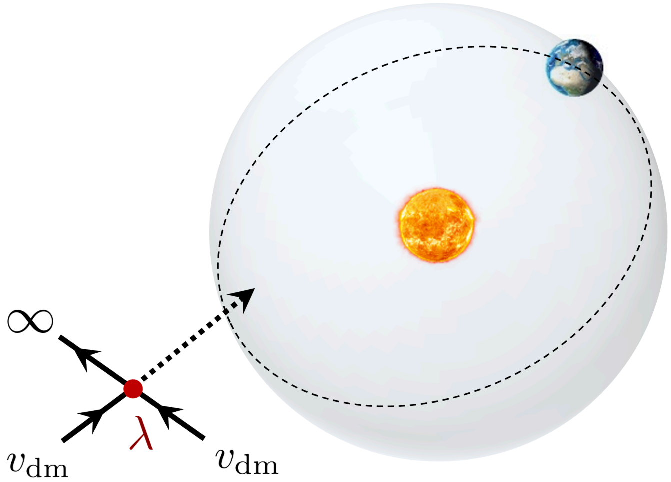
Formation of Ultralight Dark Matter Solar Halos

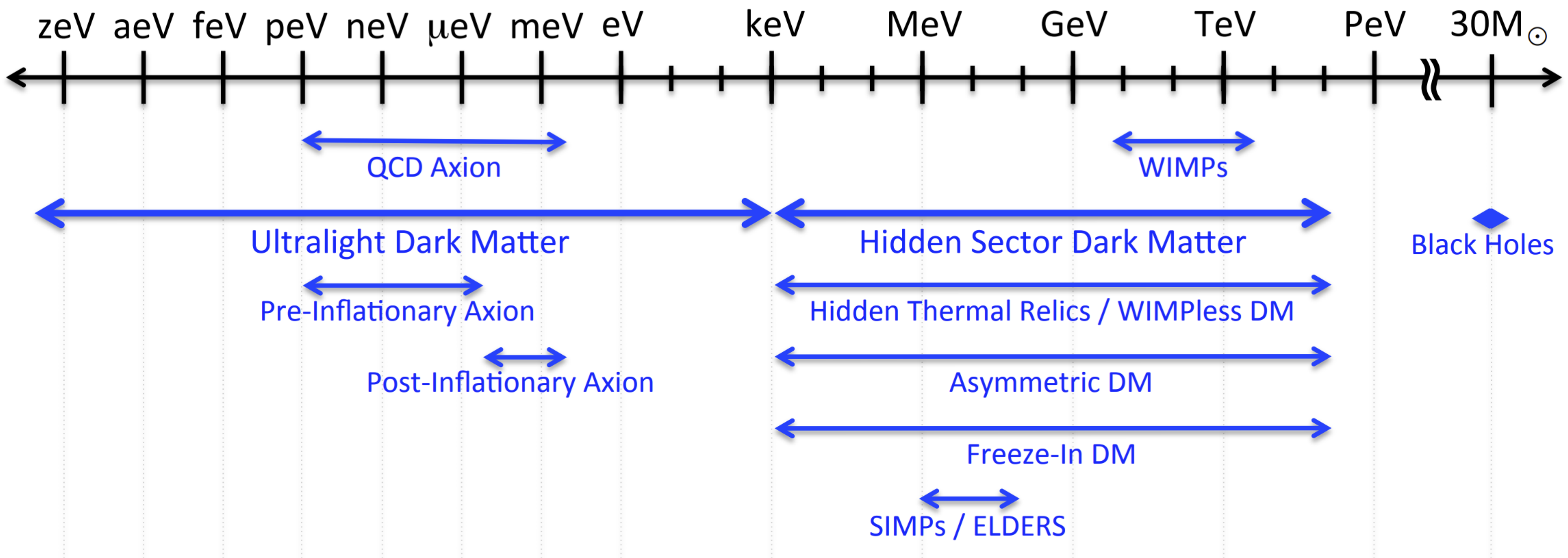


Marco Gorghetto
WEIZMANN INSTITUTE OF SCIENCE
DESY.
Alexander von Humboldt
Stiftung/Foundation

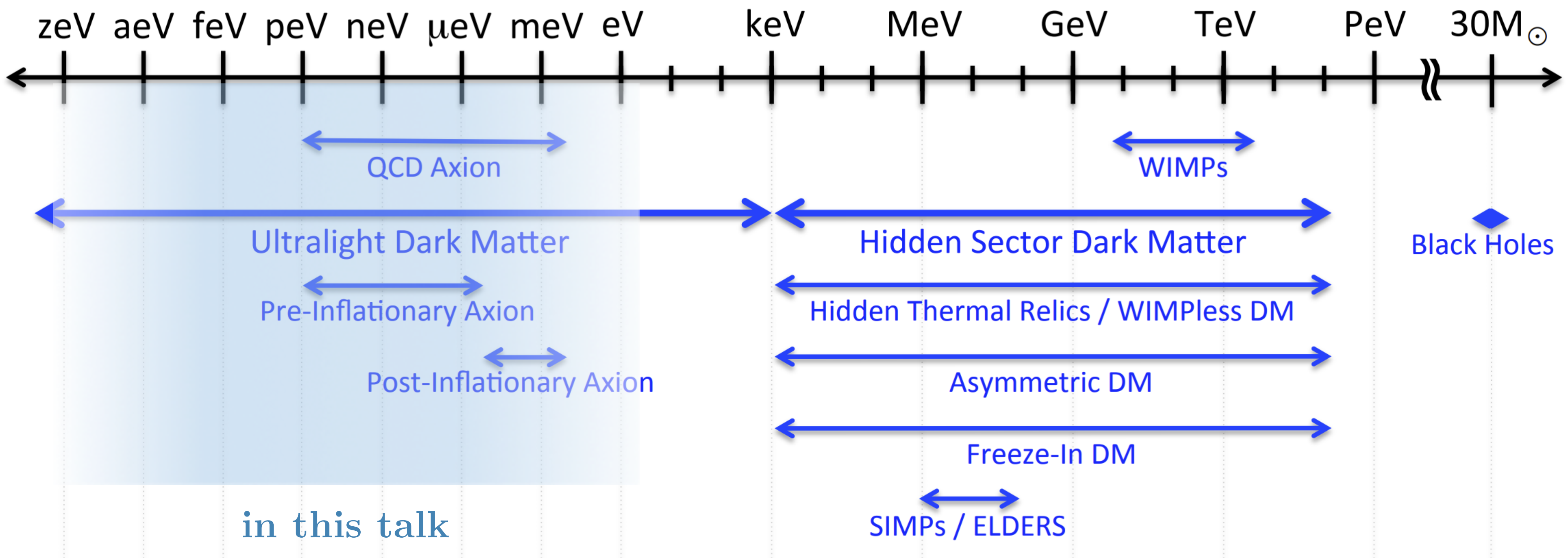
with **Budker, Eby, Jiang, Perez**

[based on 2306.12477]

Dark Matter Candidates



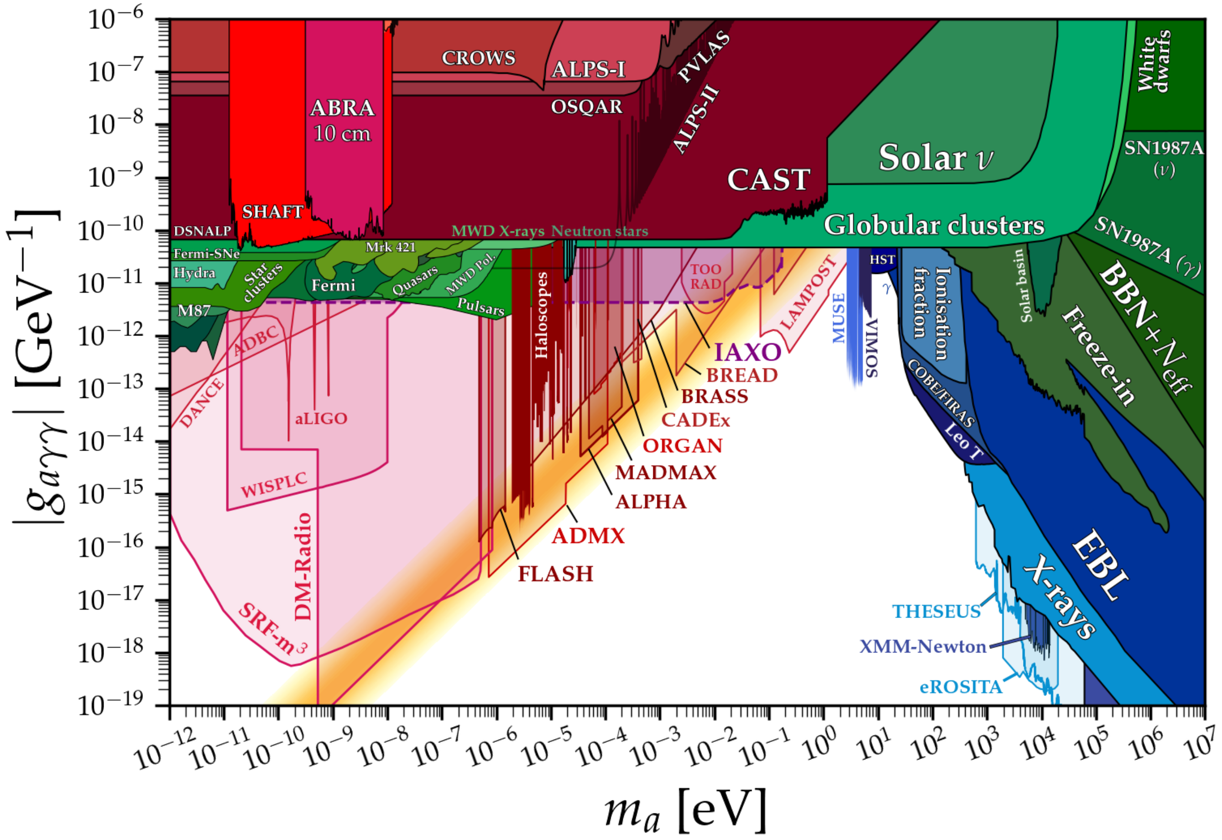
Dark Matter Candidates



- Occupation number $\gg 1 \implies$ boson, ϕ $\qquad \equiv$ number of particles per $\lambda_{\mathrm{dB}}^3 \simeq (mv_{\mathrm{dm}})^{-3}$
- Classical equations of motion
- Automatically produced as dark matter relics

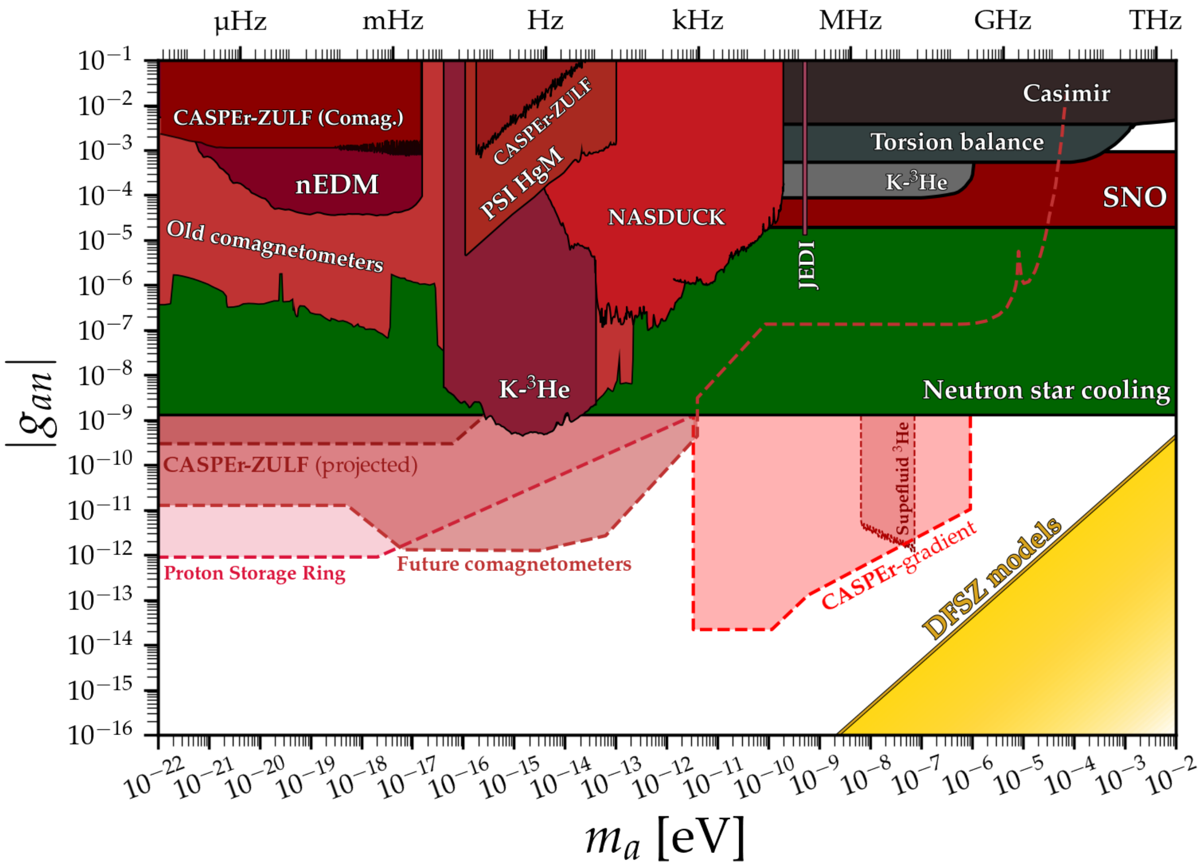
coupling to photons

$$\mathcal{L} \supset \frac{1}{4} g_{a\gamma\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$



coupling to neutrons

$$\mathcal{L} \supset g_{an} \frac{\partial_\mu \phi}{2 f_a} \bar{n} \gamma^\mu \gamma^5 n$$

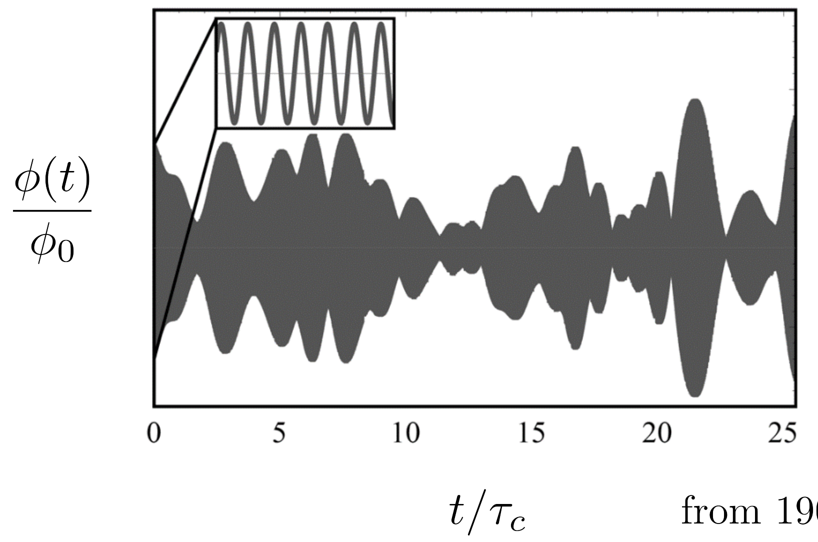


https://cajohare.github.io/AxionLimits

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun

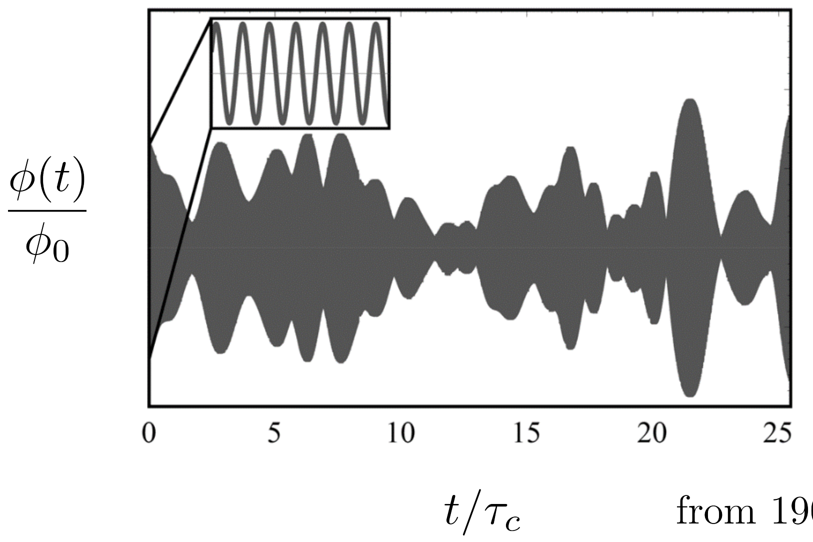
- Coherence time, τ_c



from 1905.13650

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, τ_c



from 1905.13650

In this talk:

- ϕ is the dark matter

- $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$

$\neq 0$ $|\lambda| \lll 1$

axion $\rightarrow$ $\lambda = -\frac{m^2}{f_a^2}$

$\rightarrow$ misalignment fixes $f_a = f_a(m; \theta_0)$

relation between m and f_a unfixed

Outline

- **ULDM distribution and bound states**

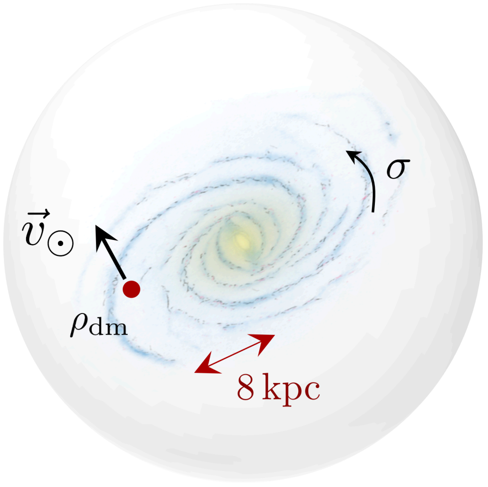
- **Formation of the gravitational atom**

- **Some implication for direct detection**

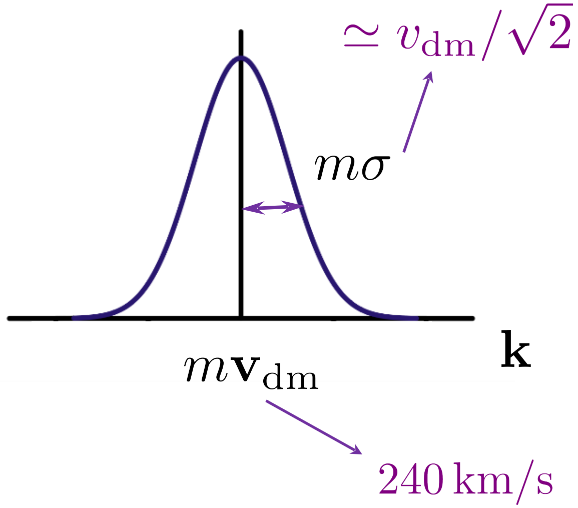
Local dark matter distribution

'Standard halo' model

- Local density $\rho_{\rm dm} \simeq 0.3 \div 0.4\,\mathrm{GeV/cm^3}$



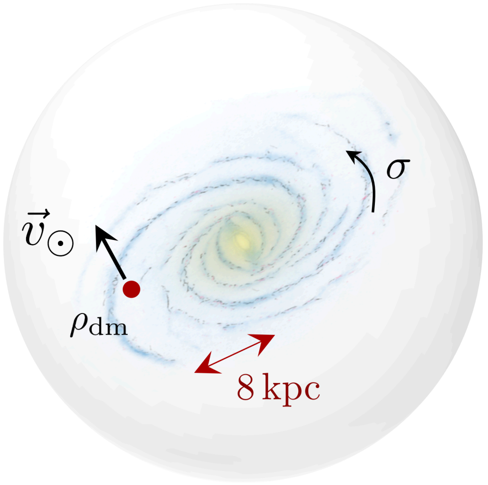
- Dark matter velocity in the frame of the Sun:
  - average $\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot \simeq 240\,\mathrm{km/s}$
  - dispersion $\sigma \simeq 160\,\mathrm{km/s}$ $\simeq v_{\rm dm}/\sqrt{2}$



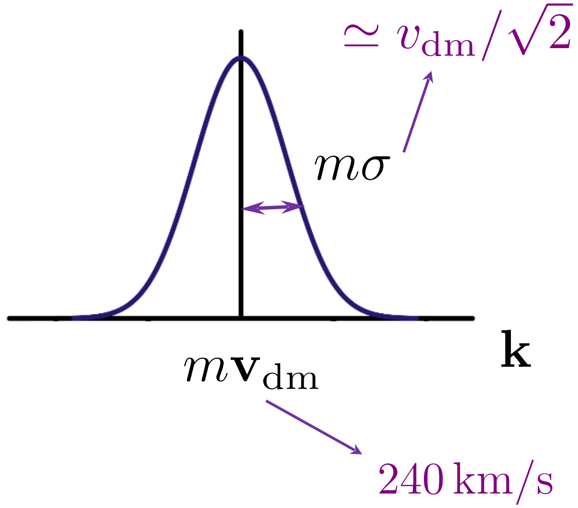
Local dark matter distribution

'Standard halo' model

- Local density $\rho_{\rm dm} \simeq 0.3 \div 0.4\,{\rm GeV/cm^3}$

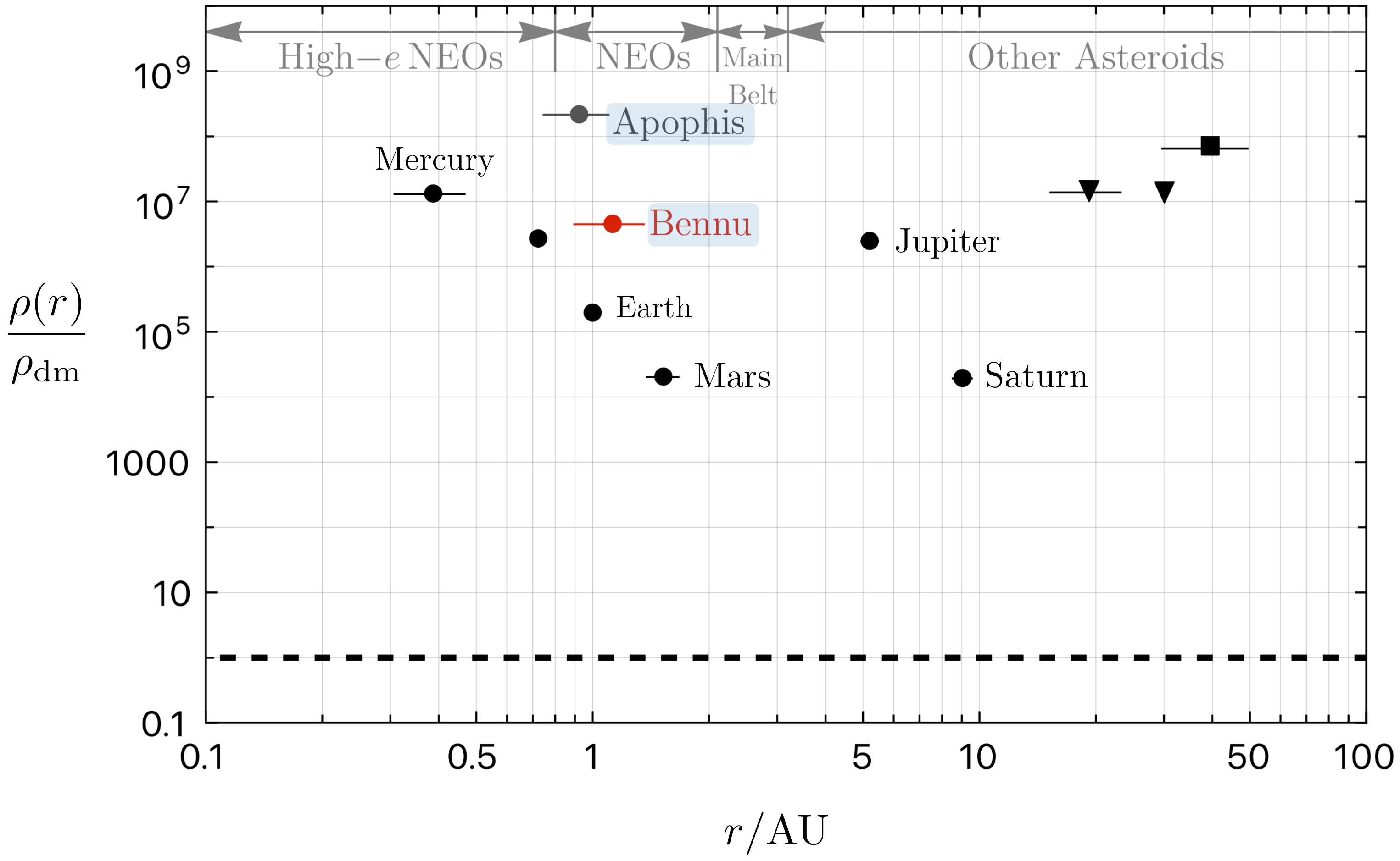


- Dark matter velocity in the frame of the Sun:
  - average $\mathbf{v}_{\rm dm} = -\mathbf{v}_\odot \simeq 240\,{\rm km/s}$
  - dispersion $\sigma \simeq 160\,{\rm km/s}$ $\simeq v_{\rm dm}/\sqrt{2}$



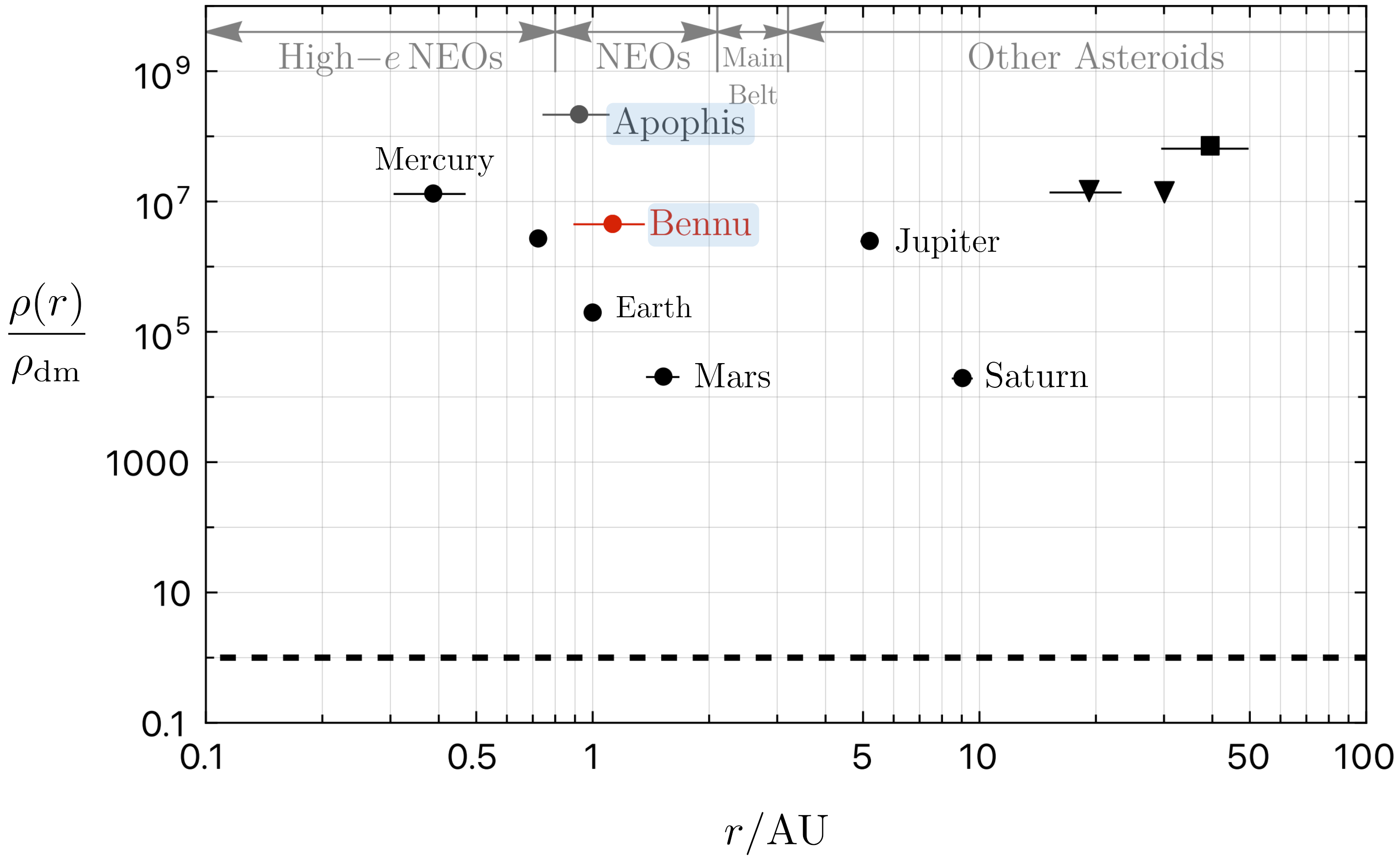
inferred from measurements on galactic scales $\longrightarrow$ insensitive to the 'very local' density

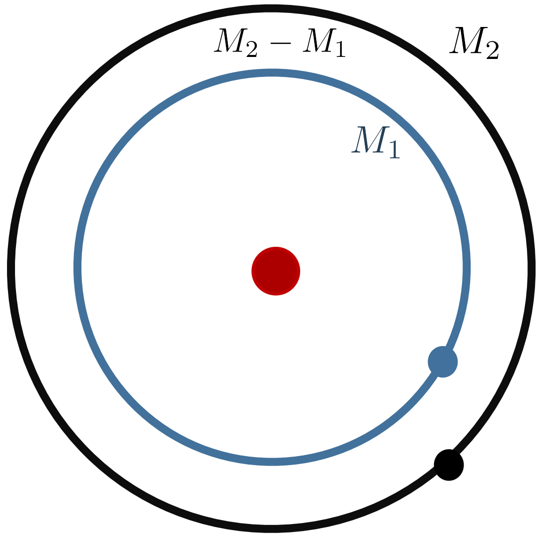
Gravitational bounds from planetary and asteroid motion



