Corrections to the thermodynamics of AdS_5 black holes and the superconformal index

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+ work to appear

Introduction

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- Use AdS/CFT to address the **microstate counting of AdS black holes**

Black hole	=	ensemble of states in quantum gravity	$\overset{\text{AdS/CFT}}{=}$	ensemble of states in the dual CFT

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- We will focus on $\text{AdS}_5/\text{CFT}_4$ and consider the **microstate counting of supersymmetric AdS_5 black holes** in minimal gauged supergravity. Dual states should exist in any holographic $\mathcal{N} = 1$ SCFT

Euclidean approach

Gibbons, Hawking

- We set ourselves in the **grand-canonical ensemble** (fixed β, Ω_i, Φ)

- The **grand-canonical partition function** $\mathcal{Z}(\beta, \Omega_i, \Phi)$ is computed by the **Euclidean path integral** with (anti-)periodic boundary conditions

$$\mathcal{Z}(\beta, \Omega_i, \Phi) = \int Dg_{\mu\nu} DA_\mu D\psi \, e^{-I[g_{\mu\nu}, A_\mu, \psi]} \overbrace{\simeq}^{\text{semiclassical approx.}} e^{-I(\beta, \Omega_i, \Phi)}$$

- $I(\beta, \Omega_i, \Phi)$ should then be identified with $\beta\times$(grand-canonical potential), leading to the **quantum statistical relation (QSR)**

$$\boxed{I = \beta E - \mathcal{S} - \beta \Omega_i J_i - \beta \Phi \, Q}$$

- By the master formula of the **AdS/CFT correspondence**:

$$\boxed{I(\beta, \Omega_i, \Phi) = -\log \mathcal{Z}_{\text{CFT}}(\beta, \Omega_i, \Phi)}$$

Review of AdS_5 black holes

We work with **minimal gauged supergravity in 5d**

$$\mathcal{L} = R + 12g^2 - \frac{1}{4}F^2 - \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda$$

The **most general AdS_5 black hole** depends on **four parameters**:

$$E, J_1, J_2, Q \qquad \leftrightarrow \qquad \beta, \Omega_1, \Omega_2, \Phi$$

Chong, Cvetic, Lu, Pope

These quantities obey the **first law of black hole mechanics**

$$\mathrm{d}E = T\mathrm{d}\mathcal{S} + \Omega_1 \mathrm{d}J_1 + \Omega_2 \mathrm{d}J_2 + \Phi\, \mathrm{d}Q$$

as well as the **quantum statistical relation**

$$I = \beta E - \mathcal{S} - \beta\Omega_1 J_1 - \beta\Omega_2 J_2 - \beta\Phi\, Q$$

Reaching the BPS locus

supersymmetry $\not\Rightarrow$ **extremality**

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It is crucial to reach the BPS locus following a **supersymmetric trajectory**

1. Supersymmetric solution if $E - gJ_1 - gJ_2 - \sqrt{3}\,Q = 0$
2. BPS (supersymmetric + extremal) limit: $\beta \to \infty$

Cabo-Bizet, Cassani, Martelli, Murthy

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The BPS charges must satisfy an **additional constraint**:

$$\boxed{\left(\frac{2\sqrt{3}}{g}Q^* + \frac{\pi}{2Gg^3}\right)\left(\frac{4}{g^2}Q^{*\,2} - \frac{\pi}{Gg^3}(J_1^* + J_2^*)\right) = \left(\frac{2}{\sqrt{3}\,g}Q^*\right)^3 + \frac{2\pi}{Gg^3}J_1^* J_2^*}$$

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In the BPS limit, the **chemical potentials** are **frozen** to the following values

$$\beta^* \to \infty\,, \qquad \Omega_1^* = \Omega_2^* = g\,, \qquad \Phi^* = \sqrt{3}$$

which coincide with the coefficients appearing in the **superalgebra**

$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} \propto E - gJ_1 - gJ_2 - \sqrt{3}\,Q = 0$$

