Flavour in ALP effective field theories

Sophie Renner
University of Glasgow

Based on work with M. Bauer, M. Neubert, M. Schnubel and A. Thamm
2012.12272, 2102.13112, 2110.10698

University of Glasgow

COSMIC WISPERS, BARI 2023

UK Research and Innovation

ALPs, generally

Any dynamics with a spontaneously broken approximate global symmetry will produce light spinless particles

Analogy: QCD pions

Λ_{QCD} $\sim$ GeV — $p, n, \ldots$

m_π — π

Pions are pseudo goldstone bosons of an approximate spontaneously broken symmetry

BSM physics

Λ_{UV} $\gtrsim$ TeV — $??$

m_a — a

ALP is a pseudo-goldstone boson (PNGB)

Pseudoscalar gauge singlet

Mass much below scale of BSM physics

Many motivated explicit models: e.g. QCD axion, dark sector models, flavon models, composite Higgs models, ...

ALPs, generally II

If we are hunting for new particles beyond the Standard Model, a few options...

All new particles are heavy ($m \gg v$)? → SM EFT (or similar)

One or more light ($m \lesssim v$) BSM particles?

+ possibly higher spin...

spin 0

ALP
(Goldstone boson explains lightness)

spin 1/2

RH neutrino/heavy neutral lepton

spin 1

Dark photon/Z'

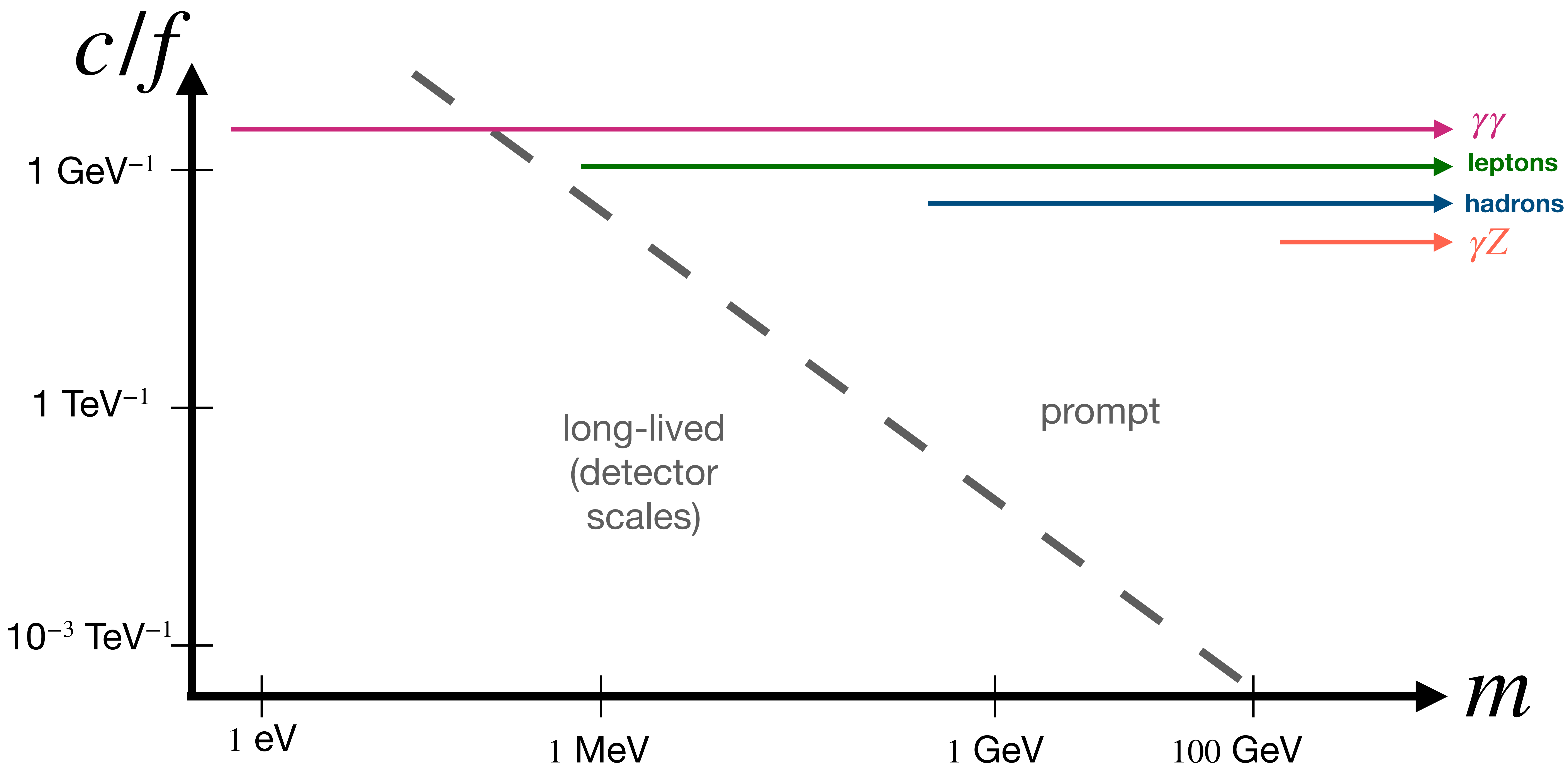
spin 3/2

e.g. gravitino/composite dark sector resonance

Cover all bases: allow all couplings

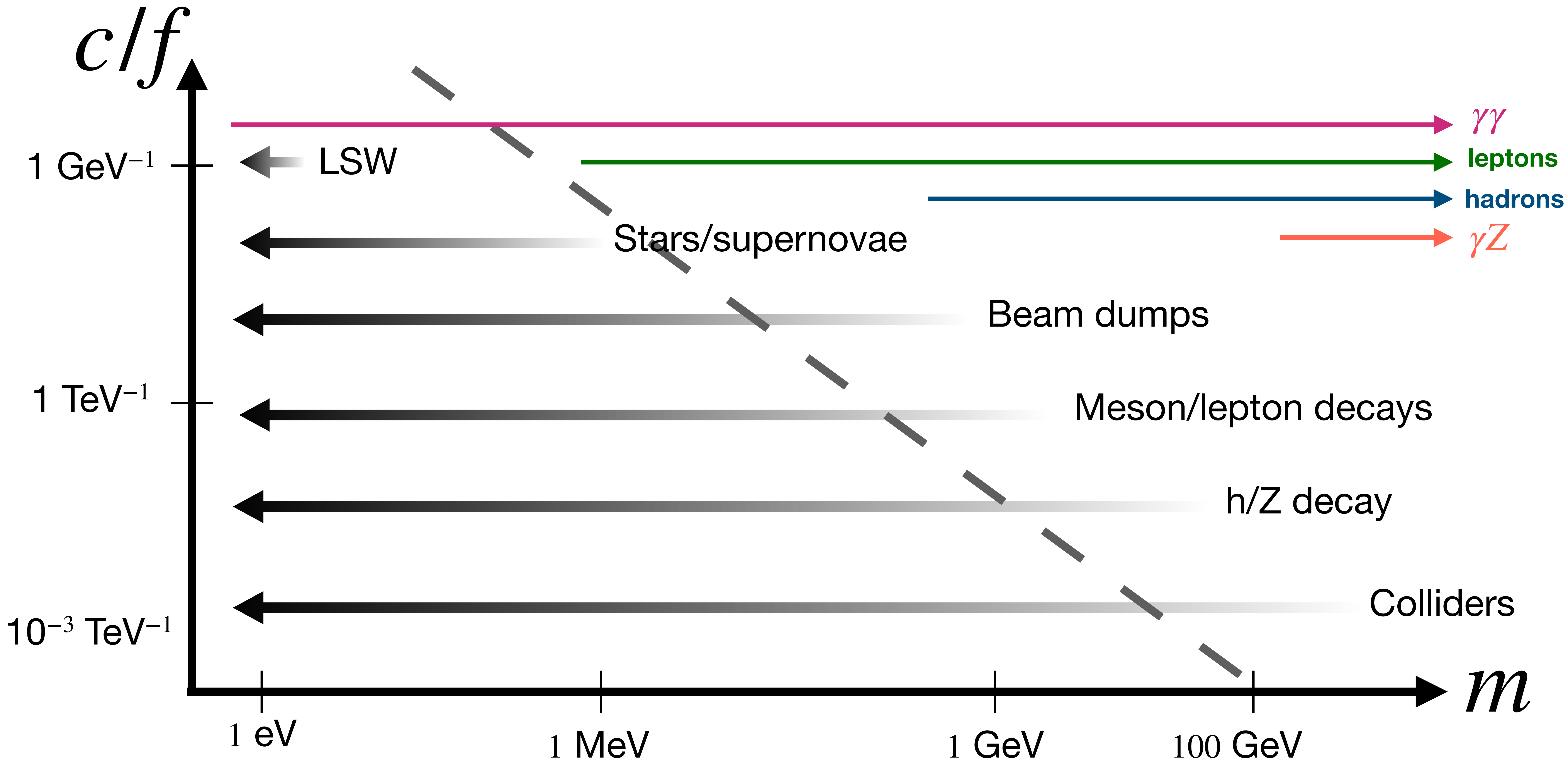
ALP pheno at a glance

Decay modes & decay length depend on mass and coupling(s)



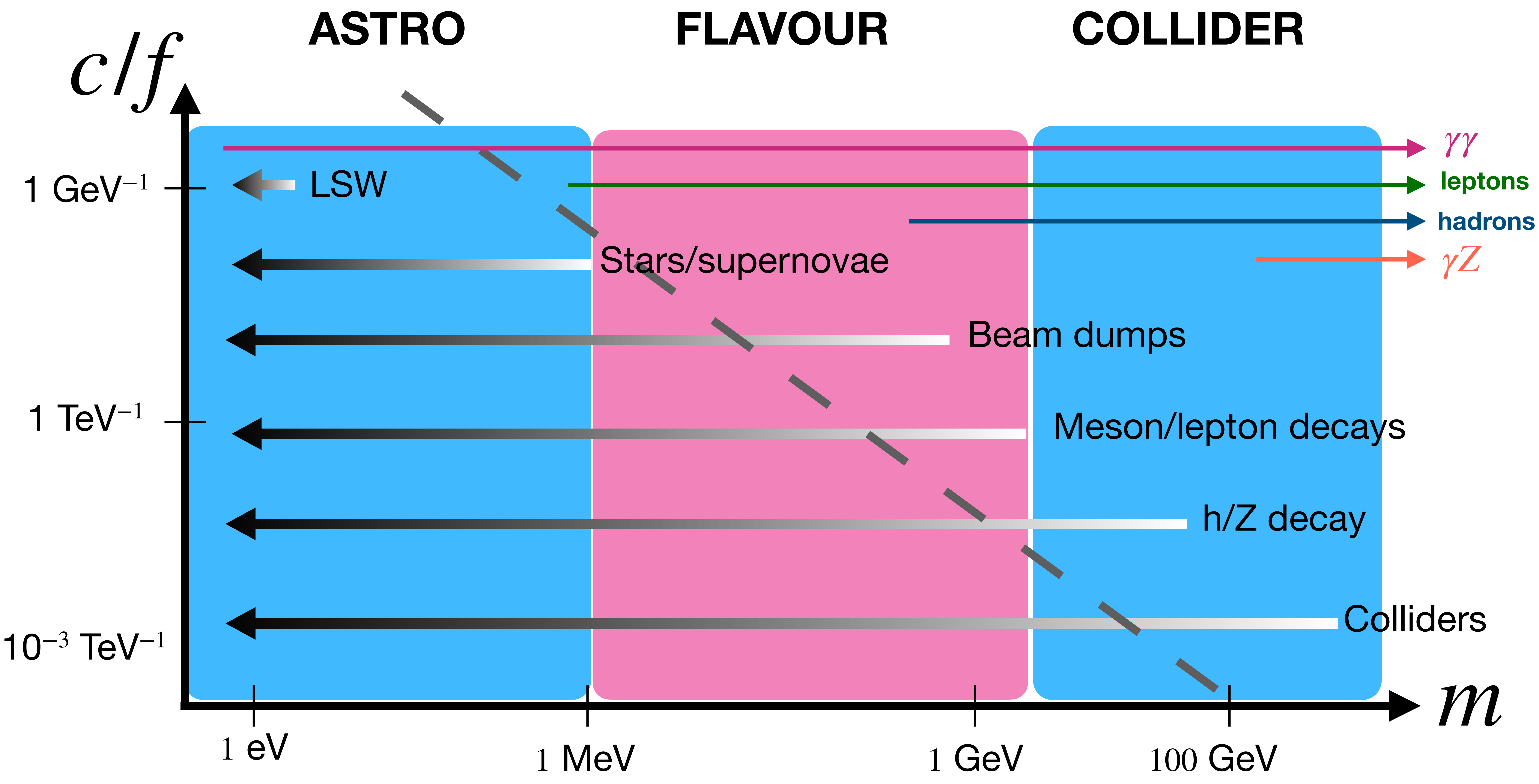
ALP pheno at a glance

Production modes depend on mass



ALP pheno at a glance

Where can measurements of flavour-changing processes play a role?

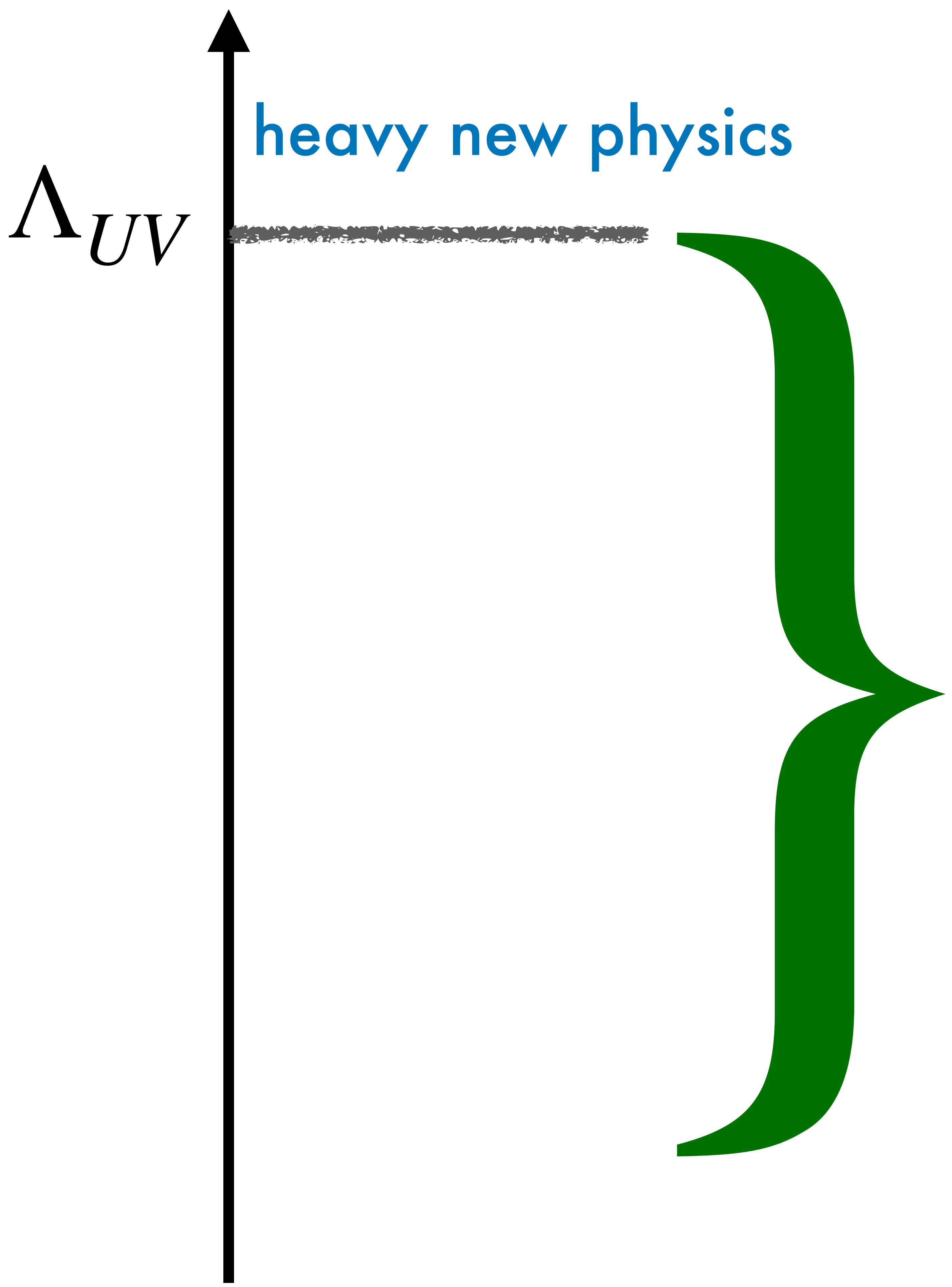


Outline

- ALP effective field theories
- The importance of renormalisation group running of the couplings
- ALPs in quark flavour processes & how they constrain the parameter space
- Lepton flavour violating ALPs

ALP effective field theories

Don't need to know the details of the UV physics to study ALP phenomenology



Particle content is **SM+ALP**
Physics is described by an
effective field theory

<u>Conditions on EFT Lagrangian:</u>

1) Invariant under symmetries of the theory
2) ALP is pseudoscalar under CP
3) Invariant under ALP shift symmetry
   $a \to a + \sigma$, broken only by ALP mass term

