Spectroscopy and Physics of the Beautiful Tetraquarks

Ahmed Ali
DESY

INFN Super-B Workshop, Frascati, Tuesday, 05.04.2011

Overview

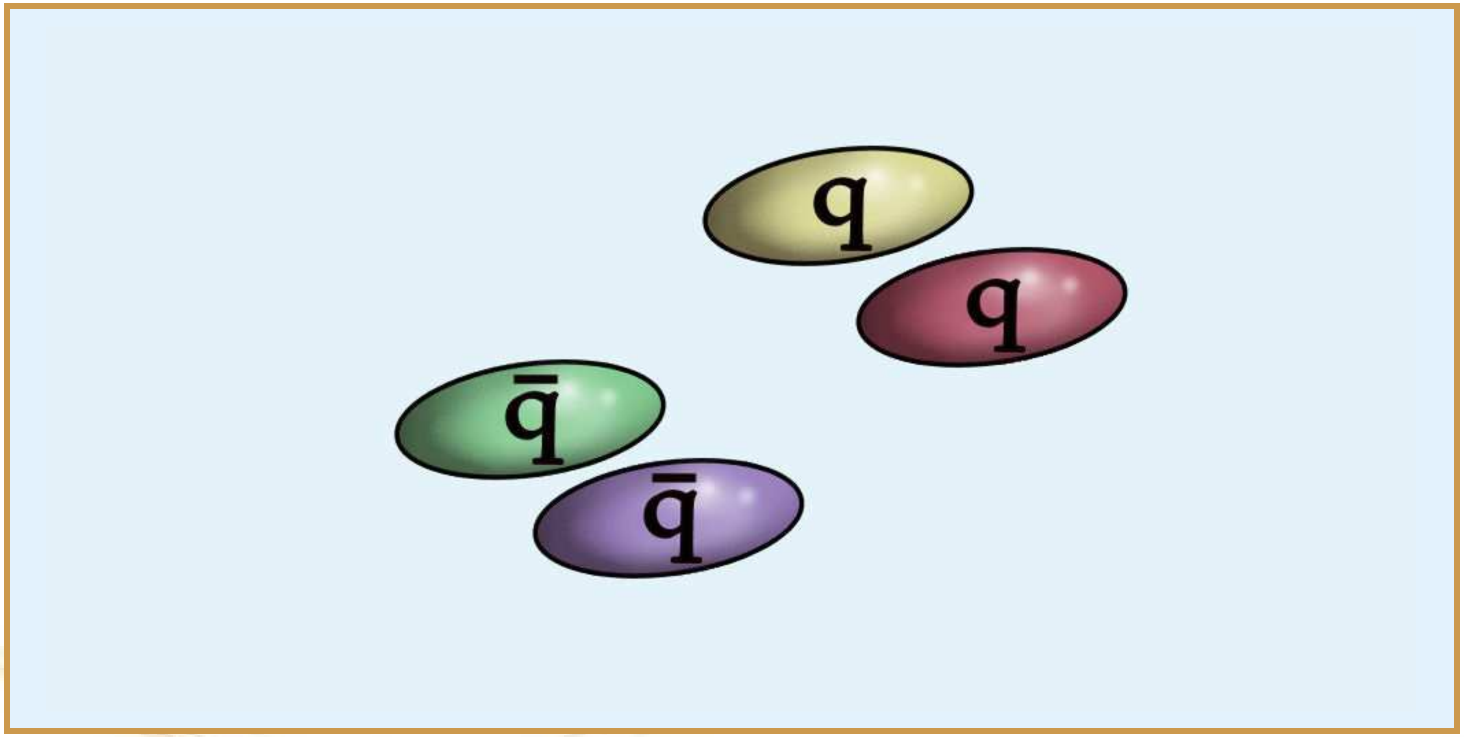
- Tetraquarks and Diquarks: Introduction
- Experimental evidence of exotic states and interpretations
- Calculation of tetraquark masses $M_{[bq][\bar{b}\bar{q}']}$
- Production & decays of $J^{PC} = 1^{--}$ tetraquarks $Y_{[bq][\bar{b}\bar{q}]}$
- Search for $Y_{[bq][\bar{b}\bar{q}]}$ in the BaBar R_b energy scan
- Interpretation of $Y_b(10890)$ as a tetraquark and analysis of the Belle data
- Summary and outlook

Tetraquarks and Diquarks: Introduction

- A basic question in hadron physics:
  Are there additional structures beyond the $(q\bar{q})$ *mesons and* (qqq) *baryons?*
- If not, why not?
- If yes, what are they? and where are they?
- In this talk, we argue that tetraquarks (bound states of diquarks antidiquarks) exist in nature
- We outline the phenomenology; analyse current data to search for them and suggest future experiments

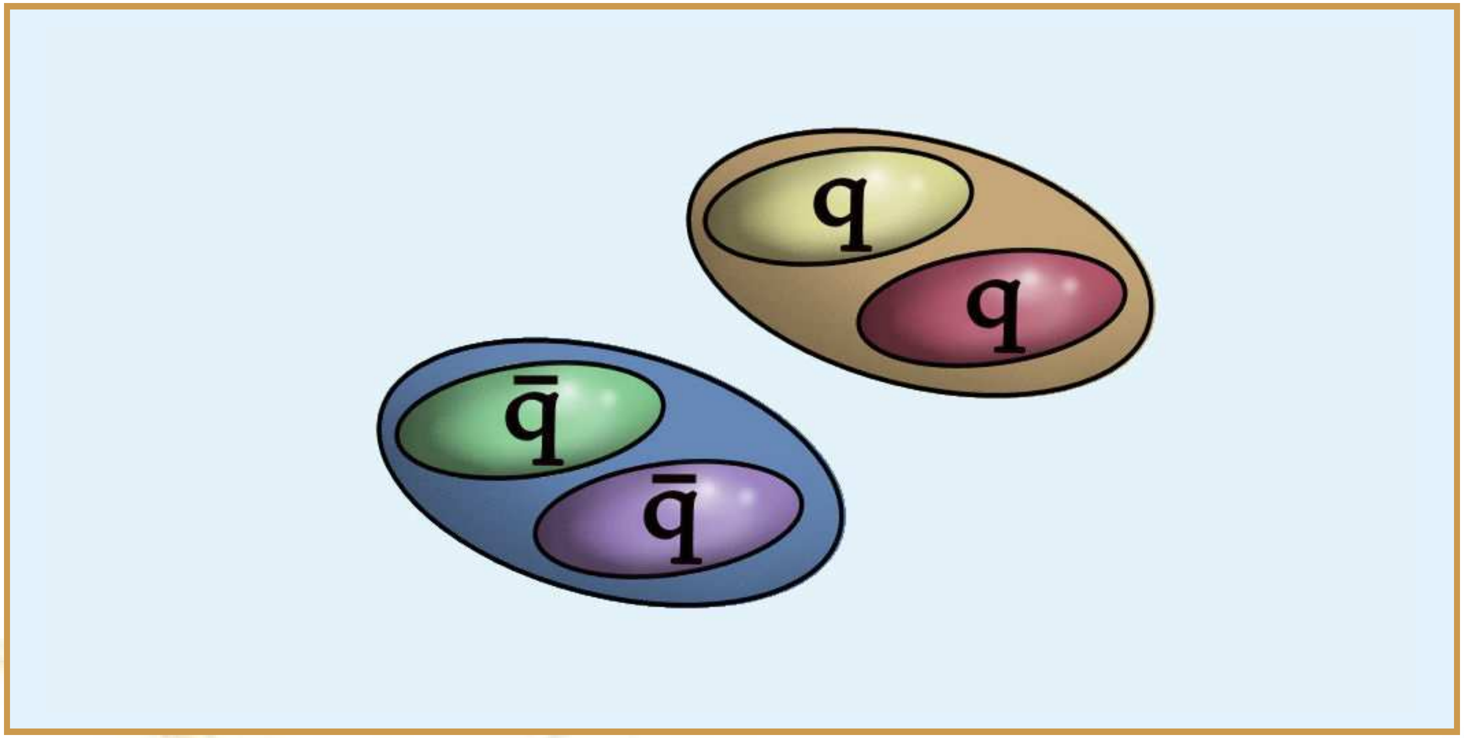
Tetraquark constituents

Tetraquarks consist of 4 quarks



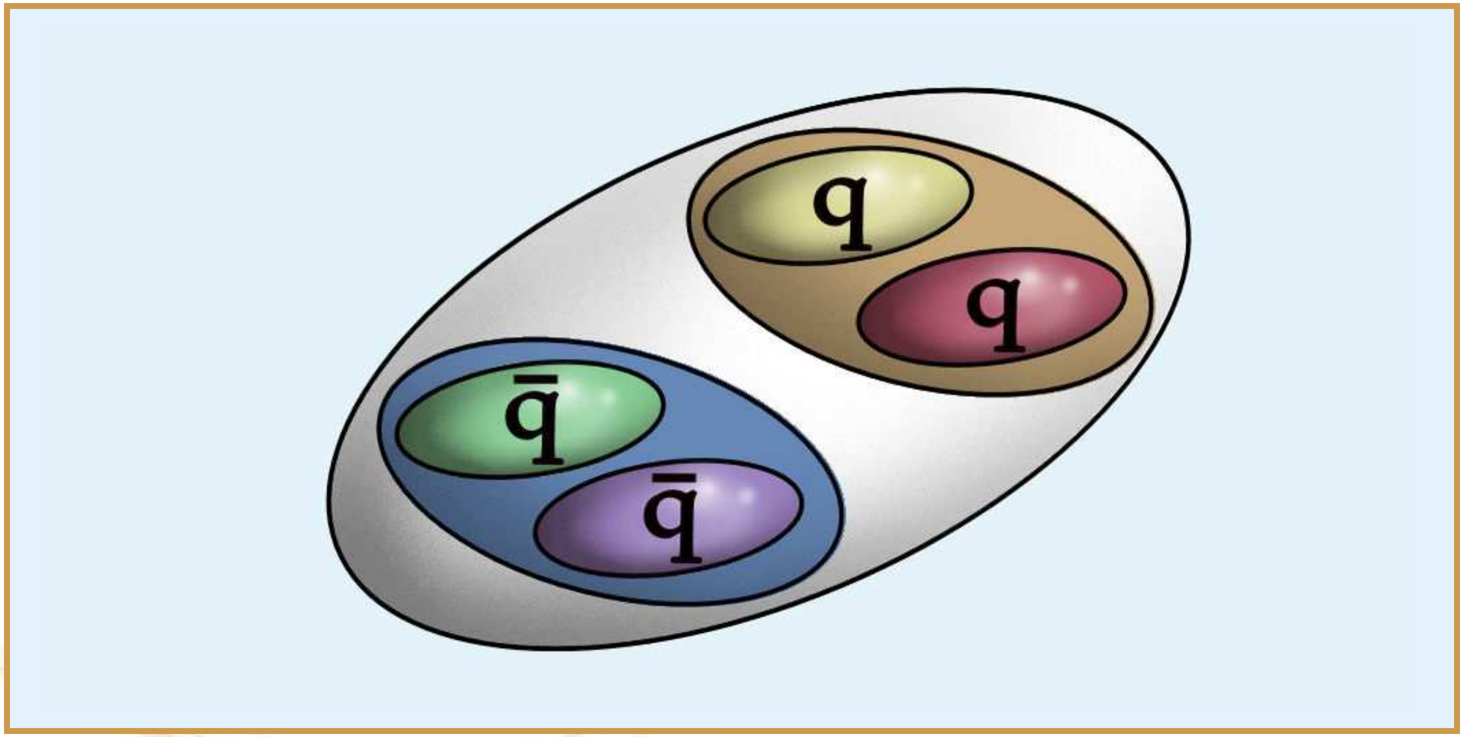
Tetraquark constituents

paired as colored diquarks $[qq]$ and antidiquarks $[\bar{q}\bar{q}]$



Tetraquark constituents

bound by QCD forces in a colorless hadron: tetraquark



Tetraquarks vs. hadronic molecules

- Two different 4-quark hadrons, seemingly similar

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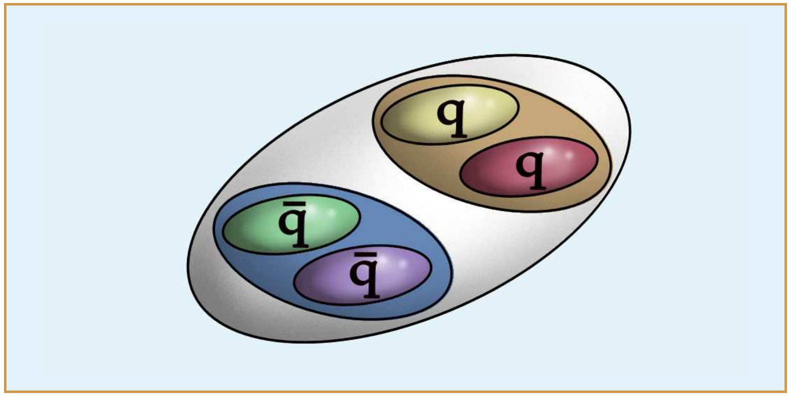
Tetraquarks:

- Diquarks and Antidiquarks are colored

⇒ Strongly bound by QCD forces



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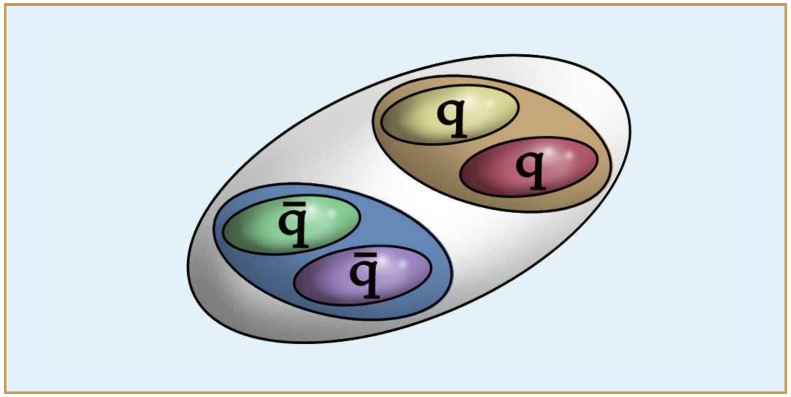
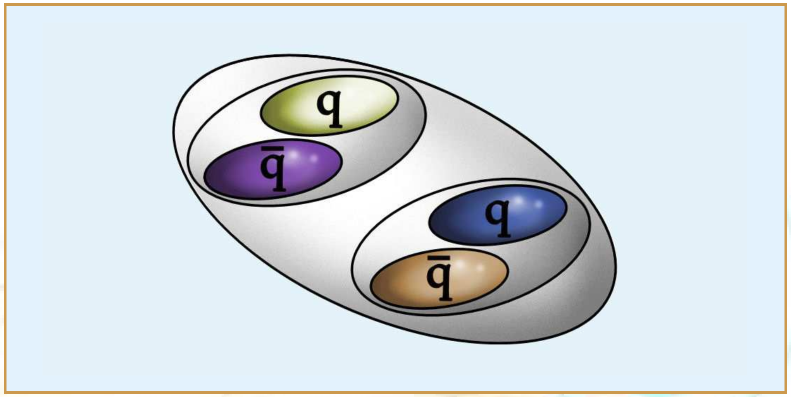


