PRIN meeting: "String Theory as a bridge between Gauge Theories and Quantum Gravity"
Turin - February 24th, 2023

Fate of Radiating Black Holes With Minimum Mass in Einstein-dilaton-Gauss-Bonnet Theory of Gravity

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Introduction

Despite its formal elegance, in **General Relativity (GR)** has deep theoretical problems that require to modify it in the high energy/curvature regime.

Introduction of a *fundamental length scale* $\tilde{\ell}$:

$$\ell >> \tilde{\ell}$$ Theory reduces to GR

$$\ell \lesssim \tilde{\ell}$$ Modifications dominate

There are theories in which this brings to a minimum value of the BH mass.

How does this affect Hawking evaporation?

Outline of the presentation

➔ Einstein-dilaton-Gauss-Bonnet (EdGB) theory of gravity

➔ Numerical setup and nonlinear stability of the BH solutions

➔ Dynamics past the minimum mass driven by Hawking’s evaporation

Einstein-dilaton-Gauss-Bonnet (EdGB) theory of gravity

Einstein-dilaton-Gauss-Bonnet (EdGB) Theory of Gravity

Action of the theory:

$$S = \frac{1}{16\pi}\int_{\Omega} d^4x \sqrt{-g}\left\{\underbrace{\mathcal{R}}_{\text{Einstein-Hilbert term}} - \underbrace{(\nabla\phi)^2}_{\text{Dilaton}} + \underbrace{2F[\phi]\mathcal{G}}_{\text{Coupling term}}\right\} + \underbrace{S_m}_{\text{Matter term}}$$

Gauss-Bonnet invariant $\mathcal{G} = \frac{1}{4}\delta^{\mu\nu\alpha\beta}_{\rho\sigma\lambda\omega}\mathcal{R}^{\rho\sigma}{}_{\mu\nu}\mathcal{R}^{\lambda\omega}{}_{\alpha\beta}$

Coupling function $F[\phi] = \lambda e^{-\gamma\phi}$

[Gross-Witten, *Nucl. Phys. B* (1986)]

Einstein-dilaton-Gauss-Bonnet (EdGB) Theory of Gravity

$$S = \frac{1}{16\pi}\int_{\Omega} d^4x\,\sqrt{-g}\left\{\mathcal{R} - (\nabla\phi)^2 + 2F[\phi]\mathcal{G}\right\} + S_m$$

Coupling function $$F[\phi] = \lambda e^{-\gamma\phi}$$

Some properties:

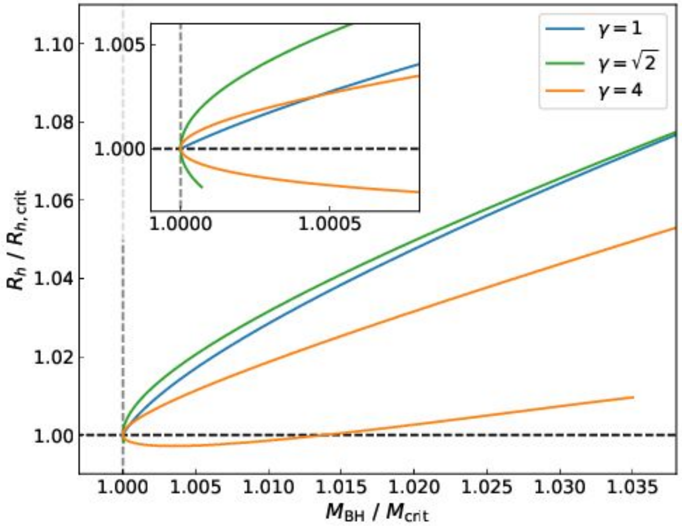
- $[\lambda] = M^2$: λ sets the scale below which modifications dominate
- λ is **not** (necessarily) plankian
- BHs have a nontrivial profile of the scalar field outside them and a curvature singularity at finite radius.

[Kanti+ *Phys. Rev. D* (1996), Alexeyev-Pomazanov *Phys. Rev. D* (1997)]

Black Holes (BHs) in EdGB Gravity

Spherically symmetric BHs in this theory have a minimum mass solution that can be either singular or regular on the horizon.

$$F[\phi] = \lambda e^{-\gamma\phi}$$



For large enough γ this critical configuration is regular on the horizon and divides the domain of existence of BH solutions in two branches.

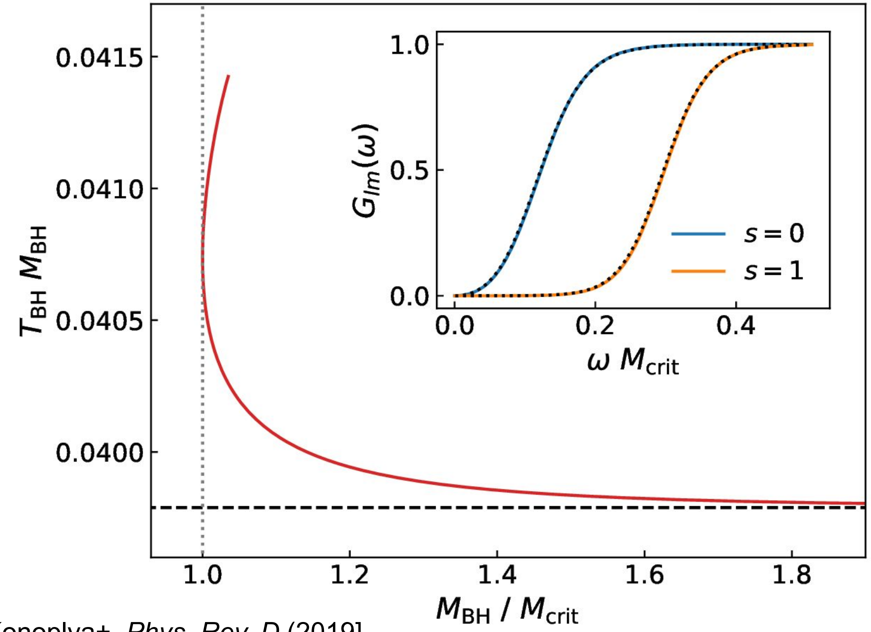
[Torii+ *Phys. Rev. D* (1997), Guo+ *Prog. Theor. Phys.* (2008), Blázquez-Salcedo+ *Phys. Rev. D* (2017)]

The upper branch is linearly stable while the lower branch is linearly unstable.