ALP effective field theories

Don't need to know the details of the UV physics to study ALP phenomenology

heavy new physics

Λ_{UV}

Full SM + ALP

$SU(3)_c \times SU(2)_L \times U(1)_Y$

m_W

SM$-h, W, Z, t$ + ALP

$SU(3)_c \times U(1)_{em}$

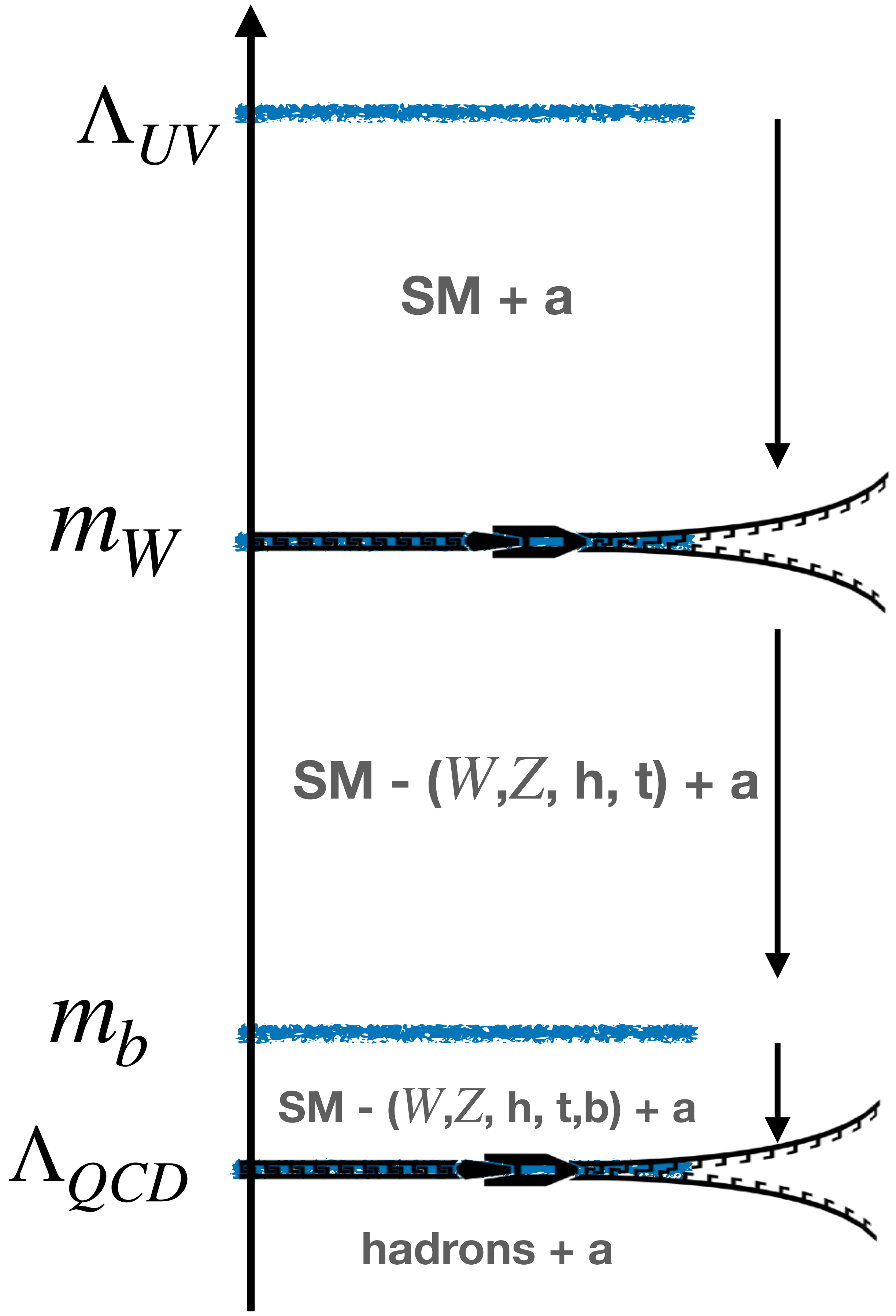
Λ_{QCD}

Chiral Perturbation theory + ALP

<u>*Conditions on EFT Lagrangian:*</u>

1) Invariant under symmetries of the theory
2) ALP is pseudoscalar under CP
3) Invariant under ALP shift symmetry $a \to a + \sigma$, broken only by ALP mass term

From the EFT to observables



ALP couplings determined by physics at Λ_{UV}

To make connection with observables, need to run and match to scale of measurement

Choi, Im, Park, Yun, 1708.00021
Chala, Guedes, Ramos, Santiago 2012.09017
Bauer, Neubert, SR, Schnubel, Thamm, 2012.12272
Bonilla, Brivio, Gavela, Sanz 2107.11392

ALP EFT above the EW scale

ALP-SM couplings begin at dimension 5

$F = Q, u, d, L, e$

$$\mathcal{L}_{\text{eff}}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial^\mu a}{f} \sum_F \bar{\psi}_F \boldsymbol{c}_F \gamma_\mu \psi_F$$
$$+ c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$\Lambda_{UV} = 4\pi f$

$$c_{\gamma\gamma} = c_{WW} + c_{BB}\,, \qquad c_{\gamma Z} = c_w^2\, c_{WW} - s_w^2\, c_{BB}\,, \qquad c_{ZZ} = c_w^4\, c_{WW} + s_w^4\, c_{BB}$$

Then the ALP pheno depends on $m_a, f, \mathbf{c}_F, c_{XX}, c_\phi$

hermitian matrices in flavour space

Also a series of higher dimensional operators:

SMEFT at dim 6:

2499 parameters (for B and L conserving)

+ ALP at dim 6:

$+ c_{aH}(H^\dagger H)(\partial_\mu a)(\partial^\mu a)$

+ dim 7, 8, ...

1 loop RG above EW scale

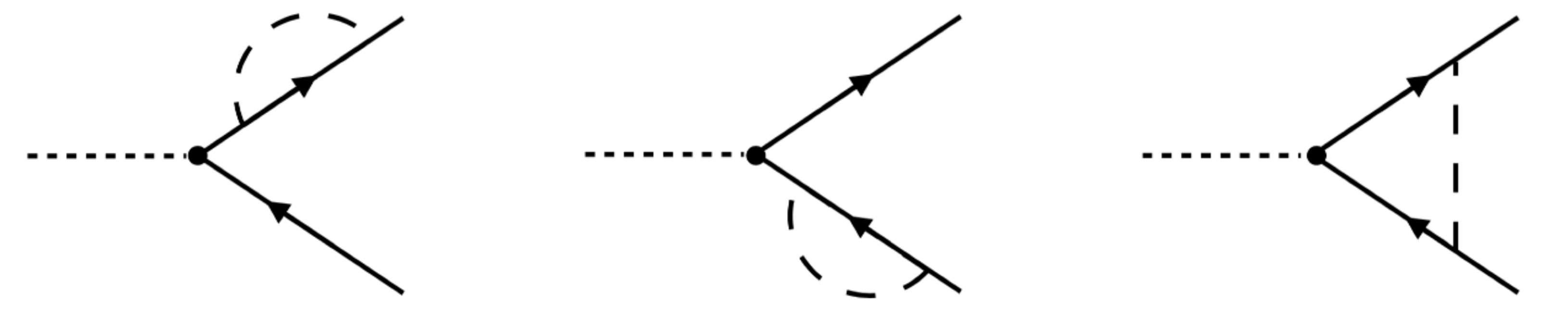
No running for gauge couplings

$$\frac{d}{d\ln\mu}\, c_{VV}(\mu) = 0\,; \quad V = G, W, B$$

Chetyrkin, Kniehl, Steinhauser, Bardeen 1998

Yukawa interactions

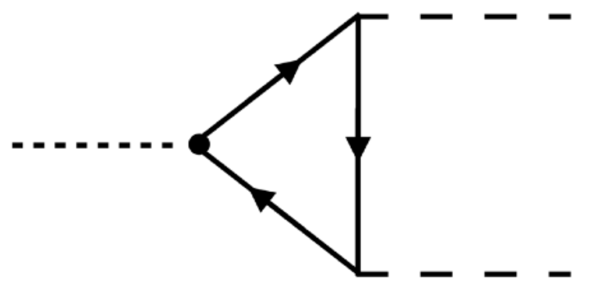
Fermion couplings:

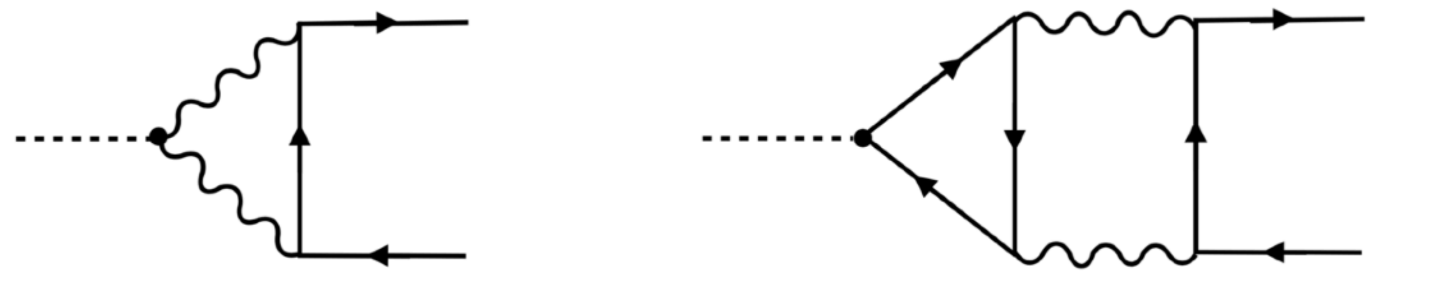


Choi, Im, Park, Yun (2017)
Martin Camalich, Pospelov, Vuong, Ziegler, Zupan (2020)

Important for flavour changing effects!

Important because produces effects in RGEs of all diagonal fermionic couplings, with large coefficients





$$q = u, d$$

$$\frac{d}{d\ln\mu}\, \boldsymbol{c}_Q(\mu) = \frac{1}{32\pi^2}\left\{\mathbf{Y}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_d\mathbf{Y}_d^\dagger, \boldsymbol{c}_Q\right\} - \frac{1}{16\pi^2}\left(\mathbf{Y}_u\,\boldsymbol{c}_u\mathbf{Y}_u^\dagger + \mathbf{Y}_d\,\boldsymbol{c}_d\mathbf{Y}_d^\dagger\right) + \left[\frac{\beta_Q}{8\pi^2}\,X - \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} - \frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_Q^2\,\tilde{c}_{BB}\right]\mathbb{1}\,,$$

$$\frac{d}{d\ln\mu}\, \boldsymbol{c}_q(\mu) = \frac{1}{16\pi^2}\left\{\mathbf{Y}_q^\dagger\mathbf{Y}_q, \boldsymbol{c}_q\right\} - \frac{1}{8\pi^2}\,\mathbf{Y}_q^\dagger\boldsymbol{c}_Q\mathbf{Y}_q + \left[\frac{\beta_q}{8\pi^2}\,X + \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} + \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_q^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

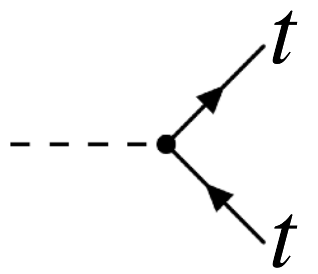
$$X = \mathrm{Tr}\left[3\boldsymbol{c}_Q\left(\mathbf{Y}_u\mathbf{Y}_u^\dagger - \mathbf{Y}_d\mathbf{Y}_d^\dagger\right) - 3\boldsymbol{c}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u + 3\boldsymbol{c}_d\mathbf{Y}_d^\dagger\mathbf{Y}_d - \boldsymbol{c}_L\mathbf{Y}_e\mathbf{Y}_e^\dagger + \boldsymbol{c}_e\mathbf{Y}_e^\dagger\mathbf{Y}_e\right]$$

RGEs for the lepton couplings are analogous

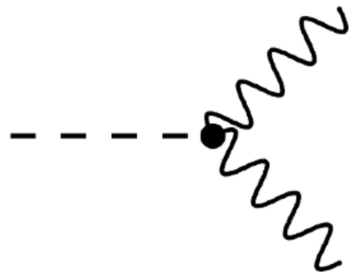
Bauer, Neubert, SR, Schnubel, Thamm 2012.12272

1 loop RG above EW scale: summary

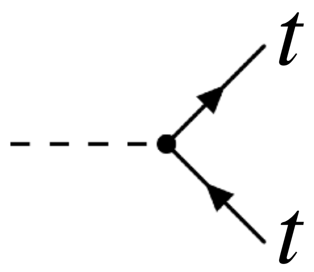
Flavour-diagonal quark couplings (especially to tops)



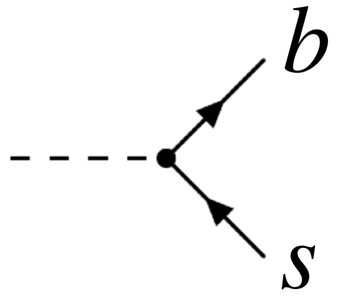
Couplings to gauge bosons



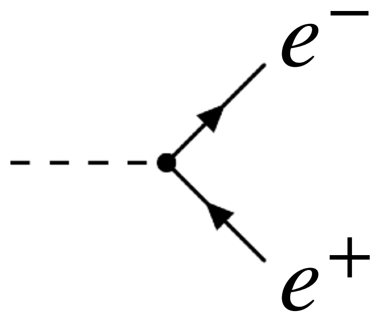
Flavour-diagonal fermion couplings (especially to tops)



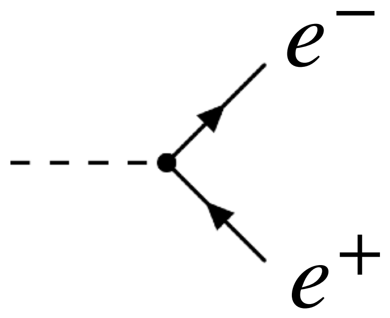
Flavour changing quark couplings



Flavour-diagonal fermion couplings



Flavour-diagonal fermion couplings

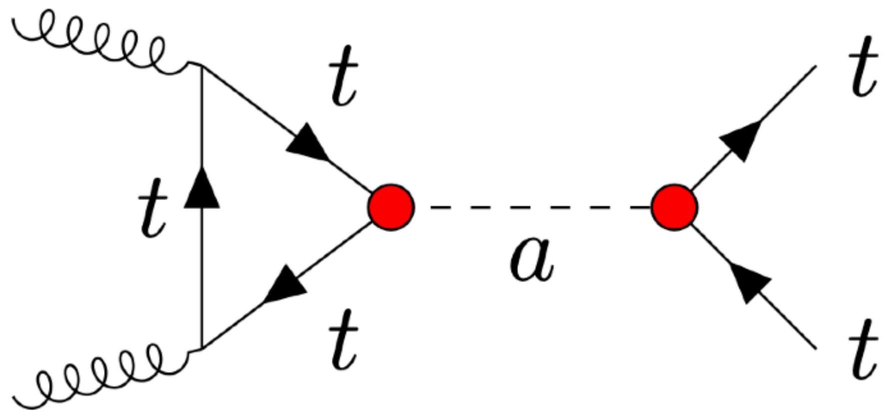


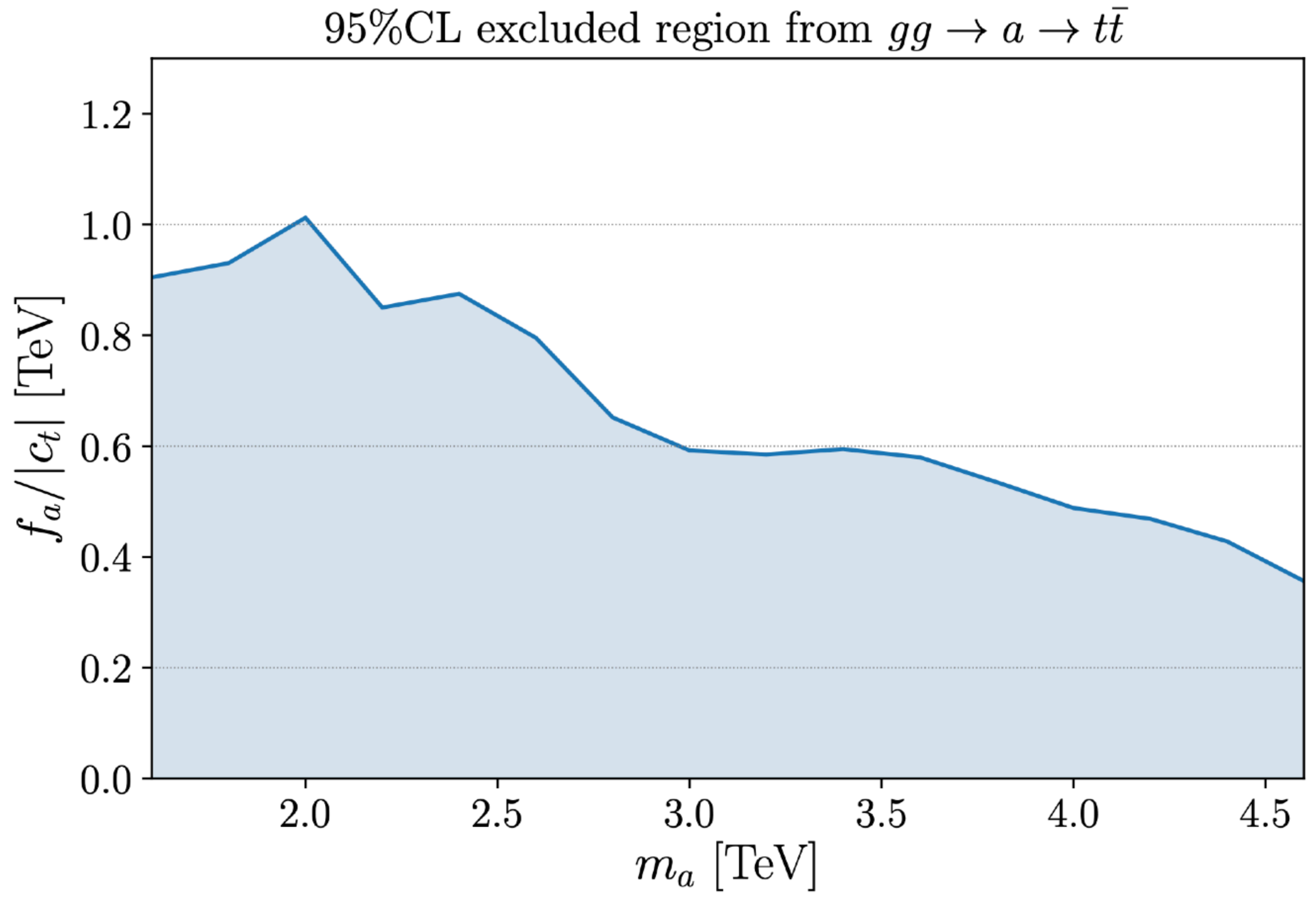
If running over a large enough scale, resummation causes multiple hops

Example: ALP with couplings only to tops at high scale

Bonilla, Brivio, Gavela, Sanz 2107.11392
(see also Bruggisser, Grabitz, Westhoff 2308.11703)

