# Holograpy with heavy states as a tool to study black holes

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# The framework

I'll focus on the traditional $\text{AdS}_{d+1}/\text{CFT}_d$ setup in the regime where

- the central charge $c$ is large $\Rightarrow$ the bulk is semiclassical
- the CFT is strongly coupled $\Rightarrow$ there is a large gap between the (super)gravity and string modes

I am particularly interested in the heavy sector $\frac{\langle O_H|\hat{\Delta}|O_H\rangle}{\langle O_H|O_H\rangle} \sim \mathcal{O}(c)$

For simplicity, I'll take $d = 2$ and work with type IIB on $\text{AdS}_3 \times S^3 \times T^4$

- The CFT enjoys an enhanced superconformal symmetry
- the supergravity description is easier than in the $\text{AdS}_5$ case
- all the interesting questions about (large) black holes remain

Strominger Vafa 9601029

The general question: what can we learn about black holes (BHs) by probing heavy states (rather than the other way around)?

# The key ingredients

The relevant CFT has a free locus (very much as $\mathcal{N} = 4$ at $g_{YM} = 0$). In this case it reduces to a collection of $N$ quadruplets of free fields (four boson and four fermions) with a $\mathcal{S}_N$ permutation gauge symmetry

There is a 20-dimensional space of deformations preserving the $(4,4)$ superconformal symmetry. The $\frac{1}{2}$-BPS states are protected. When they are light ($\Delta \sim \mathcal{O}(c^0)$), they are in one-to-one correspondence with supergravity excitations of $\mathrm{AdS}_3 \times S^3 \times T^4$

We can construct a $O_H$ by binding many light states. Example: if $O_L$ is a light $\frac{1}{2}$-BPS state, consider $O_H = O_L^p$ with $\frac{p}{N} \sim 0.5$. Main goals:

- construct the dual geometries and see how well they approximate a BTZ black hole (both with and without supersymmetry)
- make the definition of this class of operators precise
- study the correlators $\langle \bar{O}_H(\infty) O_H(0) \bar{O}_L(1) O_L(z,\bar{z}) \rangle$: how do they compare with the BH result $\langle \bar{O}_L(1) O_L(z,\bar{z}) \rangle_{BH}$?

# References

Background results

1503.01463, 1607.03908, 1711.10474, 1812.08761, 2112.03287, . . . :
Superstrata geometries with and without susy

1507.00945, 1705.09250 1710.06820, 2007.12118, . . . :
$AdS_3$ heavy-light correlators

in (various) collaboration with: I. Bena, A. Bombini, N. Ceplak, A. Galliani, B. Ganchev, S. Giusto, A. Houppe, M. Hughes, E. Martinec, E. Moscato, M. Shigemori, D. Turton, N. Warner

Work in progress with B. Ganchev, S. Giusto, A. Houppe, N. Warner (non-BPS geometries) and S. Giusto, C. Iossa (HHLL correlators)

# The plan

I'll introduce increasingly BH-like families of heavy states

(1) states obtained from the identity (locally AdS states)

(2) 1/2-BPS and 1/4-BPS "graviton gas" states (asymptotycally AdS)

(3) non-BPS "graviton gas" states (asymptotycally AdS)

Quadratic fluctuations around any such solution capture a Heavy-Light holographic correlator (HHLL)

$$\langle \bar{O}_L(1) O_L(z,\bar{z}) \rangle_{ds^2_H} \longleftrightarrow \langle \bar{O}_H(\infty) O_H(0) \bar{O}_L(1) O_L(z,\bar{z}) \rangle \equiv \mathcal{C}_H(z,\bar{z})$$

I will focus the discussion of this correlator on case (2). Interesting limits:

- in the light regime "$p \to 1$", one obtains the LLLL result. The first $AdS_3$ correlator was derived in this way! Giusto, RR, Wen 1812.06479
- in the limit "$p \to N$" the geometry becomes that of a BH. What happens to the HHLL correlator?

# The locally AdS states (1)

The simplest example Maldacena, Maoz . . .

$$|0\rangle_{\mathrm{NS}} \xrightarrow{\text{spectral flow}} |N/2, N/2\rangle_R$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\mathrm{AdS}_3 \times S^3 \qquad \phi \to \phi - \tau\,,\ \psi \to \psi - \sigma \qquad \mathrm{AdS}_3 \times' S^3$$

with $(\phi, \psi)$ $S^3$ coordinates and $(\tau, \sigma)$ $\mathrm{AdS}_3$ coordinates

At the free locus we have $|N/2, N/2\rangle_R = |++\rangle^N$, i.e. the state is made out of $N$ identical RR ground states. . . atypical!