[Torii-Maeda *Phys. Rev. D* (1998)]

Hawking's Evaporation in EdGB gravity

Hawking's evaporation: $$\frac{dM}{dt} = -\frac{1}{2\pi}\sum_{lm}\int d\omega \frac{\omega G_{lm}(\omega)}{e^{\omega/T_{\rm BH}} \pm 1}$$

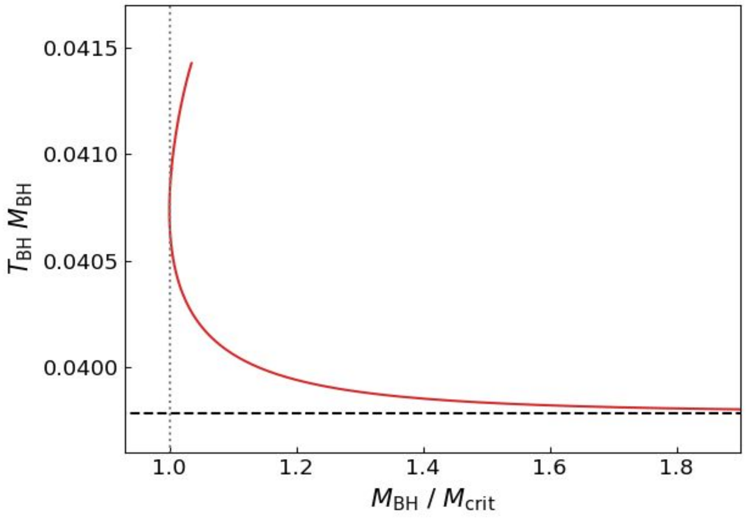


[Konoplya+, *Phys. Rev. D* (2019)]

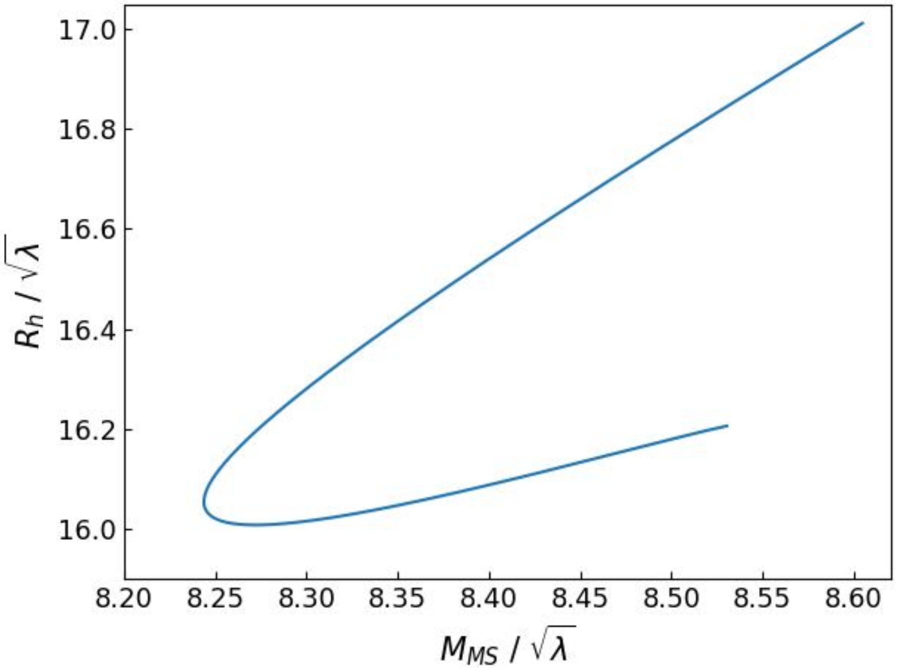
$$\left.\frac{dM}{dt}\right|_{\rm crit} \leq 0$$

Critical BHs emit Hawking radiation

Critical BH Configuration and Hawking Radiation



Evolution under Hawking's evaporation



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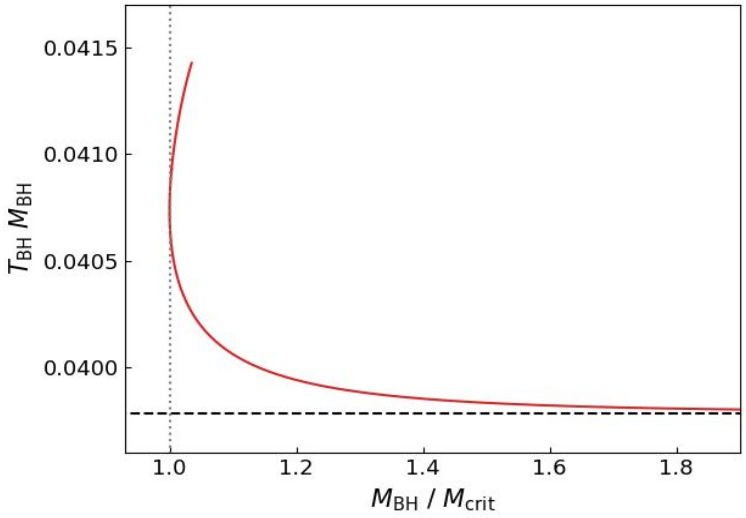

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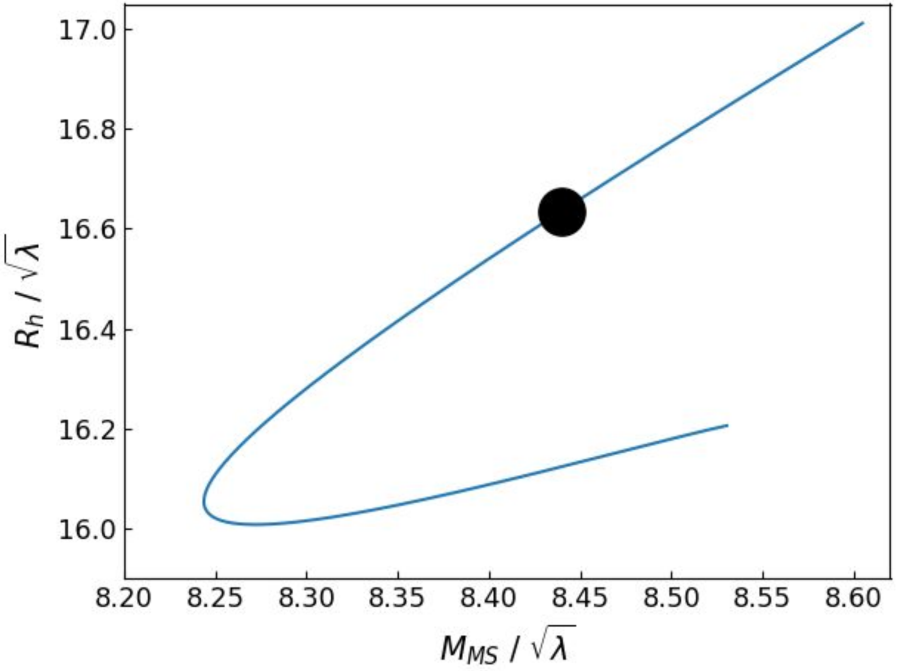

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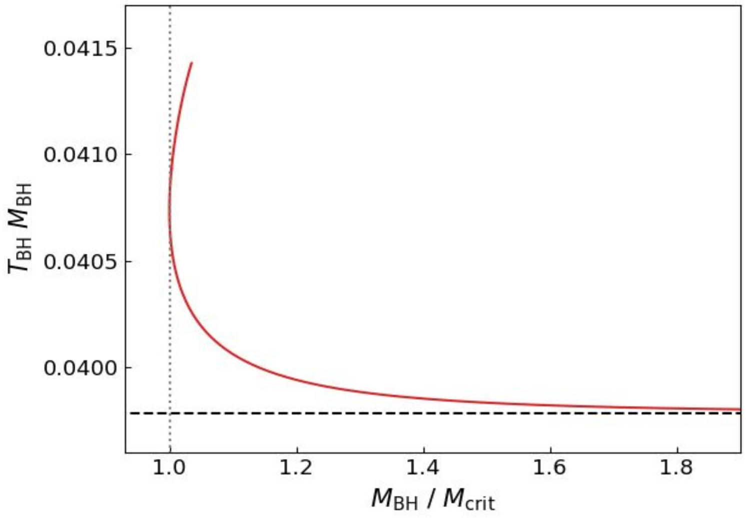

