Lattice simulations and QCD axion’s properties

(news from TWEXT)

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COSMIC WISPERS

CA21106

1° GENERAL MEETING

Bari (IT) 5-8 September 2023

LAVA lattice virtual academy

FLORENTINA STUDIORUM UNIVERSITAS

STRONG 2020

CINECA

COST EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

The axion Lagrangian:

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{4}\, a\, g^0_{a\gamma\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a}\, j^\mu_{a,0}$$

QCD EM Other SM

For the axion to solve the Strong CP problem of QCD, QCD coupling should dominate the potential

This talk:

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta\ \frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Focus of the talk : What the Lattice has done so far,
What the Lattice can do more — IF useful!

Physical quantities

Axion mass : several lattice studies

Axion potential: exploratory studies

Axion Thermal rates (k) : advocated by
Notari, Rompineve, Villadoro 2211.03799
Lattice technology available,
but: demanding studies

Strong CP problem and the QCD axion

Focus on the θ term – CP violating

$$Q = \sum q(x)$$

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a ,$$

$$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = q(x)$$

Topological charge density

GCPF and θ

The GCPF of QCD is now a function of θ:

$$\mathcal{Z}(\theta, T) = \int \mathcal{D}[\Phi]\, e^{-T \sum_t \int d^3x \mathcal{L}(\theta)} = e^{-VF(\theta,T)}.$$

The energy density $F(\theta, T)$ is related with the probability of finding configurations with given topological charge $Q = \int d^4x\, q(x)$:

$$P_Q = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} e^{-VF(\theta)},$$

so the coefficients C_n of the Taylor expansion

related with axion potential —-> $$F(\theta, T) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{2n!} C_n$$

are given by the cumulants of the topological charge:

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

Topology and the Strong CP problem

How 'large' is θ ?

The QCD Lagrangian admits a CP violating term

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a,$$

$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$ is the topological charge density $q(x)$,
$\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, and $\theta\, q(x)$ is known as the θ-term.
Without the θ-term strong interactions conserve CP. With the θ-term the neutron acquires an electric dipole moment d_n:
with QCD sum rules: $d_n = 2.4 \times 10^{-16}\theta$ e cm
chiral perturbation theory: $d_n = 3.3 \times 10^{-16}\theta$ e cm
Experiments: $|d_n| < 1.8 \times 10^{-26}$ e cm at a 90% C.L,

$$\theta < 0.5 \times 10^{-10}.$$

Solution of the strong CP problem: the axion

Suppose θ were a dynamical parameter: in such a case, dynamics would force its value to zero, thus solving the strong CP problem. Postulate Axion! a pseudo-Goldstone boson of a spontaneously broken symmetry known as the Peccei-Quinn (PQ) symmetry, which couples to the QCD topological charge, with a coupling suppressed by a scale f_A.
Axion field $a(x) = f_A\theta(x)$ is now a space-time dependent θ parameter.
The axion–QCD Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \partial^2_\mu a^2 + \frac{a}{f_A}\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a.$$

Assume a shift symmetry: $a \to a + \alpha$. The θ dependence has been traded with a dependence on the axion field, whose minimum is at zero:

This solves the strong CP problem. !

The axion mass

At leading order in $1/f_A$ – well justified as $f_A \gtrsim 4 \times 10^8$ GeV – the axion can be treated as an external source, and its mass is given by

$$m_A^2(T) f_A^2 = \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0} \equiv \chi_{top}(T) \ .$$

At zero and low temperature, chiral perturbation theory gives:

$$m_A^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_A^2},$$

In very brief summary, the essence of this discussion is the close relation between axion mass and topological susceptibility:

$$m_A^2 f_A^2 = \chi_{top},$$

Constraints on axion density:

Topological susceptibility around and above the critical temperature

T_c Peccei Quinn $\gtrsim 10^7$ -10^8 GeV

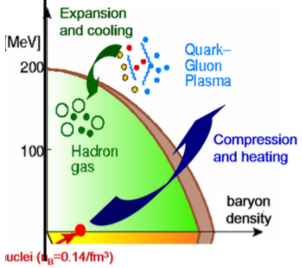
T_c Electroweak $\simeq$ 160 GeV (SM)

Temperature

Time from Big Bang

$m_a(T) = \sqrt{\chi(T)}/f_a$

Quark Gluon Plasma: Topology



T_c Peccei Quinn $\gtrsim 10^7$ -10^8 GeV

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Time from Big Bang

Temperature

Hubble parameter
$H(T) \simeq T^2/M_P$

Axions's freezout

3H(T) = m_a(T)