[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

Gravitational bounds from planetary and asteroid motion

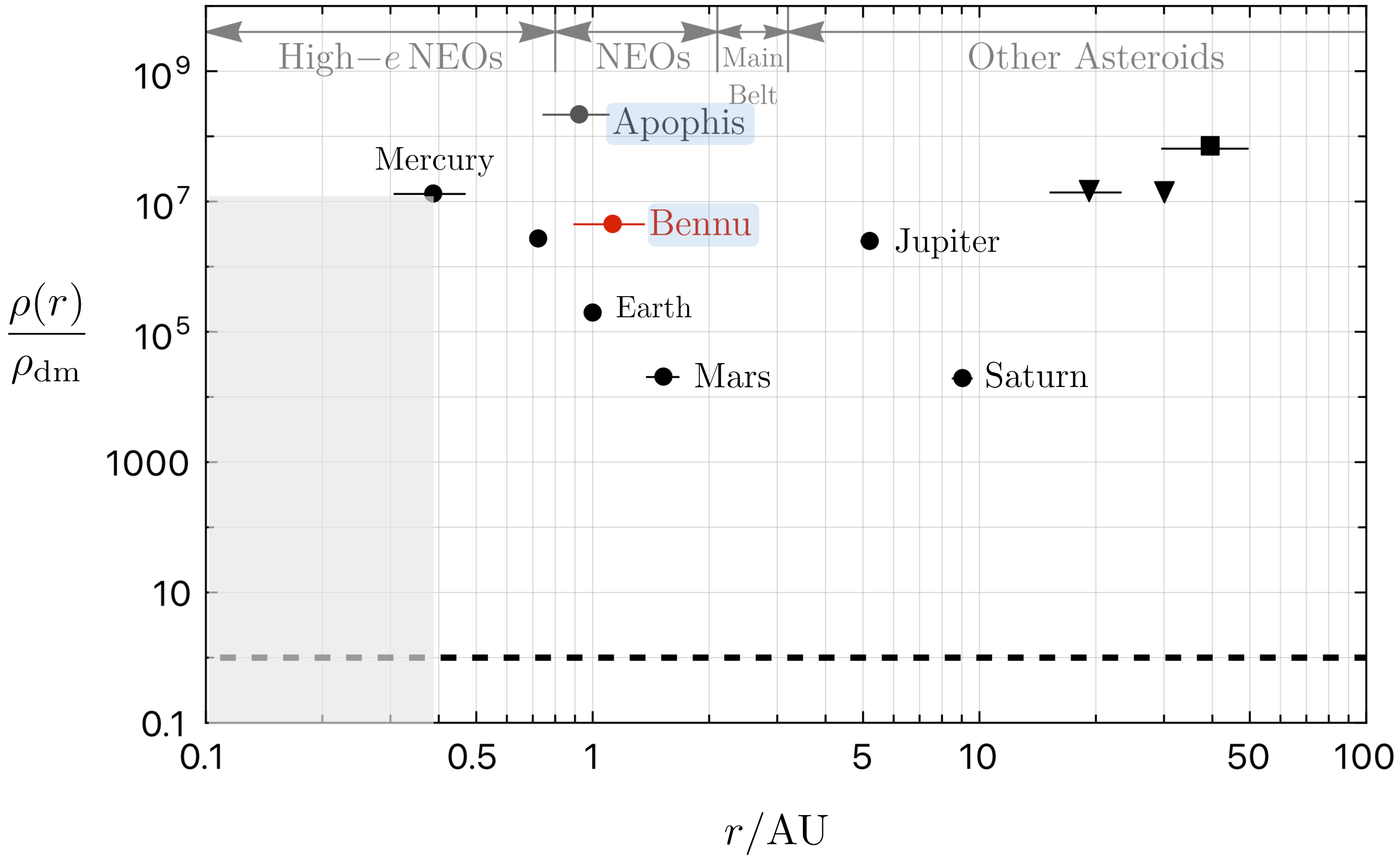


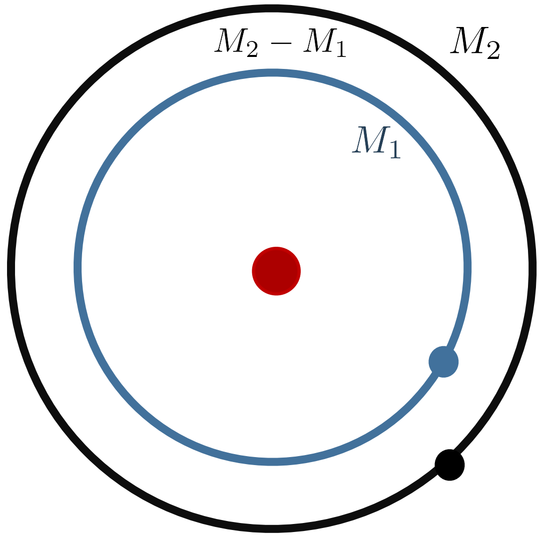


orbit comparison bounds $M_2 - M_1$

[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

Gravitational bounds from planetary and asteroid motion

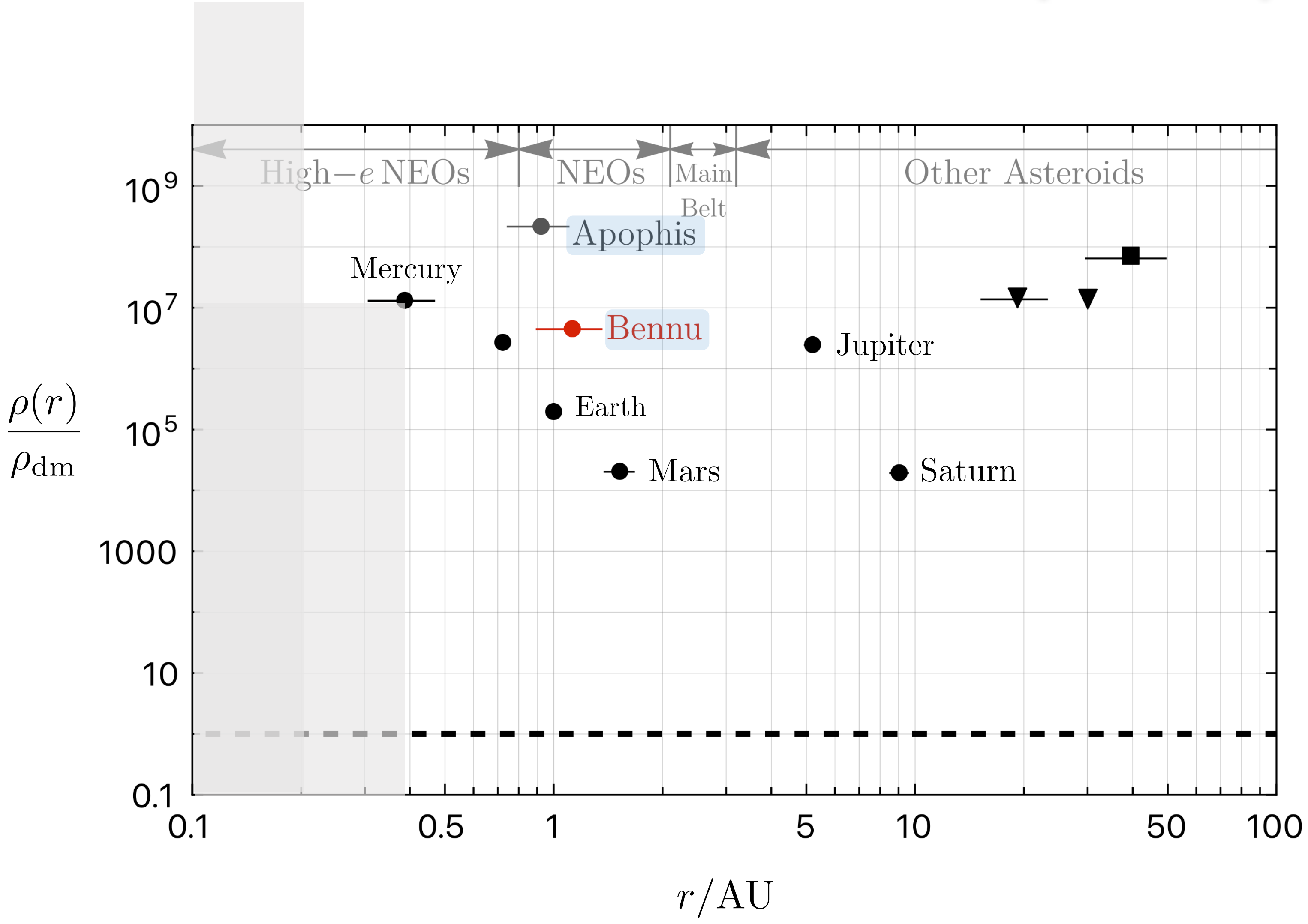


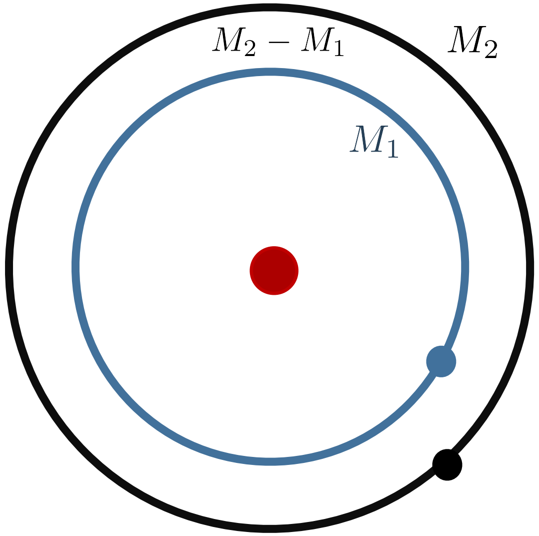


orbit comparison bounds $M_2 - M_1$

[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

Gravitational bounds from planetary and asteroid motion

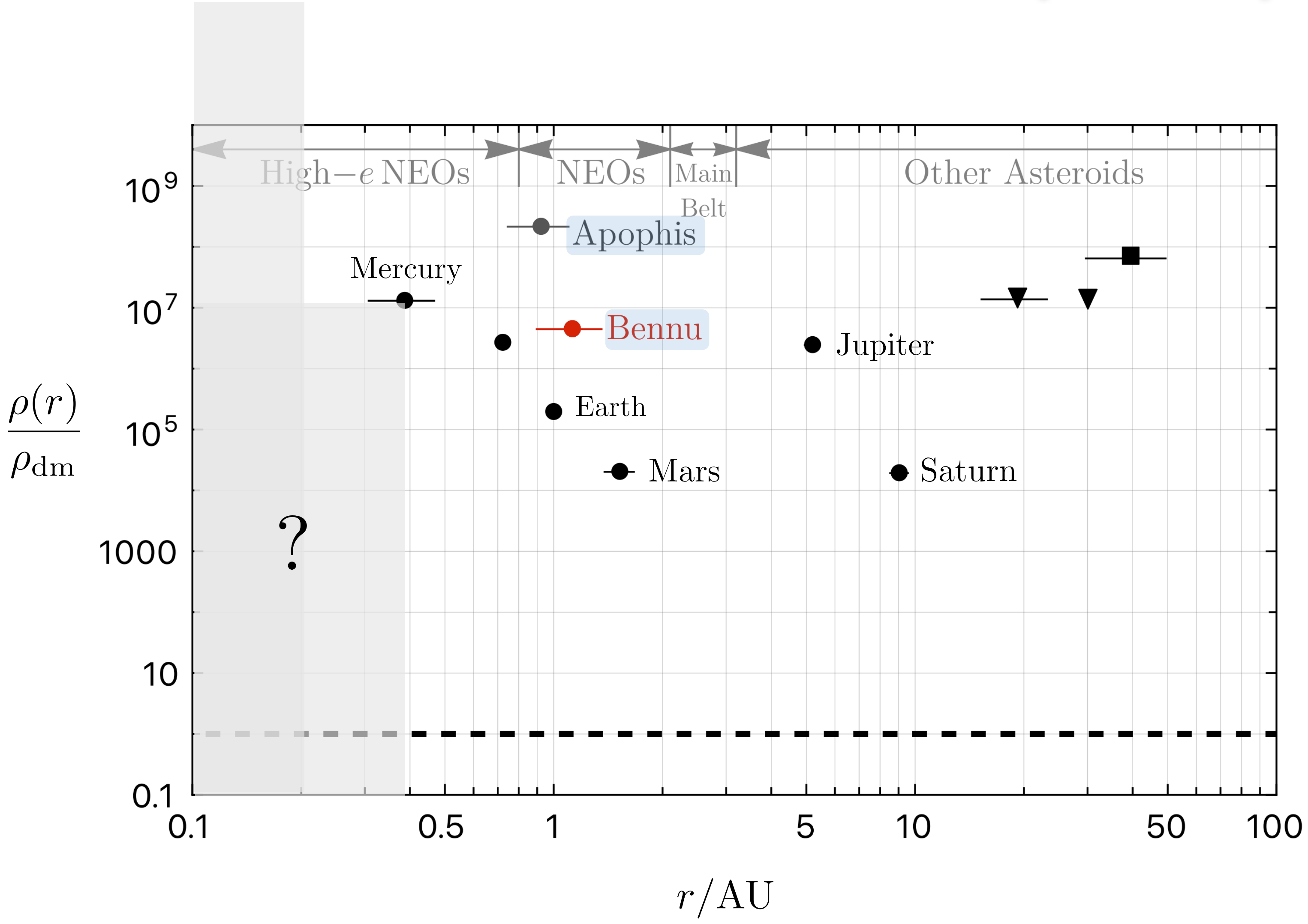


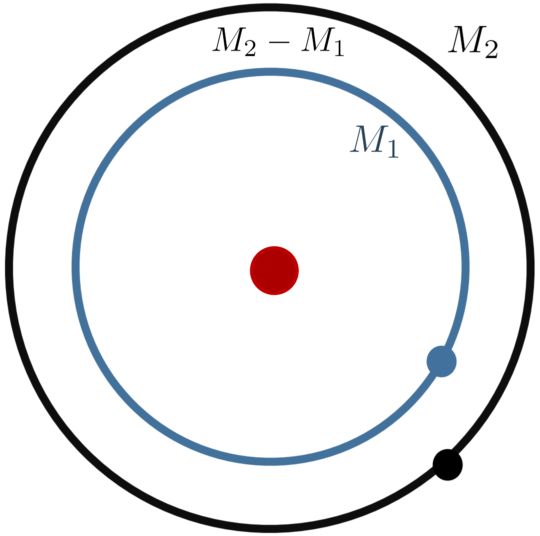


orbit comparison bounds $M_2 - M_1$

[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

Gravitational bounds from planetary and asteroid motion





orbit comparison bounds $M_2 - M_1$

- processes inducing the capture of ULDM?

[Pitjev and Pitjeva '13; Tsai et al '22; Gron and Soleng '96; Anderson et al. '95]

ULDM bound states

EoM: $$(g^{\mu\nu} D_\mu \partial_\nu + m^2)\phi = -\frac{1}{6}\lambda\phi^3 + \cdots$$

$$g_{00} = 1 + 2\Phi$$

$$\simeq \Phi_{\rm ex} = -\frac{GM}{r}$$

external mass, e.g. Sun

ULDM bound states

EoM: $$(g^{\mu\nu} D_\mu \partial_\nu + m^2)\phi = -\frac{1}{6}\lambda\phi^3 + \cdots$$

$$g_{00} = 1 + 2\Phi$$

$$\simeq \Phi_{\text{ex}} = -\frac{GM}{r}$$

external mass, e.g. Sun

$$\phi \equiv \frac{1}{\sqrt{2m}}(\psi e^{-imt} + \text{c.c.})$$

non-relativistic field

ULDM bound states

EoM: $(g^{\mu\nu} D_\mu \partial_\nu + m^2)\phi = -\frac{1}{6}\lambda\phi^3 + \cdots$

$g_{00} = 1 + 2\Phi$

$\simeq \Phi_{\rm ex} = -\frac{GM}{r}$

external mass, e.g. Sun

$\phi \equiv \frac{1}{\sqrt{2m}}(\psi e^{-imt} + \text{c.c.})$

non-relativistic field

Schroedinger: $\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\psi = g|\psi|^2\psi + \cdots$

$-m\Phi$

ULDM bound states

EoM: $$(g^{\mu\nu} D_\mu \partial_\nu + m^2)\phi = -\frac{1}{6}\lambda\phi^3 + \cdots$$

$$g_{00} = 1 + 2\Phi$$

$$\simeq \Phi_{\rm ex} = -\frac{GM}{r}$$

external mass, e.g. Sun

$$\phi \equiv \frac{1}{\sqrt{2m}}(\psi e^{-imt} + \text{c.c.})$$

non-relativistic field

Schroedinger: $$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\psi = g|\psi|^2\psi + \cdots$$

$$\frac{\alpha}{r} = -m\Phi$$

- gravitational coupling

$$\boxed{\alpha \equiv GMm}$$

- dimensionful self-coupling

$$\boxed{g \equiv \frac{\lambda}{8m^2}} = -\frac{1}{8f_a^2}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{\overset{\displaystyle -m\Phi}{\big\downarrow}}{\frac{\alpha}{r}}\right)\psi = g|\psi|^2\psi \;\; + \cdots$$

- hydrogen atom on the Sun

$\lvert\psi\rvert^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{\overset{-m\Phi}{\downarrow}}{\frac{\alpha}{r}}\right)\psi = g|\psi|^2\psi \; + \cdots$$

- hydrogen atom on the Sun

$\lvert\psi\rvert^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

density $\quad \rho \equiv m|\psi|^2$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{\displaystyle -m\Phi}{\overset{\downarrow}{\frac{\alpha}{r}}}\right)\psi = \cancel{g|\psi|^2\psi} + \cdots \quad = 0$$

- hydrogen atom on the Sun

$\|\psi\|^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

density $\quad \rho \equiv m|\psi|^2 \quad \underset{\ll}{\text{if locally}} \quad \rho_{\text{crit}} \equiv \dfrac{m^2\Phi}{|g|} = \dfrac{8m^4\Phi}{|\lambda|}$

self-interactions negligible
$\rightarrow$ free EoM

solutions:
bound
unbound

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{-m\Phi}{\overset{\downarrow}{\frac{\alpha}{r}}}\right)\psi = \cancel{g|\psi|^2\psi} + \cdots \quad = 0$$

- hydrogen atom on the Sun

$\lvert\psi\rvert^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

density $\quad \rho \equiv m|\psi|^2 \quad \overset{\text{if locally}}{\ll} \quad \rho_{\text{crit}} \equiv \frac{m^2\Phi}{|g|} = \frac{8m^4\Phi}{|\lambda|}$

self-interactions negligible
$\rightarrow$ free EoM

solutions:
- bound
- unbound

- Ground state $\quad \psi = \psi_{100}(\vec{x})e^{-i\omega_1 t}$

$$\psi_{100}(\vec{x}) \rightarrow \propto e^{-\frac{r}{R_\star}} \qquad \omega_1 \rightarrow -\frac{m\alpha^2}{2}$$

binding energy

'gravitational' Bohr radius

$$R_\star = \frac{1}{m\alpha} = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \overset{-m\Phi}{\frac{\alpha}{r}}\right)\psi = \cancel{g|\psi|^2\psi} \;+ \cdots \quad = 0$$

$$\frac{1}{R_\star^2 m} \simeq \frac{\alpha}{R_\star}$$

- hydrogen atom on the Sun

$\lvert\psi\rvert^2$	$\longleftrightarrow$	number density (could be large)	$\longleftrightarrow$	QM probability density
$\alpha = GMm$		gravitational coupling		fine structure constant

density $\quad \rho \equiv m|\psi|^2 \quad \underset{\ll}{\text{if locally}} \quad \rho_{\text{crit}} \equiv \frac{m^2\Phi}{|g|} = \frac{8m^4\Phi}{|\lambda|}$

self-interactions negligible
$\to$ free EoM

solutions:
bound
unbound

- Ground state $\quad \psi = \psi_{100}(\vec{x})e^{-i\omega_1 t}$

$$\psi_{100}(\vec{x}) \propto e^{-\frac{r}{\boxed{R_\star}}} \qquad \omega_1 \to -\frac{m\alpha^2}{2}$$

binding energy

'gravitational' Bohr radius

$$R_\star = \frac{1}{m\alpha} = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$

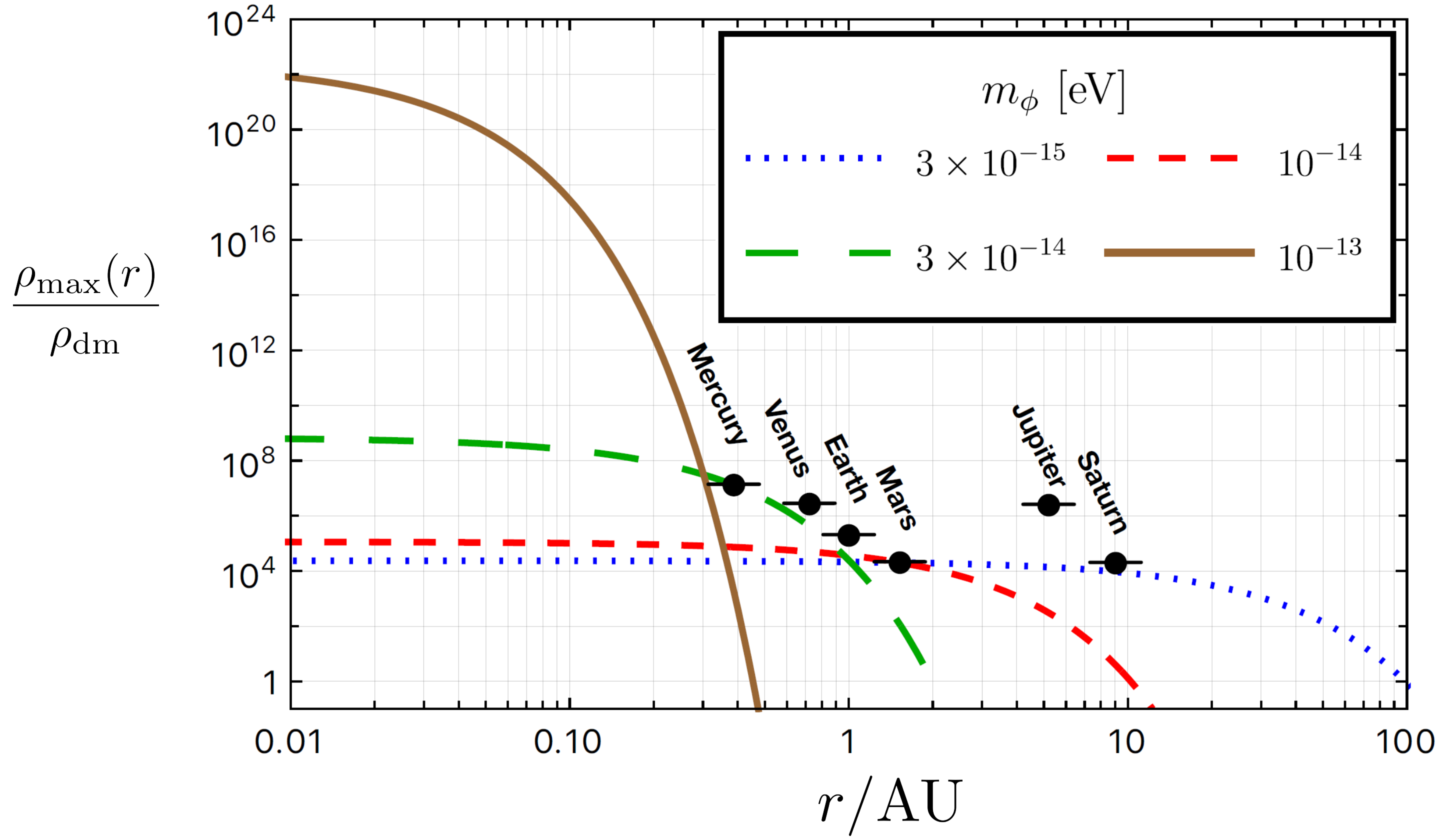
$$\psi = \sqrt{\frac{\overbrace{M_\star}^{\text{bound mass}}}{\pi R_\star^3 m}}\, e^{-\frac{r}{R_\star}}\, e^{-i\omega_1 t}$$

$$R_\star = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$

bound mass

$$\psi = \sqrt{\frac{M_\star}{\pi R_\star^3 m}}\, e^{-\frac{r}{R_\star}}\, e^{-i\omega_1 t}$$

$$R_\star = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$



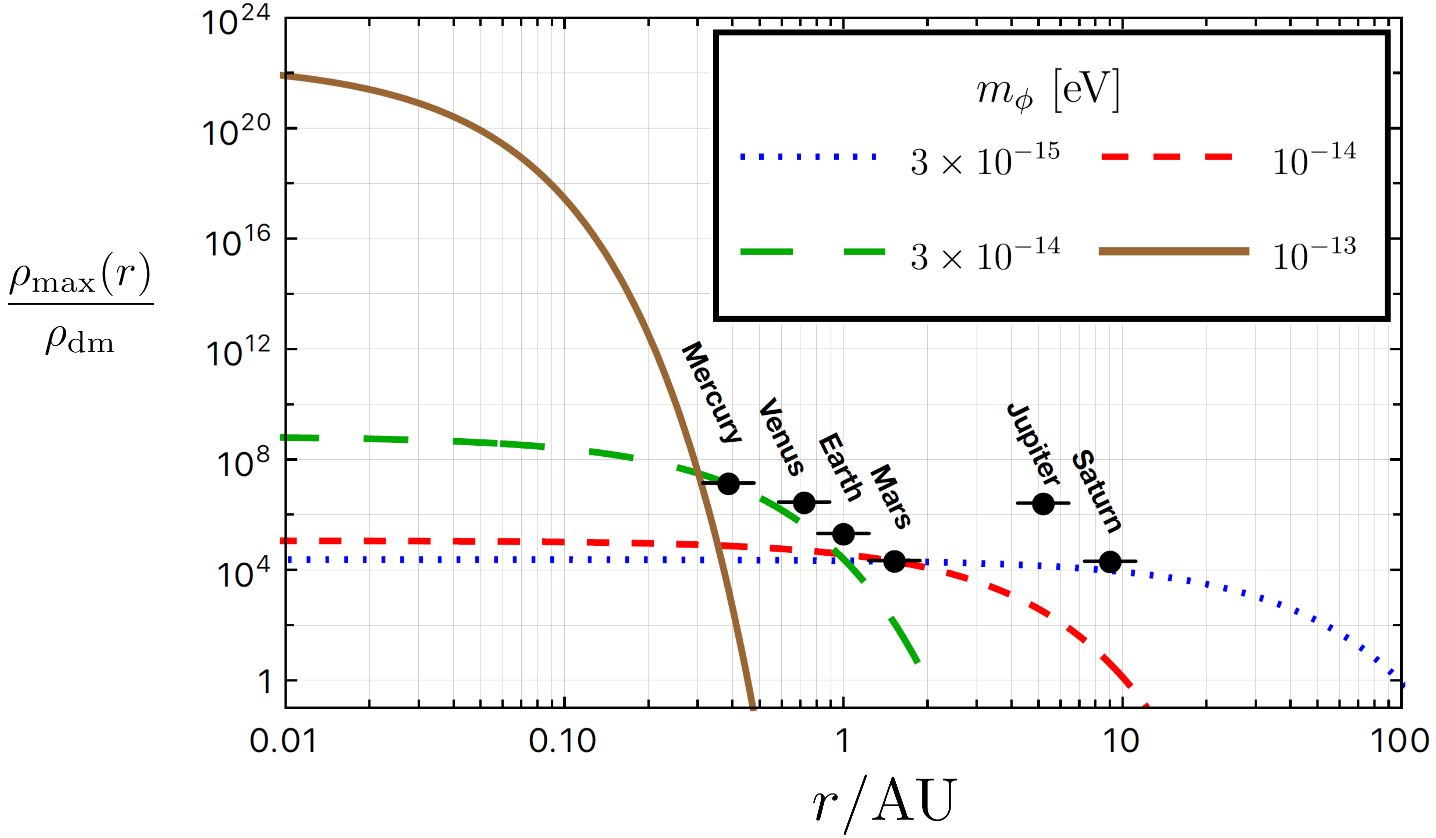
m/eV : 2×10^{-13} 1.3×10^{-14} 3×10^{-15}

$R_\star$: $R_\odot$ AU Saturn orbit

bound mass $\lll M_\odot$

$$\psi = \sqrt{\frac{\boxed{M_\star}}{\pi R_\star^3 m}}\, e^{-\frac{r}{R_\star}} e^{-i\omega_1 t}$$

$$R_\star = 1\,\mathrm{AU}\left[\frac{1.3\cdot 10^{-14}\,\mathrm{eV}}{m}\right]^2\left[\frac{M_\odot}{M}\right]$$



m/eV : 2×10^{-13} 1.3×10^{-14} 3×10^{-15}

$R_\star$: $R_\odot$ AU Saturn orbit

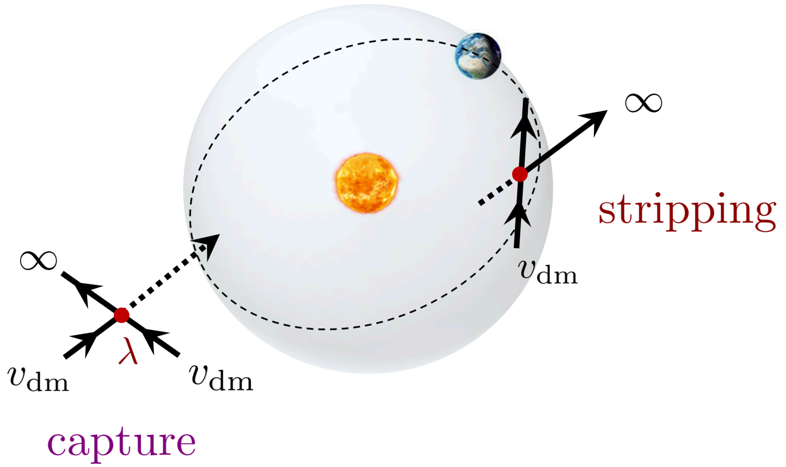
Formation of the gravitational atom

1) initially, DM in the continuum: $M_\star = 0$

Formation of the gravitational atom

1) initially, DM in the continuum: $M_\star = 0$

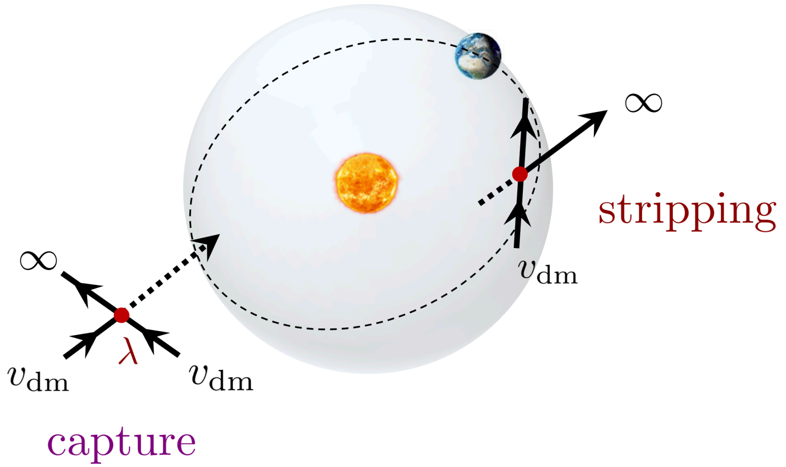
2) $$\frac{dM_\star}{dt} = \text{(capture)} - \text{(stripping)}$$



Formation of the gravitational atom

1) initially, DM in the continuum: $M_\star = 0$

2) $$\frac{dM_\star}{dt} = \text{(capture)} - \text{(stripping)}$$
$$= C + \Gamma_1 M_\star - \Gamma_2 M_\star$$



C = direct capture

Γ_1 = stimulated capture $\longleftrightarrow$ Bose enhancement

Γ_2 = stripping, via inverse process

all > 0 and $\propto g^2 \propto \lambda^2$

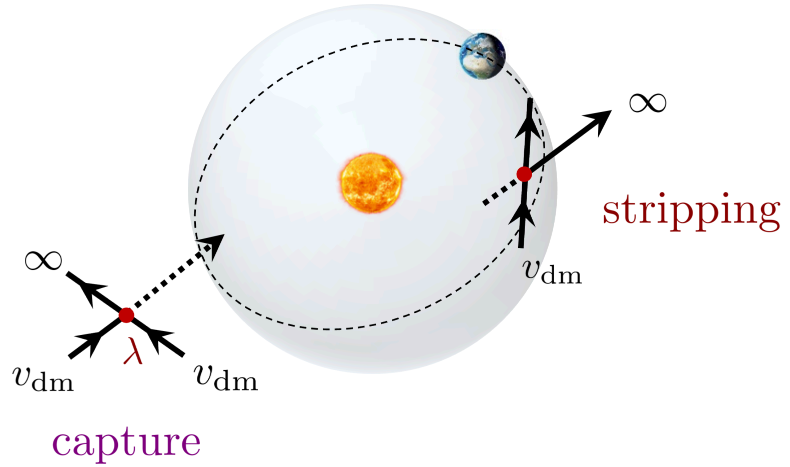
Formation of the gravitational atom

1) initially, DM in the continuum: $M_\star = 0$

$$2) \quad \frac{dM_\star}{dt} = (\text{capture}) - (\text{stripping})$$

$$= C + \Gamma_1 M_\star - \Gamma_2 M_\star$$

$$= C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma \gtrless 0} M_\star$$

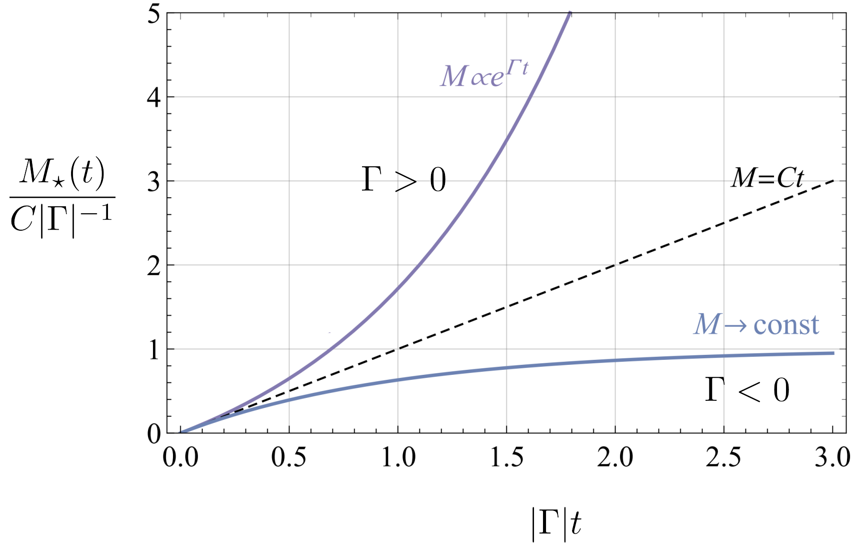


C = direct capture

Γ_1 = stimulated capture $\quad \longleftrightarrow \quad$ Bose enhancement

Γ_2 = stripping, via inverse process

all > 0 and $\propto g^2 \propto \lambda^2$



For $g = 0$, DM in the galaxy halo is

$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} a(\mathbf{k}) e^{-i\omega_k t} \psi_{\mathbf{k}}(\mathbf{x})$$

For $g = 0$, DM in the galaxy halo is

$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} a(\mathbf{k}) e^{-i\omega_k t} \psi_{\mathbf{k}}(\mathbf{x})$$

unbound solutions of the atom:
'scattering states' or 'waves', $\mathbf{k}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi_{\mathbf{k}} = 0$$

For $g = 0$, DM in the galaxy halo is

$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} a(\mathbf{k}) e^{-i\overset{\frac{k^2}{2m}}{\overbrace{\omega_k}} t} \boxed{\psi_{\mathbf{k}}(\mathbf{x})}$$

unbound solutions of the atom:
'scattering states' or 'waves', $\mathbf{k}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi_{\mathbf{k}} = 0$$

For $g = 0$, DM in the galaxy halo is

$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} \underbrace{a(\mathbf{k})}_{\text{momentum distribution}} e^{-i\overbrace{\omega_k}^{\frac{k^2}{2m}} t} \psi_{\mathbf{k}}(\mathbf{x})$$

unbound solutions of the atom:
‘scattering states’ or ‘waves’, $\mathbf{k}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi_{\mathbf{k}} = 0$$

For $g = 0$, DM in the galaxy halo is

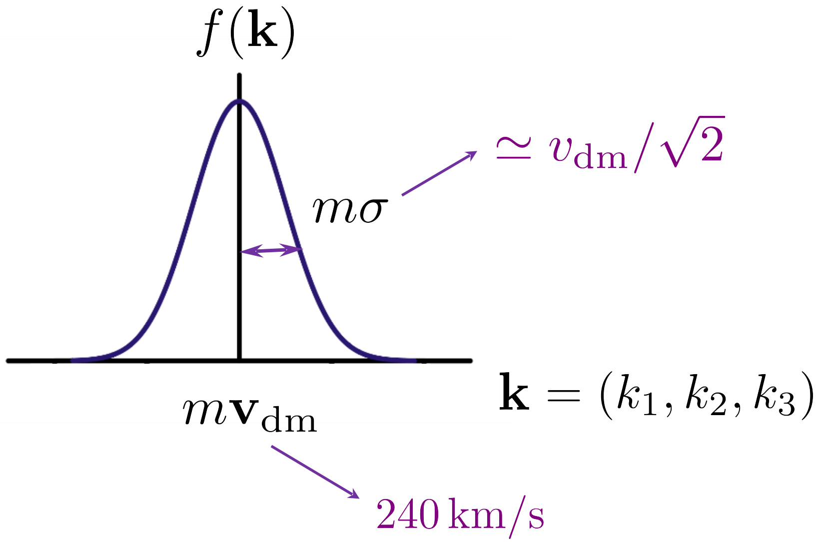
$$\psi_w(t, \mathbf{x}) \equiv \int \frac{d^3k}{(2\pi)^3} \underbrace{a(\mathbf{k})}_{\text{momentum distribution}} e^{-i\overset{\frac{k^2}{2m}}{\omega_k} t} \psi_{\mathbf{k}}(\mathbf{x})$$

unbound solutions of the atom:
'scattering states' or 'waves', $\mathbf{k}$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi_{\mathbf{k}} = 0$$