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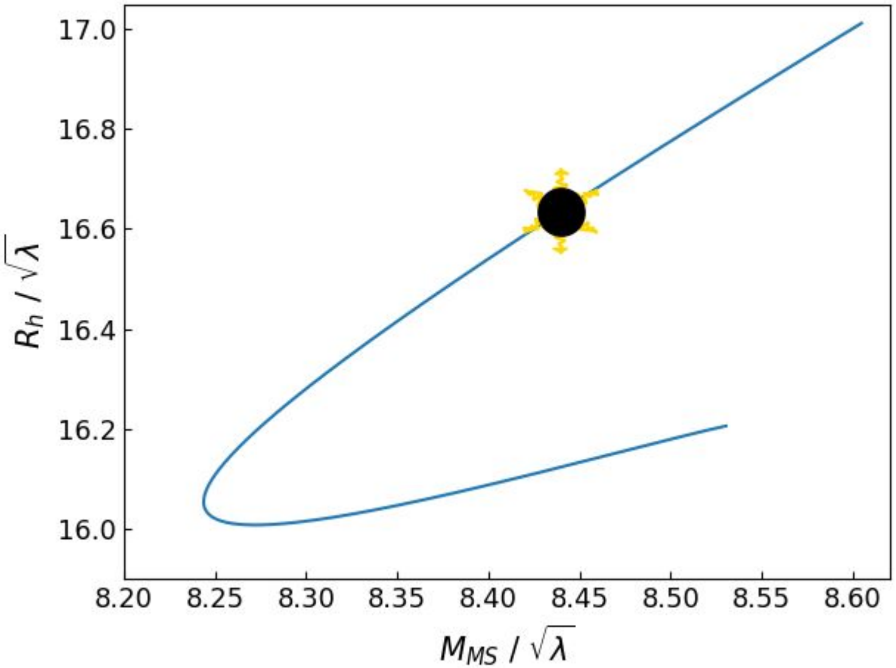

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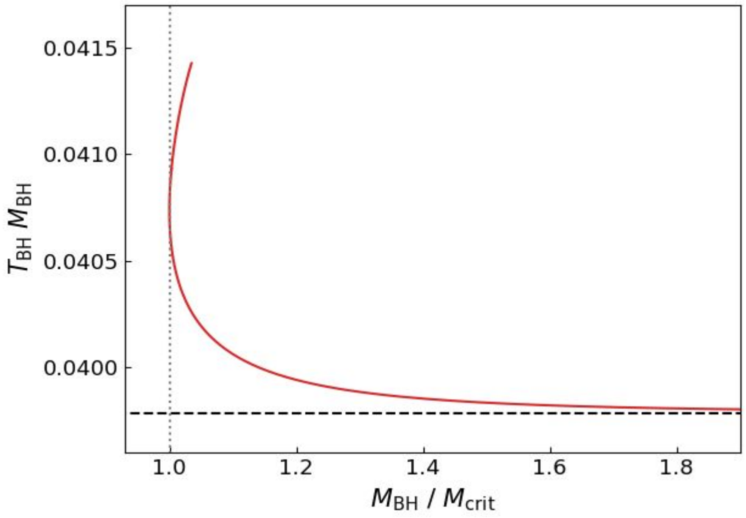

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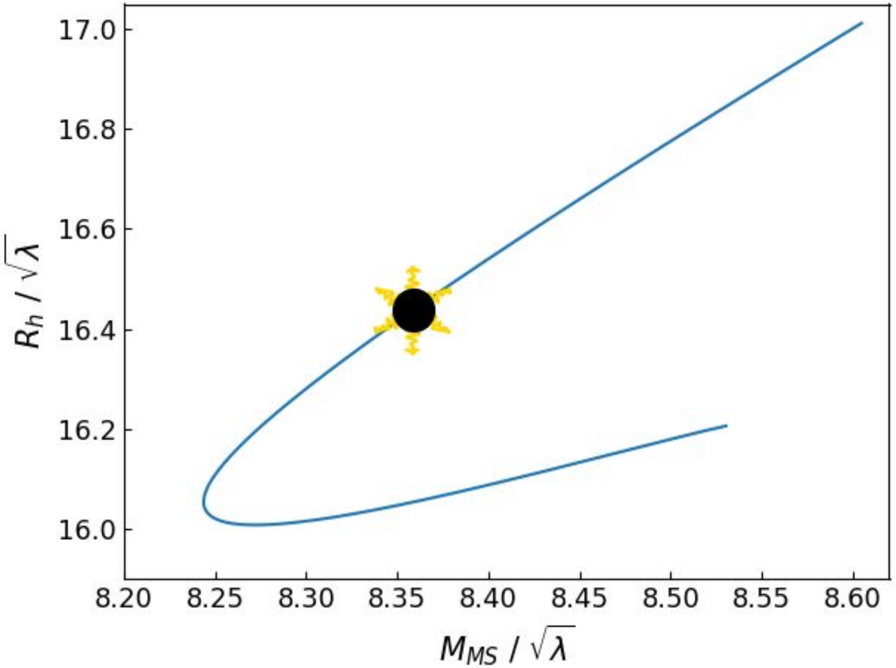

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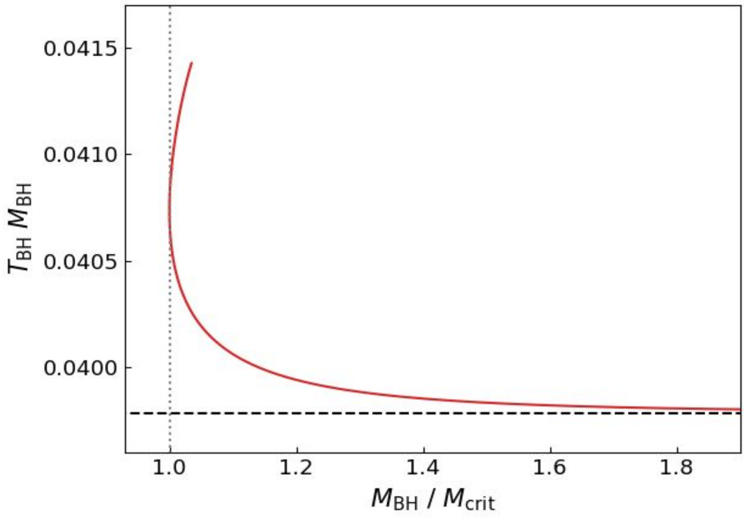