$m_a(T) = \sqrt{\chi(T)}/f_a$

From models:
T at freezout
about five times Tc

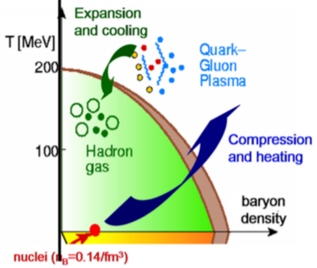
Quark Gluon Plasma:
Topology

Axions' mass
and density
today

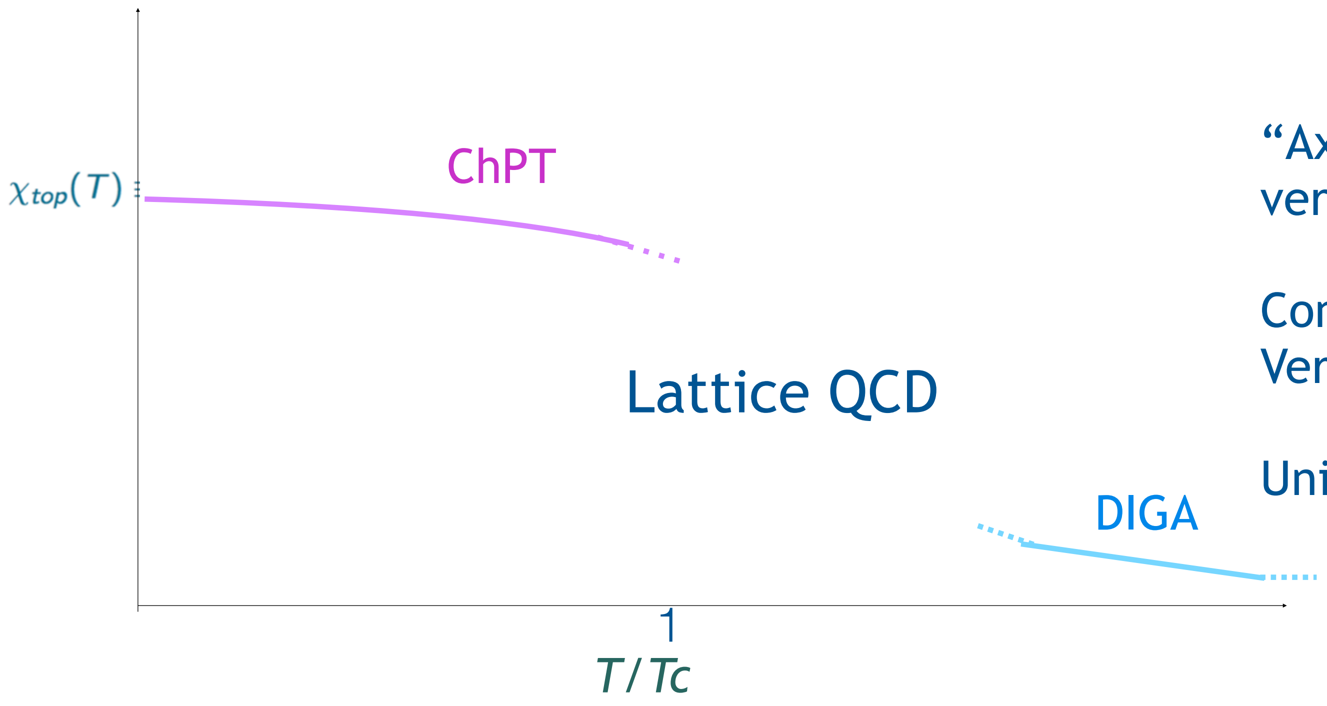
After freezout $\frac{n_a}{s}$ constant

$$\rho_{a,0} = \frac{n_a}{s} m_a s_0$$

Wantz, Shellard 2010



What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$



"Axion mass
very small

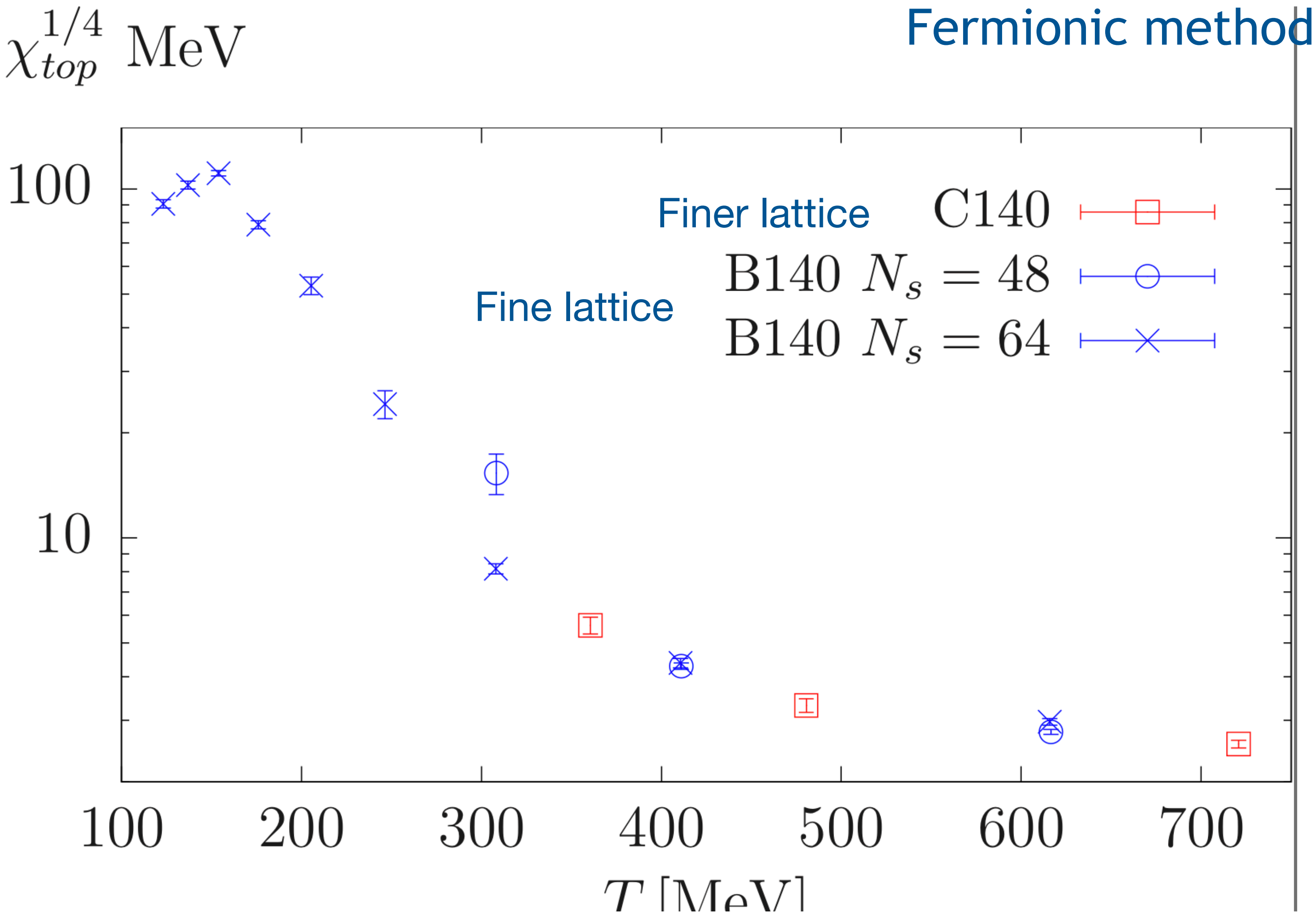
Compton length
Very large

Universe very small.."

$$\chi(T) \sim T^4 \left(\frac{m}{T}\right)^{N_f} e^{-8\pi^2/g^2(T)} \sim m^{N_f} T^{4-\frac{11}{3}N_c-\frac{1}{3}N_f}$$