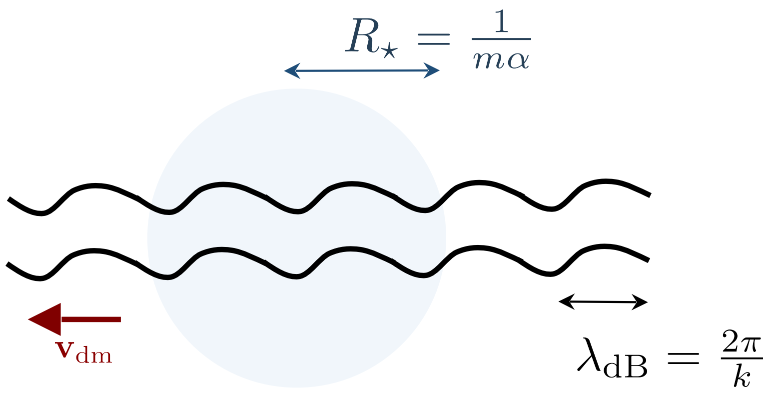
- $a(\mathbf{k})$, standard halo model

$$\langle a^*(\mathbf{k}) a(\mathbf{k}') \rangle = (2\pi)^3 f(\mathbf{k}) \delta(\mathbf{k} - \mathbf{k}')$$



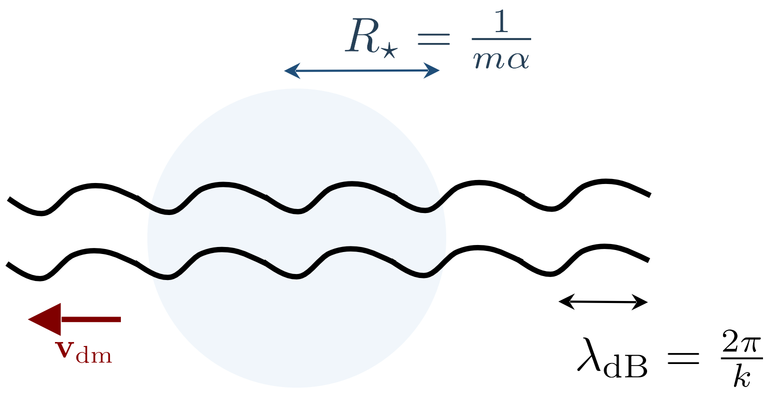
$$f(\mathbf{k}) = \frac{\overset{0.4\,\mathrm{GeV/cm^3}}{\rho_{\rm dm}}}{\sigma^3 m^4} e^{-\frac{(\mathbf{k} - m\mathbf{v}_{\rm dm})^2}{2m^2\sigma^2}}$$

- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$



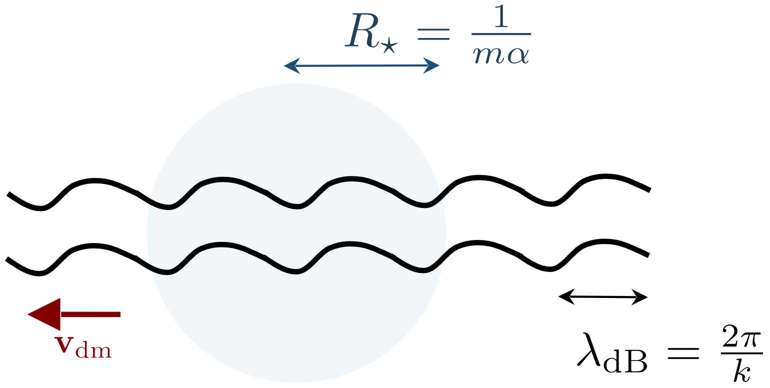
- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{k}\frac{1}{R_\star}$$



- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

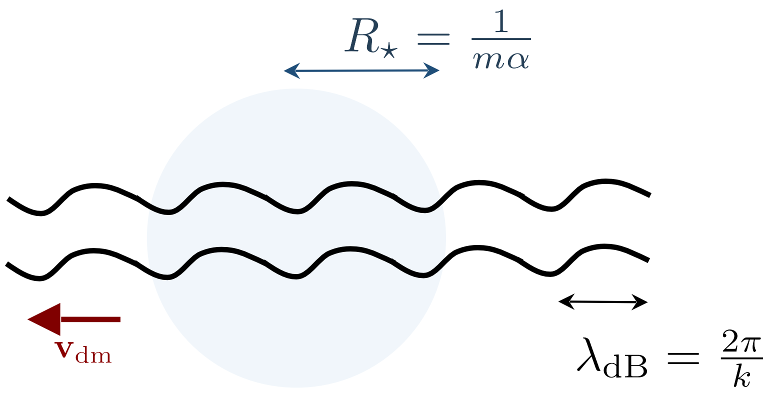
$$\boxed{\xi_{\text{foc}}(k) \equiv \frac{\lambda_{\text{dB}}}{R_\star}} = \frac{2\pi}{\underbrace{k}_{mv}} \frac{1}{\underbrace{R_\star}_{(m\alpha)^{-1}}} = \frac{2\pi\alpha}{v}$$



- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{\underset{\swarrow\, mv}{k}}\,\frac{1}{\underset{\searrow\,(m\alpha)^{-1}}{R_\star}} = \frac{2\pi\alpha}{v}$$

$$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$$



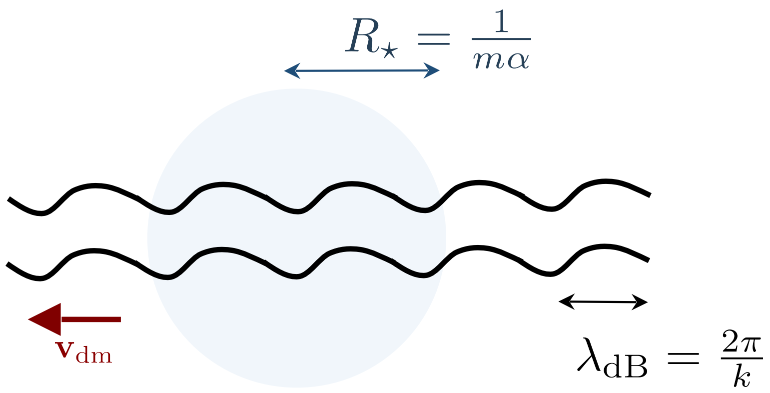
$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

large velocity

- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{\underset{mv}{k}}\,\frac{1}{\underset{(m\alpha)^{-1}}{R_\star}} = \frac{2\pi\alpha}{v}$$

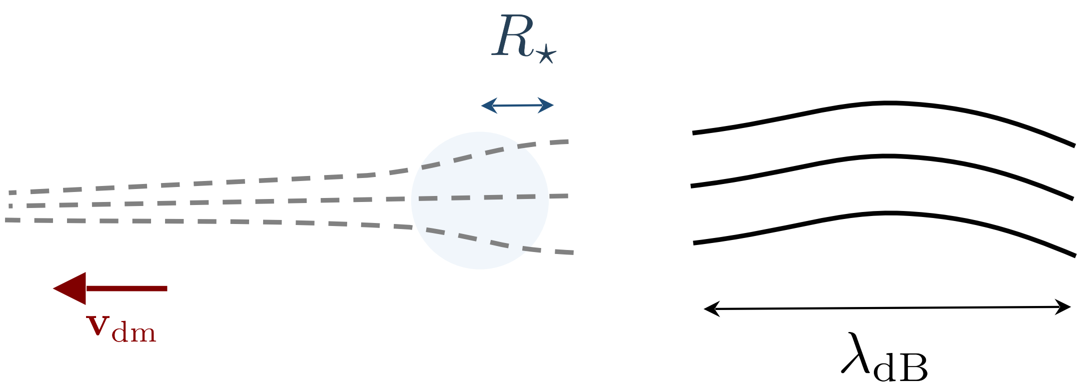
$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$



$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

large velocity

$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$



small velocity

- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \frac{2\pi}{k}\,\frac{1}{R_\star} = \frac{2\pi\alpha}{v}$$

$$k \to mv \qquad R_\star \to (m\alpha)^{-1}$$

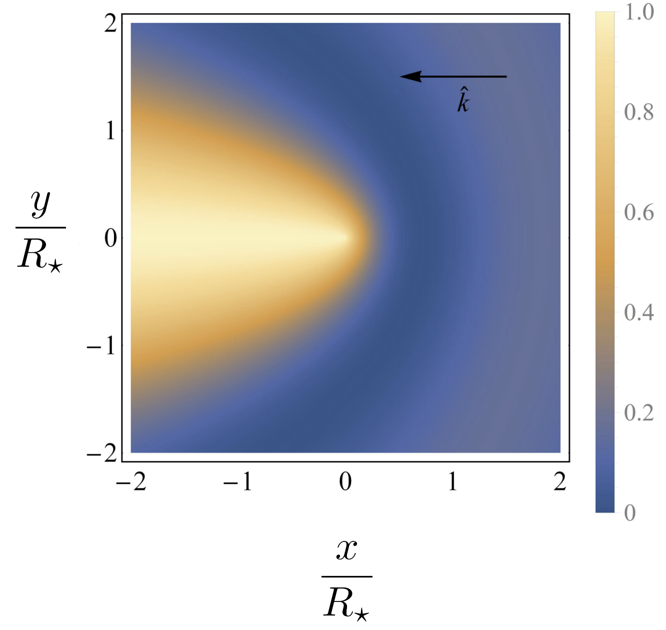
$$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$$

$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$$

gravitational focusing

$$|\psi_{\mathbf{k}}|^2 \to$$



- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \underbrace{\frac{2\pi}{k}}_{mv}\,\underbrace{\frac{1}{R_\star}}_{(m\alpha)^{-1}} = \frac{2\pi\alpha}{v}$$

$$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$$

$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

energy $\gg$ binding energy

$$\frac{mv^2}{2} \qquad\qquad -\frac{m\alpha^2}{2}$$

$$\dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star$$

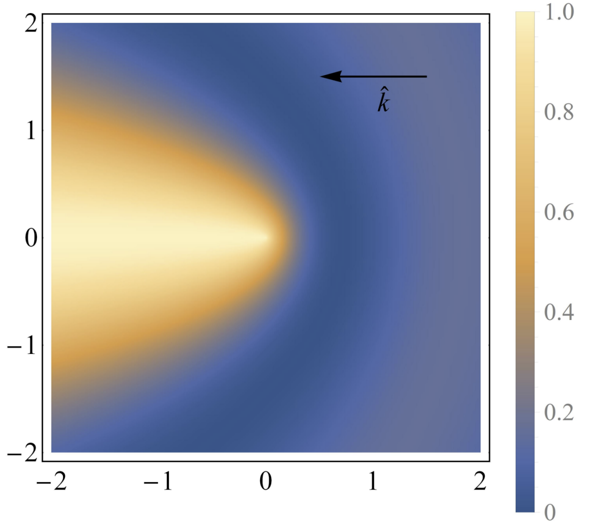
$$\Gamma_1 < \Gamma_2$$

stripping dominates

$$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$$

gravitational focusing

$$|\psi_{\mathbf{k}}|^2 \to$$



- Scattering states $\psi_{\mathbf{k}}(\mathbf{x})$

$$\boxed{\xi_{\rm foc}(k) \equiv \frac{\lambda_{\rm dB}}{R_\star}} = \underbrace{\frac{2\pi}{k}}_{mv}\ \underbrace{\frac{1}{R_\star}}_{(m\alpha)^{-1}} = \frac{2\pi\alpha}{v}$$

$$\boxed{\xi_{\rm foc}(k) \ll 1 \quad \leftrightarrow v \gg 2\pi\alpha}$$

$$\psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}$$

energy $\gg$ binding energy

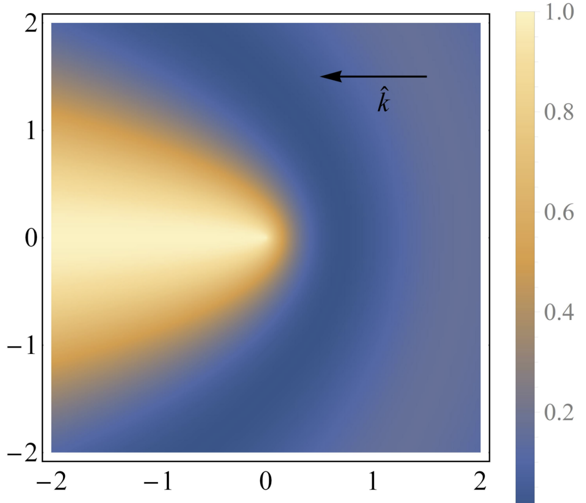
$$\frac{mv^2}{2} \qquad\qquad -\frac{m\alpha^2}{2}$$

$$\boxed{\xi_{\rm foc}(k) \gg 1 \quad \leftrightarrow v \ll 2\pi\alpha}$$

gravitational focusing

$$|\psi_{\mathbf{k}}|^2 \to$$

$\frac{y}{R_\star}$



$\frac{x}{R_\star}$

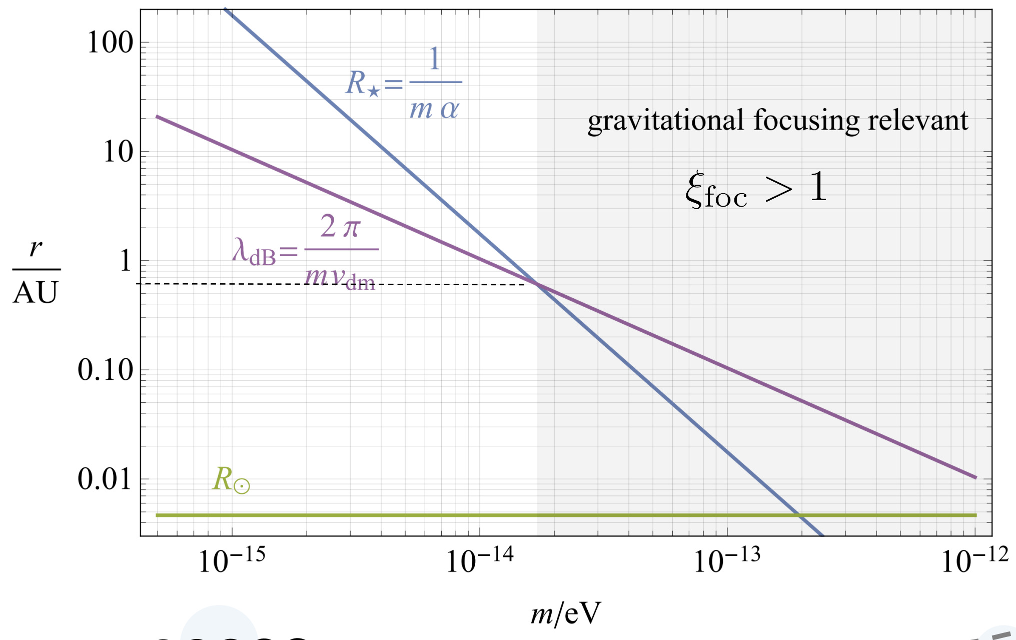
$$\dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star$$

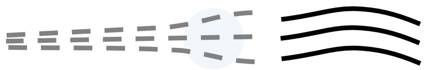
$\Gamma_1 < \Gamma_2$

stripping dominates

$\Gamma_1 > \Gamma_2$

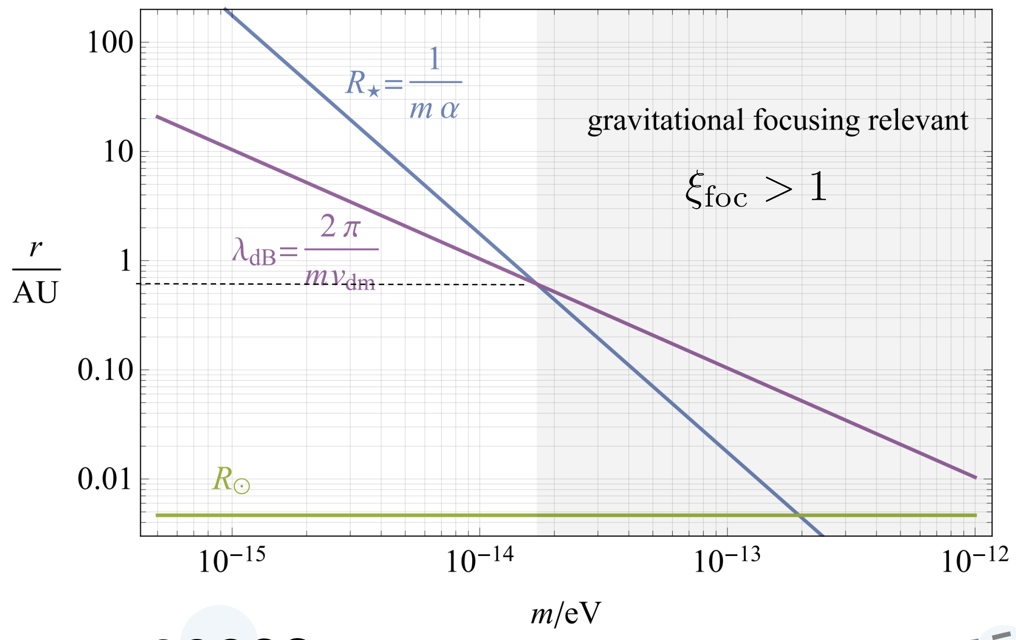
stimulated capture dominates

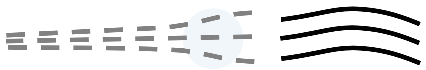




in the Solar System $$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \simeq \left[\frac{m}{1.7\times 10^{-14}\,{\rm eV}}\right]\left[\frac{M}{M_\odot}\right]\left[\frac{240\,{\rm km/s}}{v_{\rm dm}}\right] \gtrsim 1$$

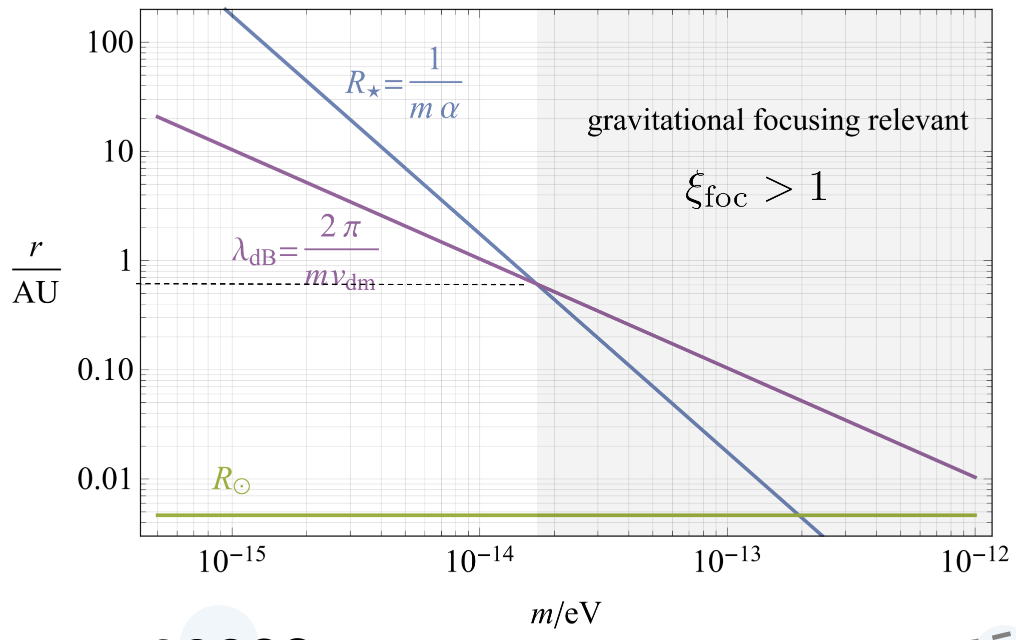
i.e. if $m \gtrsim 1.7\cdot 10^{-14}\,{\rm eV}$ $\qquad R_\star \lesssim {\rm AU}!$

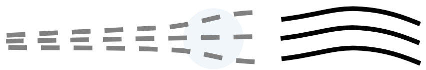




in the Solar System $\quad \xi_{\rm foc} = \dfrac{2\pi\alpha}{v_{\rm dm}} \simeq \left[\dfrac{m}{1.7\times 10^{-14}\,\mathrm{eV}}\right]\left[\dfrac{M}{M_\odot}\right]\left[\dfrac{240\,\mathrm{km/s}}{v_{\rm dm}}\right] \gtrsim 1$

i.e. if $m \gtrsim 1.7\cdot 10^{-14}\,\mathrm{eV}$ $\qquad R_\star \lesssim \mathrm{AU}!$





in the Solar System $\quad \xi_{\mathrm{foc}} = \frac{2\pi\alpha}{v_{\mathrm{dm}}} \simeq \left[\frac{m}{1.7\times 10^{-14}\,\mathrm{eV}}\right]\left[\frac{M}{M_\odot}\right]\left[\frac{240\,\mathrm{km/s}}{v_{\mathrm{dm}}}\right] \gtrsim 1$

i.e. if $m \gtrsim 1.7\cdot 10^{-14}\,\mathrm{eV}$ $\qquad R_\star \lesssim \mathrm{AU}!$

Bound state formation: derivation

$$S_{\mathrm{non-rel}} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\mathrm{ex}}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\mathrm{int}}}\right]$$

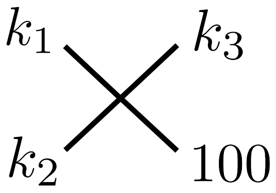
Bound state formation: derivation

$$S_{\mathrm{non-rel}} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\mathrm{ex}}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\mathrm{int}}}\right]$$

Bound state formation: derivation

$$S_{\rm non-rel} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\rm int}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$



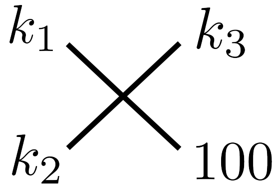
$$\mathcal{A} \;=\; \langle k_3\, nlm|\, T[\hat{H}_{int}]\, |k_1 k_2\rangle$$

Bound state formation: derivation

$$S_{\rm non-rel} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\rm int}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$

$$\hat{\psi}(x)\hat{\psi}(x)\,|k_1 k_2\rangle = 2\psi_{\mathbf{k_1}}(\mathbf{x})\psi_{\mathbf{k_2}}(\mathbf{x})e^{-i(\omega_{k_1}+\omega_{k_2})t}\,|0\rangle$$

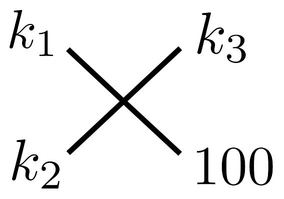


$$\mathcal{A} = \langle k_3\, nlm|\, T[\hat{H}_{int}]\,|k_1 k_2\rangle = 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

Bound state formation: derivation

$$S_{\rm non-rel} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\rm int}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$



$$\hat{\psi}(x)\hat{\psi}(x)\,|k_1 k_2\rangle = 2\psi_{\mathbf{k_1}}(\mathbf{x})\psi_{\mathbf{k_2}}(\mathbf{x})e^{-i(\omega_{k_1}+\omega_{k_2})t}\,|0\rangle$$

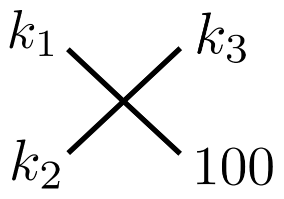
$$\mathcal{A} \;=\; \langle k_3\, nlm|\, T[\hat{H}_{int}]\,|k_1 k_2\rangle \;=\; 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

$$\mathcal{M} = \int d^3x\, \psi^*_{100}\psi^*_{\mathbf{k_3}}\psi_{\mathbf{k_1}}\psi_{\mathbf{k_2}}$$

Bound state formation: derivation

$$S_{\mathrm{non-rel}} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\mathrm{ex}}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\mathrm{int}}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$



$$\hat{\psi}(x)\hat{\psi}(x)\,|k_1 k_2\rangle = 2\psi_{\mathbf{k_1}}(\mathbf{x})\psi_{\mathbf{k_2}}(\mathbf{x})e^{-i(\omega_{k_1}+\omega_{k_2})t}\,|0\rangle$$

$$\mathcal{A} \;=\; \langle k_3\, nlm|\, T[\hat{H}_{int}]\,|k_1 k_2\rangle \;=\; 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

$$\Delta\omega \equiv \omega_{k_1} + \omega_{k_2} - \omega_{k_3} - \omega_1$$

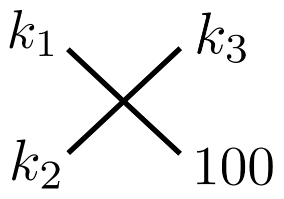
$$= \frac{k_1^2}{2m} + \frac{k_2^2}{2m} - \frac{k_3^2}{2m} + \frac{m\alpha^2}{2}$$

$$\mathcal{M} = \int d^3x\, \psi_{100}^*\psi_{\mathbf{k_3}}^*\psi_{\mathbf{k_1}}\psi_{\mathbf{k_2}}$$

Bound state formation: derivation

$$S_{\rm non-rel} = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^*\psi - \psi^*\dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \underbrace{\frac{1}{2}g|\psi|^4}_{\mathcal{H}_{\rm int}}\right]$$

$$\boxed{k_1 + k_2 \to k_3 + 100}$$



$$\hat{\psi}(x)\hat{\psi}(x)\,|k_1 k_2\rangle = 2\psi_{\mathbf{k_1}}(\mathbf{x})\psi_{\mathbf{k_2}}(\mathbf{x})e^{-i(\omega_{k_1}+\omega_{k_2})t}\,|0\rangle$$

$$\mathcal{A} \;=\; \langle k_3\, nlm|\, T[\hat{H}_{int}]\,|k_1 k_2\rangle \;=\; 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

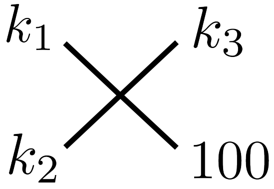
$$\Delta\omega \equiv \omega_{k_1} + \omega_{k_2} - \omega_{k_3} - \omega_1$$

$$= \frac{k_1^2}{2m} + \frac{k_2^2}{2m} - \frac{k_3^2}{2m} + \frac{m\alpha^2}{2}$$

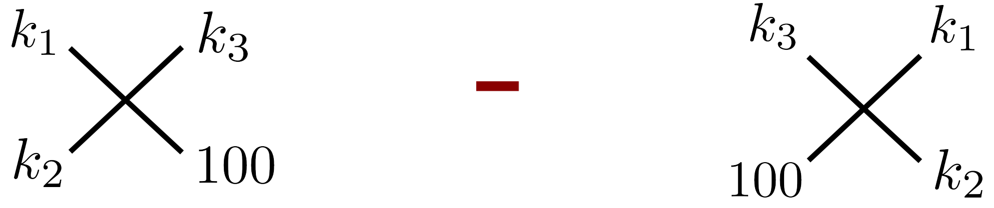
$$\mathcal{M} \;=\; \int d^3x\, \psi^*_{100}\psi^*_{\mathbf{k_3}}\psi_{\mathbf{k_1}}\psi_{\mathbf{k_2}}$$

$$P_{k_1+k_2\to k_3+100} \;=\; (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$

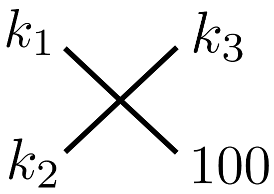


$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$

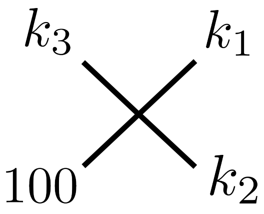


subtract inverse process

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$



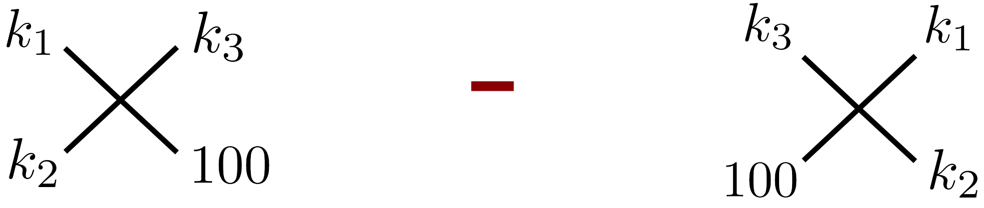
−



subtract inverse process

$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$

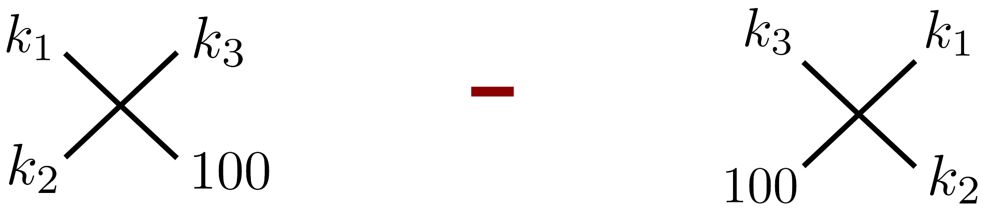


subtract inverse process

$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

initial state density

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$



subtract inverse process

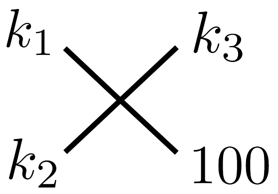
Bose enhancement:

$$P_{\text{indist}} = (N+1)P_{\text{dist}}$$

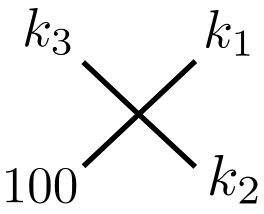
$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

initial state density

$$\boxed{P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2}$$



–



subtract inverse process

Bose enhancement:

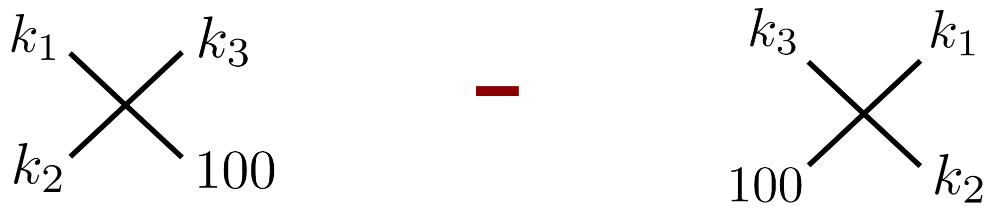
$P_{\text{indist}} = (N+1)P_{\text{dist}}$

$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

initial state density

$$= 2g^2\int [dk_1][dk_2][dk_3]\left\{f(\mathbf{k_1})f(\mathbf{k_2})\rho(\mathbf{k_3}) + N_0\left[f(\mathbf{k_1})f(\mathbf{k_2}) - 2f(\mathbf{k_2})f(\mathbf{k_3})\right]\right\}(2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$



subtract inverse process

Bose enhancement:

$$P_{\text{indist}} = (N+1)P_{\text{dist}}$$

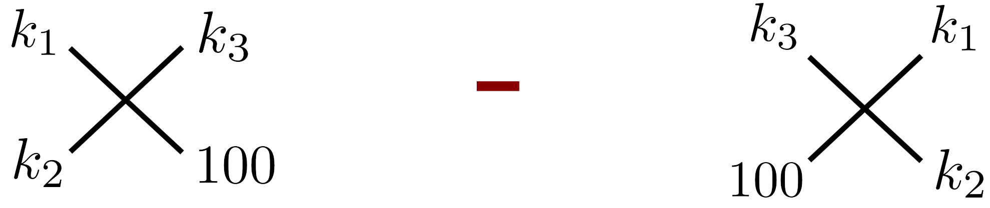
$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

initial state density

$$= 2g^2\int [dk_1][dk_2][dk_3]\left\{f(\mathbf{k_1})f(\mathbf{k_2})\rho(\mathbf{k_3}) + N_0\left[f(\mathbf{k_1})f(\mathbf{k_2}) - 2f(\mathbf{k_2})f(\mathbf{k_3})\right]\right\}(2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

$$M_\star = mN_0 \quad \rightarrow \qquad \dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star$$

$$P_{k_1+k_2\to k_3+100} = (2\pi)\delta(\Delta\omega)4g^2|\mathcal{M}|^2$$



subtract inverse process

Bose enhancement:

$P_{\text{indist}} = (N+1)P_{\text{dist}}$

$$\frac{dN_0}{dt} = \frac{4g^2}{2}\int [dk_1][dk_2][dk_3]\left[(f(\mathbf{k_3})+1)(N_0+1)f(\mathbf{k_1})f(\mathbf{k_2}) - f(\mathbf{k_3})N_0(f(\mathbf{k_1})+1)(f(\mathbf{k_2})+1)\right](2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

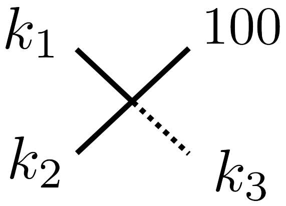
initial state density

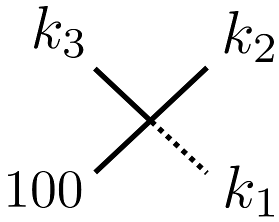
$$= 2g^2\int [dk_1][dk_2][dk_3]\left\{f(\mathbf{k_1})f(\mathbf{k_2})\rho(\mathbf{k_3}) + N_0\left[f(\mathbf{k_1})f(\mathbf{k_2}) - 2f(\mathbf{k_2})f(\mathbf{k_3})\right]\right\}(2\pi)\delta(\Delta\omega)|\mathcal{M}|^2$$

$M_\star = mN_0 \quad \rightarrow \qquad \dot{M}_\star = C + (\Gamma_1 - \Gamma_2)M_\star$

stimulated capture stripping

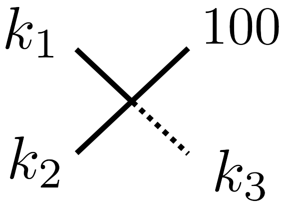
$$\Gamma \equiv \Gamma_1 - \Gamma_2 = g^2 \int [dk_{1,2,3}] \delta(\Delta\omega) |\mathcal{M}|^2 \{ \; f(\mathbf{k_1}) f(\mathbf{k_2}) - 2 f(\mathbf{k_2}) f(\mathbf{k_3}) \; \}$$

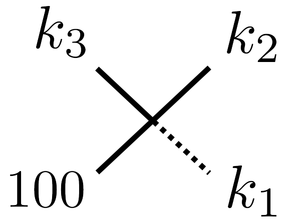




stimulated capture   stripping

$$\Gamma \equiv \Gamma_1 - \Gamma_2 = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2\{\ f(\mathbf{k_1})f(\mathbf{k_2}) - 2f(\mathbf{k_2})f(\mathbf{k_3})\ \}$$