This construction can be generalised to obtain states with a momentum charge (i.e. different holomorphic and antiholomorphic dimensions $h \neq \bar{h}$)
Giusto, Mathur, Saxena, . . .

These geometries become non-trivial when extended beyond the decoupling limit Martinec, Massai Turton; Bufalini, Iguri, Kovensky, Turton; . . .

# The graviton gas (2): CFT side

If $O_k$ is a anti-CPO of dimension $k$ one can consider its descendants

1711.10474;1812.08761

$$O_{k,m,n,q} \equiv (J_0^+)^m (L_{-1})^n (G^{+1}_{-\frac{1}{2}} G^{+2}_{-\frac{1}{2}})^q \, O_k$$

Spectral flow maps $O_{k,m,n,q}$ to a D1-D5-P state with $h > \bar{h} = \frac{c}{24}$

By using $O_{k,m,n,q}$ (also of different types) we can build "semi-classical" multi-particle states characterised by the continuous parameters $B_i$

$$|B_1, B_2, \ldots\rangle_{\mathrm{NS}} \equiv \sum_{p_1, p_2, \ldots} (B_1 O_{k_1,m_1,n_1,q_1})^{p_1} (B_2 O_{k_2,m_2,n_2,q_2})^{p_2} \ldots \, |0\rangle_{\mathrm{NS}}$$

When $B_i^2 \sim N \gg 1$, these are coherent-like states as the sums over $p_i$-sum are peaked for $p_i \approx B_i^2/k_i$

What is the gravitational description of $|B_1, B_2, \ldots\rangle_{\mathrm{NS}}$?

# The graviton gas (2): bulk side

AdS/CFT relates operators and sugra fields: $O_{k,m,n,q} \longleftrightarrow \phi_{k,m,n,q}$

At linear order in $B_i$, $|B_1,\ldots\rangle_{\mathrm{NS}}$ is a perturbation of the vacuum

$$|0\rangle_{\mathrm{NS}} + \sum_i B_i\, O_{k_i,m_i,n_i,q_i}\, |0\rangle_{\mathrm{NS}} \longleftrightarrow \mathrm{AdS}_3 \times S^3 + \sum_i B_i\, \phi_{k_i,m_i,n_i,q_i}$$

where $\phi_{k_i,m_i,n_i,q_i}$ solves the linearised sugra eqs. around $\mathrm{AdS}_3 \times S^3$

The "superstratum" approach provides an algorithm to extend the linear solutions to exact solutions valid for $B_i^2 \sim N$. The key points:

- The susy eqs. can be written in a "linear" form Bena, Giusto, Shigemori, Warner; 1306.1745
- The non-linear extension requires an ansatz: ambiguities are fixed by imposing regularity and input form the CFT 1503.01463;...; Heidmann, Warner
- Precision holography provides a posteriori checks of the non-linear completion and the holographic interpretation Kanitscheider, Skenderis, Taylor; 1507.00945; Giusto, Rawash, Turton

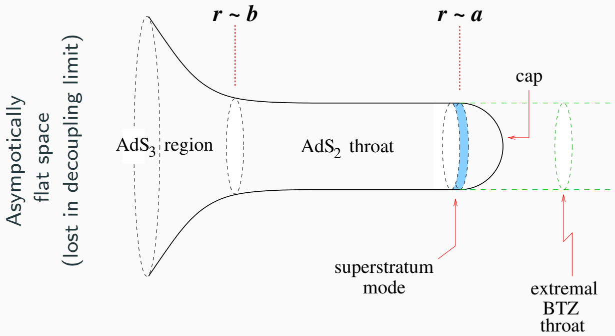
# An interesting example (e1)

We can consider a state built with just one type* of constituents $O_{1,0,1,0}$

* See later for more details

A cartoon of the dual sugra solution ("$AdS_3$" part) looks as follows

1607.03908




The dashed green lines represent the traditional black hole picture and $b^2 \sim |B|^2$ (and $a^2 + \frac{b^2}{2} = a_0^2$, where $a_0^2 = \frac{R_{AdS}^4}{R_y^2}$)

# Another interesting example (e2)

The solution above fits in a 3D gauge supergravity (after reducing on $S^3$)
Mayerson, Walker, Warner

It is part of a family of 3d gauges sugra solutions, including one with enhanced symmetries in the scalar sector (the "special locus")
Ganchev, Houppe, Warner

What is the CFT interpretation of this family and the two main examples? The subtle point lies in the definition $O_L^p$

(e1) Start from the free locus: $O_L^p$ is defined by placing the excitations $O_L$ in different copies of the orbifold CFT