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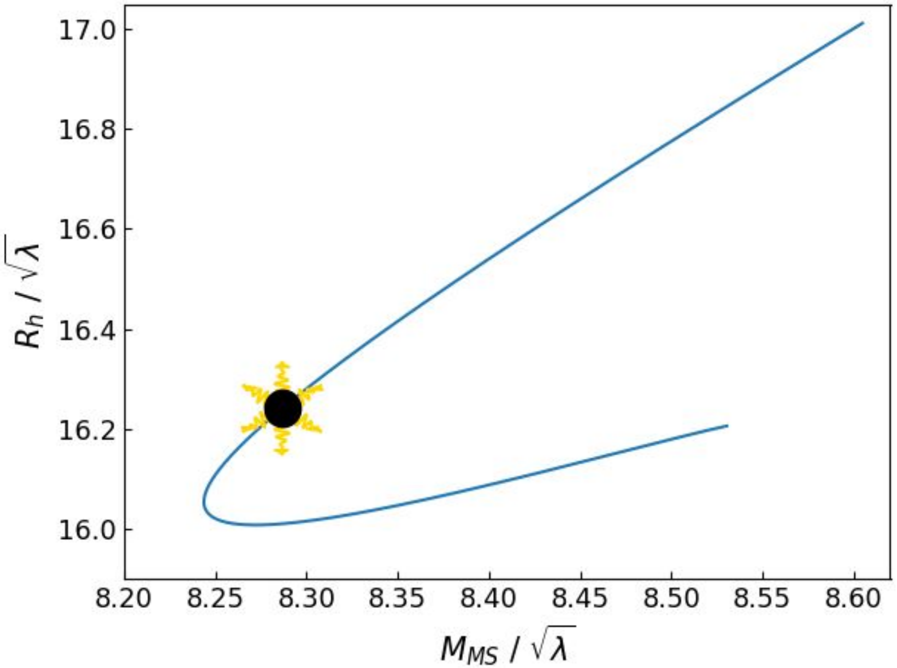

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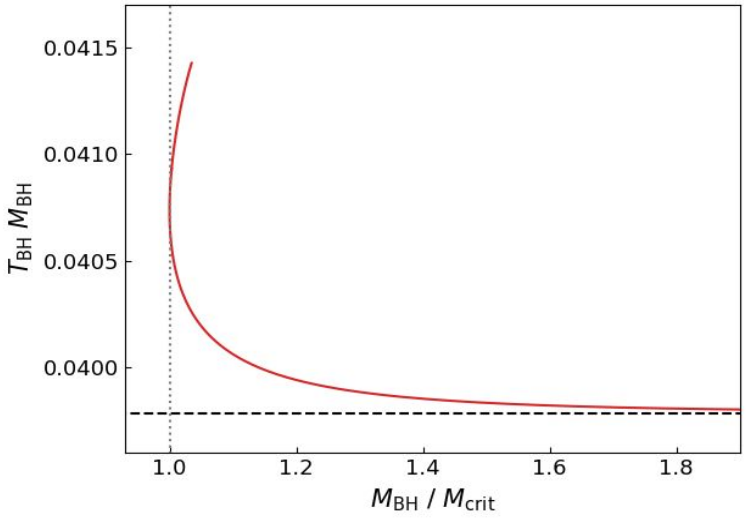

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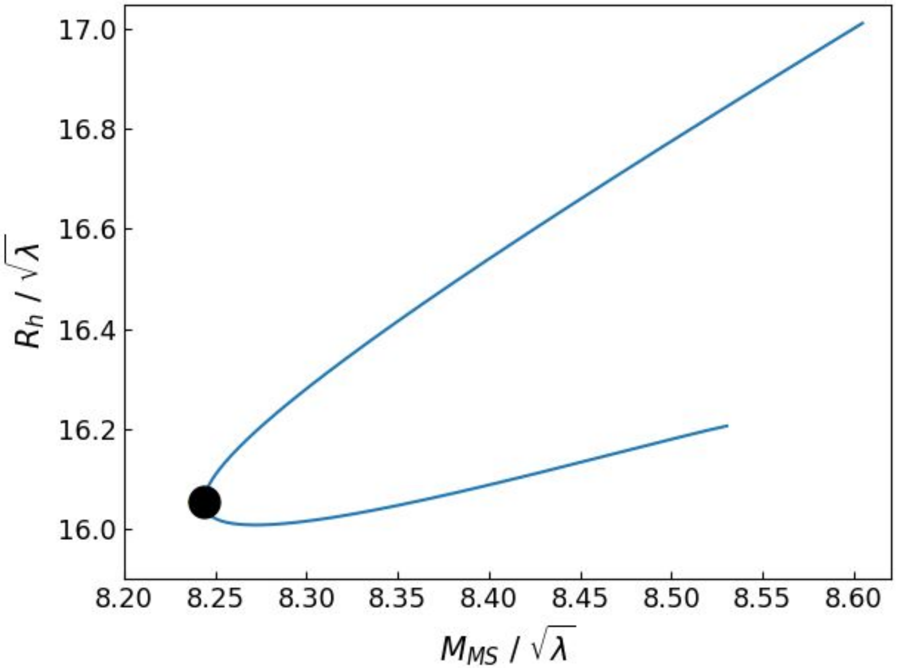

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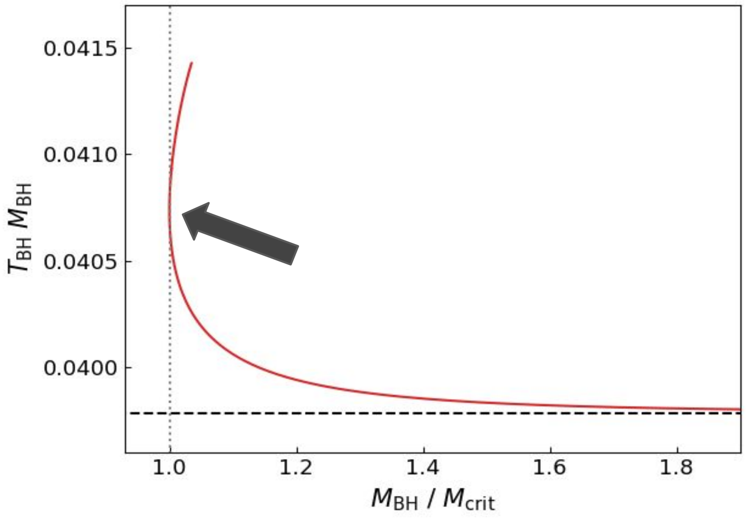

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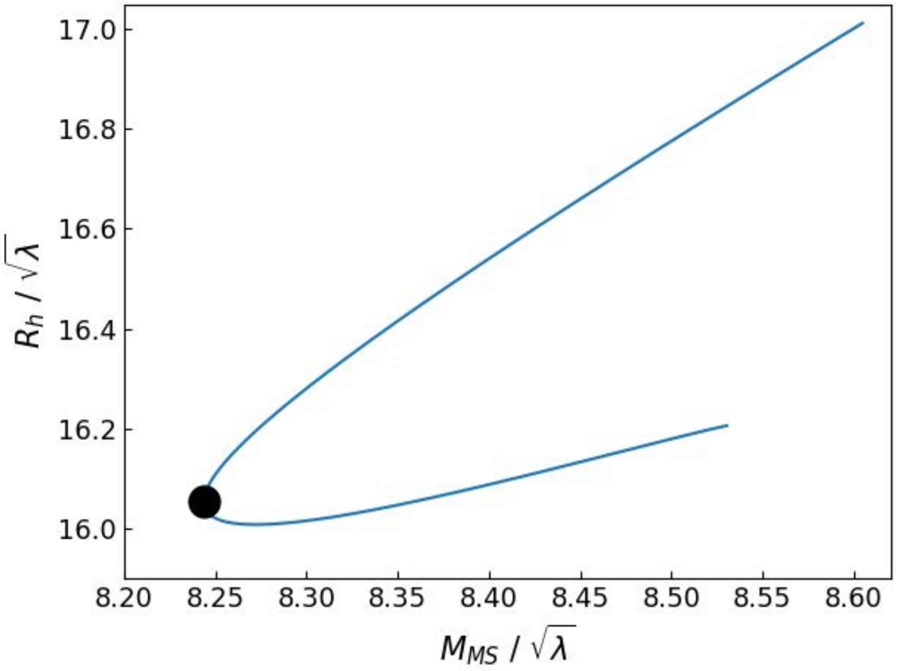