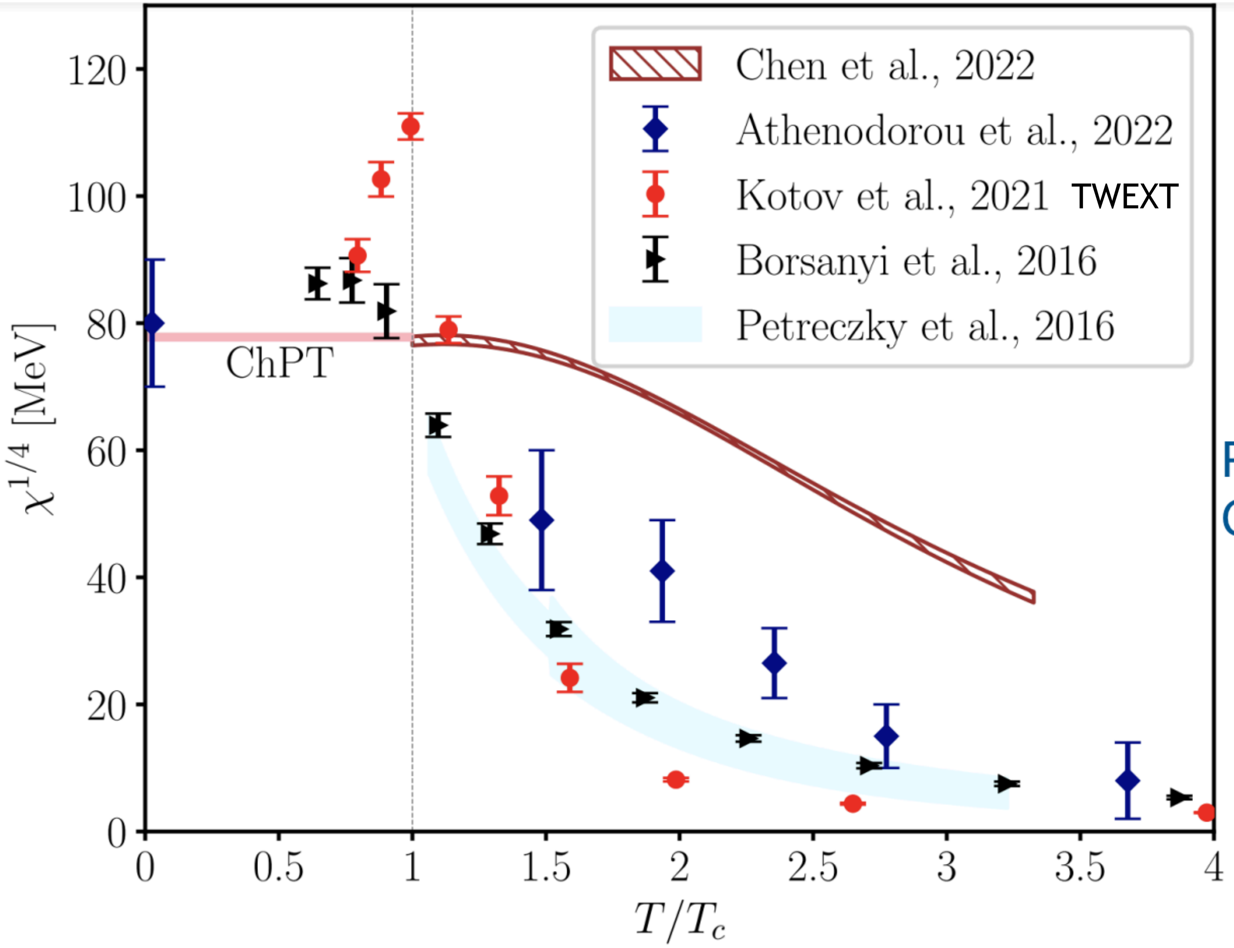
Systematics from twisted mass Wilson fermions 2+1+1 flavours

Fermionic method



TWEXT - Kotov MpL Trunin (2021 + in progress)

QCD - summary



Plot by
Claudio Bonanno

Na6-Strong2020, MpL et al, 2023

From exponent d to axion mass in three steps

$$\chi_{top} \simeq A T^{-d}$$

$$d = (6.26, 6.88, 7.52, 7.48)$$

$$m_\pi = (470, 370, 260, 210)\ \text{MeV}$$

Time from Big Bang

Temperature

Hubble parameter
$H(T) \simeq T^2/M_P$

Axions's freezout

1. 3H(T) = m_a(T)