(e2) Start from the Generalised Free Field picture: $O_L^p$ is defined by taking the OPE of $p$ single-particle states

Both cases yield BPS states (just two different basis in the same space). The same distinction applies to the descendants $O_{k,m,n,q}$

# Precision holography

The solution (e1) contains normalisable deviations from AdS $\sim B^2$

They correspond to the expectation values of light operators $O'_{\Delta=2}$ that do *not* enter in the definition of the heavy operator

A non-renormalisation theorem allows for a precise comparison between the sugra and the free-locus results with perfect agreement Rawash, Turton

The CFT input in this calculation is the correlator $\langle O'_{\Delta=2} | O_1^2 \rangle$ where $O_1^2$ is taken from $O_H$. This vanishes with the definition (e2).

This explains why there are no normalisable deviations from AdS corresponding to operators with $\Delta = 2$ in the solution (e2)

The special locus describes bound states obtained by taking the standard OPE of many copies of the same single particle constituent

# The non-BPS case

So far we discussed bound states of mutually BPS objects: the total dimension is simply the sum of the dimensions of the constituents

Is it possible to generalise this construction to globally non-BPS object? The key idea is the following:

- $L_{-1}^{n+m}\bar{L}_{-1}^{m}O_1$ is a protected state also when $m \neq 0$: at linear level it corresponds to a supergravity fluctuation
- the multiparticle state $(L_{-1}^{n+m}\bar{L}_{-1}^{m}O_1)^p$ is non-BPS, but should exist in supergravity: the same non-linear completion as before

The problem is hard. We work with the 3d gauged sugra and in perturbation theory (in the number if constituents in $O_H$)

- When working at the special locus, we can find regular solutions at all orders we tested (up to order 10)
- Not an isolated example: we can build other families of non-BPS solutions based on more types of constituents  to appear

# Holography with non-BPS states

Is it possible to make any holographic check in the non-BPS world?

There are no non-renormalisation theorems, so one should focus on the internal consistency and on realising general CFT properties

An interesting observable is the anomalous dimensions of $O_H$: physically they are due to the non-trivial interaction between the constituents

We can obtain them in two ways: extracting the stress-tensor from the solution and looking at the phase of the field related to $O_1$

- they agree (a non-trivial check!)
- they start at quadratic order (i.e. involves pairs of constituents)
- the leading correction is negative: it represents a binding energy
- In presence of different types of constituents, we can see the effects of interactions between triplet of constituents

# 4-point correlators: generalities

So far we focused on constructing sugra solutions dual to heavy states

Most of the interesting dynamics is encoded in the perturbations around the geometries. The first step is to study the quadratic fluctuations

We consider perturbations $O_L$ that are described by a scalar field in 6D

From now on we focus on the BPS-solutions of type (e1)

Technically, we need to derive the regular, non-normalisable solution that at the boundary ($\rho \to \infty$) scales as

$$z = e^{i(\tau+\sigma)} \qquad\qquad \text{vev of } O_L(z,\bar{z})$$

$$\phi_\Delta(\rho; z, \bar{z}) \stackrel{\rho\to\infty}{\longrightarrow} \delta(z-1)\,\rho^{\Delta-2} + b(z,\bar{z})\,\rho^{-\Delta}$$

$$\text{source for } \bar{O}_L(1)$$

# An easy connection problem

The problem becomes tractable when the 6D equation reduces to 3D. Then, in Fourier space $(\omega, \tau)$, we get an ODE (in the radial direction)

For (e1) with $n = 0$, we get the standard ${}_2F_1$ equation (as for AdS$_3$)

The key step is to impose the regularity conditions at $\rho = 0$

In the hypergeometric case we can use the known connection formulas between ${}_2F_1(\cdot, \cdot; \cdot; x)$ and ${}_2F_1(\cdot, \cdot; \cdot; 1-x)$ (with $x = \frac{\rho^2}{1+\rho^2}$)

$\mathcal{C}_H(\omega, l)$ has poles at 1710.06820

$$\omega_n = -\frac{a}{a_0}\sqrt{(|l| + 2n)^2 + \frac{b^2 l^2}{2a^2}}$$

They correspond to the average anomalous dimension $\Gamma_n$ of the $: O_H O_L :$ bound states (the binding energies mentioned before!)

This correlator contains interesting CFT data, but ...

# A difficult connection problem

...one key feature missed by the previous calculation is the long $AdS_2$ throat that develops in the $a \to 0$ limit of the $n = 1$ case

For (e1) the problem can again be reduced to 3D, but we get a Heun equation. Choosing $n = 1$, we get a reduced confluent Heun

The same problem appears in several other black hole related problems: Quasi Normal Modes, tidal response, thermal correlators: recent progress exploiting the relation to Seiberg-Witten theory and Liouville CFT

Aminov, Grassi, Hatsuda; Bianchi, Consoli, Grillo, Morales; Bonelli, Iossa, Lichtig, Tanzini; ...