Hadronic molecules:

- Bound states of uncolored mesons

⇒ Bound by pionic exchanges



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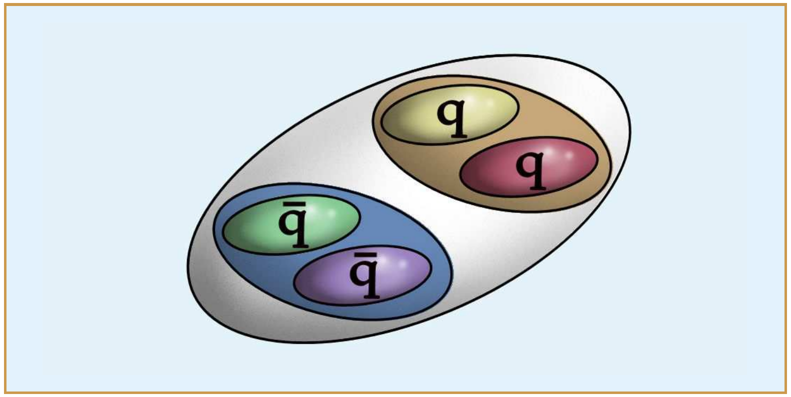

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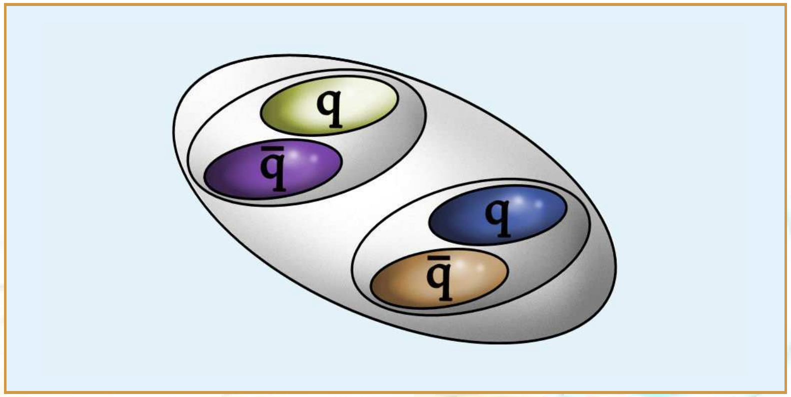


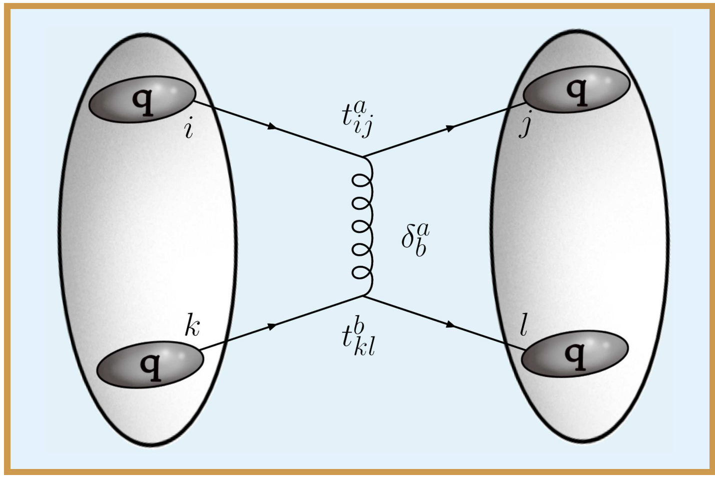
$\Rightarrow$ Very different phenomenology!

Diquarks: Color representation

- One-gluon exchange model [R. Jaffe (2005)]

$\Rightarrow$ Color factor determines binding:

- Negative sign $\Rightarrow$ Attraction



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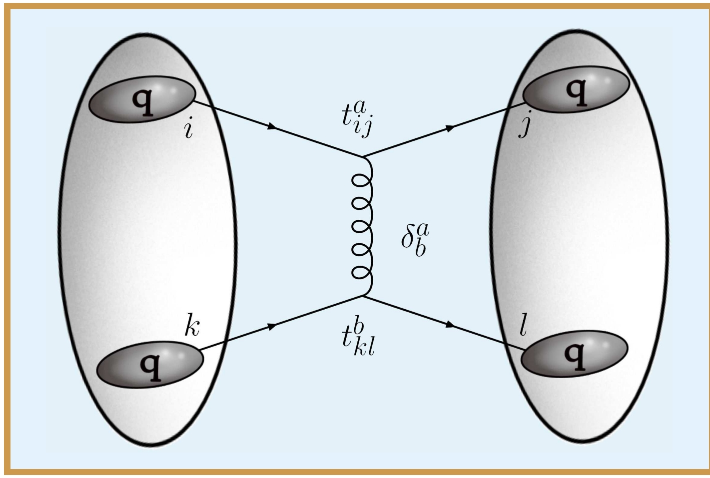

- Quarks in diquark transform as:

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

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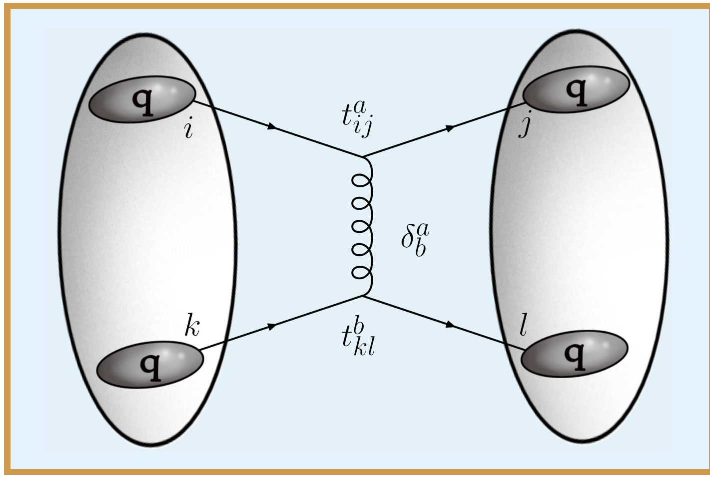

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$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{\mathbf{3}}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } \mathbf{6}}$$

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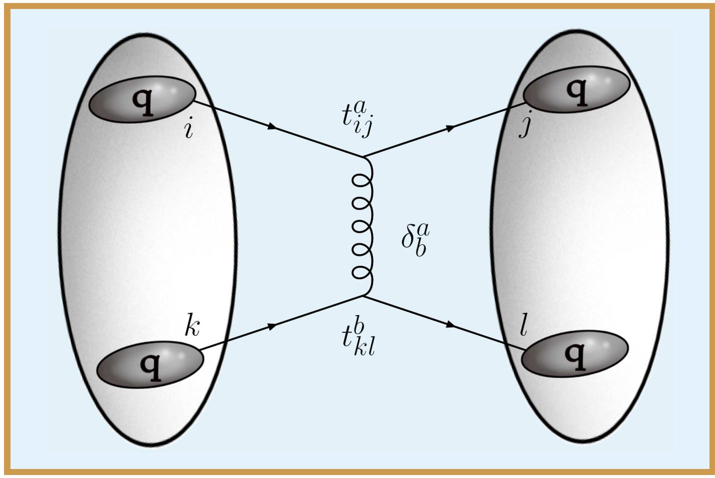

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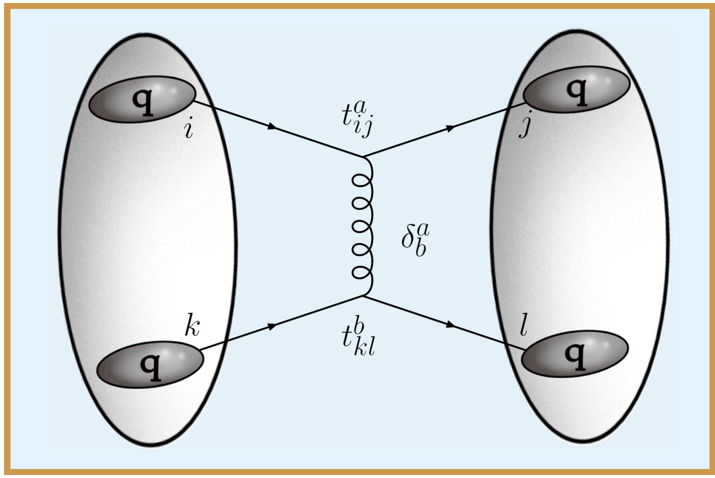

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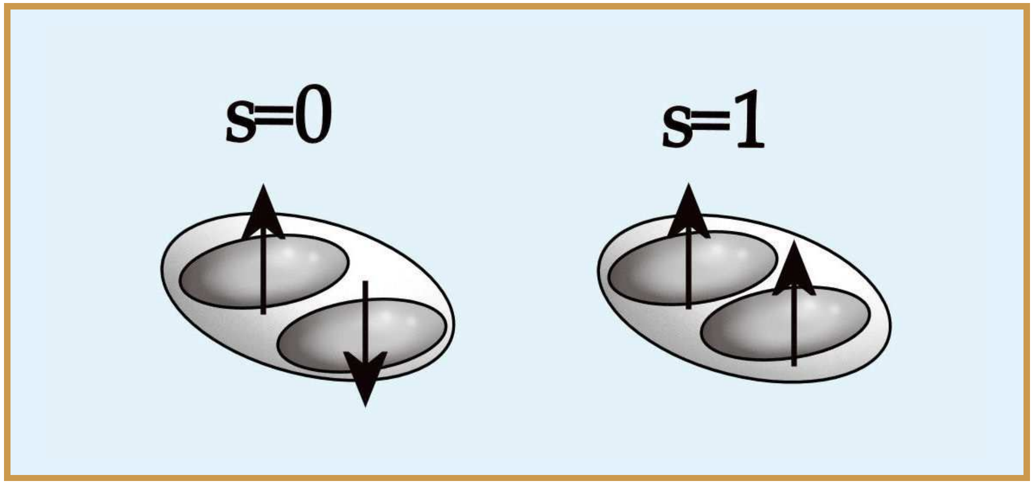

Lattice simulations of diquarks with light quarks [Alexandrou et al., PRL 97:222002 (2006)]

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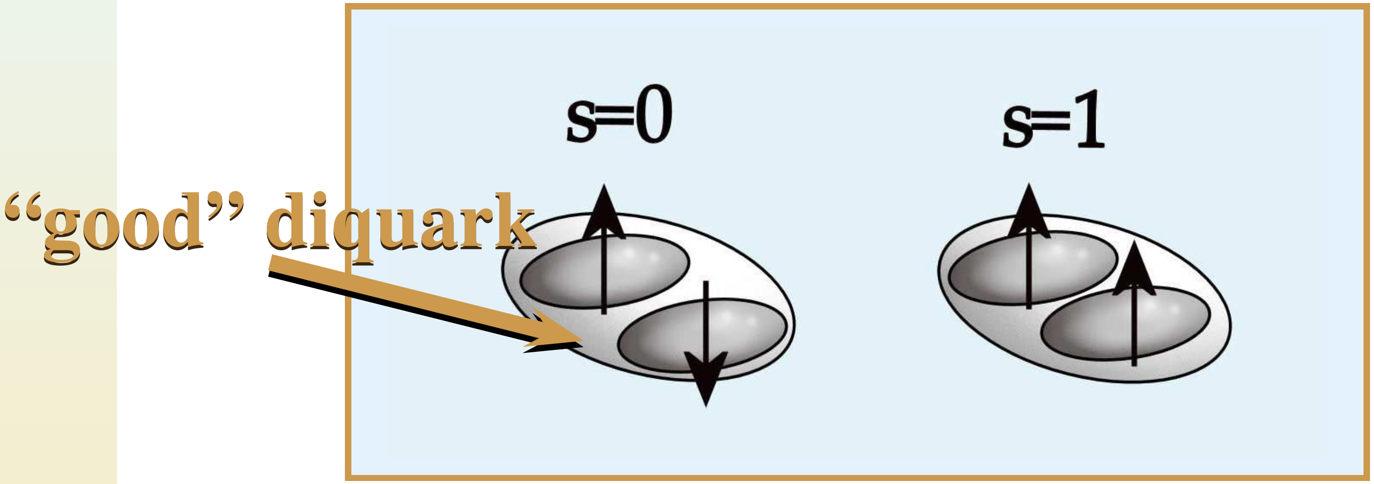