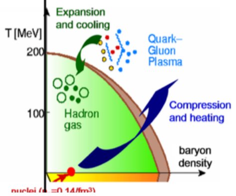
$m_a(T) = \sqrt{\chi(T)}/f_a$

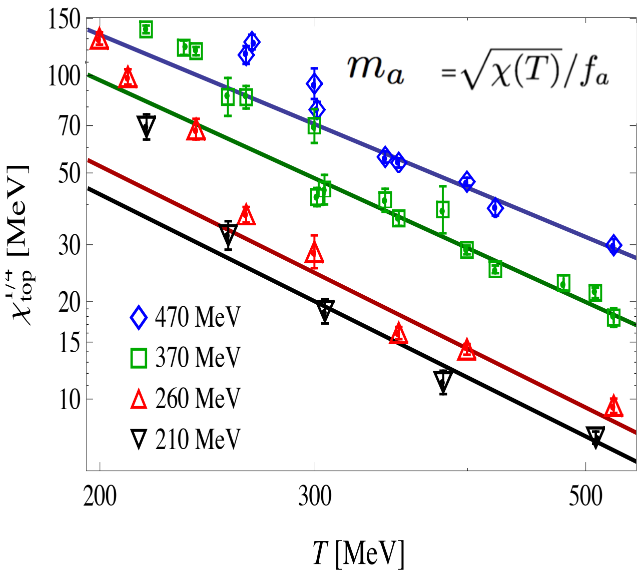
Quark Gluon Plasma: Topology

2. Axions' mass and density today

After freezout $\frac{n_a}{s}$ constant

3. $\rho_{a,0} = \frac{n_a}{s} m_a s_0$

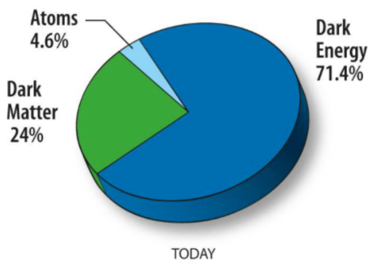


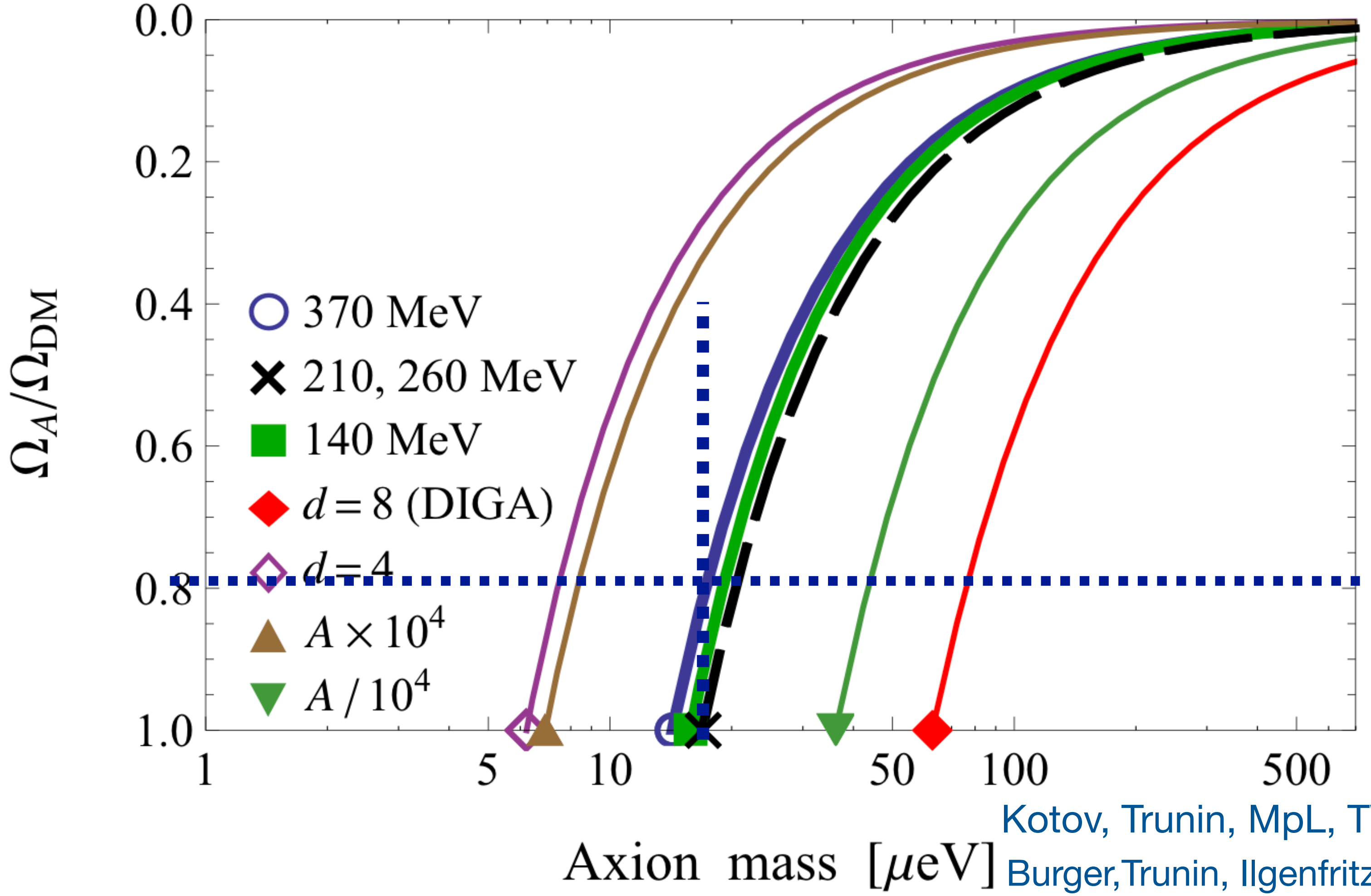


$$\rho_a(m_a) \propto m_a^{-\frac{3.053+d/2}{2.027+d/2}}$$

Limits on the (post-inflactionary) axion mass

$$\Omega_A = F(A, d, \ldots)\, m_A^{-\frac{3.053+d/2}{2.027+d/2}}$$





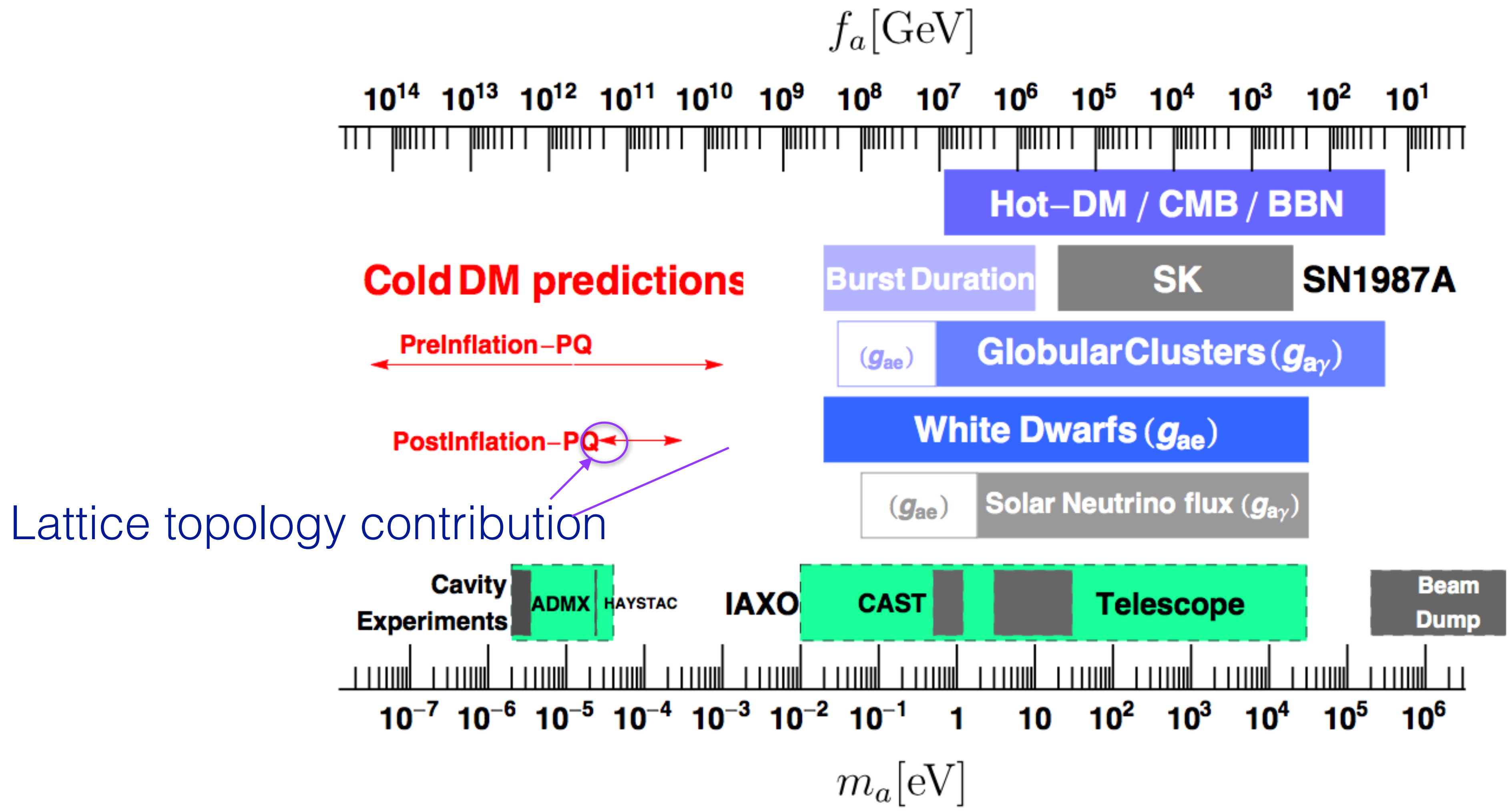
Kotov, Trunin, MpL, TWEXT, 2023, preliminary