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Using this in the QSR leads to

$$I = -S - \omega_1 J_1 - \omega_2 J_2 - \frac{2}{\sqrt{3}\,g}\varphi\, Q$$

where

$$\omega_i = \beta(\Omega_i - \Omega_i^*) \qquad \text{and} \qquad \varphi = \frac{\sqrt{3}g}{2}\beta(\Phi - \Phi^*)$$

which are **constrained** by

$$\boxed{\omega_1 + \omega_2 - 2\varphi = 2\pi i}$$

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One can also obtain a **supersymmetric first law**

$$\mathrm{d}\mathcal{S} + \omega_1\,\mathrm{d}J_1 + \omega_2\,\mathrm{d}J_2 + \frac{2}{\sqrt{3}\,g}\varphi\,\mathrm{d}Q = 0$$

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The **dependence of I on β disappears** after imposing supersymmetry

Supersymmetric action and BPS entropy

- Evaluating the Euclidean on-shell action of the CCLP black hole and **imposing supersymmetry** $(\omega_1 + \omega_2 - 2\varphi = 2\pi i)$:

$$\boldsymbol{I = \frac{16a}{27}\,\frac{\varphi^3}{\omega_1\omega_2}}$$

where $\boldsymbol{a}$ and $\boldsymbol{c = a + \dots}$ are the **superconformal anomaly coefficients**

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- The *constrained* **Legendre transform** of the supersymmetric action gives the black hole **entropy** Hosseini, Hristov, Zaffaroni; Cabo-Bizet, Cassani, Martelli, Murthy

$$\boldsymbol{\mathcal{S}^* = \pi \sqrt{3 {Q_R^*}^2 - 8a(J_1^* + J_2^*)}}$$

Kim, Lee

CFT side of the story

The relevant CFT quantity is the **superconformal index** ($\equiv$ supersymmetric partition function on $S^1 \times S^3$) on the "second sheet"

$$\mathcal{I} = \mathrm{Tr}\,(-\mathbf{1})^{\mathbf{F}} e^{-\beta\left\{\mathcal{Q},\mathcal{Q}^\dagger\right\}+(\boldsymbol{\omega_1}-2\pi i)\left(J_1+\frac{1}{2}Q\right)+\boldsymbol{\omega_2}\left(J_2+\frac{1}{2}Q\right)}$$

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This is a **refined Witten index**: only receives contributions from states annihilated by $\mathcal{Q}, \mathcal{Q}^\dagger$

Kinney, Maldacena, Minwalla, Raju; Romelsberger

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The contribution of the supersymmetric black hole can be isolated by taking a **Cardy-like limit**:

$$-\log \mathcal{I} = \frac{8\,(5a - 3c)}{27} \frac{\varphi^3}{\omega_1\omega_2} + \frac{2\,(c-a)}{3} \frac{\varphi(\omega_1^2 + \omega_2^2 - 4\pi^2)}{\omega_1\omega_2}$$
$$- \log |\mathcal{G}_{1-\mathrm{form}}| + \text{exp-terms}$$

Cassani, Komargodsky; Choi, J. Kim, S. Kim, Nahmgoong + ...

The **leading-order contribution** in the large N limit exactly **matches the SUSY on-shell action**

$$-\log \mathcal{I} = \frac{16a}{27} \frac{\varphi^3}{\omega_1 \omega_2} \qquad \Rightarrow \qquad -\log \mathcal{I} = I$$

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But the result for the index in the Cardy-like limit **holds at finite** N...

$$-\log \mathcal{I} = \frac{8(5a-3c)}{27}\frac{\varphi^3}{\omega_1\omega_2} - \frac{2(a-c)}{3}\frac{\varphi(\omega_1^2+\omega_2^2-4\pi^2)}{\omega_1\omega_2} + \dots$$

Q1: How can we **match** this result from the **gravitational side**?