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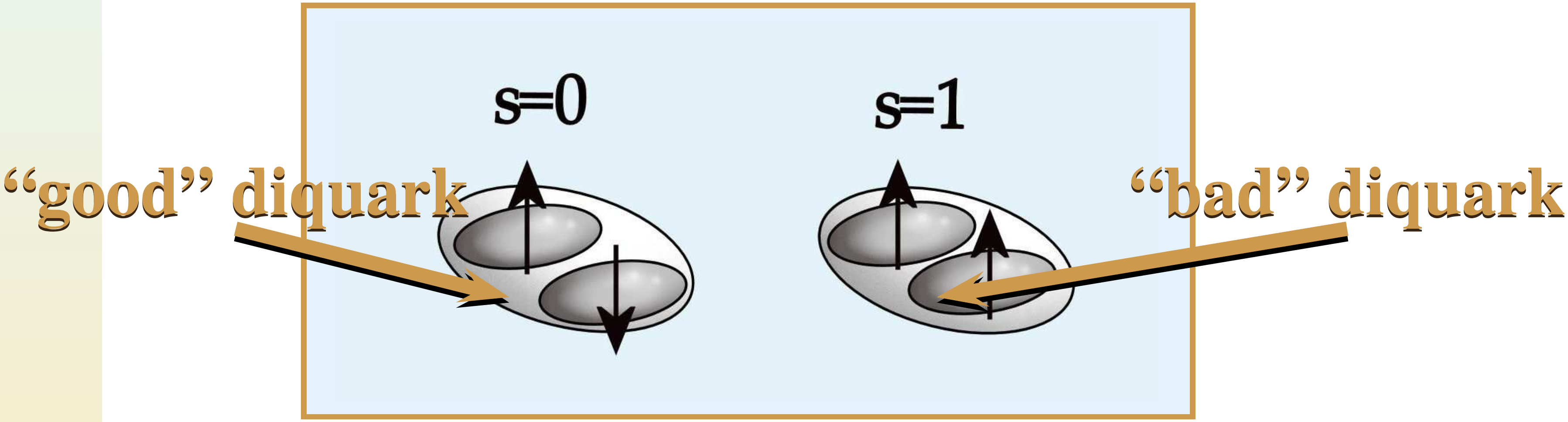

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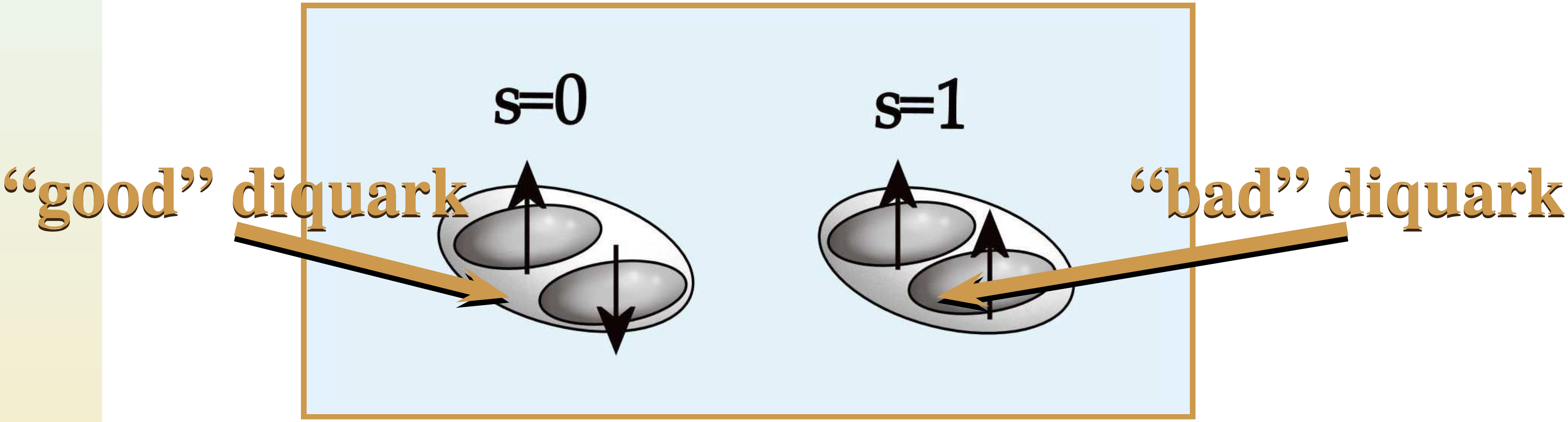

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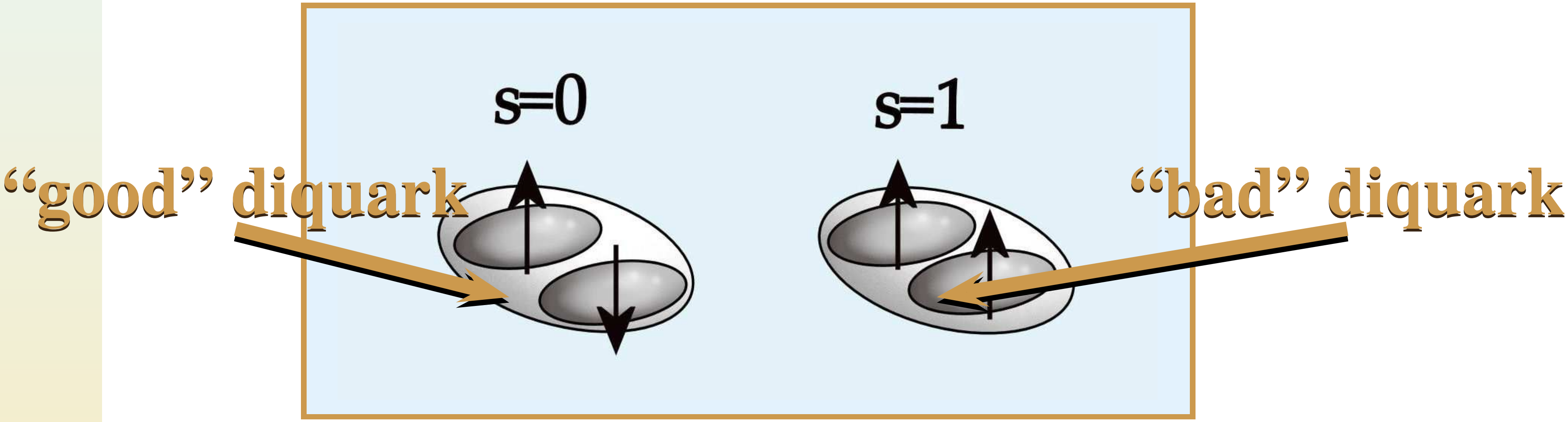

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Now: Closer look at lattice results

- Binding uarks
- No binding for "bad" spin 1 diquarks

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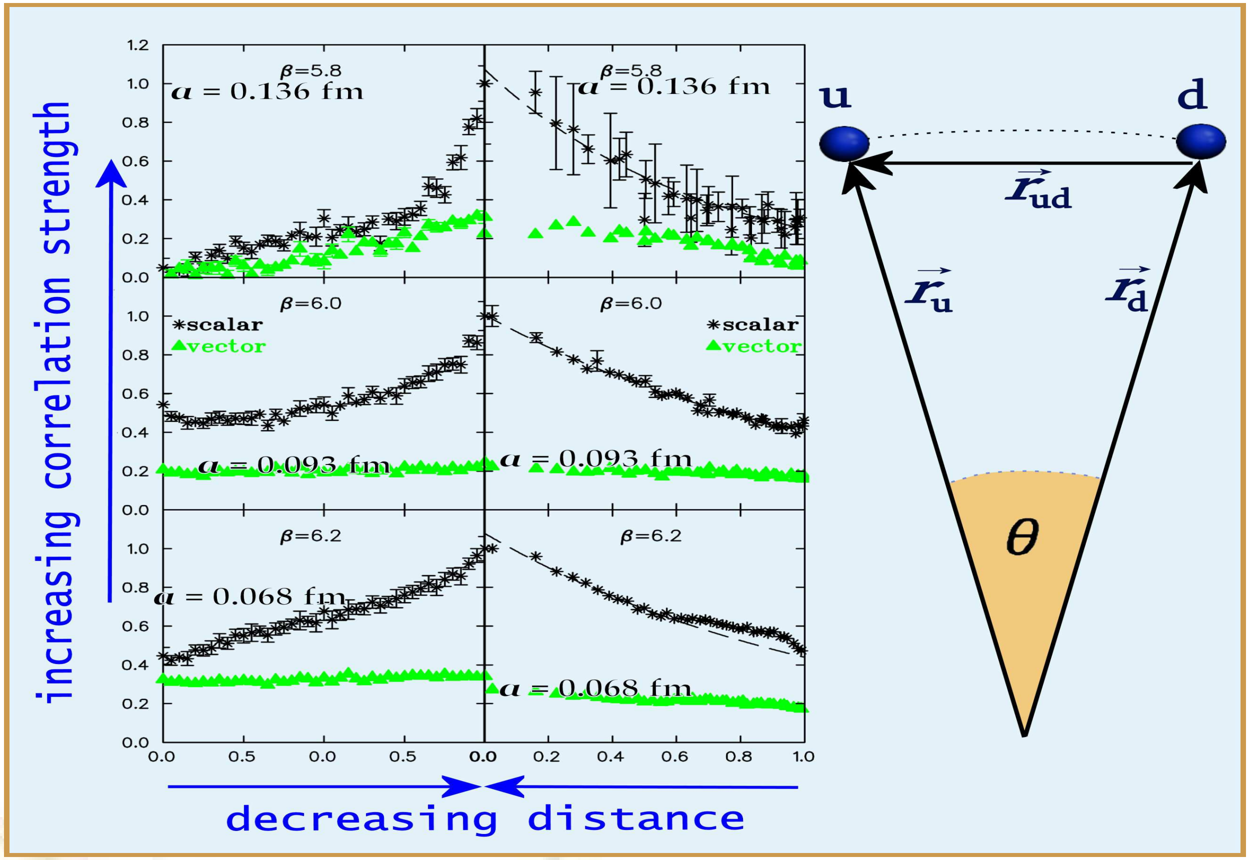
Evidence for diquarks in lattice QCD



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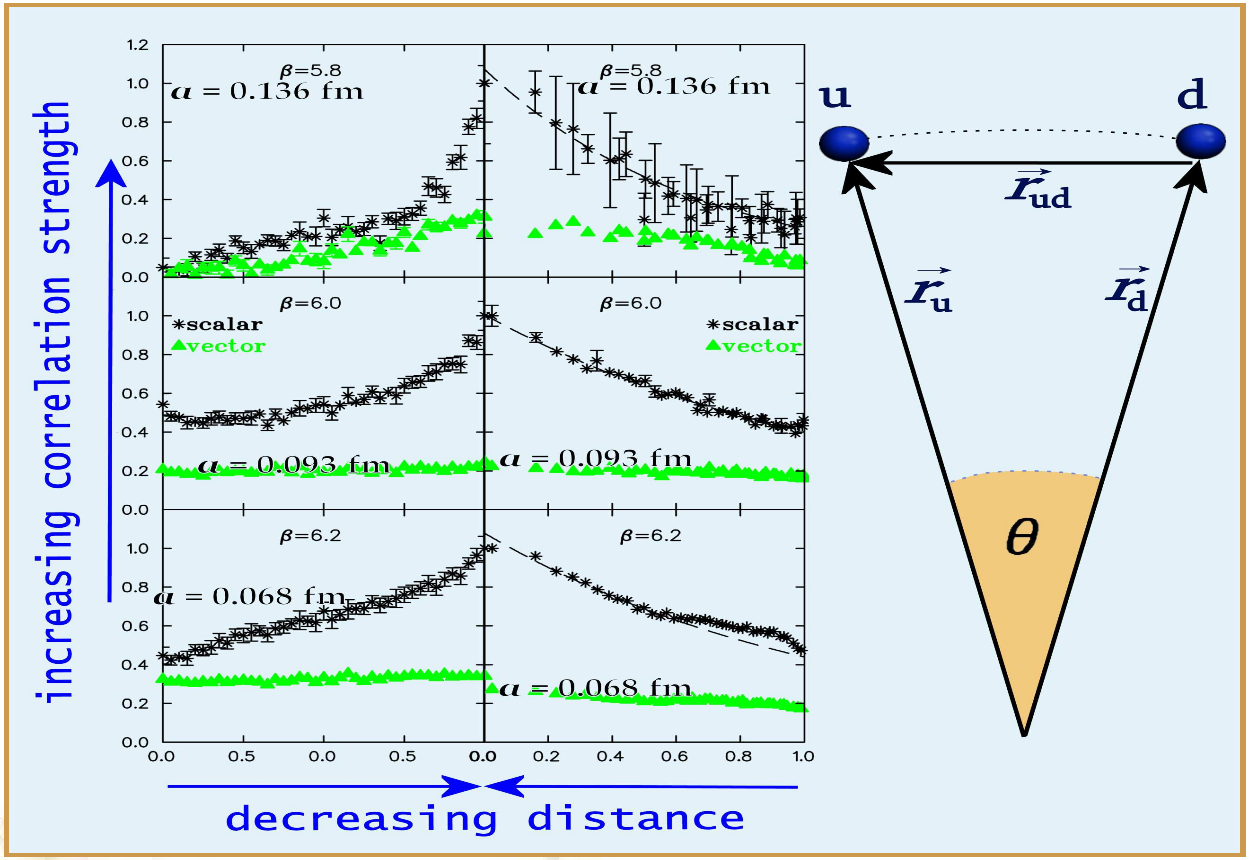