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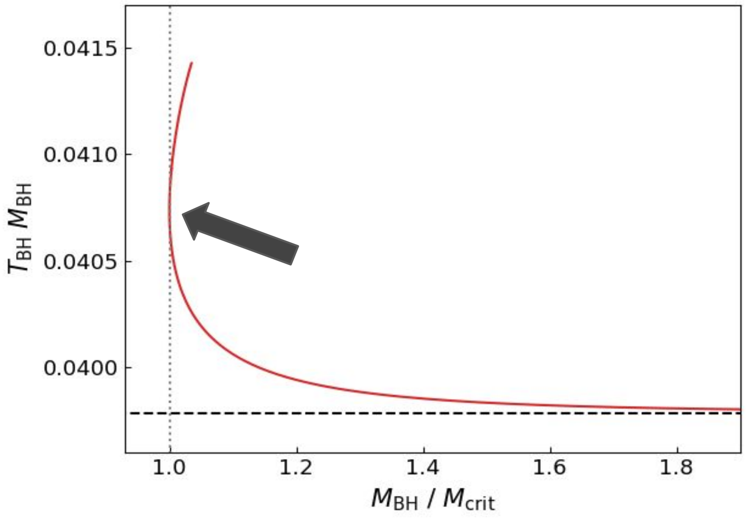

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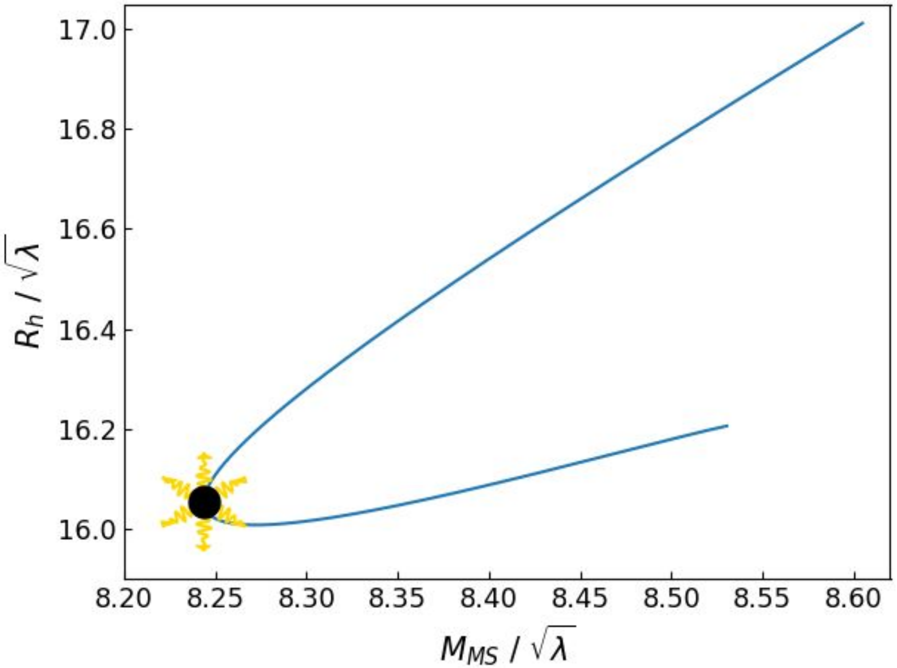

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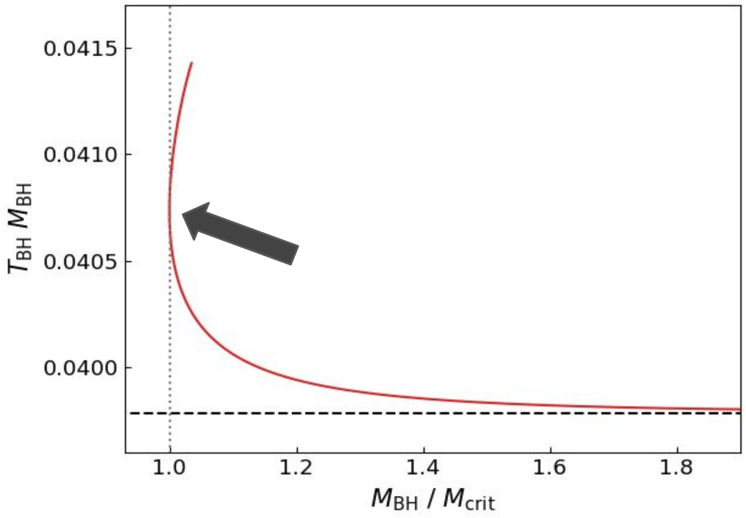

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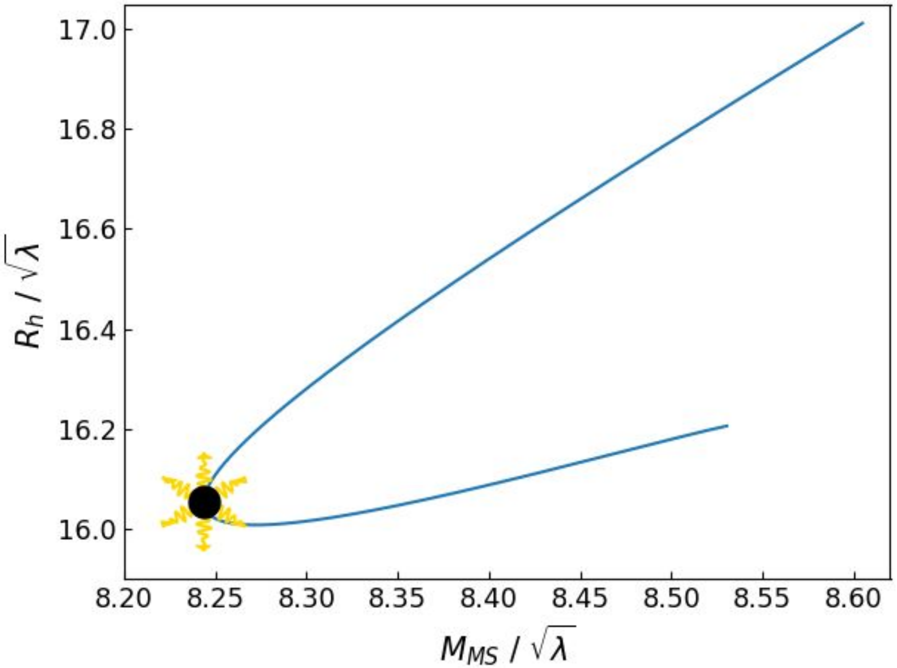


What is the fate of a critical BH after Hawking evaporation?