Q2: Does it **count** the number of **microstates of BPS black holes** beyond leading order in the large-N limit?

Plan for the rest of the talk

1. Legendre transform of the superconformal index: field theory prediction for the black hole entropy

2. Include relevant higher-derivative/quantum corrections

3. Evaluate the on-shell action for the CCLP black hole and impose supersymmetry to match the result from the index

4. Corrections to the entropy and charges and match with field theory

Legendre transform of the superconformal index

- Extremization principle:

$$\mathcal{S} = \ \text{ext}_{\{\boldsymbol{\omega_1},\boldsymbol{\omega_2},\boldsymbol{\varphi},\boldsymbol{\Lambda}\}}\left[-I - \boldsymbol{\omega_1} J_1 - \boldsymbol{\omega_2} J_2 - \boldsymbol{\varphi}\, Q_R - \boldsymbol{\Lambda}\left(\boldsymbol{\omega_1} + \boldsymbol{\omega_2} - 2\boldsymbol{\varphi} - 2\pi i\right)\right]$$

$$-\frac{\partial I}{\partial \boldsymbol{\omega_i}} = J_i + \boldsymbol{\Lambda}\,, \qquad -\frac{\partial I}{\partial \boldsymbol{\varphi}} = Q_R - 2\boldsymbol{\Lambda}\,, \qquad \boldsymbol{\omega_1} + \boldsymbol{\omega_2} - 2\boldsymbol{\varphi} = 2\pi i$$

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- I is homogeneous of degree 1

$$\mathcal{S} = 2\pi i \Lambda|_{\text{ext}}$$

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$$\mathcal{S} = 2\pi i \Lambda|_{\text{ext}}$$

- Working at linear order in $\mathbf{a} - \mathbf{c}$, one finds that Λ satisfies:

$$\Lambda^3 + p_2 \Lambda^2 + p_1 \Lambda + p_0 + \frac{p_{-1}}{\Lambda - \frac{Q_R}{2}} = 0$$

with $p_\alpha = p_\alpha(Q_R, J_1, J_2)$ real coefficients

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- Reality condition $(\operatorname{Im}\mathcal{S} = 0) \quad \Leftrightarrow \quad (\Lambda^2 + X)(\text{rest}) = 0$

Field theory predictions

The factorization condition $(\Lambda^2 + X)(\text{rest}) = 0$ must be equivalent to the **corrected non-linear constraint** among the charges

$$\left[3Q_R^* + 4\,(2\,a - c)\right]\left[3{Q_R^*}^2 - 8c\,(J_1^* + J_2^*)\right]$$
$$= {Q_R^*}^3 + 16\,(3c - 2a)\,J_1^* J_2^* + 64a\,(a-c)\,\frac{(Q_R^* + a)(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2a(J_1^* + J_2^*)}$$

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The **corrected BPS entropy**

$$S = 2\pi\sqrt{X} = \pi\sqrt{3{Q_R^*}^2 - 8a(J_1^* + J_2^*) - 16\,a(a - c)\frac{(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2a(J_1^* + J_2^*)}}$$

Cassani, AR, Turetta

The four-derivative effective action

- The **goal** is to go beyond the 2∂ approximation including 4∂ terms

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- **Methodology**

  1. Start from the **off-shell** formulation of 5d **supergravity** where 4∂ supersymmetric invariants have been worked out in the literature

  $$\mathcal{L}_{\mathrm{off-shell}} = \mathcal{L}^{(2\partial)}_{\mathrm{off-shell}} + \alpha\,(\lambda_1\,\mathcal{L}_{C^2} + \lambda_2\,\mathcal{L}_{R^2} + \cancel{\lambda_3\,\mathcal{L}_3})$$