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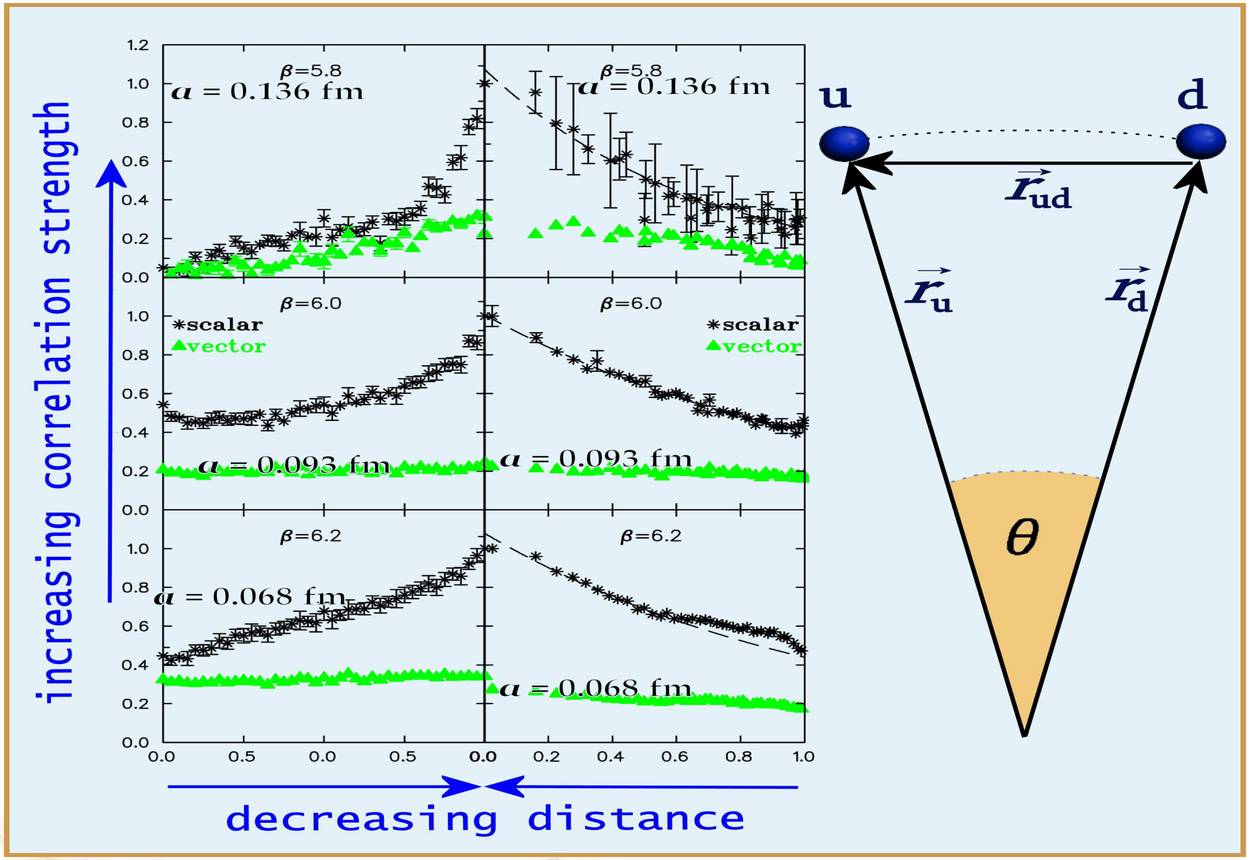

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Experimental evidence of exotic states and interpretations

- Experimental evidence exists for "Exotic States" from e^+e^-colliders and Tevatron
- Lost tribes of Charmonium? [Quigg (2004)]
- $c\bar{c}g$ Hybrids? [Close & Page (2005); Kou & Pene (2005)]
- $D\bar{D}^{(*)}$ Molecules? [Tornquist (2004); Braaten & Kusonoki (2004); Swanson (2004); Voloshin (2004); Liu et al. (2005); Rosner (2007); ...]
- Tetraquarks $[cq][\bar{c}\bar{q}]$? [Maiani et al.; Polosa et al. (2004 - 2010)]
- Recent Review on Heavy Quarkonium: [Brambilla *et al.*, EPJ, C71, 1534 (2011)]

Exotic states

Belle observations [A. Zupanc [Belle], arXiv:0910.3404 (2009)] (updated)

State	M (MeV)	Γ (MeV)	J^{PC}	Decay Modes	Production Modes	Also observed by
$\phi(2170)$	2175 ± 15	61 ± 18	1^{--}	$\phi f_0(980)$	e^+e^- (ISR) $J/\psi \to \eta Y_s(2175)$	BaBar, BESII
$X(3872)$	3871.5 ± 0.2	< 2.2	$1^{++}/2^{-+}$	$\pi^+\pi^- J/\psi$, $\gamma J/\psi$, $D\bar{D}^*$	$B \to KX(3872)$, $p\bar{p}$	BaBar CDF, D0,
$X(3915)$	3914 ± 4	28 ± 10	$0/2^{++}$	$\omega J/\psi$	$\gamma\gamma \to X(3915)$	
$\chi_{c2}(2P)$	3929 ± 5	29 ± 10	2^{++}	$D\bar{D}$	$\gamma\gamma \to Z(3940)$	
$X(3940)$	3942 ± 9	37 ± 17	$0^{?+}$	$D\bar{D}^*$ (not $D\bar{D}$ or $\omega J/\psi$)	$e^+e^- \to J/\psi X(3940)$	
$Y(4008)$	4008^{+121}_{-49}	226 ± 97	1^{--}	$\pi^+\pi^- J/\psi$	e^+e^- (ISR)	
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$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$\pi^+\pi^- \psi'$	e^+e^- (ISR)	
$Z(4050)$	4051^{+24}_{-23}	82^{+51}_{-29}	?	$\pi^\pm \chi_{c1}$	$B \to KZ^\pm(4050)$	
$Z(4250)$	4248^{+185}_{-45}	177^{+320}_{-72}	?	$\pi^\pm \chi_{c1}$	$B \to KZ^\pm(4250)$	
$Z(4430)$	4433 ± 5	45^{+35}_{-18}	?	$\pi^\pm \psi'$	$B \to KZ^\pm(4430)$	
$Y_b(10890)$	$10,888.4 \pm 3.0$	$30.7^{+8.9}_{-7.7}$	1^{--}	$\pi^+\pi^- \Upsilon(1,2,3S)$	$e^+e^- \to Y_b$	

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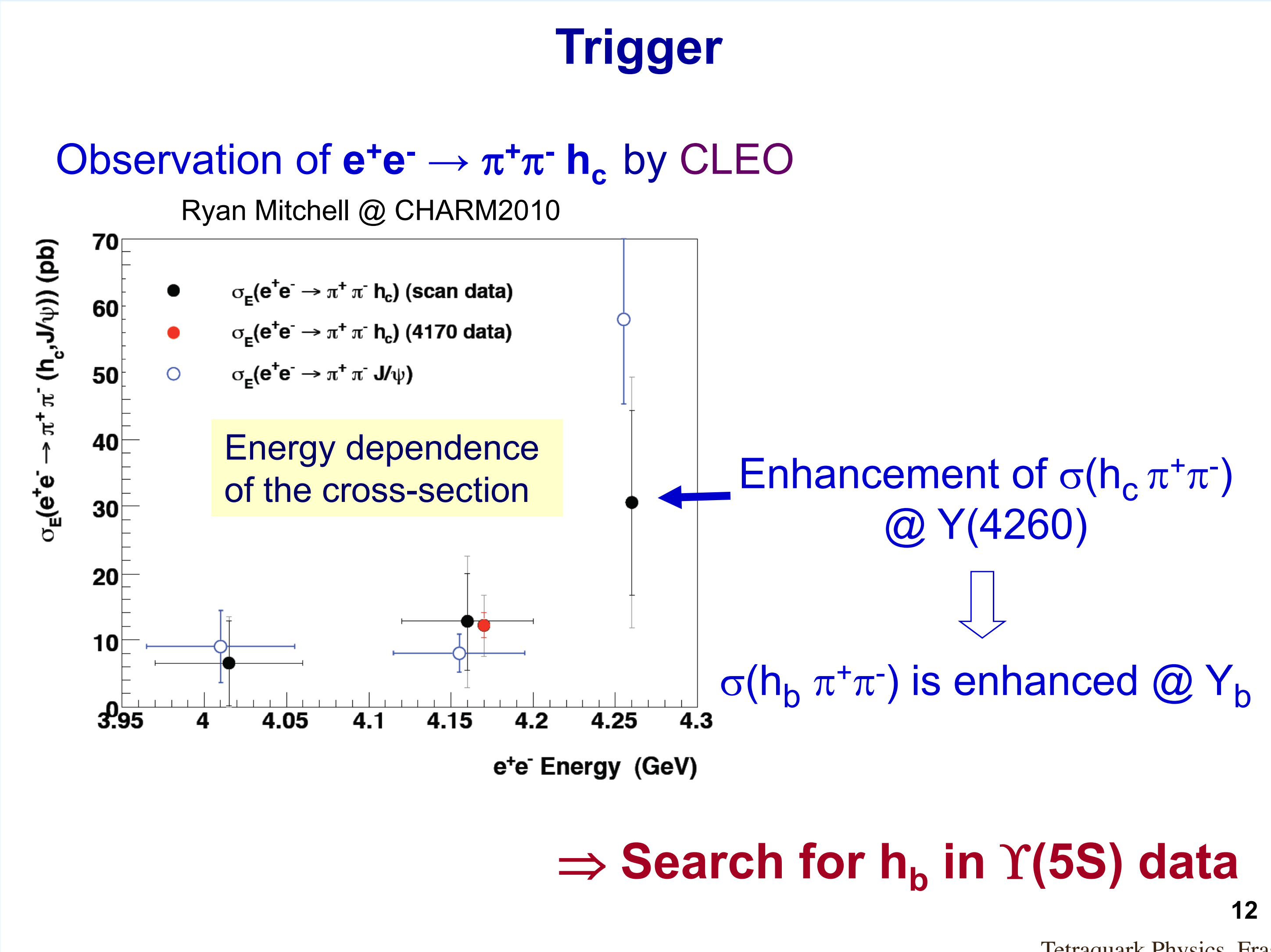
tetraquark candidate with a $b\bar{b}$ pair (... more later)

Singlet P states: $h_c(1P)$, $h_b(1P)$, $h_b(2P)$

Discovery of $h_c(1P)$ in $e^+e^- \to \pi^+\pi^- h_c(1P)$ and energy dependence



Introduction to $h_b(nP)$

$(b\bar{b})$: S=0 L=1 $J^{PC}=1^{+-}$

Expected mass

$\approx (M\chi_{b0} + 3\, M\chi_{b1} + 5\, M\chi_{b2}) / 9$

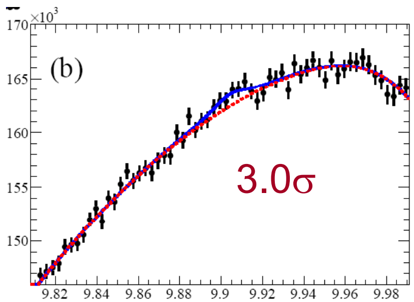
$\Delta M_{CoG} \Rightarrow$ test of hyperfine interaction

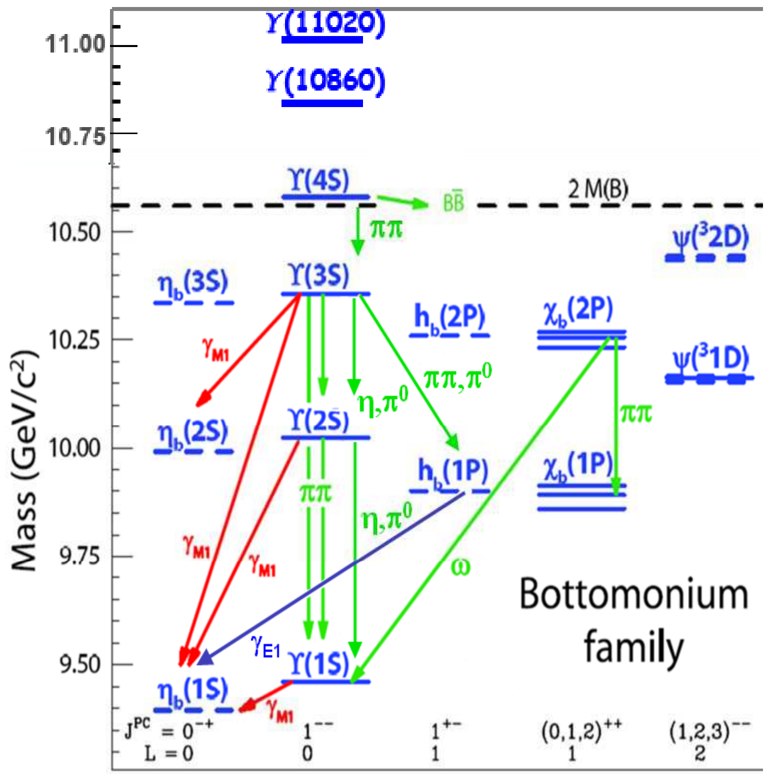
For h_c $\Delta M_{CoG} = -0.12 \pm 0.30$,
expect smaller deviation for $h_b(nP)$.

arXiv:1102.4565

Evidence from BaBar

$\Upsilon(3S) \rightarrow \pi^0 h_b(1P) \rightarrow \pi^0 \gamma\, \eta_b(1S)$





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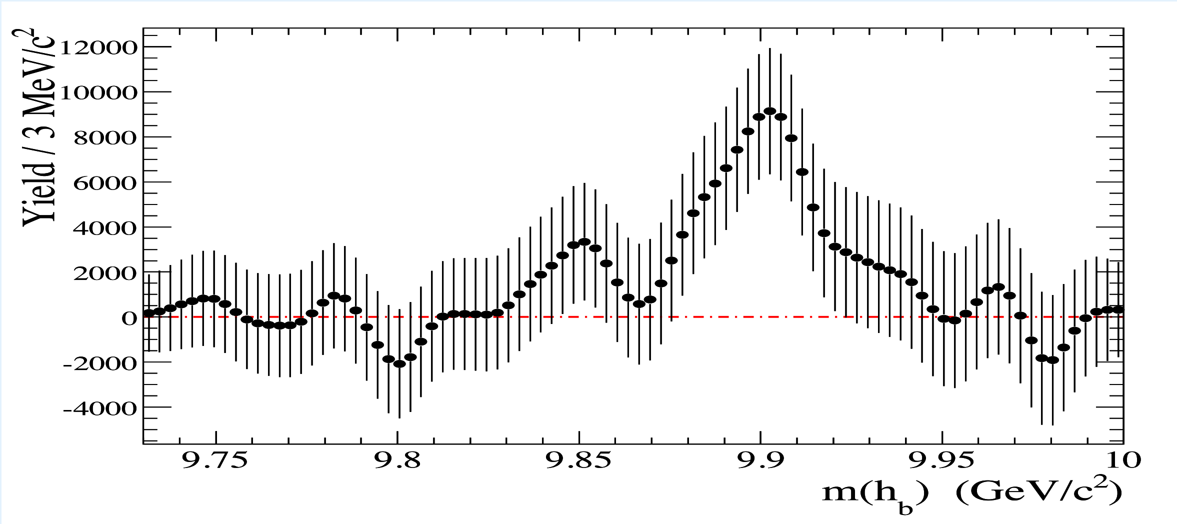
Evidence for $h_b(1P)$ in the decay $\Upsilon(3S) \to \pi^0 h_b(1P)$