In the perturbative limit $b \to 0$ (small number of constituents) we get explicit (perturbative) results from the Nekrasov partition function

We can match and generalise the $\Gamma_n$'s obtained via other approaches

to appear

The "BH"-like limit $a \to 0$ is more challenging

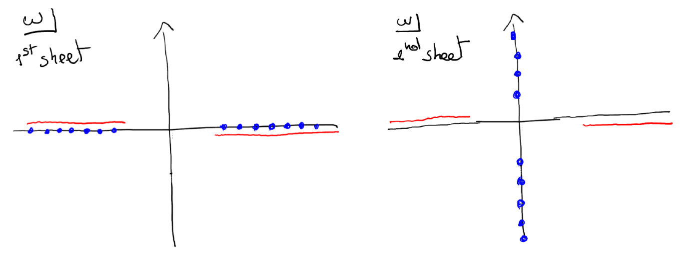
# A "BH"-like limit

An interesting regime is when $\alpha^2 = \frac{a^4}{a_0^4} - 4(1 - \frac{a^2}{a_0^2}) < 0$, i.e. $\frac{a^2}{a_0^2} \lesssim 0.828$

The light cone limit is the same as extremal BTZ

However the full correlator has an interesting analiticity structure

Studying $\mathcal{C}_H(\omega, 0)$ we obtain the following picture [preliminary]




The exact correlator has just real poles (in blue, left fig.); in the semiclassical limit the poles merge into cuts (red lines) which connect to a second sheet with imaginary poles (in blue, right fig.) in the BH regime

# Conclusions

We studied a class of tractable heavy states in a holographic CFT

- Their backreaction is described by regular solutions
- There is an explicit family that approaches extremal BTZ
- It is possible to study the dynamics around such background

We obtained interesting CFT data in the strongly coupled regime

Open questions:

- Can this approach shed light on the transition to the BH regime?
- Are there CFT constraints on HHLL correlators with pure states that have an interesting spacetime interpretation?
- What is the role of quantum corrections in the "BH"-like limit?

Lin, Maldacena, Rozenberg, Shan

# Extra slides

# An example: $k = 1,\ n \geq 1,\ m = q = 0$

It is convenient to parametrise the 6D metric as

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right]+\sqrt{\mathcal{P}}\,ds_4^2\,, \quad \mathcal{P} = Z_1 Z_2 - Z_4^2$$

and all remaining fields have analogous parametrisations

In the case $k = 1,\ n \geq 1,\ m = q = 0$ (Details are not important)

$$ds_4^2 = \frac{\Sigma\,dr^2}{r^2+a^2}+\Sigma\,d\theta^2+(r^2+a^2)\sin^2\theta\,d\phi^2+r^2\cos^2\theta\,d\psi^2 \qquad \beta = 2^{-1/2}\,a^2 R_y\,\Sigma^{-1}\left(\sin^2\theta\,d\phi-\cos^2\theta\,d\psi\right) \qquad \Sigma \equiv r^2+a^2\cos^2\theta$$

$$\hat{v}_{k,m,n} \equiv \sqrt{2}\,R_y^{-1}\,(m+n)\,v+(k-m)\phi-m\psi \qquad \Delta_{k,m,n} \equiv a^k\,r^n(r^2+a^2)^{-(k+n)/2}\cos^m\theta\,\sin^{k-m}\theta$$

$$Z_1 = \frac{Q_1}{\Sigma}+\frac{R_y^2}{2Q_5}b_{k,m,n}^2\frac{\Delta_{2k,2m,2n}}{\Sigma}\cos\hat{v}_{2k,2m,2n} \qquad Z_2 = \frac{Q_5}{\Sigma}\,, \quad Z_4 = b_{k,m,n}R_y\frac{\Delta_{k,m,n}}{\Sigma}\cos\hat{v}_{k,m,n}$$

$$\mathcal{F}_{1,0,n} = -a^{-2}\left(1-r^{2n}(r^2+a^2)^{-n}\right) \qquad \omega_{1,0,n} = 2^{-1/2}\,R_y\,\Sigma^{-1}\left(1-r^{2n}(r^2+a^2)^{-n}\right)\sin^2\theta\,d\phi$$

This class of solutions is fully regular if $a^2 + \frac{b_{1,0,n}^2}{2} = \frac{Q_1 Q_5}{R_y^2}$. For an appropriate choice of parameters it's described by the picture for (e1)