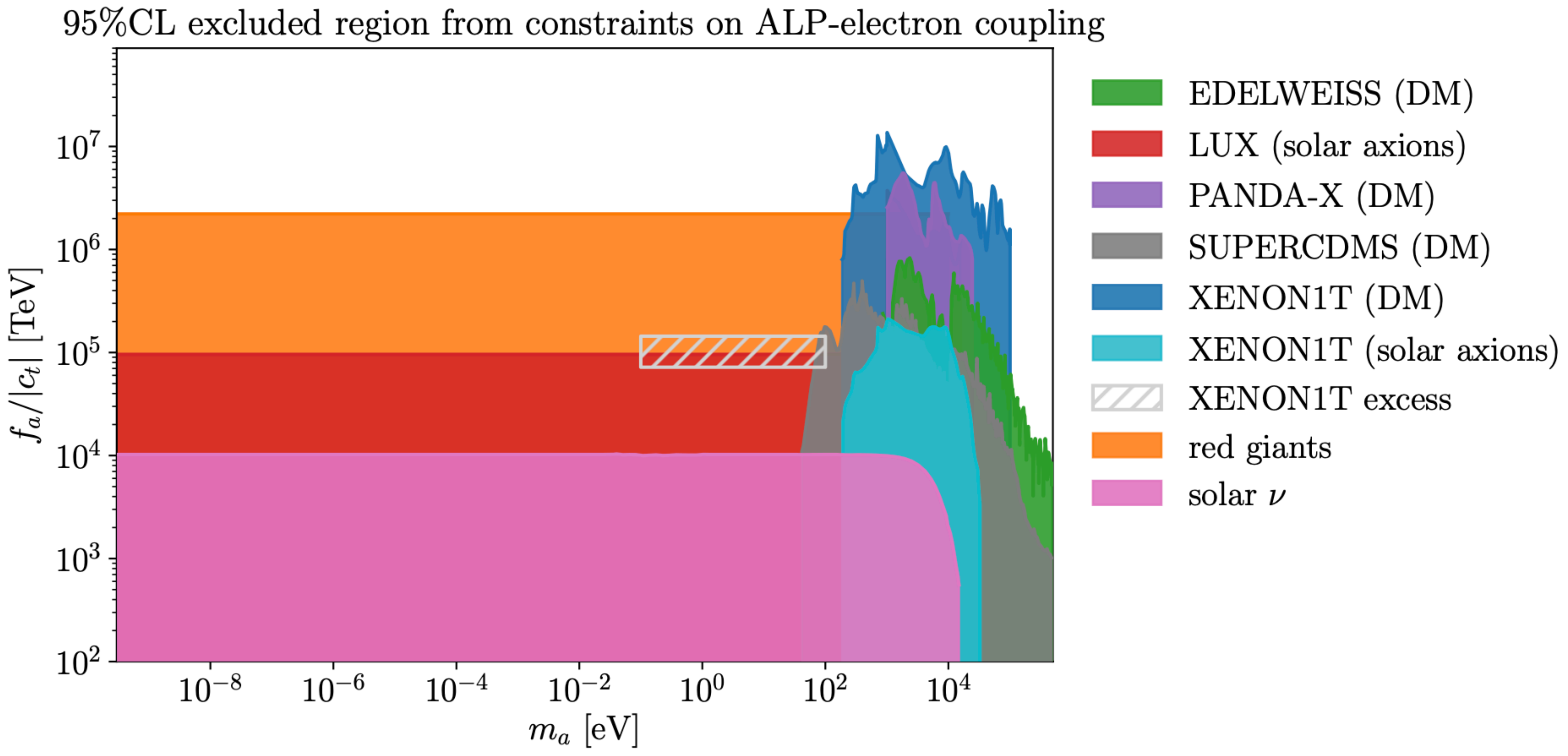
Heavy ALPs



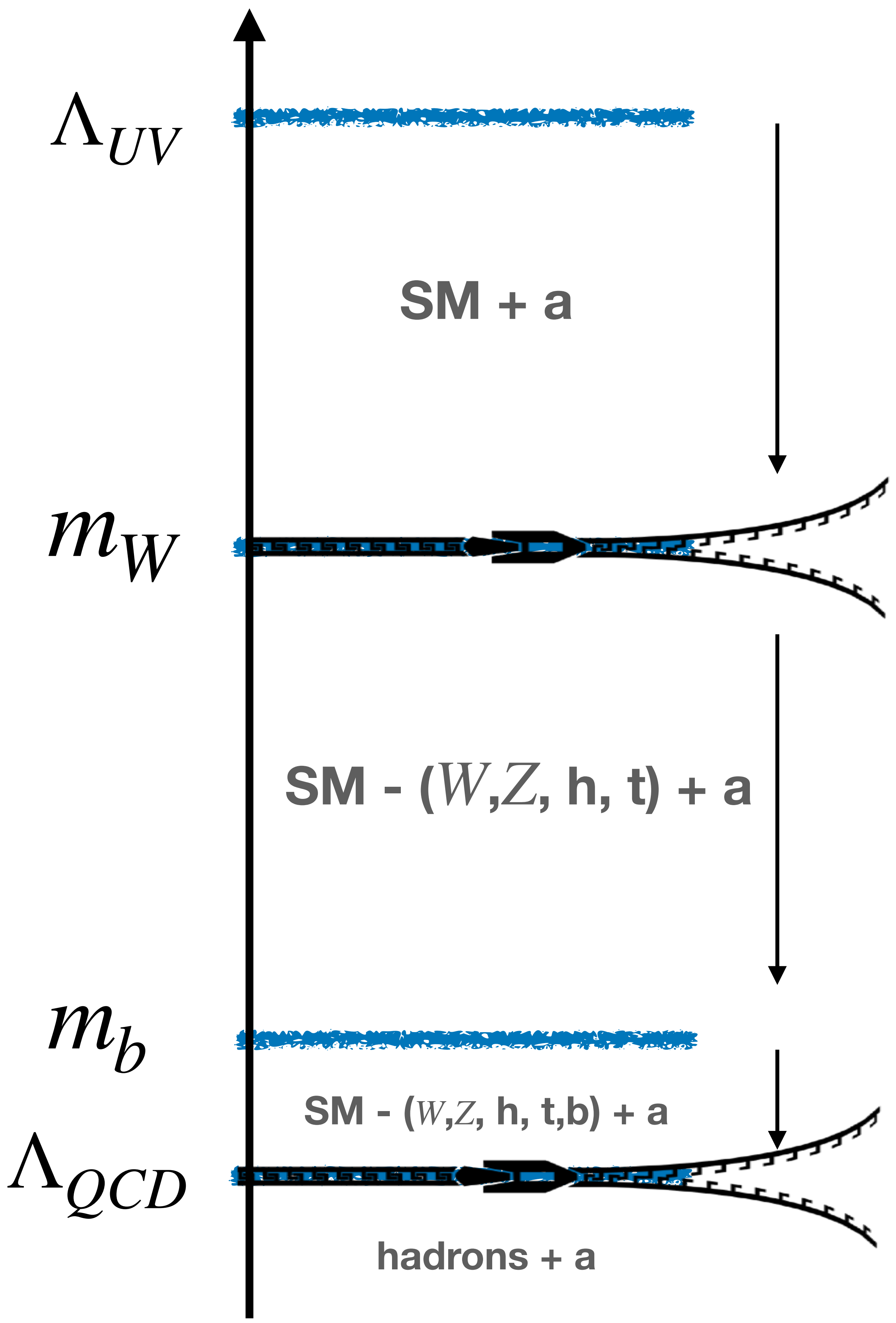


Limits from an ATLAS resonance search

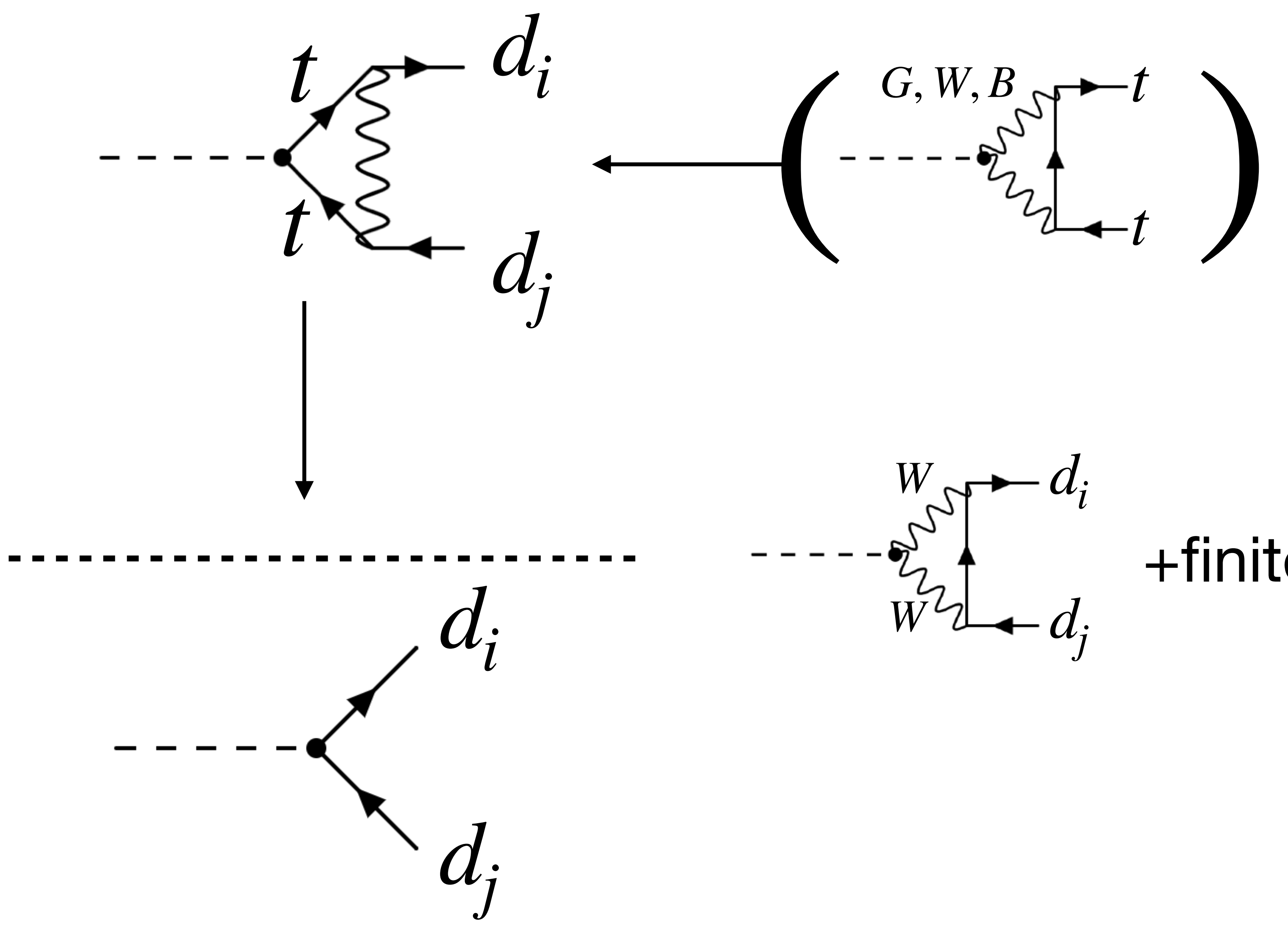
Light ALPs: constraints from loop-induced electron coupling



Flavour effects



Inevitably generate quark flavour changing effects



+finite matching contributions

Starting from flavour universal couplings at $\Lambda = 4\pi$ TeV: $\boldsymbol{c}_F \equiv c_F \mathbb{1}$

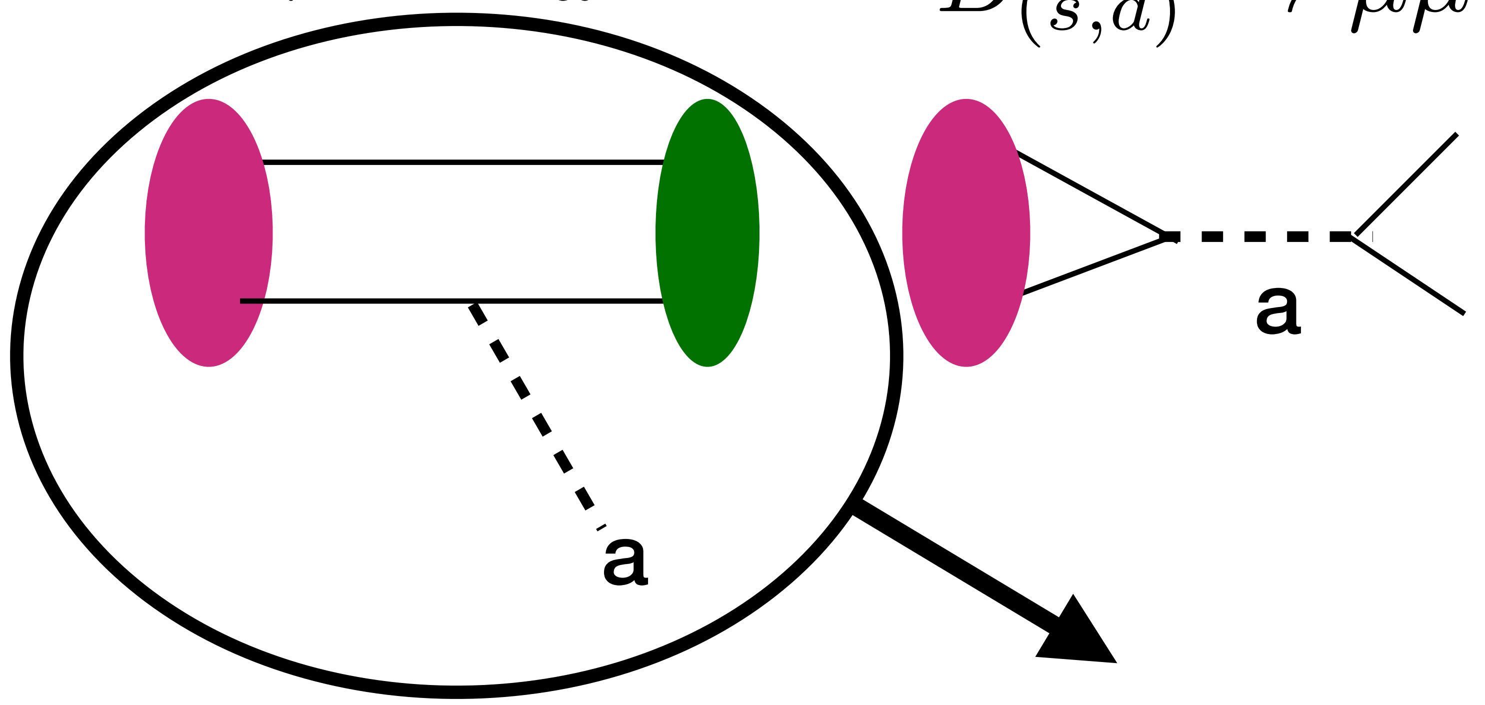
$$[k_D(m_t)]_{ij}^{\rm univ} \simeq 10^{-5}\, V_{ti}^* V_{tj} \Big[-6.1\, c_{GG} - 2.8\, c_{WW} - 0.02\, c_{BB} + 1.9\times 10^3\, c_u(\Lambda) - 9.2\, c_d(\Lambda) - 1.9\times 10^3\, c_Q(\Lambda) - 0.05\, c_e(\Lambda) + 4.2\, c_L(\Lambda) \Big]$$

ALPs in quark flavour processes

Meson decays

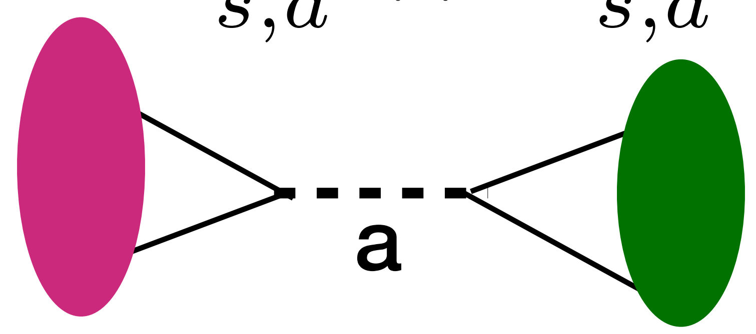
$K \to \pi a$
$B \to K^{(*)} a$

$B_{(s,d)} \to \mu\mu$



Meson mixing

$B^0_{s,d} \leftrightarrow \bar{B}^0_{s,d}$



On-shell signatures:

Long lived ALP: missing energy, monoenergetic final state meson/photon

Decaying ALP: narrow resonance in decay products

RG and matching calculations allow:

> calculate all observables in terms of fundamental lagrangian coeffs at high scale
> plot other constraints & regions of interest in same parameter space

$K \to \pi a$

Georgi, Kaplan, Randall 1986; Srednicki 1985; Bardeen, Peccei, Yanagida 1987;...

