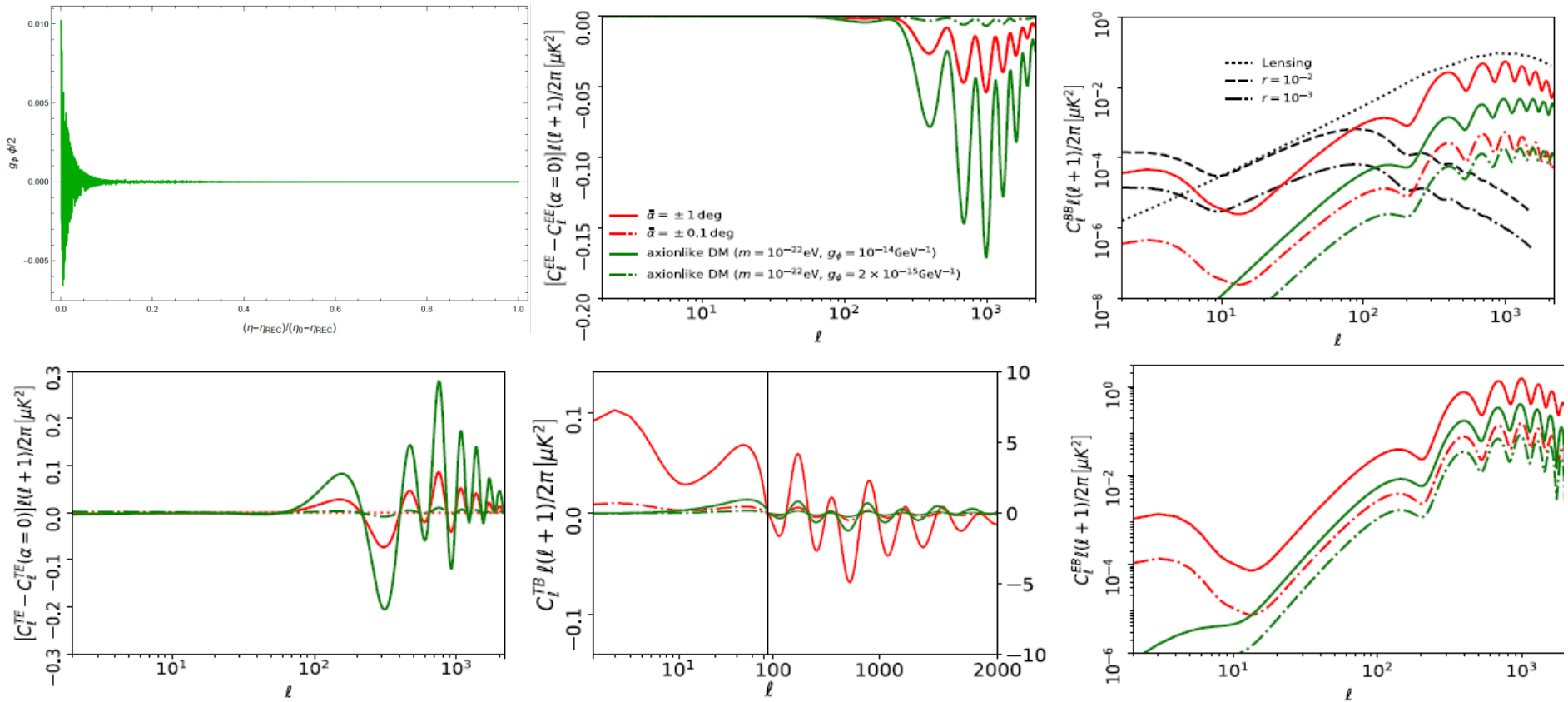
assuming:

$m = 10^{-22}$ eV and

$g_\phi = 10^{-14}$ GeV $^{-1}$ ($g_\phi = 2 \times 10^{-15}$ GeV $^{-1}$)