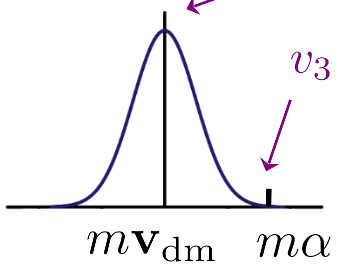




$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\mathrm{eV}$$

v_1 or v_2

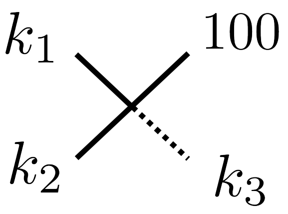


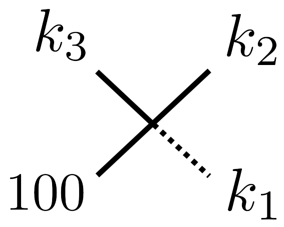
$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

stimulated capture stripping

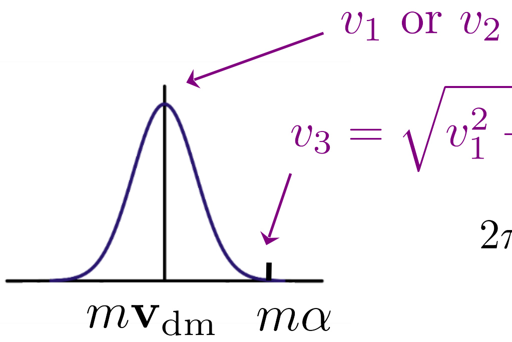
$$\Gamma \equiv \Gamma_1 - \Gamma_2 = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2\{\ f(\mathbf{k_1})f(\mathbf{k_2}) - 2f(\mathbf{k_2})f(\mathbf{k_3})\ \}$$





$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\text{eV}$$



$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

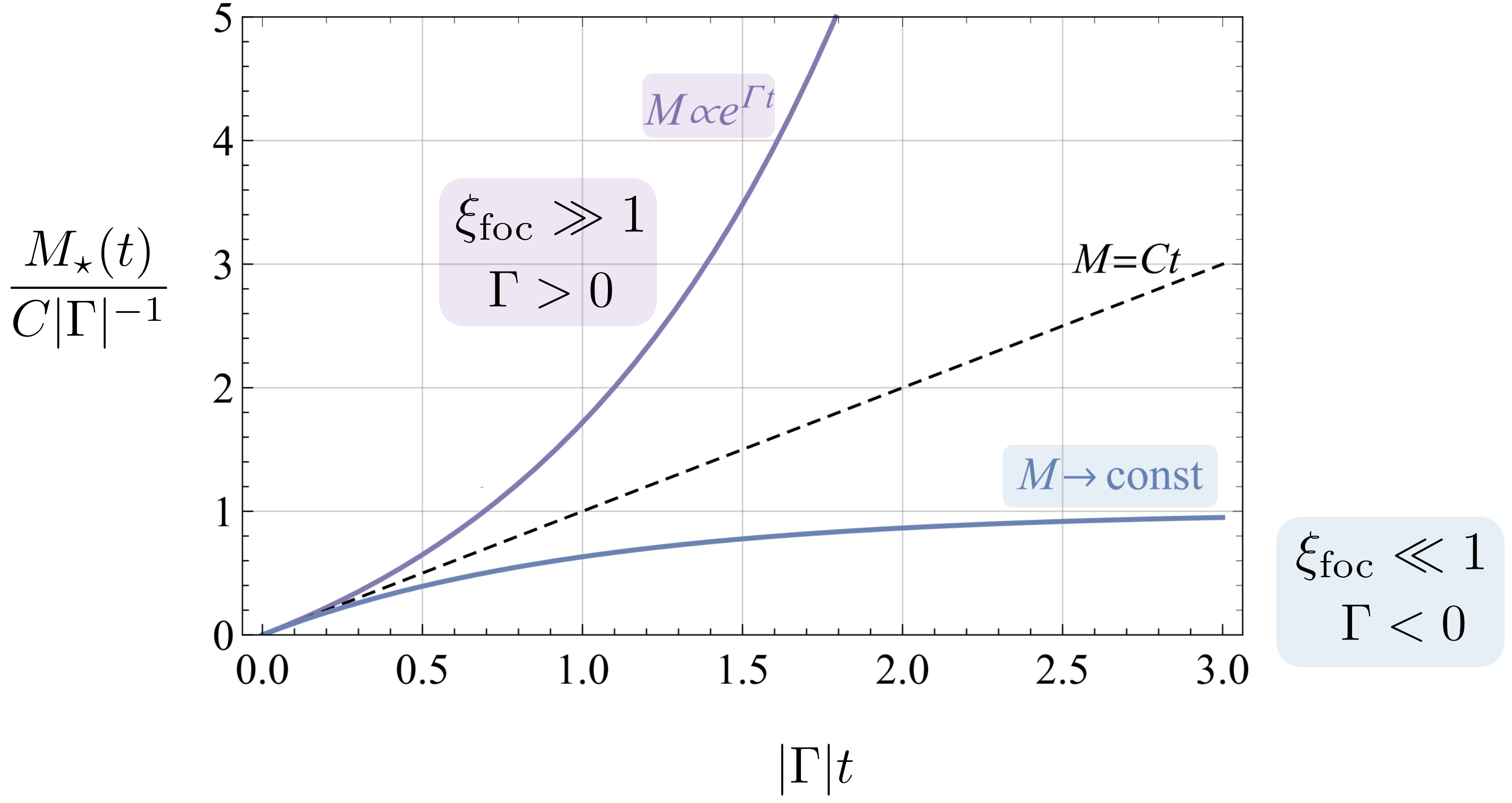
$$\Gamma_1 \simeq \text{const}$$

$$\Gamma_2 \simeq e^{-\xi_{\rm foc}}$$

$$\to \Gamma > 0$$

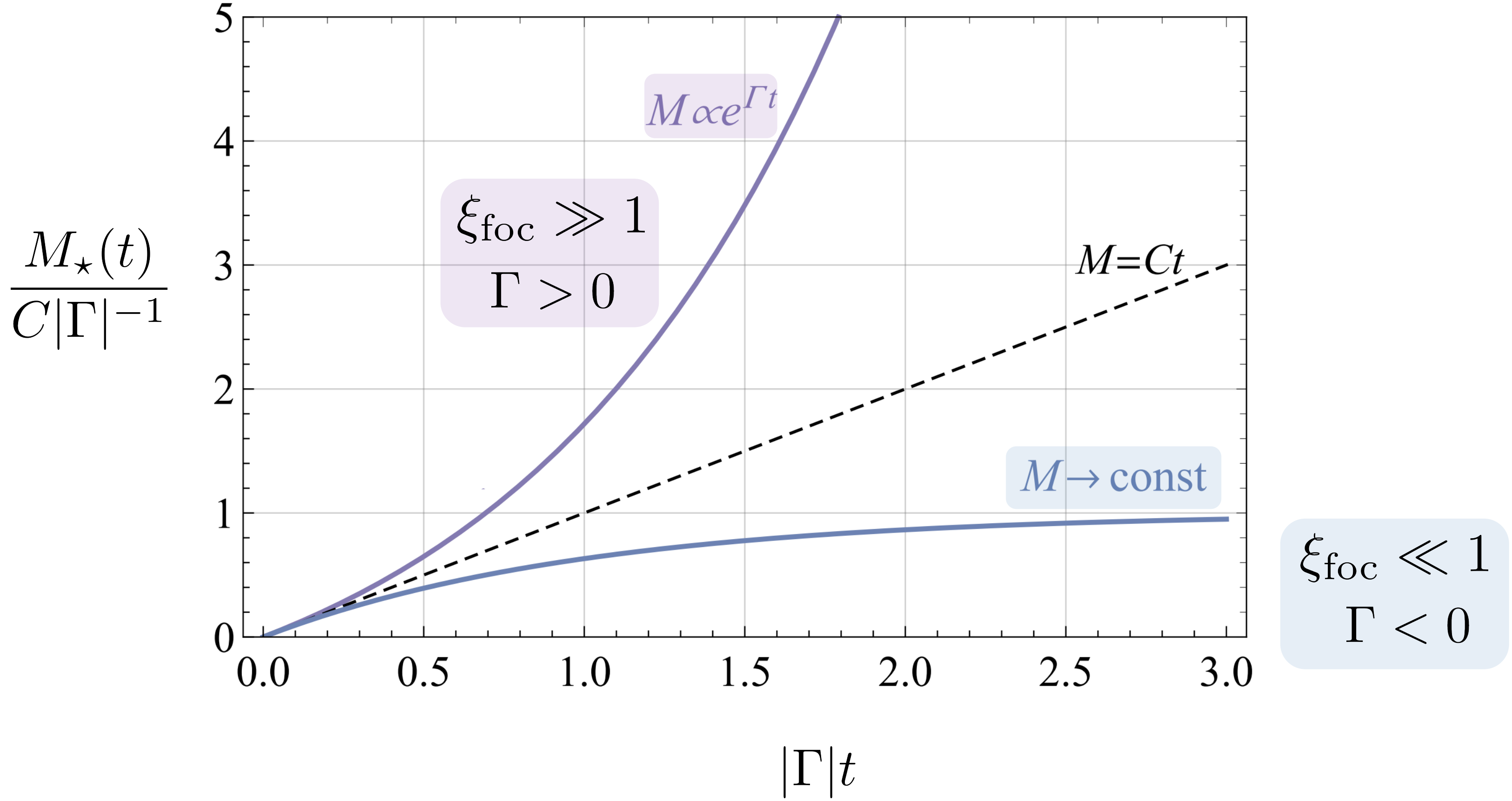
Phases of formation

$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$



$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

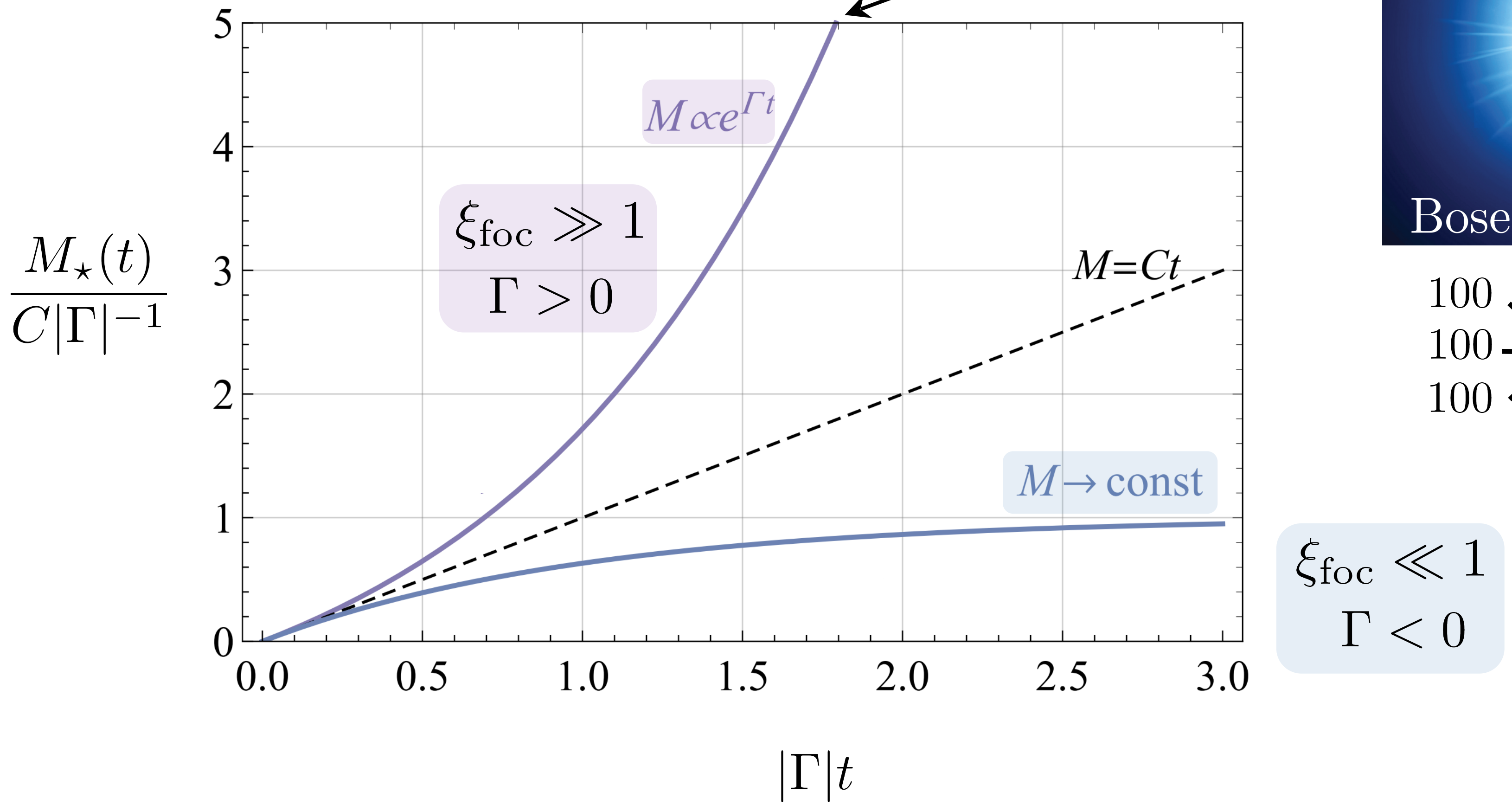
Phases of formation



$$\rho_{\rm crit} \equiv \frac{2\Phi_{\rm ex} m^2}{|g|} \simeq 2\frac{\alpha^2 m^2}{|g|} \simeq 6 \cdot 10^4 \rho_{\rm dm} \left[\frac{f_a}{5 \cdot 10^7\,{\rm GeV}}\right]^2 \left[\frac{m}{1.7 \cdot 10^{-14}\,{\rm eV}}\right]^4 \left[\frac{M}{M_\odot}\right]^2 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]$$

$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

Phases of formation



100
100 ...... relativistic
100

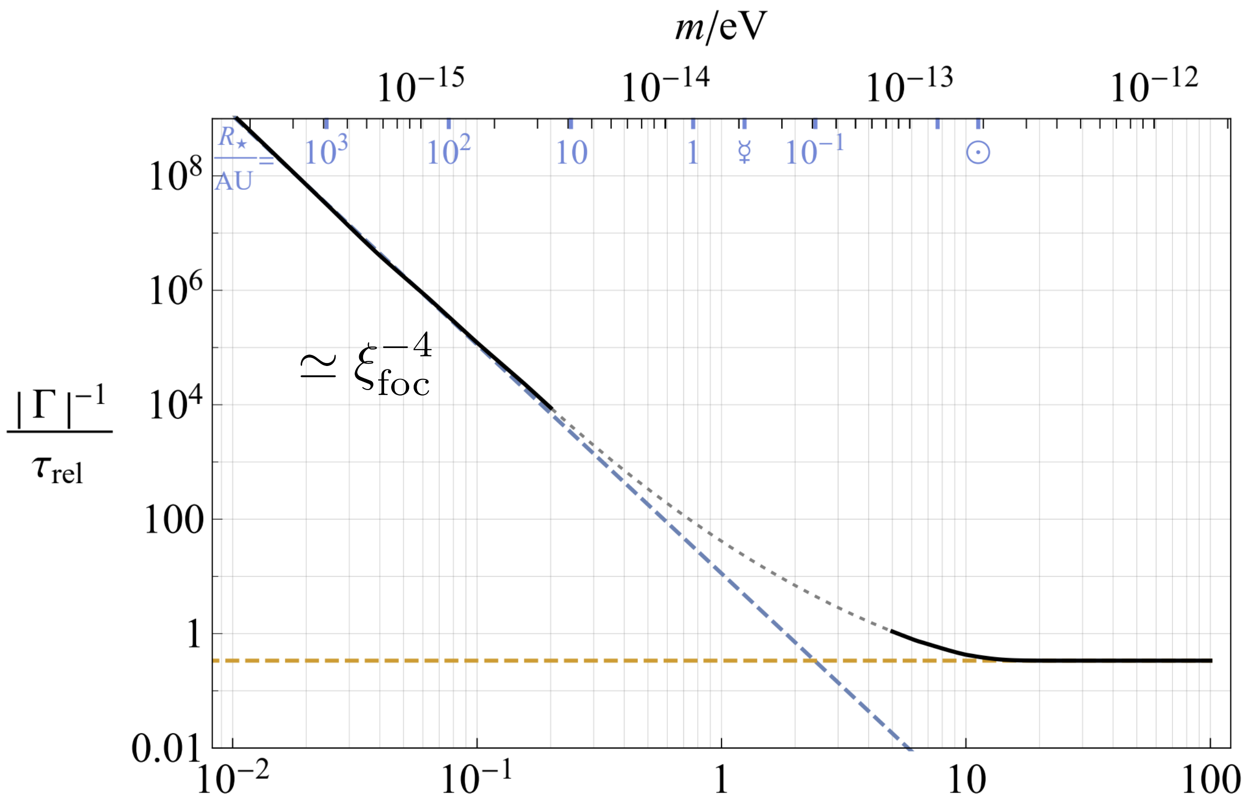
$$\rho_{\rm crit} \equiv \frac{2\Phi_{\rm ex} m^2}{|g|} \simeq 2\frac{\alpha^2 m^2}{|g|} \simeq 6 \cdot 10^4 \rho_{\rm dm} \left[\frac{f_a}{5 \cdot 10^7\,\mathrm{GeV}}\right]^2 \left[\frac{m}{1.7 \cdot 10^{-14}\,\mathrm{eV}}\right]^4 \left[\frac{M}{M_\odot}\right]^2 \left[\frac{0.4\,\mathrm{GeV/cm^3}}{\rho_{\rm dm}}\right]$$

$\Gamma^{-1} \leftrightarrow$

relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$\Gamma^{-1} \leftrightarrow$

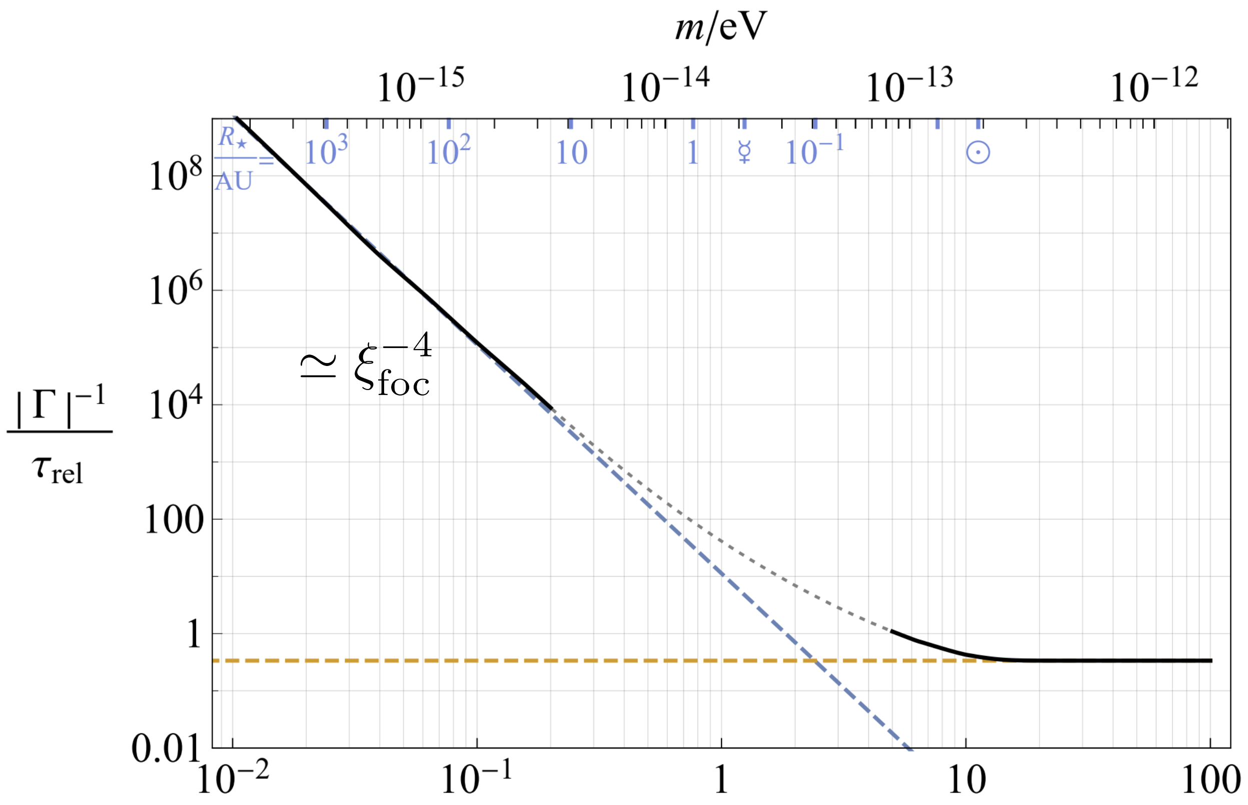
relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

$\Gamma^{-1} \leftrightarrow$

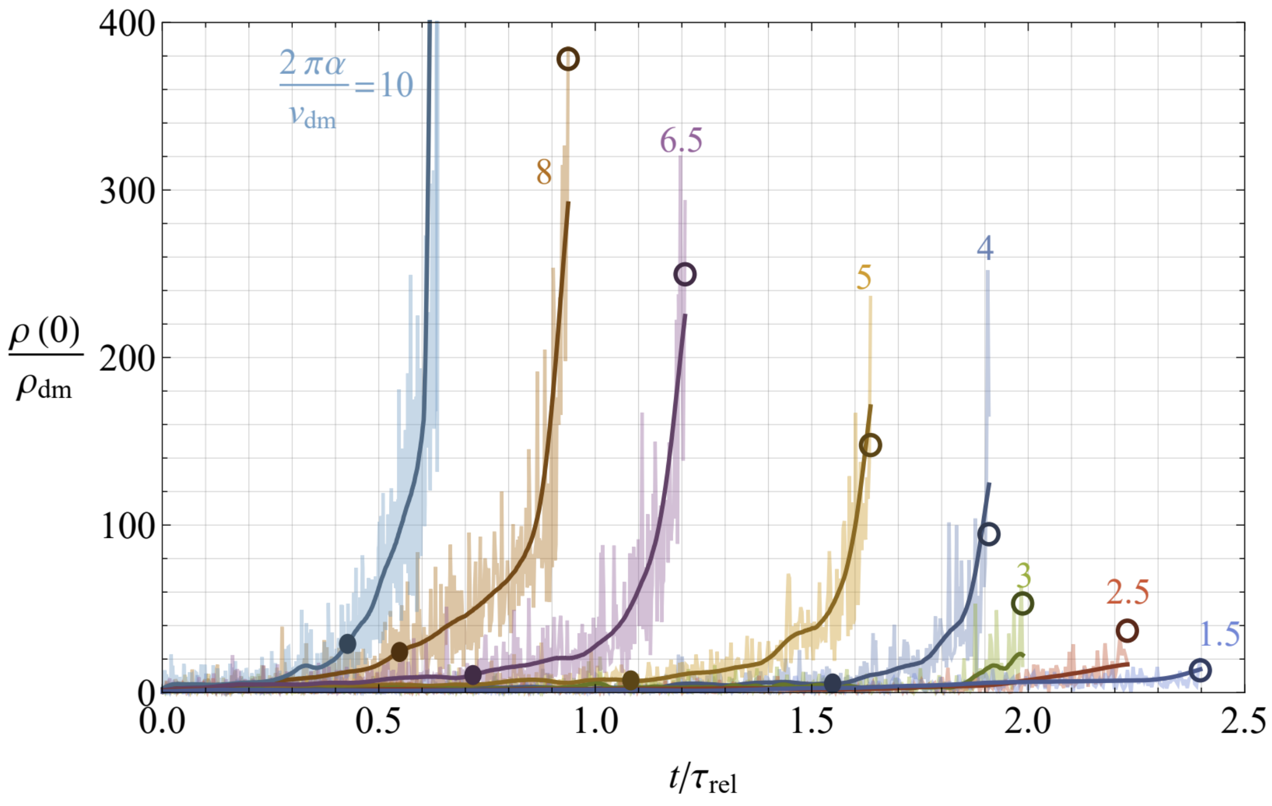
relaxation time $\quad \tau_{\rm rel} \equiv \dfrac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\dfrac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\dfrac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\dfrac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\dfrac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$

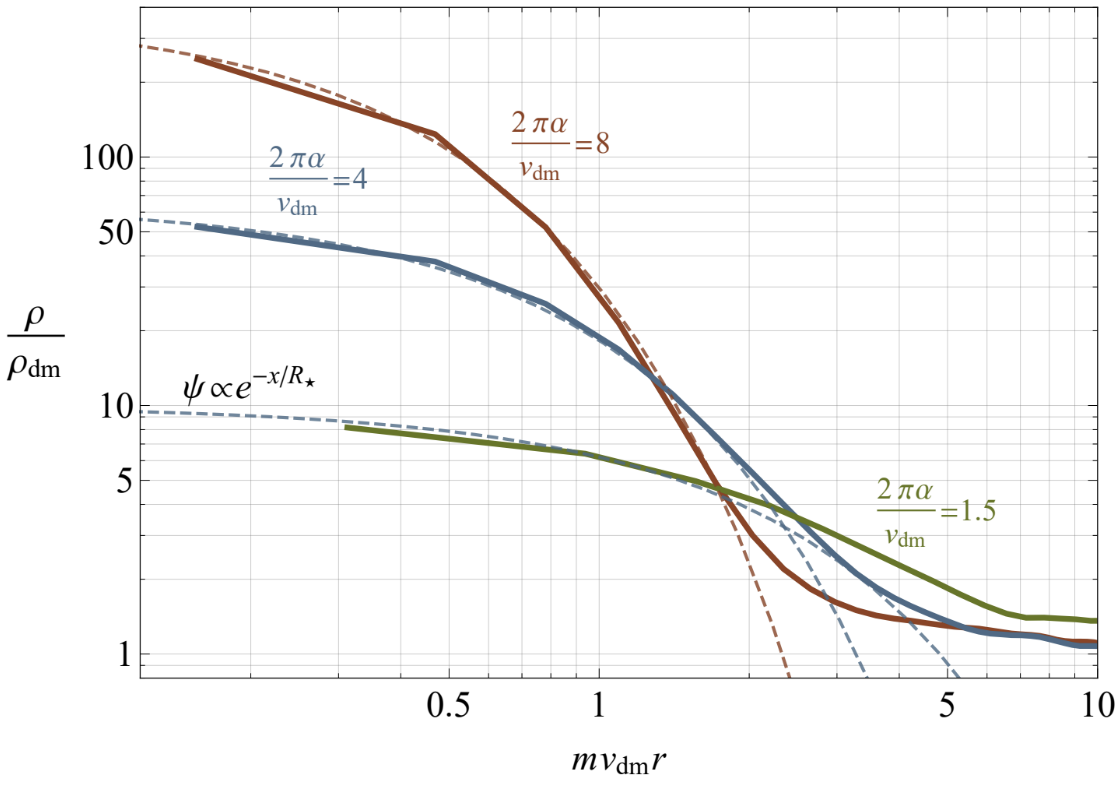


$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

- the size of λ (or f_a) only enters into Γ

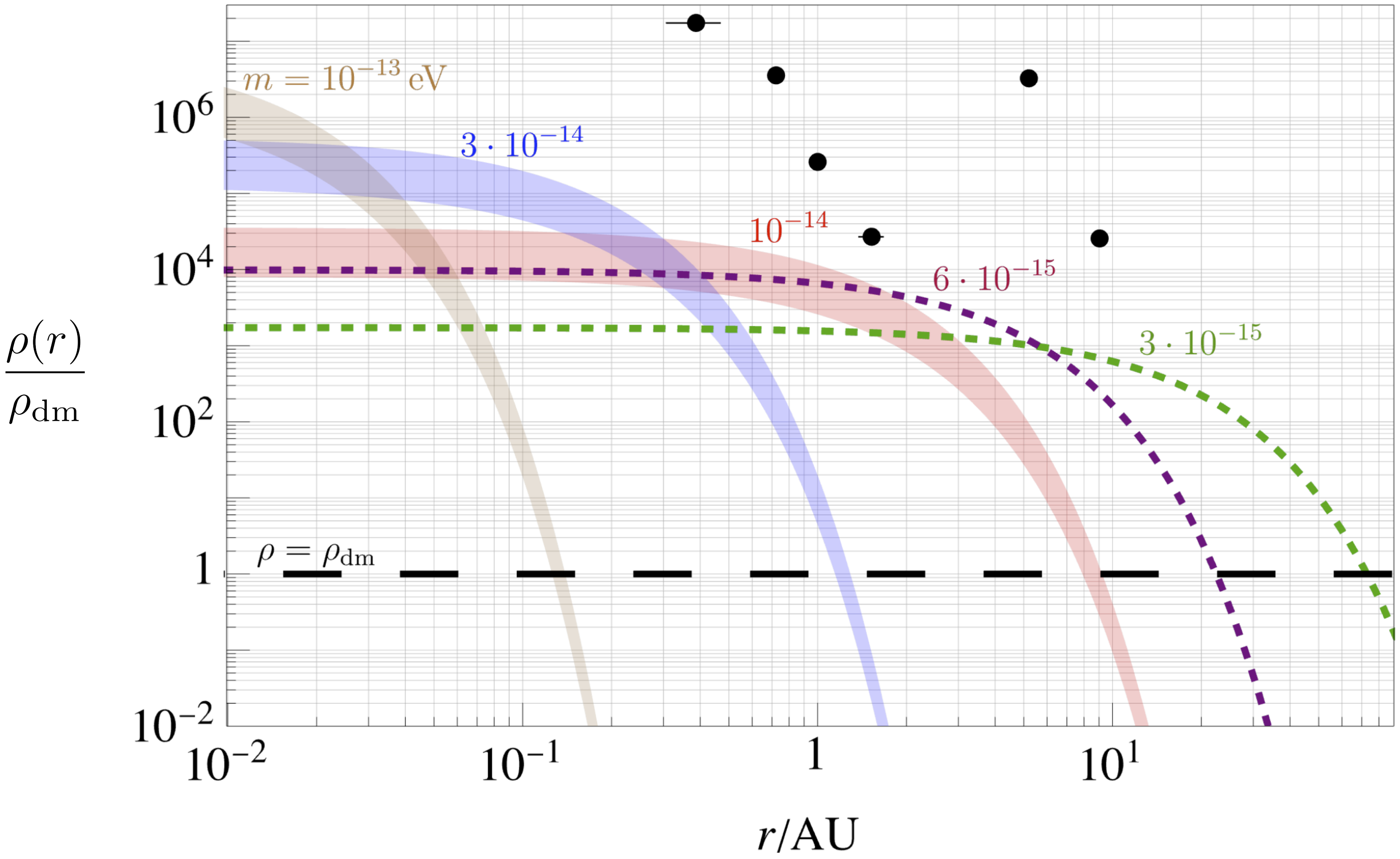
Comparison with simulations



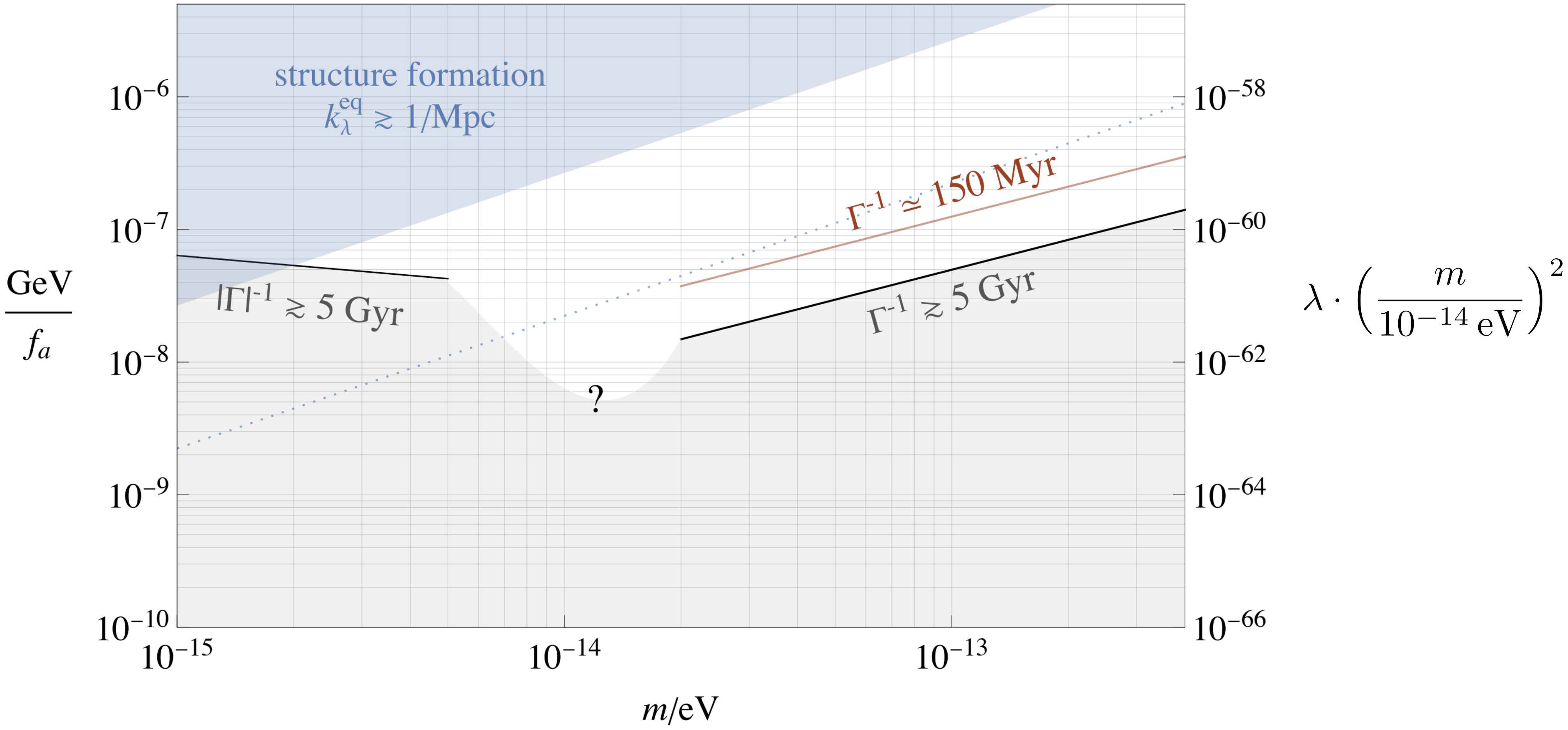


- transition around $\xi = 1$

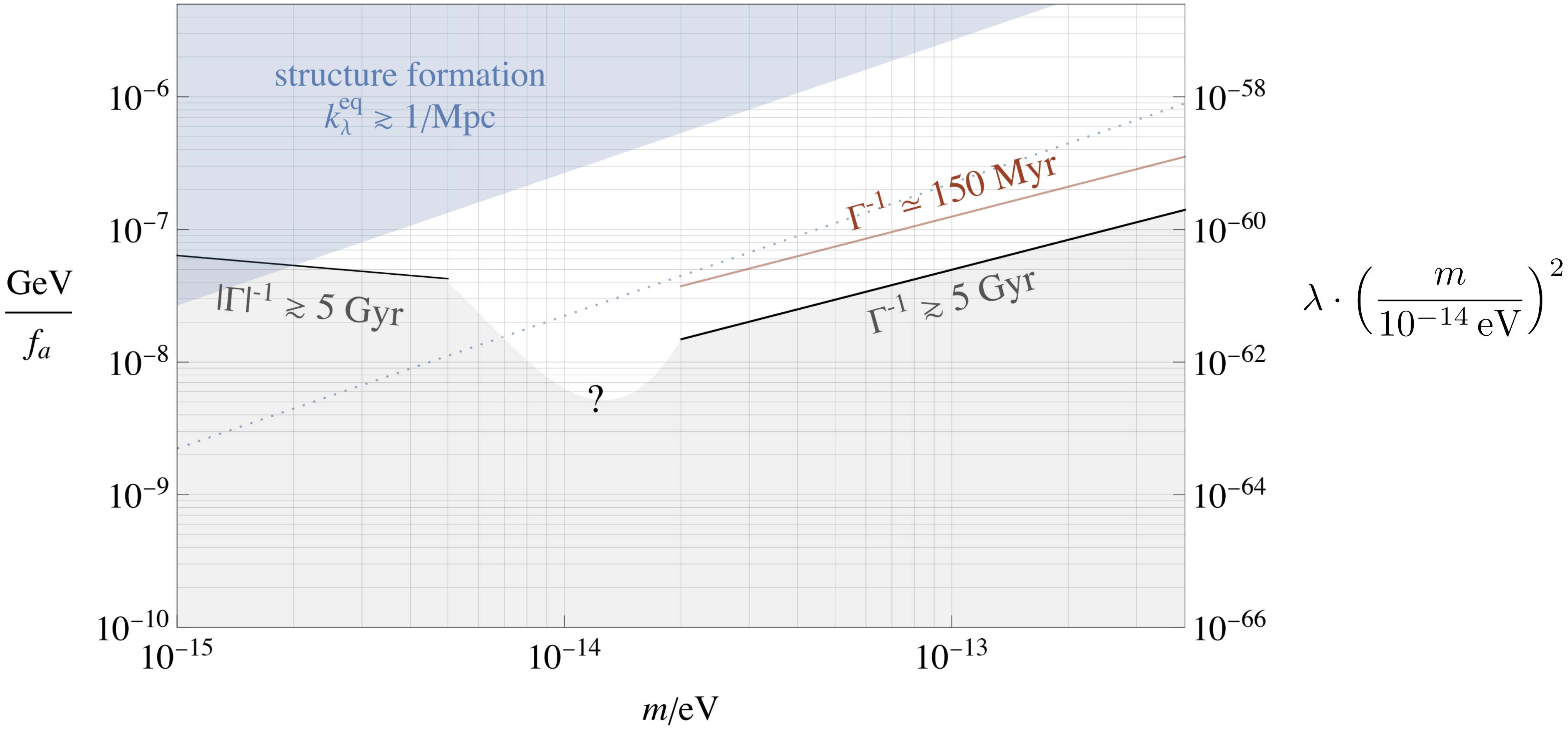
density profile after 5 Gyr



- bands have $v_{\rm dm} = 50 \div 240\,\mathrm{km/s}$
- f_a (or λ) fixed in $10^7 \div 10^8\,\mathrm{GeV}$