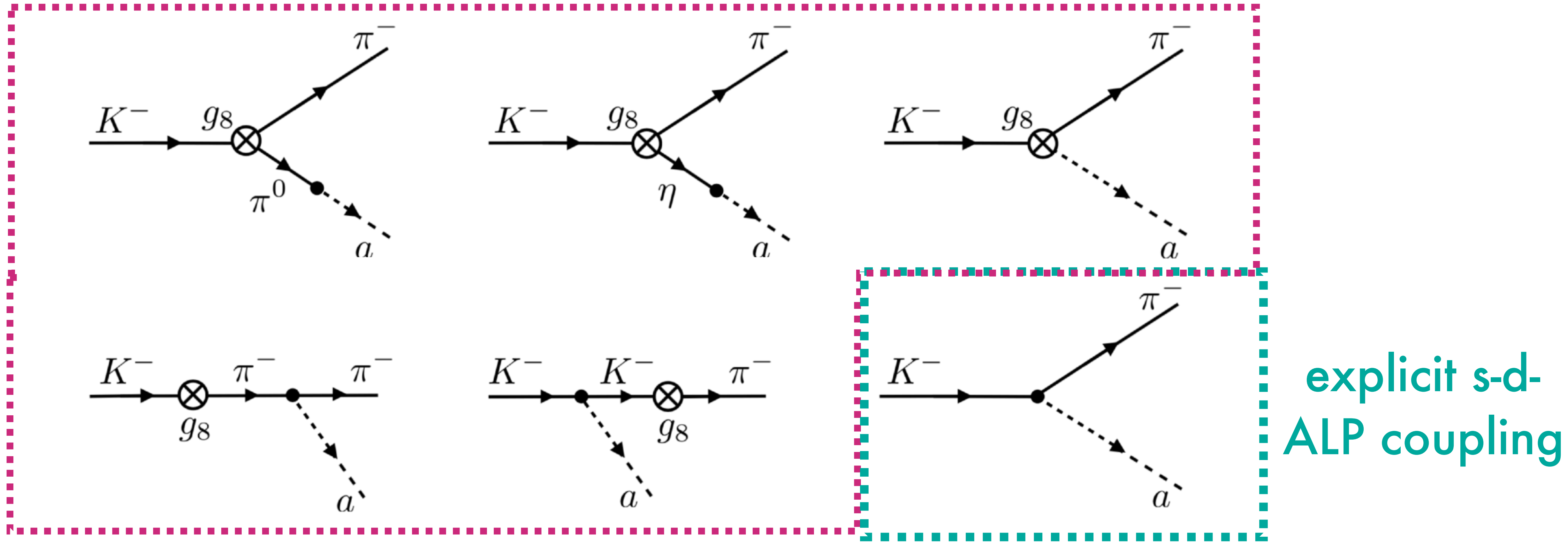
By including the ALP consistently in chiral perturbation theory, can calculate decay rate $K \to \pi a$

Two mechanisms for a flavour diagonal coupling to contribute to $K \to \pi a$:

a) contributing to flavour-diagonal parts of the diagram (flavour change through SM weak interactions)

b) running into an s-d-ALP coupling

flavour change through weak interactions $s \to \bar{u}ud$



Bauer, Neubert, SR, Schnubel, Thamm, 2102.03112
Cornella, Galda, Neubert, Wyler 2308.16903

These two mechanisms give different CKM dependences:

$K^+ \to \pi^+ a$ $\quad \propto V_{us}^* V_{ud}$ $\quad \propto V_{ts}^* V_{td}$

$K_L \to \pi^0 a$ $\quad \propto \epsilon V_{us}^* V_{ud} \sim 10^{-3} V_{us}^* V_{ud}$ $\quad \propto \mathrm{Im}[V_{ts}^* V_{td}] \sim 0.4 |V_{ts}^* V_{td}|$

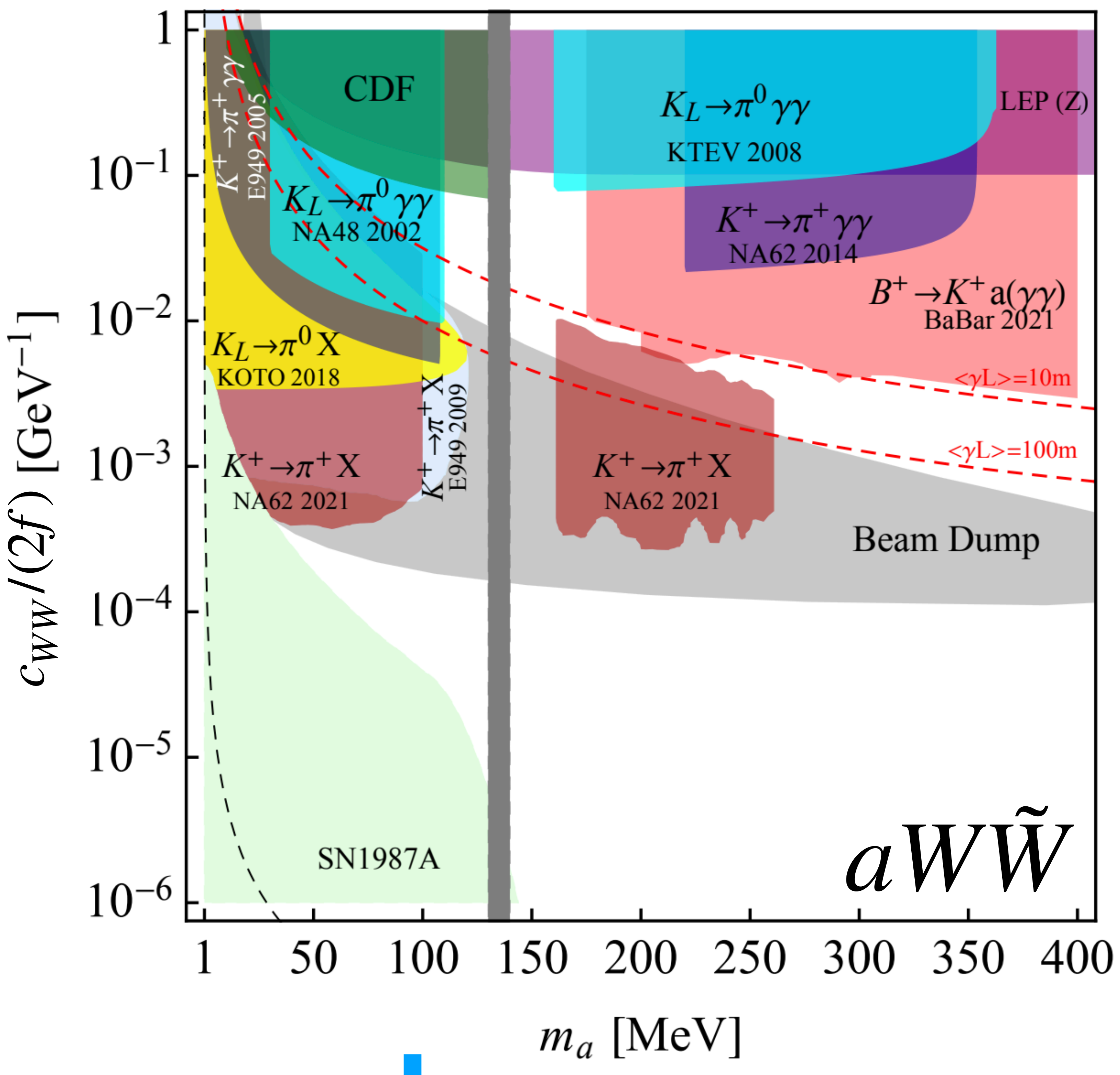
Depending on which mechanism is more important for any particular coupling, get different hierarchies of branching ratios!

Constraints from kaon decays

If light ALPs are produced in $K \to \pi a$, can either:

- escape detector (missing energy signature $K \to \pi X$)
- decay sufficiently promptly (final states such as $K \to \pi\gamma\gamma$)

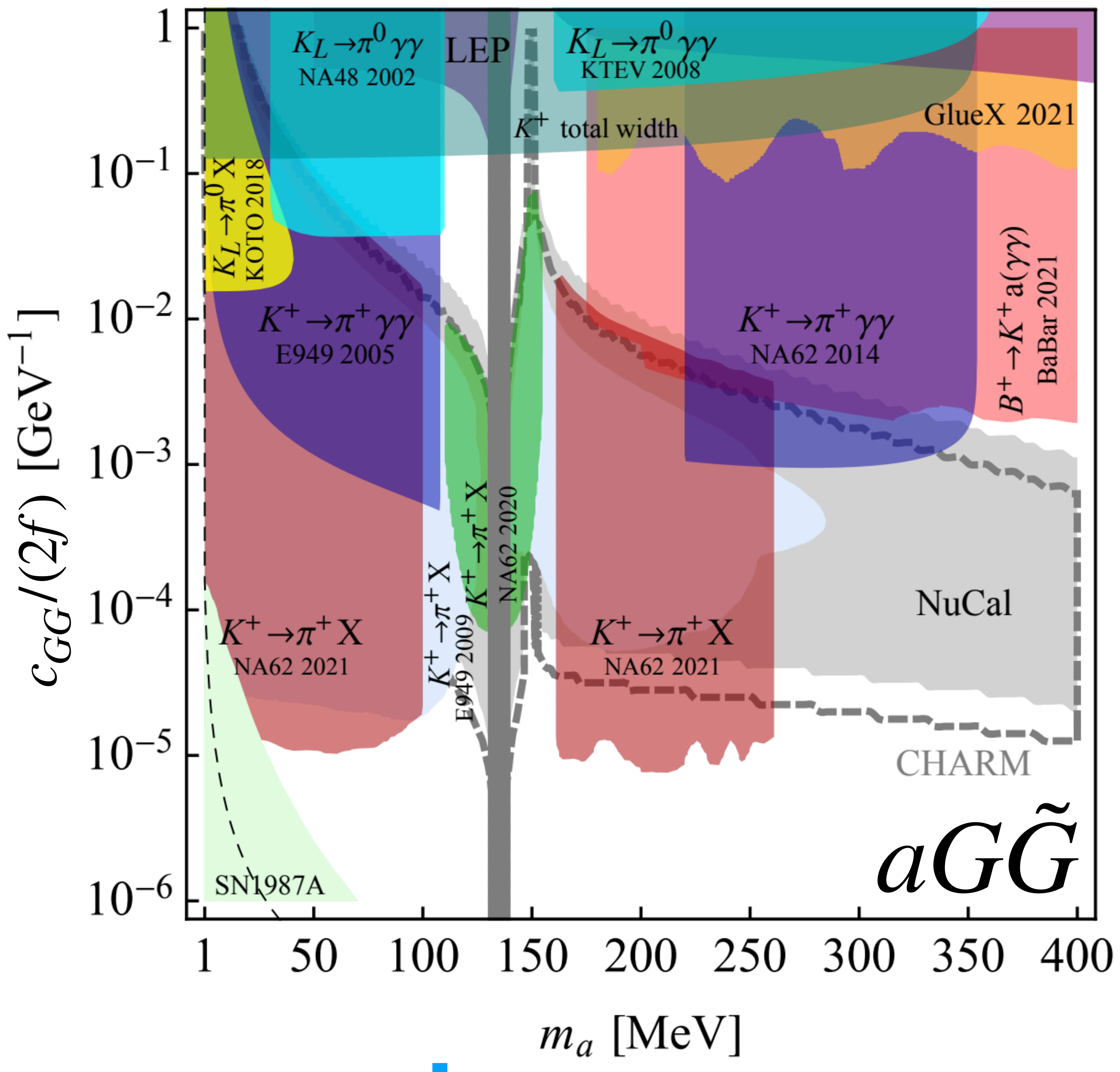
Simplified scenarios: only one coupling to WW or GG at $\Lambda_{UV} = 4\pi$ TeV



Ratio

$K^+ : K_L \sim |V_{ts}^* V_{td}| : \mathrm{Im}[V_{ts}^* V_{td}]$

2201.07805



Ratio

$K^+ : K_L \sim 1 : \epsilon$

2.2×10^{-3}

Recent study of future prospects at KOTO: 2303.01521

All flavour constraints: coupling to SU(2) gauge bosons

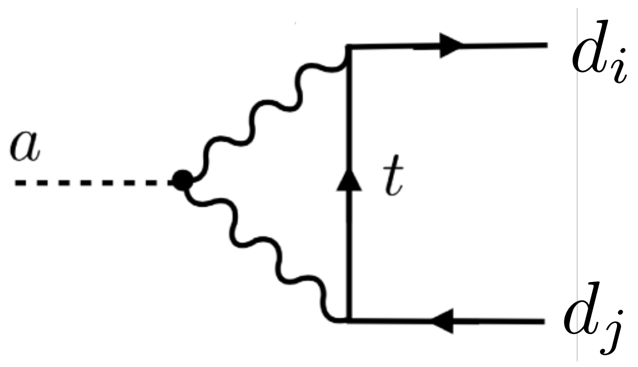
$$c_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W^A_{\mu\nu}\,\tilde{W}^{\mu\nu,A}$$

$\Lambda_{UV} = 4\pi$ TeV

Bauer, Neubert, SR, Schnubel, Thamm, 2110.10698

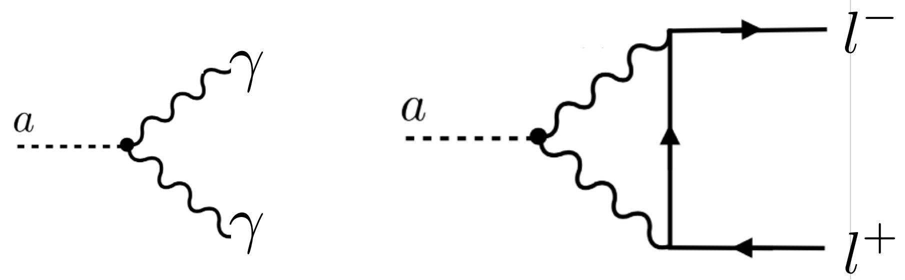
see also: Gavela, Houtz, Quilez, del Rey, Sumensari (2019)
Izaguirre, Lin, Shuve (2016); Gori, Perez, Tobioka (2020)

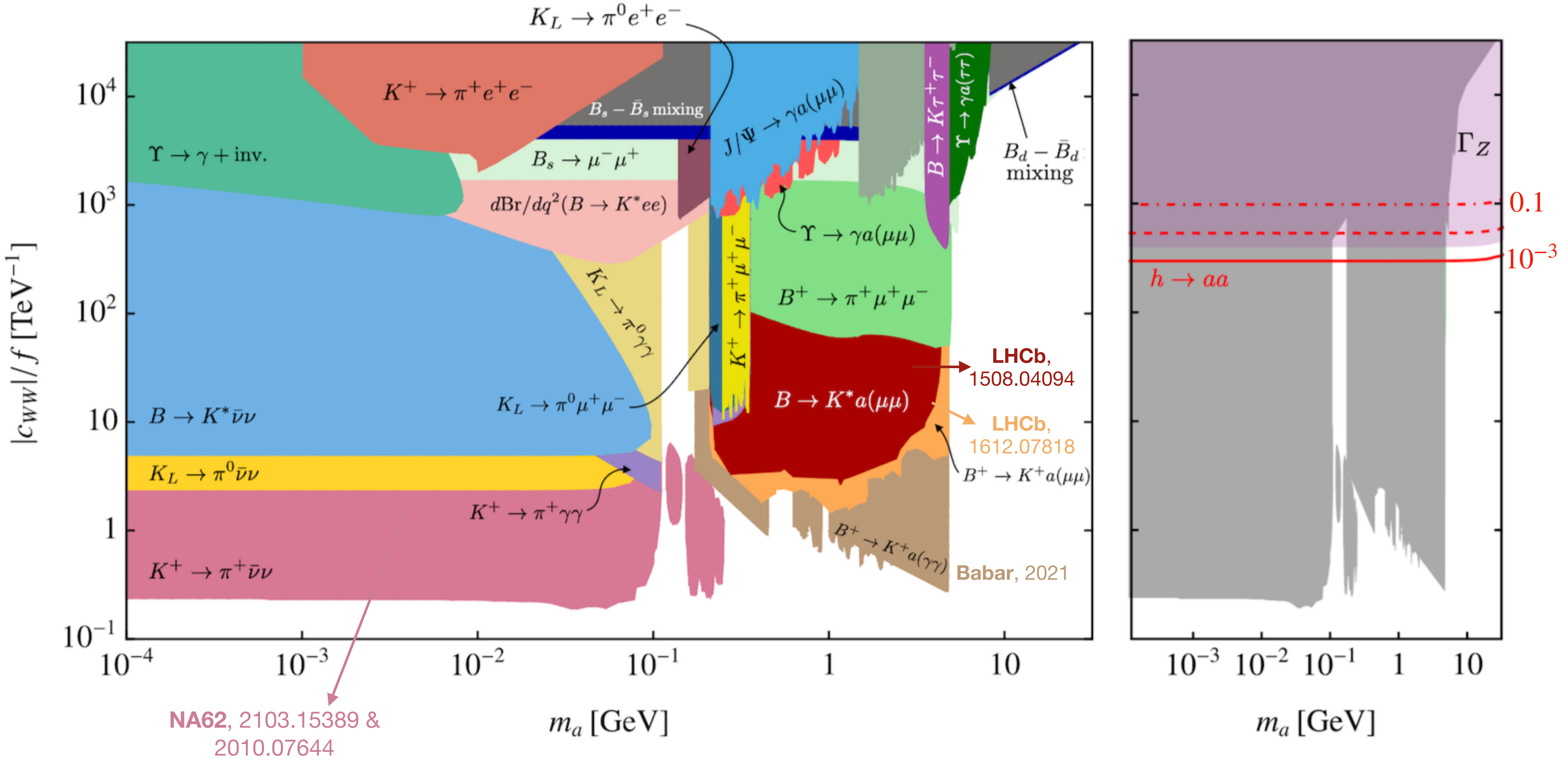
Flavour change:



Decays:

$$c_{\gamma\gamma} = c_{WW} + c_{BB}$$





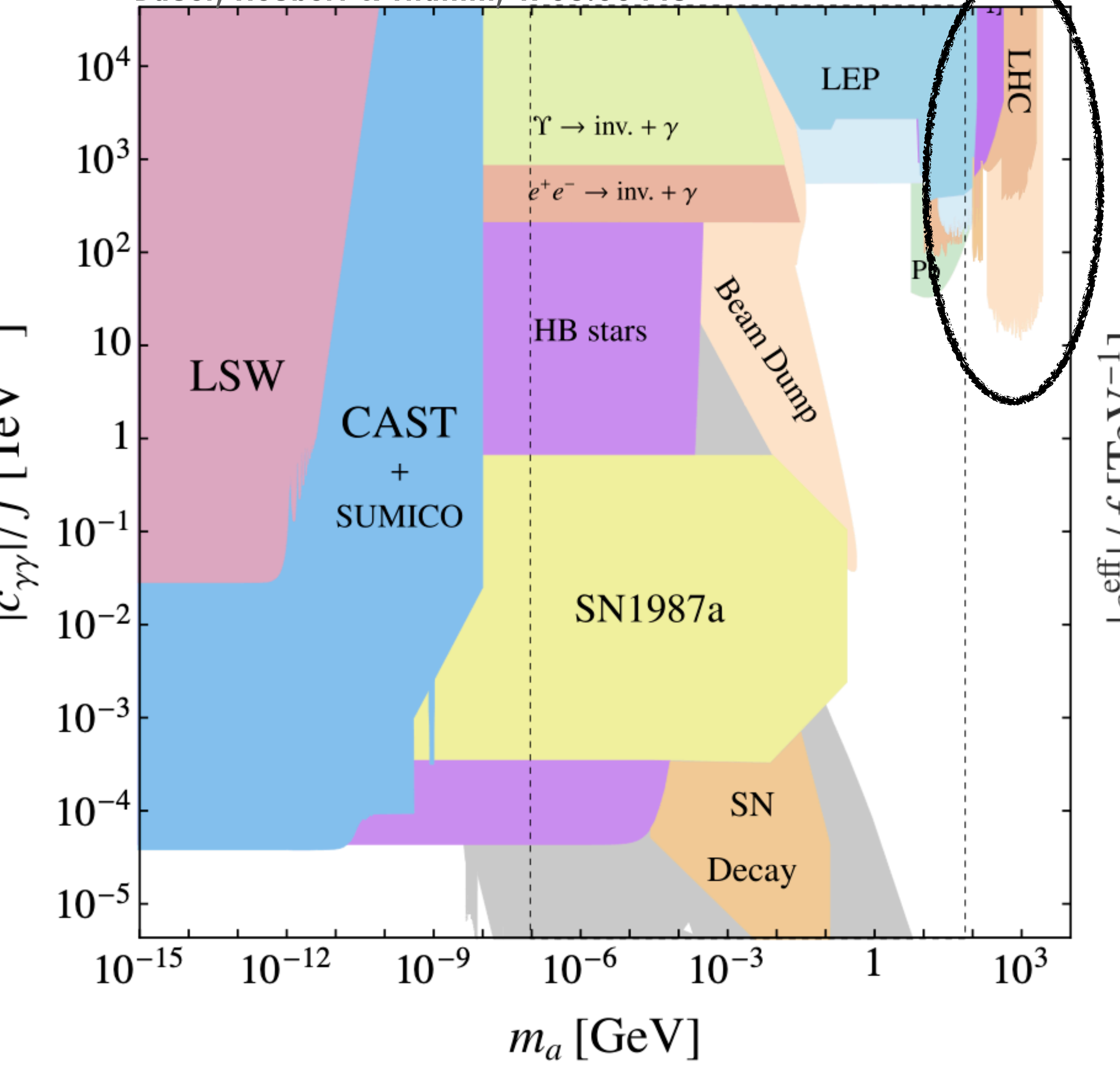
See Michele Gallinaro's talk yesterday, or CMS search for
$h \to aa \to 4\gamma$
2208.01469
Limits $\sim 10^{-2}$

