

# The QCD axion and density effects in stars

*The CA21106 1st General Meeting*

**Stefan Stelzl**  
**EPFL Lausanne**



Based on [2211.02661](#) and [2307.14418](#) and work in progress  
Reuven Balkin, Javi Serra, Konstantin Springmann,  
Michael Stadlbauer, SS and Andreas Weiler

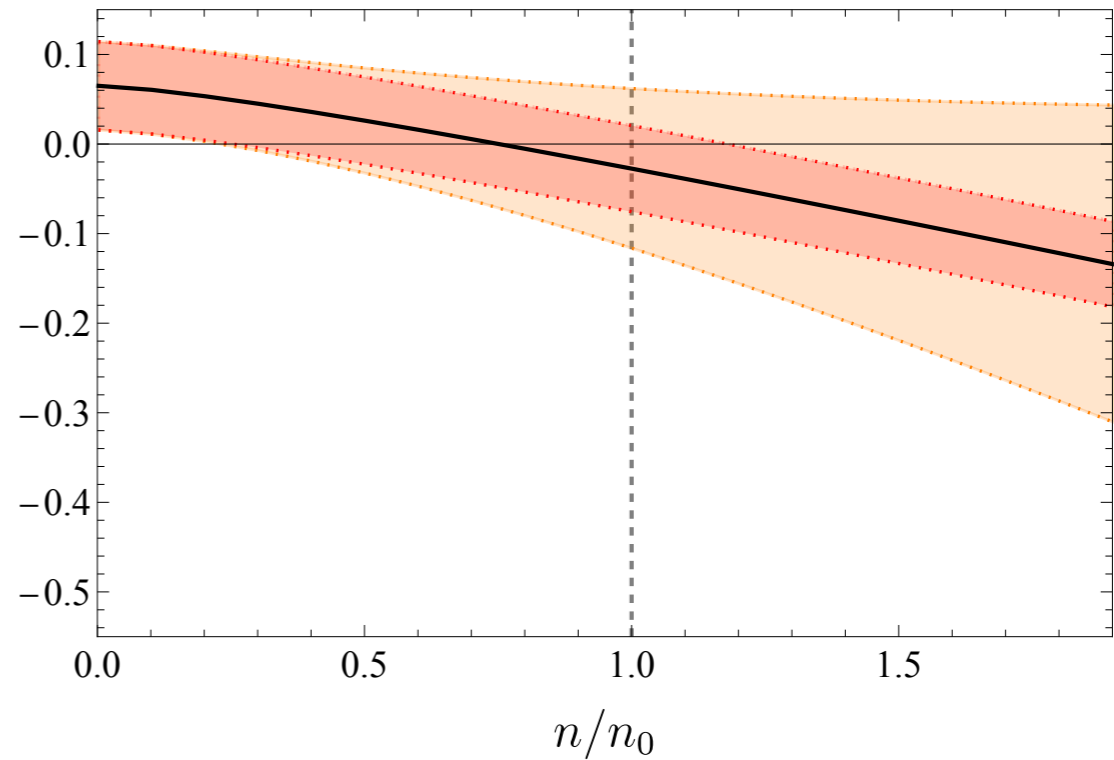
05.09.2023

# Axion - finite density effects

## QCD axion coupling to nucleons at finite density

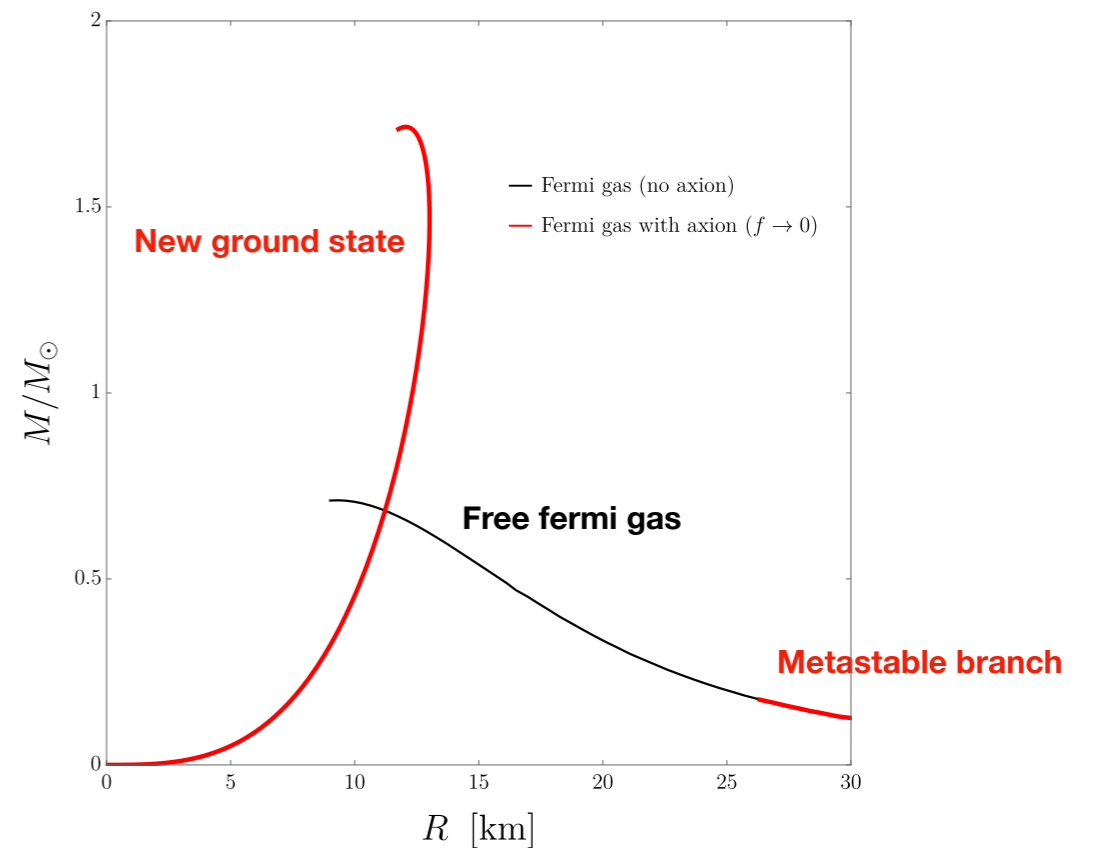
To appear soon  
and *JHEP* 07 (2020) Balkin, Serra, Springmann, Weiler

$c_n$



## Influence of light (pseudo-)scalars on structure of neutron stars

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Balkin, Serra, Springmann, SS, Weiler

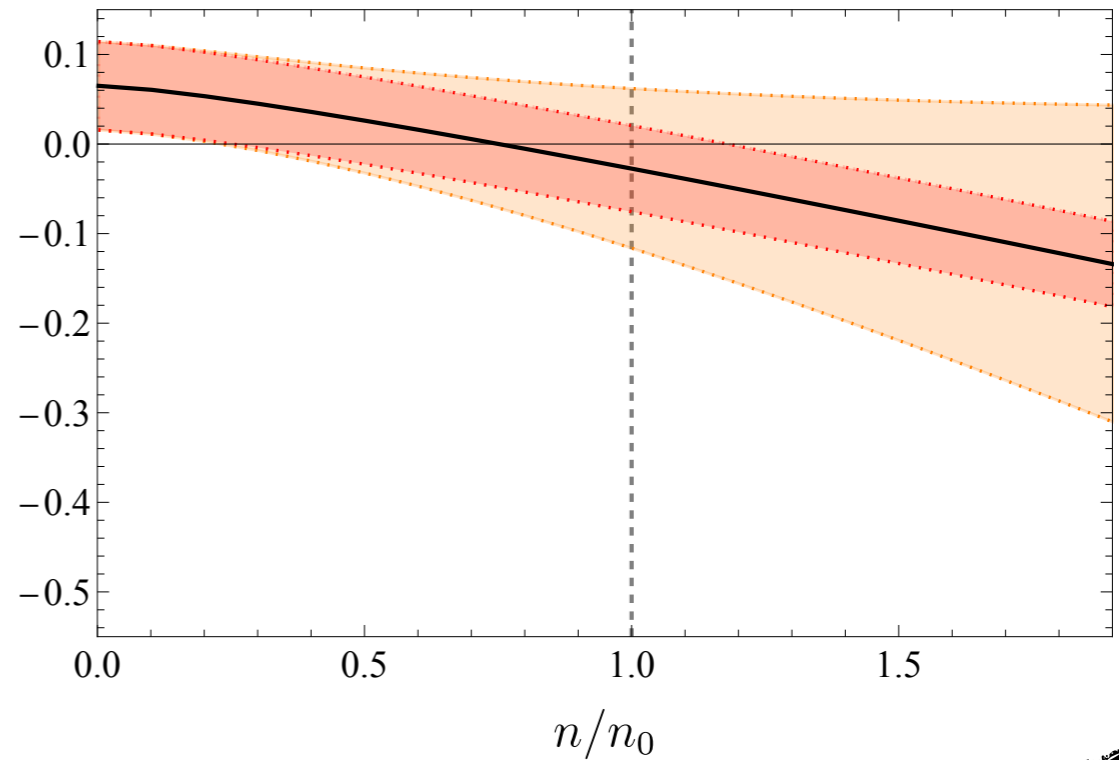


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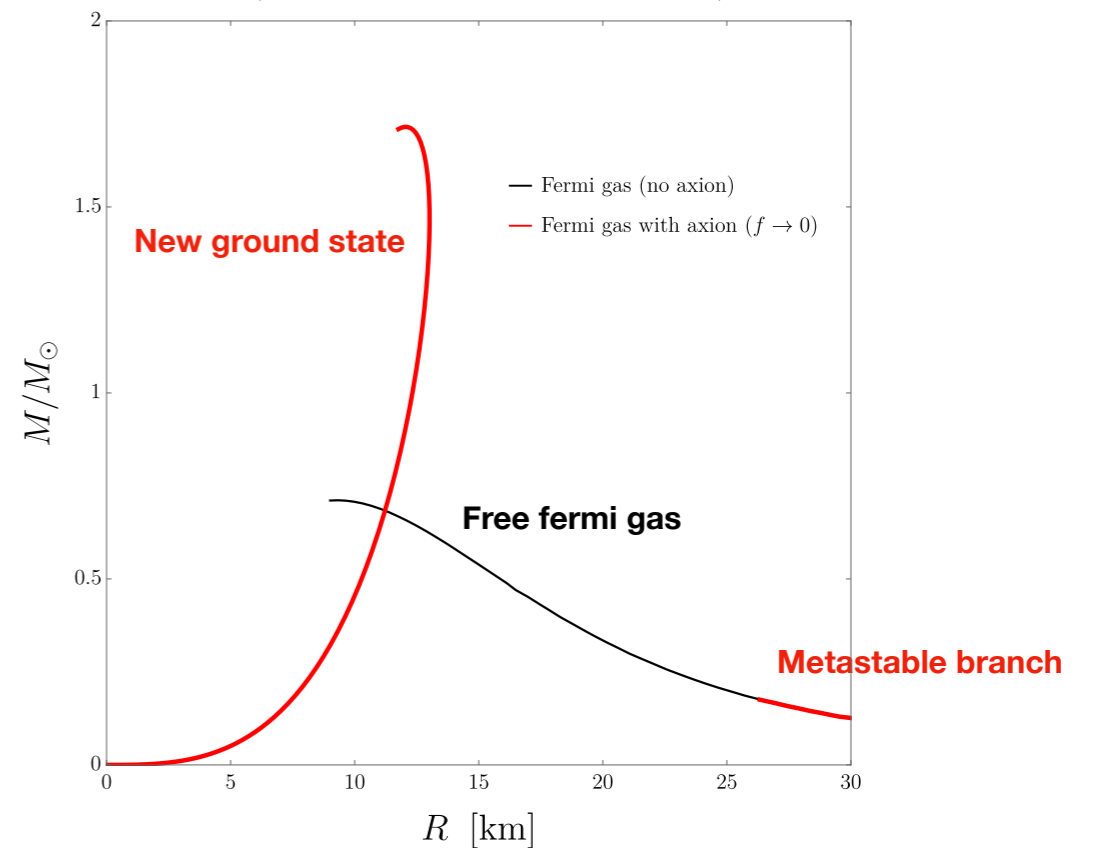
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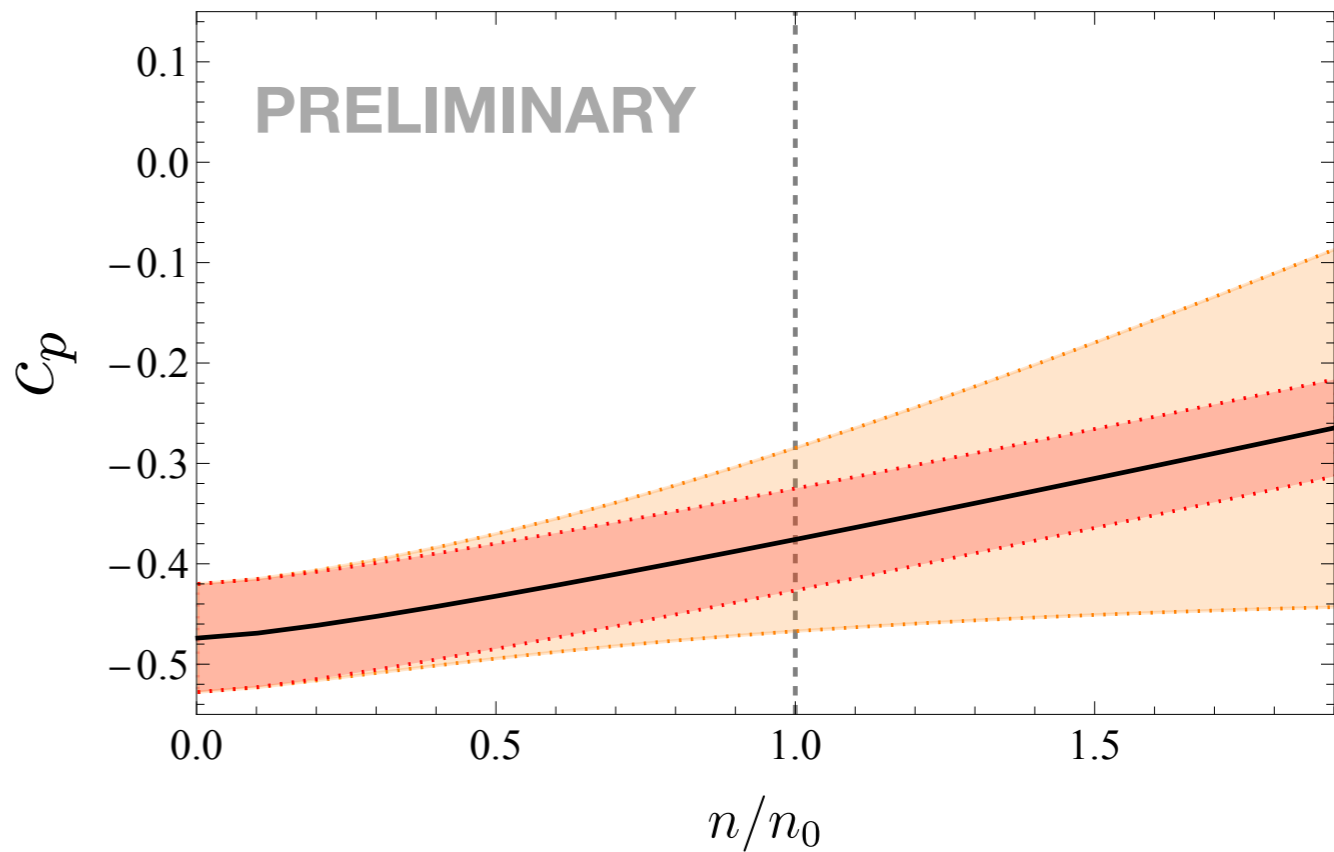
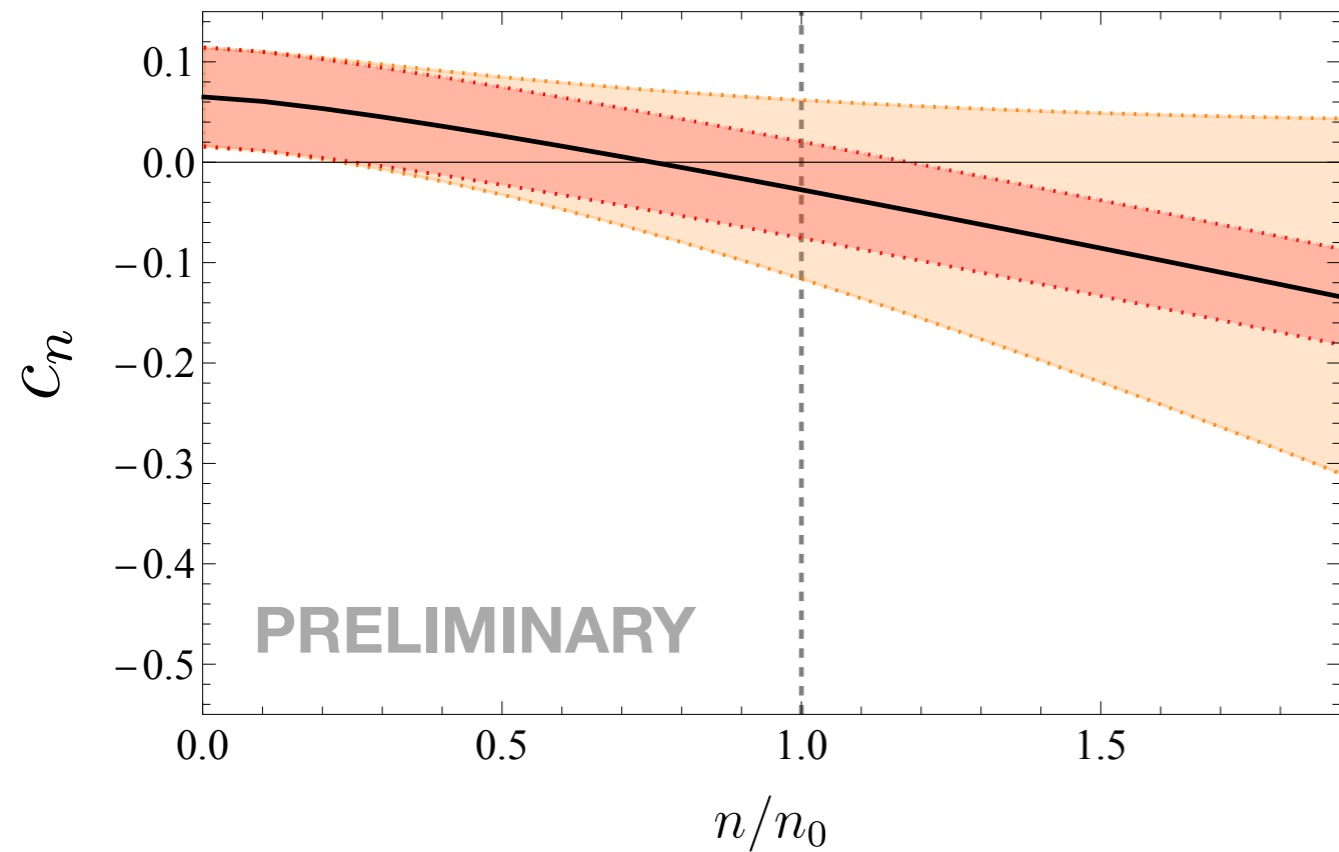
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# QCD axion couplings at finite density

example: KSVZ axion



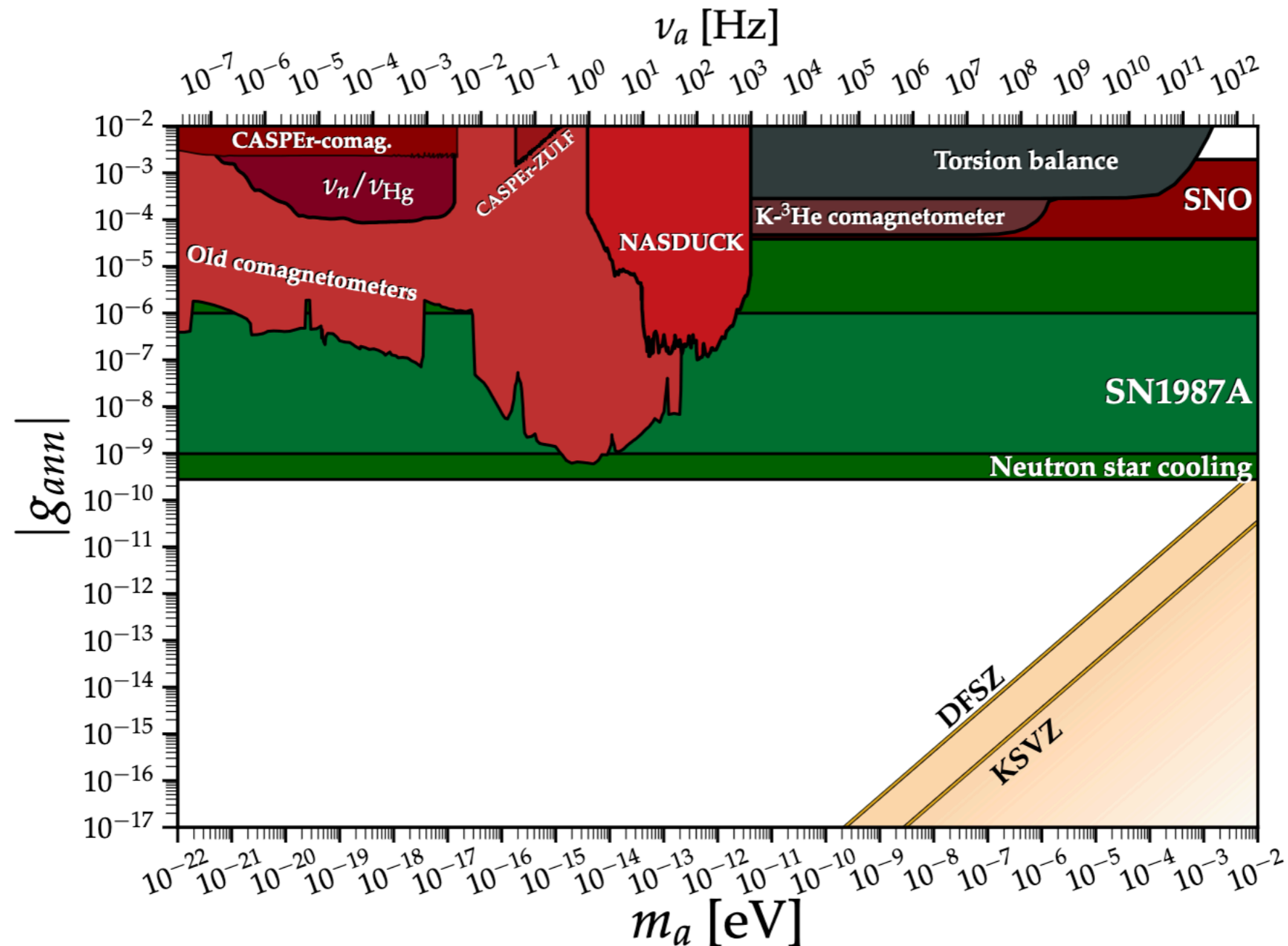
**Axion couplings change in dense objects!**