  Hanaki, Ohashi, Tachikawa
  Bergshoeff, Rosseel, Sezgin
  Ozkan, Pang

  2. **Integrate out** all the **auxiliary fields** at linear order in α

  3. Use **perturbative field redefinitions**

  $$g_{\mu\nu} \to g_{\mu\nu} + \alpha\,\Delta_{\mu\nu}\,, \qquad A_\mu \to A_\mu + \alpha\,\Delta_\mu$$

  to simplify the resulting action as much as possible

The four-derivative effective action

Cassani, AR, Turetta

- **Final result**

$$\mathcal{L} = \boldsymbol{c_0} R + 12 \boldsymbol{c_1} g^2 - \frac{\boldsymbol{c_2}}{4} F^2 - \frac{\boldsymbol{c_3}}{12\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda + \boldsymbol{\lambda_1 \alpha} \left(\mathcal{X}_{\text{GB}} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right)$$

where $\mathcal{X}_{\text{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$ and $c_i = 1 + \alpha g^2 \delta c_i$, with:

$$\delta c_0 = 4\boldsymbol{\lambda_2}\,, \qquad \delta c_1 = -10\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}\,, \qquad \delta c_2 = 4\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}\,, \qquad \delta c_3 = -12\boldsymbol{\lambda_1} + 4\boldsymbol{\lambda_2}$$

- **Remarks**
  - Together with 4∂ corrections, there are 2∂ **corrections** controlled by $\lambda_i \alpha g^2$
  - The effect of λ_2 simply reduces to a **renormalization of** G
  - It can be argued that λ_3 can be set to zero without loss of generality

Holographic dictionary

- The bulk theory is controlled by 2 dimensionless quantities

$$\frac{1}{G_{\text{eff}}\, g^3} \equiv \frac{1 + 4\lambda_2 \alpha g^2}{G g^3}\,, \qquad\qquad \lambda_1 \alpha g^2$$

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- $\mathcal{N}=1$ SCFTs have a **superconformal anomaly** controlled by 2 coefficients:

$$T_i{}^i \;=\; -\,\frac{\boldsymbol{a}}{16\pi^2}\hat{E} + \frac{\boldsymbol{c}}{16\pi^2}\hat{C}^2 \;-\; \frac{\boldsymbol{c}}{6\pi^2}\,\hat{F}^2\;,$$

$$\nabla_i J^i \;=\; \frac{\boldsymbol{c}-\boldsymbol{a}}{24\pi^2}\,\frac{1}{2}\,\epsilon^{ijkl}\hat{R}_{ijab}\hat{R}_{kl}{}^{ab} \;+\; \frac{5\boldsymbol{a}-3\boldsymbol{c}}{27\pi^2}\,\frac{1}{2}\,\epsilon^{ijkl}\hat{F}_{ij}\hat{F}_{kl}\,,$$

Anselmi, Freedman, Grisaru, Johansen; Cassani, Martelli

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Anselmi, Freedman, Grisaru, Johansen; Cassani, Martelli

- The **holographic dictionary** tells us that

$$\boxed{\boldsymbol{a} = \frac{\pi}{8Gg^3}\left(1+4\boldsymbol{\lambda_2}\alpha g^2\right) \qquad \boldsymbol{c} = \frac{\pi}{8Gg^3}\left(1+4\left(2\boldsymbol{\lambda_1}+\boldsymbol{\lambda_2}\right)\alpha g^2\right)}$$

Witten

Henningson, Skenderis

Fukuma, Matsuura, Sakai

Setting up the computation of the on-shell action at $\mathcal{O}(\alpha)$

A priori we need two ingredients to **evaluate the on-shell action at** $\mathcal{O}(\alpha)$:

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However, it is possible to show that **the two-derivative solution is enough** for evaluating the on-shell action

$$I = I^{(0)}|_{\alpha=0} + \alpha \, \cancel{\partial_\alpha I^{(0)}_{\alpha=0}} + \alpha \, I^{(1)}|_{\alpha=0} + \mathcal{O}(\alpha^2)$$

if one fixes the boundary conditions appropriately, which is automatic if we work in the **grand-canonical ensemble**

Reall, Santos

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We are then left with the only task of identifying an appropriate set of **boundary terms**