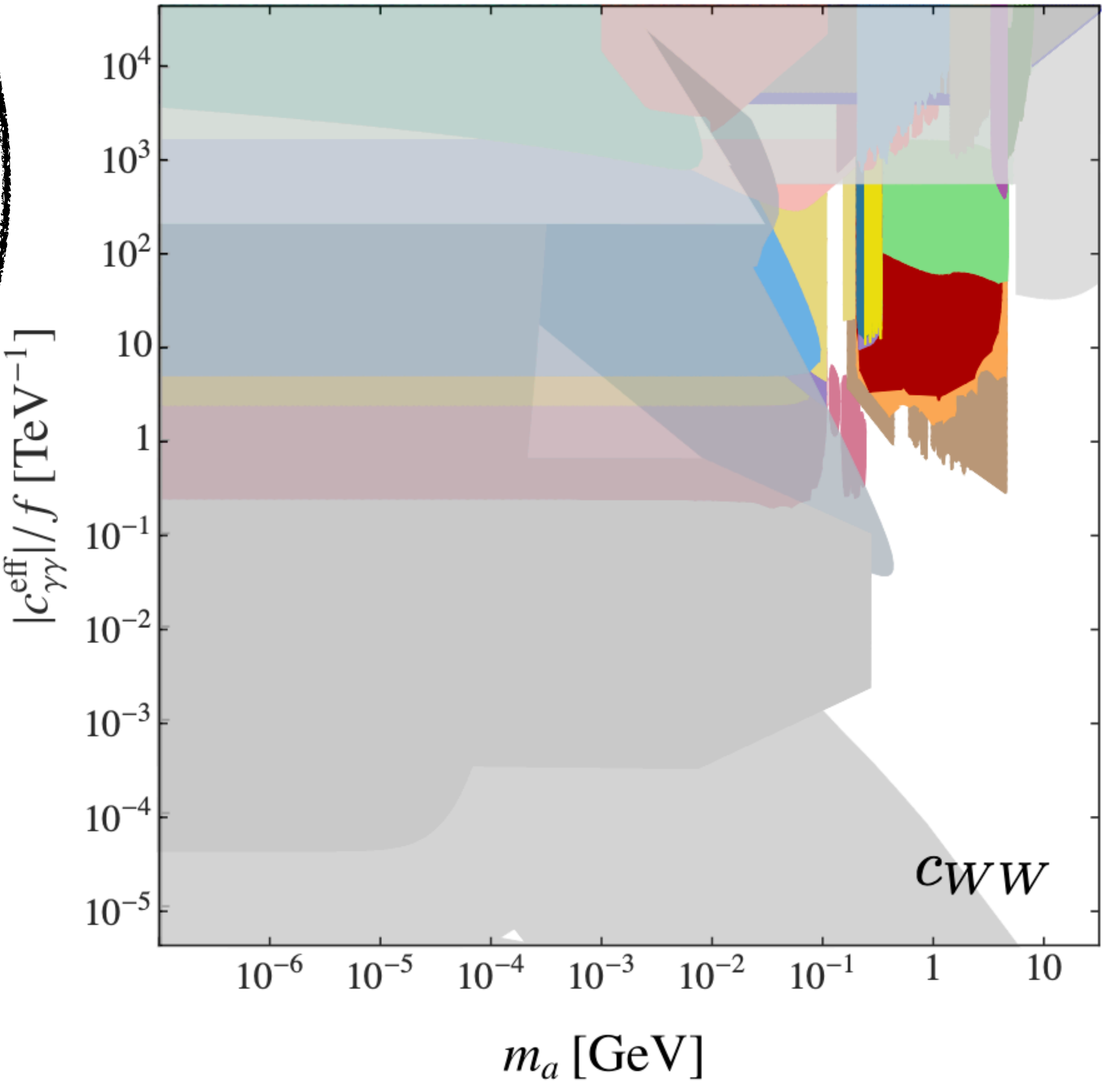
Comparison with photonic constraints

N.B. new LHC light-by-light and
higgs decay constraints in this area,
see Michele Gallinaro's talk

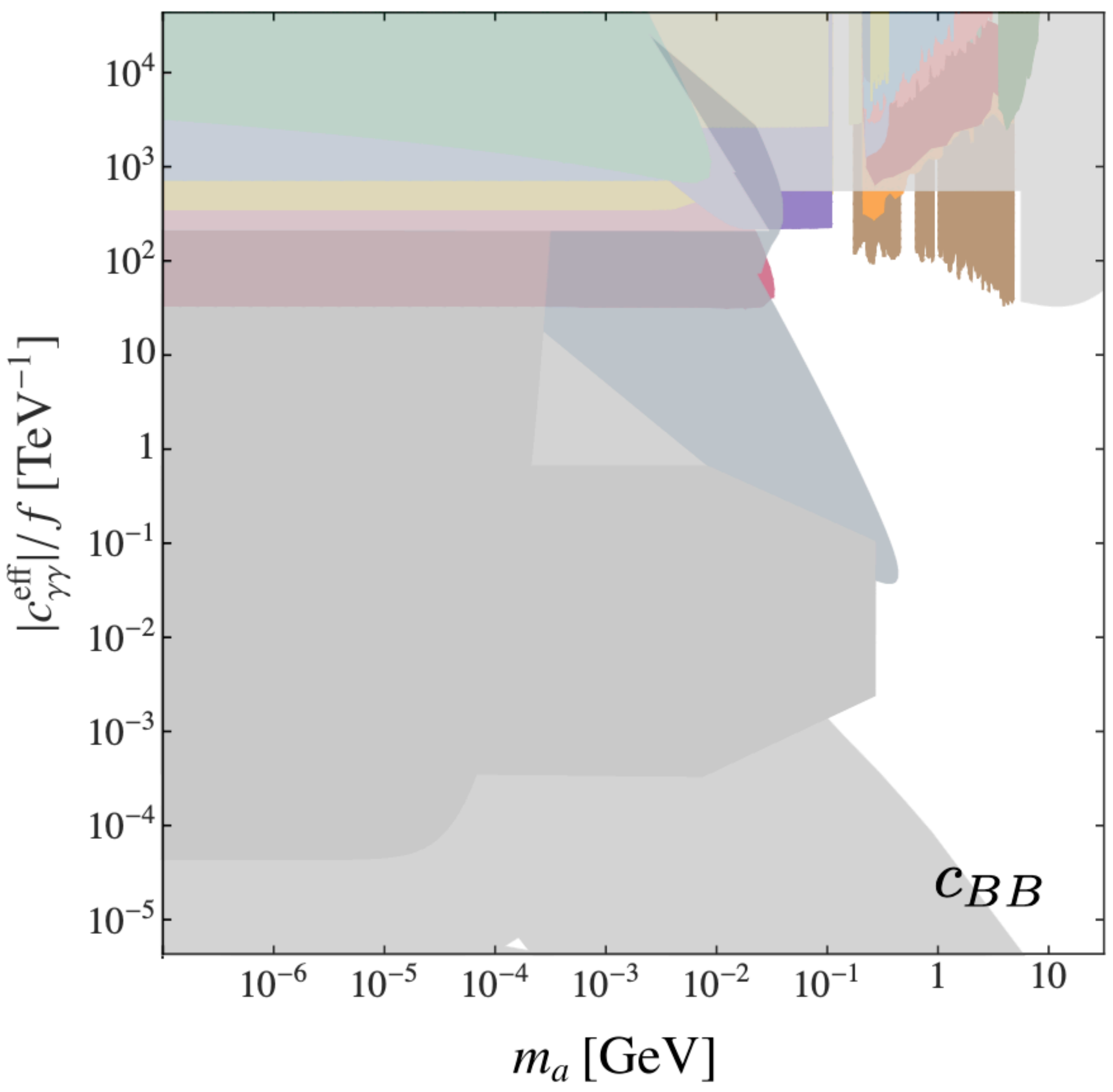
$$\boxed{c_{\gamma\gamma} = c_{WW} + c_{BB}}$$

Bauer, Neubert & Thamm, 1708.00443





Bauer, Neubert, SR, Schnubel, Thamm, 2110.10698



What about leptons?

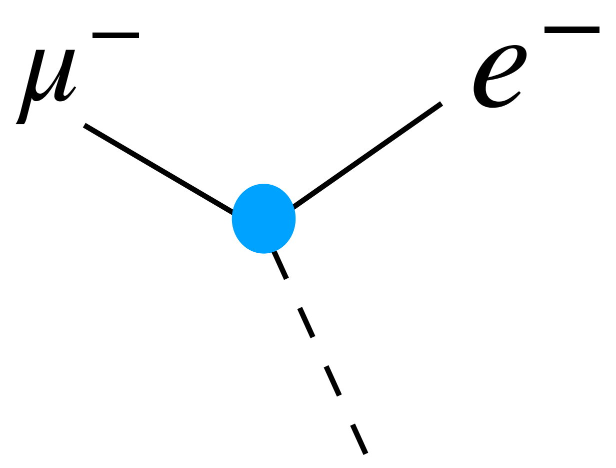
ALPs may also have lepton flavour violating (LFV) couplings

SM is lepton flavour conserving $\Longrightarrow$ unlike quark case, LFV cannot be created from RG alone

But there can still be loop level connections between LFV and flavour conserving processes

e.g.

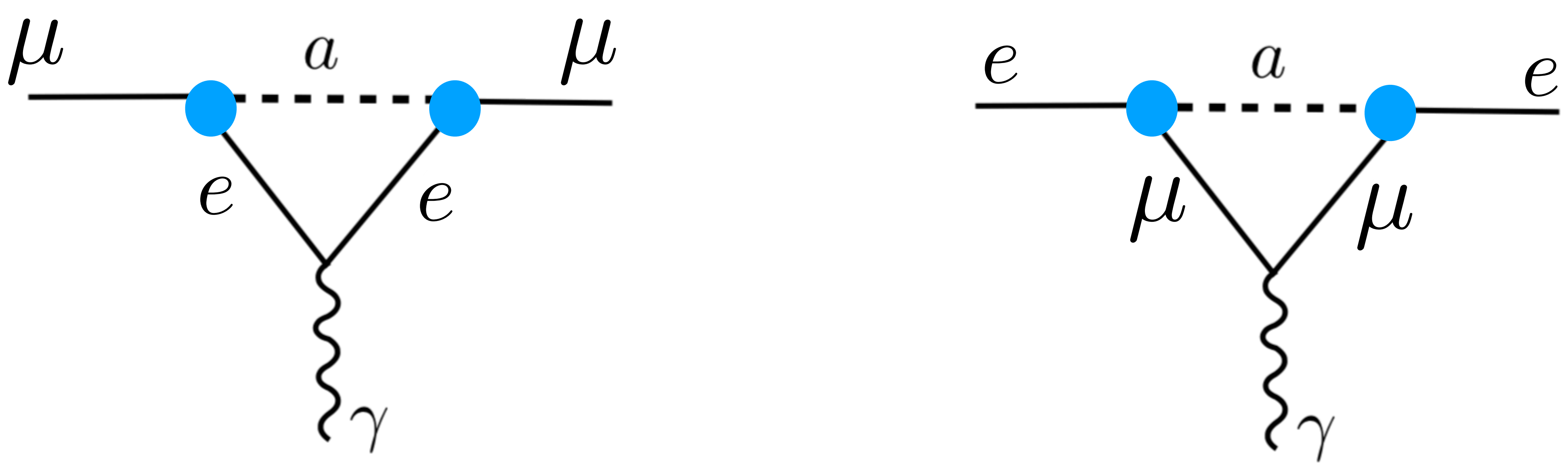
ALPs produced in $\mu \to e$ decays



$\mu \to ea$

$\Longrightarrow$

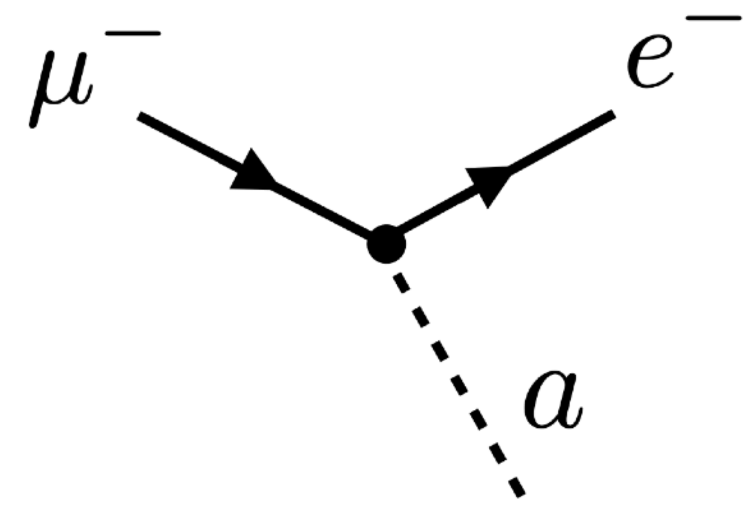
Contribution to $(g-2)_\mu$, $(g-2)_e$



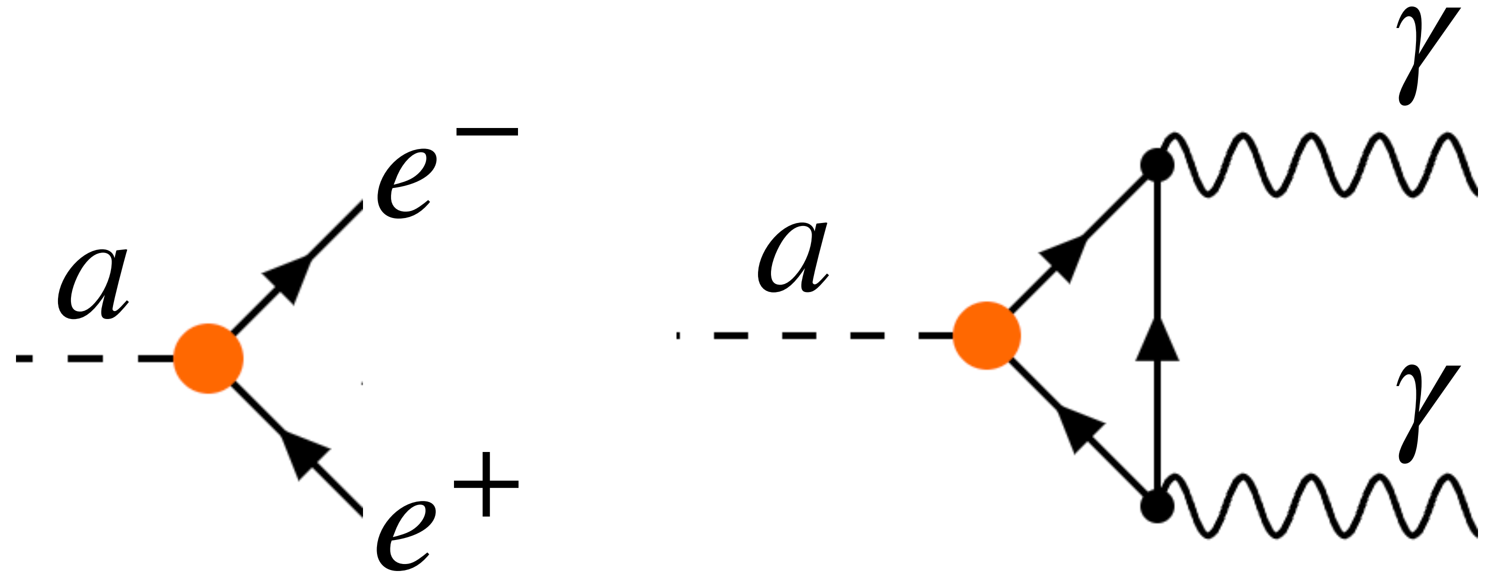
Effect of flavour conserving couplings

Simple scenario with only leptonic couplings at tree level

ALP production:



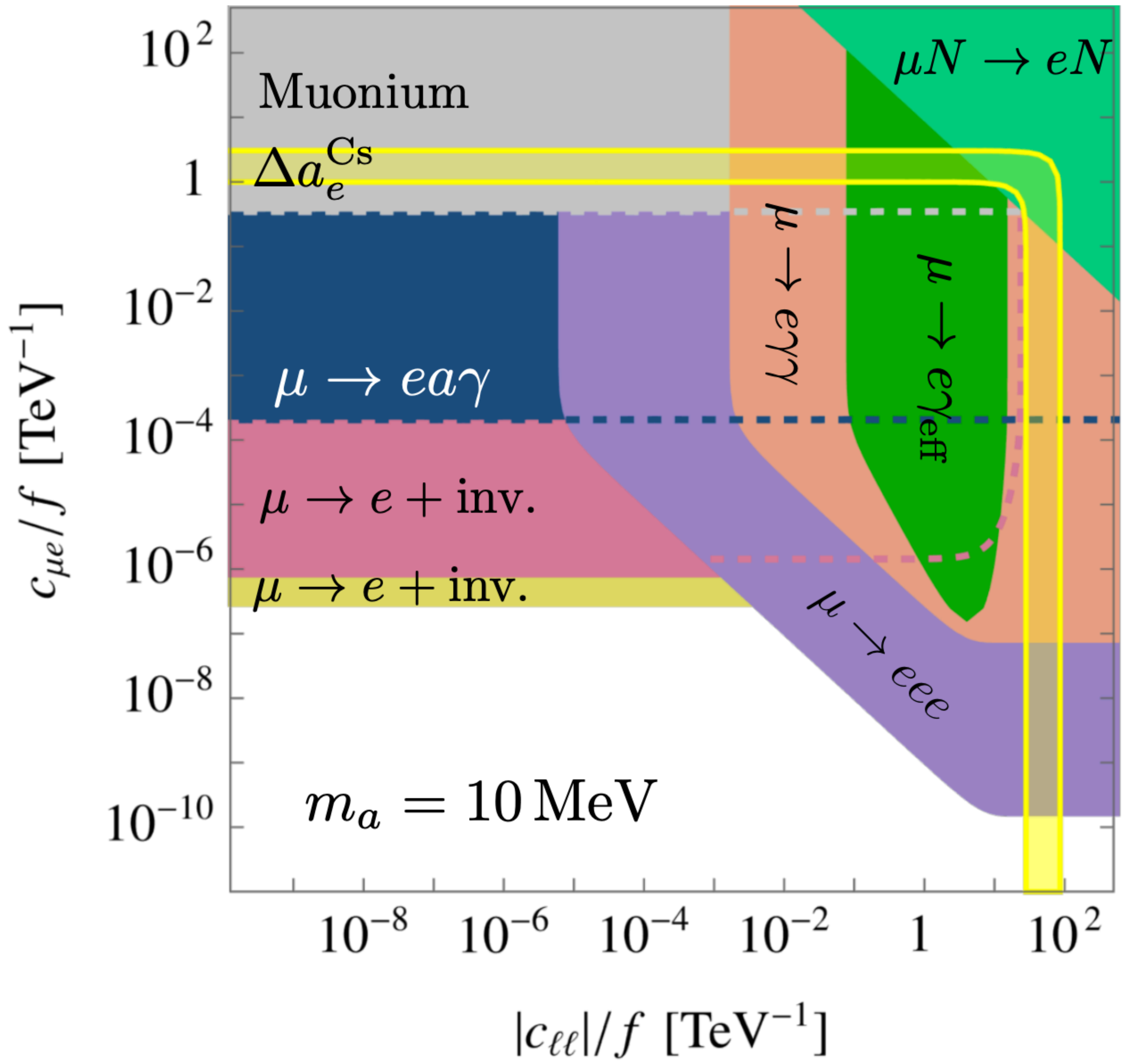
ALP decay:



$$\mathcal{L}_{\text{eff}}^{\text{LFV}} = \frac{\partial^\mu a}{f} \left(\bar{\ell}_i (k_E)_{ij} \gamma_\mu P_L \ell_j + \bar{\ell}_i (k_e)_{ij} \gamma_\mu P_R \ell_j \right)$$

$$c_{ij} \equiv \sqrt{|(k_e)_{ij}|^2 + |(k_E)_{ij}|^2} \quad i \neq j$$

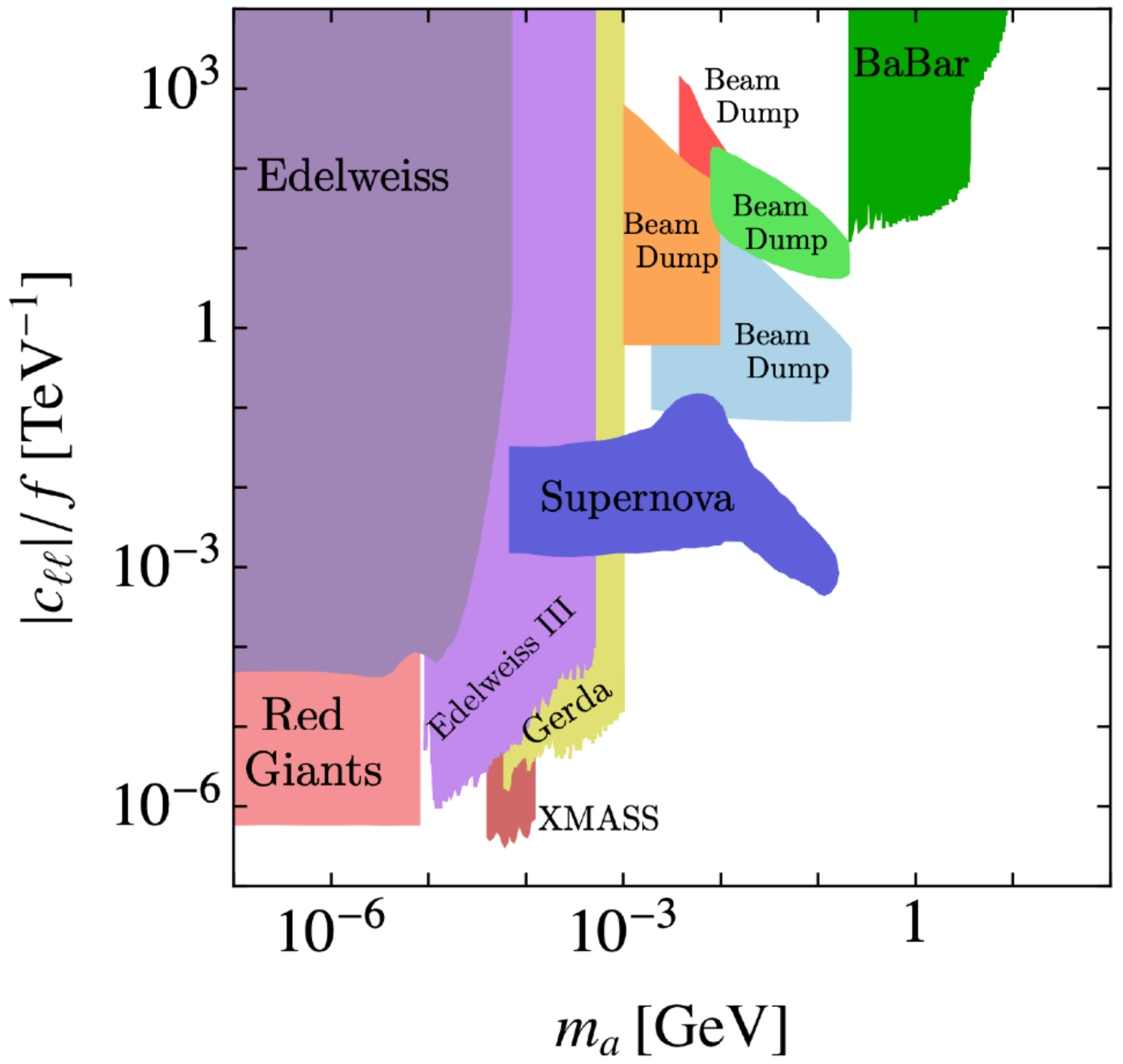
Bauer, Neubert, SR, Schnubel, Thamm, 2110.10698



Mass dependence

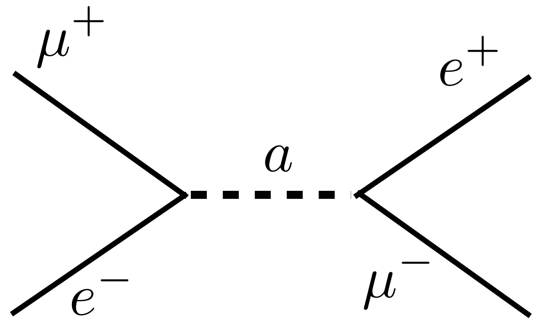
For ALP masses too heavy to be produced in muon decays, $\mu \to e\gamma$ and $\mu \to 3e$ can still be constraining

All* LFV bounds depend on choice for flavour-conserving couplings, chosen to be consistent with other bounds:

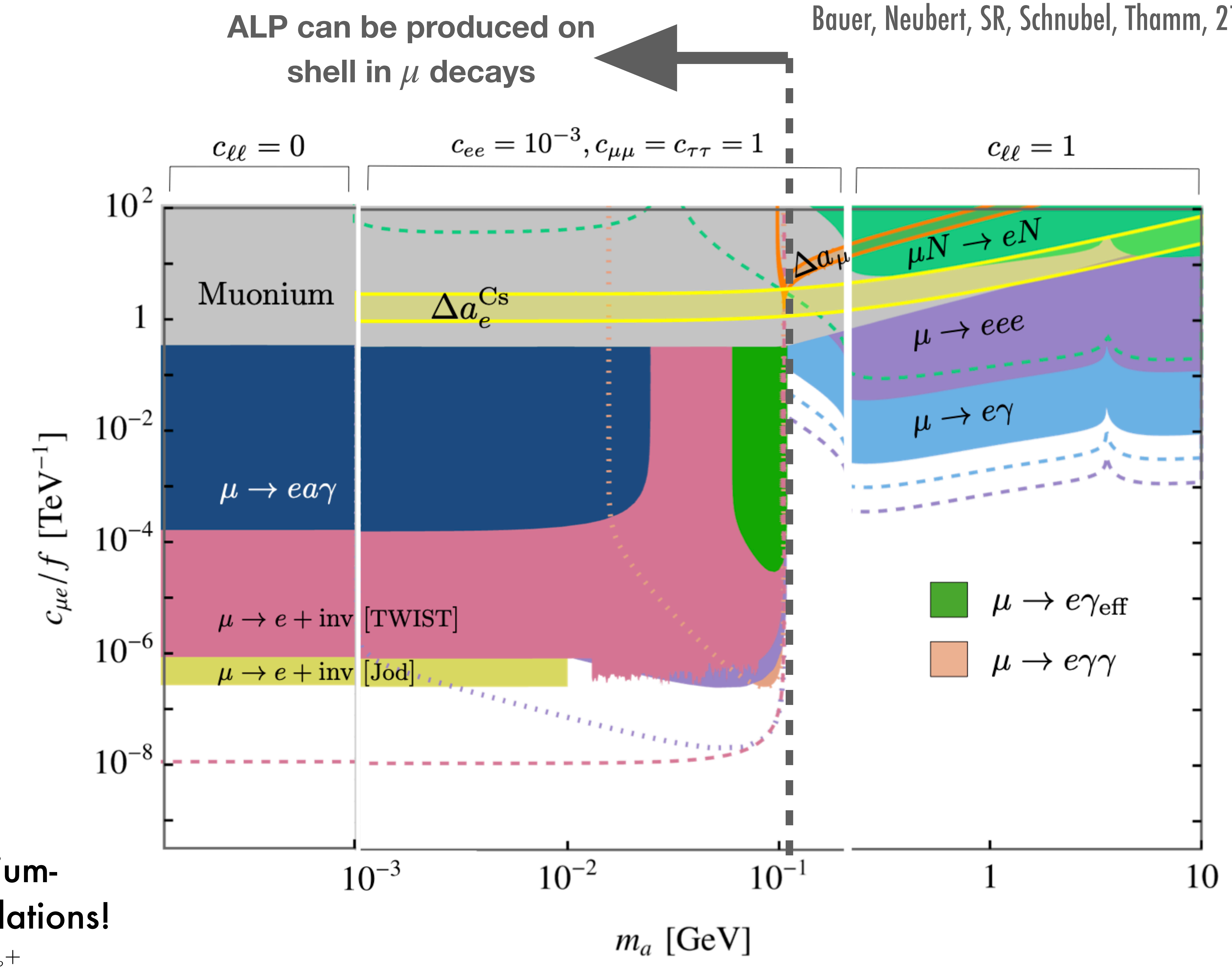


$\Rightarrow$

*except muonium-antimuonium oscillations!



Bauer, Neubert, SR, Schnubel, Thamm, 2110.10698



Projected limits: MEGII $\mu \to e + \text{inv}$ * MEGII $\mu \to e\gamma$ Mu3E $\mu \to 3e$

*Optimistic limits. Could be improved with a forward calorimeter, see 2006.04795

Summary

- Axion-like particles are a generic option for light new physics, and can be studied independently of their UV completion via EFTs
- The appropriate EFT depends on the scale of the observable
- Through running and matching from the UV scale, new couplings e.g. flavour changing effects can generically arise
- Flavour changing observables have good discovery potential for ALPs in MeV-GeV mass range

Thank you!

Which scale to run from?

Can imagine that within the EFT, $\Lambda < 4\pi f$. How much does this change things?

Running from $\Lambda = 4\pi$ TeV:

$$c_{uu,cc}(\mu_w) \simeq c_{uu,cc}(\Lambda) - 0.116\, c_{tt}(\Lambda) - \Big[6.35\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.02\, \tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

$$c_{dd,ss}(\mu_w) \simeq c_{dd,ss}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \Big[7.08\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

Running from $\Lambda = 4\pi \times 10^{12}$ TeV:

$$c_{uu,cc}(m_t) \simeq c_{uu,cc}(\Lambda) - 0.350\, c_{tt}(\Lambda) - \Big[12.6\, \tilde{c}_{GG}(\Lambda) + 0.84\, \tilde{c}_{WW}(\Lambda) + 0.10\, \tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}\,,$$

$$c_{dd,ss}(m_t) \simeq c_{dd,ss}(\Lambda) + 0.353\, c_{tt}(\Lambda) - \Big[16.8\, \tilde{c}_{GG}(\Lambda) + 1.30\, \tilde{c}_{WW}(\Lambda) + 0.07\, \tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}\,,$$

Not much!

Lepton flavour violating ALPs

$F = Q, u, d, L, e$

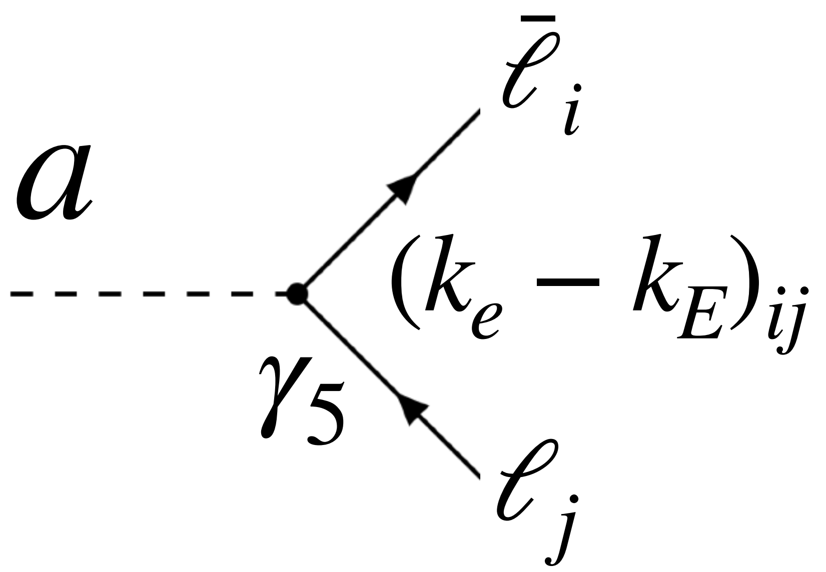
Bjorkeroth, Chun, King, 1806.00660
Bauer, Neubert, SR, Schnubel, Thamm, 1908.00008
Cornella, Paradisi, Sumensari, 1911.06279

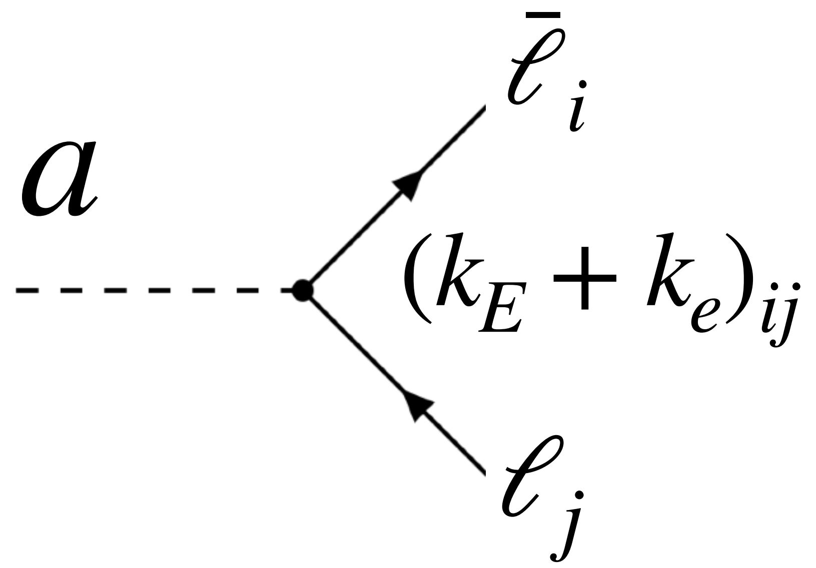
$$\mathcal{L}_{\rm eff}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{\partial^\mu a}{f}\sum_F \bar{\psi}_F \boldsymbol{c}_F \gamma_\mu \psi_F$$
$$+ c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W^A_{\mu\nu}\tilde{W}^{\mu\nu,A} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

$$\Lambda_{UV} = 4\pi f$$

Zooming in on the fermionic couplings…

$$\mathcal{L}_{\rm eff}^{\rm LFV} = \frac{\partial^\mu a}{f}\left(\bar{\ell}_i (k_E)_{ij}\gamma_\mu P_L \ell_j + \bar{\ell}_i (k_e)_{ij}\gamma_\mu P_R \ell_j\right)$$





LFV ALPs can naturally arise as PNGBs of symmetries addressing

- the strong CP problem (DFSZ axion) Calibbi, Redigolo, Ziegler, Zupan, 2006.04795
- the flavour problem (familon) Linster, Ziegler 1805.07341, Calibbi, Redigolo, Ziegler, Zupan 2006.04795
- neutrino masses (majoron) Chikashige, Mohapatra, Peccei 1981, Schechter & Valle 1982, Garcia-Cely & Heeck 1701.07209, Heeck & Patel 1909.02029

if $i = j$, only pseudoscalar coupling

Symmetries and redundancies

By doing ALP-dependent field redefinitions on the effective Lagrangian, can eliminate some operators

If we do the redefinitions:
$$\psi_F \to \exp\left(ic\,\frac{a}{f}\,\boldsymbol{Q}_F\right)\psi_F\,, \qquad \phi \to \exp\left(ic\,\frac{a}{f}\,Q_\phi\right)\phi$$

charge (matrix) under $U(1)$ symmetries of SM

Then the dimension 4 (SM) Lagrangian is unchanged

At dimension 5, all ALP couplings are shifted:

$$\begin{aligned}
&\boldsymbol{c}_F \to \boldsymbol{c}_F - c\,\boldsymbol{Q}_F\,, && c_{WW} \to c_{WW} - \frac{c}{2}\,\mathrm{Tr}\,(3\boldsymbol{Q}_Q + \boldsymbol{Q}_L)\,,\\
&c_\phi \to c_\phi - c\,Q_\phi\,, && c_{BB} \to c_{BB} + c\,\mathrm{Tr}\left(\frac{4}{3}\boldsymbol{Q}_u + \frac{1}{3}\boldsymbol{Q}_d - \frac{1}{6}\boldsymbol{Q}_Q + \boldsymbol{Q}_e - \frac{1}{2}\boldsymbol{Q}_L\right)\\
&c_{GG} \to c_{GG} + \frac{c}{2}\,\mathrm{Tr}\,(\boldsymbol{Q}_u + \boldsymbol{Q}_d - 2\boldsymbol{Q}_Q) &&
\end{aligned}$$

Five global symmetries of the SM Lagrangian $\Longrightarrow$ 5 redundant parameters

In particular, can use *hypercharge* transformation to eliminate ALP-Higgs operator:

$$\cancel{c_\phi\,\frac{\partial^\mu a}{f}\,(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)}$$

operator removed if $c = 2c_\phi$

$$\boxed{c_\phi \to c_\phi - \frac{c}{2}\,,} \qquad \boldsymbol{c}_Q \to \boldsymbol{c}_Q - \frac{c}{6}\mathbb{1}\,, \qquad \boldsymbol{c}_L \to \boldsymbol{c}_L + \frac{c}{2}\mathbb{1}\,,$$
$$\boldsymbol{c}_u \to \boldsymbol{c}_u - \frac{2c}{3}\mathbb{1}\,, \qquad \boldsymbol{c}_d \to \boldsymbol{c}_d + \frac{c}{3}\mathbb{1}\,, \qquad \boldsymbol{c}_e \to \boldsymbol{c}_e + c\mathbb{1}\,,$$

Theory at GeV scale

We have run down to the GeV scale, now we only have u,d,s quarks

General Lagrangian can be written as

$$\boxed{\begin{aligned}\mathcal{L} = &\, i\bar{q}\not{D}q - \frac{1}{4}GG - \frac{1}{4}FF + \frac{1}{2}(\partial a)^2 - \frac{1}{2}m_a^2 a^2 \\ &- \bar{q}\boldsymbol{m_q}q + \frac{\partial_\mu a}{f}\left(\bar{q}_L \boldsymbol{k_Q}\gamma^\mu q_L + \bar{q}_R \boldsymbol{k_q}\gamma^\mu q_R\right) + c_{GG}\frac{\alpha_S}{4\pi}\frac{a}{f}G\tilde{G} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F\tilde{F}\end{aligned}}$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Values of all couplings are given at the low scale $\mu \sim \text{GeV}$
(but can be found in terms of UV couplings via all the equations shown till now)

Now want to match to a chiral Lagrangian written in terms of meson fields

Georgi, Kaplan, Randall 1986; Srednicki 1985; Bardeen, Peccei, Yanagida 1987;...