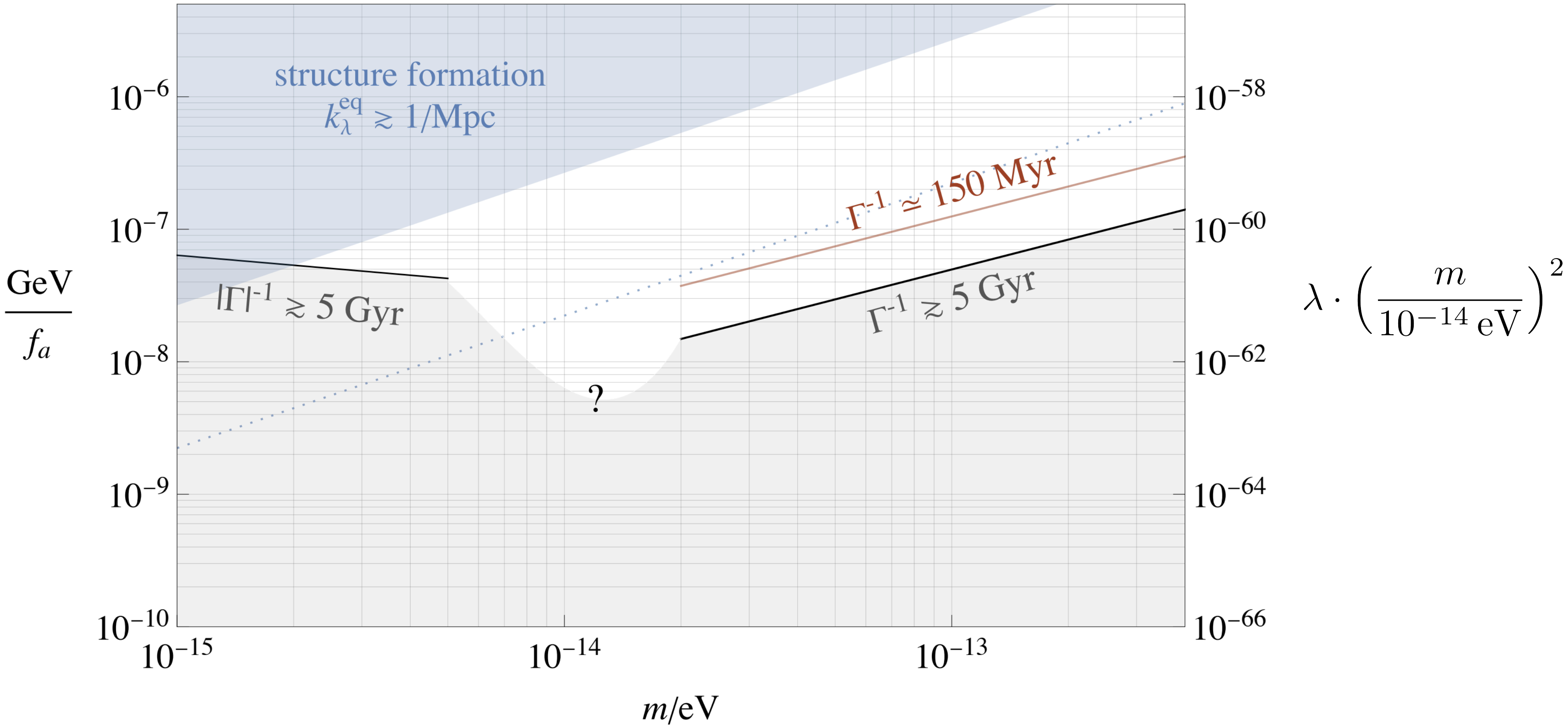


structure formation
$k_\lambda^{\mathrm{eq}} \gtrsim 1/\mathrm{Mpc}$
$\Gamma^{-1} \simeq 150$ Myr
$|\Gamma|^{-1} \gtrsim 5$ Gyr
$\Gamma^{-1} \gtrsim 5$ Gyr
?
$\frac{\mathrm{GeV}}{f_a}$
$\lambda \cdot \left(\frac{m}{10^{-14}\,\mathrm{eV}}\right)^2$
m/eV
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-15}
10^{-14}
10^{-13}
10^{-58}
10^{-60}
10^{-62}
10^{-64}
10^{-66}



$$\delta \equiv \rho/\bar{\rho} - 1$$

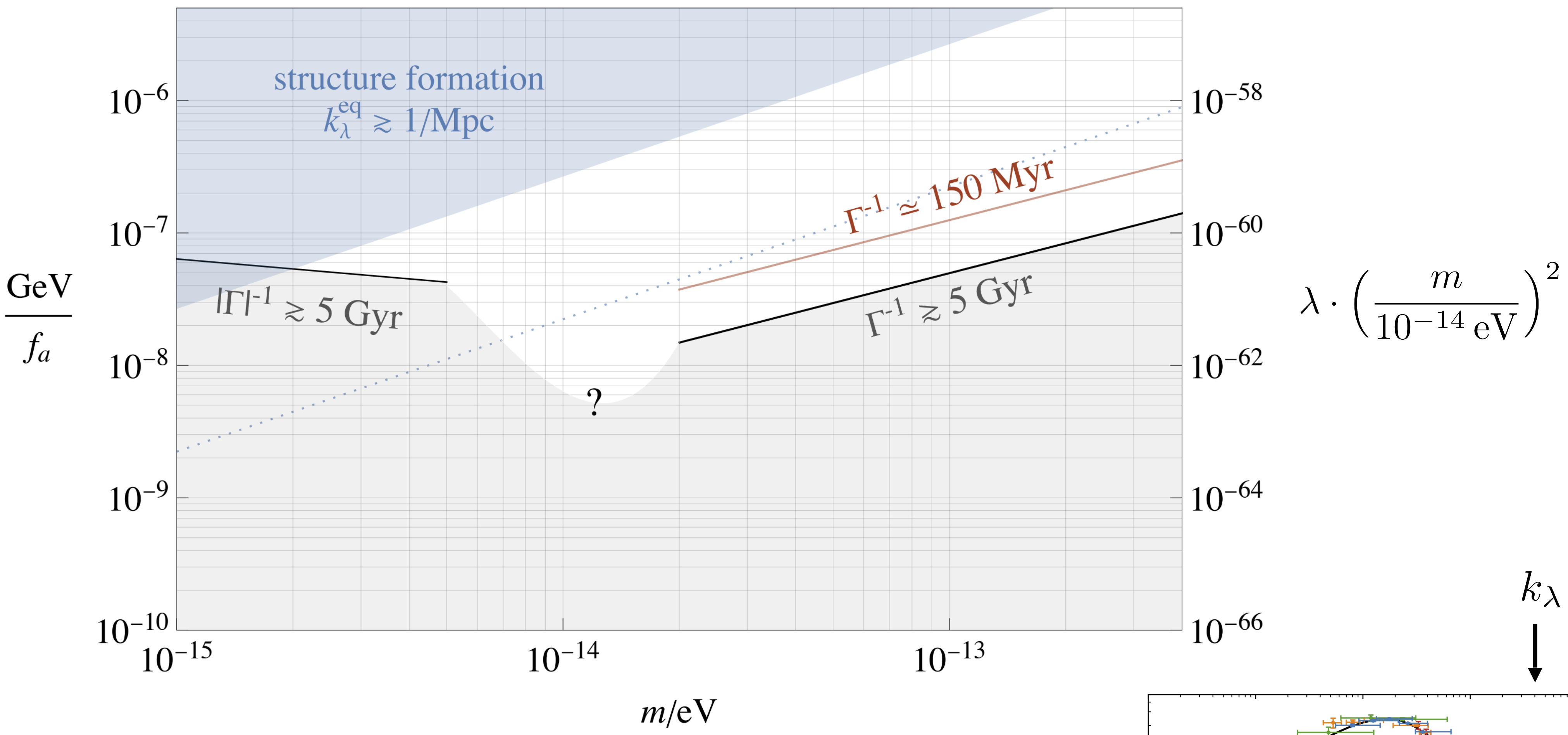
$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - \left[4\pi G\rho + \frac{k^2}{a^2}\frac{\rho}{8f_a^2 m_a^2}\right]\delta_{\mathbf{k}} = 0$$



$$\delta \equiv \rho/\bar{\rho} - 1$$

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - \underbrace{\left[4\pi G\rho + \frac{k^2}{a^2}\frac{\rho}{8f_a^2 m_a^2}\right]}_{4\pi G\rho\left[1+\left(\frac{k_{\rm today}}{k_\lambda}\frac{a_{\rm eq}}{a}\right)^2\right]}\delta_{\mathbf{k}} = 0$$

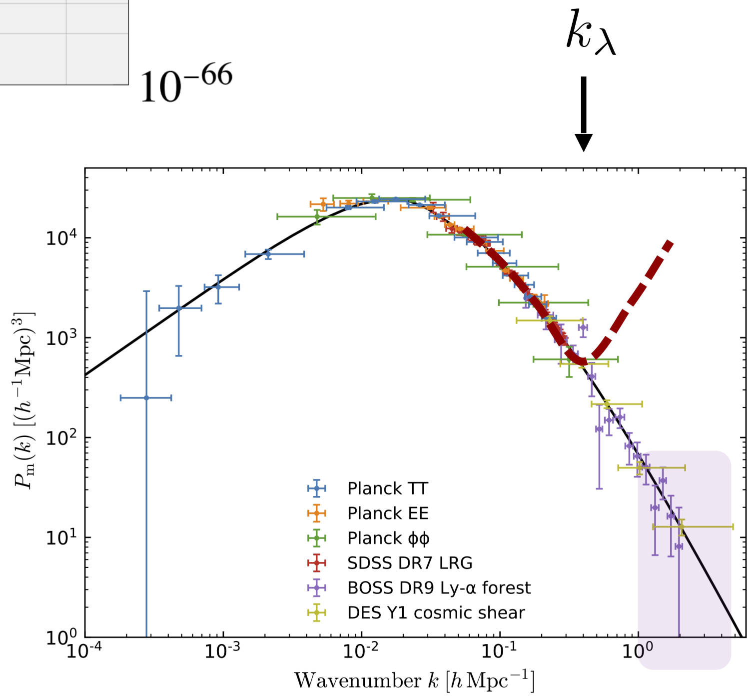
$$k_\lambda \simeq \frac{3.8}{\rm Mpc}\frac{f_a}{10^7\,{\rm GeV}}\frac{m}{10^{-14}{\rm eV}}$$



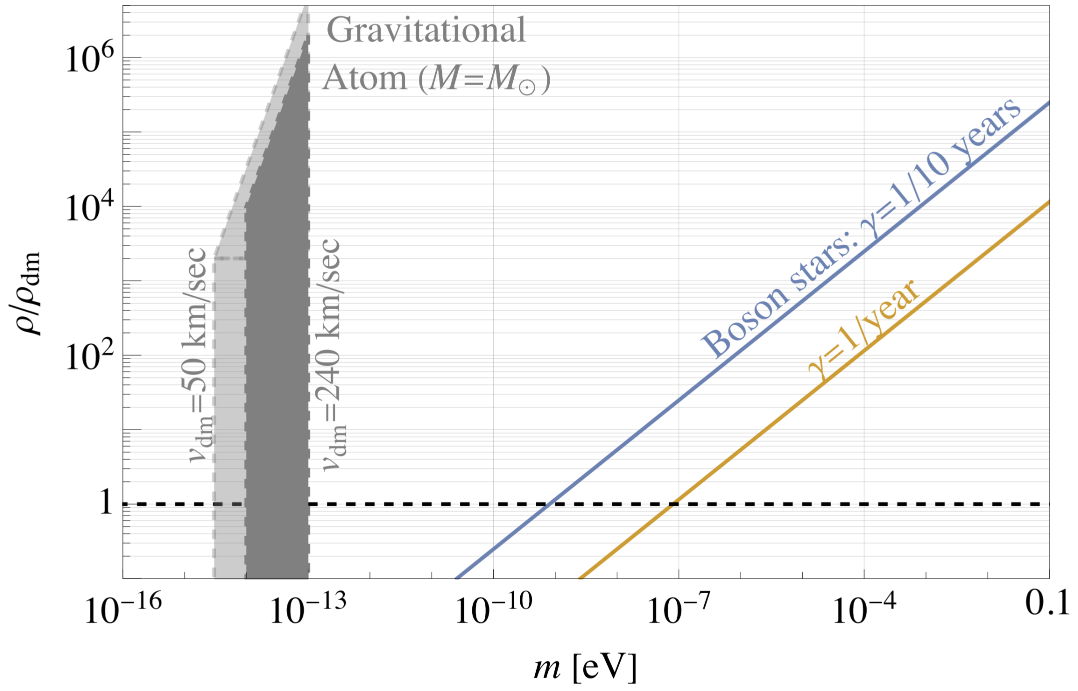
$$\delta \equiv \rho/\bar{\rho} - 1$$

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - \underbrace{\left[4\pi G\rho + \frac{k^2}{a^2}\frac{\rho}{8f_a^2 m_a^2}\right]}_{4\pi G\rho\left[1+\left(\frac{k_{\mathrm{today}}}{k_\lambda}\frac{a_{\mathrm{eq}}}{a}\right)^2\right]}\delta_{\mathbf{k}} = 0$$

$$k_\lambda \simeq \frac{3.8}{\mathrm{Mpc}}\frac{f_a}{10^7\,\mathrm{GeV}}\frac{m}{10^{-14}\mathrm{eV}} \lesssim 1\,\mathrm{Mpc}^{-1}$$



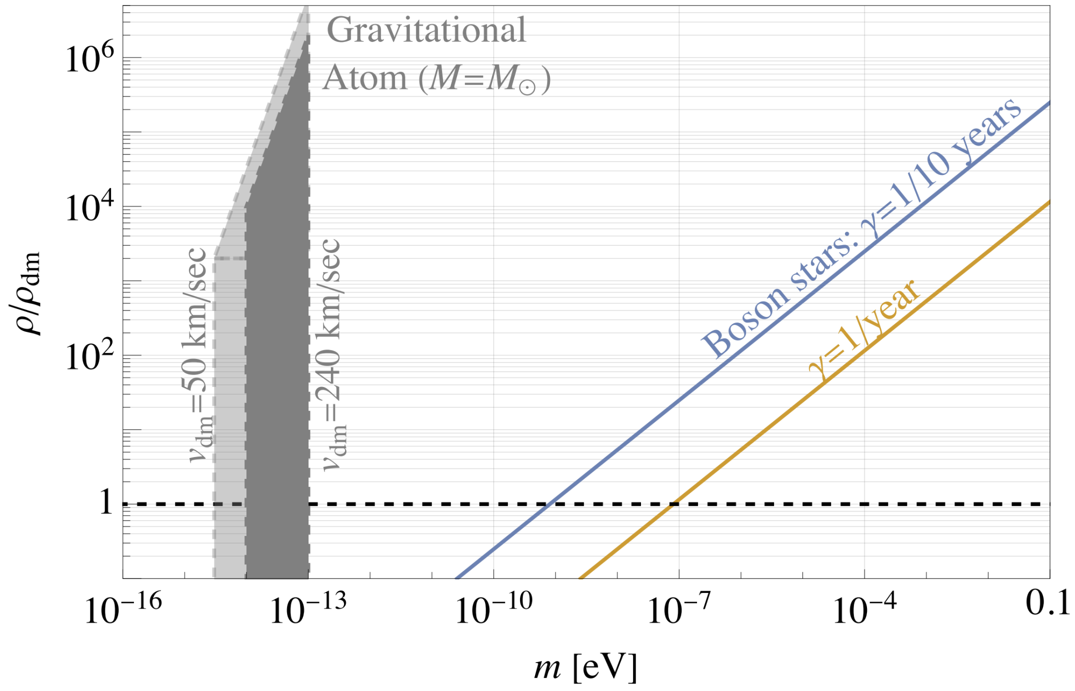
dark matter overdensity at the center (of the Sun)

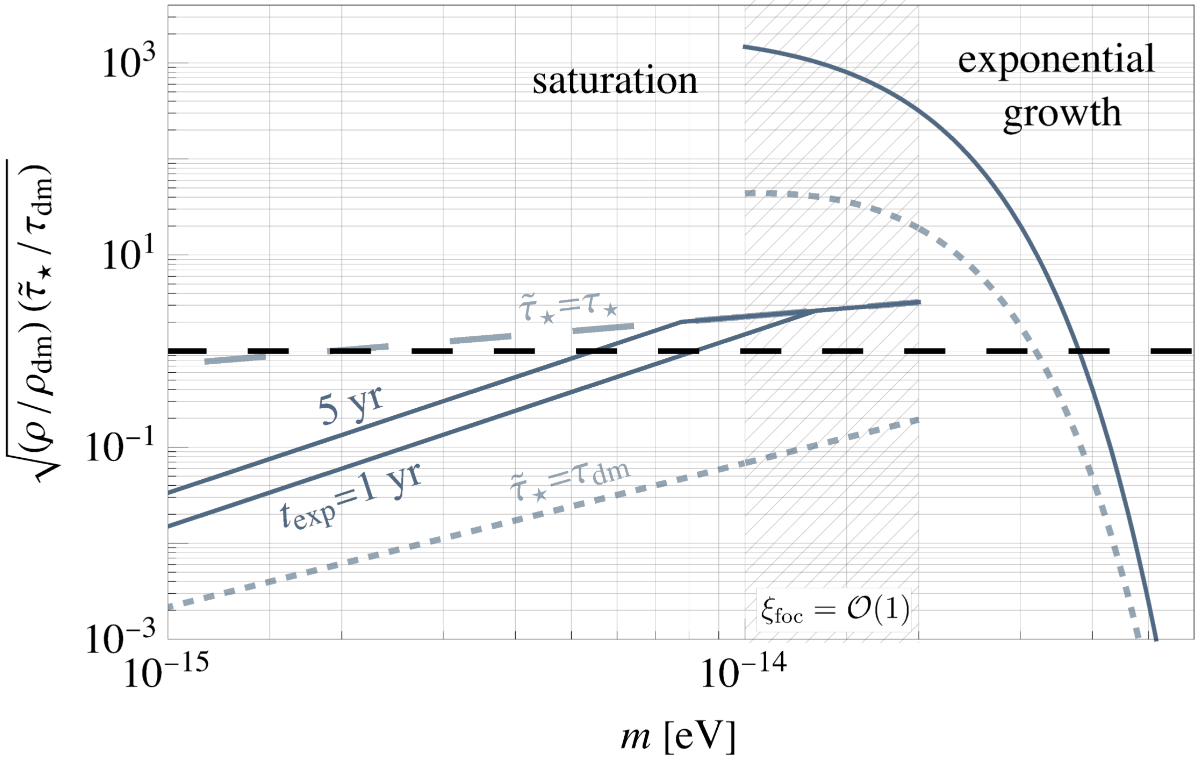


dark matter overdensity at the center (of the Sun)

effective coherence time

$$\tau_\star \gtrsim \frac{2\pi}{m\alpha^2} = \left[\frac{2\pi}{\xi_{\rm foc}}\right]^2 \tau_{\rm dm} \simeq 1\,{\rm year}\left[\frac{1.3\cdot 10^{-14}\,{\rm eV}}{m}\right]^3$$





Summary

- **Small ULDM self-interactions induce the capture of the galaxy halo DM**

  $\rightarrow$ happens efficiently when galactic DM is gravitationally focused, i.e. if $\frac{\lambda_{\rm dB}}{R_\star} = \frac{2\pi\alpha}{v_{\rm dm}} \gtrsim 1$

  $\rightarrow$ leads to exponential growth of gravitational atoms bound to the Sun for $m \gtrsim 10^{-14}\,\mathrm{eV}$

  $\rightarrow$ speed of the process depends on f_a (or λ) and ends with Bosenova explosions

Summary

- **Small ULDM self-interactions induce the capture of the galaxy halo DM**

  $\rightarrow$ happens efficiently when galactic DM is gravitationally focused, i.e. if $\frac{\lambda_{\rm dB}}{R_\star} = \frac{2\pi\alpha}{v_{\rm dm}} \gtrsim 1$

  $\rightarrow$ leads to exponential growth of gravitational atoms bound to the Sun for $m \gtrsim 10^{-14}\,\mathrm{eV}$

  $\rightarrow$ speed of the process depends on f_a (or λ) and ends with Bosenova explosions

Outlook

- direct detection on Earth, larger DM density and coherence time
- detection of Bosenova explosions
- apply to other systems, including more massive object and SMBH (smaller m)

Thanks!

Backup

Motivated example of light particles

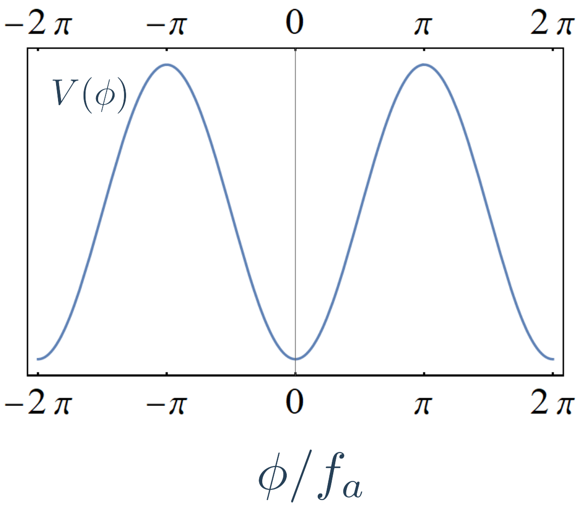
Axion or ALP, ϕ

- Approximate shift symmetry, $\phi \to \phi + c$

- Broken by $V(\phi)$, periodic of period $2\pi f_a$

  $\to$ mass $m \lll f_a$

  e.g. $V(\phi) = -m^2 f_a^2 \cos(\phi/f_a)$



Motivated example of light particles

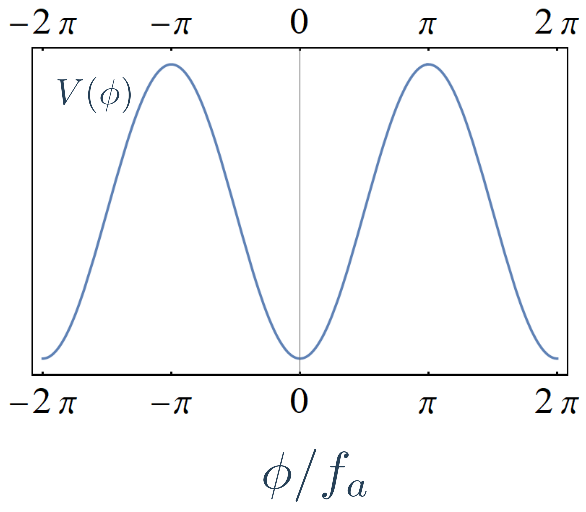
Axion or ALP, ϕ

- Approximate shift symmetry, $\phi \to \phi + c$

- Broken by $V(\phi)$, periodic of period $2\pi f_a$

  $\to$ mass $m \lll f_a$

  e.g. $V(\phi) = -m^2 f_a^2 \cos(\phi/f_a)$



$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

$$\lambda = -\frac{m^2}{f_a^2} < 0$$

$$|\lambda| \lll 1$$

attractive self-interaction

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t,\vec{x}_{\rm det}) = \phi_0 \cos(\overbrace{mt + \varphi(t)}^{\omega t})$$

Dark matter detection prospects depend on:

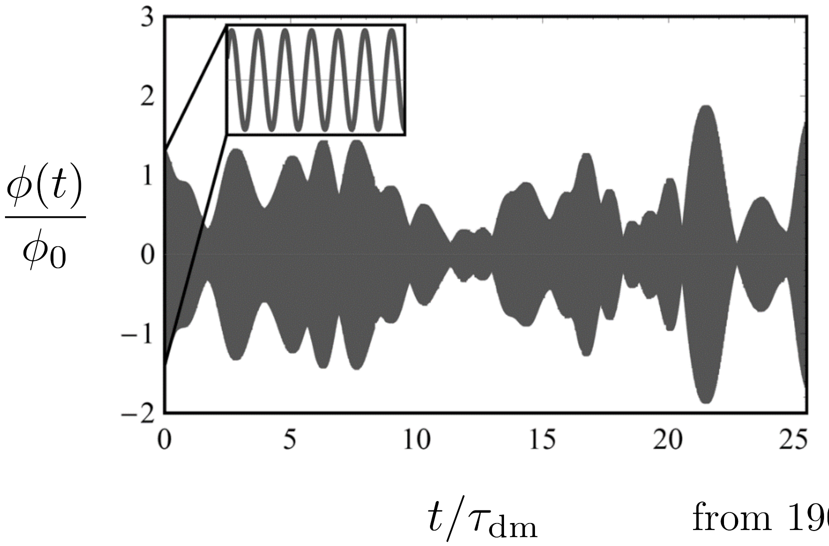
- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t,\vec{x}_{\rm det}) = \phi_0 \cos(\overbrace{mt + \varphi(t)}^{\omega t})$$

$$\propto \sqrt{\rho} \qquad\qquad m v_{\rm dm}^2 t/2 + \cdots$$

$\varphi(t)$ changes by order one at:

$$\tau_{\rm dm} \simeq \frac{2\pi}{m v_{\rm dm}^2} \simeq 1\,\text{year}\ \frac{10^{-16}\,\text{eV}}{m}\left[\frac{10^{-3}}{v_{\rm dm}}\right]^2$$

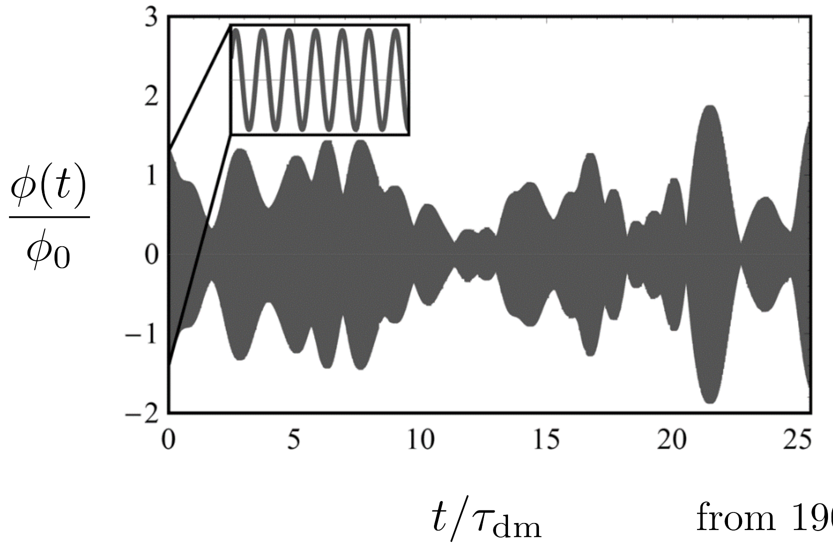


from 1905.13650

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t,\vec{x}_{\rm det}) = \underbrace{\phi_0}_{\propto\sqrt{\rho}} \cos(\overbrace{mt + \underbrace{\varphi(t)}_{mv_{\rm dm}^2 t/2+\cdots}}^{\omega t})$$



from 1905.13650

$\varphi(t)$ changes by order one at:

$$\tau_{\rm dm} \simeq \frac{2\pi}{mv_{\rm dm}^2} \simeq 1\,\text{year}\,\frac{10^{-16}\,\text{eV}}{m}\left[\frac{10^{-3}}{v_{\rm dm}}\right]^2$$

In this talk:

- ϕ is the dark matter
- $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$

$\neq 0 \qquad |\lambda| \lll 1$

axion $\rightarrow \quad \lambda = -\frac{m^2}{f_a^2}$

$\rightarrow$ misalignment fixes $f_a = f_a(m;\theta_0)$

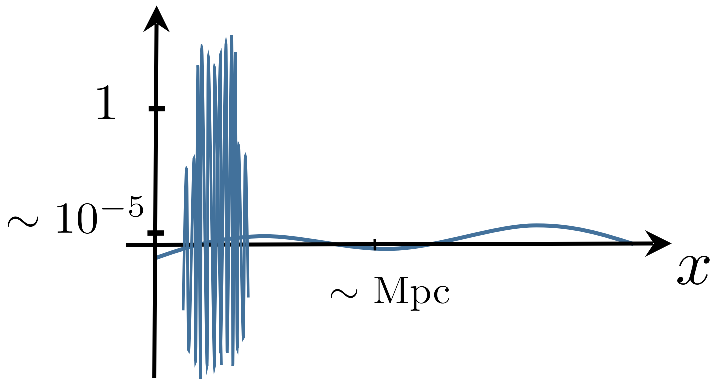
relation between m and f_a unfixed

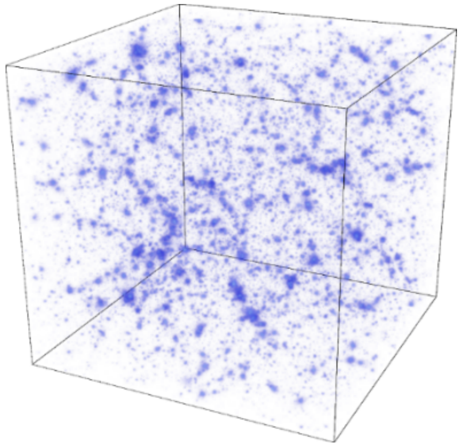
Overdensities at small scales, in addition to the galaxy halo

1) Boson stars/miniclusters $\rightarrow$ low encounter rate for $m \ll 1\,\mathrm{eV}$

$$\simeq \frac{1}{10\,\mathrm{yr}} \left[\frac{m}{1\mathrm{eV}}\right]^{\frac{1}{2}}$$

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$



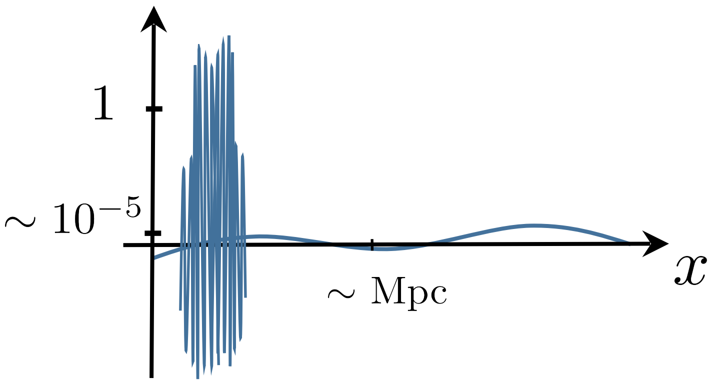


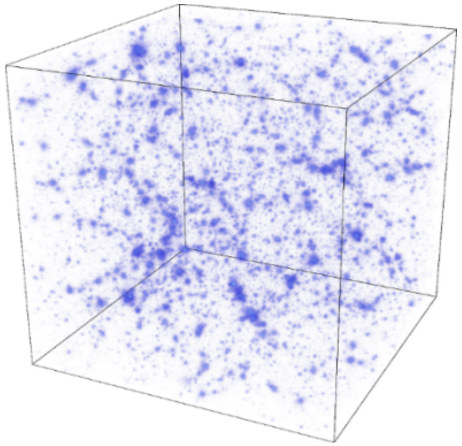
Overdensities at small scales, in addition to the galaxy halo

1) Boson stars/miniclusters $\rightarrow$ low encounter rate for $m \ll 1\,\mathrm{eV}$