**...but uncertainties become large at high densities.....**

# QCD axion couplings at finite density

## Some of the best bounds on QCD axion from SN and NS cooling

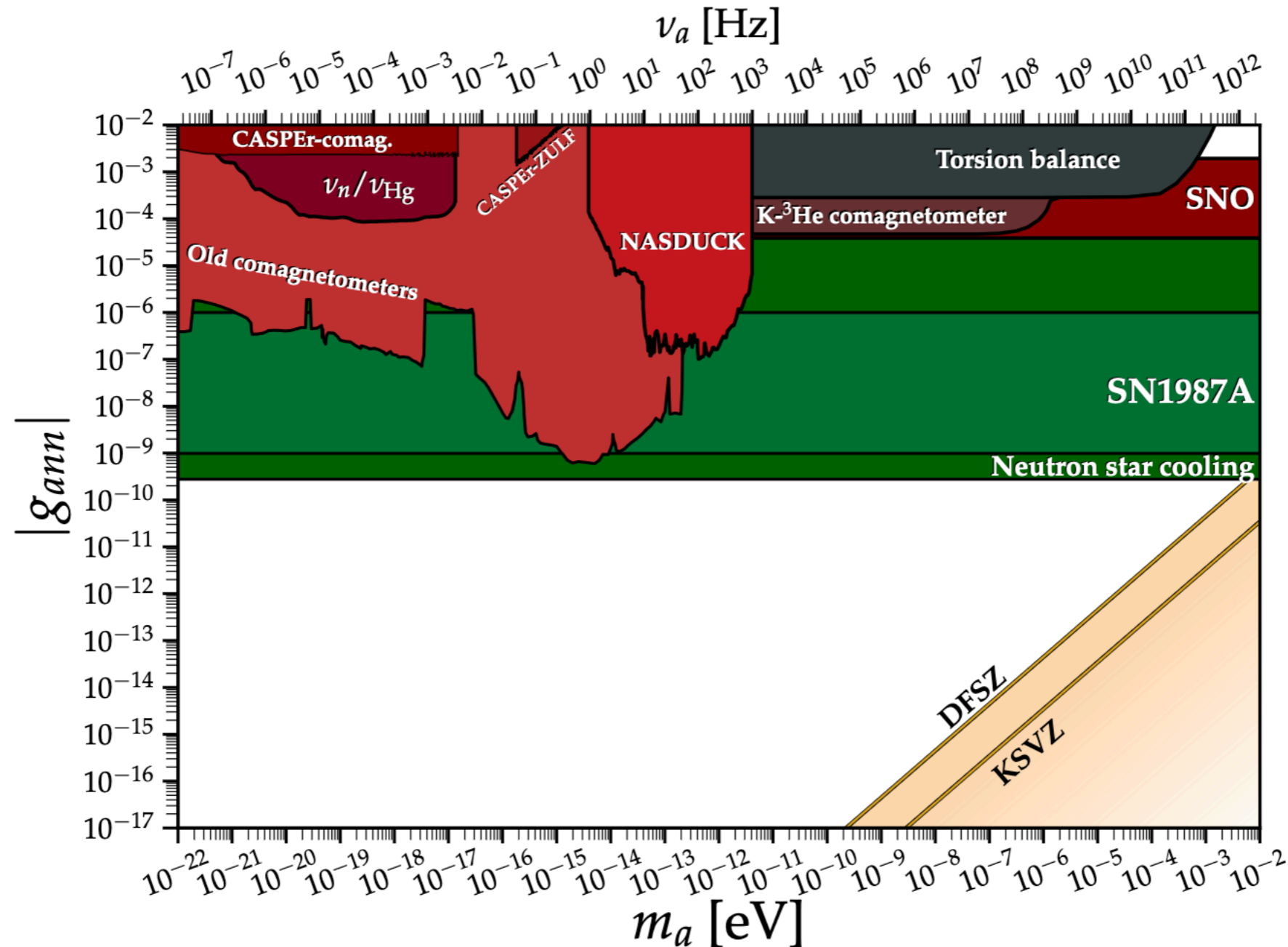
For recent bounds see e.g. Carenza et al. '19 (SN), Buschmann et al. (NS) '21



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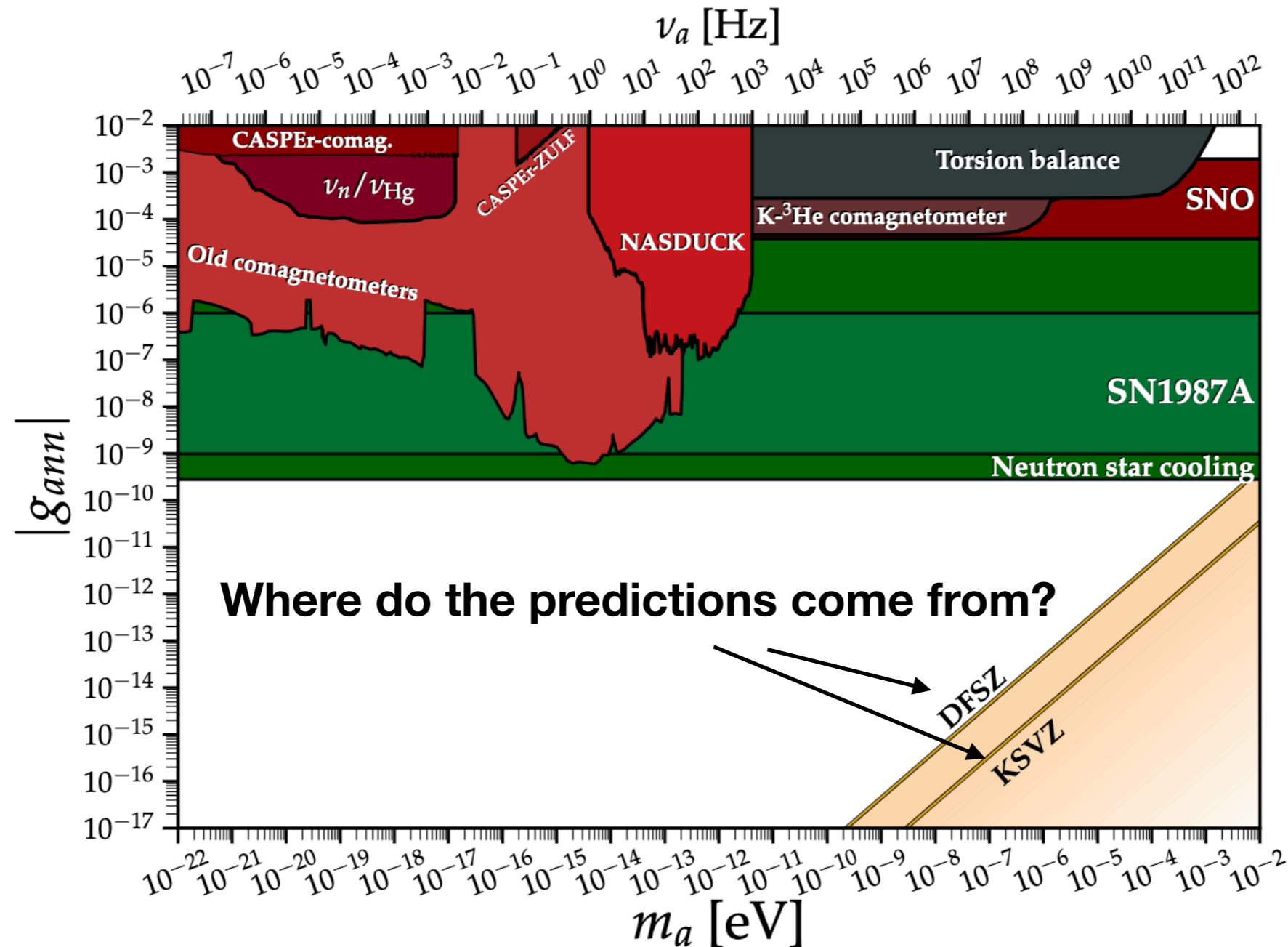
Current bounds use vacuum couplings\*

\*this is not completely true...

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# QCD axion couplings in vacuum

GG di Cortona et al. '15

**Construct low energy EFT of nucleons, pions and the QCD axion**

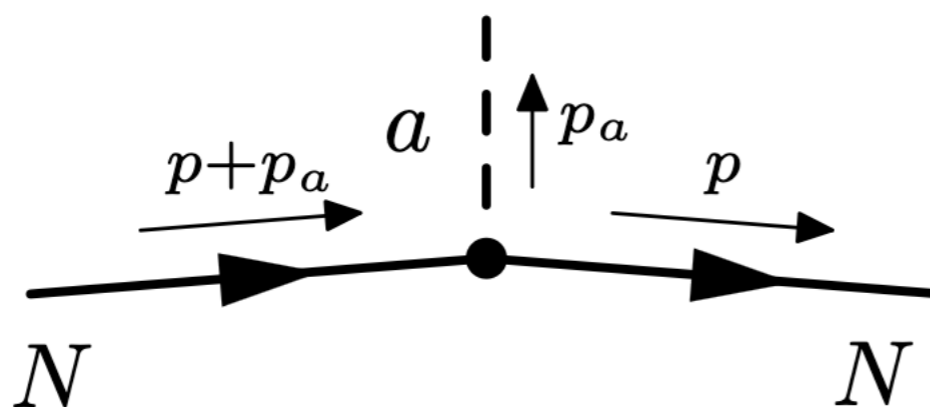


# QCD axion couplings in vacuum

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Construct low energy EFT of nucleons, pions and the QCD axion

Leading order:


$$= -\frac{1}{f_a} c_N S \cdot p_a$$

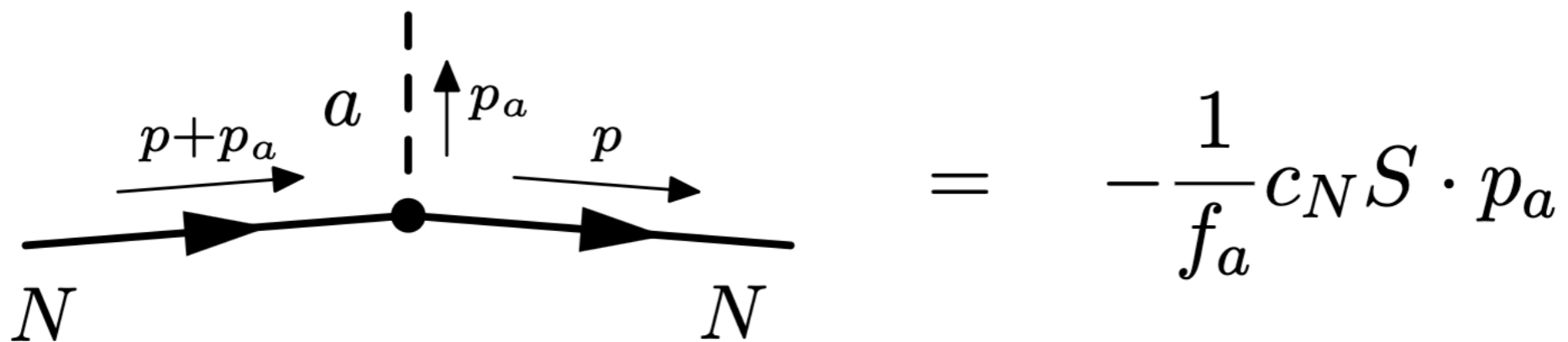
$$c_p^{\text{LO, KSVZ}} = -0.426(36), \quad c_n^{\text{LO, KSVZ}} = -0.008(29),$$

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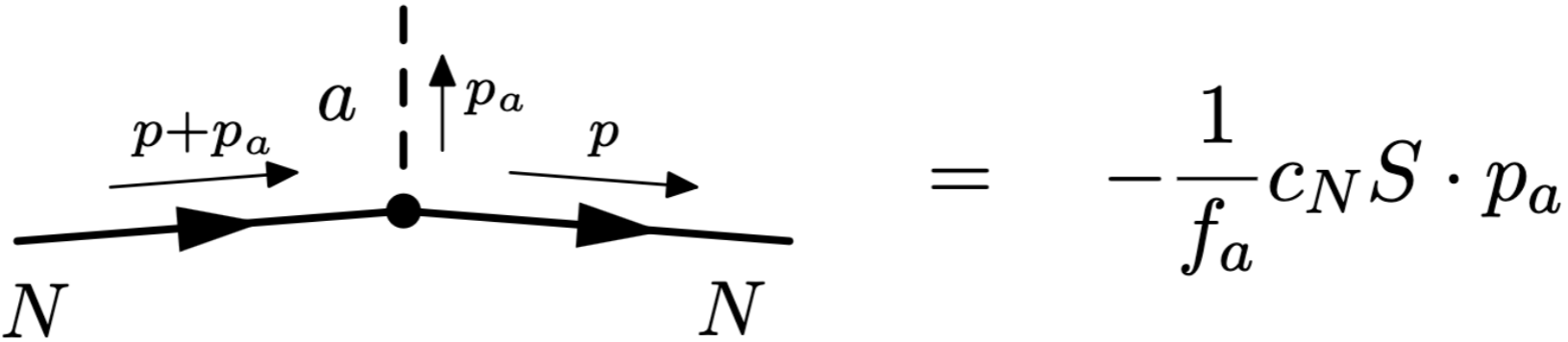
Small due to a cancellation of two unrelated terms!

# QCD axion couplings in vacuum

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Leading order:



Higher order terms also contribute

They change vacuum couplings!

See also Vonk et al. 2021

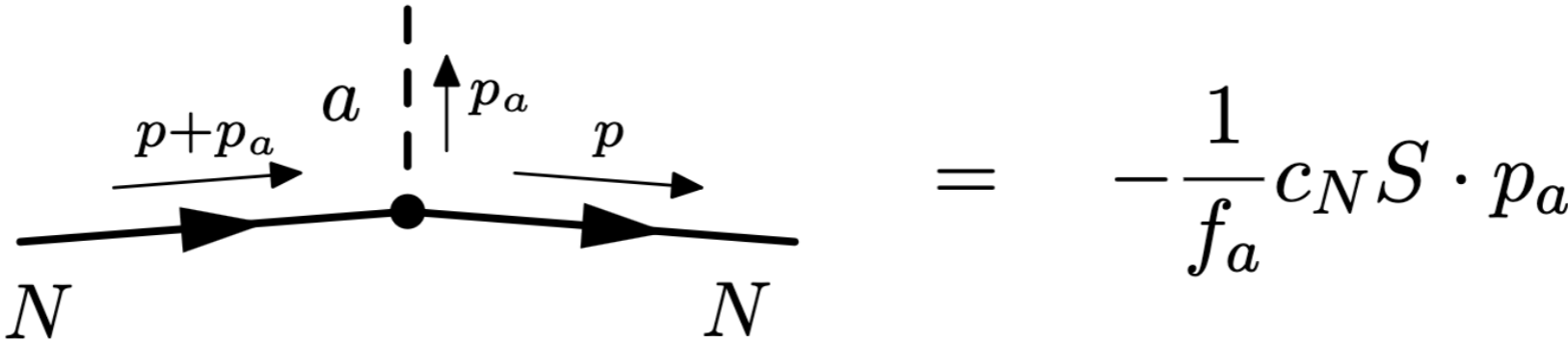
**They give rise to density corrections!**

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They give rise to density corrections!

What is the validity of this EFT?

Low-lying resonances e.g.  $\Delta(1232)$  have been integrated out...

...some terms are enhanced!