[Torii+ *Phys. Rev. D* (1997), Alexeyev+ *Class. Quant. Grav. (2002)*]

Numerical setup and nonlinear stability of the BH solutions

Numerical Setup

We use Painlevé-Gullstrand-like (PG-like) coordinates:

$$ds^2 = -\alpha^2 dt^2 + (R'(r)dr + \alpha\zeta\, dt)^2 + R(r)^2 d\Omega^2$$

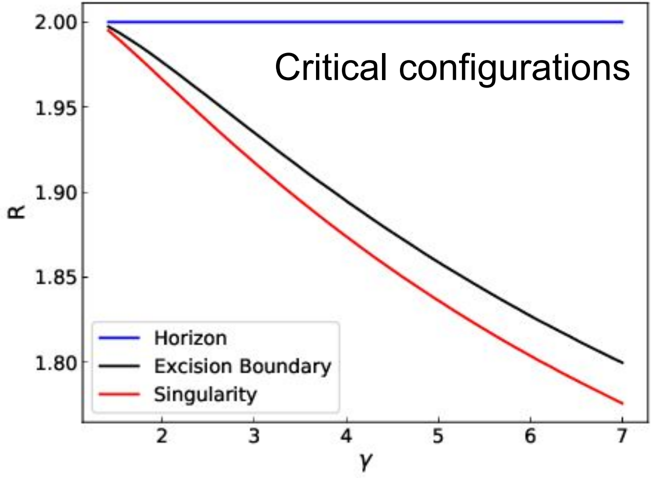
In EdGB gravity there are regions in which the system of evolution equations is not well-posed (**elliptic regions**).

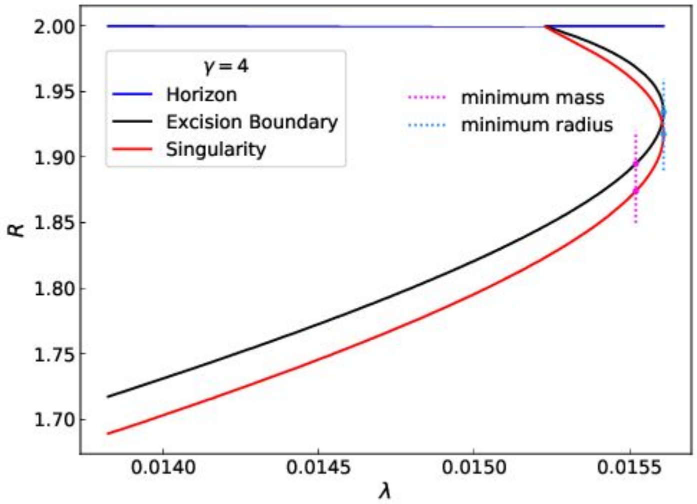
The equations can be integrated outside these regions as long as they remain inside the black hole horizon (**excision strategy**).

[Ripley-Petorius *Phys. Rev. D* (2020)]

Excision Boundary and Curvature Singularity

The excision boundary is close to the curvature singularity and slightly outside it.



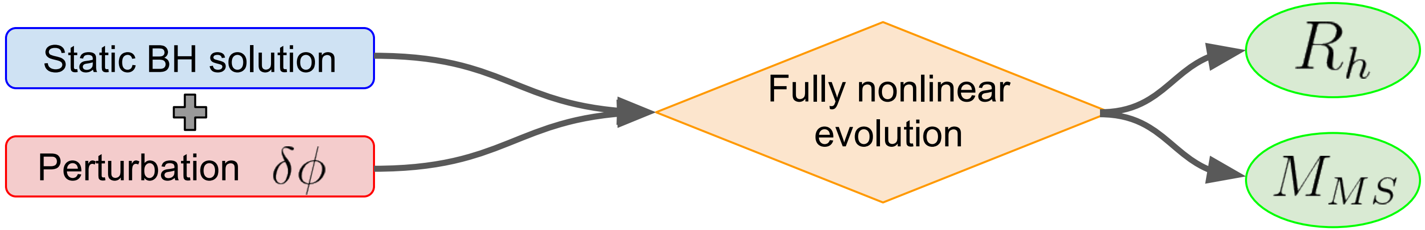


$$\gamma \uparrow \implies R_h - R_e \uparrow$$

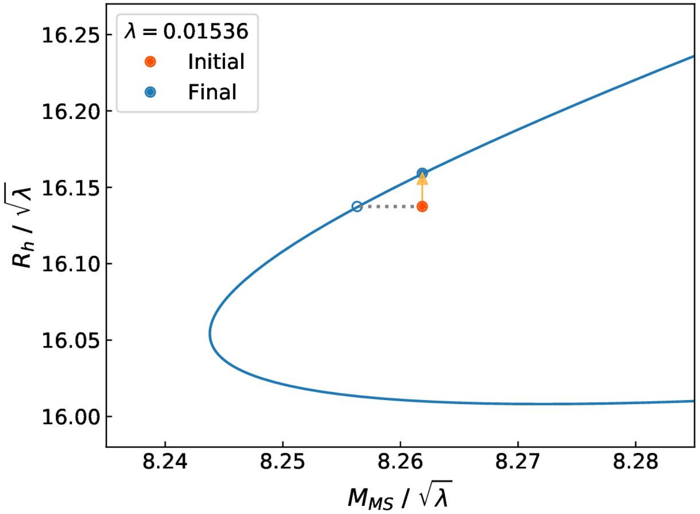
We choose $\gamma = 4$

Nonlinear Stability of the BH Solutions

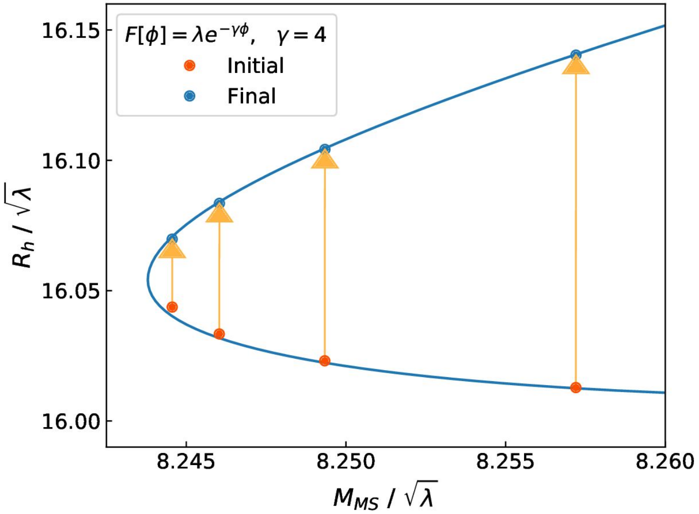
Strategy:



Upper branch



Lower branch

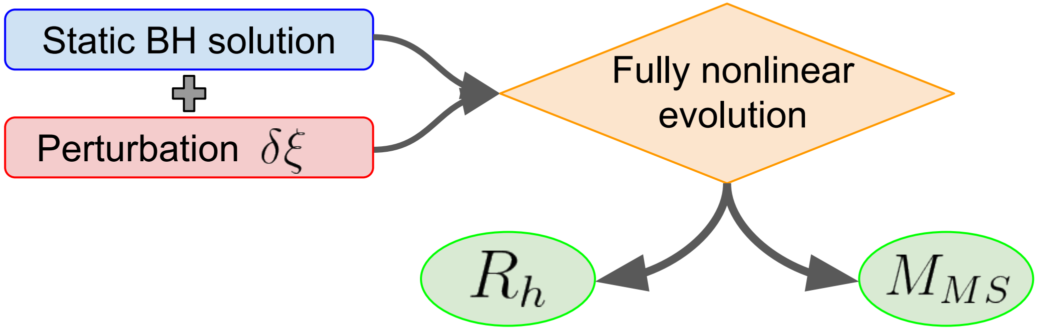


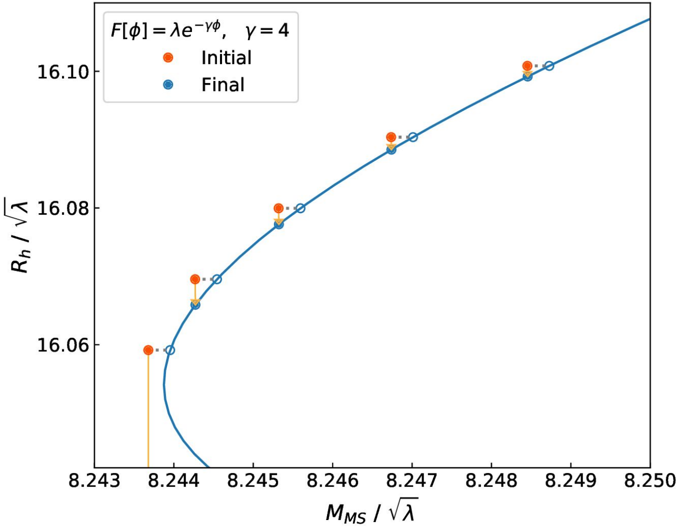
Classical Emulation of Hawking Evaporation

Idea: Introducing a scalar field with opposite kinetic term (*phantom field*) we can mimic *at the classical level* the effect of the Hawking's evaporation process.

$$S_m = \frac{1}{16\pi}\int_\Omega d^4x\,\sqrt{-g}\,\left(\nabla\xi\right)^2$$

Strategy:

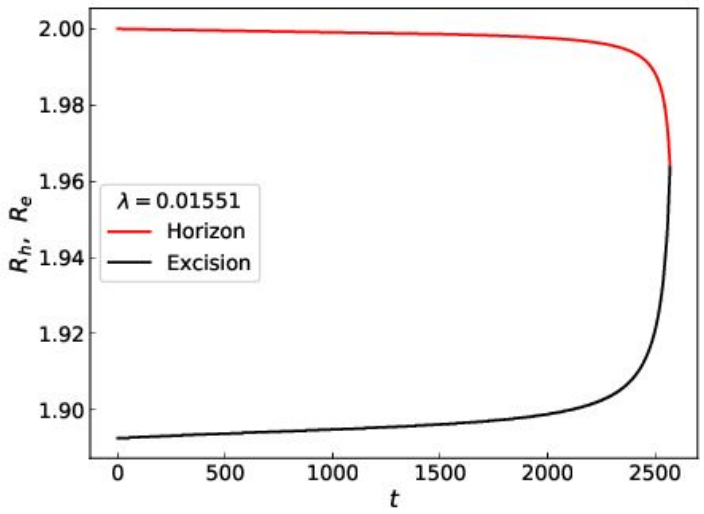




Dynamics past the minimum mass driven by Hawking’s evaporation

Apparent Horizon and Excision Boundary

Evolution of the apparent horizon and excision boundary



Formation of a naked elliptic region!

A consideration:

- At the beginning of the simulation the curvature singularity is close to the excision boundary.
- The excision boundary expands and at some point it crosses the apparent horizon.

Are we forming a naked singularity?

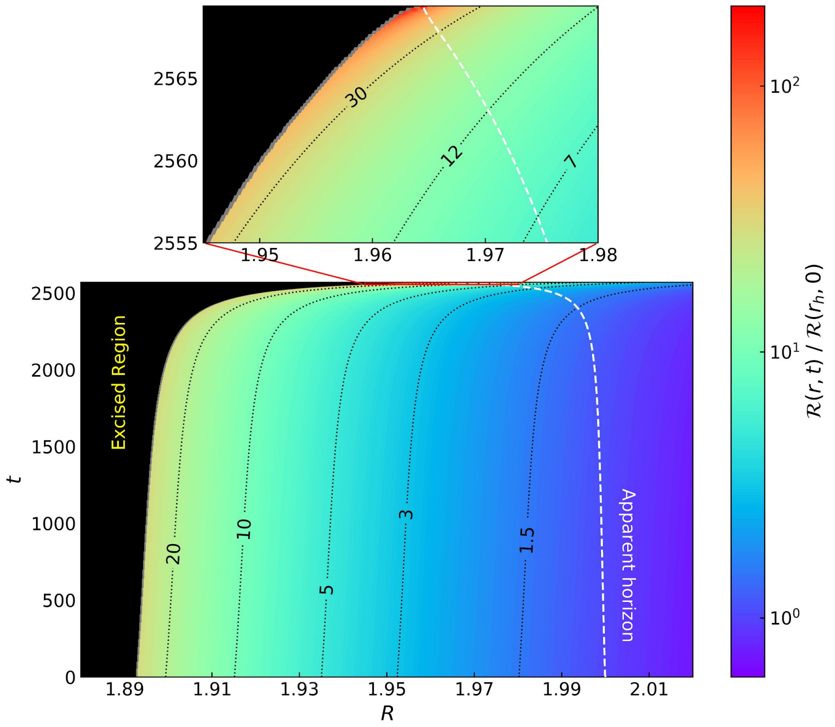
Formation of a Naked Singularity?

- The curvature at the horizon has grown by a factor 58 compared to its initial value.
- The contour lines follow the behavior of the excision boundary.

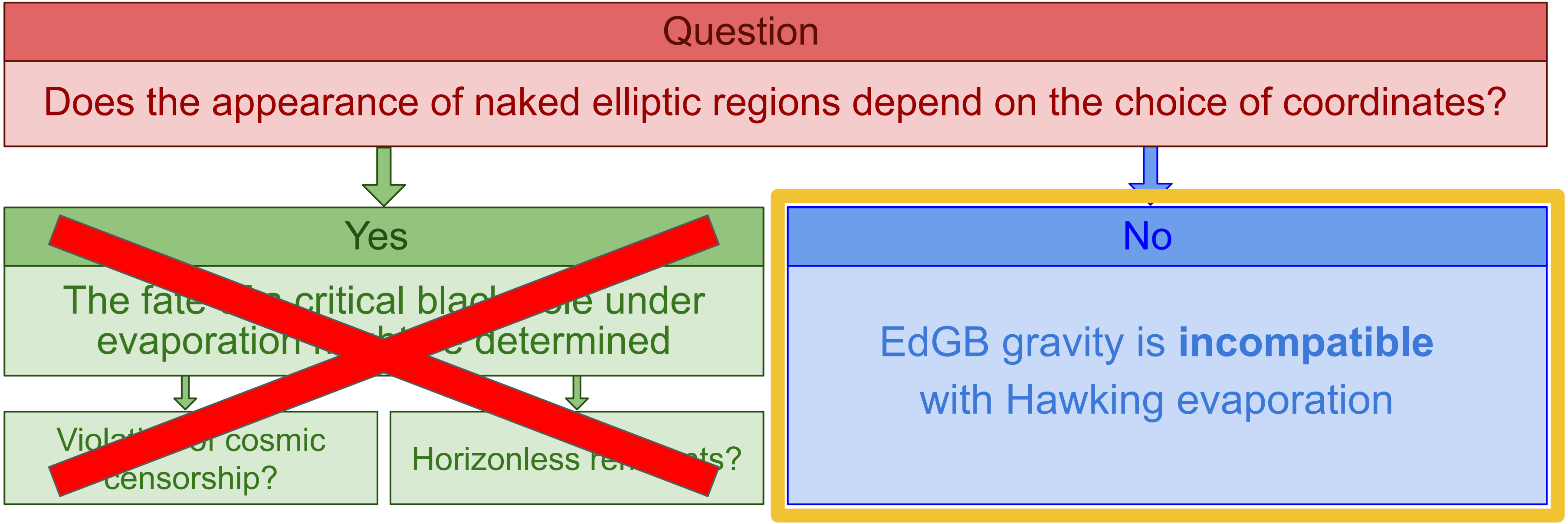
Hint to the formation of a naked singularity and violation of the weak cosmic censorship conjecture!