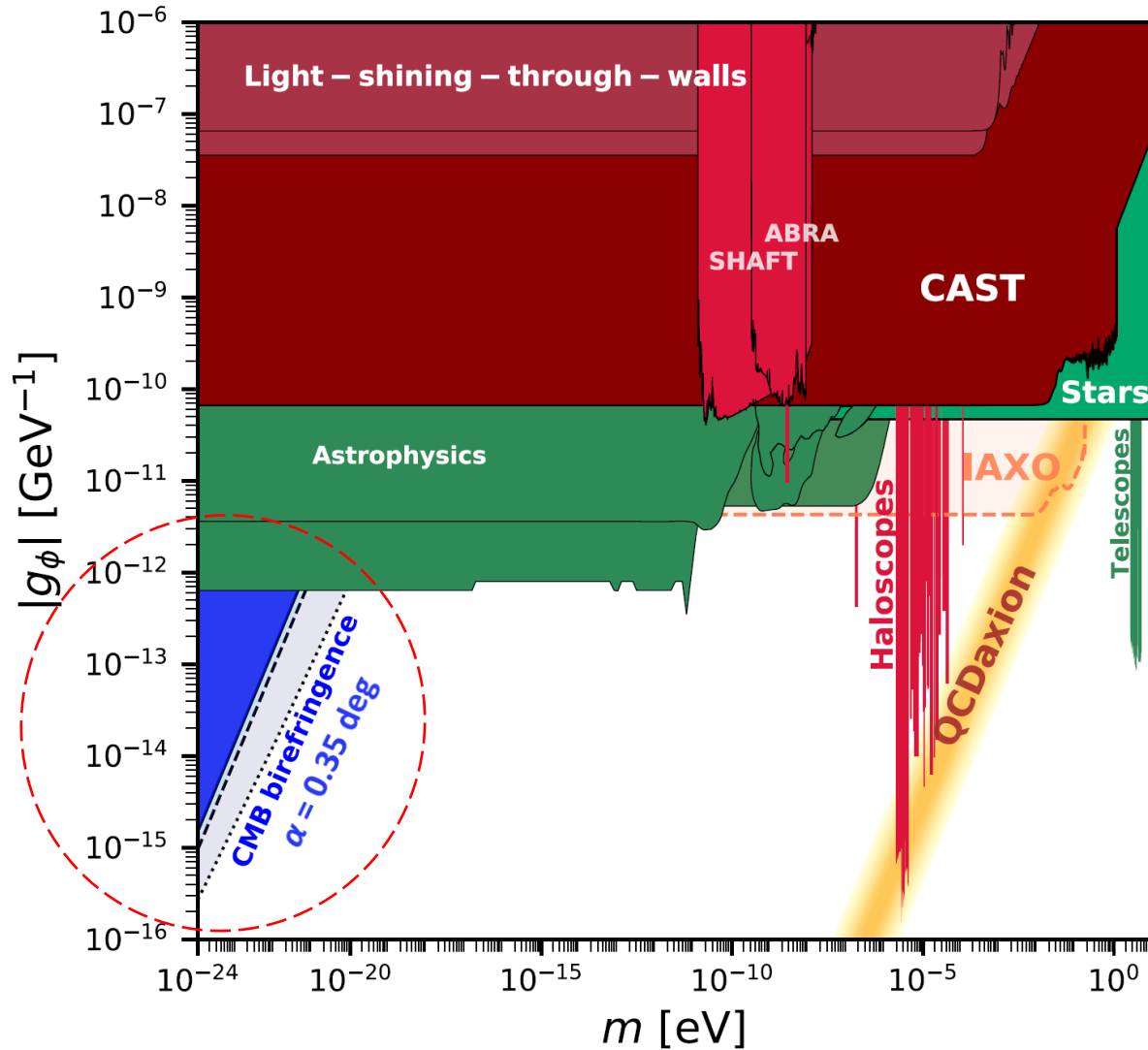


Constraints for a LiteBIRD-like mission for DM



\bar{C}_ℓ theoretical (DM) $+N_\ell$	\hat{C}_ℓ observed $+N_\ell$	χ_{eff}^2
$C_\ell(g_\phi = 1.37 \times 10^{-14} \text{GeV}^{-1})$	$C_\ell(\bar{\alpha} = 0.35 \text{ deg})$	69.8
$C_\ell(g_\phi = 8.1 \times 10^{-16} \text{GeV}^{-1})$	$C_\ell(\alpha = 0 \text{ deg})$	10.4

Constraints for axion-like DM



Conclusions

We studied the imprints of an **isotropic redshift-dependent pseudoscalar field** on CMB polarization power spectra for phenomenological and physical motivated models.

→ **Redshift evolution of the birefringence angle**

has important **effects on CMB polarization power spectra**:

- not only $\bar{\alpha} \equiv \alpha(\eta_{\text{rec}}) - \alpha(\eta_0)$ is important , but also **when** the rotation occurs;
- **different theoretical motivated redshift dependencies** of the pseudoscalar field (EDE, DE, DM) produce **different multipole dependence** for the CMB polarization power spectra;
- **isotropic birefringence** - and not only anisotropic one - can produce **different multipole dependencies** of the power spectra, this can be used to break the degeneracy with the miscalibration angle of the detector.