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Still, in our case:

- the GH term associated to the GB is well known Myers; Teitelboim, Zanelli
- GH terms associated to $C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ and to the mixed Chern-Simons term can be derived as well but do not contribute for **AlAdS$_5$** spacetimes

Grumiller, Mann, McNees; Landsteiner, Megías, Pena-Benitez

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Boundary counterterms

- same as in the 2∂ theory (only the GB term diverges in the 4∂ sector)

Matching the superconformal index

- We can now calculate the **on-shell action** of the CCLP black hole. After imposing **supersymmetry** $(\omega_1 + \omega_2 - 2\varphi = 2\pi i)$ we find:

$$I = \frac{2\pi}{27Gg^3}\left(1 - 4(3\lambda_1 - \lambda_2)\alpha g^2\right)\frac{\varphi^3}{\omega_1\omega_2} + \frac{2\pi\alpha\lambda_1}{3Gg}\frac{\varphi\left(\omega_1^2 + \omega_2^2 - 4\pi^2\right)}{\omega_1\omega_2}$$

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Cassani, AR, Turetta

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- What about the **black hole entropy**?

The corrected BPS entropy and charges

Cassani, AR, Turetta

- **Corrected charges** from I (assuming first law and QSR):

$$E = \frac{\partial I}{\partial \beta} \ , \qquad J_1 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_1} \ , \qquad J_2 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_2} \ , \qquad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi}$$

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- **Corrected BPS entropy** (from QSR and taking BPS limit):

$$\mathcal{S}^* = \pi\sqrt{3{Q_R^*}^2 - 8a(J_1^* + J_2^*) - 16\,a(a-c)\frac{(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2a(J_1^* + J_2^*)}}$$

(also: Bobev, Dimitrov, Reys, Vekemans)

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- **Corrected non-linear relation between the charges:**

$$\boldsymbol{\left[3Q_R^* + 4\,(2\,a - c)\right]\left[3{Q_R^*}^2 - 8c\,(J_1^* + J_2^*)\right]}$$
$$\boldsymbol{= {Q_R^*}^3 + 16\,(3c - 2a)\,J_1^* J_2^* + 64a\,(a-c)\frac{(Q_R^* + a)(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2a(J_1^* + J_2^*)}}$$

Wald entropy from the near-horizon geometry

- We want to compute the **BPS entropy** using **Wald's formula**
- **Corrected solution needed** but the **near-horizon geometry** is enough
- Focus on the $J_1^* = J_2^* (\equiv J^*)$ case (**Gutowski-Reall black hole**), much simpler

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$$\begin{aligned} \mathrm{d}s^2 &= v_1 \left(-\varrho^2 \mathrm{d}t^2 + \frac{\mathrm{d}\varrho^2}{\varrho^2} \right) + \frac{v_2}{4} \left[\sigma_1^2 + \sigma_2^2 + v_3 \left(\sigma_3 + w\, \varrho\, \mathrm{d}t \right)^2 \right] \\ A &= e\, \varrho\, \mathrm{d}t + p\, \left(\sigma_3 + w\, \varrho\, \mathrm{d}t \right) \end{aligned}$$

where

$$v_i = v_i^{\mathrm{GR}} + \alpha\, \delta v_i\,, \quad w = w^{\mathrm{GR}} + \alpha\, \delta w\,, \quad e = e^{\mathrm{GR}} + \alpha\, \delta e\,, \quad p = p^{\mathrm{GR}} + \alpha\, \delta p$$

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Solving the corrected EOMs boils down to a linear system of algebraic eqs

$$\mathcal{M} \mathcal{X} = \mathcal{N}\,, \qquad \mathcal{X} = \mathcal{X}^H + \mathcal{X}^P$$

- $\mathcal{X}^H$ is the **homogeneous solution**, fixed by **boundary conditions**
- $\mathcal{X}^P$ is a **particular solution**, it contains the **"new physics"**