Yield as a function of the assumed h_b mass: [BaBar-PUB-10/032]



- Search for $h_b(1P)$ spin-singlet partner of $\chi_b(1P)$:

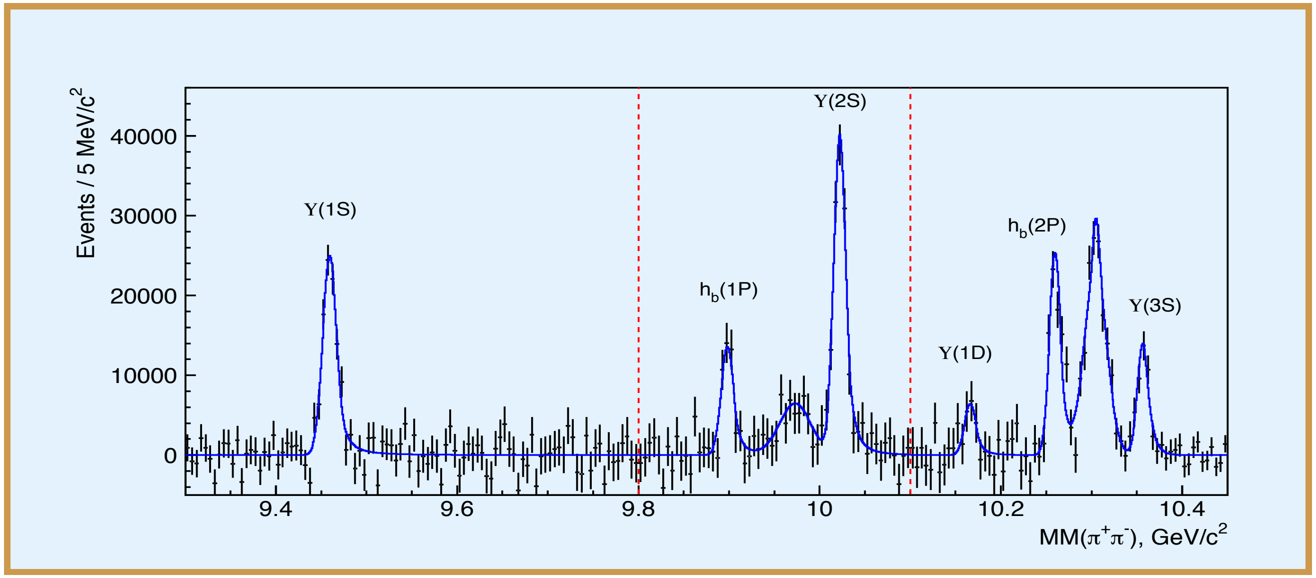
$$e^+e^- \to \Upsilon(3S) \to \pi^0 h_b(1P),\ h_b(1P) \to \gamma\eta_b(1S)$$

$$\mathcal{B}(\Upsilon(3S) \to \pi^0 h_b(1P)) \times \mathcal{B}(h_b(1P) \to \gamma\eta_b) = (3.7 \pm 1.1 \pm 0.7) \times 10^{-4}$$

- Consistent with theoretical estimate: 4×10^{-4} Godfrey (2005)

Observation of $h_b(1P)$ and $h_b(2P)$ bottomonium states

$MM(\pi^+\pi^-)$ spectrum: [Adachi *et al*. (Belle), arxiv:1103.3419]



- Search for $e^+e^- \to h_b(nP)\pi^+\pi^-$ near the $\Upsilon(5S)$:

$$M[h_b(1P)] = (9898.25 \pm 1.06^{+1.03}_{-1.07})\,\mathrm{Me}V; \quad M[h_b(2P)] = (10259.76 \pm 0.64^{+1.43}_{-1.03})\,\mathrm{Me}V$$

$$\frac{\sigma(e^+e^- \to h_b(1P)\pi^+\pi^-)}{\sigma(e^+e^- \to \Upsilon(2S)\pi^+\pi^-)} = 0.407{\pm}0.079^{+0.043}_{-0.070}; \quad \frac{\sigma(e^+e^- \to h_b(2P)\pi^+\pi^-)}{\sigma(e^+e^- \to \Upsilon(2S)\pi^+\pi^-)} = 0.78{\pm}0.09^{+0.22}_{-0.10}$$

- $\sigma(e^+e^- \to \Upsilon(2S)\pi^+\pi^-) = 4.82^{+0.77}_{-0.62}$ pb [Belle, PRD 82, 091106]

- All X-sections are larger by 2 orders of magnitude compared to the QCD multipole estimates!

Calculation of tetraquark masses $M_{[bq][\bar{b}\bar{q}']}$

- Constituent Diquark Hamiltonian Model [N. Drenska, R. Faccini, A.D. Polosa, PLB 669 (2008) 160]
- Spectroscopic estimates presented here are based on [A. A., C. Hambrock,I. Ahmed and J. Aslam, PLB 684, 28 (2010)]
- For similar estimaes, see also [N. Drenska et al., arXiv:1006.2741; D. Ebert et al., Mod. Phys. Lett. A **24**, 567 (2009); Z.G. Wang, Eur. Phys. J. C **67**, 411 (2010)]

Diquarks

- Interpolating diquark operators:

"good": 0^+ $\quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}(\bar{b}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{i_c}^\beta \gamma_5 b^\gamma)$

"bad": 1^+ $\quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}(\bar{b}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{i_c}^\beta \vec{\gamma} b^\gamma)$

α, β, γ: $SU(3)_C$ indices

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$\Rightarrow$ NR limit: States parametrized by Pauli matrices :

$$\text{“good”:} \quad 0^+ \quad \Gamma^0 = \frac{\sigma_2}{\sqrt{2}}$$

$$\text{“bad”:} \quad 1^+ \quad \vec{\Gamma} = \frac{\sigma_2 \vec{\sigma}}{\sqrt{2}}$$

Tetraquark states

Characterized by the diquark and antidiquark spins s_Q and $s_{\bar{Q}}$and the tetraquark total angular momentum J

$$\left|Y_{[bq]}\right\rangle = \left|s_Q, s_{\bar{Q}};\; J\right\rangle$$

$\Rightarrow$ Tetraquark matrix representation:

$$
\begin{aligned}
\left|0_Q, 0_{\bar{Q}};\; 0_J\right\rangle &= \Gamma^0 \otimes \Gamma^0, \\
\left|1_Q, 1_{\bar{Q}};\; 0_J\right\rangle &= \frac{1}{\sqrt{3}}\Gamma^i \otimes \Gamma_i, \\
\left|0_Q, 1_{\bar{Q}};\; 1_J\right\rangle &= \Gamma^0 \otimes \Gamma^i, \\
\left|1_Q, 0_{\bar{Q}};\; 1_J\right\rangle &= \Gamma^i \otimes \Gamma^0, \\
\left|1_Q, 1_{\bar{Q}};\; 1_J\right\rangle &= \frac{1}{\sqrt{2}}\varepsilon^{ijk}\Gamma_j \otimes \Gamma_k
\end{aligned}
$$

Hamiltonian

- States need to diagonalize Hamiltonian:

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL}$$

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constituent mass

with

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with qq **spin coupling** $q\bar{q}$ **spin coupling**

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{bq})_{\bar{3}}[(\mathbf{S}_b \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{b}} \cdot \mathbf{S}_{\bar{q}})]$$

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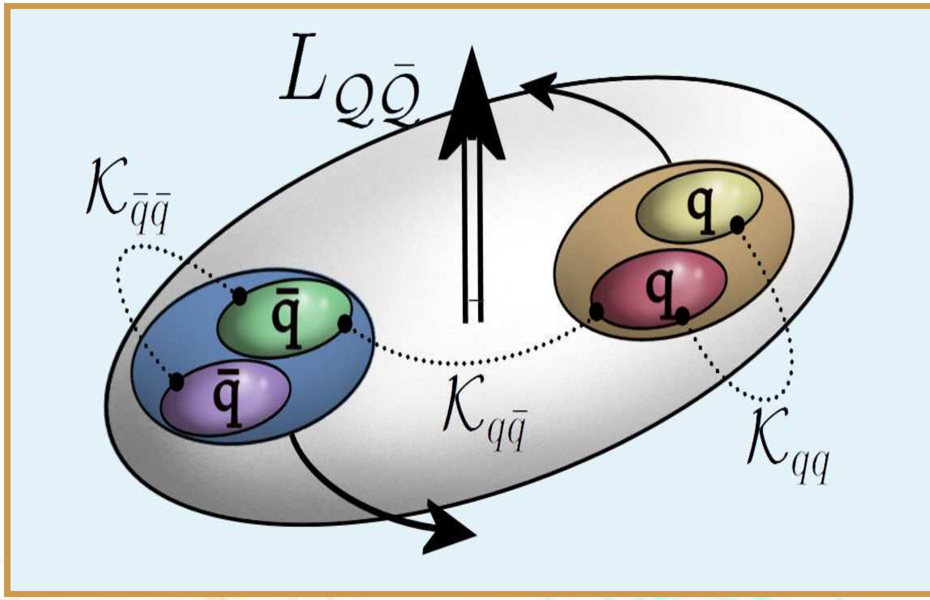

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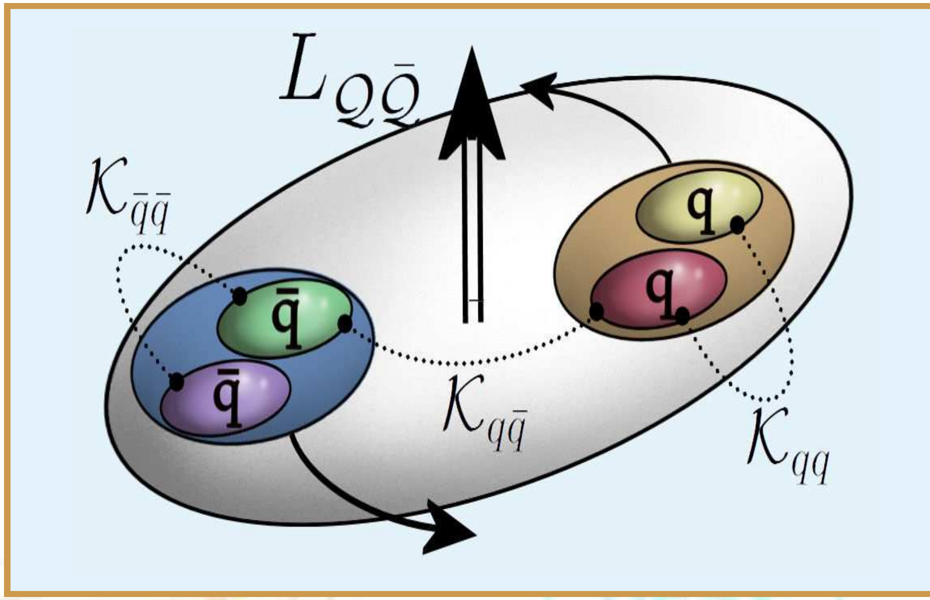

Hamiltonian

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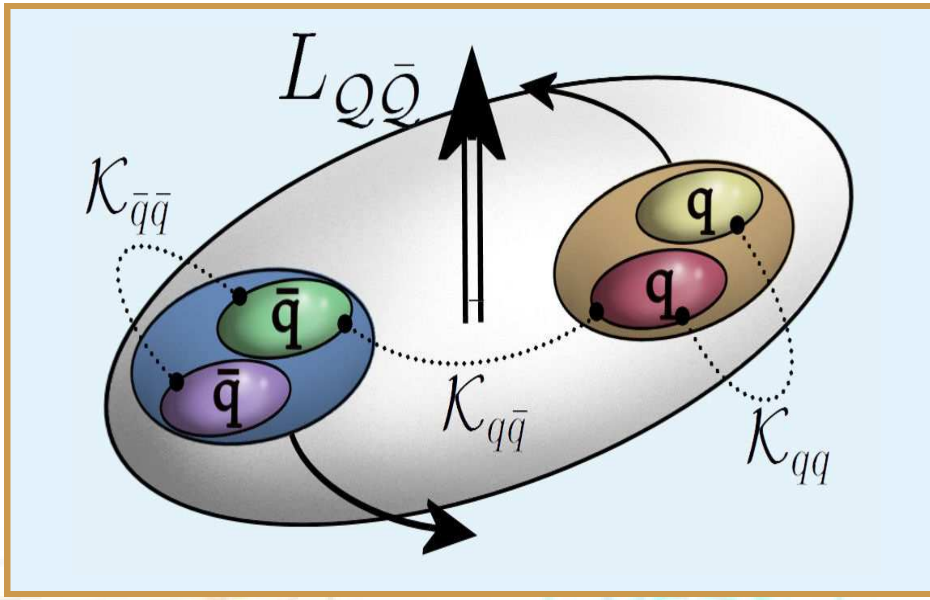
$$H_{LL} = B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1)}{2}$$



Example 1^{++} state

- $[\bar{b}\bar{q}][bq]$ state:

$$|1^{++}\rangle = \frac{1}{\sqrt{2}}\left(|0_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\ 1_J\rangle + |1_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\ 1_J\rangle\right)$$