Spurion analysis

Want to ensure that the chiral lagrangian has all the same symmetry properties

Lagrangian again:

$$\mathcal{L} = i\bar{q}\not{D}q - \frac{1}{4}GG - \frac{1}{4}FF + \frac{1}{2}(\partial a)^2 - \frac{1}{2}m_a^2 a^2$$
$$- \bar{q}\boldsymbol{m_q}q + \frac{\partial_\mu a}{f}\left(\bar{q}_L \boldsymbol{k_Q}\gamma^\mu q_L + \bar{q}_R \boldsymbol{k_q}\gamma^\mu q_R\right) + c_{GG}\frac{\alpha_S}{4\pi}\frac{a}{f}G\tilde{G} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F\tilde{F}$$

Ignoring the gauge kinetic terms, can rewrite this in terms of spurionic background fields

$$\boxed{\mathcal{L} = i\bar{q}\not{\partial}q - \bar{q}\left(\underline{\boldsymbol{s}} - i\underline{\boldsymbol{p}}\gamma_5\right)q + \bar{q}_L\underline{\boldsymbol{l_\mu}}\gamma^\mu q_L + \bar{q}_R\underline{\boldsymbol{r_\mu}}\gamma^\mu q_R + \underline{\theta_G}\frac{\alpha_S}{4\pi}G\tilde{G} + \underline{\theta_\gamma}\frac{\alpha}{4\pi}F\tilde{F}}$$

Now the whole Lagrangian has local $U(3)_L \times U(3)_R$ invariance under which:

$$\hat{q}_L = \boldsymbol{V_L}q_L = e^{i\boldsymbol{\beta_L}}q_L$$
$$\hat{q}_R = \boldsymbol{V_R}q_R = e^{i\boldsymbol{\beta_R}}q_R$$

...if the background fields transform as:

$$\hat{\boldsymbol{l}}_\mu = \boldsymbol{V_L}\boldsymbol{l_\mu}\boldsymbol{V_L^\dagger} + i\boldsymbol{V_L}\partial_\mu\boldsymbol{V_L^\dagger}$$
$$\hat{\boldsymbol{r}}_\mu = \boldsymbol{V_R}\boldsymbol{r_\mu}\boldsymbol{V_R^\dagger} + i\boldsymbol{V_R}\partial_\mu\boldsymbol{V_R^\dagger}$$
$$(\hat{\boldsymbol{s}} + i\hat{\boldsymbol{p}}) = \boldsymbol{V_R}(\boldsymbol{s} + i\boldsymbol{p})\boldsymbol{V_L^\dagger}$$
$$(\hat{\boldsymbol{s}} - i\hat{\boldsymbol{p}}) = \boldsymbol{V_L}(\boldsymbol{s} - i\boldsymbol{p})\boldsymbol{V_R^\dagger}$$
$$\hat{\theta}_G = \theta_G - \frac{1}{2}\mathrm{Tr}[\boldsymbol{\beta_R} - \boldsymbol{\beta_L}]$$
$$\hat{\theta}_\gamma = \theta_\gamma - N_c\mathrm{Tr}[(\boldsymbol{\beta_R} - \boldsymbol{\beta_L})\boldsymbol{Q}^2]$$

Chiral Lagrangian

Construct a chiral lagrangian out of the meson octet and the spurions

$$\boldsymbol{\Sigma} = \exp\left(\frac{i\sqrt{2}}{f_\pi}\boldsymbol{\lambda^a}\pi^a\right) \qquad \hat{\boldsymbol{\Sigma}} = \boldsymbol{V_L}\boldsymbol{\Sigma}\boldsymbol{V_R}^\dagger$$

The covariant derivative is $D_\mu\boldsymbol{\Sigma} = \partial_\mu\boldsymbol{\Sigma} - i\boldsymbol{l_\mu}\boldsymbol{\Sigma} + i\boldsymbol{\Sigma}\boldsymbol{r_\mu}$ Gasser & Leutwyler, 1985

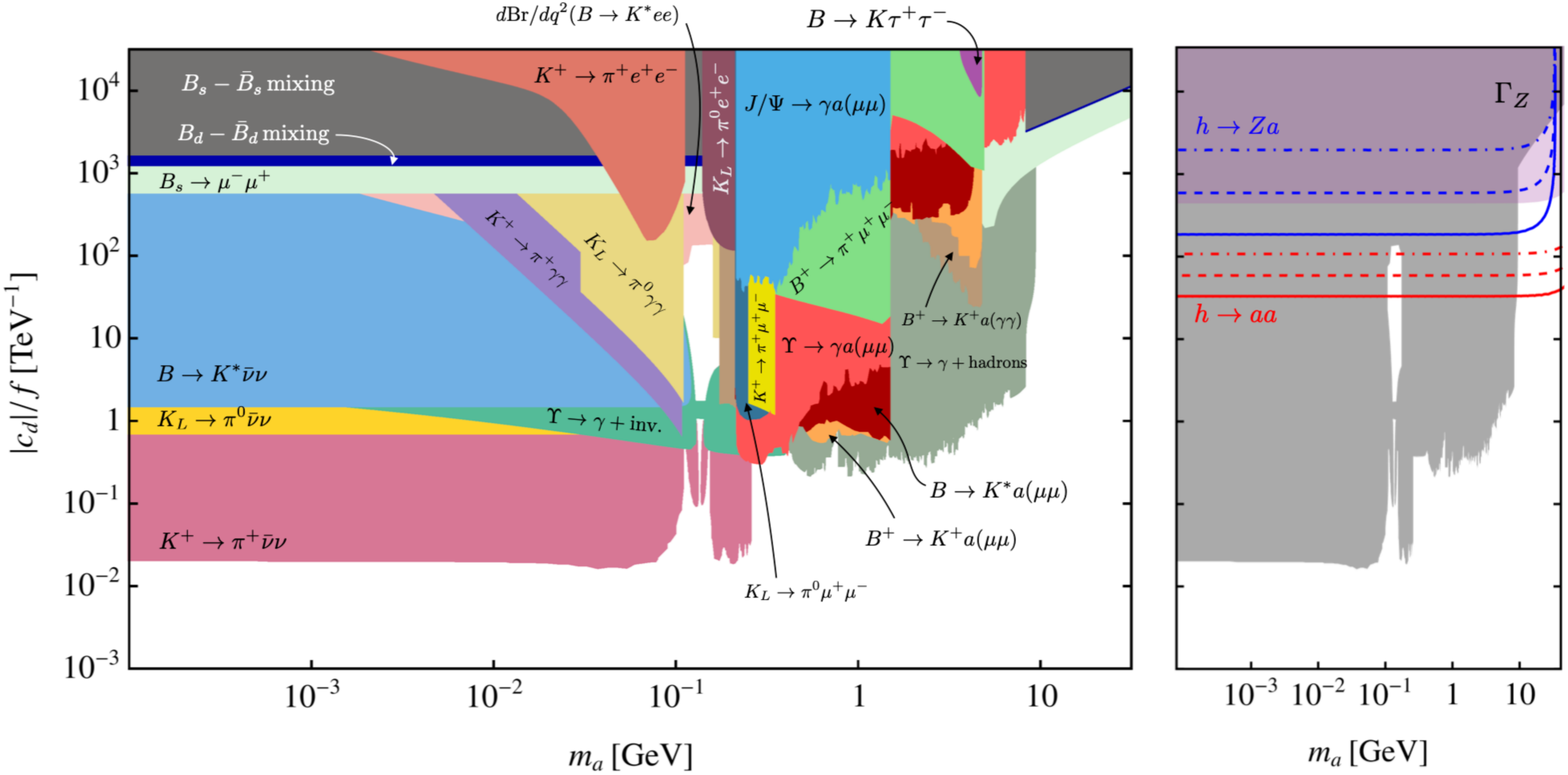
$$= \partial_\mu\boldsymbol{\Sigma} - ieA_\mu[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(e^{-i\boldsymbol{\kappa}c_{GG}a/f}\left(\boldsymbol{k_Q} - \boldsymbol{\kappa}c_{GG}\right)e^{i\boldsymbol{\kappa}c_{GG}a/f}\boldsymbol{\Sigma} - \boldsymbol{\Sigma}e^{i\boldsymbol{\kappa}c_{GG}a/f}\left(\boldsymbol{k_q} + \boldsymbol{\kappa}c_{GG}\right)e^{-i\boldsymbol{\kappa}c_{GG}a/f}\right)$$

The Lagrangian can be written in terms of these as

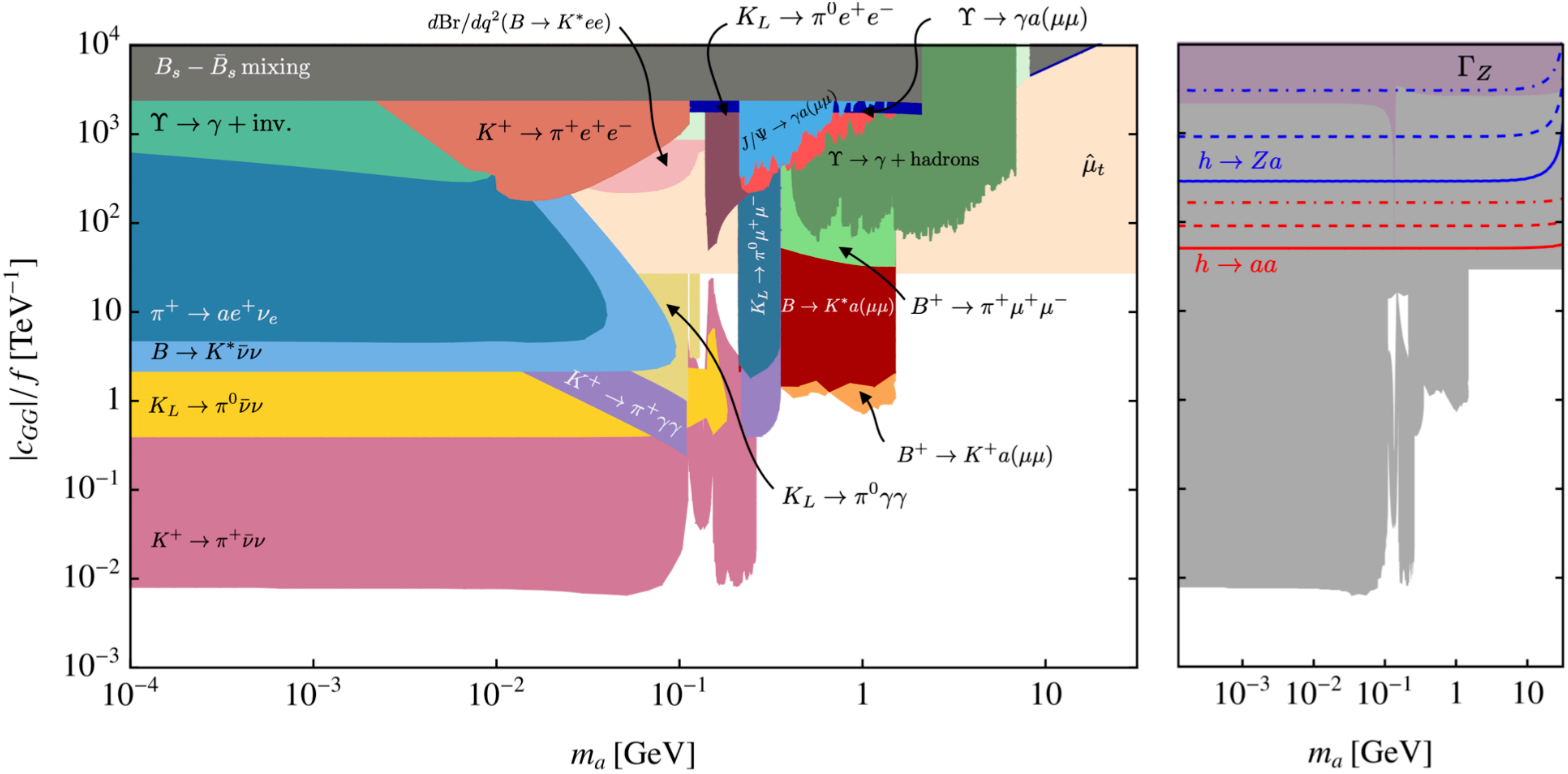
fixed empirically by pion mass

$$\mathcal{L}_{\rm eff}^\chi = \frac{f_\pi^2}{8}\,\mathrm{Tr}\left[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4}B_0\,\mathrm{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \mathrm{h.c.}\right]$$
$$+ \frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\,,$$

Terms in first line contain mass- and kinetic-mixing between ALP and neutral mesons



dBr/dq^2(B → K*ee)
B → Kτ+τ−
Bs − B̄s mixing
Bd − B̄d mixing
Bs → μ−μ+
K+ → π+e+e−
KL → π0e+e−
J/Ψ → γa(μμ)
K+ → π+γγ
KL → π0γγ
B+ → π+μ+μ−
B+ → K+a(γγ)
K+ → π+μ+μ−
Υ → γa(μμ)
Υ → γ + hadrons
B → K*ν̄ν
KL → π0ν̄ν
Υ → γ + inv.
B → K*a(μμ)
B+ → K+a(μμ)
K+ → π+ν̄ν
KL → π0μ+μ−
|cd|/f [TeV−1]
ma [GeV]
ΓZ
h → Za
h → aa



dBr/dq²(B → K*ee)
K_L → π⁰e⁺e⁻
Υ → γa(μμ)
B_s − B̄_s mixing
Υ → γ + inv.
K⁺ → π⁺e⁺e⁻
J/Ψ → γa(μμ)
Υ → γ + hadrons
μ̂_t
K_L → π⁰μ⁺μ⁻
B → K*a(μμ)
B⁺ → π⁺μ⁺μ⁻
π⁺ → ae⁺ν_e
B → K*ν̄ν
K_L → π⁰ν̄ν
K⁺ → π⁺γγ
B⁺ → K⁺a(μμ)
K_L → π⁰γγ
K⁺ → π⁺ν̄ν
|c_GG| / f [TeV⁻¹]
m_a [GeV]
Γ_Z
h → Za
h → aa
m_a [GeV]

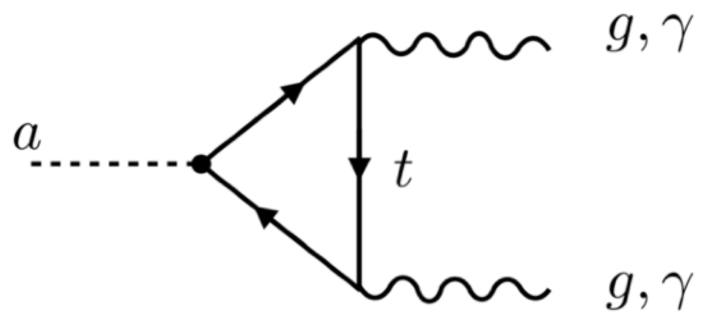
Matching at the EW scale

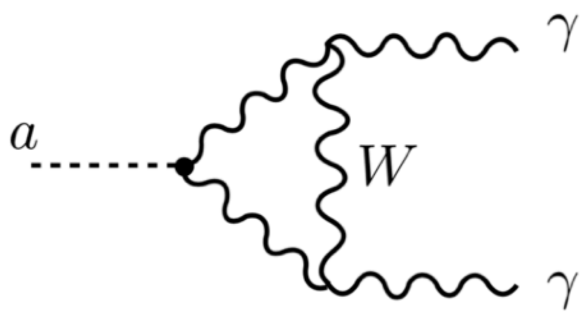
Integrating out t, W, Z, h to find EFT coefficients below EW scale

$$\mathcal{L}_{\rm eff}(\mu_w) = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \mathcal{L}_{\rm ferm}(\mu_w) + c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

$$\mathcal{L}_{\rm ferm}(\mu_w) = \frac{\partial^\mu a}{f}\Big[\bar{u}_L \boldsymbol{k}_U \gamma_\mu u_L + \bar{u}_R \boldsymbol{k}_u \gamma_\mu u_R + \bar{d}_L \boldsymbol{k}_D \gamma_\mu d_L + \bar{d}_R \boldsymbol{k}_d \gamma_\mu d_R + \bar{\nu}_L \boldsymbol{k}_\nu \gamma_\mu \nu_L + \bar{e}_L \boldsymbol{k}_E \gamma_\mu e_L + \bar{e}_R \boldsymbol{k}_e \gamma_\mu e_R\Big]\,.$$

$$c_{\gamma\gamma} = c_{WW} + c_{BB}$$



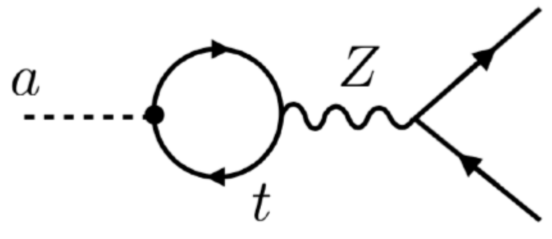


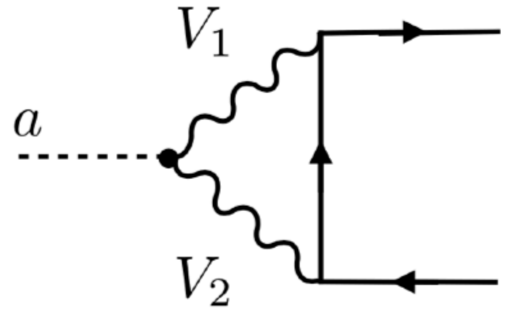
$\sim m_a^2/m_t^2,\; m_a^2/m_W^2$

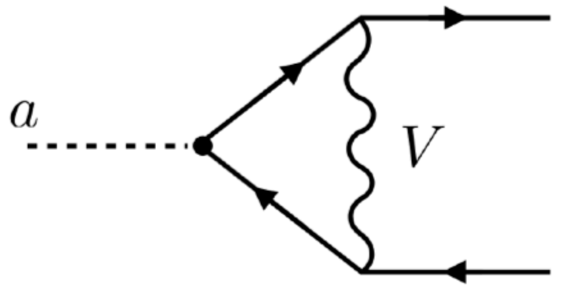
Bauer, Neubert, Thamm 2017

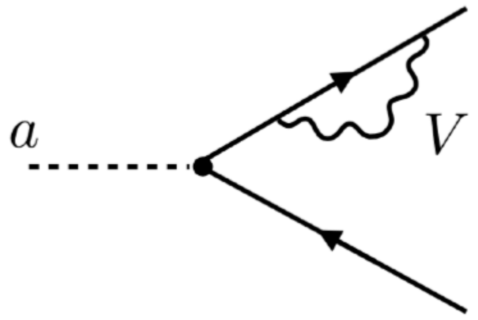
No loop matching contributions to gauge couplings for a light ALP