# QCD axion couplings at finite density

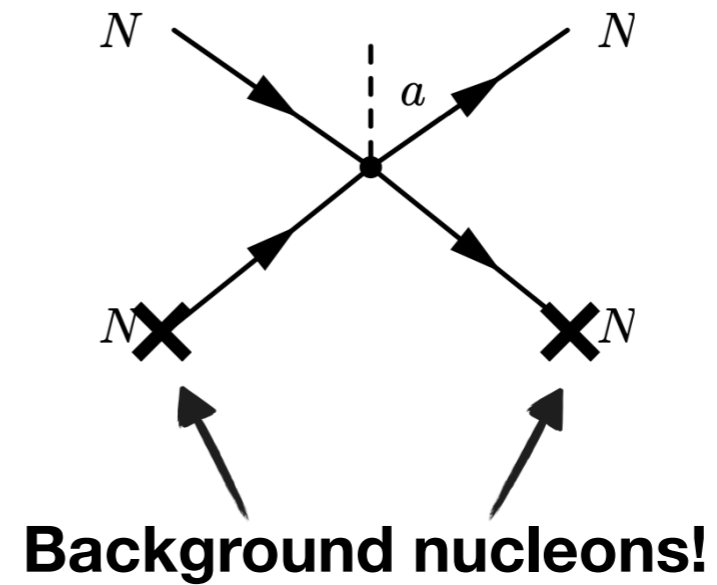
**Schematic example:**

$$\mathcal{L}_{\pi NN}^{(2)} = \frac{c_D}{2f_\pi^2 \Lambda_\chi} (\bar{N}N) (\bar{N}S \cdot uN)$$

# QCD axion couplings at finite density

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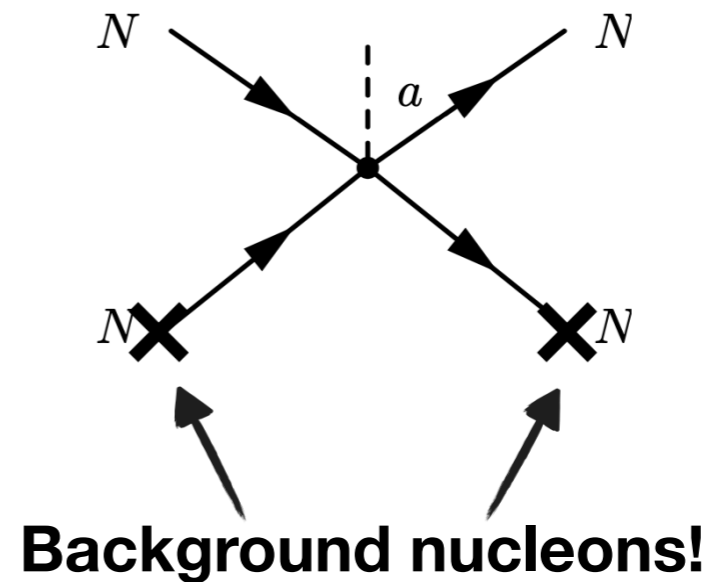
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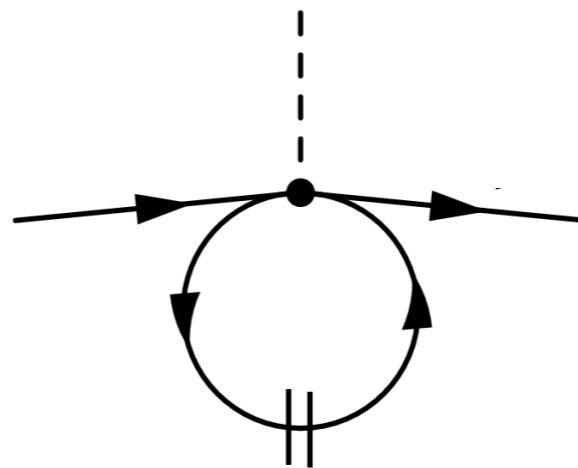
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More concretely:



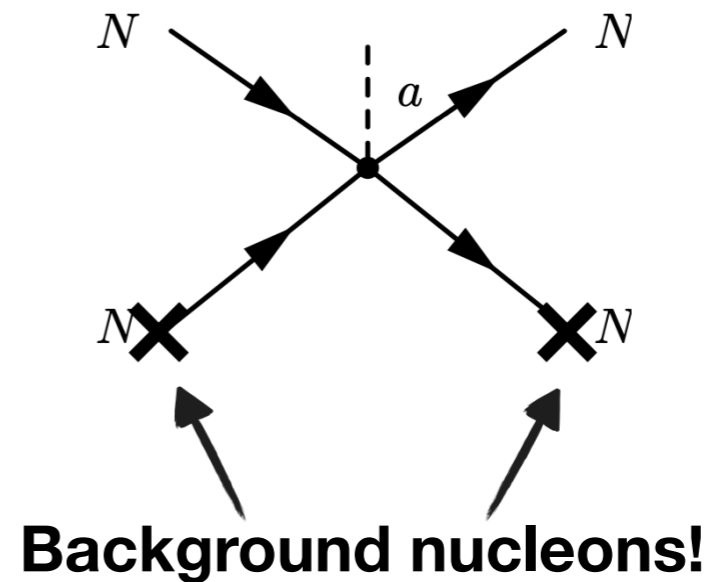
With finite density propagator:

$$iG(k) = \frac{i}{k^0 + i\epsilon} - 2\pi\delta(k^0)\theta(k_f - |\vec{k}|)$$

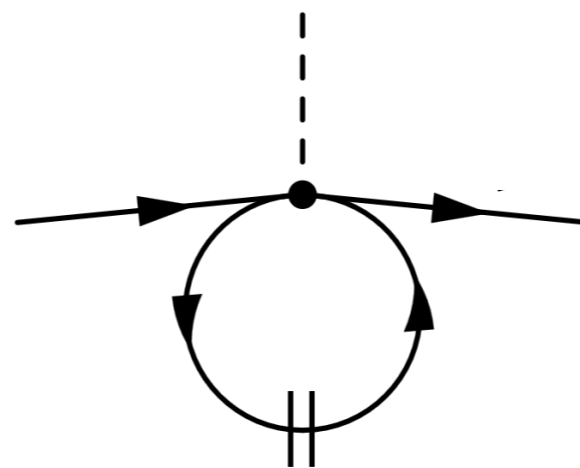
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Filled 'Fermi sea'

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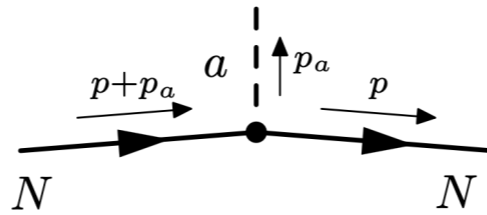
NR fermion propagator  $\longrightarrow$



# QCD axion couplings at finite density

Systematic expansion in nucleon momenta:  $\left(\frac{k}{4\pi f_\pi}\right)^\nu \rightarrow \left(\frac{k_f}{4\pi f_\pi}\right)^\nu$

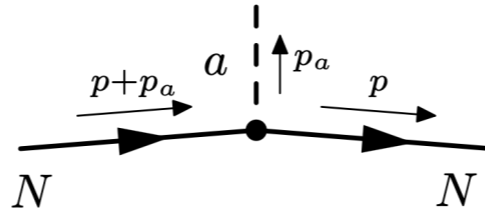
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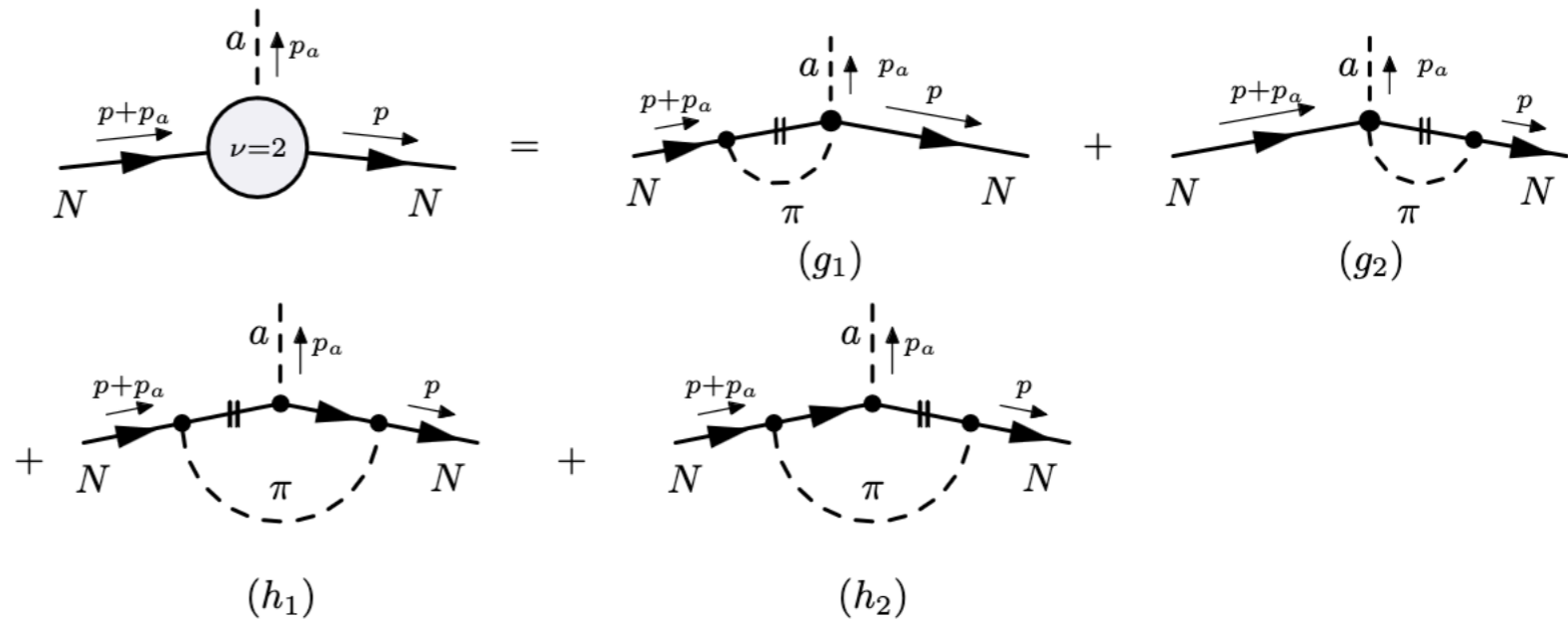
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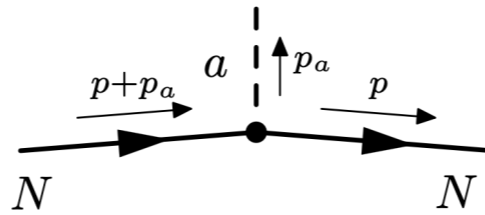
$$\left(\frac{k_f}{4\pi f_\pi}\right)^2$$



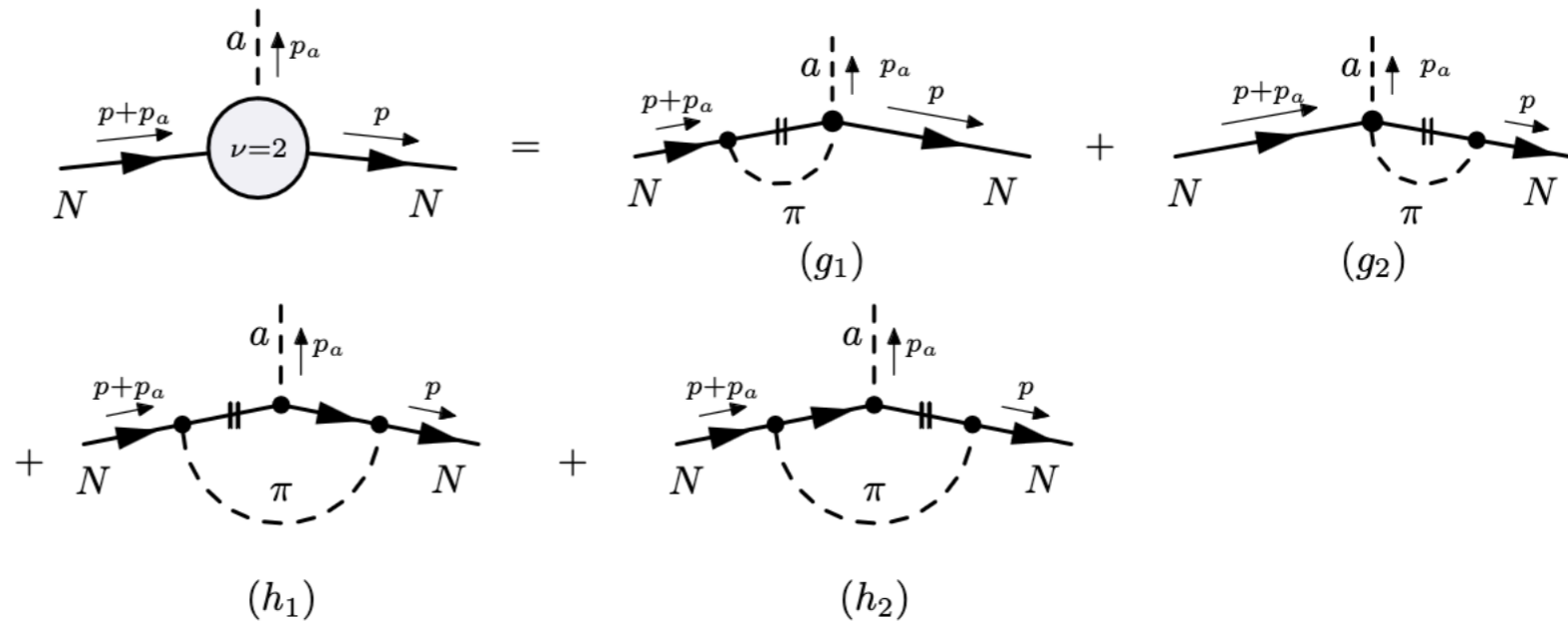
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$$\left(\frac{k_f}{4\pi f_\pi}\right)^2$$



Naively this is suppressed in the chiral expansion...

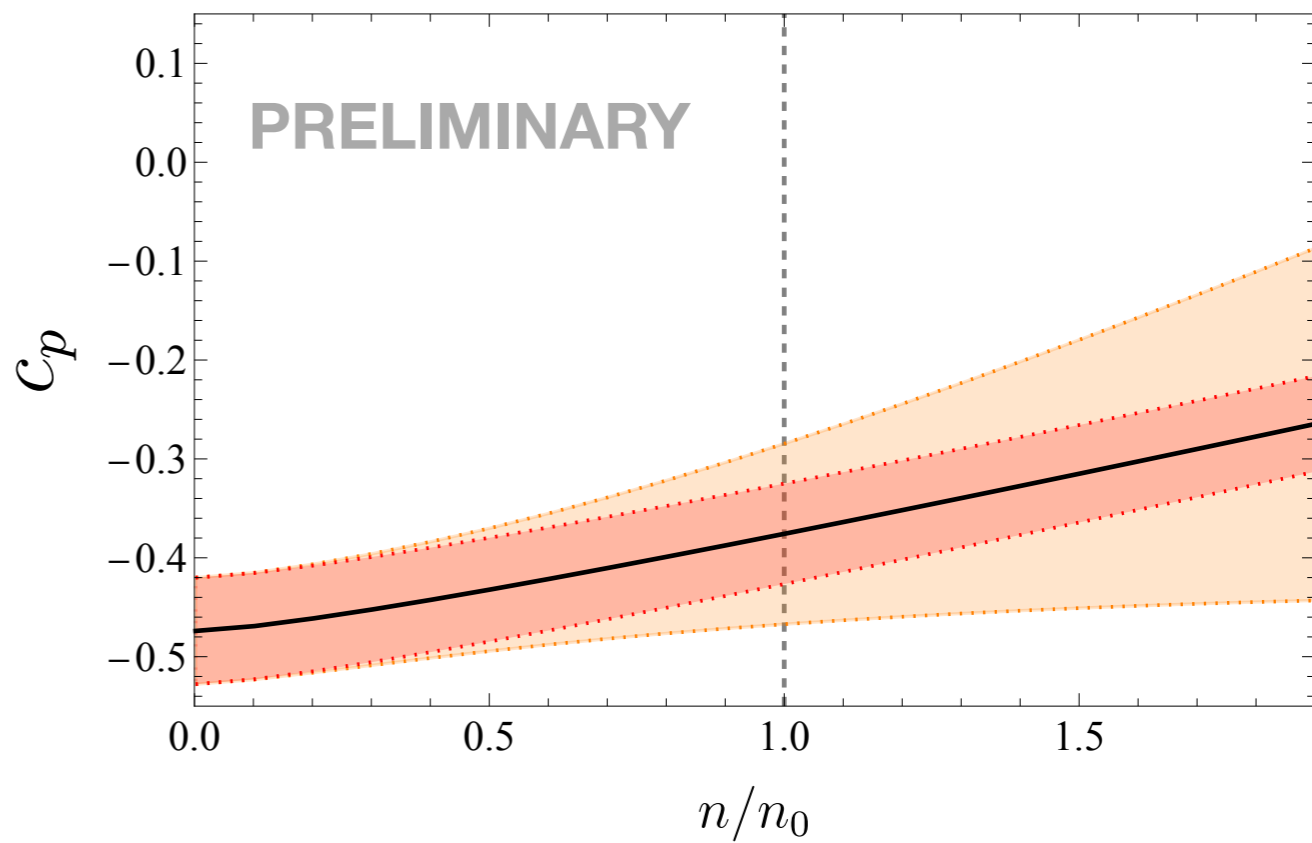
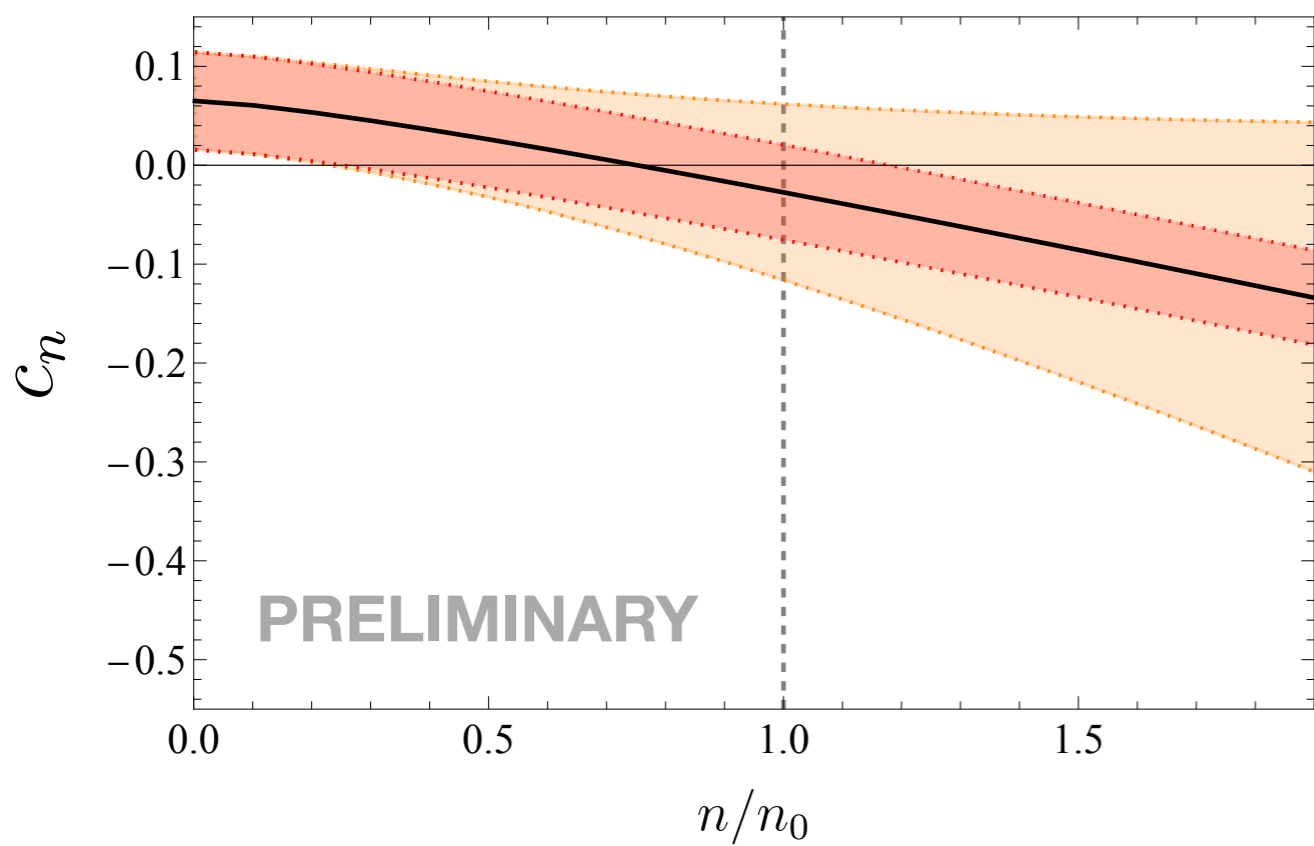
... but due to  $\Delta(1232)$  resonances some corrections are large!