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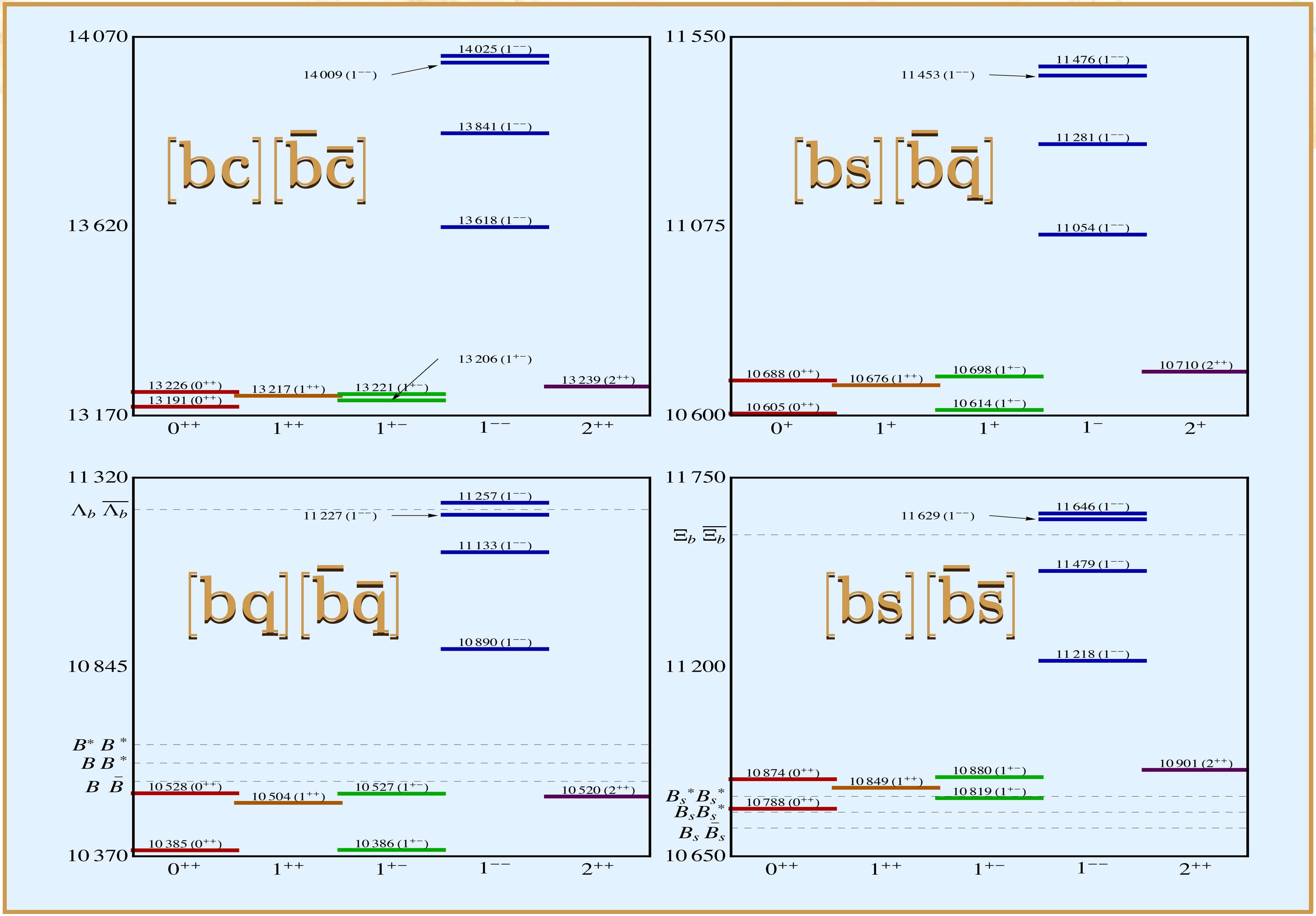
- Example parameters from known hadron spectrum:

$m_{[bq]}$	$(\mathcal{K}_{bq})_{\bar{3}}$	$\mathcal{K}_{b\bar{q}}$	$\mathcal{K}_{q\bar{q}}$	$\mathcal{K}_{b\bar{b}}$	$M(1^{++})$
5250 MeV	6 MeV	6 MeV	80 MeV	9 MeV	10533 MeV



[A. A., C. Hambrock, I. Ahmed and M. Aslam, Phys. Lett. B **684**, 28 (2010)]

Rich spectroscopy of $b\bar{b}$ tetraquarks at the B/Super-B Factories

- One expects 40 tetraquark states of the type $[bq][\bar{b}\bar{q}]$ $(q = u, d, c, s)$, with well-defined $J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 1^{--}, 2^{++}$ and 10 states of the type $[bs][\bar{b}\bar{d}]$ with $J^P = 0^+, 1^+, 1^-, 2^+$ in the mass range $10.3 - 14.1$ **GeV**!

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- Of these 16 have $J^{PC} = 1^{--}$, having masses from 10.890 **GeV**, called $Y_b(10890)$, to about 14.1 **GeV**, $Y_{[bc][\bar{b}\bar{c}]}(14030)$, which can be directly produced in e^+e^- annihilation at the B/Super-B factories

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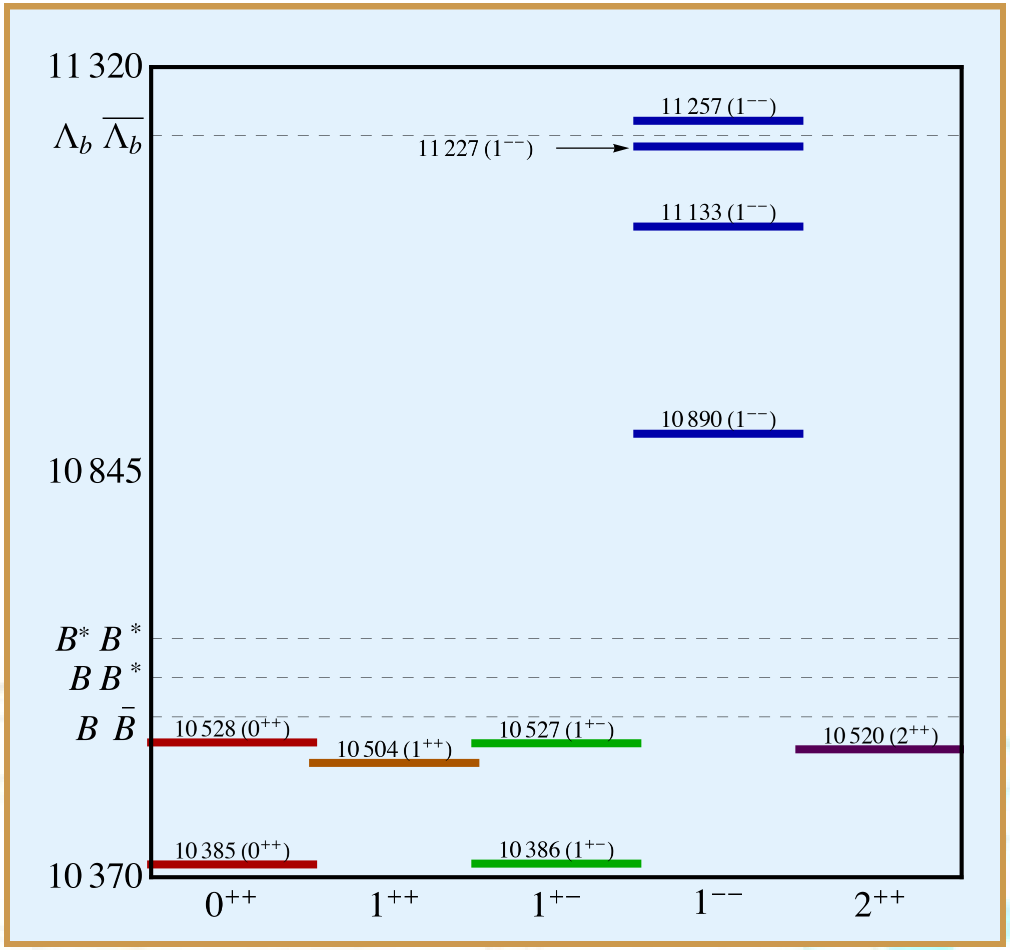
But. . .

Mass spectrum of the $[bq][\bar{b}\bar{q}]$ tetraquarks

Heavy-light $[bq][\bar{b}\bar{q}]$ $(q = u, d)$ tetraquarks



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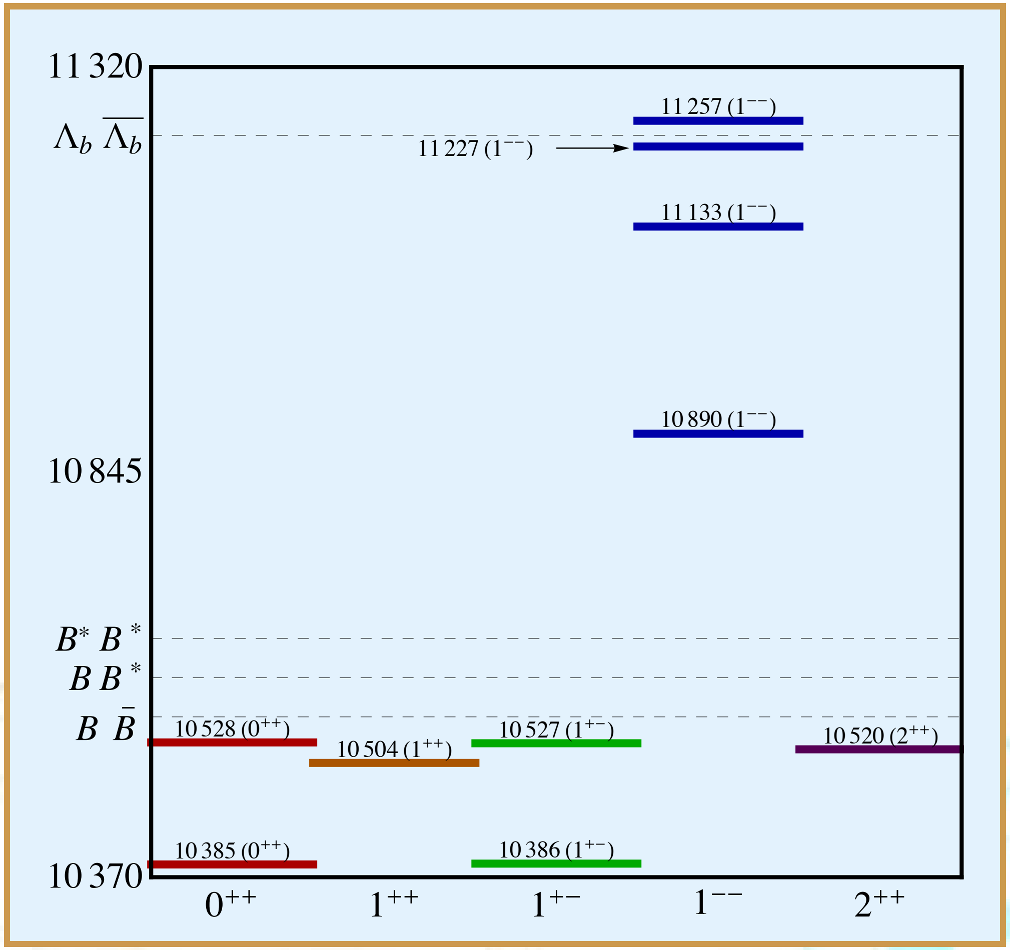

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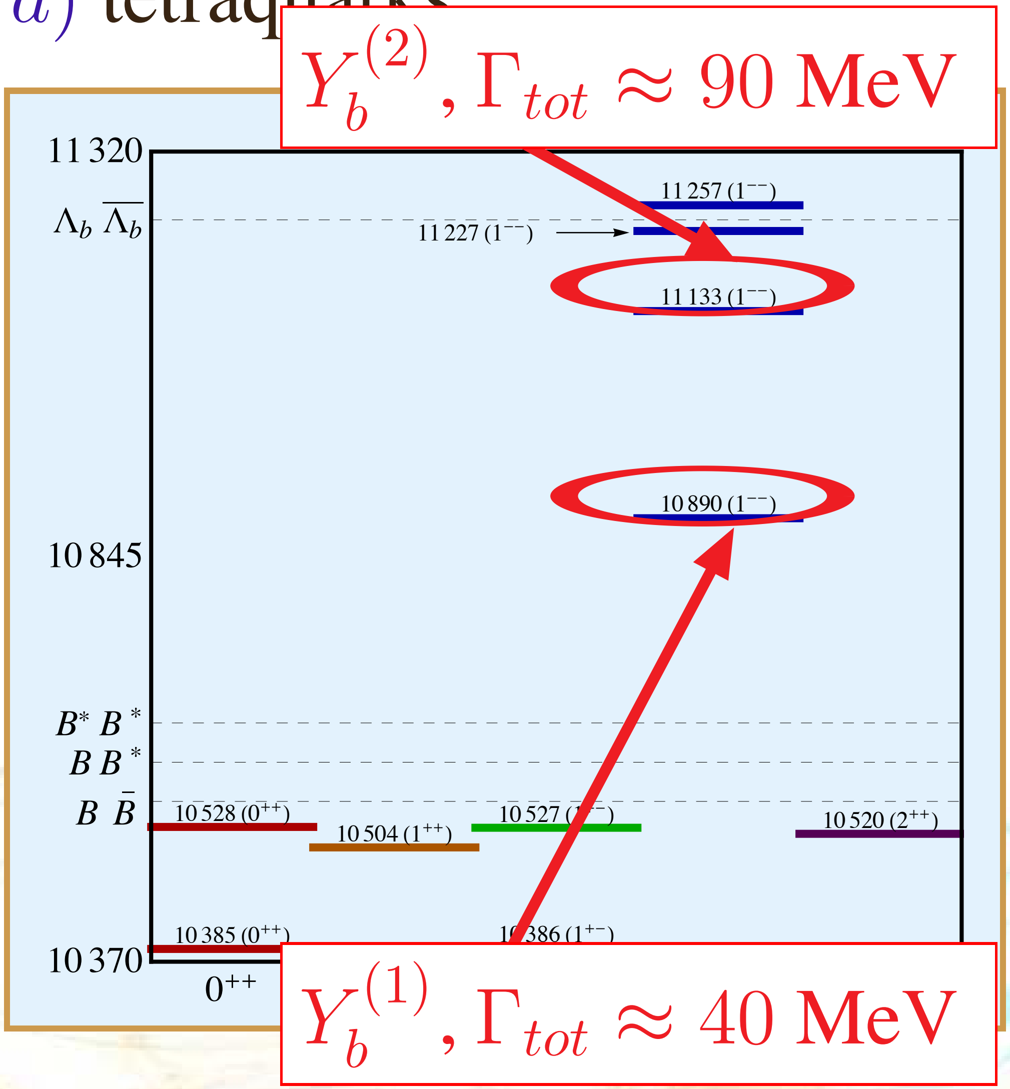