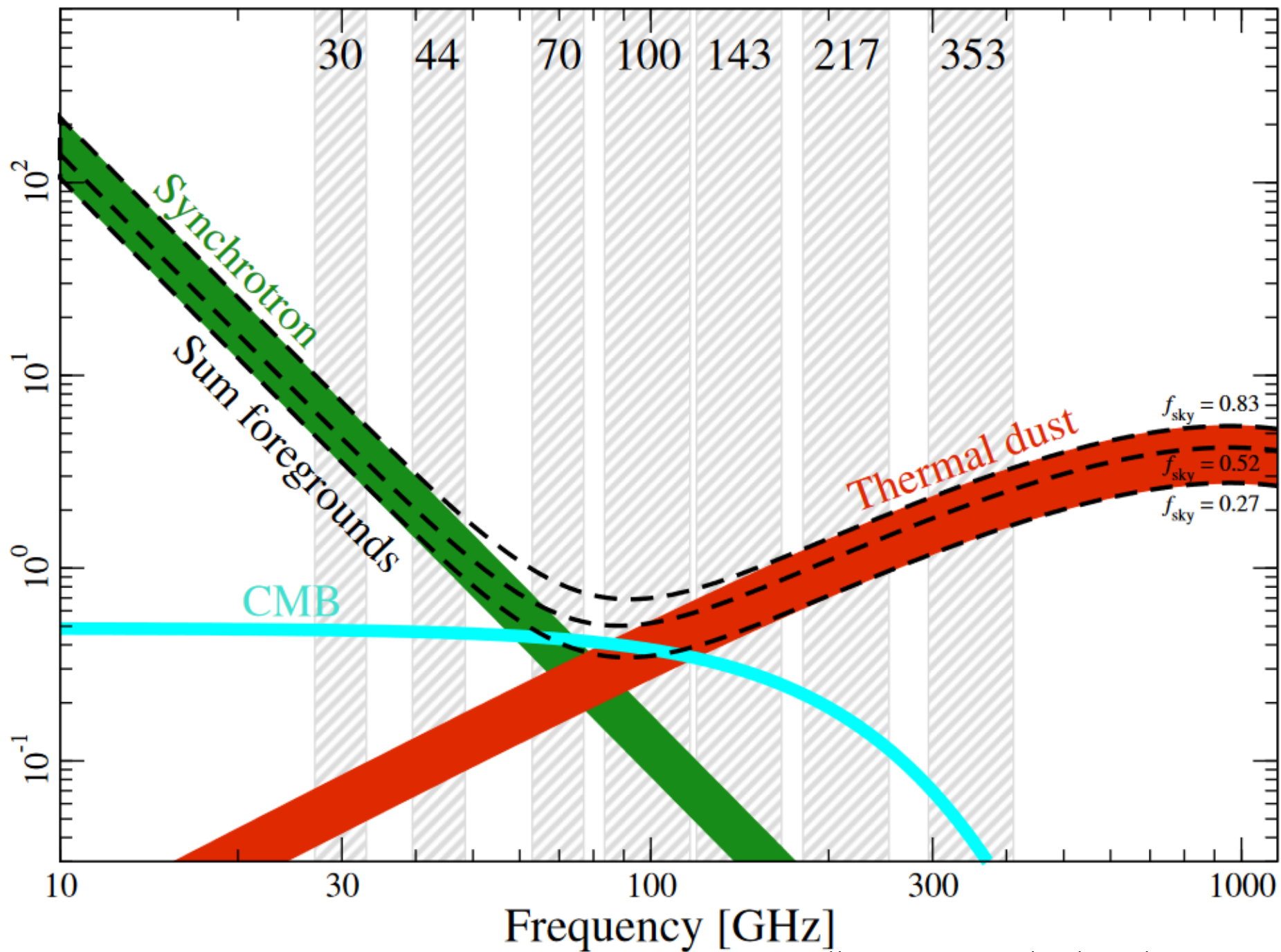
For more details see:

Redshift evolution of cosmic birefringence in CMB anisotropies

M.G., F. Finelli and D. Paoletti

Phys.Rev.D **107** (2023) 8, 083529 (arXiv 2301.07971 [astro-ph.CO])

Rms polarization amplitude [μK]



Cosmic Birefringence from the *Planck* Data Release 4

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 R. B. Barreiro¹, H. K. Eriksen³, K. M. Górski,^{9,10} R. Keskitalo^{11,12}, E. Komatsu^{13,14}
 E. Martínez-González,¹ D. Scott⁶, P. Vielva¹ and I. K. Wehus³