This happens the first time at  $\nu = 3$

# Results

Example 1:

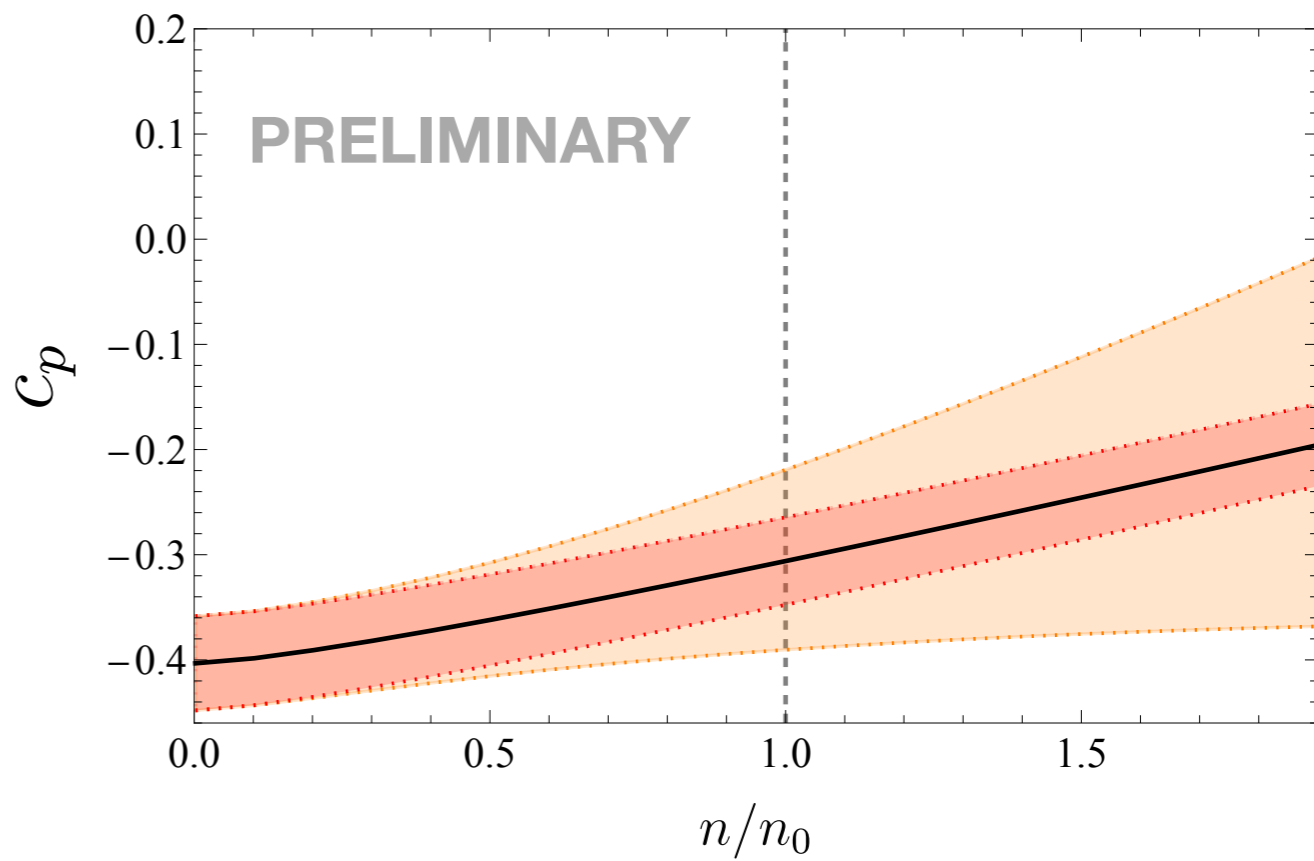
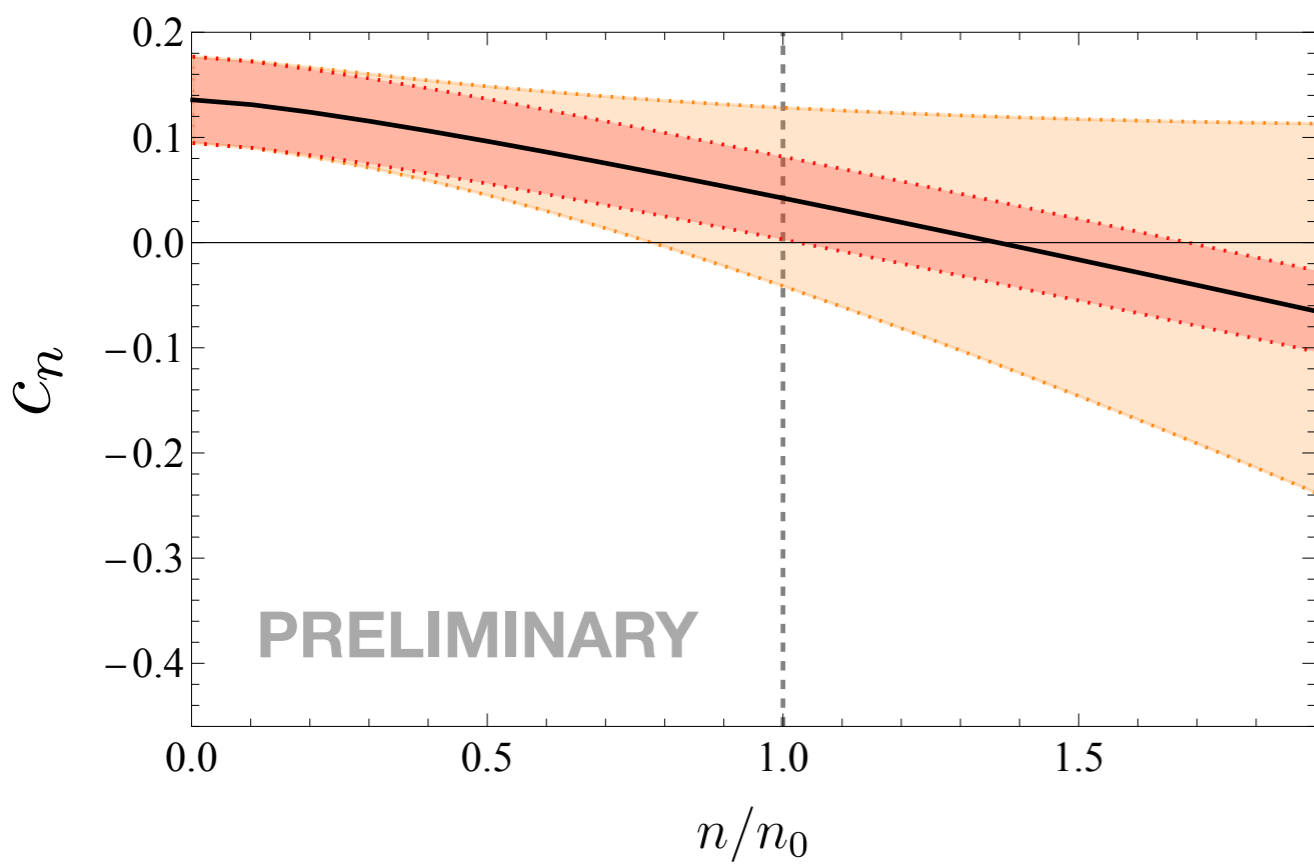
**KSVZ axion**



# Results

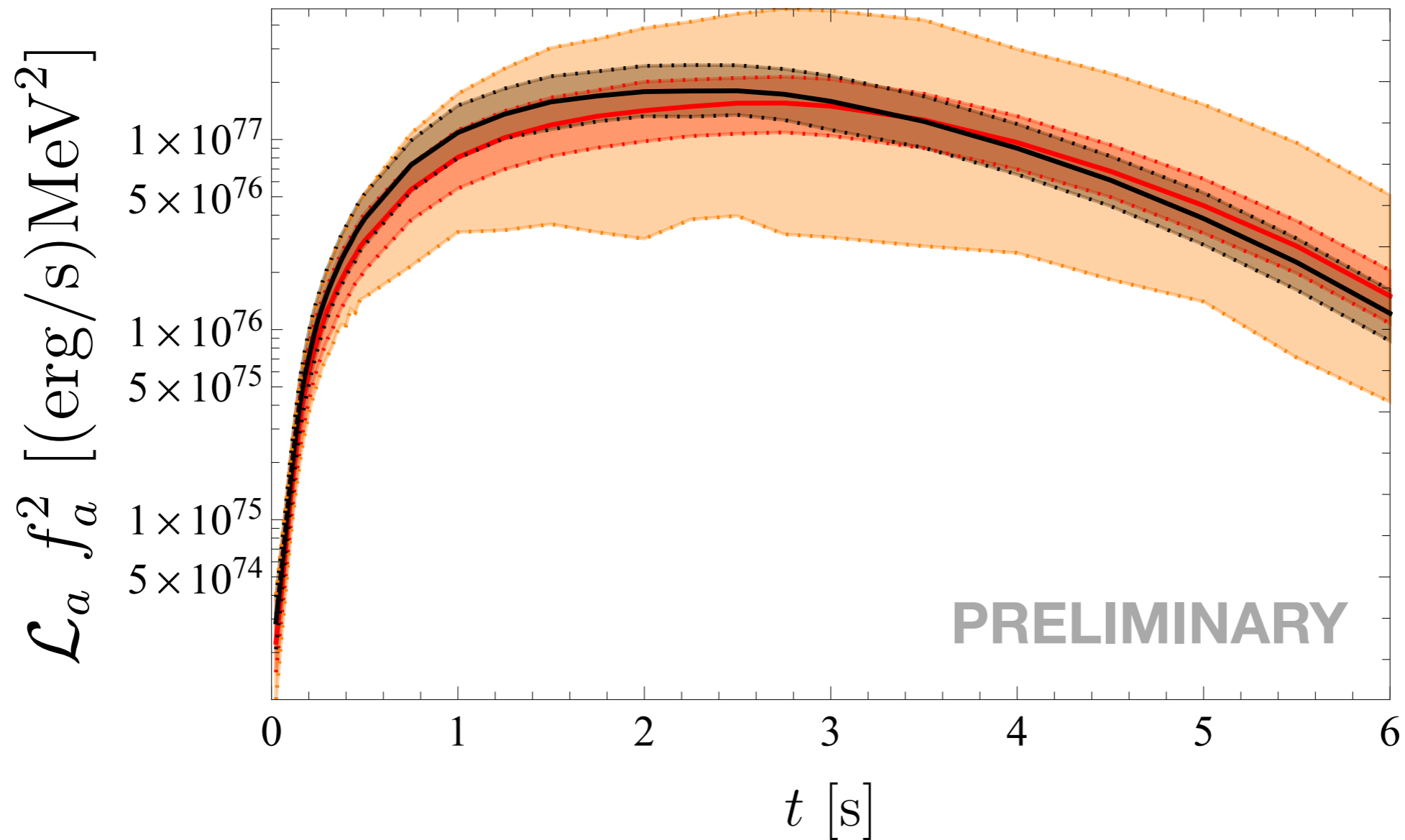
Example 2:

**DFSZ axion**  $\sin^2(\beta) = 1/2$



# Implications for supernova bound

example: KSVZ axion



Supernova profile from <https://wwwmpa.mpa-garching.mpg.de/ccsnarchive/>

**Changes the SN bound by O(1)**

**Adds large uncertainty**

# Implications for neutron star cooling

High densities inside NS of  $n \sim O(\text{few})n_0$

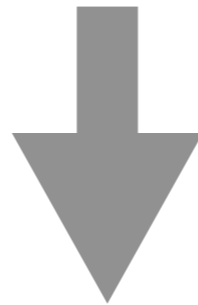
**ChPT expansion breaks down at these densities!**

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High densities inside NS of  $n \sim O(\text{few})n_0$

ChPT expansion breaks down at these densities!

No way to consistently calculate the axion couplings

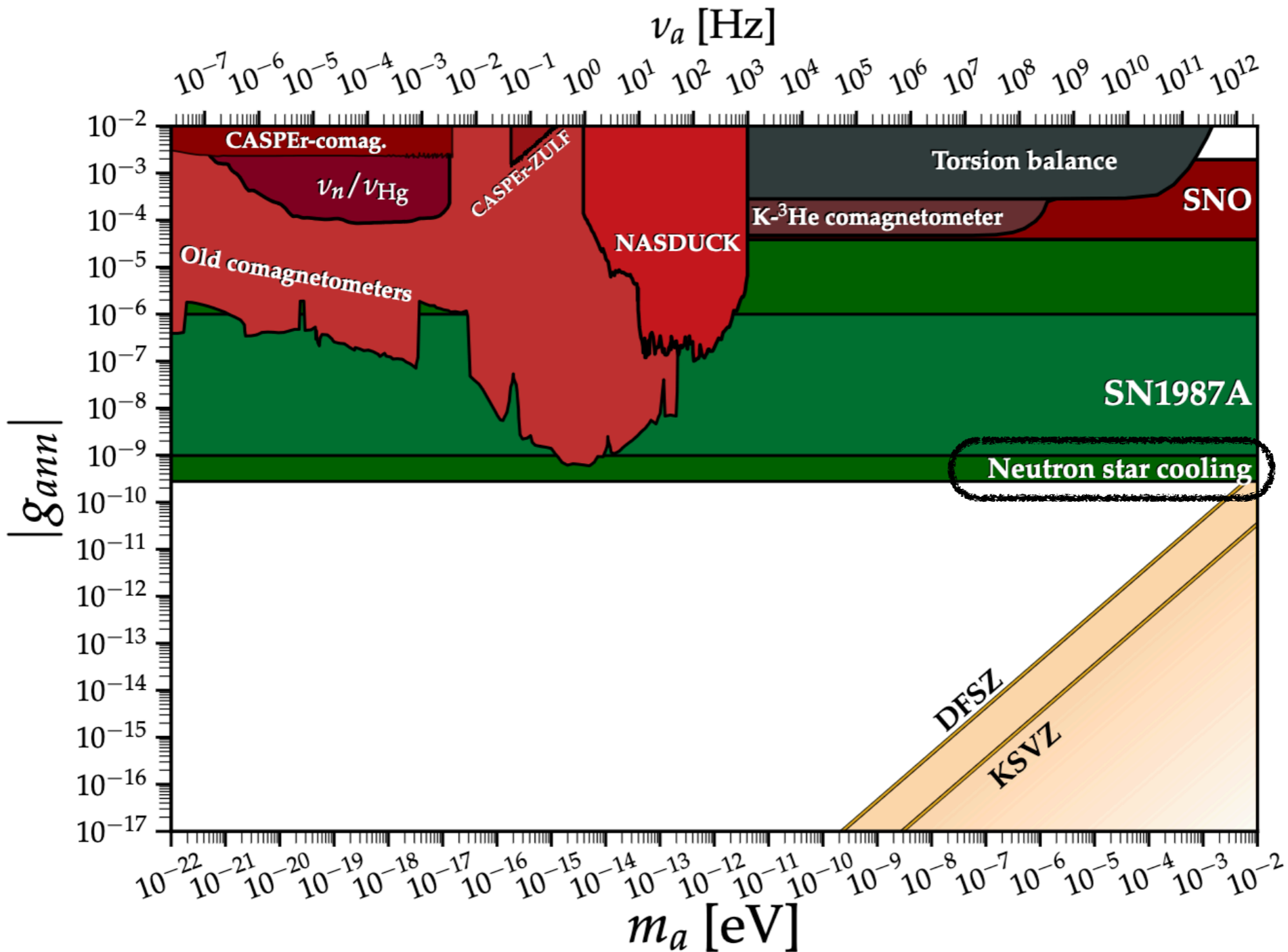


No distinction between models possible, only order of magnitude estimates

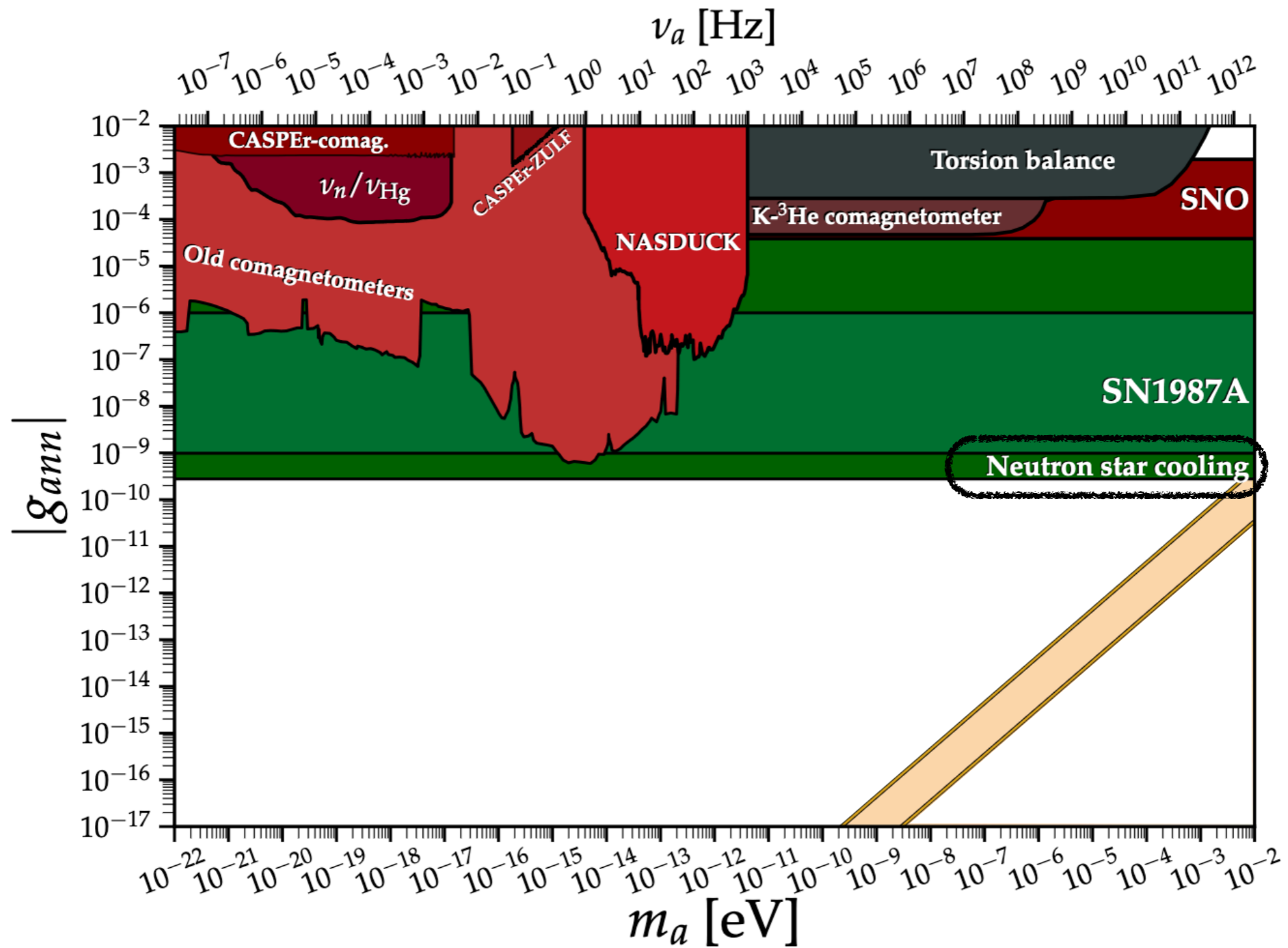
e.g.  $|c_N| \sim O(0.3) \pm 0.3$



# Implications for neutron star cooling



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# Astrophobic axions or a model-independent SN bound

di Luzio et al. '15

**Some aspects of 'astrophobic axion' survive at finite density, only subleading corrections**

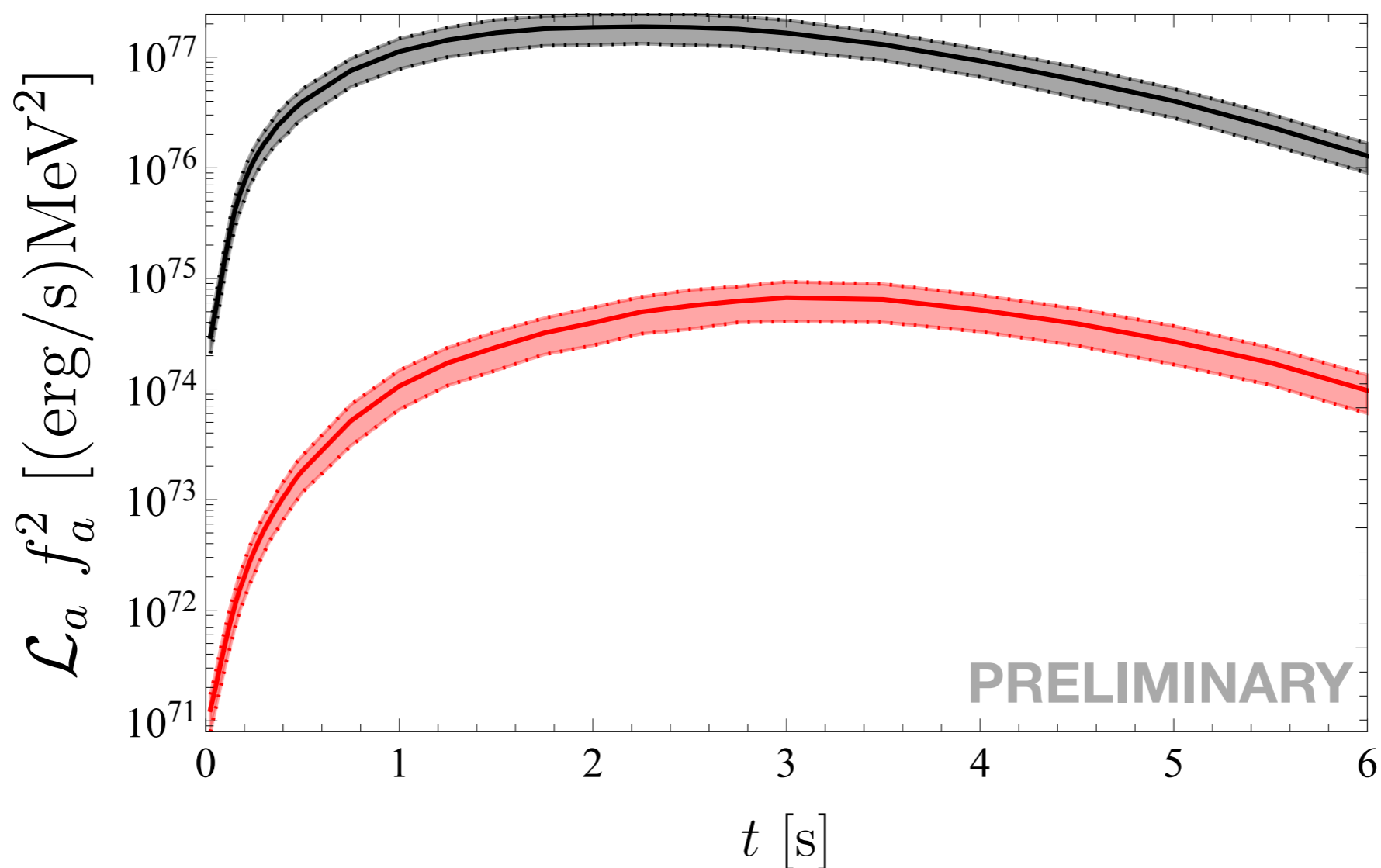
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For current best bound see Lucente et al. '22