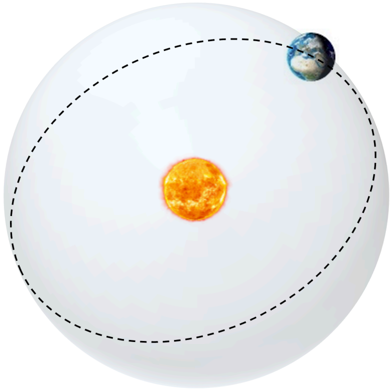
$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

$$\simeq \frac{1}{10\,\mathrm{yr}} \left[\frac{m}{1\mathrm{eV}}\right]^{\frac{1}{2}}$$





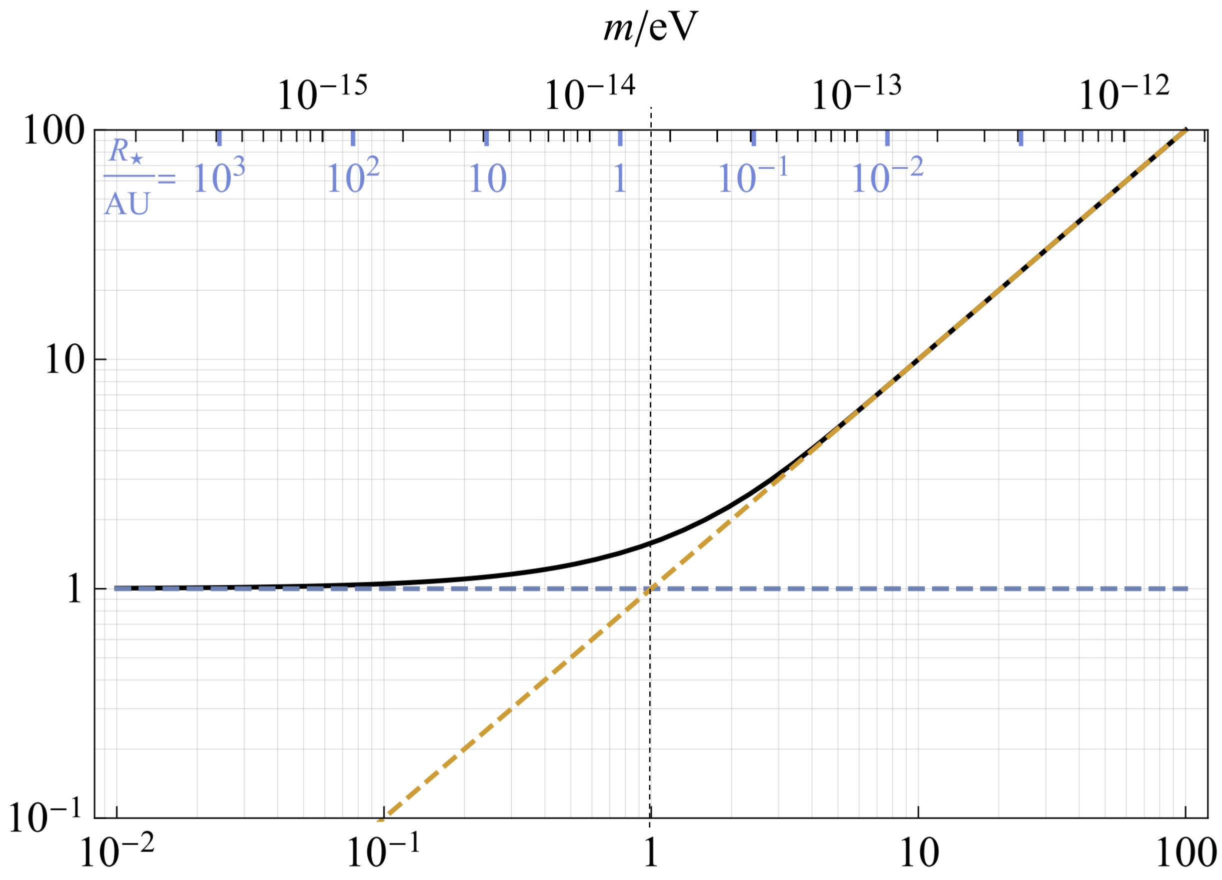
2) Dark matter bound to the Sun in a ‘halo’ with large overdensity



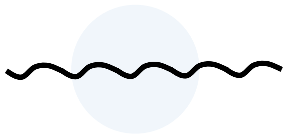
[Budker et al, ’19]

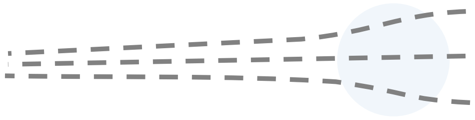
overdensity at the center

$|\psi_{\mathbf{k}}(0)|^2$



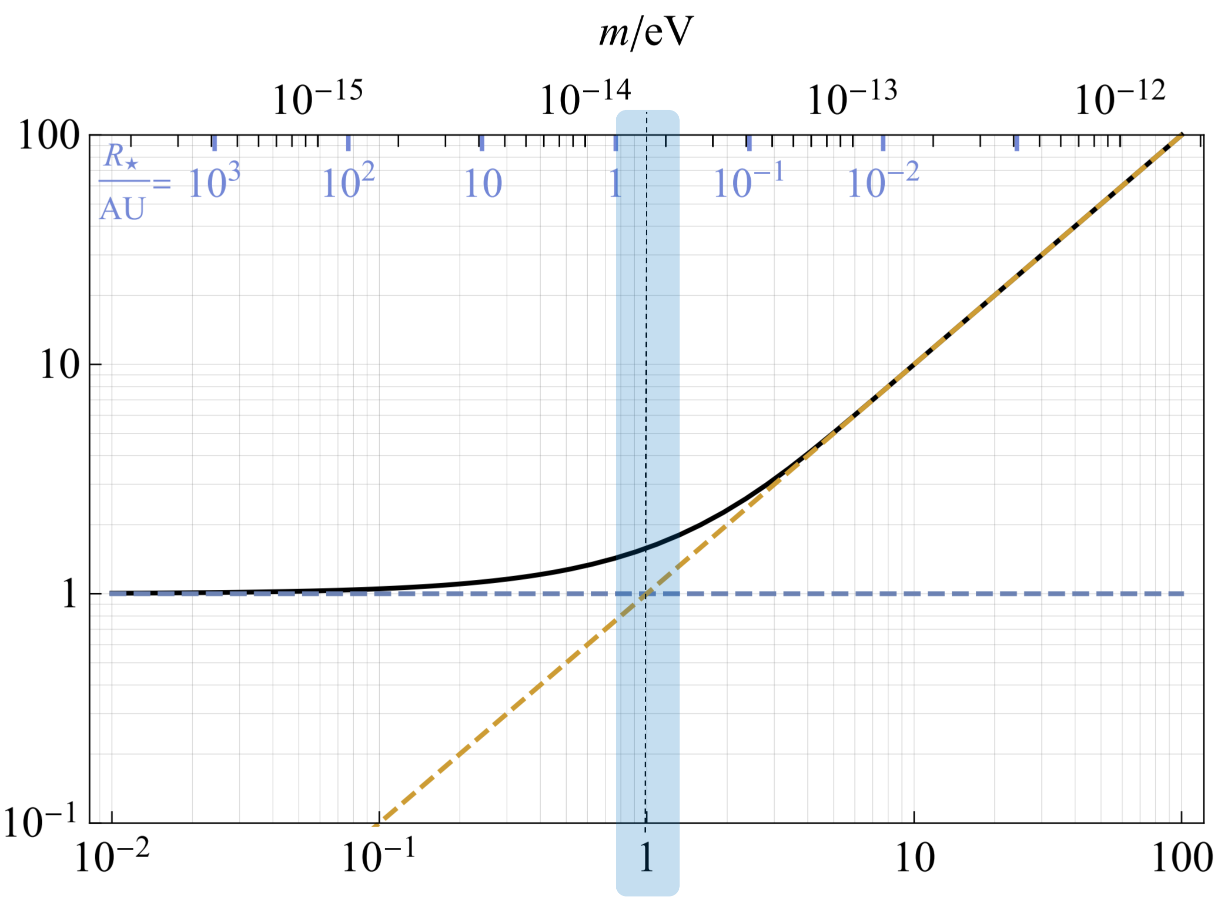
$$\xi_{\mathrm{foc}}(k) = \frac{2\pi}{kR_\star} = \frac{2\pi\alpha}{v}$$



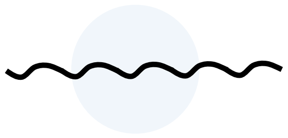


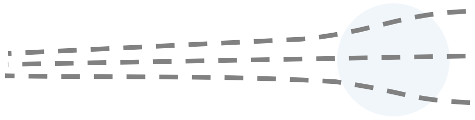
overdensity at the center

$|\psi_{\mathbf{k}}(0)|^2$



$$\xi_{\rm foc}(k) = \frac{2\pi}{kR_\star} = \frac{2\pi\alpha}{v}$$



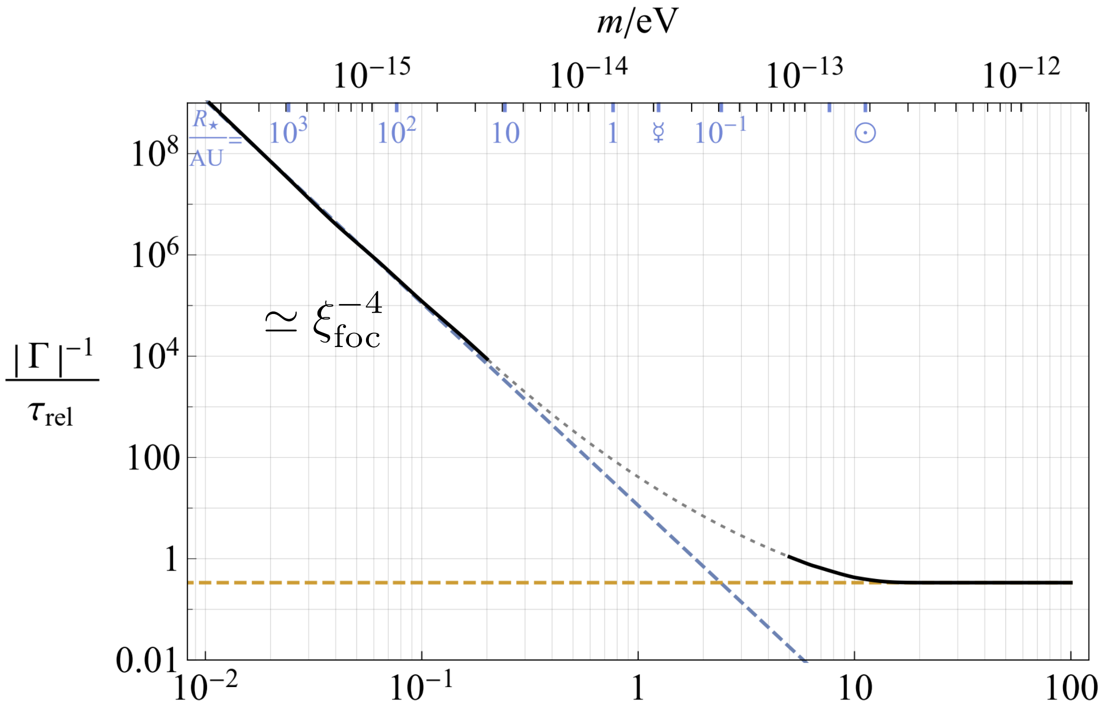


$\Gamma^{-1} \leftrightarrow$

relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr}\left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$\Gamma^{-1} \leftrightarrow$

relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

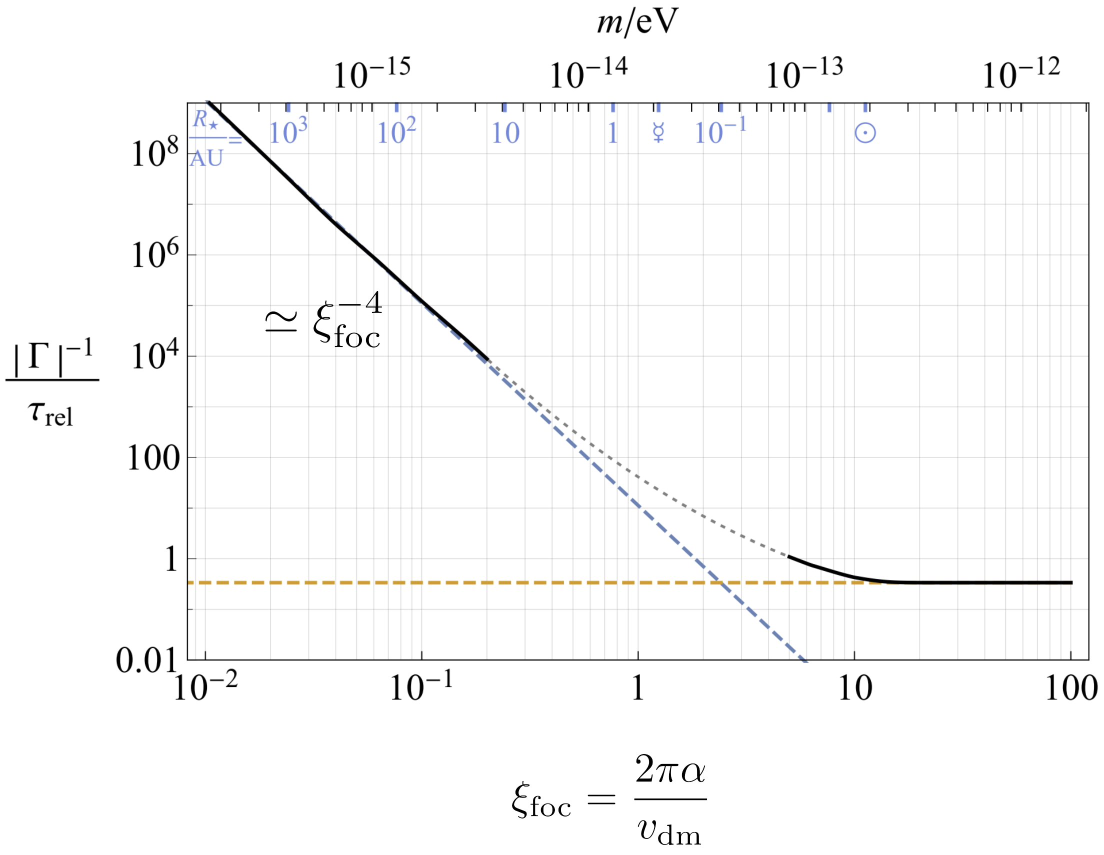


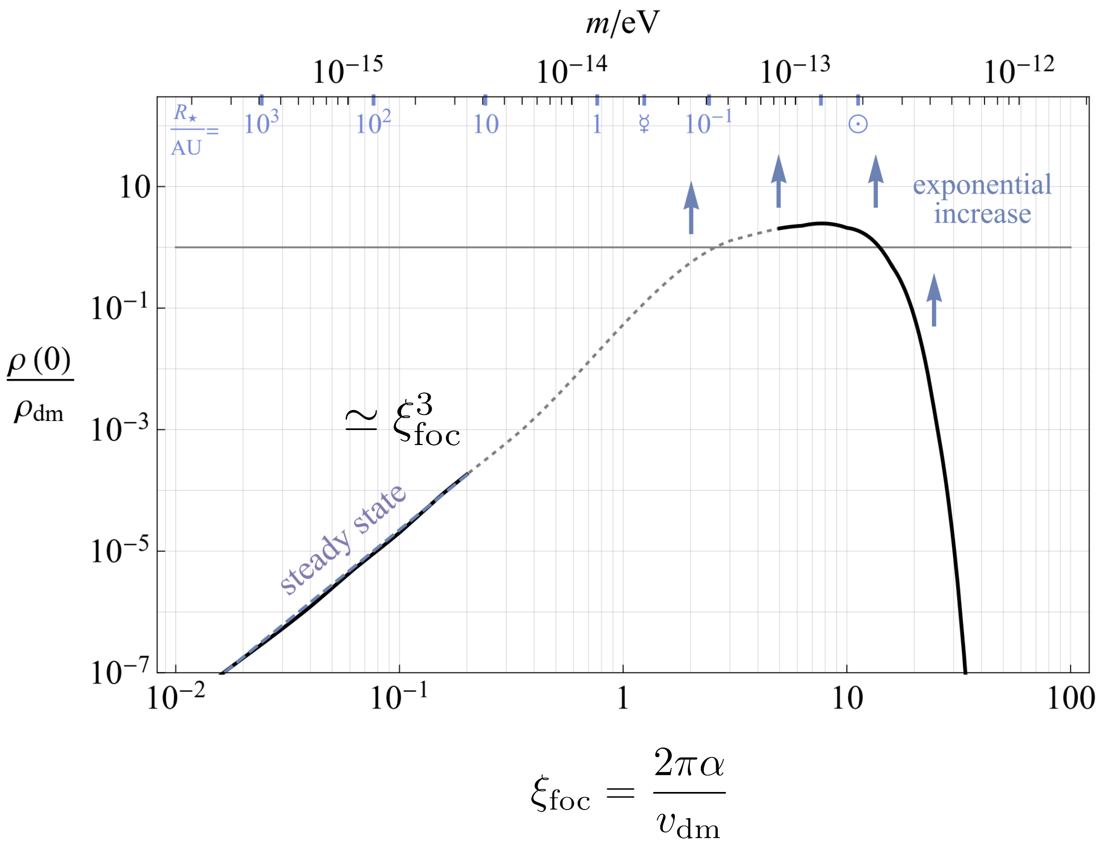
$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

- the size of λ (or f_a) only enters into Γ

$\Gamma^{-1} \leftrightarrow$

relaxation time $\quad \tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$





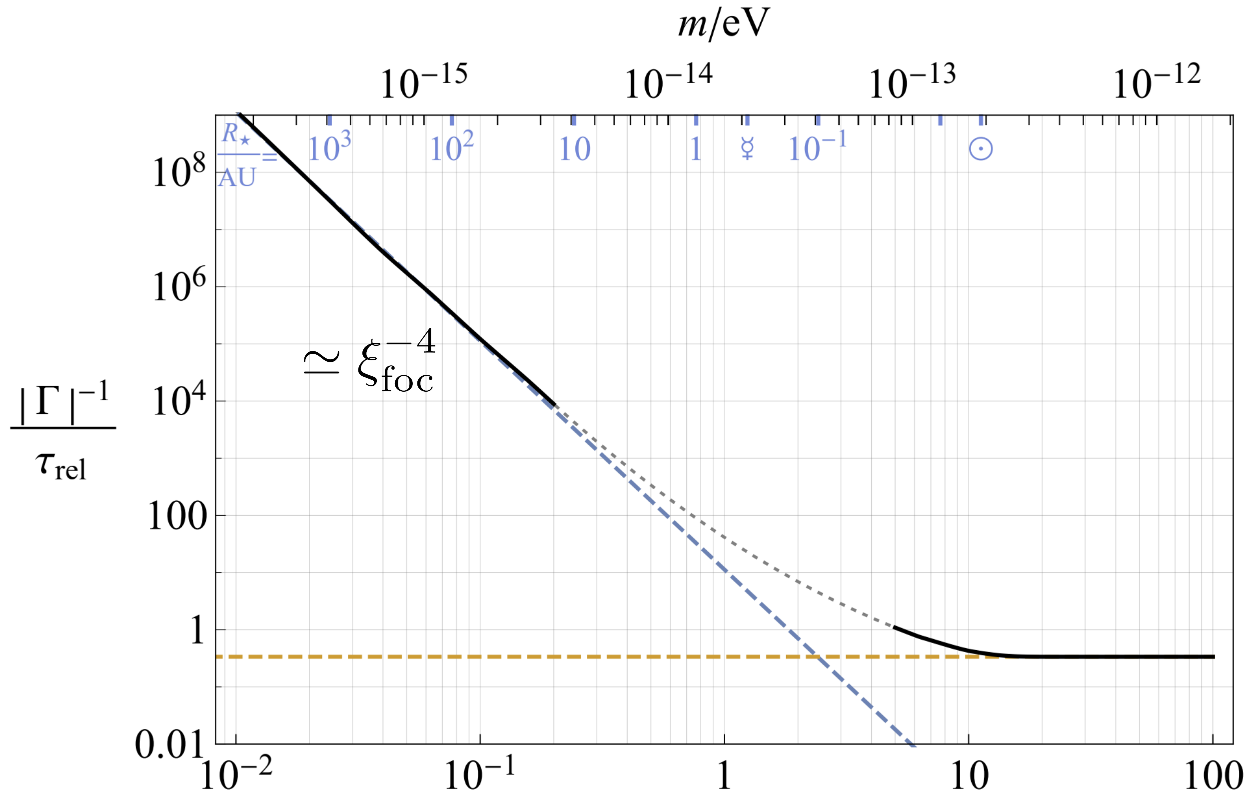
- the size of λ (or f_a) only enters into Γ

$\Gamma^{-1} \leftrightarrow$

relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$\Gamma^{-1} \leftrightarrow$

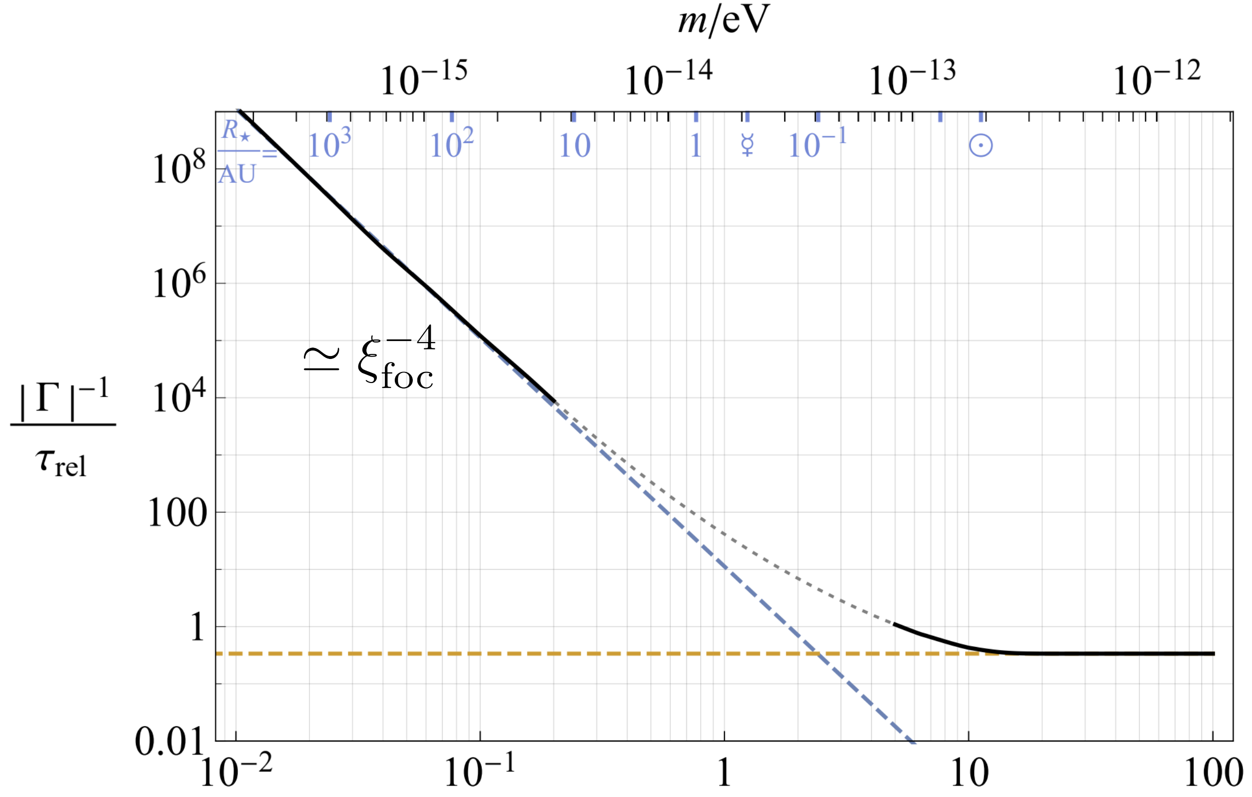
relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

$\Gamma^{-1} \leftrightarrow$

relaxation time $\qquad \tau_{\rm rel} \equiv \dfrac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr}\left[\dfrac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\dfrac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\dfrac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\dfrac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}}$$

Large occupation numbers

Quantum mechanics with potential $V(x)$

- $\hat{x}, \hat{p}$ position and momentum operators
- $|\psi\rangle$ state
- average values defined as e.g. $\langle p\rangle \equiv \langle\psi|\hat{p}|\psi\rangle$

Large occupation numbers

Quantum mechanics with potential $V(x)$

- $\hat{x}, \hat{p}$ position and momentum operators
- $|\psi\rangle$ state
- average values defined as e.g. $\langle p \rangle \equiv \langle \psi | \hat{p} | \psi \rangle$

'average values follow the classical equations of motion (Newton eq.)'

$$\begin{cases} m\dot{x} = p \\ \dot{p} = -V'(x) \end{cases}$$

Ehrenfest Th: $$m\frac{d}{dt}\langle x \rangle = \langle p \rangle \qquad \frac{d}{dt}\langle p \rangle = -\langle V'(x) \rangle$$

- $A(x, p)$ observable $$\frac{d}{dt}\langle A \rangle = \frac{1}{i\hbar}\langle [A, H] \rangle$$

Large occupation numbers

Quantum mechanics with potential $V(x)$

- $\hat{x}, \hat{p}$ position and momentum operators
- $|\psi\rangle$ state
- average values defined as e.g. $\langle p\rangle \equiv \langle\psi|\hat{p}|\psi\rangle$

'average values follow the classical equations of motion (Newton eq.)'

$$\begin{cases} m\dot{x} = p \\ \dot{p} = -V'(x) \end{cases}$$

Ehrenfest Th: $m\dfrac{d}{dt}\langle x\rangle = \langle p\rangle \qquad \dfrac{d}{dt}\langle p\rangle = -\langle V'(x)\rangle$

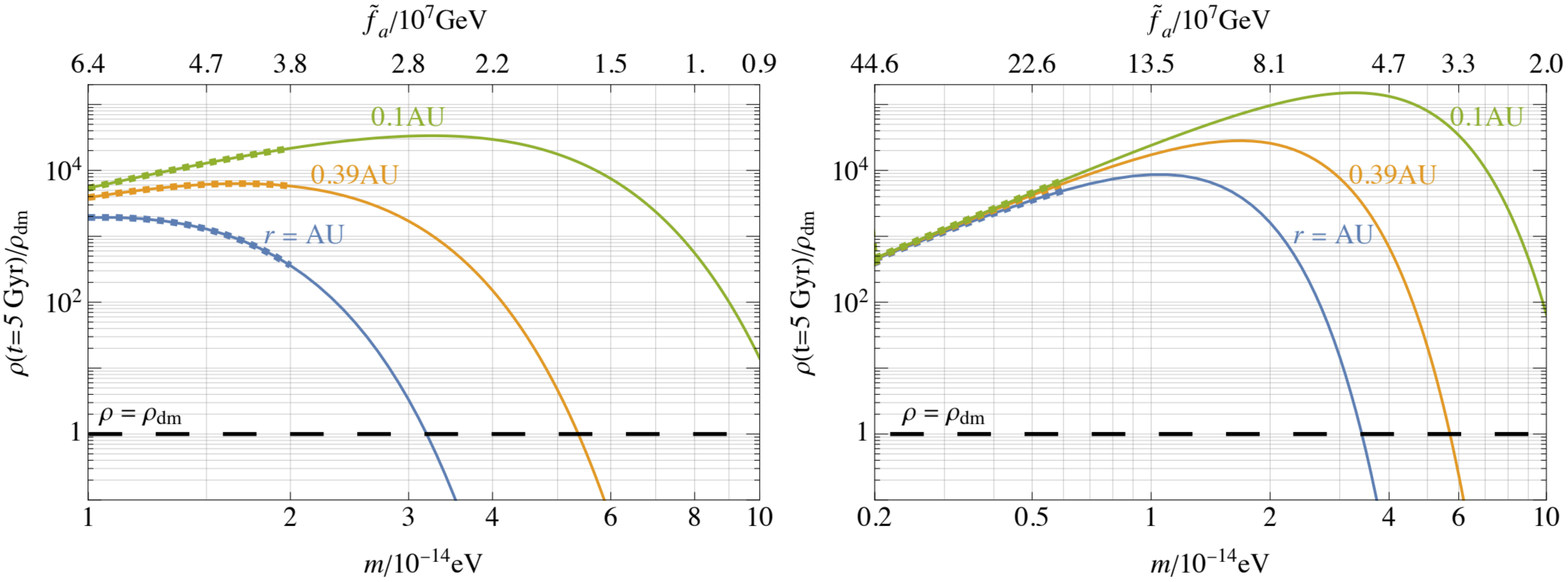
- $A(x,p)$ observable $\qquad \dfrac{d}{dt}\langle A\rangle = \dfrac{1}{i\hbar}\langle[A,H]\rangle$

- Occupation number operator $\hat{N} \longrightarrow$ Occupation number in $|\psi\rangle$: $N \equiv \langle\hat{N}\rangle$

- Variance from the mean: $\qquad \langle A^2\rangle - \langle A\rangle^2 \propto \dfrac{1}{N^{\#>0}}$

'classical equations of motion are a good approximation of the full quantum evolution'

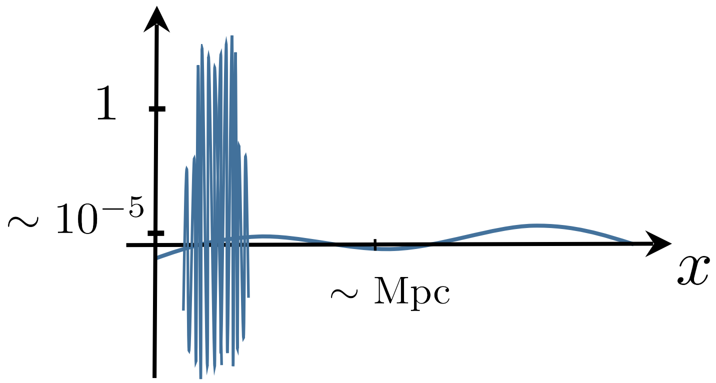
Solar halo dark matter overdensity



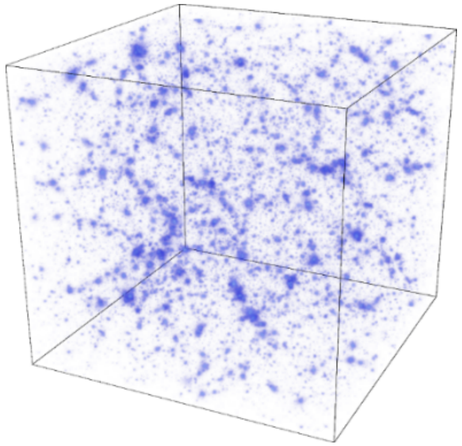
Overdensities at small scales, in addition to the galaxy halo

1) Boson stars/miniclusters $\rightarrow$ low encounter rate for $m \ll 1\,\mathrm{eV}$

$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$



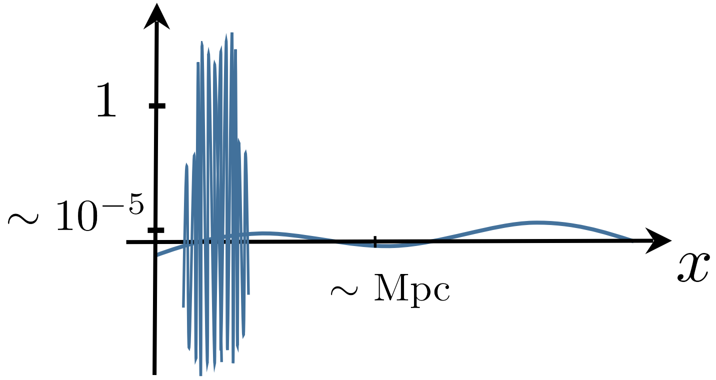
$$\simeq \frac{1}{10\,\mathrm{yr}} \left[\frac{m}{1\mathrm{eV}}\right]^{\frac{1}{2}}$$



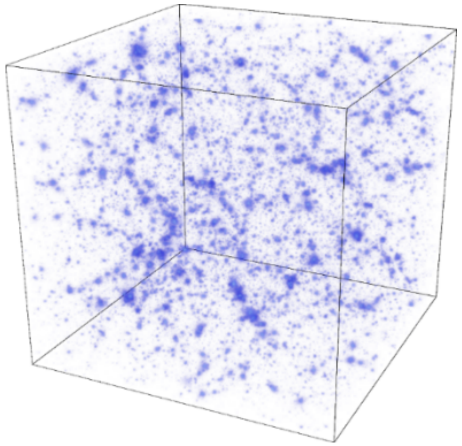
Overdensities at small scales, in addition to the galaxy halo

1) Boson stars/miniclusters $\rightarrow$ low encounter rate for $m \ll 1\,\mathrm{eV}$

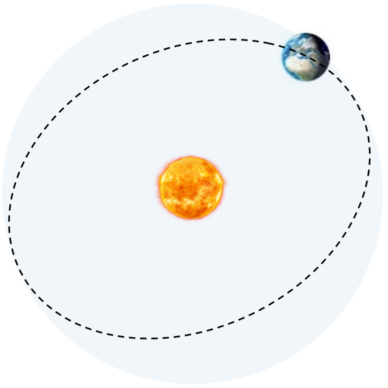
$$\delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$



$$\simeq \frac{1}{10\,\mathrm{yr}} \left[\frac{m}{1\mathrm{eV}}\right]^{\frac{1}{2}}$$



2) Dark matter bound to the Sun in a 'halo' with large overdensity



[Budker et al, '19]

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t, \vec{x}_{\rm det}) = \phi_0 \cos(mt + \varphi(t))$$

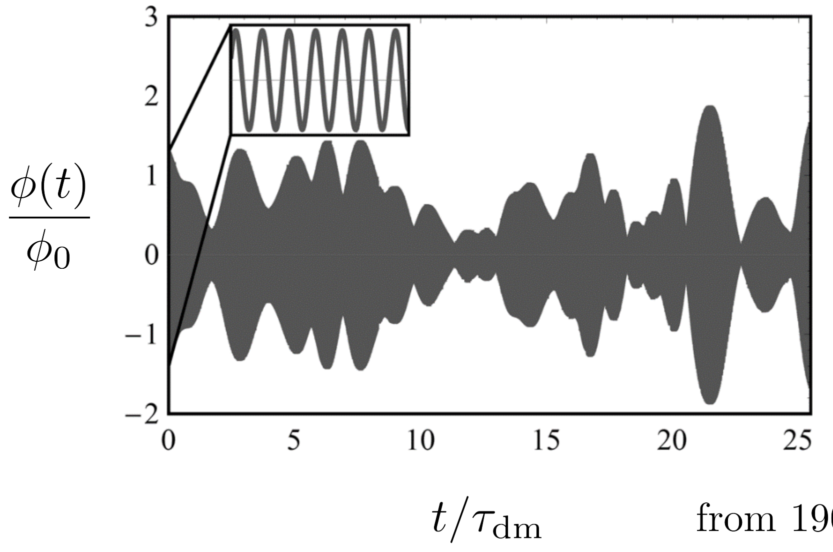
Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t, \vec{x}_{\rm det}) = \underbrace{\phi_0}_{\propto \sqrt{\rho}} \cos(mt + \underbrace{\varphi(t)}_{m v_{\rm dm}^2 t/2 + \cdots})$$

phase changes of order one at:

$$\tau_{\rm dm} \simeq \frac{2\pi}{m v_{\rm dm}^2} \simeq 1\,\text{year}\, \frac{10^{-16}\,\text{eV}}{m} \left[\frac{10^{-3}}{v_{\rm dm}}\right]^2$$