Burger,Trunin, Ilgenfritz,Mueller-Preussker,MpL 2019

$$\Omega_a = \frac{\rho_{a,0}}{\rho_c}$$

Example: if axions constitute 80% DM, our results give a lower bound for the axion mass of $\simeq 30\mu\mathrm{eV}$

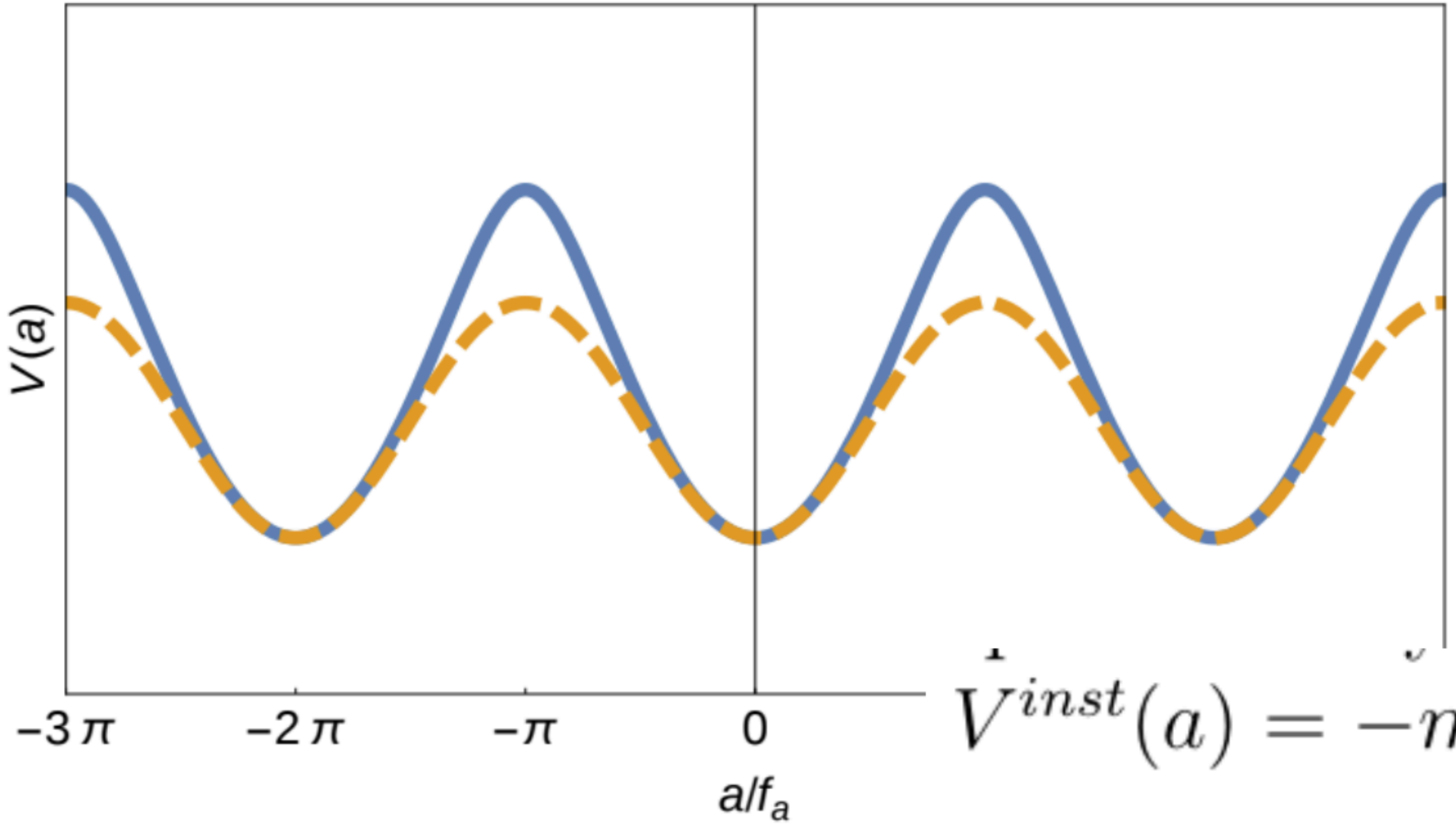
Limits on the post-inflationary axion mass



Beyond topological susceptibility

Higher moments — towards axion potential

Grilli di Cortona, Hardy, Vega, Villadoro et al. 1511.02867



$$V^{inst}(a) = -m_a^2 f_a^2 \cos(a/f_a)$$

Beyond topological susceptibility

Higher moments — towards axion potential

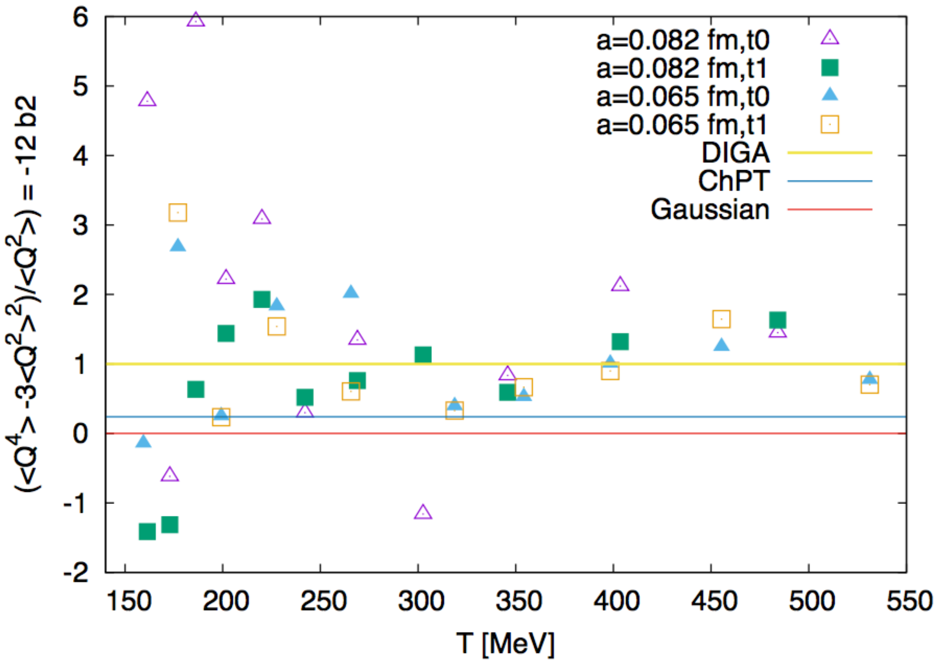
$$F(\theta, T) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{2n!} C_n$$