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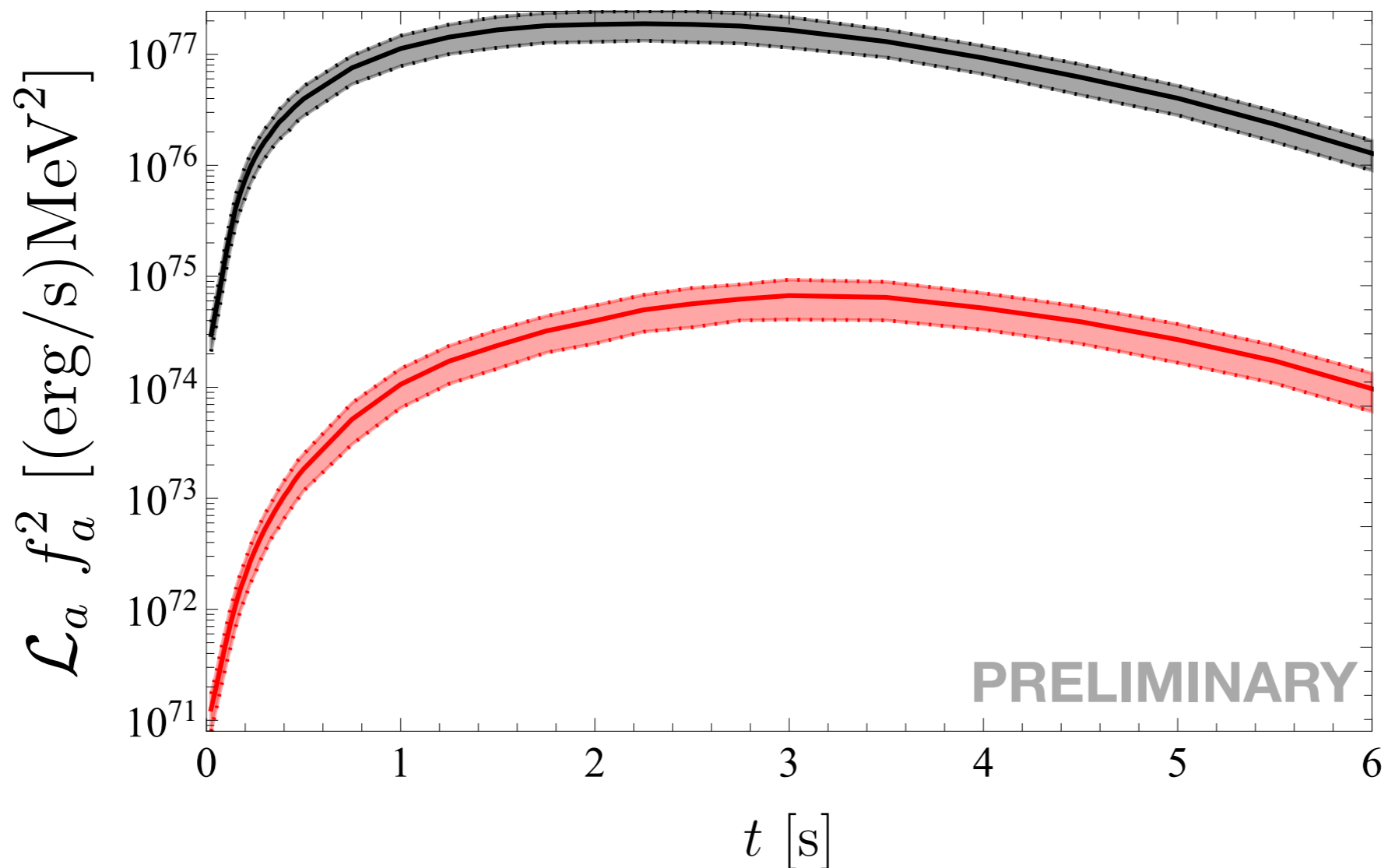
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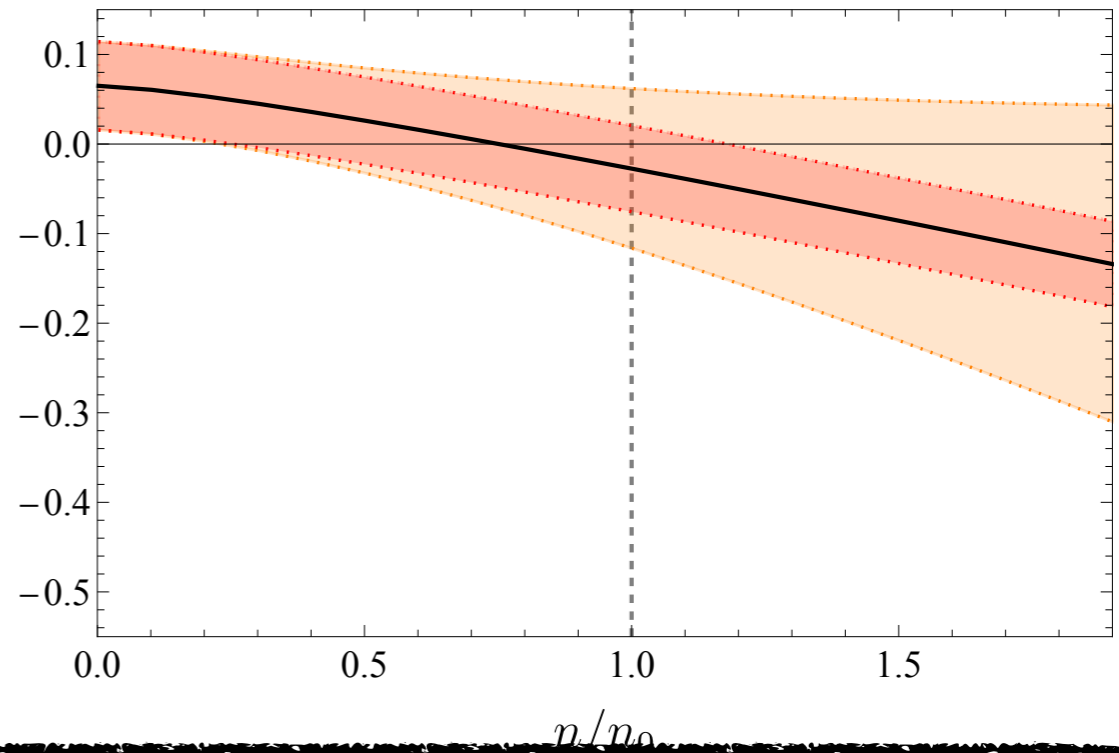
Leads to  $f_a \gtrsim O(5) \times 10^7 \text{ GeV}$

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## QCD axion coupling to nucleons at finite density

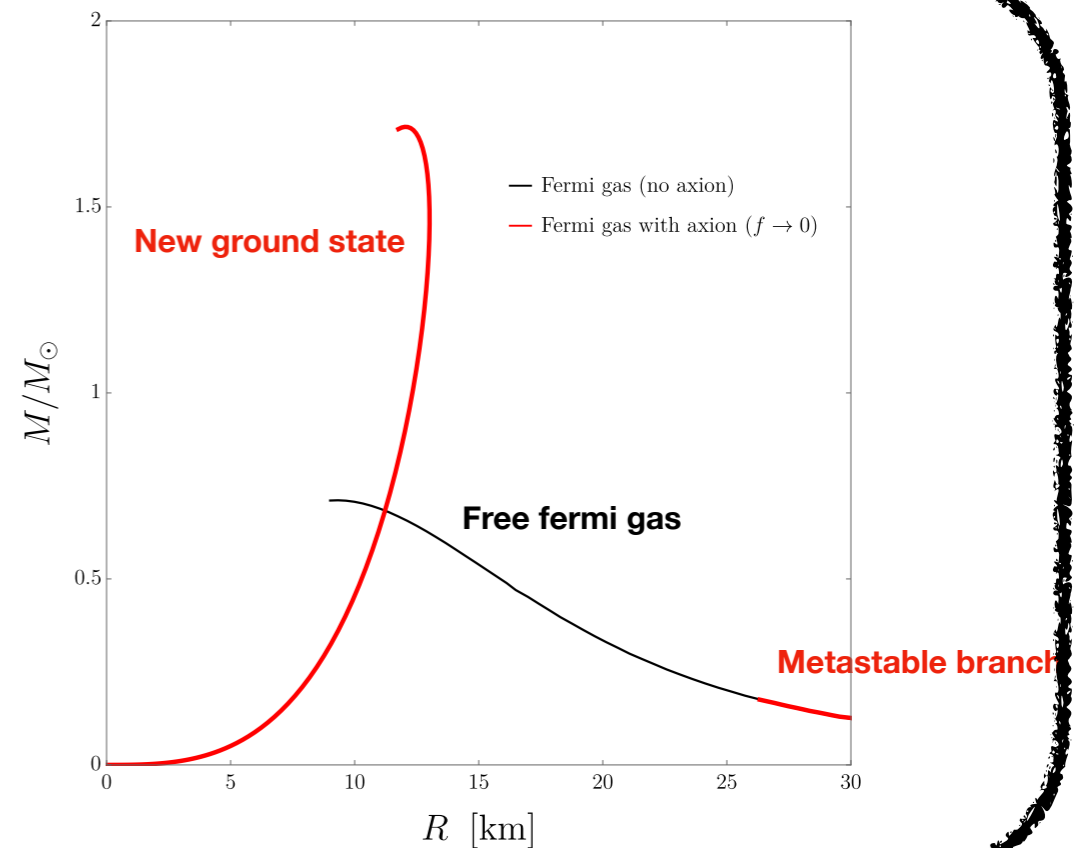
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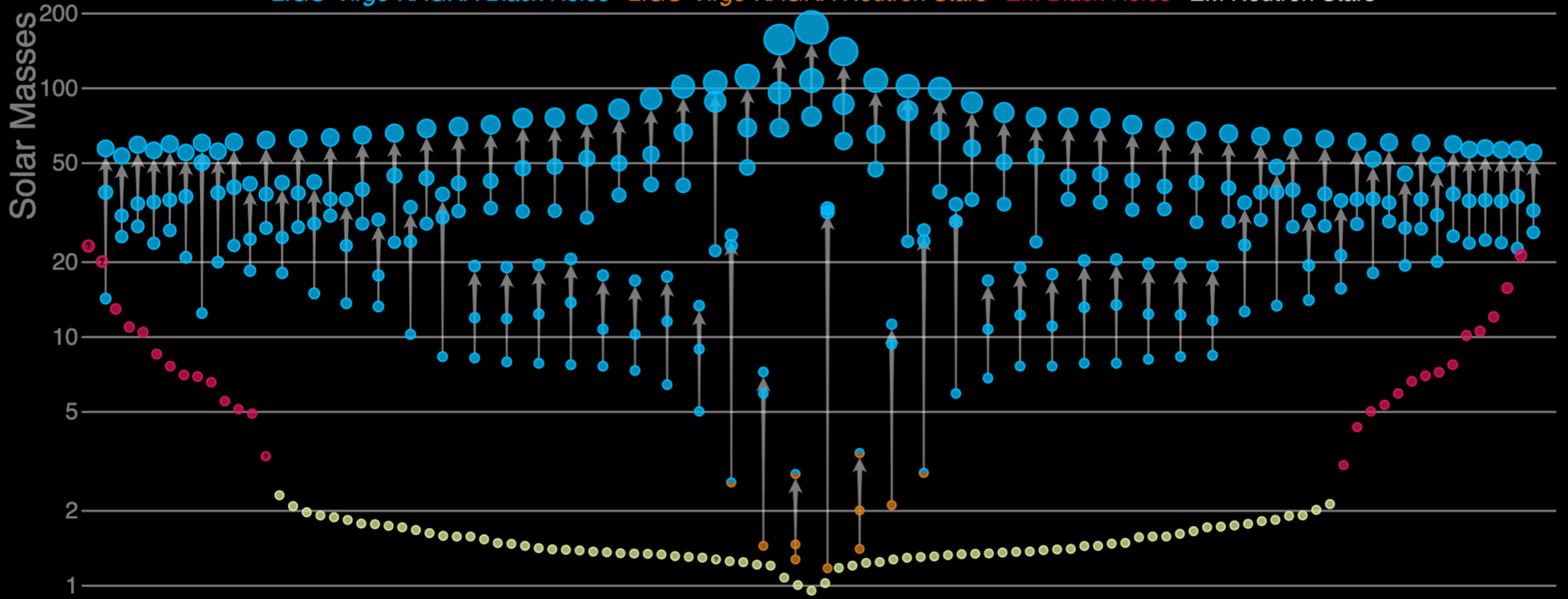


# Heavy neutron stars from light scalars

... or how axions (ALPs) change stars

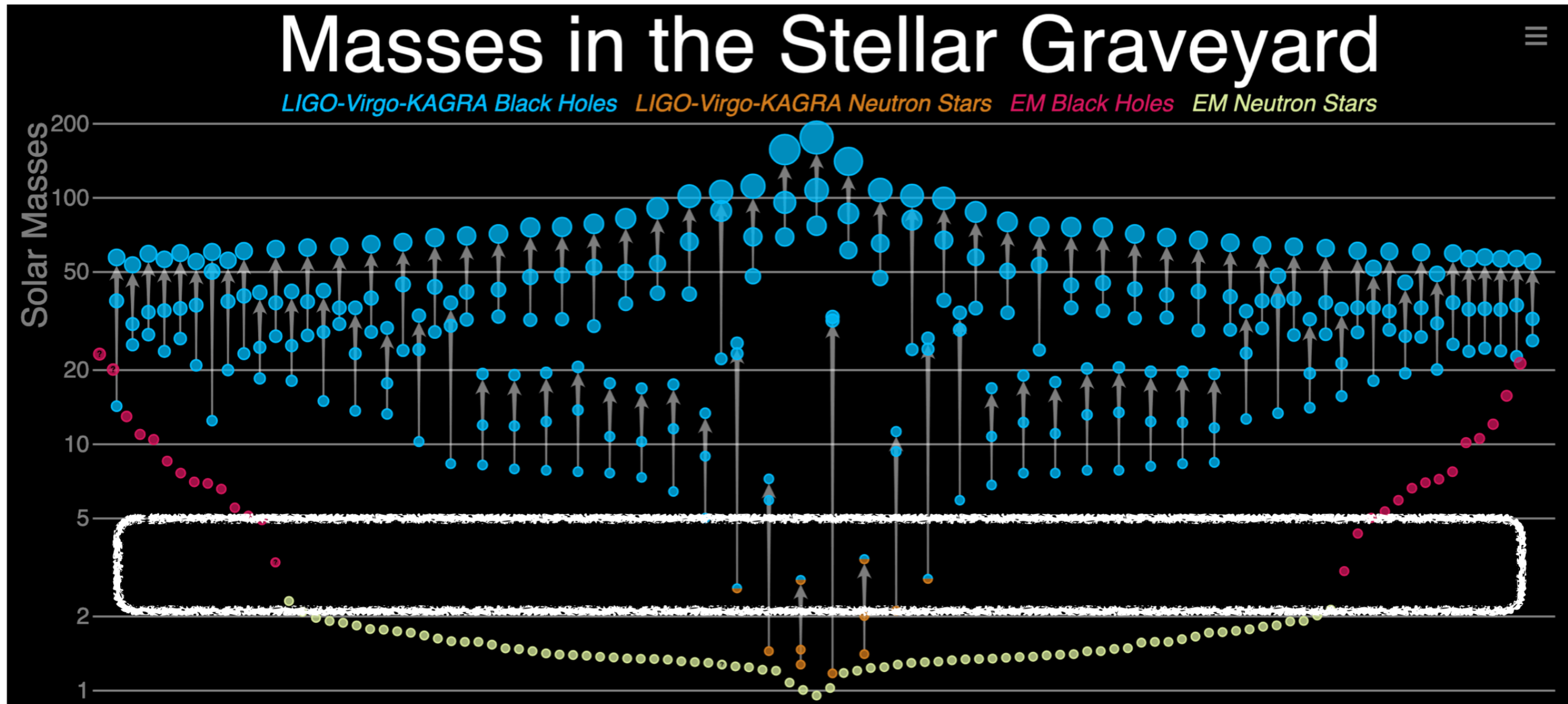
## Masses in the Stellar Graveyard

*LIGO-Virgo-KAGRA Black Holes* *LIGO-Virgo-KAGRA Neutron Stars* *EM Black Holes* *EM Neutron Stars*



# Heavy neutron stars from light scalars

... or how axions (ALPs) change stars



Can new (light) physics make neutron stars heavier?