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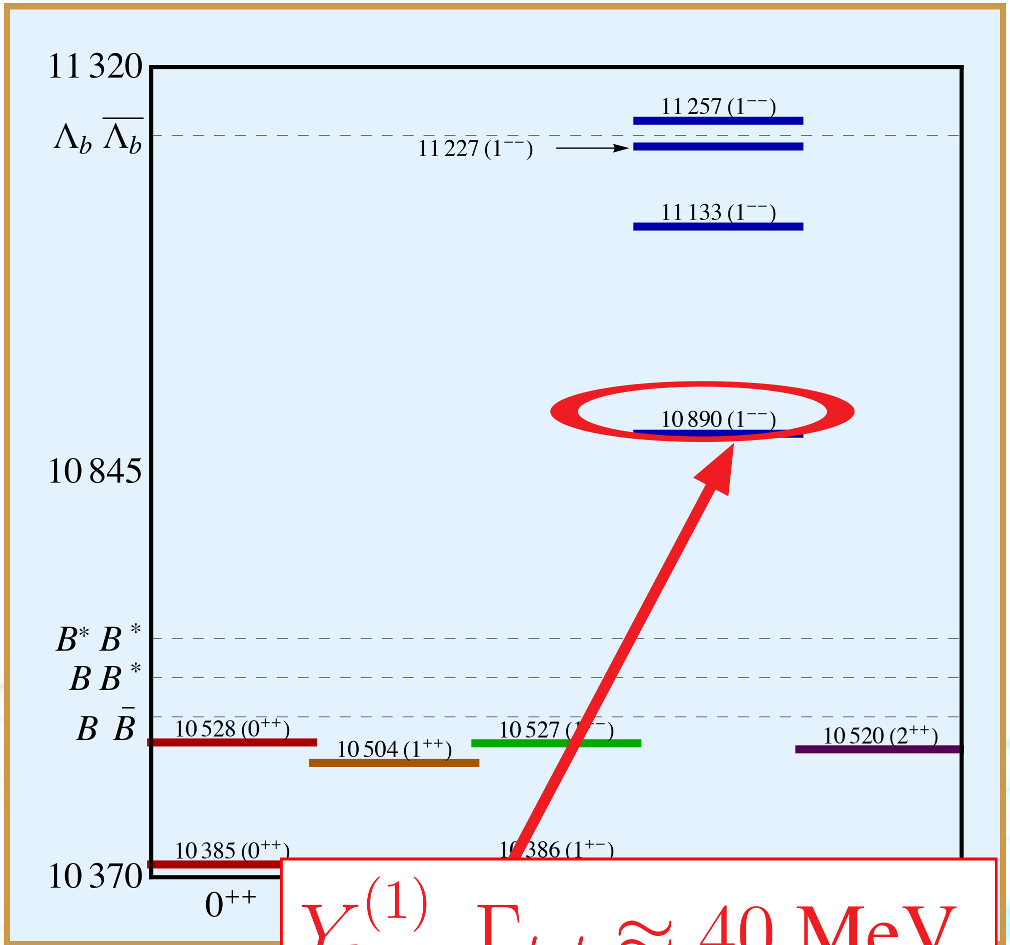
- $J^{PC} = 1^{--}$
  $\Rightarrow$ can be produced in e^+e^- annihilation
- Two of them in the range of BaBar and Belle E_{CM}
- $Y_b^{(1)}$ only composed of “good” diquarks

  $\Rightarrow$ This is Belle’s $Y_b(10890)$!



$Y_b^{(1)}$, $\Gamma_{tot} \approx 40$ MeV

Isospin breaking

- $Y_b^{(1)}$ mass eigenstates:

$$\begin{aligned} Y_{[b,l]} &= \cos\theta\, Y_{[bu]} + \sin\theta\, Y_{[bd]} \\ Y_{[b,h]} &= -\sin\theta\, Y_{[bu]} + \cos\theta\, Y_{[bd]} \end{aligned}$$

- Isospin mass breaking:

$$M(Y_{[b,h]}) - M(Y_{[b,l]}) = (7 \pm 3)\ \cos(2\theta)\ \text{MeV}$$

- Effective diquark charge:

$$\begin{aligned} Q_{Y_{[b,l]}} &= \frac{1}{3}\cos\theta - \frac{2}{3}\sin\theta \\ Q_{Y_{[b,h]}} &= -\frac{1}{3}\sin\theta - \frac{2}{3}\cos\theta \end{aligned}$$

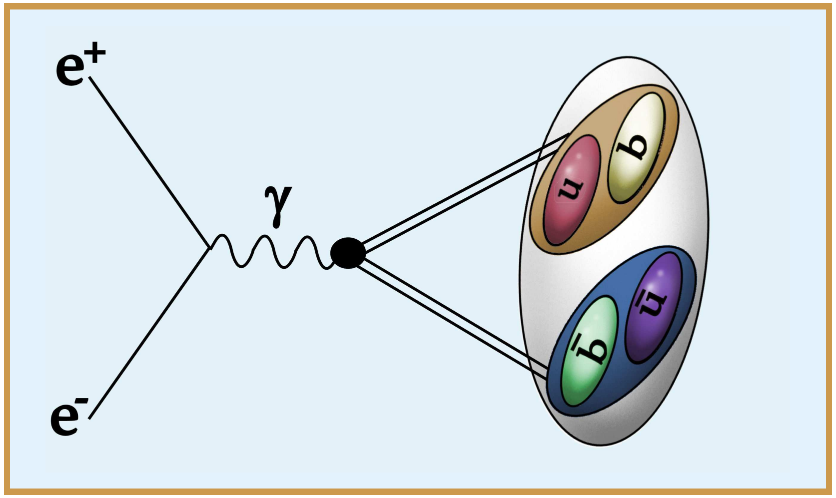
- $\theta = -45^\circ \quad \Rightarrow$ Isospin eigenstates

Y_b production

- Van Royen-Weisskopf formula $\Rightarrow \Gamma(1^{--} \rightarrow e^+e^-)$

Assumption: Point-like diquarks

[A. A., C. Hambrock and S. Mishima, PRL 106:092002 (2011)]



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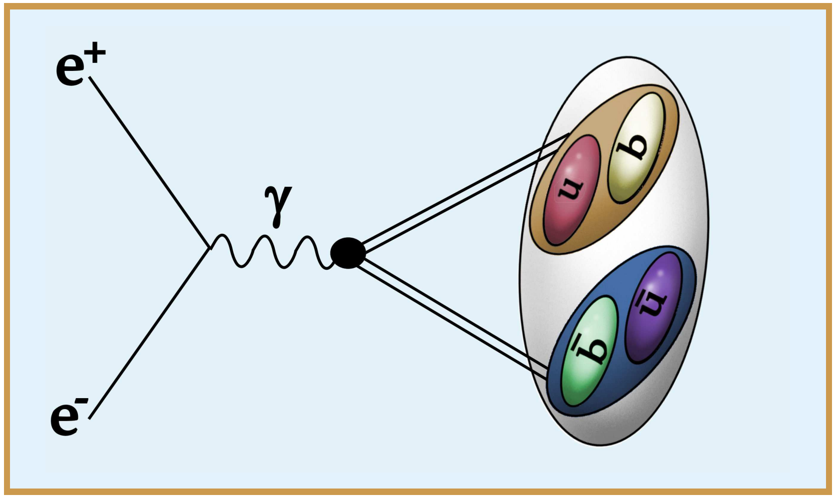

$$\Gamma_{ee}(Y_{[b,l/h]}) = \frac{24\alpha^2 Q^2_{[b,l/h]}}{M^4_{Y_{[b,l/h]}}} \kappa^2 \left| R^{(1)}_{11}(0) \right|^2$$

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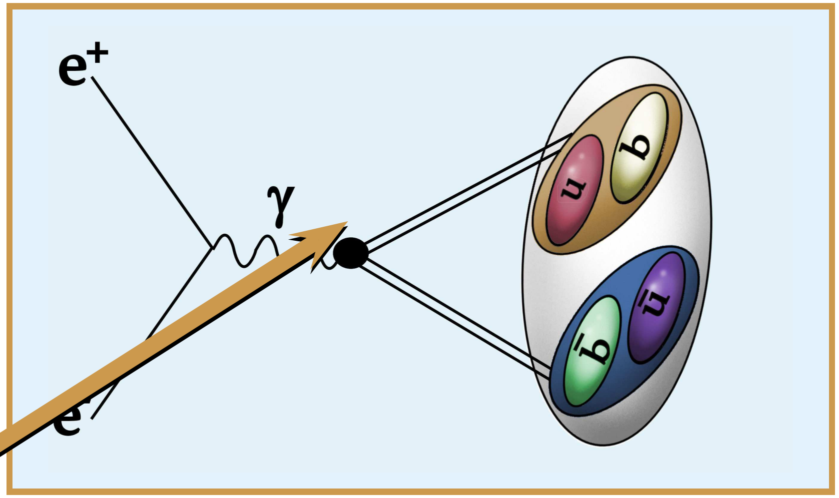

diquark charge

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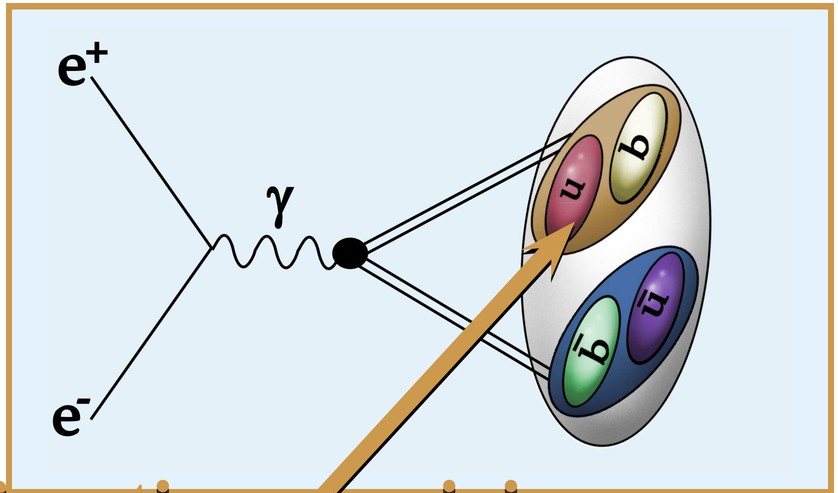

radial tetraquark wave function at origin

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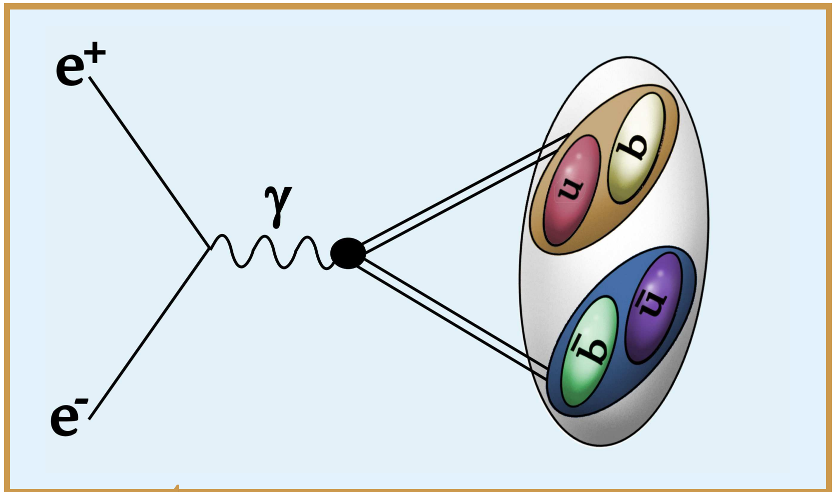

[A. A., C. Hambrock and S. Mishima, PRL 106:092002 (2011)]

hadronic size parameter

$$\Gamma_{ee}(Y_{[b,l/h]}) = \frac{24\alpha^2 Q^2_{[b,l/h]}}{M^4_{Y_{[b,l/h]}}} \kappa^2 \left| R^{(1)}_{11}(0) \right|^2$$

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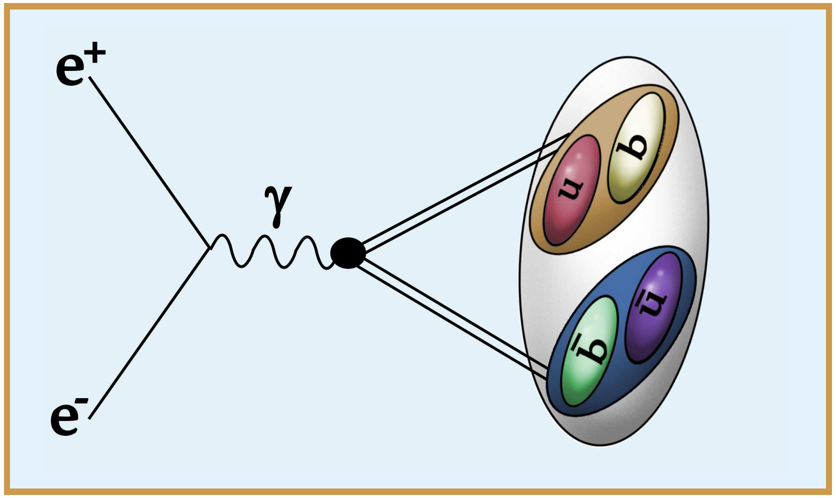

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$\Rightarrow$ Suppressed $\mathcal{O}(10)$ vs bottomonia

$\Rightarrow$ Production ratio: $\Gamma_{Y_{[b,l]}} / \Gamma_{Y_{[b,h]}} = \left(\frac{1-2\tan\theta}{2+\tan\theta}\right)^2$

Dominant Y_b decays

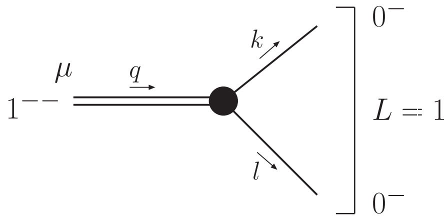
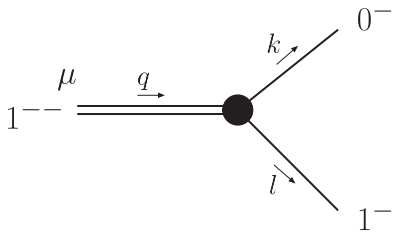
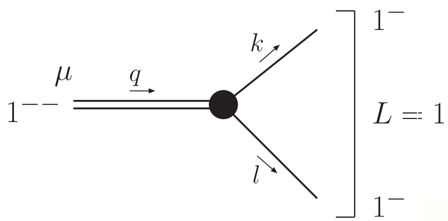
channel