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

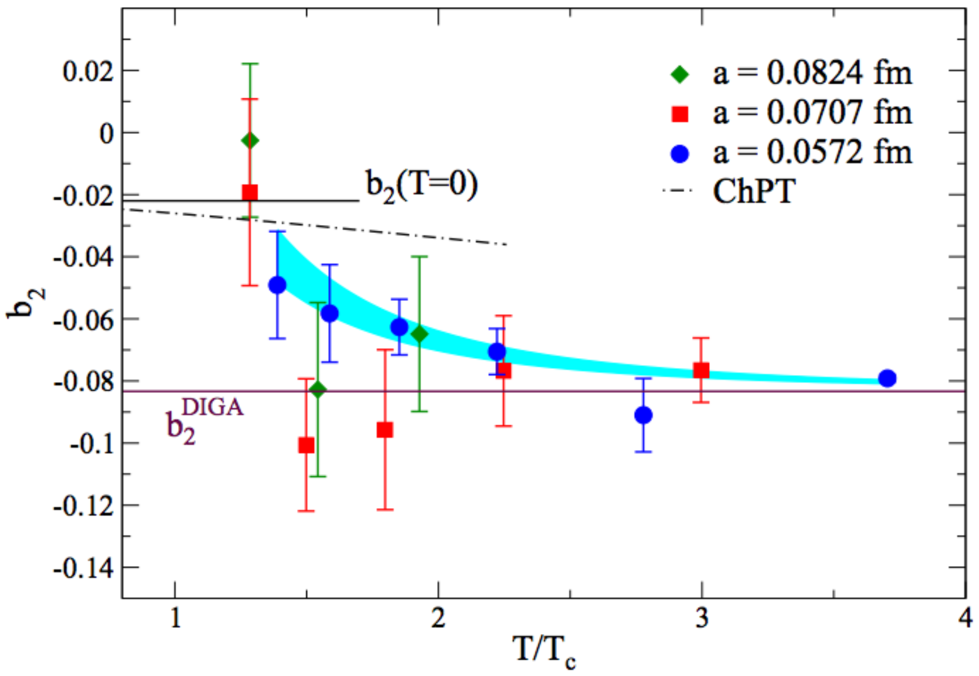
$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} .$$

Quartic interaction term in the potential

Bonati, d'Elia, Vicari 1301.7640

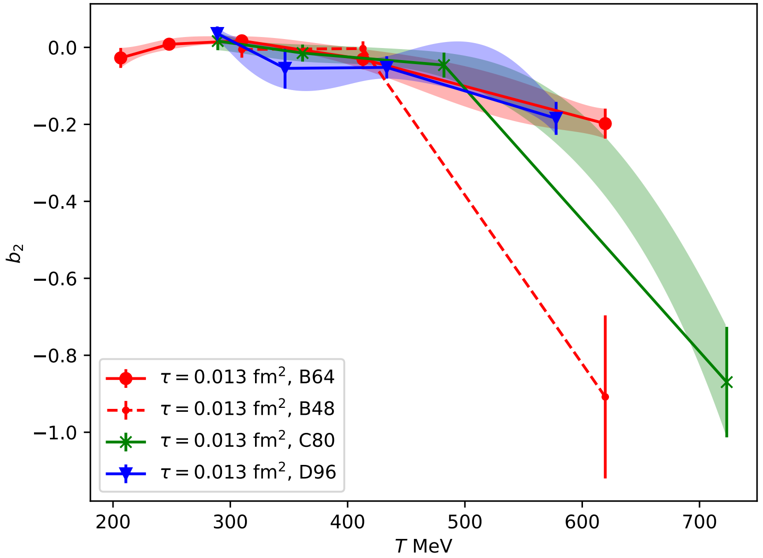


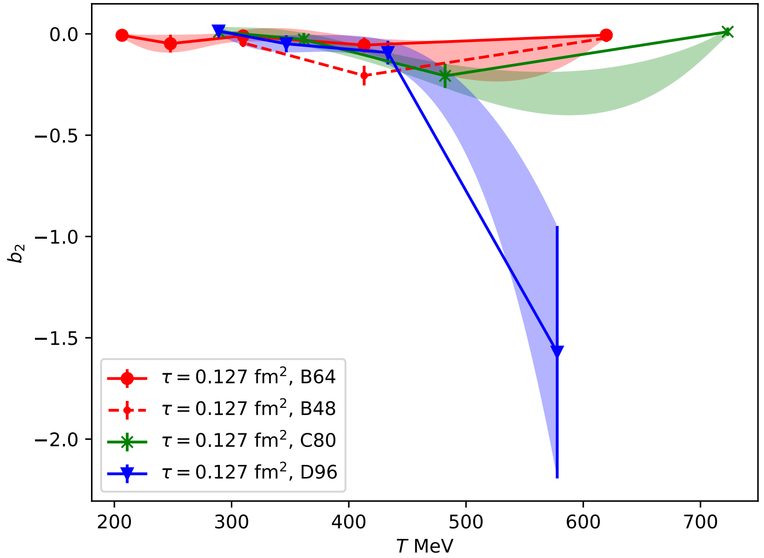
Ilgenfritz, MpL, Mueller-Preussker, Trunin (2018)

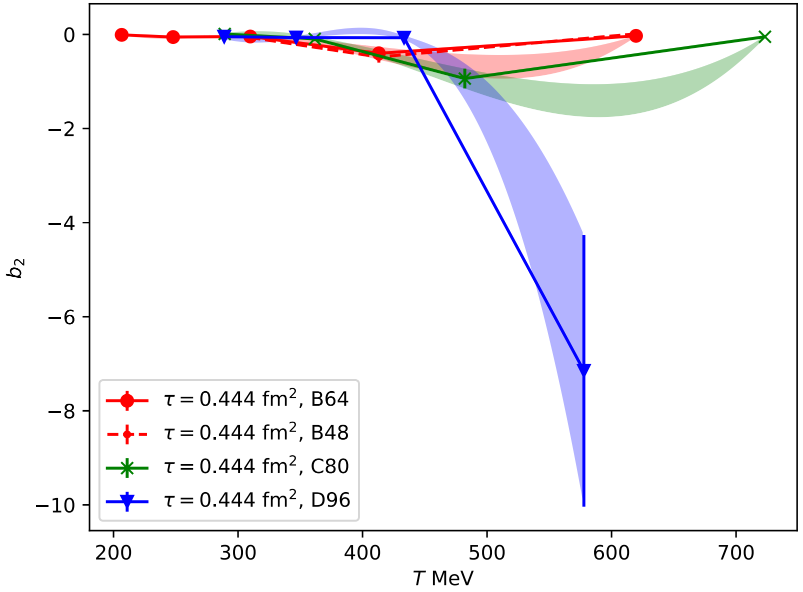


Bonati et al. (2016)

b2, TWEXT, in progress — preliminary







Axion momentum dependent thermal width ?