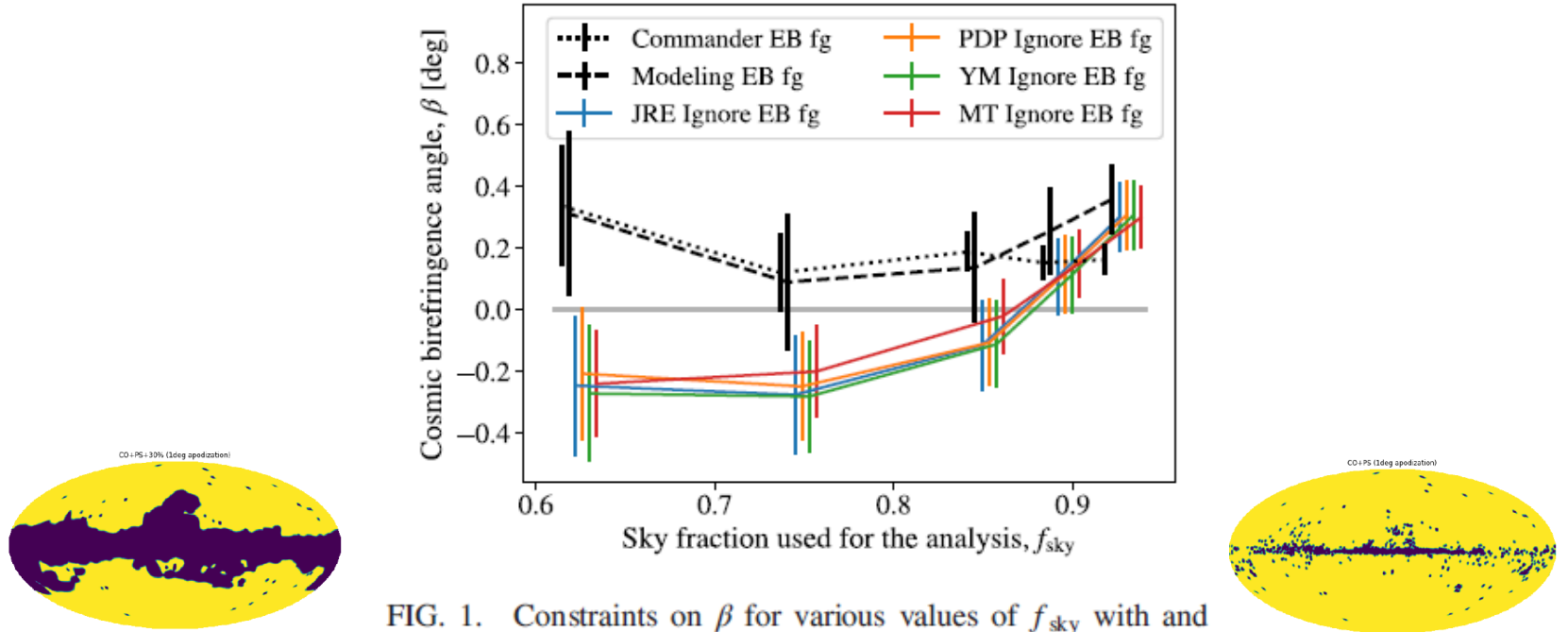


FIG. 1. Constraints on β for various values of f_{sky} with and without accounting for the foreground *EB* correlations. For the former, the dashed and dotted lines show corrections using the filament model [Eq. (2)] and the COMMANDER sky model, respectively. For the latter, the results of four pipelines (JRE, PDP, YM, MT) are shown.

α [deg]: *isotropic* and *z dependent*

Boltzmann equation for linear polarization **with cosmic birefringence** (Liu, Lee and Ng [PRL 2006], Finelli and Galaverni [PRD 2009]):

$$\Delta'_{Q\pm iU}(k, \eta) + ik\mu\Delta_{Q\pm iU}(k, \eta) = -n_e\sigma_T a(\eta) \left[\Delta_{Q\pm iU}(k, \eta) + \sum_m \sqrt{\frac{6\pi}{5}} \pm 2 Y_2^m S_P^{(m)}(k, \eta) \right] \mp i2\alpha'(\eta)\Delta_{Q\pm iU}(k, \eta)$$

we follow the **line-of-sight** strategy [Seljak and Zaldarriaga (1996)]

$$C_\ell^{EE/BB/EB} = (4\pi)^2 \frac{9(\ell+2)!}{16(\ell-2)!} \int k^2 dk [\Delta_{E/B/E}(k, \eta_0)\Delta_{E/B/B}(k, \eta_0)]$$

where the **source terms for scalar perturbations** are modified:

$$\Delta_E(k, \eta_0) = \int_0^{\eta_0} d\eta \boxed{g(\eta)} S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \boxed{\cos 2[\alpha(\eta) - \alpha(\eta_0)]}$$

$$\Delta_B(k, \eta_0) = \int_0^{\eta_0} d\eta \boxed{g(\eta)} S_P^{(0)}(k, \eta) \frac{j_\ell(k\eta_0 - k\eta)}{(k\eta_0 - k\eta)^2} \boxed{\sin 2[\alpha(\eta) - \alpha(\eta_0)]}$$