Wald entropy

Wald formula:

$$\boldsymbol{\mathcal{S} = -2\pi \int_{\Sigma} \mathrm{d}^3 x \sqrt{\gamma}\, \frac{\delta S}{e \delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma}}$$

where $\epsilon_{\mu\nu}$ is the binormal at the horizon

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Using the **corrected near-horizon solution** and expressing the result in terms of the charges, one gets

$$\boldsymbol{\mathcal{S}^* = \pi\sqrt{3{Q_R^*}^2 - 16 a J^*}}$$

which agrees with the previous expressions when $\boldsymbol{J_1^* = J_2^* \equiv J^*}$

Note that all the corrections are encoded in $\boldsymbol{a, c}$

Cassani, AR, Turetta

Charges from the near-horizon

Surface charges

Make use of covariant phase-space formalism to construct a $(d-1)$-form current $\mathbf{j}_{\xi,\chi}$ such that:

1. $\mathbf{j}_{\xi,\chi}$ is conserved, $\mathbf{j}_{\xi,\chi} = \mathrm{d}\mathbf{k}_{\xi,\chi}$
2. $\mathbf{j}_{\xi,\chi}$ vanishes on-shell

when ξ, χ are **field symmetries** ($\equiv \delta_\xi g_{\mu\nu} = 0$ and $\delta_{\xi,\chi} A_\mu = 0$)

Barnich, Brandt, Henneaux

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$\mathbf{k}_{\xi,\chi}$ can be constructed from the Noether-Wald surface charge $\mathbf{Q}_{\xi,\chi}$ as follows:

$$\mathbf{k}_{\xi,\chi} = \mathbf{Q}_{\xi,\chi} - \mathbf{\Xi}_{\xi,\chi} \qquad \mathrm{d}\mathbf{\Xi}_{\xi,\chi} = \iota_\xi \mathbf{L} + \chi \mathbf{\Delta}$$

and crucially satisfies

$$\boxed{\int_{\mathcal{C}} \mathrm{d}\mathbf{k}_{\xi,\chi} = 0 \quad \Rightarrow \quad \int_{\partial\mathcal{M}\cap\mathcal{C}} \mathbf{k}_{\xi,\chi} = \int_{\mathcal{H}} \mathbf{k}_{\xi,\chi}}$$

Cassani, AR, Turetta (to appear)

Applications

- **Electric charge** $(\xi = 0, \chi = 1)$: $Q = \int_{\mathcal{H}} \mathbf{k}_{0,1}$
  - It corresponds to the Page charge of the solution
  - It matches the charge extracted from the on-shell action (up to a topological contribution)

- **Angular momenta** $(\xi = \partial_{\phi_i}, \chi = 0)$: $J_i = \int_{\mathcal{H}} \mathbf{k}_{\partial\phi_i,0}$
  - It fully matches the expression extracted from the on-shell action

- Derivation of the **quantum statistical relation** and **first law**

Our main results

- Construction of a **four-derivative extension of 5d minimal gauged supergravity** that captures the corrections to the index

- We have evaluated the **on-shell action** of the CCLP black hole **at linear order** in the corrections, showing that it **matches the CFT prediction** when **supersymmetry** is imposed

$$\boxed{I = -\log \mathcal{I}}$$

- We have computed the **corrections to the BPS entropy** using different methods, showing that all of them agree

- We have shown that the **index counts** the number of black hole **microstates** beyond leading order in the large-N limit approx.

Open questions and future directions

- Better understanding of the BPS limit

- What information can be extracted from the near-horizon?

Cassani, AR, Turetta (to appear)

- Extension to matter-coupled supergravities. Multi-charge case.

- Thermodynamics of near-BPS AdS_5 black holes. Corrections to the mass gap? CFT description?

Boruch, Heydemann, Iliesiu, Turiaci

- Possible stringy origins of the gravitational EFT?

Bilal, Chu
Liu, Minasian

Thank you!