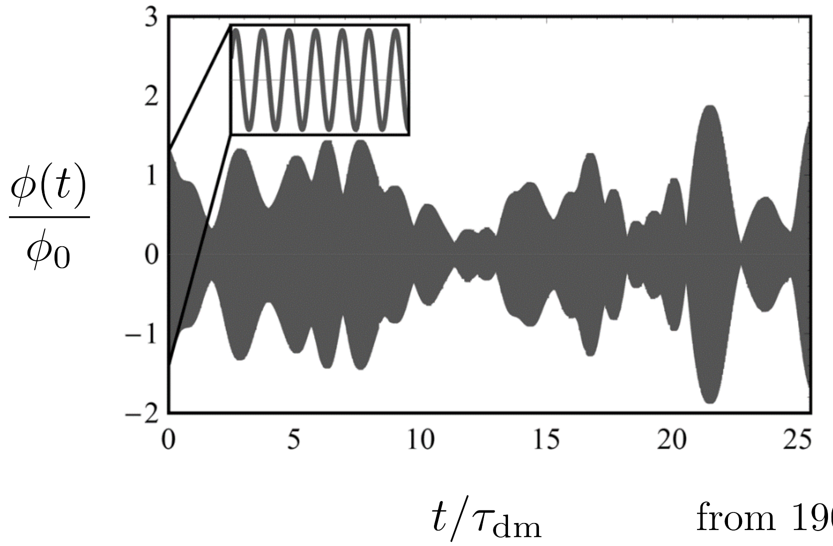
from 1905.13650

Dark matter detection prospects depend on:

- Local density ρ in the neighborhood of the Sun
- Coherence time, $\tau_{\rm dm}$

$$\phi(t,\vec{x}_{\rm det}) = \phi_0 \cos(mt + \varphi(t))$$

$\phi_0 \propto \sqrt{\rho}$ $\qquad$ $\varphi(t) = m v_{\rm dm}^2 t/2 + \cdots$



from 1905.13650

phase changes of order one at:

$$\tau_{\rm dm} \simeq \frac{2\pi}{m v_{\rm dm}^2} \simeq 1\,\text{year}\,\frac{10^{-16}\,\text{eV}}{m}\left[\frac{10^{-3}}{v_{\rm dm}}\right]^2$$

In this talk:

- ϕ is the dark matter
- $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$

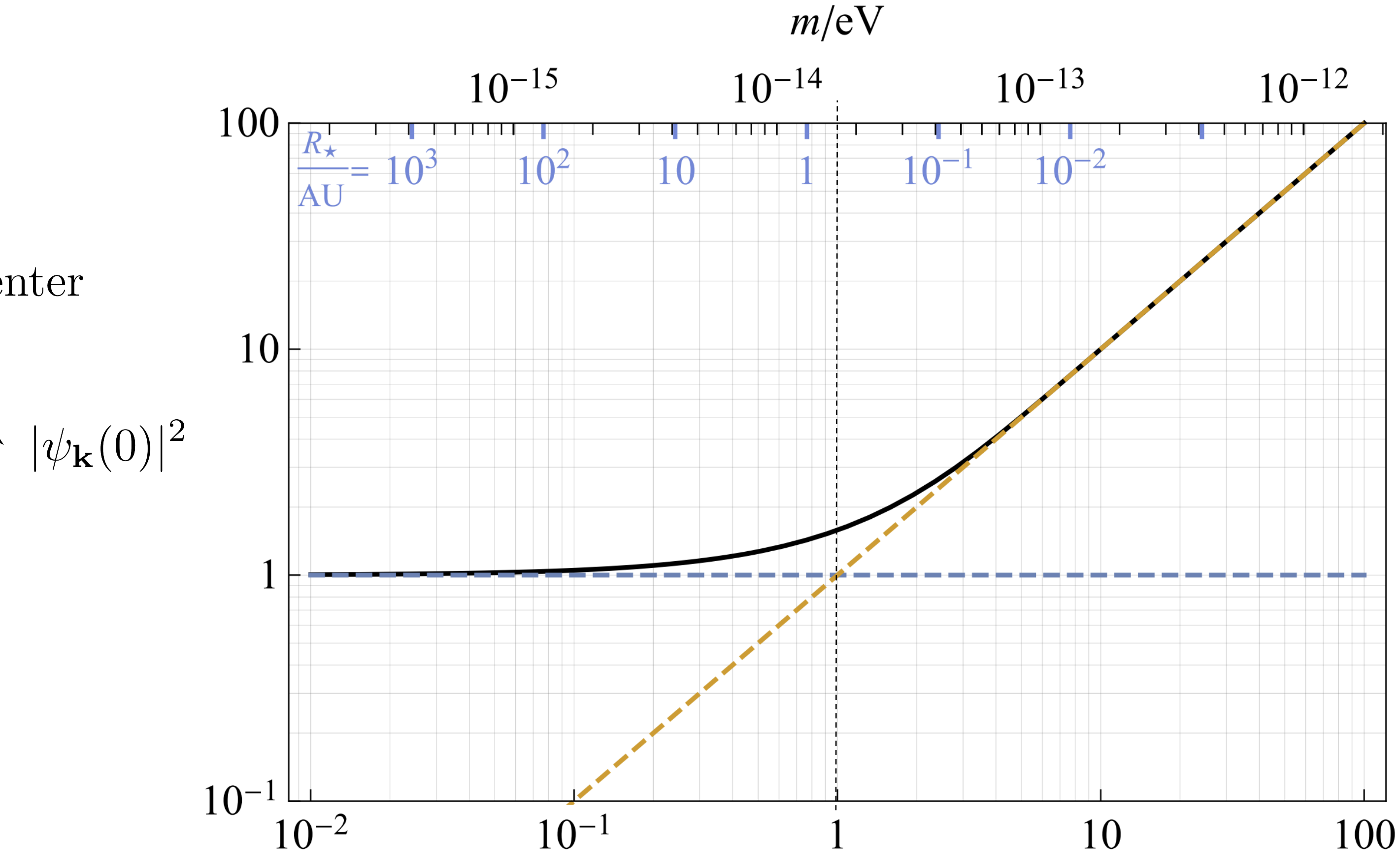
$\lambda \neq 0$ $\qquad$ $|\lambda| \lll 1$

axion $\rightarrow$ $\quad \lambda = -\frac{m^2}{f_a^2}$

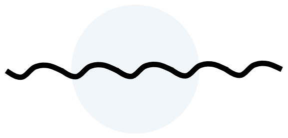
$\rightarrow$ misalignment fixes $f_a = f_a(m;\theta_0)$

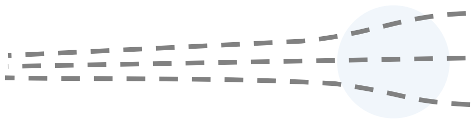
relation between m and f_a unfixed

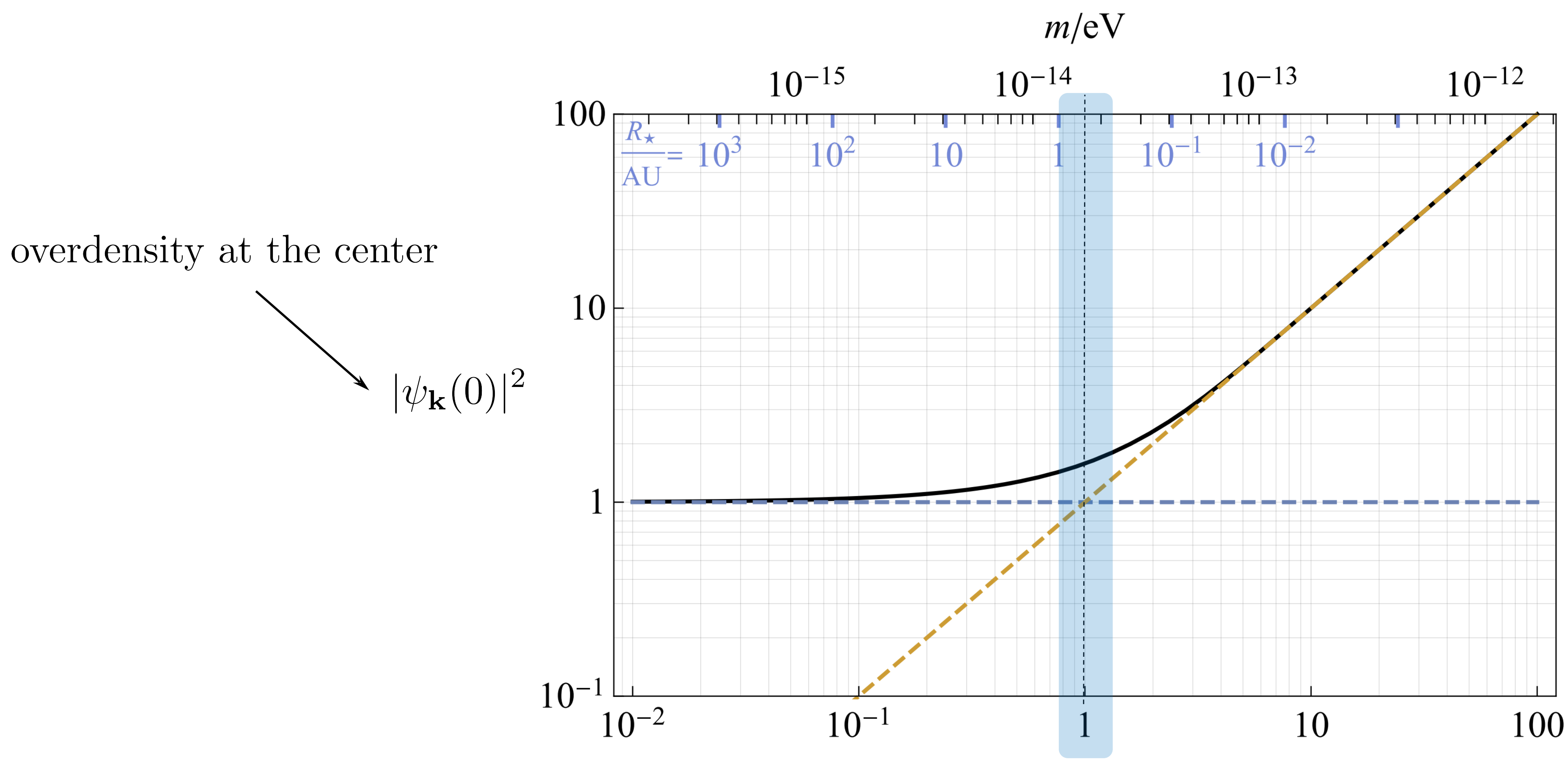
overdensity at the center



$$\xi_{\mathrm{foc}}(k) = \frac{2\pi}{kR_\star} = \frac{2\pi\alpha}{v}$$







$$\xi_{\rm foc}(k) = \frac{2\pi}{kR_\star} = \frac{2\pi\alpha}{v}$$

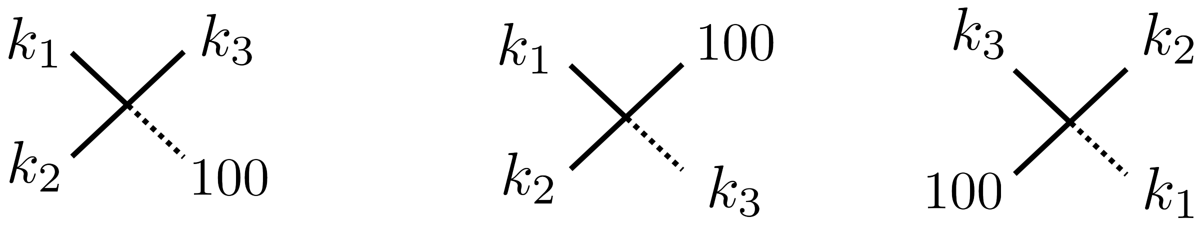
Scattering states

$$\psi_{\mathbf{k}} = e^{i\mathbf{k}\cdot\mathbf{x}}\,\Gamma\left[1-\frac{i}{kR_\star}\right]e^{\frac{\pi}{2kR_\star}}\,{}_1F_1\left[\frac{i}{kR_\star},1,i(kr-\mathbf{k}\cdot\mathbf{x})\right]$$

$$\begin{cases} \psi_{\mathbf{k}} \to e^{i\mathbf{k}\cdot\mathbf{x}}\,, & \xi_{\rm foc}(k) \ll 1\,. \\ \psi_{\mathbf{k}} \to e^{i\varphi(k)}\sqrt{\frac{2\pi}{kR_\star}}J_0\left[2\sqrt{\frac{r}{R_\star}(1-\hat{k}\cdot\hat{x})}\,\right]\,, & \xi_{\rm foc}(k) \gg 1 \end{cases}$$

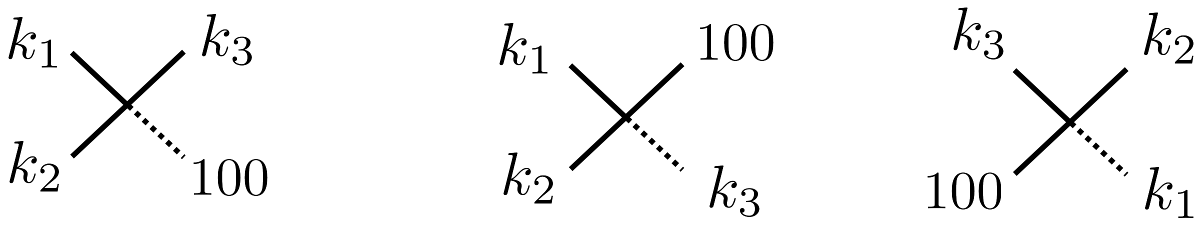
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\ \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\,\delta(\Delta\omega)\,|\mathcal{M}|^2 \Big\{\ \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ ,\ \ \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ \Big\}$$



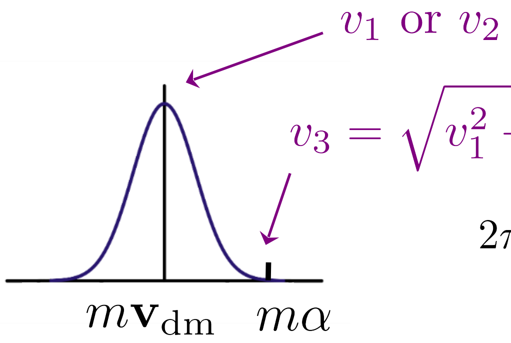
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{\ \underbrace{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}^{\text{capture}}\,,\ \ \underbrace{\rho(\mathbf{k_1})\rho(\mathbf{k_2})}^{\text{stimulated capture}} - \underbrace{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}^{\text{stripping}}\ \}$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\mathrm{eV}$$

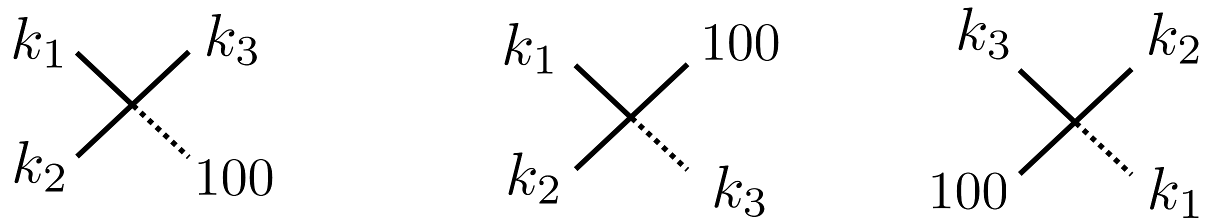


$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

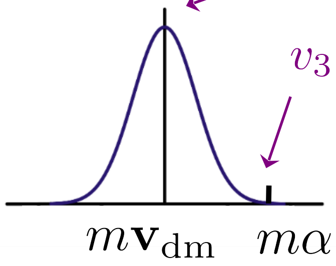
$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}] \delta(\Delta\omega) |\mathcal{M}|^2 \Big\{ \underbrace{m\, \rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}^{\text{capture}} \,, \quad \overbrace{\rho(\mathbf{k_1})\rho(\mathbf{k_2})}^{\text{stimulated capture}} - \overbrace{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}^{\text{stripping}} \Big\}$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\text{eV}$$

v_1 or v_2



$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

$$\Gamma_1 \simeq \text{const}$$

$$\Gamma_2 \simeq e^{-\xi_{\rm foc}}$$

$$\to \Gamma > 0$$

Quantum scattering and Bose Enhancement

$$S = -\int dt\, d^3x \left[\frac{i}{2}(\dot{\psi}^* \psi - \psi^* \dot{\psi}) + \frac{1}{2m}|\nabla\psi|^2 + m\Phi_{\rm ex}|\psi|^2 + \frac{1}{2} g |\psi|^4 \right]$$

$$k_1 + k_2 \to k_3 + nlm$$

$$\mathcal{A} = \langle k_3\, nlm |\, T[\hat{H}_{int}]\, | k_1 k_2 \rangle = 2g(2\pi)\delta(\Delta\omega)\mathcal{M}$$

$$\text{probability} = (2\pi)\delta(\Delta\omega) 4g^2 |\mathcal{M}|^2.$$

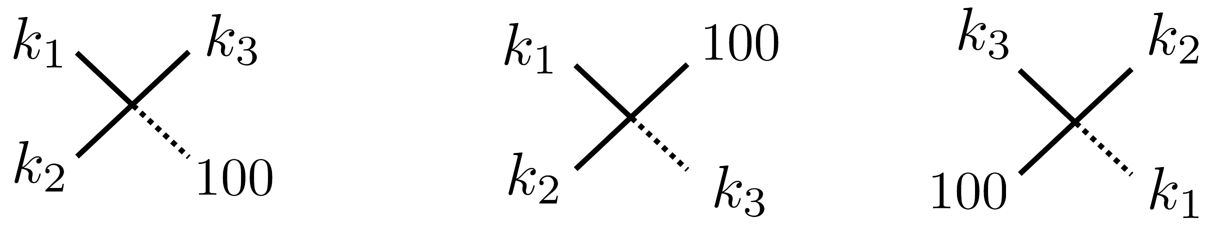
$$\dot{N} = \frac{4g^2}{2} \int [dk_1][dk_2][dk_3] \left[(\rho(\mathbf{k_3}) + 1)(N_0 + 1)\rho(\mathbf{k_1})\rho(\mathbf{k_2}) - \rho(\mathbf{k_3}) N_0 (\rho(\mathbf{k_1}) + 1)(\rho(\mathbf{k_2}) + 1) \right] (2\pi)\delta(\Delta\omega) |\mathcal{M}|^2$$

$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\ \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{ \quad m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})\,, \quad \rho(\mathbf{k_1})\rho(\mathbf{k_2}) - 2\rho(\mathbf{k_2})\rho(\mathbf{k_3}) \quad \}$$

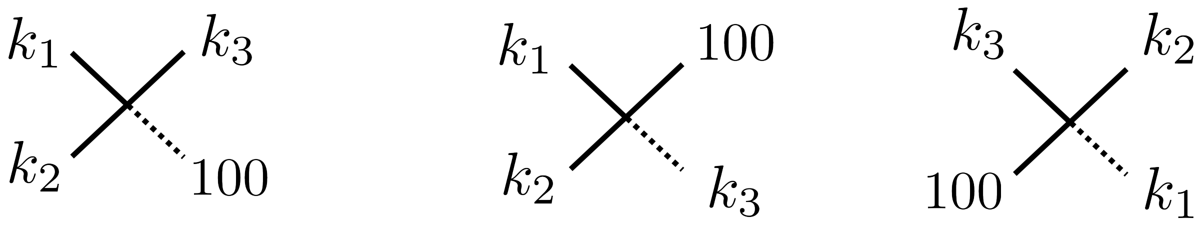
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\ \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \Big\{\ \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\,,\ \ \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ \Big\}$$



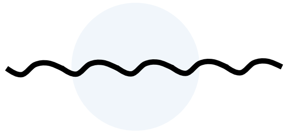
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{ \;\; \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\,, \;\; \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})} \;\; \}$$



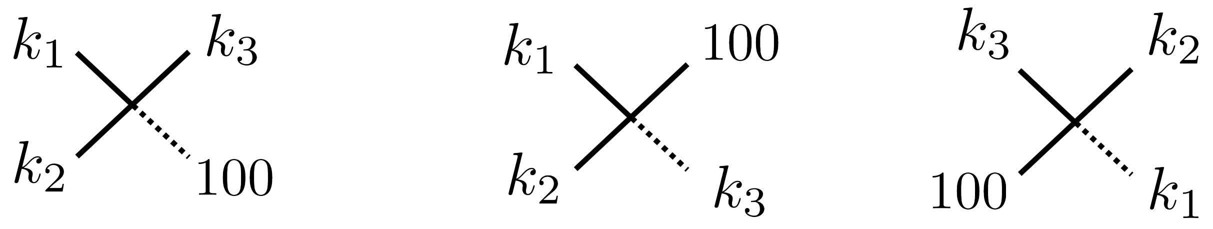
$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \ll 1$$

$$m \lesssim 10^{-14}\,\text{eV}$$



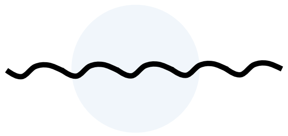
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

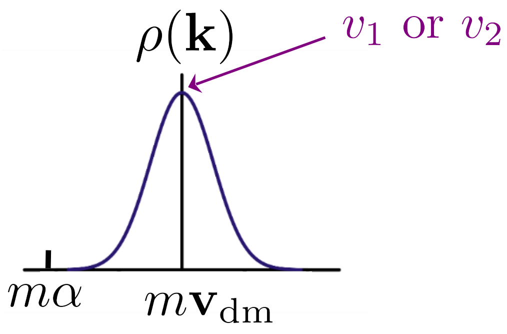
$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \Big\{\ \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\,,\quad \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ \Big\}$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \ll 1$$

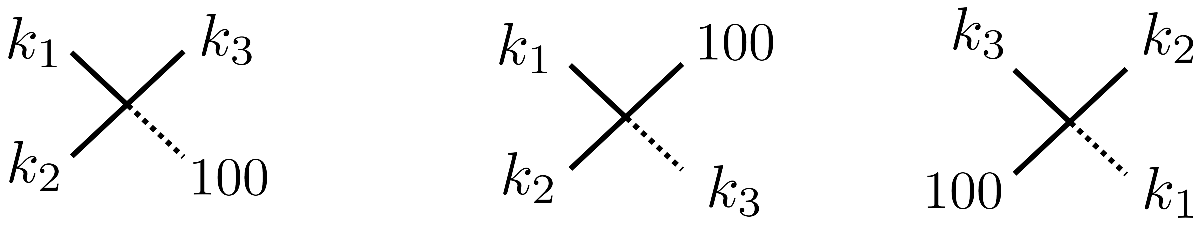
$$m \lesssim 10^{-14}\,\mathrm{eV}$$





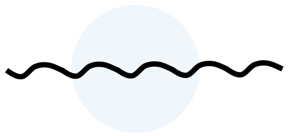
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

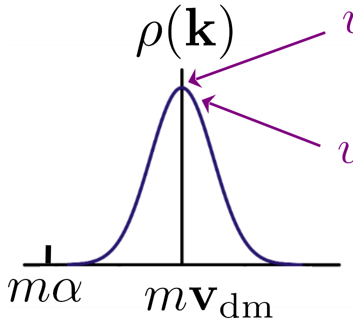
$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{ \; \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})} \,, \;\; \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})} \; \}$$



$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \ll 1$$

$$m \lesssim 10^{-14}\,\mathrm{eV}$$





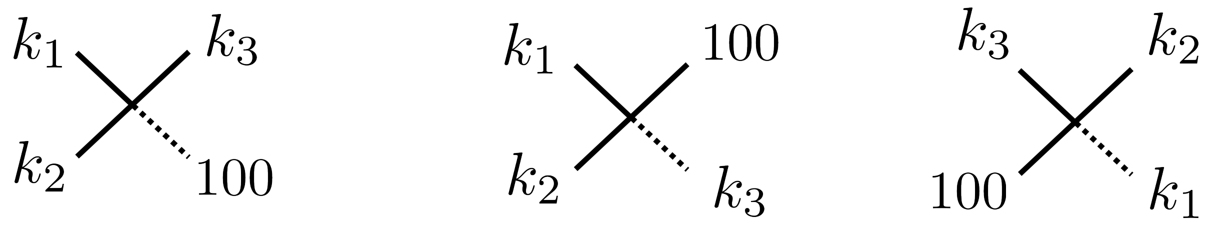
v_1 or v_2

$v_3 = \sqrt{v_1^2 + v_2^2 + \cancel{\alpha^2}}$

$2\pi\alpha \ll v_{\rm dm}$

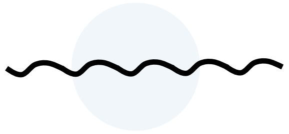
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\ \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \Big\{\ \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\,,\ \ \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ \Big\}$$

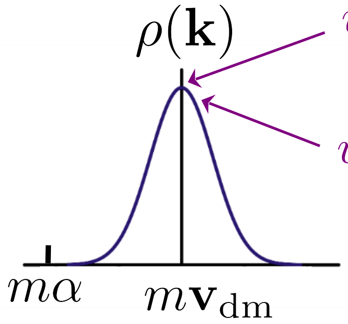


$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \ll 1$$

$$m \lesssim 10^{-14}\,\text{eV}$$



$$\mathcal{M} = \int d^3x\, \underset{\displaystyle \downarrow \atop \displaystyle e^{-\frac{r}{R_\star}}}{\psi^*_{100}}\psi^*_{\mathbf{k_3}}\psi_{\mathbf{k_1}}\underset{\displaystyle \downarrow \atop \displaystyle e^{i\mathbf{k}\cdot\mathbf{x}}}{\psi_{\mathbf{k_2}}}$$



v_1 or v_2

$$v_3 = \sqrt{v_1^2 + v_2^2 + \cancel{\alpha^2}}$$

$$2\pi\alpha \ll v_{\rm dm}$$

$$\mathcal{M} \propto \xi^4_{\rm foc} \ll 1$$

$$\Gamma_2 \simeq 2\Gamma_1 \quad \rightarrow \quad \Gamma < 0$$

Bound state formation: classical perturbation theory

- for $g \neq 0$, $\psi_w(t, \mathbf{x})$ is not an exact solution
- if $\rho < \rho_{\rm crit}$ the self-interaction term $g|\psi|^2\psi$ is perturbative

$$\rightarrow \quad \dot{M}_\star = C + (\Gamma_1 - \Gamma_2) M_\star$$

$$\psi = \psi^{(0)} + \delta\psi$$

Bound state formation: classical perturbation theory

- for $g \neq 0, \quad \psi_w(t, \mathbf{x})$ is not an exact solution
- if $\rho < \rho_{\text{crit}}$ the self-interaction term $g|\psi|^2\psi$ is perturbative

$$\rightarrow \boxed{\dot{M}_\star = C + (\Gamma_1 - \Gamma_2) M_\star}$$

$$\psi = \psi^{(0)} + \underbrace{\delta\psi}_{\text{perturbation due to self-interactions}}$$

$$\psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t} \psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100} e^{-i\omega_1 t}}_{\text{bound state}}$$

Bound state formation: classical perturbation theory

- for $g \neq 0$, $\psi_w(t, \mathbf{x})$ is not an exact solution
- if $\rho < \rho_{\rm crit}$ the self-interaction term $g|\psi|^2\psi$ is perturbative

$$\rightarrow \boxed{\dot{M}_\star = C + (\Gamma_1 - \Gamma_2) M_\star}$$

$$\psi = \psi^{(0)} + \underbrace{\delta\psi}_{\text{perturbation due to self-interactions}}$$

$$\psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t} \psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100} e^{-i\omega_1 t}}_{\text{bound state}}$$

$\mathcal{O}(\lambda^0)$:

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi^{(0)} = 0$$

Bound state formation: classical perturbation theory

- for $g \neq 0$, $\psi_w(t, \mathbf{x})$ is not an exact solution
- if $\rho < \rho_{\text{crit}}$ the self-interaction term $g|\psi|^2\psi$ is perturbative

$$\rightarrow \boxed{\dot{M}_\star = C + (\Gamma_1 - \Gamma_2) M_\star}$$

$$\psi = \psi^{(0)} + \underbrace{\delta\psi}_{\text{perturbation due to self-interactions}}$$

$$\psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t} \psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star} \psi_{100} e^{-i\omega_1 t}}_{\text{bound state}}$$

$\mathcal{O}(\lambda^0)$:

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \psi^{(0)} = 0$$

$\mathcal{O}(\lambda)$:
source for $\delta\psi$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r} \right) \delta\psi = g|\overbrace{\psi^{(0)}}^{\cancel{\psi^{(0)}} + \delta\psi}|^2 \overbrace{\psi^{(0)}}^{\psi^{(0)} + \cancel{\delta\psi}}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\delta\psi = g|\psi^{(0)}|^2\psi^{(0)} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\delta\psi = g|\psi^{(0)}|^2\psi^{(0)} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

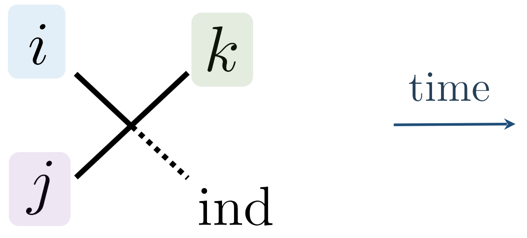
$$\supset g\psi_i\psi_j\psi_k^* e^{-i\overbrace{(\omega_i+\omega_j-\omega_k)}^{\omega_{\text{ind}}}t}$$

$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\delta\psi = g|\psi^{(0)}|^2\psi^{(0)}$$

$$\psi^{(0)} = \underbrace{\int [dk]\, f(\mathbf{k})\, e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}\,e^{-i\omega_1 t}}_{\text{bound state}}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i\overbrace{(\omega_i+\omega_j-\omega_k)}^{\omega_{\text{ind}}}t}$$

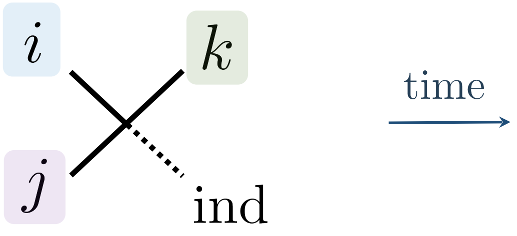
$$\omega_i + \omega_j = \omega_k + \omega_{\text{ind}}$$



$$\left(i\partial_t + \frac{\nabla^2}{2m} + \frac{\alpha}{r}\right)\delta\psi = g|\psi^{(0)}|^2\psi^{(0)} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i\overbrace{(\omega_i+\omega_j-\omega_k)}^{\omega_{\text{ind}}}t}$$

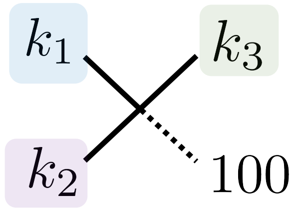
$$\omega_i + \omega_j = \omega_k + \omega_{\text{ind}}$$



- 3 relevant types of terms:

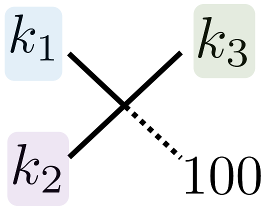
  (a) $\psi_{i,j,k} =$ waves

$$\supset g f(\mathbf{k}_1) f(\mathbf{k}_2) f^*(\mathbf{k}_3)\psi_{\mathbf{k}_1}\psi_{\mathbf{k}_2}\psi_{\mathbf{k}_3}^* e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_{k_3})t}$$



capture

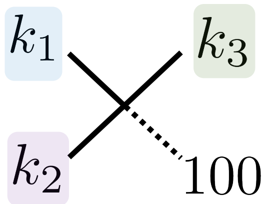
$$\supset g f(\mathbf{k}_1) f(\mathbf{k}_2) f^*(\mathbf{k}_3) \psi_{\mathbf{k}_1} \psi_{\mathbf{k}_2} \psi^*_{\mathbf{k}_3} e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_{k_3})t}$$



capture

$$\dot{M}_\star \sim m \frac{d}{dt} \int d^3x |\delta\psi|^2 \sim$$

$$\supset g f(\mathbf{k}_1) f(\mathbf{k}_2) f^*(\mathbf{k}_3) \psi_{\mathbf{k}_1} \psi_{\mathbf{k}_2} \psi^*_{\mathbf{k}_3} e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_{k_3})t}$$