# Heavy neutron stars from light scalars

**How would NSs look like if neutrons would be lighter?**

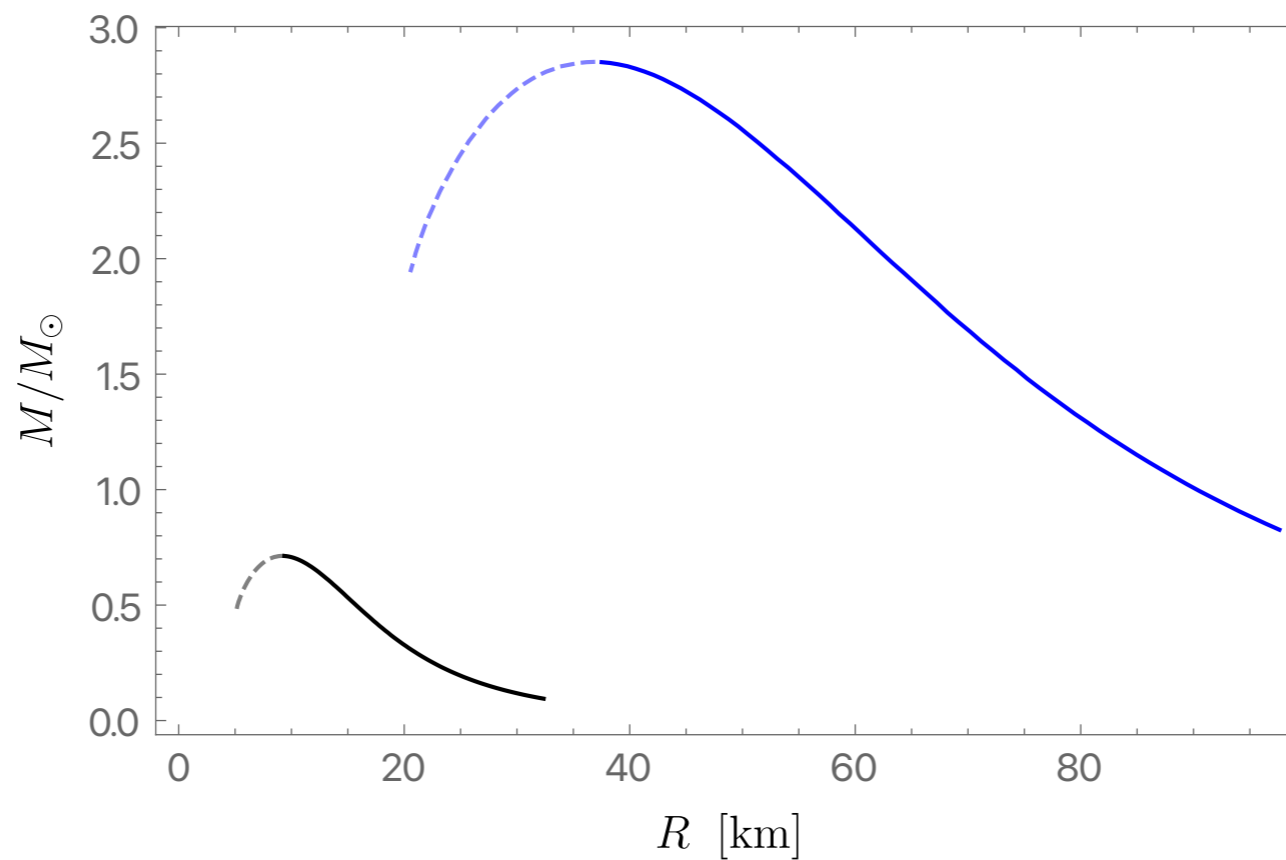
Toy model: Free Fermi gas of neutrons

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—  $m = 1 \text{ GeV}$       —  $m = 0.5 \text{ GeV}$



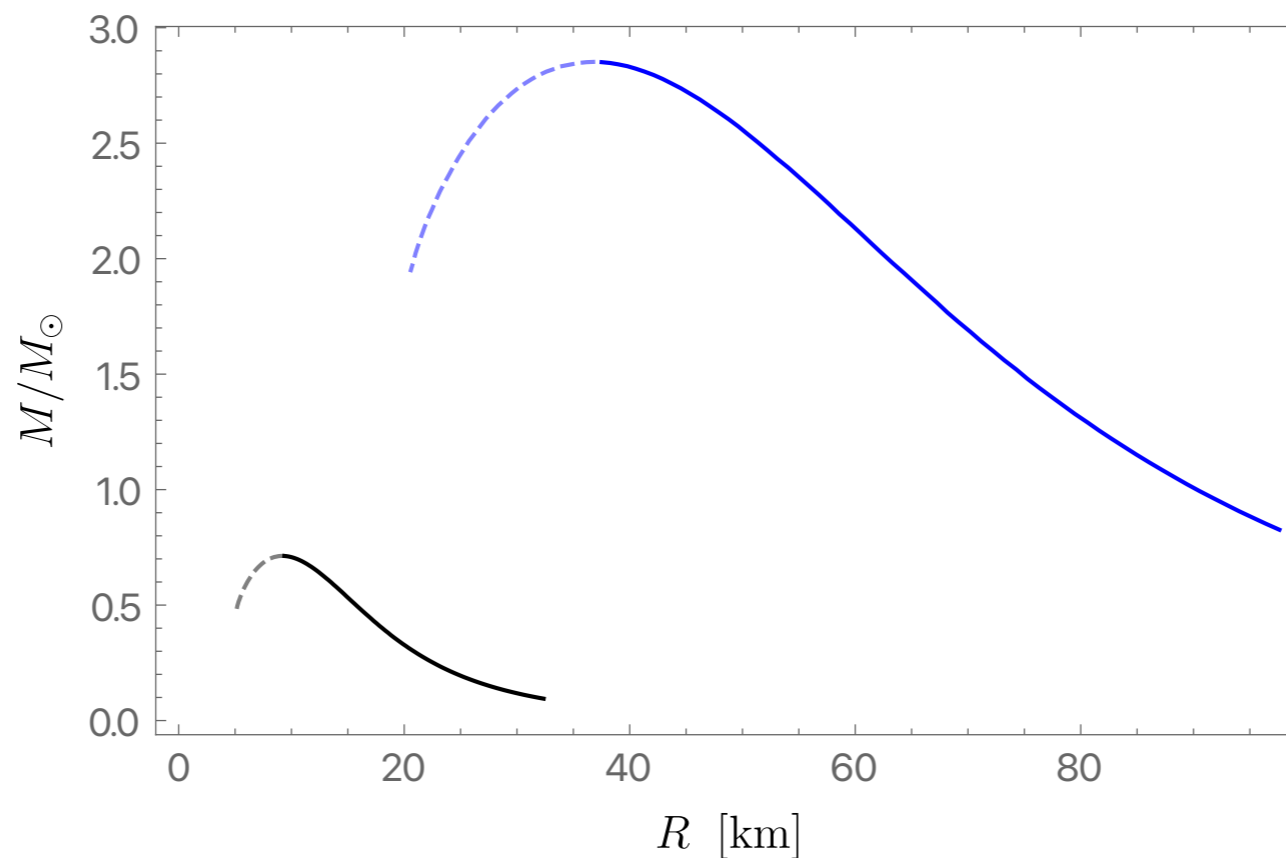
$$\frac{M_{\max}(m_1)}{M_{\max}(m_2)} = \left(\frac{m_2}{m_1}\right)^2$$

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Toy model: Free Fermi gas of neutrons

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**Why? In NR limit: at fixed energy density, need more neutrons!  $\epsilon \sim nm_N$**

# Heavy neutron stars from light scalars

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**Field dependent nucleon mass**  $m_N(\phi) = 1 - g \left( \cos \left( \frac{\phi}{f} \right) - 1 \right)$

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**This happens for the vanilla QCD axion with  $g \sim 0.025$**

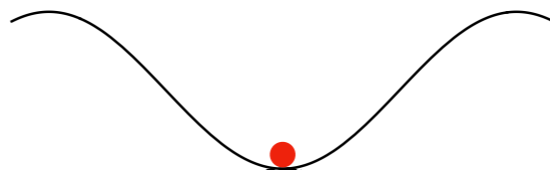
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Potential can get destabilized



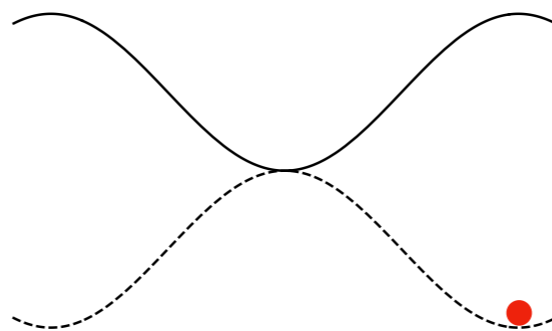
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$$n \gtrsim \Lambda^4 / gm_N$$



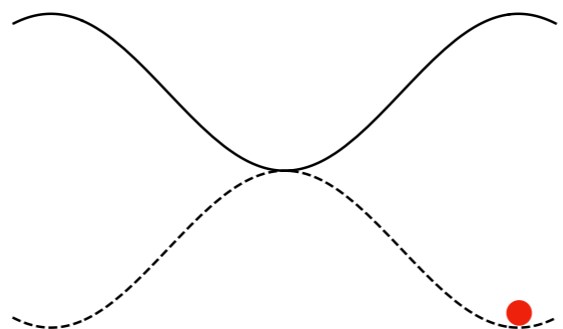
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**ALP develops a non-trivial expectation value around dense objects!**

# Fermi gas - axion system

**Consists of complicated coupled equations of gravity and scalar field EOMs.**

**... can be solved numerically** 

# Fermi gas - axion system

Consists of complicated coupled equations of gravity and scalar field EOMs.

... can be solved numerically ✓

... but there is a simplifying limit:

## Hierarchy of scales

$$R \gg m_{\phi}^{-1}(n) \sim \frac{\pi f}{\sqrt{gm_N n - \Lambda^4}}$$

Size of star

size of scalar field

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All gradient effects of the axion become negligible

Opposite hierarchy of scales prevents us from seeing it on earth:  $R_{\text{nucleus}} \ll m_{\phi}^{-1}$

# Axion Fermi gas equation of state

**Axion follows its potential**  $\frac{\partial V}{\partial \phi} + n \frac{\partial m_N}{\partial \phi} = 0$



$$\phi(n)$$

# Axion Fermi gas equation of state

**Axion follows its potential**

$$\frac{\partial V}{\partial \phi} + n \frac{\partial m_N}{\partial \phi} = 0$$



$$\phi(n)$$

**Usual free fermi gas  
with field dependent mass**



$$\varepsilon(n, \phi) = \varepsilon_N(n, \phi) + V(\phi)$$

$$p(n, \phi) = p_N(n, \phi) - V(\phi)$$

**Equation of state**

# Axion Fermi gas equation of state

## Equation of state

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$$p(n, \phi) = p_N(n, \phi) - V(\phi)$$

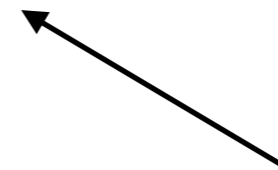
### Competing effects:

More pressure at same energy density



Mass reduction: stiffens the equation of state

Potential: softens the equation of state



Less pressure at same energy density



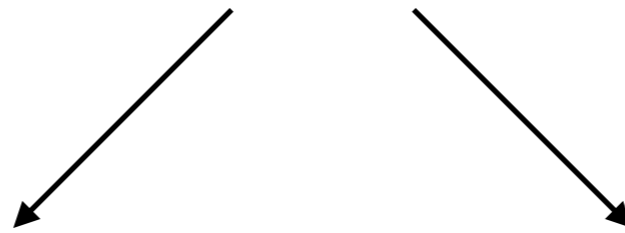
# Axion Fermi gas equation of state

## Equation of state

$$\varepsilon(n, \phi) = \varepsilon_N(n, \phi) + V(\phi)$$

$$p(n, \phi) = p_N(n, \phi) - V(\phi)$$

Parameter space splits in two regions



**Coexistence region**

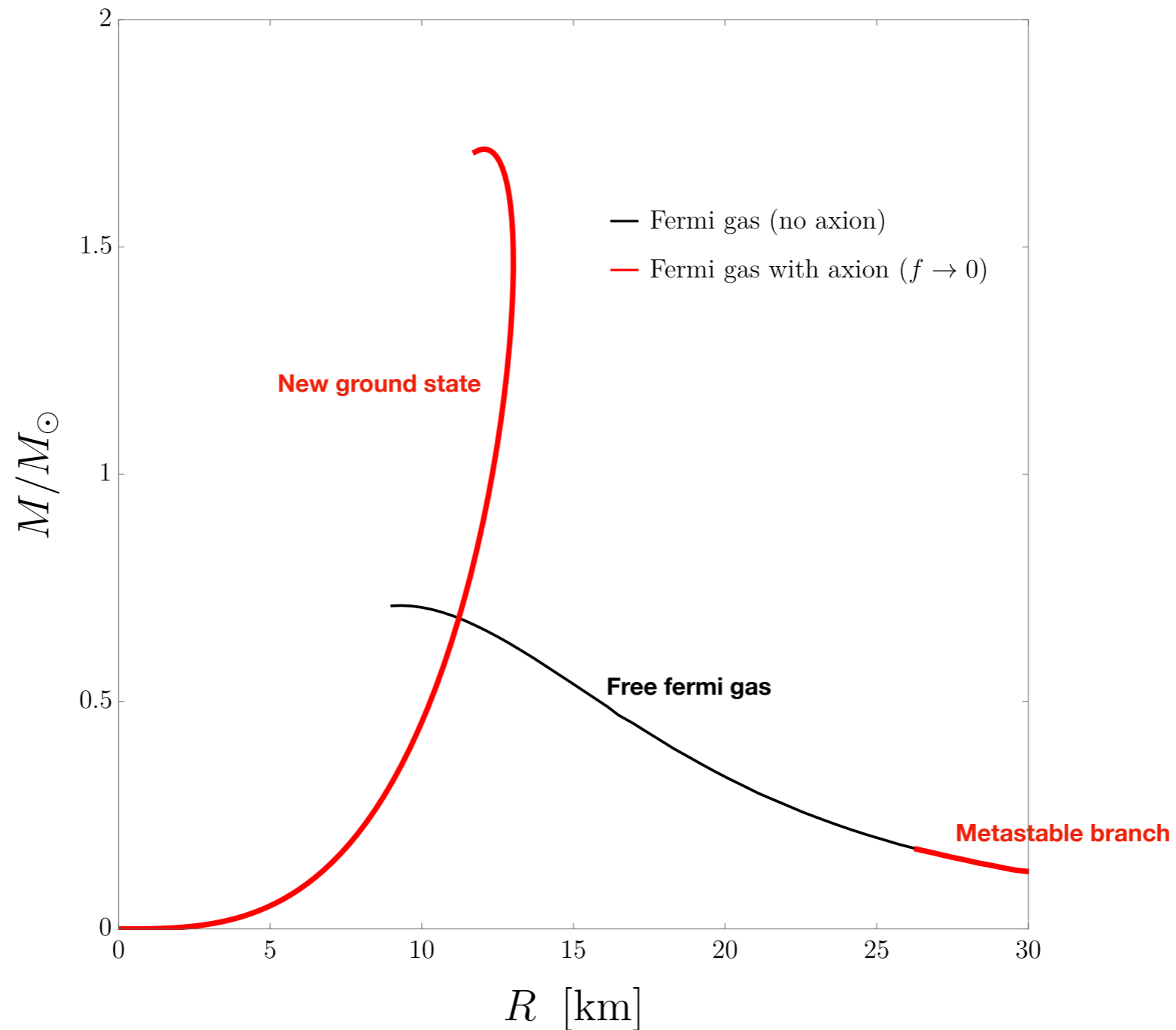
**New ground state of matter!**

**1st order PT inside star**

**Energy per particle:**

$$\frac{\varepsilon}{n} < m_N$$

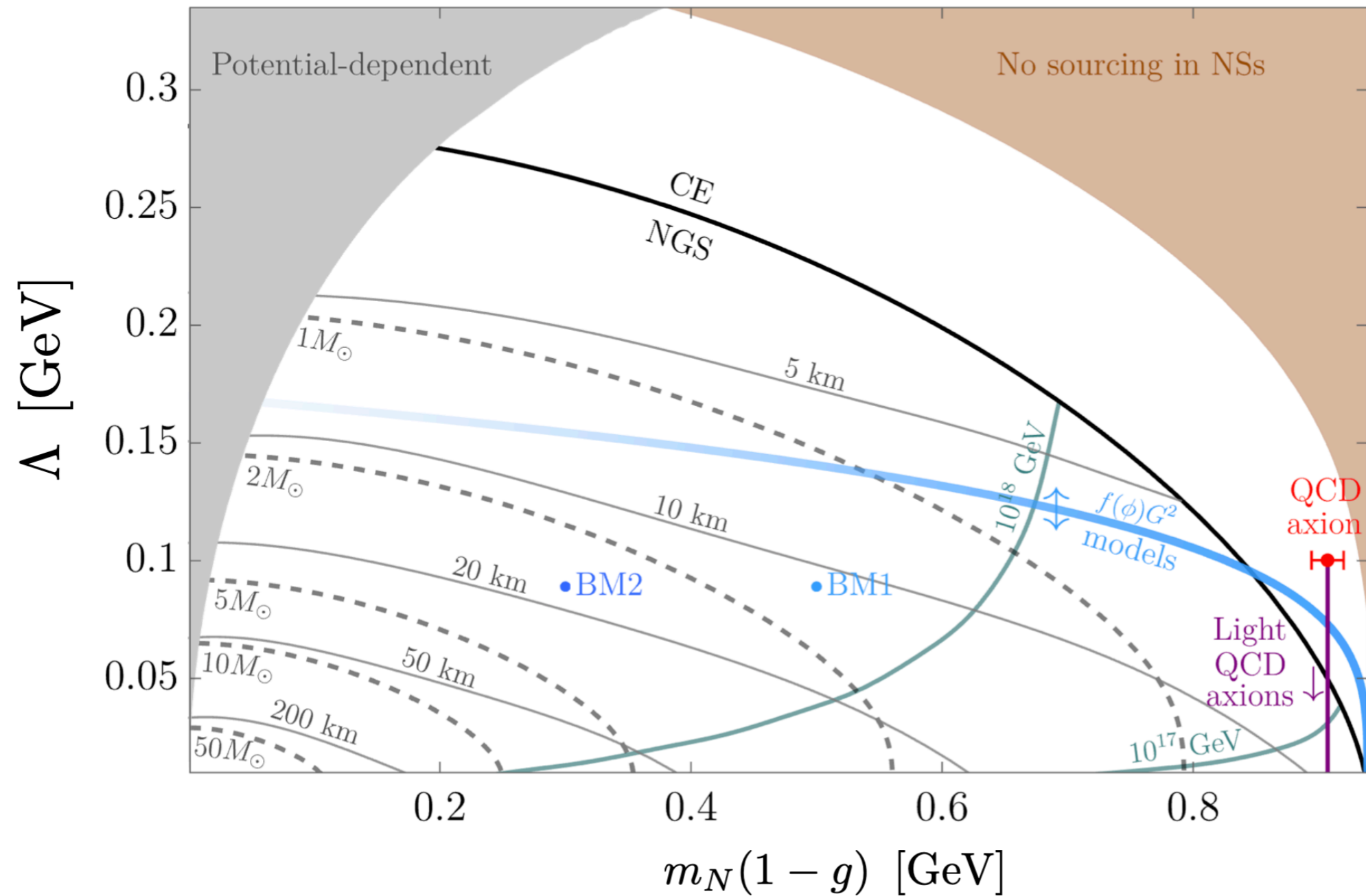
# Heavy neutron stars from light scalars



**New ground state: Energy per particle less than for separated nucleons  $m_N$**

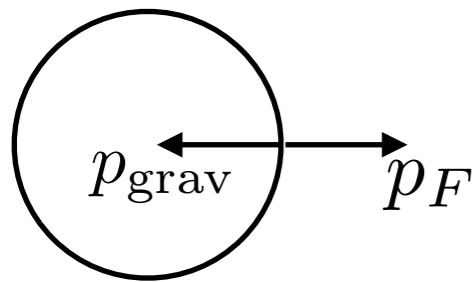
In absence of gravity bound by ALP field!