The challenge of real time

Thermal rates : Fourier transform of two point functions

$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}\,,$$

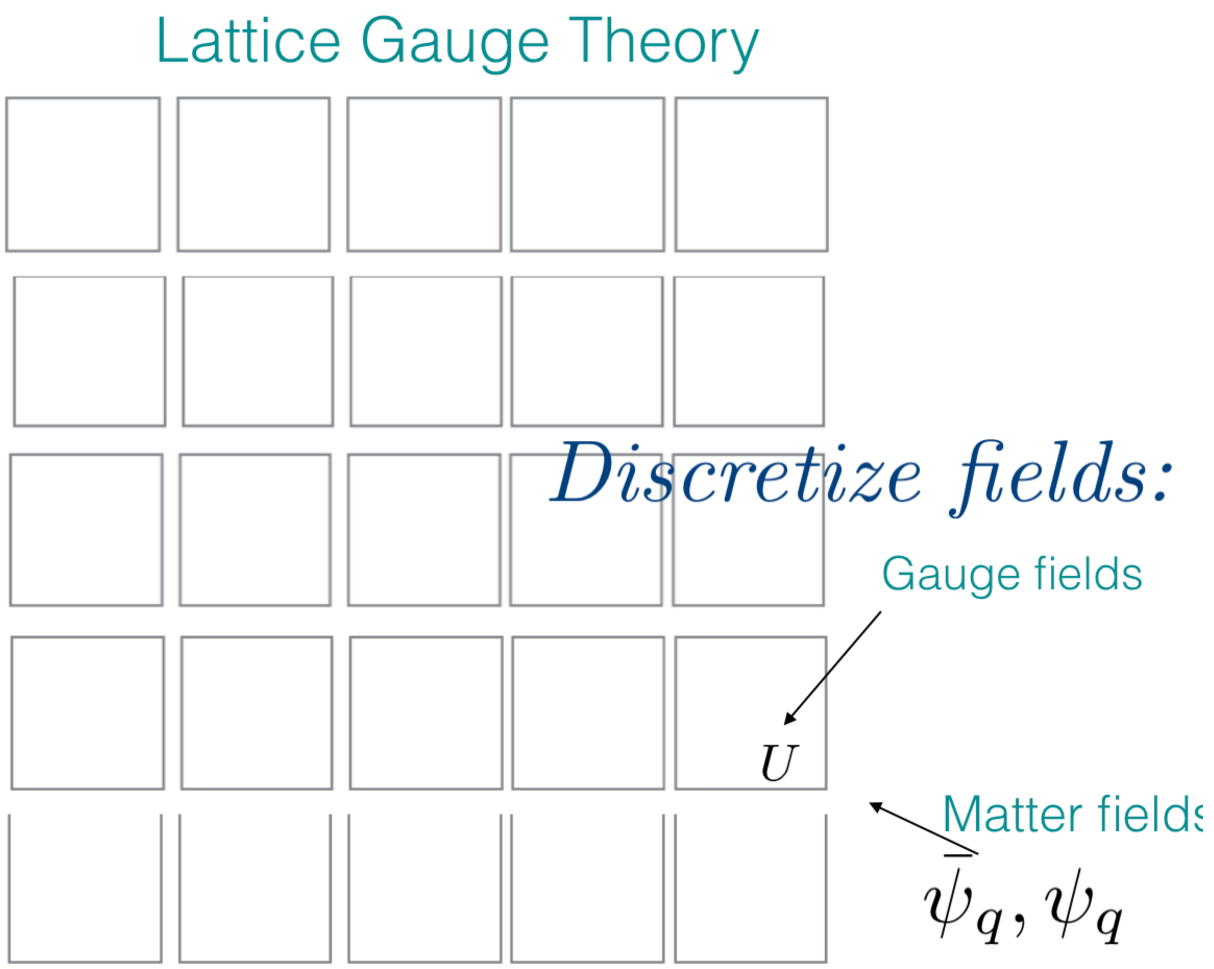
Creation and destruction of axions

$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle\,,$$

Real time quantities

Computational Strategy:

1) Rotation to imaginary time + discretisation



Computational Strategy:

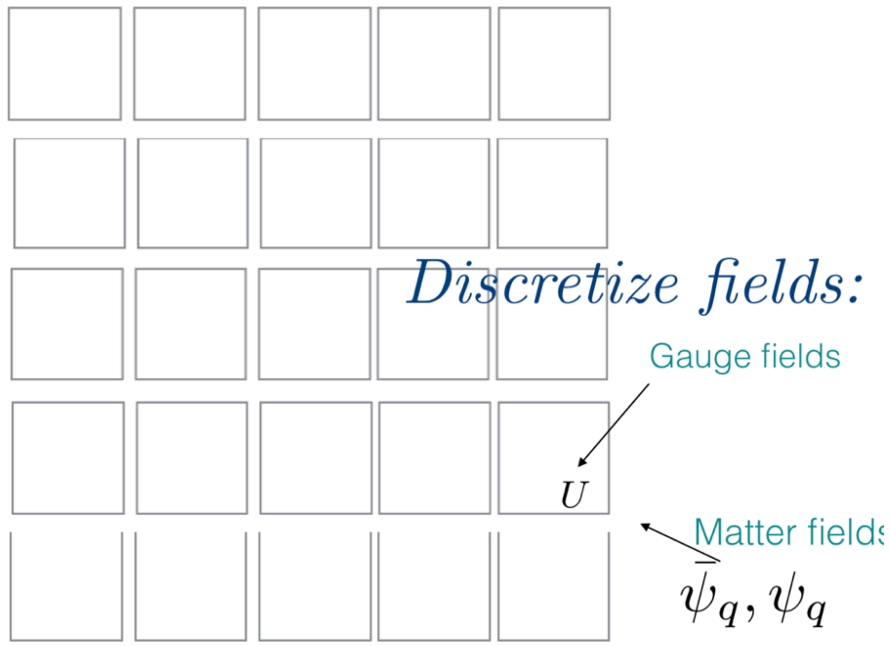
1) Rotation to imaginary time + discretisation