… elliptic region?

Ricci Scalar



Gauge Dependence of naked Elliptic Regions



Abhishek Hegade+, *Phys. Rev. D* (2023):

In EdGB gravity the elliptic regions are gauge invariant

Formation of a Naked Elliptic Region: Event Horizon

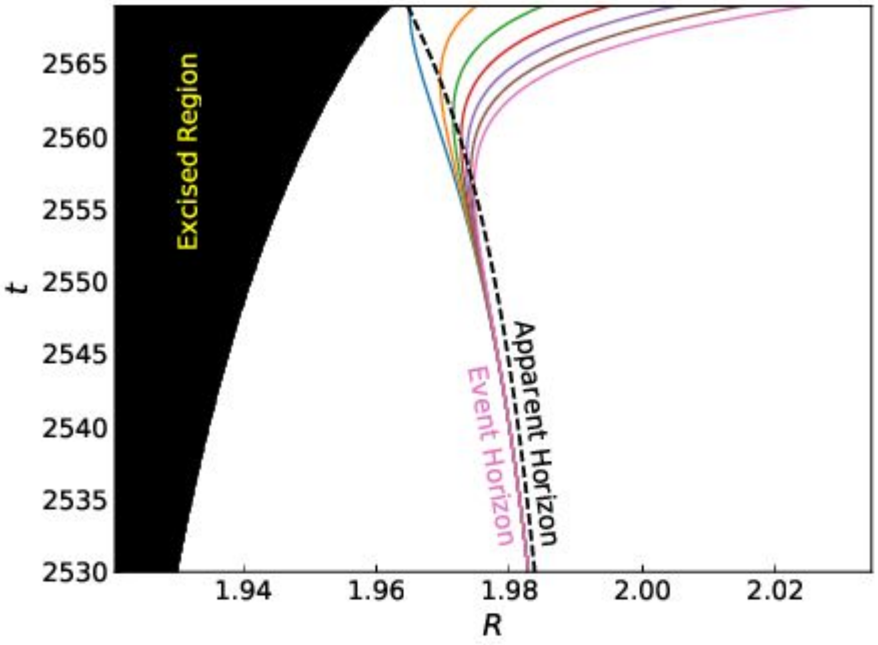
During the last stages of evolution the event horizon is ***inside*** the apparent horizon and shrinks!

The formation of a naked elliptic region is gauge independent

Is this a proof?

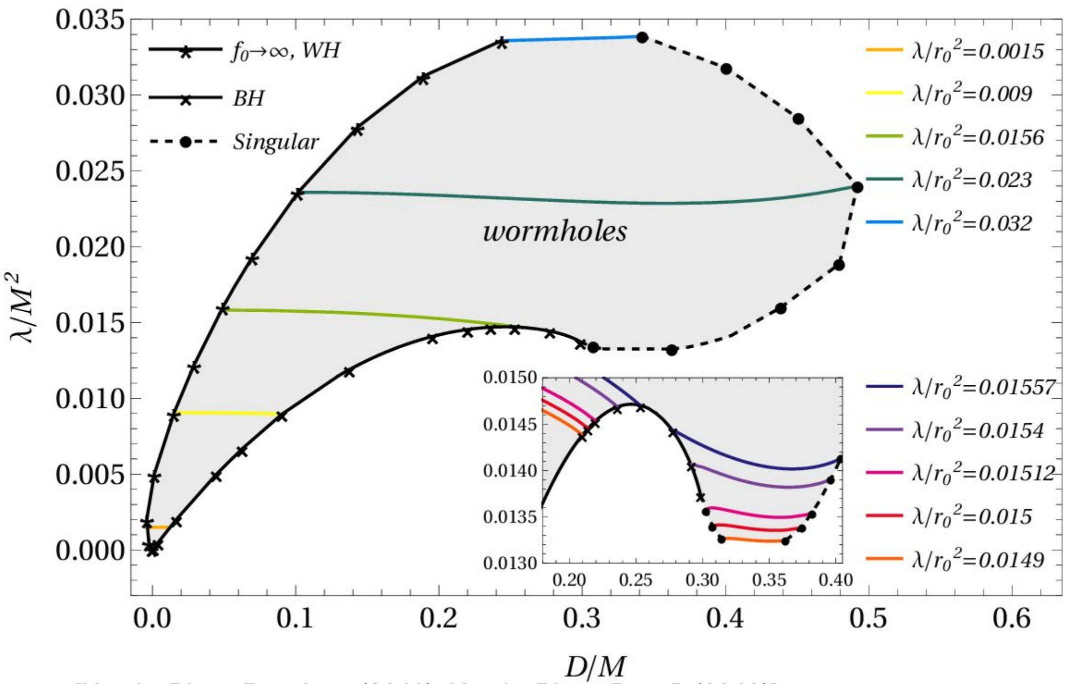
No, but…

Null ray tracing



Alternative Scenario: Wormholes

In the parameter space BH solutions coexist with wormhole solutions.



[Kanti+ *Phys. Rev. Lett.* (2011), Kanti+ *Phys. Rev. D* (2012)]

1. Can a critical BH transit toward a wormhole solution?

2. Wormholes is EdGB gravity have matter at the throat: can Hawking particles provide the correct matter content for forming a wormhole?

Take-home message

- ➔ In EdGB gravity there exists a BH configuration with minimum mass.
- ➔ The minimum mass BH is subject to Hawking evaporation.
- ➔ When simulating the Hawking evaporation process we observed a runaway instability due to the intrinsic (classical) dynamics of the theory.
- ➔ A naked elliptic region forms.
- ➔ Possible scenarios: violation of the weak cosmic censorship conjecture, transition toward a (horizonless) wormhole solution.
- ➔ These behaviors are interesting *per se* and might appear also in other modified theories of gravity that predict a minimum-mass BH solution

Future prospects

- Inclusion of higher order terms in the action (*e.g.* $A[\phi](\nabla\phi)^4$)

- Different model with second order field equations such as

$$\mathcal{L} = f(\mathcal{R}) - (\nabla\phi)^2 + F[\phi]\mathcal{G}$$

- Simulation of the transition between a BH and a wormhole configuration

Thank You!