# Heavy neutron stars from light scalars



Future GW experiments can distinguish BHs and other compact objects

# White dwarfs as a probe of light QCD axions



**What is a WD?**

Electron degeneracy pressure vs gravity

**White dwarfs are much less dense than NSs**



**EOS is much better understood!**



**Can be used to test models that change the structure of WDs**

# White dwarfs as a probe of light QCD axions

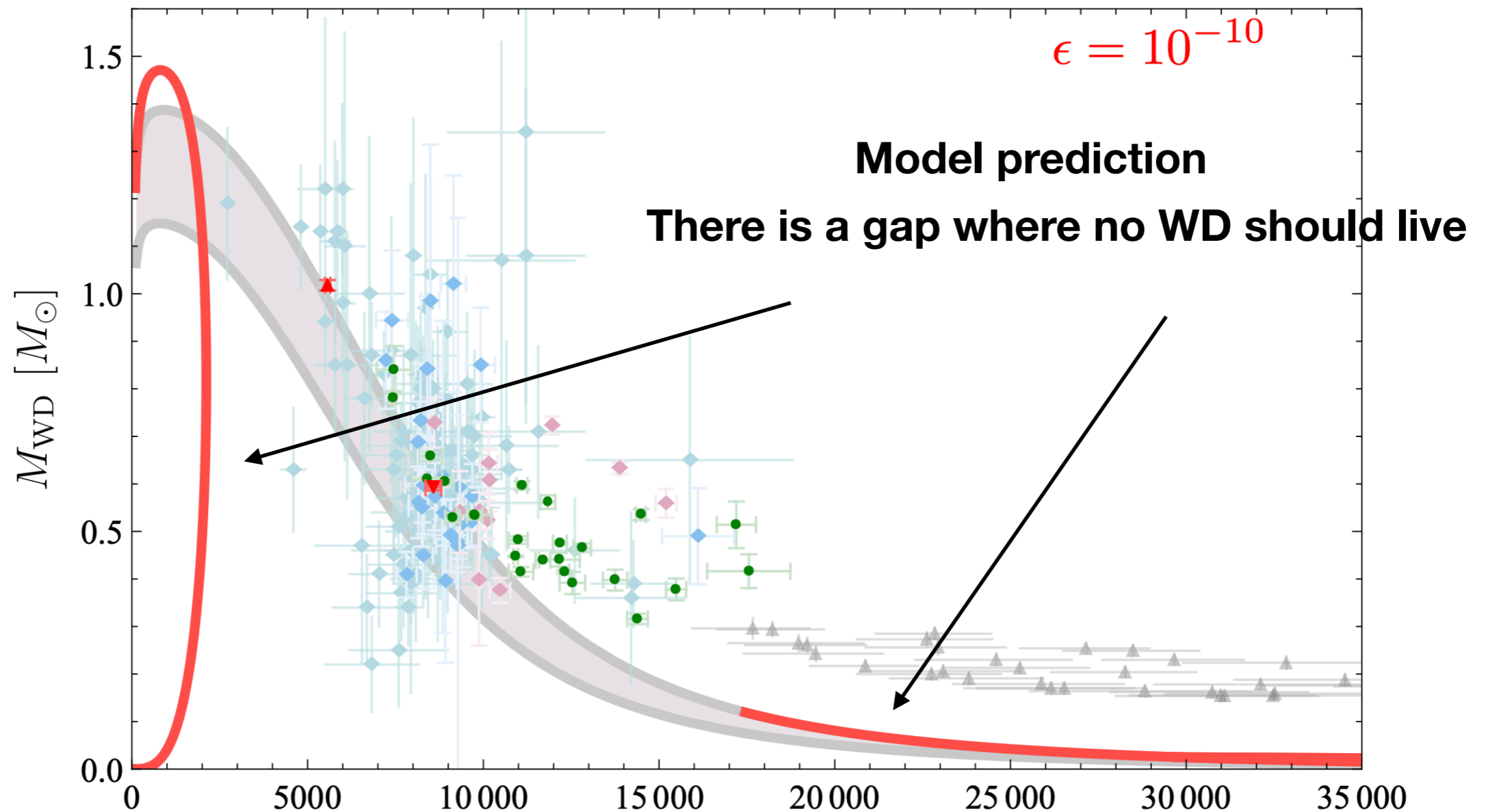
**Light QCD axion:**

$$V(\phi) = -\epsilon m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2 \left( \frac{\phi}{2f} \right)} \quad \text{with } \epsilon \ll 1$$

# White dwarfs as a probe of light QCD axions

Light QCD axion:

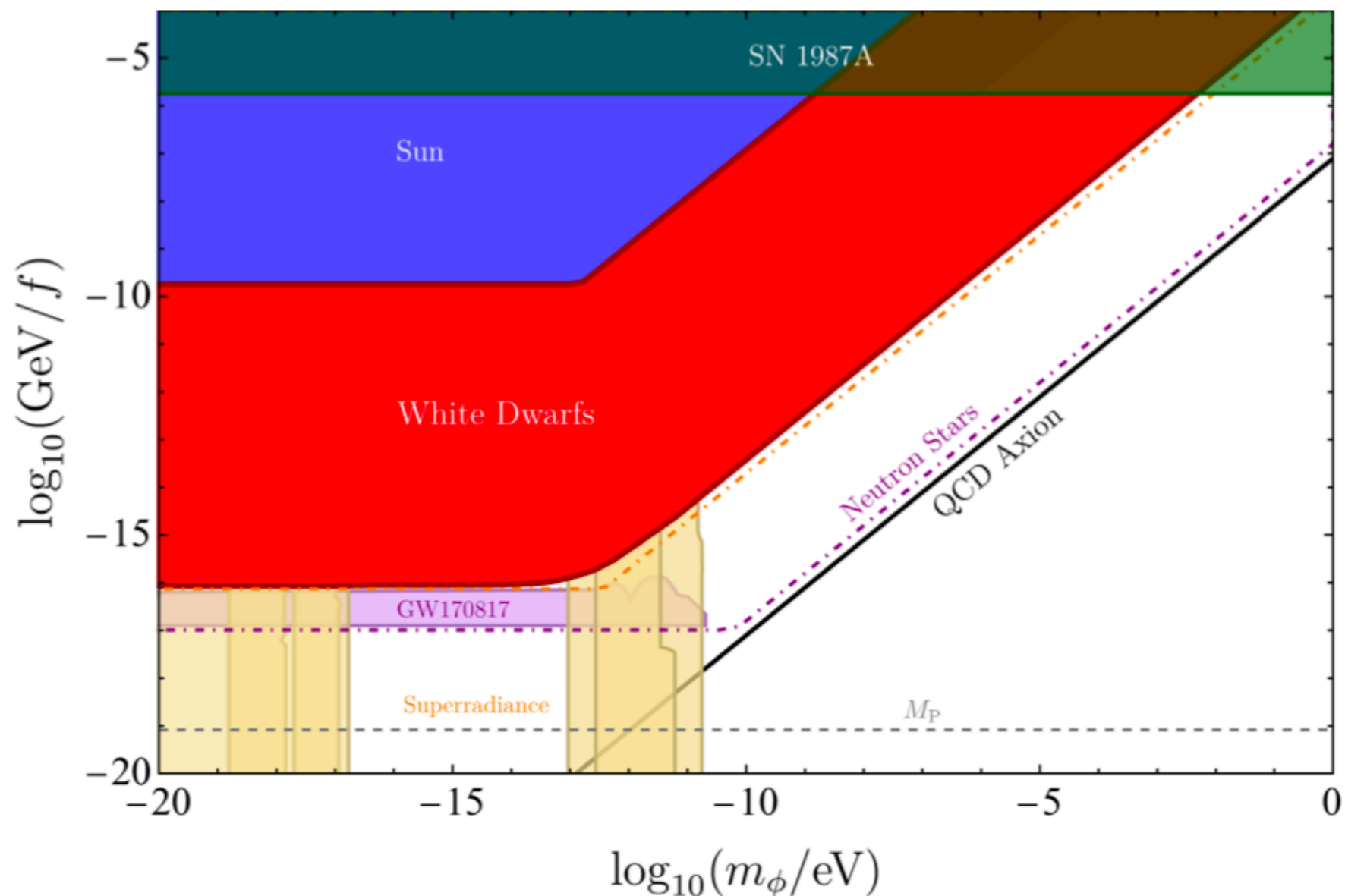
$$V(\phi) = -\epsilon m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\phi}{2f}\right)} \quad \text{with } \epsilon \ll 1$$



# White dwarfs as a probe of light QCD axions

Light QCD axion:

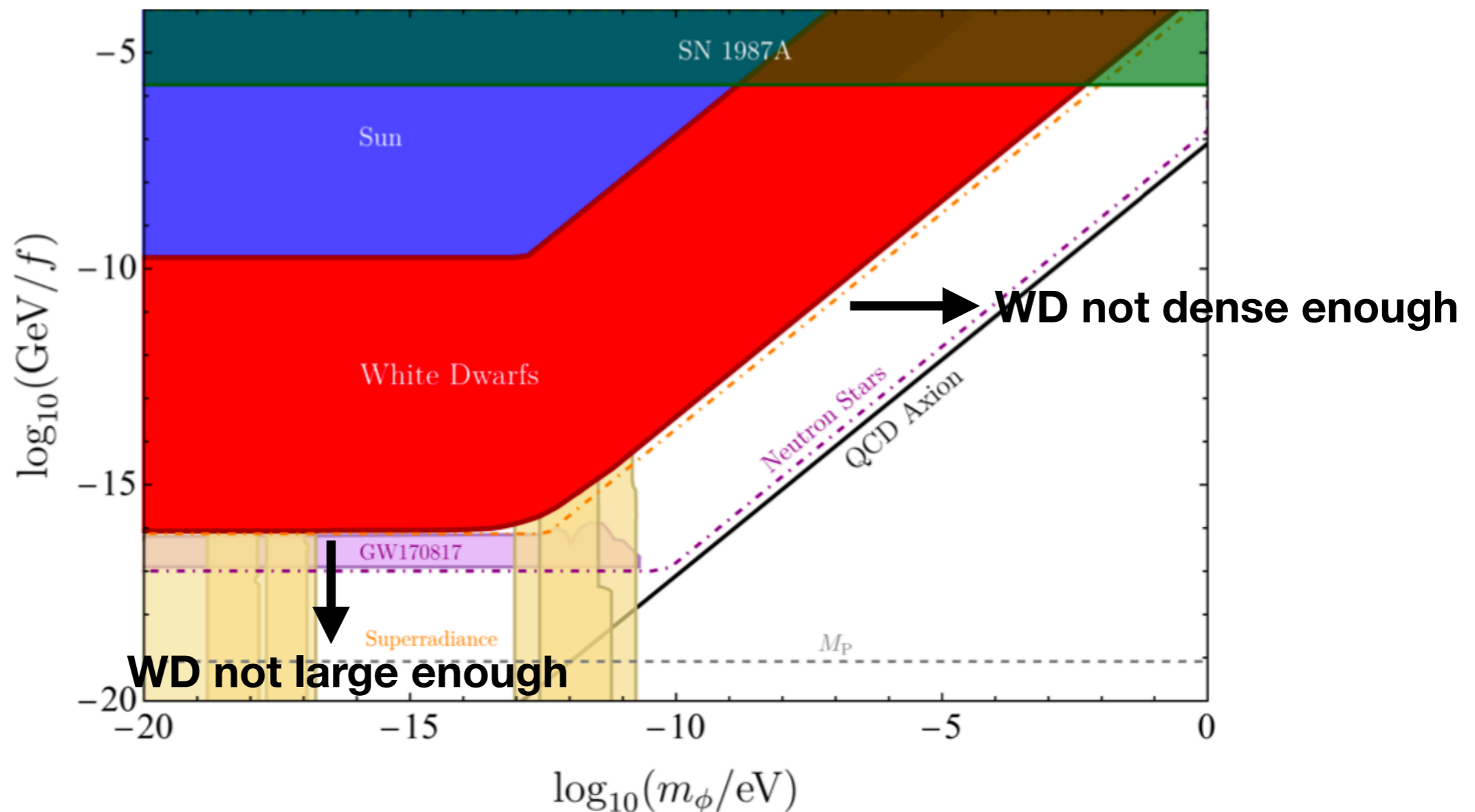
$$V(\phi) = -\epsilon m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\phi}{2f}\right)} \quad \text{with } \epsilon \ll 1$$



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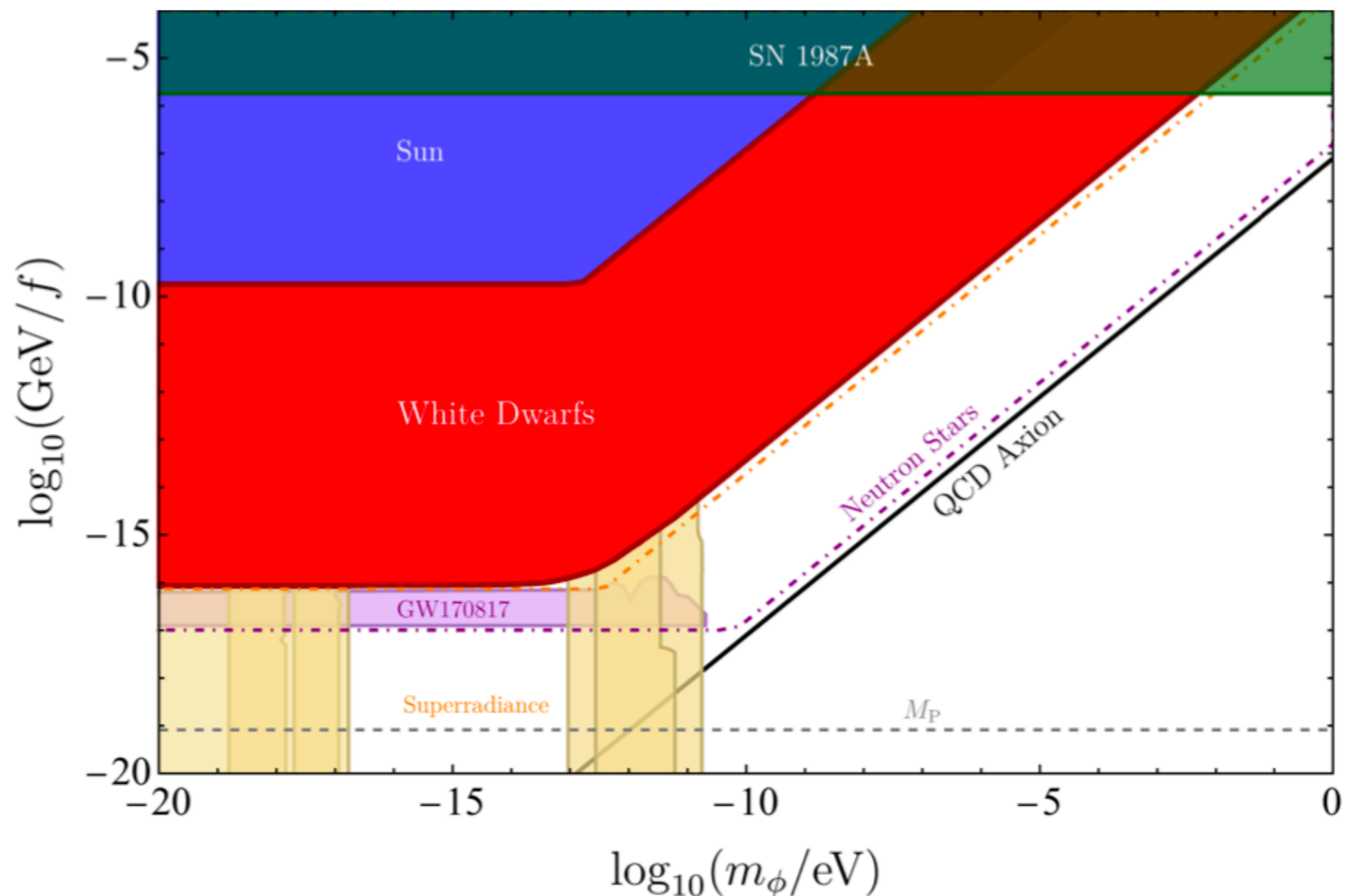




# White dwarfs as a probe of light QCD axions

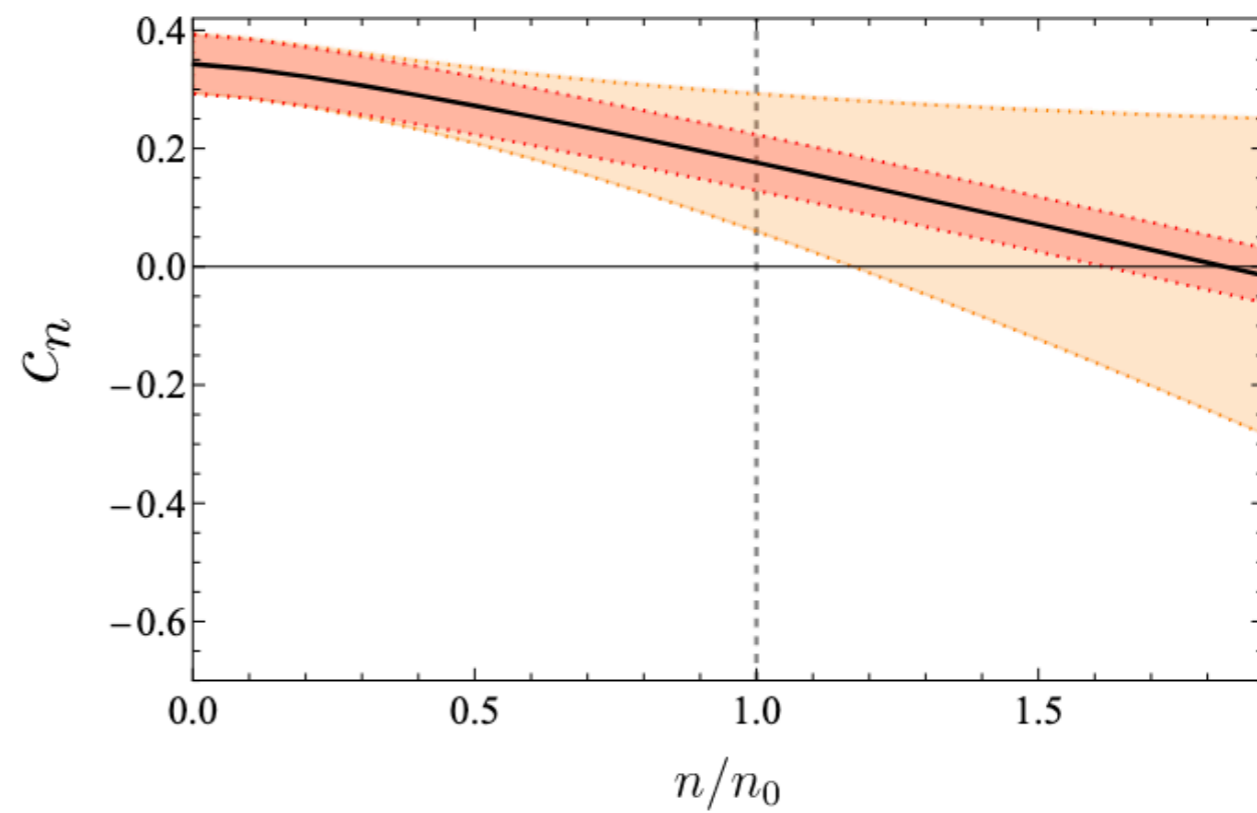
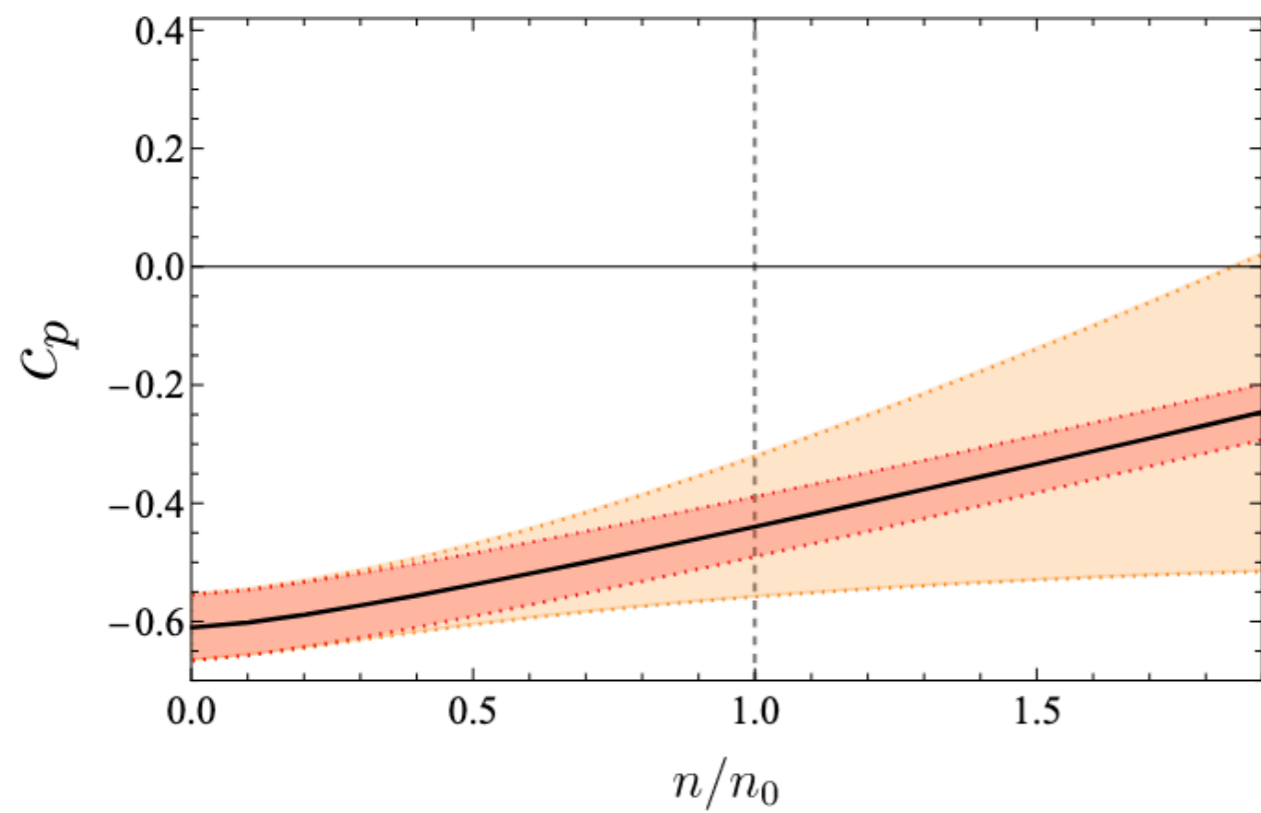
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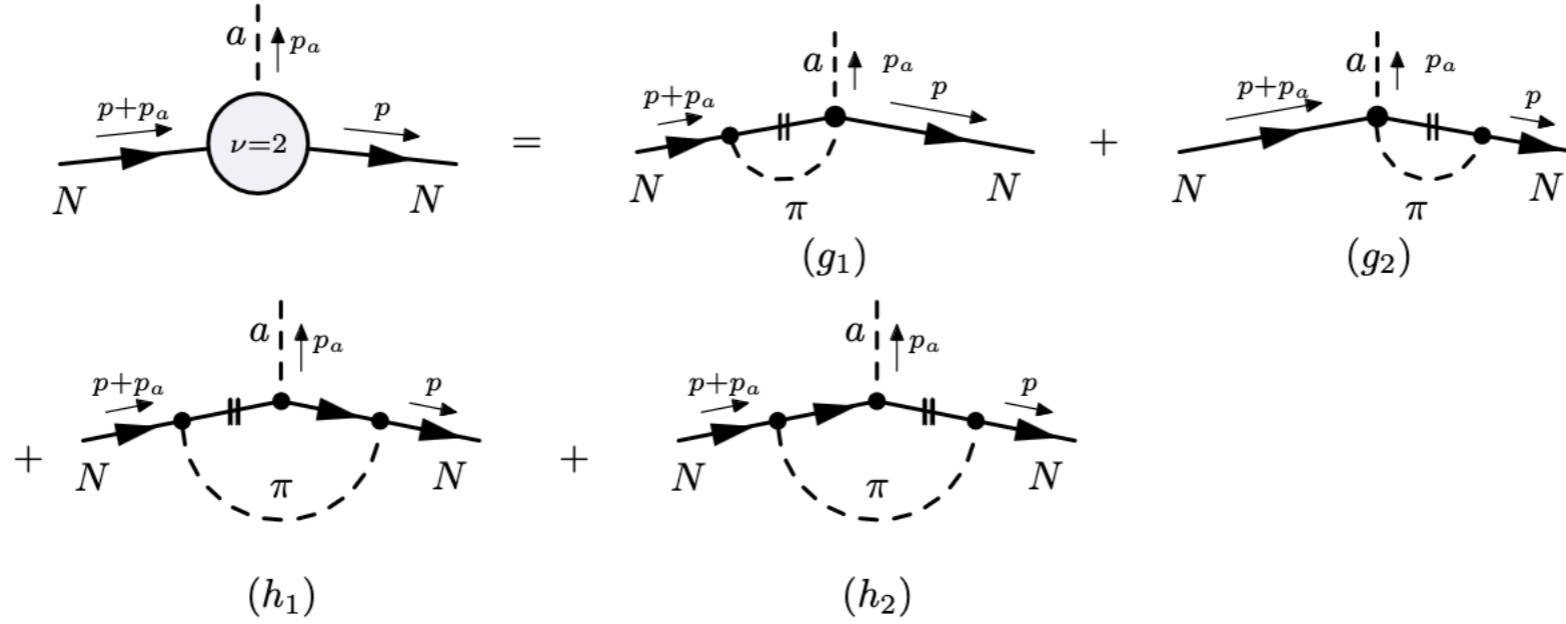