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S}$$

2) Monte Carlo Simulation

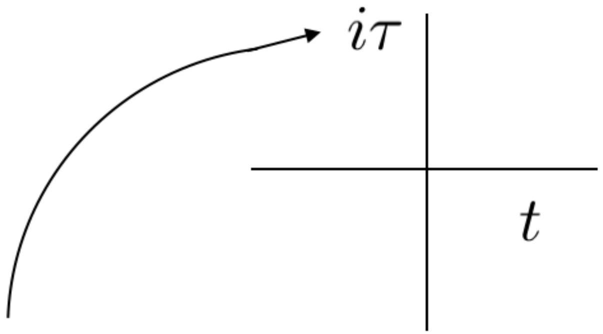
Performing the integration

Lattice Gauge Theory



$$\langle 0 | \mathcal{O} | 0 \rangle = \frac{\sum_\alpha \mathcal{O}_\alpha e^{-S_\alpha}}{\sum_\alpha e^{-S_\alpha}}.$$

Objects of interest: Spectral functions



Computed on the lattice: Euclidean (imaginary) Time Correlators

Functions of real, continuous frequency Spectral functions

Integral inverse transform

Euclidean Time Correlators

Analytic continuation

Fourier transform

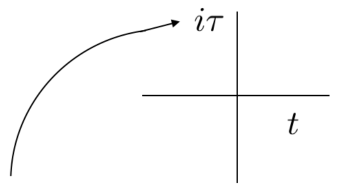
Euclidean correlator in imaginary (Matsubara) frequency space

$$D(\tau) = \int_0^\infty \frac{e^{-\tau\omega} + e^{-(\beta-\tau)\omega}}{1 - e^{-\beta\omega}} S(\omega) d\omega$$

Euclidean time correlators

Spectral function

Objects of interest: Spectral functions



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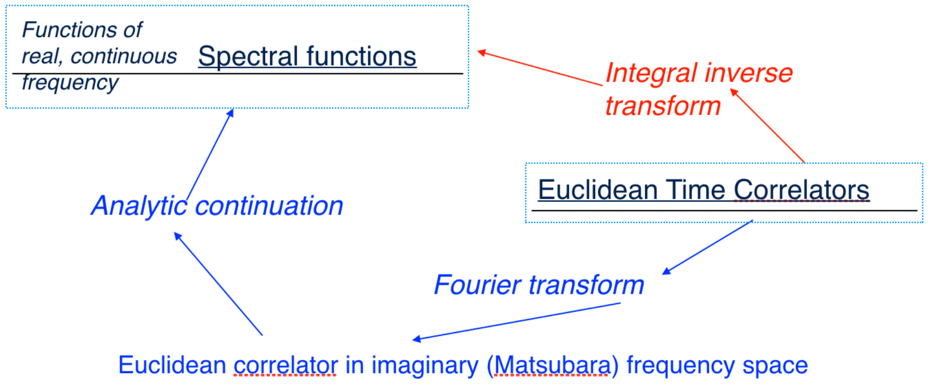
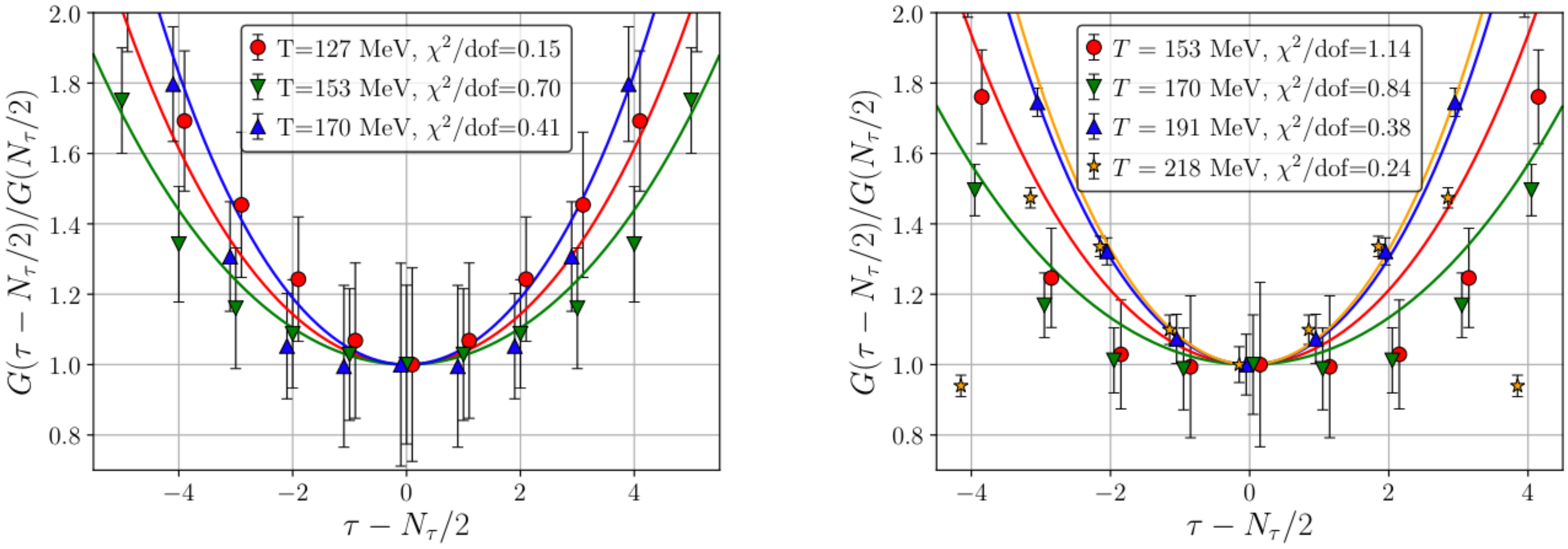


Topological charge correlators: very noisy!



TWEXT, Kotov, MpL, Trunin 1903.05633

Summary, questions:

Axion mass : incremental improvements - requirements on the accuracy?

Axion potential: exploratory studies — useful? In which temperature range?

Axion Thermal rates (k) : advocated by Notari, Rompineve, Villadoro 2211.03799
Lattice technology available, demanding studies

Resurrecting hot axions & physics around Tc???

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+ Topics for general discussion: computational issues, codes, AI in physics, open access…


Thanks !