But one loop matching for fermionic couplings









Running below EW scale

Only from diagrams involving photons and gluons

$$\frac{d}{d\ln\mu}\boldsymbol{k}_q(\mu) = -\frac{d}{d\ln\mu}\boldsymbol{k}_Q(\mu) = \left(\frac{\alpha_s^2}{\pi^2}\,\tilde{c}_{GG} + \frac{3\alpha^2}{4\pi^2}\,Q_q^2\,\tilde{c}_{\gamma\gamma}\right)\mathbb{1}$$

$$\frac{d}{d\ln\mu}\boldsymbol{k}_e(\mu) = -\frac{d}{d\ln\mu}\boldsymbol{k}_E(\mu) = \frac{3\alpha^2}{4\pi^2}\,\tilde{c}_{\gamma\gamma}\,\mathbb{1}\,,$$

As for the theory above EW scale, gauge couplings do not run

Overall, the most important qualitative RG effects happen above the EW scale, and are mostly due to the top Yukawa

These can affect pheno, bounds & model discrimination for the QCD axion, see:

Choi, Im, Kim, Seong 2106.05816
Di Luzio, Mescia, Nardi, Okawa 2205.15326
Di Luzio, Giannotti, Mescia, Nardi, Okawa, Piazza, 2305.11958

Theory at GeV scale

We have run down to the GeV scale, now we only have u,d,s quarks

General Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = & \, i\bar{q}\not{D}q - \frac{1}{4}GG - \frac{1}{4}FF + \frac{1}{2}(\partial a)^2 - \frac{1}{2}m_a^2 a^2 \\ & - \bar{q}\boldsymbol{m_q}q + \frac{\partial_\mu a}{f}\left(\bar{q}_L \boldsymbol{k_Q}\gamma^\mu q_L + \bar{q}_R \boldsymbol{k_q}\gamma^\mu q_R\right) + c_{GG}\frac{\alpha_S}{4\pi}\frac{a}{f}G\tilde{G} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F\tilde{F}\end{aligned}$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

Values of all couplings are given at the low scale $\mu \sim \mathrm{GeV}$
(but can be found in terms of UV couplings via all the equations shown till now)

Now want to match to a chiral Lagrangian written in terms of meson fields

Georgi, Kaplan, Randall 1986; Srednicki 1985; Bardeen, Peccei, Yanagida 1987;...

Ignoring the gauge kinetic terms, can rewrite the lagrangian in terms of spurionic background fields

$$\mathcal{L} = i\bar{q}\not{\partial}q - \bar{q}\left(\boldsymbol{s} - i\boldsymbol{p}\gamma_5\right)q + \bar{q}_L \boldsymbol{l_\mu}\gamma^\mu q_L + \bar{q}_R \boldsymbol{r_\mu}\gamma^\mu q_R + \theta_G \frac{\alpha_S}{4\pi}G\tilde{G} + \theta_\gamma \frac{\alpha}{4\pi}F\tilde{F}$$

Now the whole Lagrangian has local $U(3)_L \times U(3)_R$ invariance under assumptions for how the spurions transform

Breaking the symmetry

For ease of matching, we rotate away the gluon coupling via the general chiral rotation

$$\boldsymbol{V_L} = \exp\left[-i\boldsymbol{\kappa} c_{GG}\frac{a}{f}\right]$$

$$\boldsymbol{V_R} = \exp\left[i\boldsymbol{\kappa} c_{GG}\frac{a}{f}\right]$$

*but to not break charge, $[Q, \kappa] = 0$

$\boldsymbol{\kappa}$ is a 3x3 matrix

Any choice(*) will work as long as

$$\mathrm{Tr}[\boldsymbol{\kappa}] = 1$$

We assume it's diagonal, but keeping the entries of kappa free will allow crosschecks of final results

By doing this we induce effects proportional to c_{GG} in other terms in the Lagrangian

We break the symmetry by setting the spurions to their rotated values

$$\hat{\boldsymbol{l}}_{\boldsymbol{\mu}} = \frac{1}{2}eA_\mu Q + \frac{1}{2}g_s G^a_\mu T^a + \frac{\partial_\mu a}{f} e^{-i\boldsymbol{\kappa} c_{GG} a/f}\left(\boldsymbol{k_Q} - \boldsymbol{\kappa} c_{GG}\right) e^{i\boldsymbol{\kappa} c_{GG} a/f}$$

$$\hat{\boldsymbol{r}}_{\boldsymbol{\mu}} = \frac{1}{2}eA_\mu Q + \frac{1}{2}g_s G^a_\mu T^a + \frac{\partial_\mu a}{f} e^{i\boldsymbol{\kappa} c_{GG} a/f}\left(\boldsymbol{k_q} + \boldsymbol{\kappa} c_{GG}\right) e^{-i\boldsymbol{\kappa} c_{GG} a/f}$$

$$\hat{\boldsymbol{s}} - i\hat{\boldsymbol{p}} = e^{-i\boldsymbol{\kappa} c_{GG} a/f}\boldsymbol{m_q} e^{-i\boldsymbol{\kappa} c_{GG} a/f} = \hat{\boldsymbol{m}}_{\boldsymbol{q}}$$

$$\hat{\theta}_G = 0$$

$$\hat{\theta}_\gamma = \frac{a}{f}\left(c_{\gamma\gamma} - 2N_c c_{GG}\mathrm{Tr}[\boldsymbol{\kappa}\boldsymbol{Q}^2]\right)$$

Chiral Lagrangian

Construct a chiral lagrangian out of the meson octet and the spurions

$$\boldsymbol{\Sigma} = \exp\left(\frac{i\sqrt{2}}{f_\pi}\boldsymbol{\lambda^a}\pi^a\right) \qquad \hat{\boldsymbol{\Sigma}} = \boldsymbol{V_L}\boldsymbol{\Sigma}\boldsymbol{V_R^\dagger}$$

The covariant derivative is $D_\mu\boldsymbol{\Sigma} = \partial_\mu\boldsymbol{\Sigma} - i\boldsymbol{l_\mu}\boldsymbol{\Sigma} + i\boldsymbol{\Sigma}\boldsymbol{r_\mu}$ Gasser & Leutwyler, 1985

$$= \partial_\mu\boldsymbol{\Sigma} - ieA_\mu[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(e^{-i\boldsymbol{\kappa} c_{GG} a/f}\left(\boldsymbol{k_Q} - \boldsymbol{\kappa} c_{GG}\right)e^{i\boldsymbol{\kappa} c_{GG} a/f}\boldsymbol{\Sigma} - \boldsymbol{\Sigma} e^{i\boldsymbol{\kappa} c_{GG} a/f}\left(\boldsymbol{k_q} + \boldsymbol{\kappa} c_{GG}\right)e^{-i\boldsymbol{\kappa} c_{GG} a/f}\right)$$

The Lagrangian can be written in terms of these as

fixed empirically by pion mass

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\text{Tr}\left[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4}B_0\,\text{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\right]$$
$$+\frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\,,$$

Terms in first line contain mass- and kinetic-mixing between ALP and neutral mesons

Weak interactions in the chiral picture

The leading part of the weak interaction Lagrangian is

$$\mathcal{L}_{\text{weak}}^{s\,\to\,d} = -\frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, \underbrace{g_8}_{\approx 5} \left[L_\mu L^\mu\right]^{32}$$

Cirigliano, Ecker, Neufeld, Pich 2004

This is the $SU(3)_L$ octet operator, there are also 27-plet operators but their coefficients are suppressed in comparison ($\Delta I = 1/2$ rule)

where the left handed current L_μ is the thing that couples to the spurion $\boldsymbol{l}_\mu$

(chPT version of $\bar{q}_L \gamma^\mu q_L$)

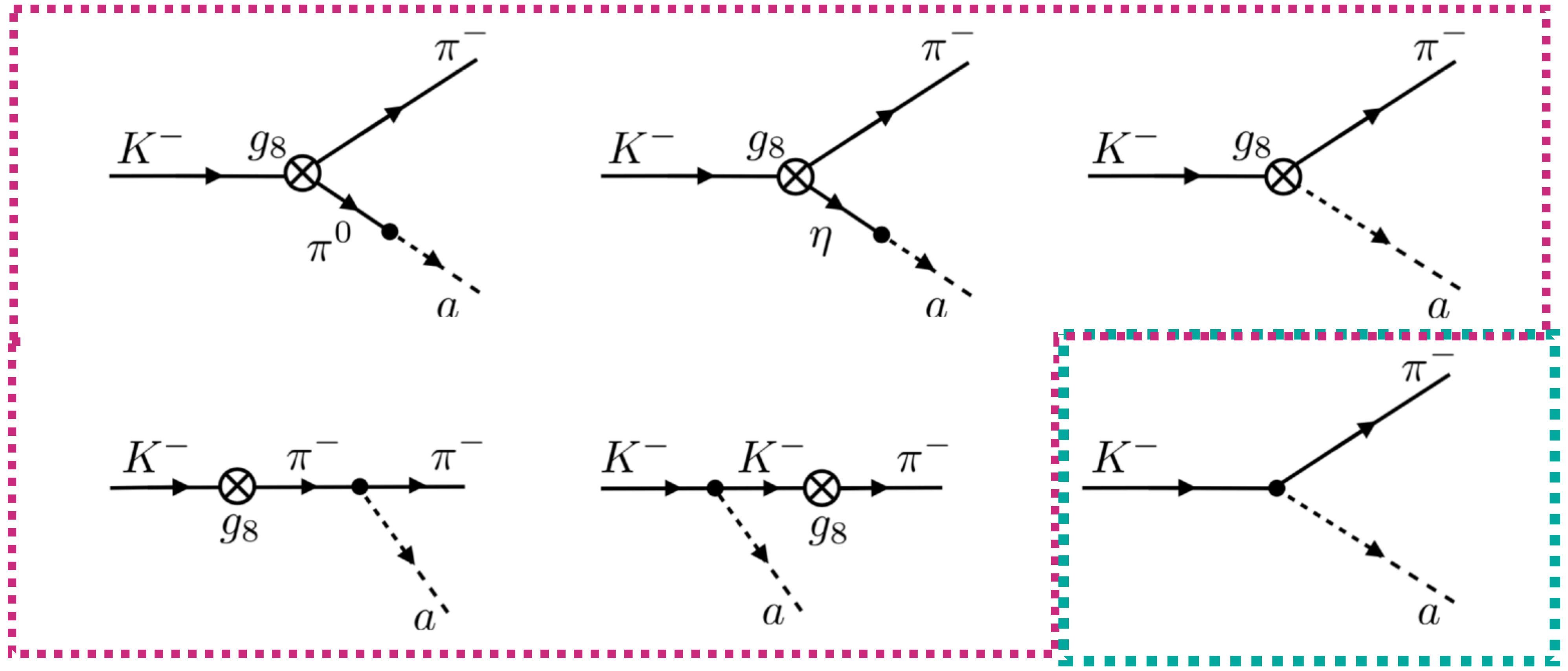
$$\begin{aligned} L_\mu^{ji} &= -\frac{i f_\pi^2}{4}\, e^{i(\kappa_{q_j} - \kappa_{q_i}) c_{GG} \frac{a}{f}} \left[\boldsymbol{\Sigma} \left(\boldsymbol{D}_\mu \boldsymbol{\Sigma}\right)^\dagger\right]_{ji} \\ &\ni -\frac{i f_\pi^2}{4} \left[1 + i(\kappa_{q_j} - \kappa_{q_i}) c_{GG} \frac{a}{f}\right] \left[\boldsymbol{\Sigma}\, \partial_\mu \boldsymbol{\Sigma}^\dagger\right]_{ji} + \boxed{\frac{f_\pi^2}{4} \frac{\partial^\mu a}{f} \left[\hat{\boldsymbol{k}}_Q - \boldsymbol{\Sigma}\, \hat{\boldsymbol{k}}_q \boldsymbol{\Sigma}^\dagger\right]_{ji}} \end{aligned}$$

piece from ALP terms in covariant derivative

$K^+ \to \pi^+ a$ amplitude

General result for an ALP

flavour change through weak interactions $s \to \bar{u}ud$



explicit s-d-ALP coupling

$$i\mathcal{A}(K^- \to \pi^- a) = \frac{N_8}{4f}\Bigg[16 c_{GG}\,\frac{(m_K^2 - m_\pi^2)(m_K^2 - m_a^2)}{4m_K^2 - m_\pi^2 - 3m_a^2} + (2c_{uu} + c_{dd} + c_{ss})\,(m_K^2 - m_\pi^2)$$
$$- (2c_{uu} + c_{dd} - 3c_{ss})\, m_a^2 + 6\,(c_{uu} + c_{dd} - 2c_{ss})\,\frac{m_a^2\,(m_K^2 - m_a^2)}{4m_K^2 - m_\pi^2 - 3m_a^2}$$
$$+ \big([k_d + k_D]_{11} - [k_d + k_D]_{22}\big)\,(m_K^2 + m_\pi^2 - m_a^2)\Bigg]$$
$$- \frac{m_K^2 - m_\pi^2}{2f}\,[k_d + k_D]_{12}\,,$$

If we assume flavour universal, flavour conserving couplings at low scale:

NA62 upper limit on $BR(K^+ \to \pi^+ X) < 6 \times 10^{-11}$ $\implies$ $\frac{1}{f}\left|2c_{GG} + c_{uu} + c_{dd}\right| \lesssim \frac{1}{58\,\text{TeV}}$

NA62, 2103.15389

for a light, long-lived ALP

Bauer, Neubert, SR, Schnubel, Thamm 2102.13112

$K \to \pi a$ in terms of high scale couplings

Amplitude on previous slide was written in terms of couplings at the scale $\mu_0 \sim 2\,\mathrm{GeV}$

What does it mean in terms of couplings defined at $\Lambda = 4\pi$ TeV?

Bauer, Neubert, SR, Schnubel, Thamm, 2102.03112

$K^+ \to \pi^+ a$

$$i\mathcal{A}(K^- \to \pi^- a) = 10^{-11}\,\mathrm{GeV}\left[\frac{1\,\mathrm{TeV}}{f}\right]$$

$$\times\Big\{e^{i\beta}\Big[-0.21\,c_{GG} - \underline{0.10\,c_{WW}} - 6.4\times10^{-4}\,c_{BB} + 67c_u(\Lambda) - 0.32\,c_d(\Lambda) - 66\,c_Q(\Lambda) - 1.9\times10^{-3}\,c_e(\Lambda) + 0.15\,c_L(\Lambda)\Big]$$

flavour change via ALP-s-d interactions $\propto V_{ts}^* V_{td}$

$$+\,e^{i\delta_8}\Big[3.4\,c_{GG} - \underline{7.5\times10^{-4}\,c_{WW}} - 7.5\times10^{-5}\,c_{BB} + 1.6\,c_u(\Lambda) + 1.5\,c_d(\Lambda) - 3.1\,c_Q(\Lambda) - 2.2\times10^{-4}\,c_e(\Lambda) + 1.2\times10^{-3}\,c_L(\Lambda)\Big]\Big\}$$

flavour change through weak interactions $s \to \bar{u}ud$ $\propto V_{us}^* V_{ud}$

$K_L \to \pi^0 a$

$$i\mathcal{A}(K_L \to \pi^0 a) = 10^{-11}\,\mathrm{GeV}\left[\frac{1\,\mathrm{TeV}}{f}\right]$$

$$\times\Big\{i\,e^{i\xi_\epsilon}\Big[0.083\,c_{GG} + 0.037\,c_{WW} + 2.5\times10^{-4}\,c_{BB} - 26\,c_u(\Lambda) + 0.12\,c_d(\Lambda) + 26\,c_Q(\Lambda) + 7.4\times10^{-4}\,c_e(\Lambda) - 0.056\,c_L(\Lambda)\Big]$$

$\propto \mathrm{Im}[V_{ts}^* V_{td}] \sim 0.4\,|V_{ts}^* V_{td}|$

$$+\,e^{i(\delta_8+\phi_\epsilon)}\Big[7.7\times10^{-3}\,c_{GG} - 1.8\times10^{-6}\,c_{WW} - 7.8\times10^{-8}\,c_{BB} + 5.8\times10^{-4}\,c_u(\Lambda) + 7.4\times10^{-3}\,c_d(\Lambda) - 8.0\times10^{-3}\,c_Q(\Lambda) - 2.4\times10^{-7}\,c_e(\Lambda) + 2.8\times10^{-6}\,c_L(\Lambda)\Big]\Big\}$$

$\propto \epsilon V_{us}^* V_{ud} \sim 10^{-3} V_{us}^* V_{ud}$

Different hierarchies for different ALP couplings between charged and neutral modes

QCD axion vs ALPs

$$m_a = 5.691(51)\,\mu\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)$$