# Backup Slides

# DFSZ axion with $\sin^2 \beta = 0.1$



## Example calculation of finite density loops



$$\begin{aligned}
 (h_1) + (h_2) &= \int \frac{d^4 k}{(2\pi)^4} \left[ -\frac{g_A}{2f_\pi} \vec{\sigma} \cdot (\vec{k} - \vec{p}) \tau^a \right] \left[ \frac{i}{k^0} - 2\pi \delta(k^0) \theta(k_f - |\vec{k}|) \right] \left[ \frac{c_N}{2f_a} \vec{\sigma} \cdot \vec{p}_a \right] \\
 &\times \left[ \frac{i}{k^0 + p_a^0} - 2\pi \delta(k^0 + p_a^0) \theta(k_f - |\vec{k} + \vec{p}_a|) \right] \left[ \frac{g_A}{2f_\pi} \vec{\sigma} \cdot (\vec{k} - \vec{p}) \tau^b \right] \left[ \frac{-i\delta^{ab}}{m_\pi^2 - (k - p)^2} \right].
 \end{aligned}$$

## Backup - Coupled system NS - scalar

$$p' + \phi' \left( \frac{dV}{d\phi} \right) = -\frac{(\epsilon + p) e^\sigma}{2r} \left[ 1 - e^{-\sigma} + \kappa r^2 \left( p + \frac{e^{-\sigma}}{2} (\phi')^2 \right) \right],$$

$$\sigma' = \kappa r e^\sigma \left[ \epsilon + \frac{e^{-\sigma}}{2} (\phi')^2 \right] - \frac{e^\sigma - 1}{r},$$

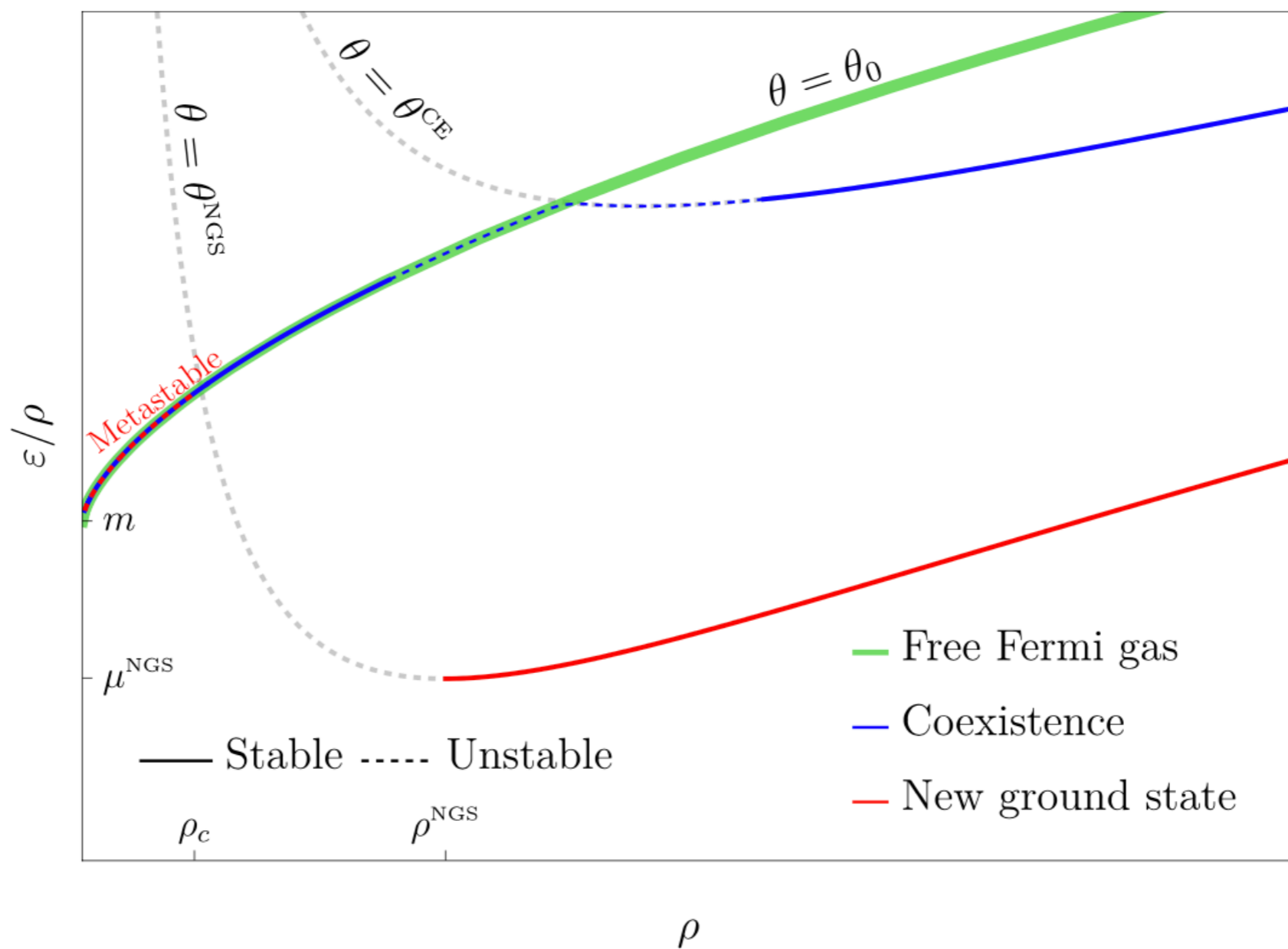
$$\phi'' + \frac{2}{r} \left[ \frac{1 + e^\sigma}{2} + \frac{\kappa r^2 e^\sigma}{4} (p - \epsilon) \right] \phi' = e^\sigma \frac{dV}{d\phi}.$$

**Simplify in negligible gradient limit to normal TOV equations**

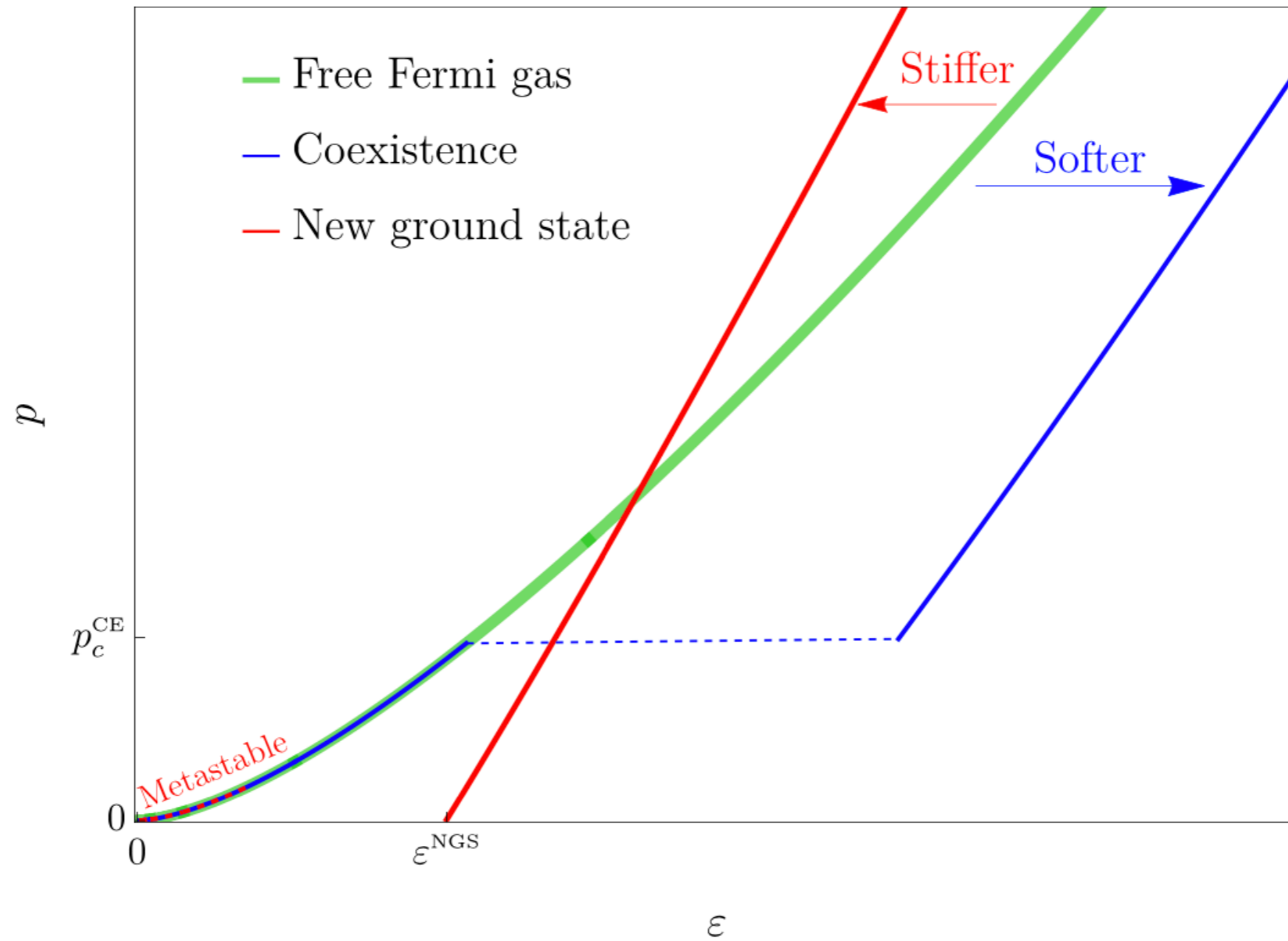
$$p' = -\frac{(p + \epsilon)}{M_{\text{P}}^2 r^2} \left( 1 - \frac{2M}{r M_{\text{P}}^2} \right)^{-1} (4\pi r^3 p + M),$$

$$M' = 4\pi r^2 \epsilon,$$

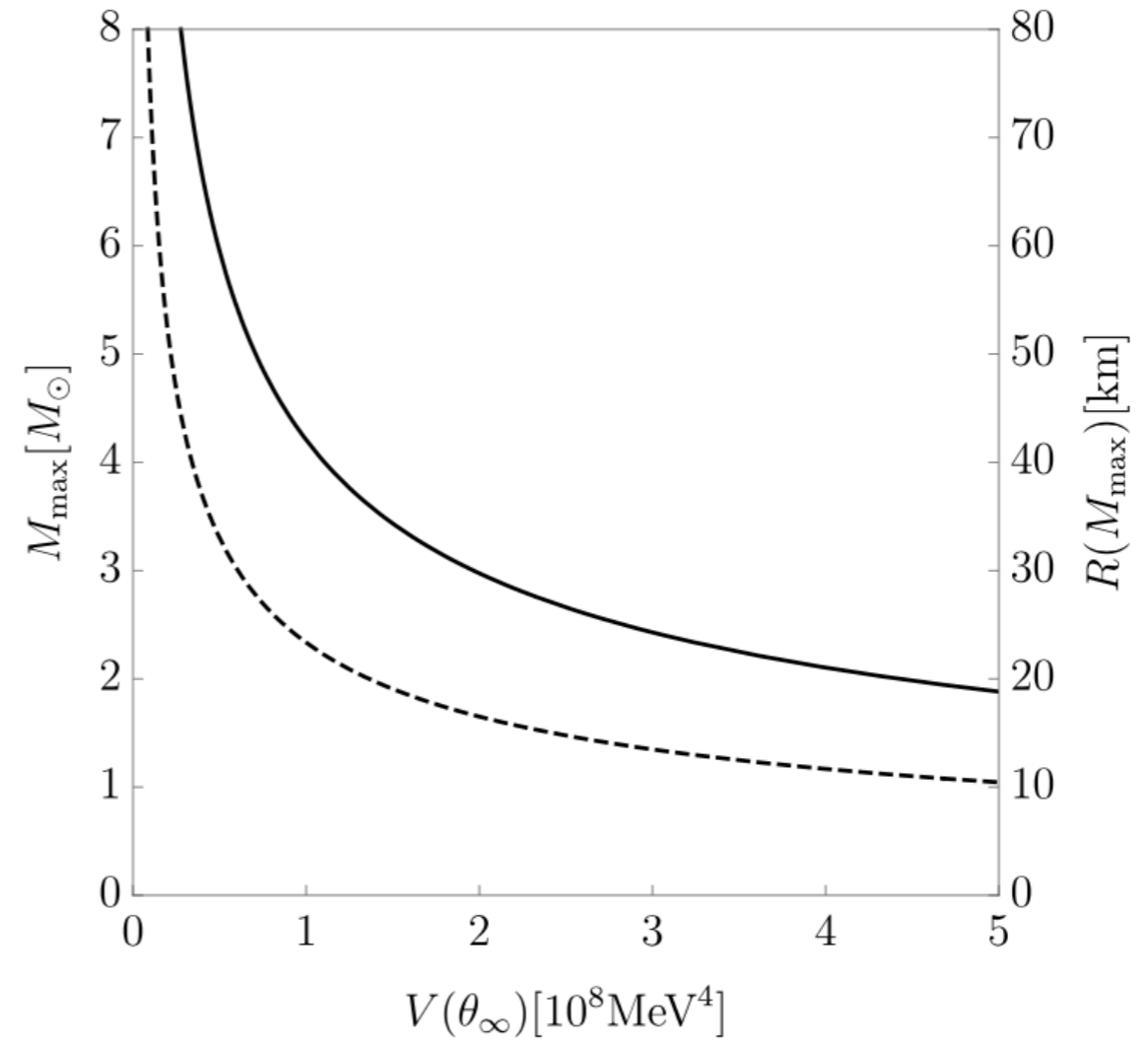
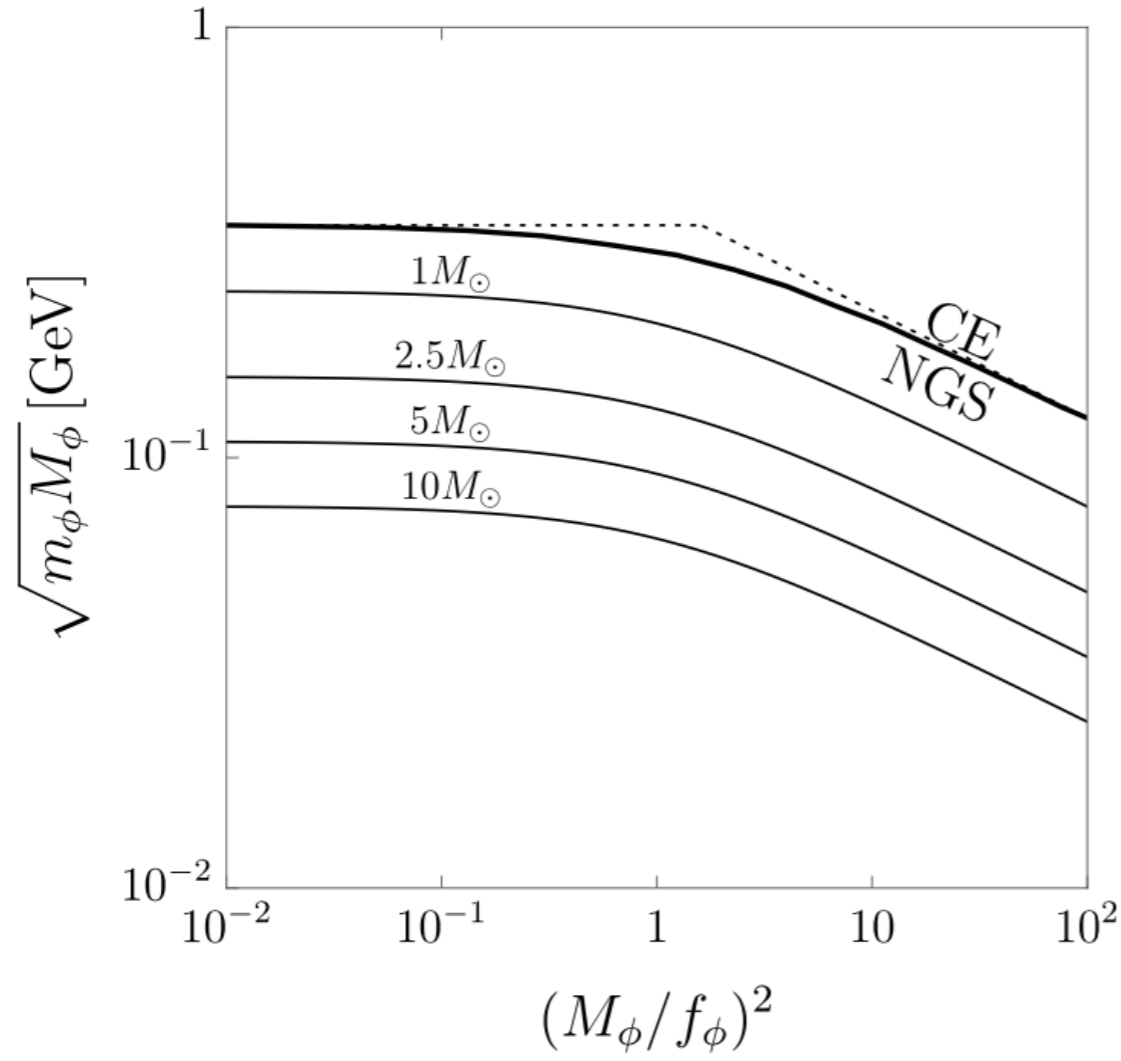
# Energy per particle



## Pressure over energy density (EOS)



# Not limited to periodic functions - quadratic coupling





# Global view of stellar remnants with new ground state