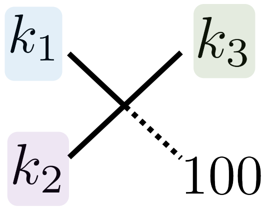


capture

$$\dot{M}_\star \sim m \frac{d}{dt} \int d^3x |\delta\psi|^2 \sim$$

$$\sim m g^2 \int [dk_{1,2,3}] \rho(\mathbf{k}_1) \rho(\mathbf{k}_2) \rho(\mathbf{k}_3) |\mathcal{M}|^2 \delta(\Delta\omega) \qquad \equiv C$$

$$\supset g f(\mathbf{k}_1) f(\mathbf{k}_2) f^*(\mathbf{k}_3) \psi_{\mathbf{k}_1} \psi_{\mathbf{k}_2} \psi^*_{\mathbf{k}_3} e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_{k_3})t}$$



capture

$$\dot{M}_\star \sim m \tfrac{d}{dt} \int d^3x |\delta\psi|^2 \sim$$

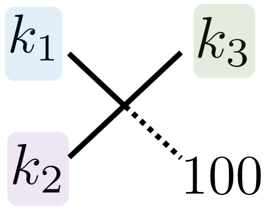
$$\sim m g^2 \int [dk_{1,2,3}] \rho(\mathbf{k}_1) \rho(\mathbf{k}_2) \rho(\mathbf{k}_3) |\mathcal{M}|^2 \delta(\Delta\omega) \qquad \equiv C$$

$$\mathcal{M} = \int d^3x \, \psi^*_{100} \psi^*_{\mathbf{k_3}} \psi_{\mathbf{k_1}} \psi_{\mathbf{k_2}}$$

$$\Delta\omega \equiv \omega_{k_1} + \omega_{k_2} - \omega_{k_3} - \omega_1$$

$$= \frac{k_1^2}{2m} + \frac{k_2^2}{2m} - \frac{k_3^2}{2m} + \frac{m\alpha^2}{2}$$

$$\supset g f(\mathbf{k}_1) f(\mathbf{k}_2) f^*(\mathbf{k}_3) \psi_{\mathbf{k}_1} \psi_{\mathbf{k}_2} \psi^*_{\mathbf{k}_3} e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_{k_3})t}$$



capture

$$\dot{M}_\star \sim m \frac{d}{dt} \int d^3x |\delta\psi|^2 \sim$$

Bose enhancement

$$\sim m g^2 \int [dk_{1,2,3}] \rho(\mathbf{k}_1) \rho(\mathbf{k}_2) \rho(\mathbf{k}_3) |\mathcal{M}|^2 \delta(\Delta\omega) \qquad \equiv C$$

$$\mathcal{M} = \int d^3x \, \psi^*_{100} \psi^*_{\mathbf{k_3}} \psi_{\mathbf{k_1}} \psi_{\mathbf{k_2}}$$

$$\Delta\omega \equiv \omega_{k_1} + \omega_{k_2} - \omega_{k_3} - \omega_1$$

$$= \frac{k_1^2}{2m} + \frac{k_2^2}{2m} - \frac{k_3^2}{2m} + \frac{m\alpha^2}{2}$$

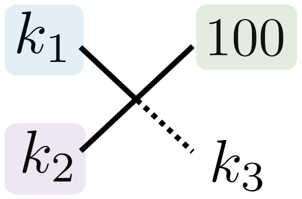
$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

(b) $\psi_{i,j} = \text{waves}, \ \psi_k = \text{bound}$



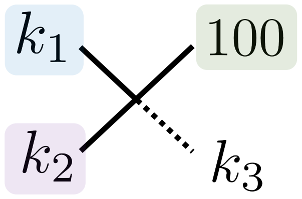
stimulated capture

$$\supset g\sqrt{N_\star}f(\mathbf{k}_1)f(\mathbf{k}_2)\psi_{\mathbf{k}_1}\psi_{\mathbf{k}_2}\psi_{100}^* e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_1)t}$$

$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

(b) $\psi_{i,j}$ = waves, ψ_k = bound



stimulated capture

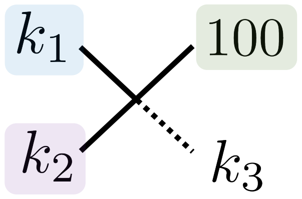
$$\supset g\sqrt{N_\star}\, f(\mathbf{k}_1) f(\mathbf{k}_2)\psi_{\mathbf{k}_1}\psi_{\mathbf{k}_2}\psi^*_{100} e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_1)t}$$

$$\dot{M}_\star \sim m g^2 N_\star \int [dk_{1,2,3}]\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)|\mathcal{M}|^2\delta(\Delta\omega) \equiv \Gamma_1 \underset{\substack{\downarrow \\ mN_\star}}{M_\star}$$

$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

(b) $\psi_{i,j}$ = waves, ψ_k = bound



stimulated capture

$$\supset g\sqrt{N_\star}\, f(\mathbf{k}_1) f(\mathbf{k}_2)\psi_{\mathbf{k}_1}\psi_{\mathbf{k}_2}\psi^*_{100}\, e^{-i(\omega_{k_1}+\omega_{k_2}-\omega_1)t}$$

$$\dot{M}_\star \sim m g^2 N_\star \int [dk_{1,2,3}]\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)|\mathcal{M}|^2\delta(\Delta\omega) \equiv \Gamma_1 M_\star$$

$$\downarrow$$

$$mN_\star$$

$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t}$$

$$\psi^{(0)} = \int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}} + \sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}$$

waves

bound state

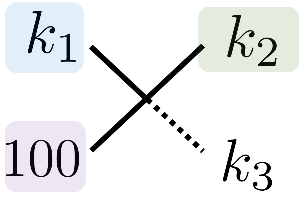
$$g|\psi^{(0)}|^2\psi^{(0)}$$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

$g|\psi^{(0)}|^2\psi^{(0)}$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \longleftarrow \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

(c) $\psi_{i,k}$ = waves, ψ_j = bound



stripping

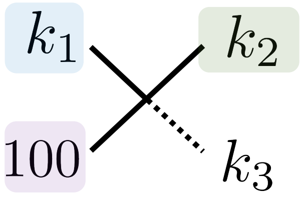
$+2 \leftrightarrow 1$

$$\supset 2g\sqrt{N_\star}f(\mathbf{k}_1)f^*(\mathbf{k}_2)\psi_{\mathbf{k}_1}\psi_{100}\psi_{\mathbf{k}_2}^* e^{-i(\omega_{k_1}-\omega_{k_2}+\omega_1)t}$$

$g|\psi^{(0)}|^2\psi^{(0)}$

$$\supset g\psi_i\psi_j\psi_k^* e^{-i(\omega_i+\omega_j-\omega_k)t} \qquad \longleftarrow \qquad \psi^{(0)} = \underbrace{\int [dk] f(\mathbf{k}) e^{-i\omega_k t}\psi_{\mathbf{k}}}_{\text{waves}} + \underbrace{\sqrt{N_\star}\,\psi_{100}e^{-i\omega_1 t}}_{\text{bound state}}$$

(c) $\psi_{i,k}$ = waves, ψ_j = bound



stripping

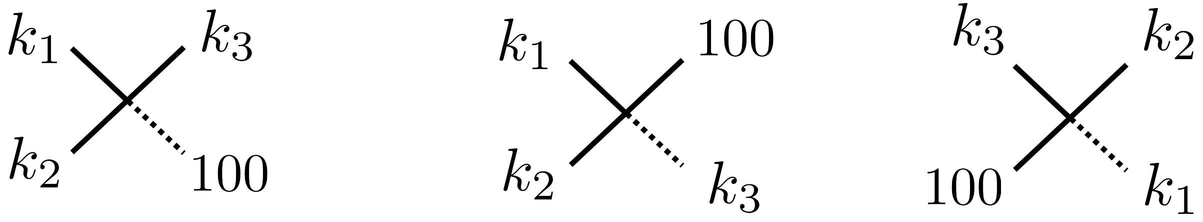
$+2 \leftrightarrow 1$

$$\supset 2g\sqrt{N_\star}\, f(\mathbf{k}_1) f^*(\mathbf{k}_2)\psi_{\mathbf{k}_1}\psi_{100}\psi^*_{\mathbf{k}_2} e^{-i(\omega_{k_1}-\omega_{k_2}+\omega_1)t}$$

$$\dot{M}_\star \sim -2mg^2N_\star \int [dk_{1,2,3}]\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)|\mathcal{M}|^2\delta(\Delta\omega) \equiv -\Gamma_2 M_\star$$

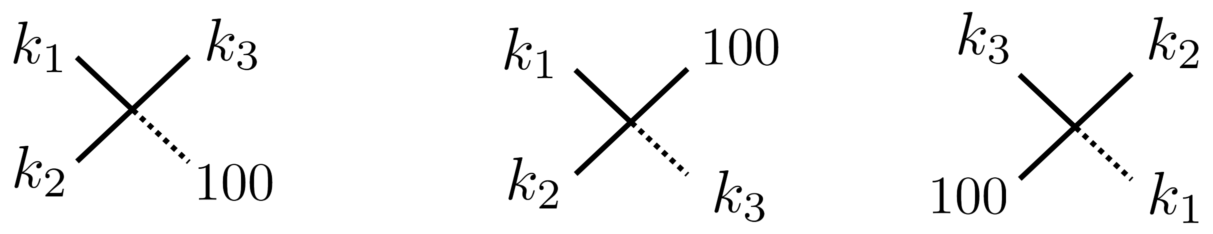
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\,C,\ \Gamma_1 - \Gamma_2\,\} = g^2 \int [dk_{1,2,3}]\,\delta(\Delta\omega)\,|\mathcal{M}|^2 \{\ \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\,,\ \ \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} \overset{\text{stripping}}{- 2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}\ \}$$



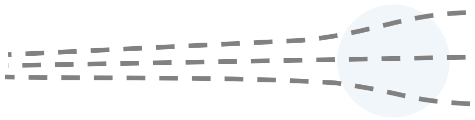
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\, \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\delta(\Delta\omega)|\mathcal{M}|^2 \{ \; \overset{\text{capture}}{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})} \,, \;\; \overset{\text{stimulated capture}}{\rho(\mathbf{k_1})\rho(\mathbf{k_2})} - \overset{\text{stripping}}{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})} \; \}$$

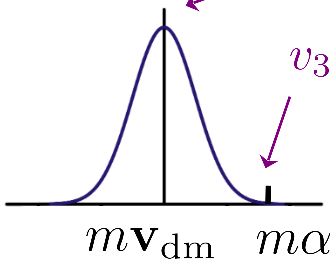


$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\mathrm{eV}$$



v_1 or v_2

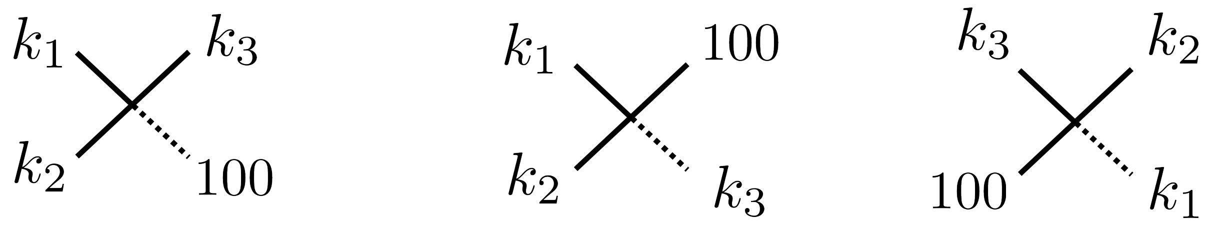


$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

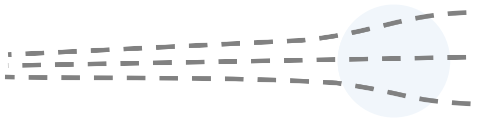
$$\dot{M}_\star = C + \underbrace{(\Gamma_1 - \Gamma_2)}_{\Gamma} M_\star$$

$$\{\, C,\ \Gamma_1 - \Gamma_2 \,\} = g^2 \int [dk_{1,2,3}]\,\delta(\Delta\omega)\,|\mathcal{M}|^2 \{\ \underbrace{m\,\rho(\mathbf{k_1})\rho(\mathbf{k_2})\rho(\mathbf{k_3})}_{\text{capture}}\,,\ \ \underbrace{\rho(\mathbf{k_1})\rho(\mathbf{k_2})}_{\text{stimulated capture}} - \underbrace{2\rho(\mathbf{k_2})\rho(\mathbf{k_3})}_{\text{stripping}}\ \}$$



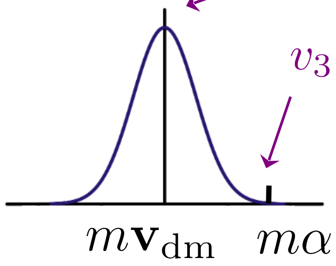
$$\xi_{\rm foc} = \frac{2\pi\alpha}{v_{\rm dm}} \gg 1$$

$$m \gtrsim 10^{-14}\,\text{eV}$$



$$\mathcal{M} = \int d^3x\, \psi^*_{100}\psi^*_{\mathbf{k_3}}\psi_{\mathbf{k_1}}\psi_{\mathbf{k_2}}$$

v_1 or v_2



$$v_3 = \sqrt{v_1^2 + v_2^2 + \alpha^2} \gtrsim \alpha$$

$$2\pi\alpha \gg v_{\rm dm}$$

$$\mathcal{M} \simeq \text{const}$$

$$\Gamma_1 \simeq \text{const}$$

$$\Gamma_2 \simeq e^{-\xi_{\rm foc}}$$

$$\to \Gamma > 0$$

$\Gamma^{-1} \leftrightarrow$

relaxation time $$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$\Gamma^{-1} \leftrightarrow$

relaxation time

$$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2 \rho_{\rm dm}^2} \simeq 9\,{\rm Gyr} \left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4 \left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3 \left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2 \left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$$\tau_{\rm rel}^{-1} \simeq n \Sigma\, v_{\rm dm} \times \frac{n}{(m v_{\rm dm})^3}$$

$\frac{\rho_{\rm dm}}{m}$

$g^2 m^2$

$2 \to 2$ cross section

occupation number

Dilute atoms

$$\mathcal{M}_{k_1+k_2\to k_3+100} = \int d^3x\, e^{i(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)\cdot\mathbf{x}}\, \frac{e^{-r/R_\star}}{\sqrt{\pi R_\star^3}} = \frac{8\sqrt{\pi R_\star^3}}{\left[1+(\Delta k\, R_\star)^2\right]^2}$$

$$\Gamma =$$

$$\frac{64 g^2 \rho_{\rm dm}^2 \alpha^4}{(\pi m v_{\rm dm}^2)^3} \int dp_1 dp_2\, d\cos\theta_1\, d\cos\theta_2\, d\cos\theta_3\, d\phi_1 d\phi_2 \frac{p_1^2 p_2^2 p_3}{(1+\Delta p^2)^4} \left[e^{-\left(\frac{\alpha}{v_{\rm dm}}\mathbf{p}_1-\hat{z}\right)^2} - 2e^{-\left(\frac{\alpha}{v_{\rm dm}}\mathbf{p}_3-\hat{z}\right)^2}\right] e^{-\left(\frac{\alpha}{v_{\rm dm}}\mathbf{p}_2-\hat{z}\right)^2}$$

Dilute atoms

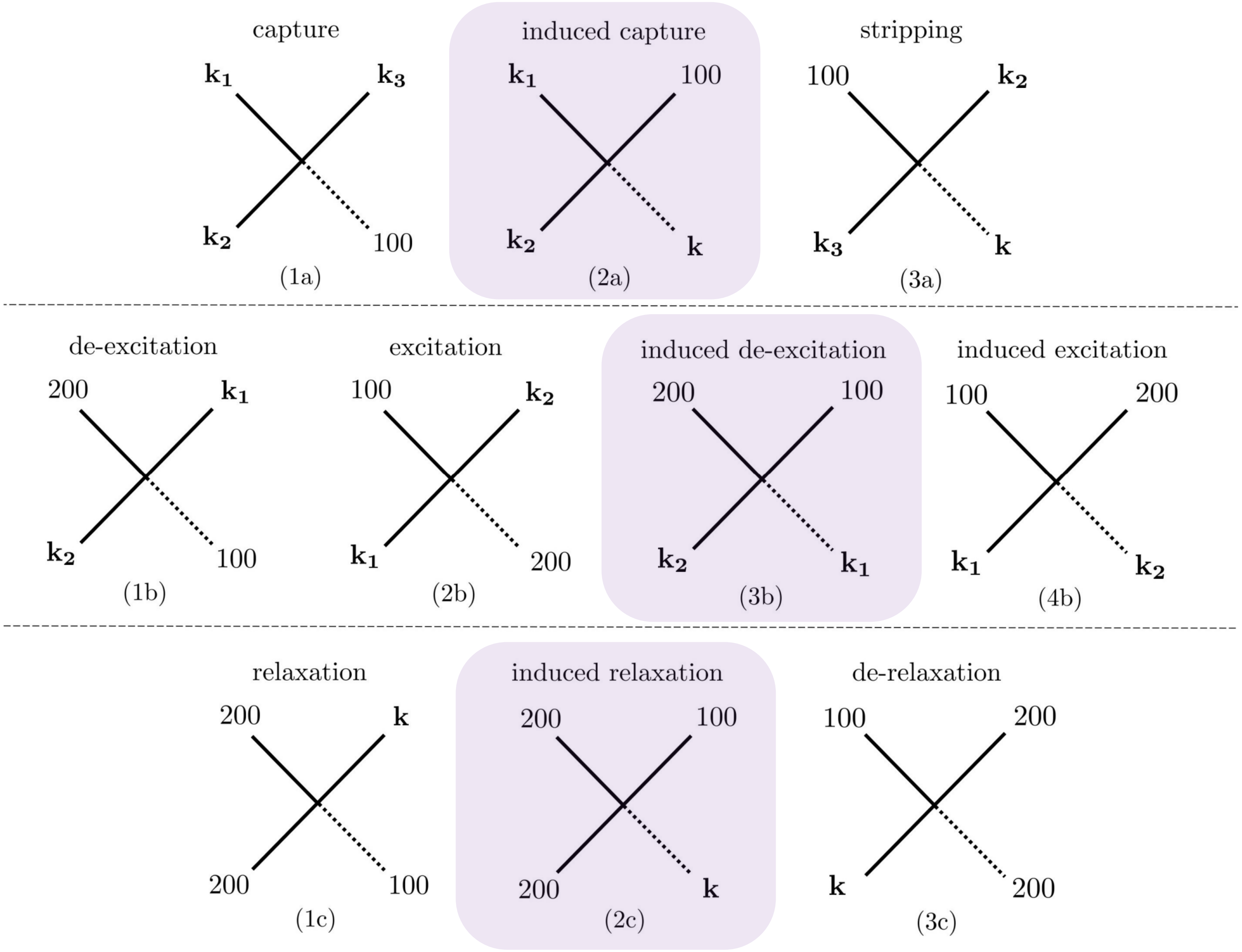
$$\frac{\rho(r=0)}{\rho_{\rm dm}} \simeq \frac{0.4\xi_{\rm foc}^3}{\pi^{5/2}} \simeq 5\cdot 10^{-5}\left[\frac{M}{M_\odot}\,\frac{m}{2\cdot 10^{-15}\,{\rm eV}}\,\frac{240\,{\rm km/\,s}}{v_{\rm dm}}\right]^3$$

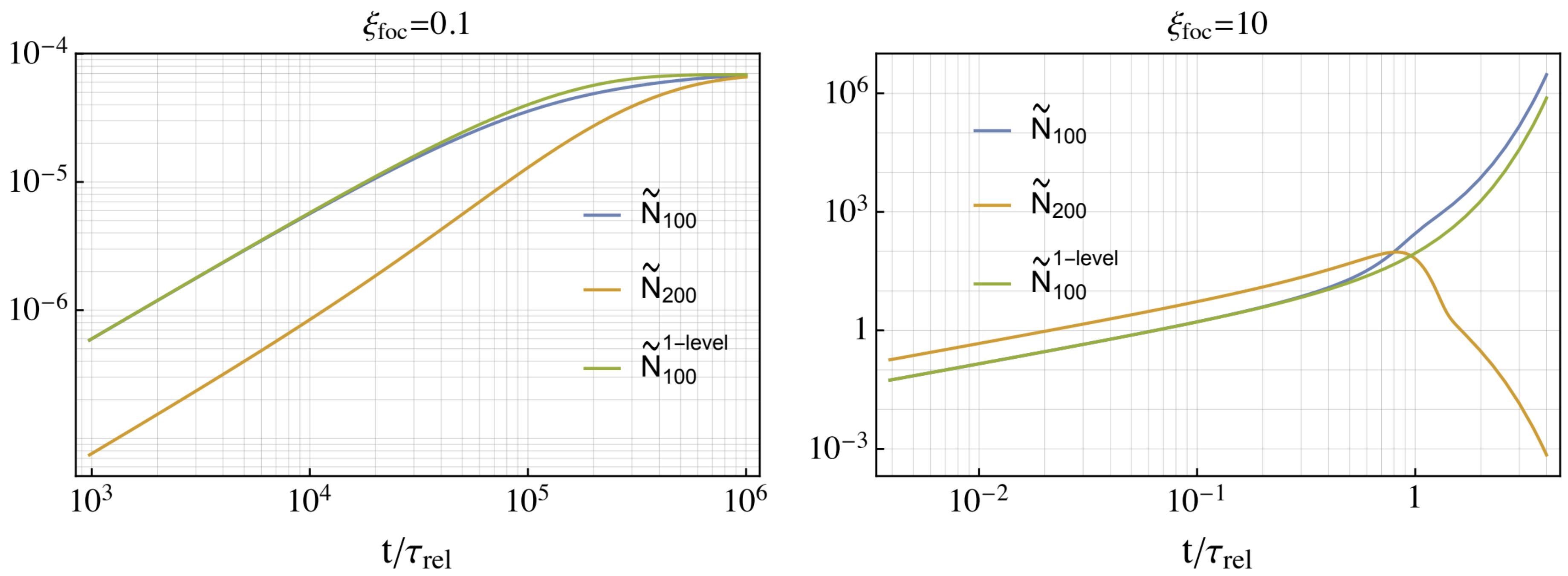
$$\tau_\star \gtrsim \frac{2\pi}{m\alpha^2} = \left[\frac{2\pi}{\xi_{\rm foc}}\right]^2 \tau_{\rm dm} \simeq 1\,{\rm year}\left[\frac{1.3\cdot 10^{-14}\,{\rm eV}}{m}\right]^3$$

Excited states and two level system

$$\dot{N}_{100} = 2g^2 \int [dk_1][dk_2][dk_3]|\mathcal{M}_{k_1+k_2\to k_3+100}|^2(2\pi)\delta(\omega_{k_1}+\omega_{k_2}-\omega_1-\omega_{k_3})$$
$$\{\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)\rho(\mathbf{k}_3)+N_{100}[\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)-2\rho(\mathbf{k}_2)\rho(\mathbf{k}_3)]\}$$
$$+\,4g^2\int [dk_1][dk_2]|\mathcal{M}_{k_1+100\to k_2+200}|^2(2\pi)\delta(\omega_{k_1}+\omega_1-\omega_2-\omega_{k_2})$$
$$\{N_{100}N_{200}[\rho(\mathbf{k}_2)-\rho(\mathbf{k}_1)]-\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)[N_{100}-N_{200}]\}$$
$$+\,2g^2\int [dk]|\mathcal{M}_{200+200\to 100+k}|^2(2\pi)\delta(2\omega_2-\omega_1-\omega_k)\{\rho(\mathbf{k})N_{200}^2+N_{100}N_{200}^2-2N_{200}N_{100}\rho(\mathbf{k})\}\,.$$

$$\dot{N}_{200} = 2g^2 \int [dk_1][dk_2][dk_3]|\mathcal{M}_{k_1+k_2\to k_3+200}|^2(2\pi)\delta(\omega_{k_1}+\omega_{k_2}-\omega_2-\omega_{k_3})$$
$$\{\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)\rho(\mathbf{k}_3)+N_{200}[\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)-2\rho(\mathbf{k}_2)\rho(\mathbf{k}_3)]\}$$
$$-\,4g^2\int [dk_1][dk_2]|\mathcal{M}_{k_1+100\to k_2+200}|^2(2\pi)\delta(\omega_{k_1}+\omega_1-\omega_2-\omega_{k_2})$$
$$\{N_{100}N_{200}[\rho(\mathbf{k}_2)-\rho(\mathbf{k}_1)]-\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)[N_{100}-N_{200}]\}$$
$$-\,4g^2\int [dk]|\mathcal{M}_{200+200\to 100+k}|^2(2\pi)\delta(2\omega_2-\omega_1-\omega_k)\{\rho(\mathbf{k})N_{200}^2+N_{100}N_{200}^2-2N_{200}N_{100}\rho(\mathbf{k})\}\,,$$


capture
$\mathbf{k_1}$ $\mathbf{k_3}$ $\mathbf{k_2}$ 100
(1a)
induced capture
$\mathbf{k_1}$ 100 $\mathbf{k_2}$ $\mathbf{k}$
(2a)
stripping
100 $\mathbf{k_2}$ $\mathbf{k_3}$ $\mathbf{k}$
(3a)
de-excitation
200 $\mathbf{k_1}$ $\mathbf{k_2}$ 100
(1b)
excitation
100 $\mathbf{k_2}$ $\mathbf{k_1}$ 200
(2b)
induced de-excitation
200 100 $\mathbf{k_2}$ $\mathbf{k_1}$
(3b)
induced excitation
100 200 $\mathbf{k_1}$ $\mathbf{k_2}$
(4b)
relaxation
200 $\mathbf{k}$ 200 100
(1c)
induced relaxation
200 100 200 $\mathbf{k}$
(2c)
de-relaxation
100 200 $\mathbf{k}$ 200
(3c)



ξ_{foc}=0.1
10^{-4}
10^{-5}
10^{-6}
10^3
10^4
10^5
10^6
t/τ_{rel}
$\tilde{N}_{100}$
$\tilde{N}_{200}$
$\tilde{N}_{100}^{1\text{-level}}$
ξ_{foc}=10
10^6
10^3
1
10^{-3}
10^{-2}
10^{-1}
1
t/τ_{rel}
$\tilde{N}_{100}$
$\tilde{N}_{200}$
$\tilde{N}_{100}^{1\text{-level}}$

$$\mathcal{M}_{k_1+k_2\to k_3+100} = \frac{2\sqrt{2}\pi}{\sqrt{k_1k_2k_3}} \underbrace{\int d^3\zeta\, e^{-\zeta}\, J_0\left[2\xi(1-\hat{k}_1\cdot\hat{\zeta})\right] J_0\left[2\zeta(1-\hat{k}_2\cdot\hat{\zeta})\right] J_0\left[2\zeta(1-\hat{k}_3\cdot\hat{\zeta})\right]}_{F[\hat{k}_i\cdot\hat{k}_j]}$$

$$\Gamma = \frac{8g^2\rho_{\rm dm}^2}{\pi^2 m^3 v_{\rm dm}^2}\int dp_1 dp_2\, d\cos\theta_1\, d\cos\theta_2\, d\cos\theta_3\, d\phi_1 d\phi_2\, p_1p_2 F^2[\hat{p}_i\cdot\hat{p}_j]\left[e^{-(\mathbf{p}_1-\hat{z})^2} - 2e^{-(\mathbf{p}_3-\hat{z})^2}\right] e^{-(\mathbf{p}_2-\hat{z})^2}$$

$$\tau_{\rm rel} \equiv \frac{m^3 v_{\rm dm}^2}{g^2\rho_{\rm dm}^2} \simeq 9\,{\rm Gyr}\left[\frac{f_a}{10^8\,{\rm GeV}}\right]^4\left[\frac{m}{10^{-14}\,{\rm eV}}\right]^3\left[\frac{0.4\,{\rm GeV/cm^3}}{\rho_{\rm dm}}\right]^2\left[\frac{v_{\rm dm}}{240\,{\rm km/s}}\right]^2$$

$$\mathscr{L}_\phi = \frac{d_\alpha}{4M_{\rm Pl}}\phi\, F^{\mu\nu}F_{\mu\nu}$$

$$\frac{\delta\alpha}{\alpha_0} \simeq \frac{d_\alpha\phi}{M_{\rm Pl}} \simeq 6\times10^{-16} d_\alpha \left(\frac{10^{-15}\,{\rm eV}}{m_\phi}\right)\sqrt{\frac{\rho_\phi}{\rho_{\rm dm}}}$$

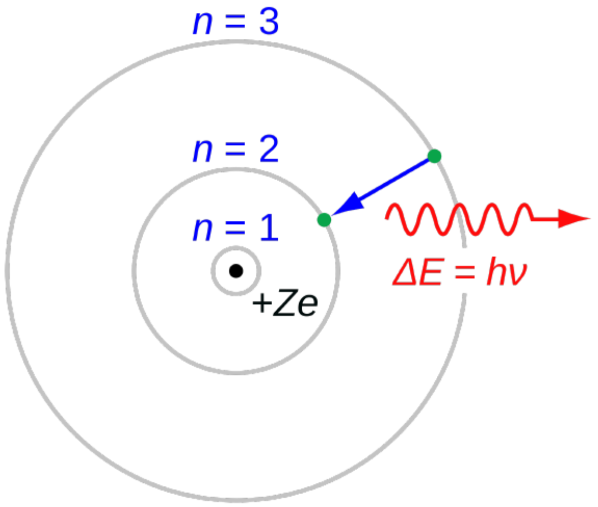
c.f. $\mathscr{L}_{\rm em} = \frac{\alpha}{4}F^{\mu\nu}F_{\mu\nu}$

w/ oscillations at frequency $\omega_\phi \simeq m_\phi \simeq {\rm few\ Hz}\left(\frac{m_\phi}{10^{-15}\,{\rm eV}}\right)$

Derevianko + Pospelov
(1311.1244)

Arvanitaki, Huang, Van Tilburg
(1405.2925)

Stadnik and Flambaum
(1412.7801, 1503.08540)
...

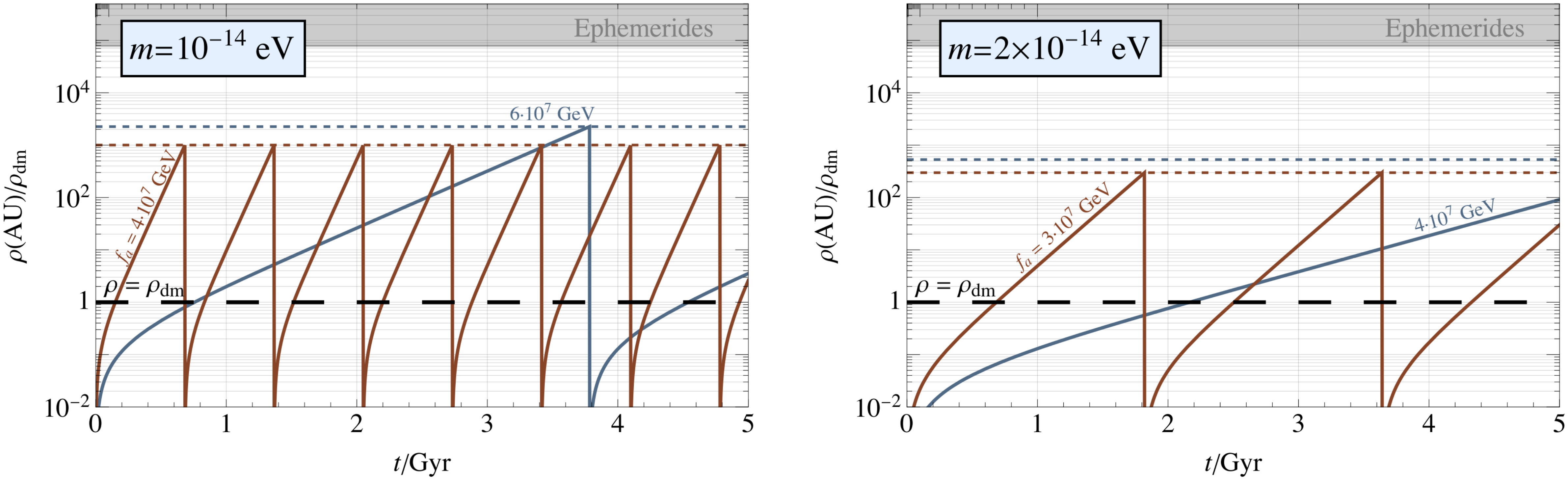


Modern optical clocks have achieved

$$\frac{\delta\alpha}{\alpha_0} \lesssim 10^{-18}$$

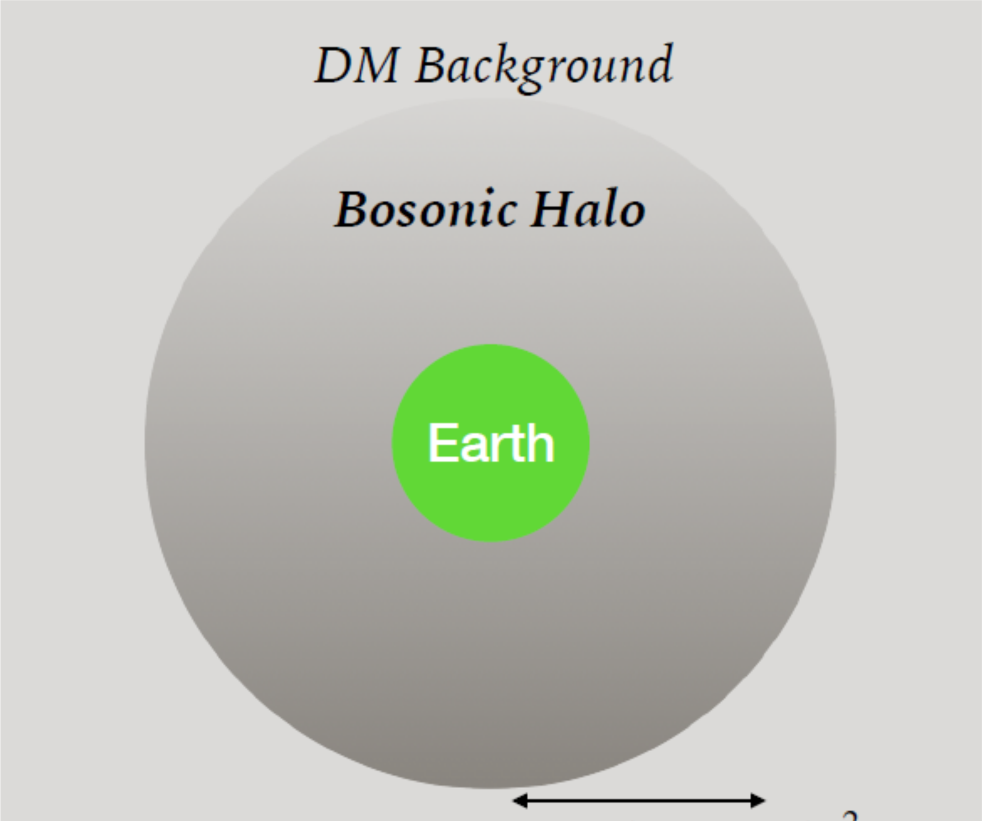
Future nuclear clock expected to reach

$$\frac{\delta\alpha}{\alpha_0} \lesssim 10^{-23}$$



Ephemerides
m=10^{-14} eV
$6{\cdot}10^7$ GeV
$f_a = 4{\cdot}10^7$ GeV
$\rho = \rho_{\rm dm}$
$\rho(\mathrm{AU})/\rho_{\rm dm}$
t/Gyr
Ephemerides
m=2×10^{-14} eV
$f_a = 3{\cdot}10^7$ GeV
$4{\cdot}10^7$ GeV
$\rho = \rho_{\rm dm}$
$\rho(\mathrm{AU})/\rho_{\rm dm}$
t/Gyr

Halo supported by Earth
"Earth Halo"



$$R_\star \simeq R_E \left(\frac{10^{-9}\,\mathrm{eV}}{m_\phi} \right)^2$$