$B\bar{B}$

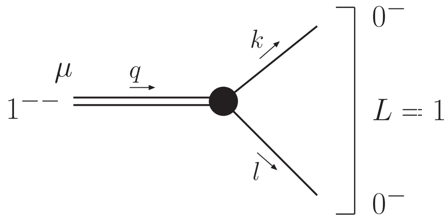
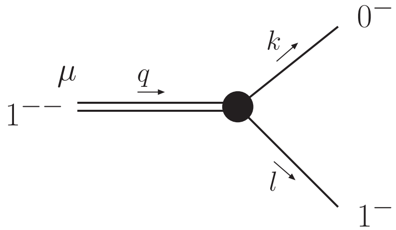
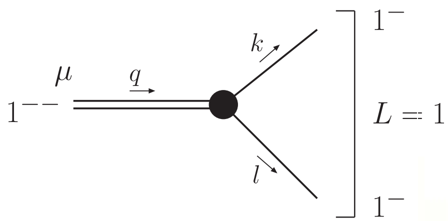
$B\bar{B}^*$

$B^*\bar{B}^*$

Dominant Y_b decays

channel	diagram
$B\bar{B}$	
$B\bar{B}^*$	
$B^*\bar{B}^*$	

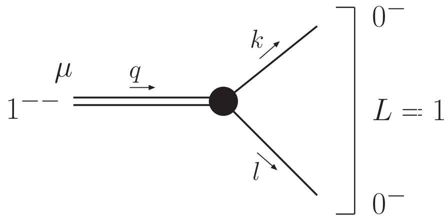
Dominant Y_b decays

channel	diagram	vertex
$B\bar{B}$		$\hat{=}\, F(k^\mu - l^\mu)$
$B\bar{B}^*$		$\hat{=}\, \frac{F}{M}\epsilon^{\mu\nu\rho\sigma}k_\rho l_\sigma$
$B^*\bar{B}^*$		$\hat{=}\, F(g^{\mu\rho}(q+l)^\nu - g^{\mu\nu}(k+q)^\rho + g^{\rho\nu}(q+k)^\mu)$

Dominant Y_b decays

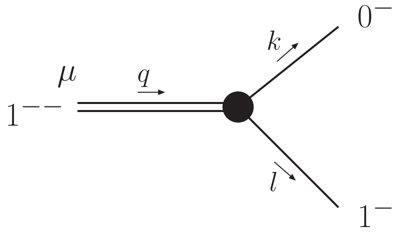
channel | diagram | vertex | width

$B\bar{B}$



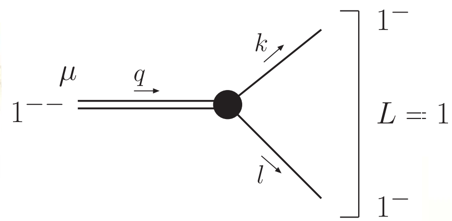
$$\widehat{=} F(k^\mu - l^\mu) \quad \Rightarrow \Gamma = \frac{F^2|\vec{k}|^3}{2M^2\pi}$$

$B\bar{B}^*$



$$\widehat{=} \frac{F}{M}\epsilon^{\mu\nu\rho\sigma}k_\rho l_\sigma \quad \Rightarrow \Gamma = \frac{F^2|\vec{k}|^3}{4M^2\pi}$$

$B^*\bar{B}^*$

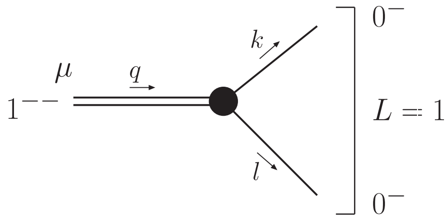


$$\widehat{=} \begin{array}{l} F(g^{\mu\rho}(q+l)^\nu \\ \quad -g^{\mu\nu}(k+q)^\rho \\ \quad +g^{\rho\nu}(q+k)^\mu) \end{array} \quad \Rightarrow \Gamma = \frac{F^2|\vec{k}|^3(48|\vec{k}|^4 - 104M^2|\vec{k}|^2 + 27M^4)}{2\pi(M^3 - 4|\vec{k}|^2 M)^2}$$

Dominant Y_b decays

channel — diagram — vertex — width

$B\bar{B}$



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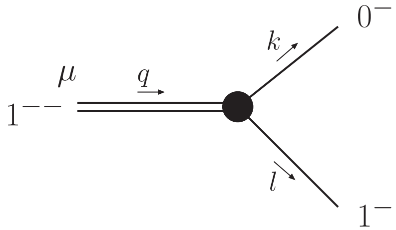
$B\bar{B}^*$



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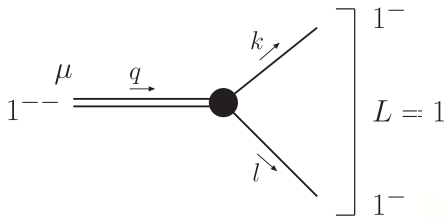

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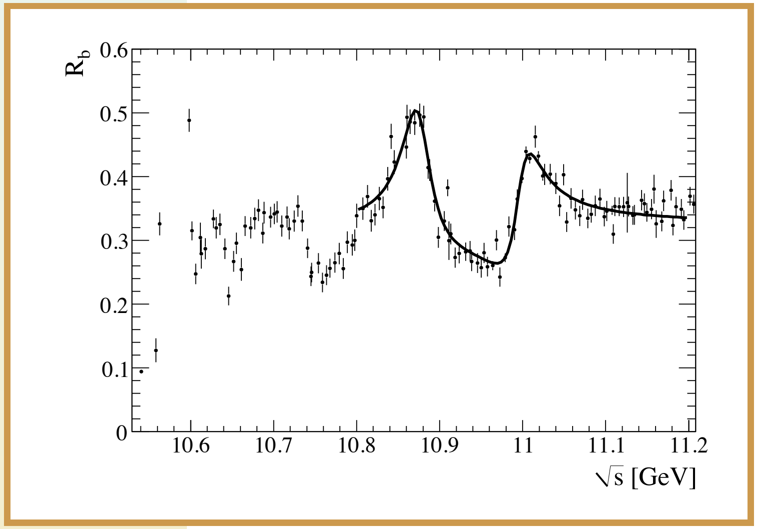
- Couplings estimated from $\Upsilon(5S)$ decays

$$\Rightarrow \Gamma_{tot}(Y_b^{(1)}) \approx \kappa^2 40 \text{ MeV}, \quad \Gamma_{tot}(Y_b^{(2)}) \approx \kappa^2 90 \text{ MeV}$$

Fit to BaBar data

[A. A., C. Hambrock,I. Ahmed and J. Aslam, PLB 684, 28 (2010)]

BaBar fit



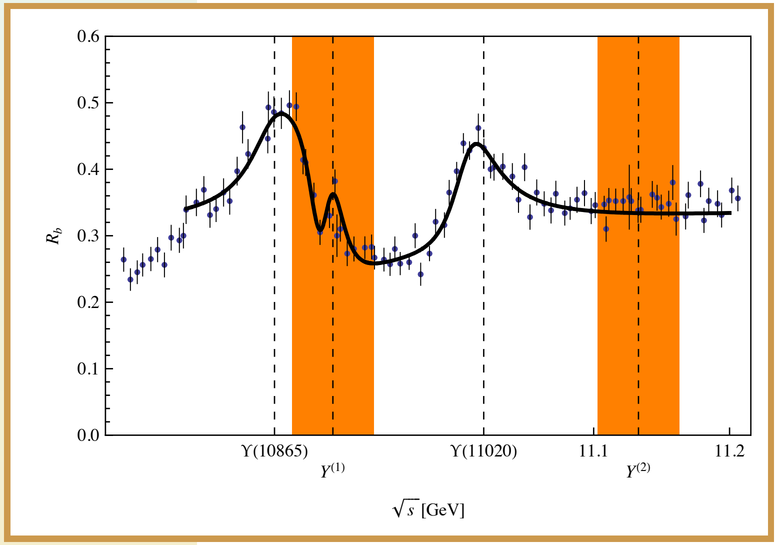
$\chi^2/\text{d.o.f.} \approx 2$

[Phys. Rev. Lett. **102**, 012001 (2009)]

Model function :

$$\sigma(e^+e^- \to b\bar{b}) = |A_{nr}|^2 + \Big| A_r + A_{10860} e^{i\phi_{10860}} BW(M_{10860}, \Gamma_{10860}) + A_{11020} e^{i\phi_{11020}} BW(M_{11020}, \Gamma_{11020}) \Big|^2$$

BaBar fit



$\chi^2/\text{d.o.f.} = 88/67$

Model function modified :

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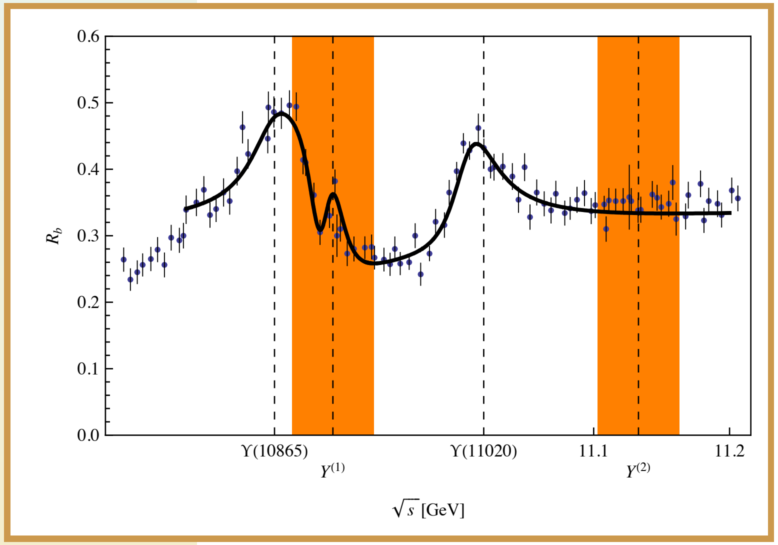

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	M [MeV]	Γ [MeV]	φ [rad.]
$\Upsilon(5S)$	10864 ± 5	46 ± 8	1.3 ± 0.3
$\Upsilon(6S)$	11007 ± 0.3	40 ± 2	0.88 ± 0.06
$Y_{[b,l]}$	$10900 - \Delta M/2 \pm 2$	28 ± 2	1.3 ± 0.2
$Y_{[b,h]}$	$10900 + \Delta M/2 \pm 2$	28 ± 2	1.9 ± 0.2

$$\Delta M = 5.6 \pm 2.8 \text{ MeV}$$
$$\Gamma_{ee}(Y_{[b,l]}) = 45 \pm 15 \text{ eV}$$
$$\Gamma_{ee}(Y_{[b,h]}) = 40 \pm 15 \text{ eV}$$

- Structure seen in inclusive BaBar data

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- But not conclusive:
  - Suppression $\mathcal{O}(10)$ vs $\Upsilon(5S)$
  - Mass splitting $\approx$ BaBar binning
  - Overlapping resonances

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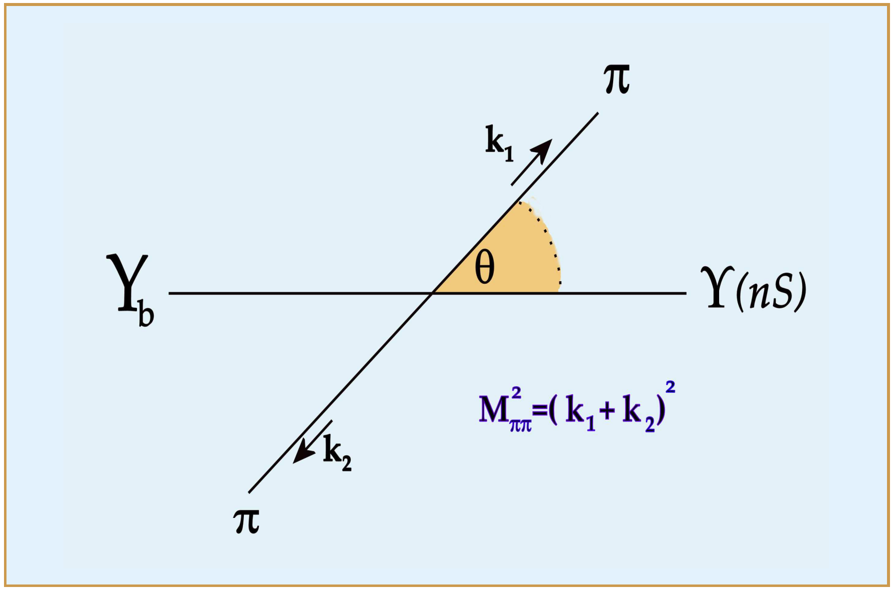
Exclusive data more promising

Exclusive Belle data

- Observed anomaly: Explanation

Enigmatic Belle data



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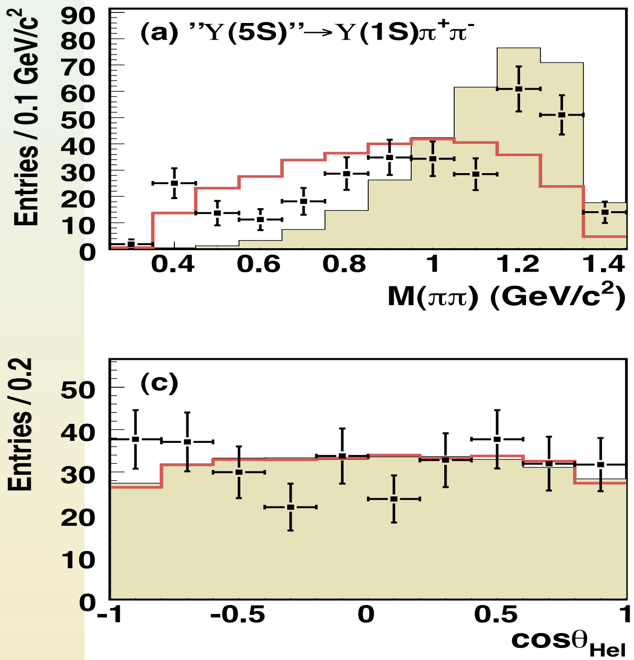

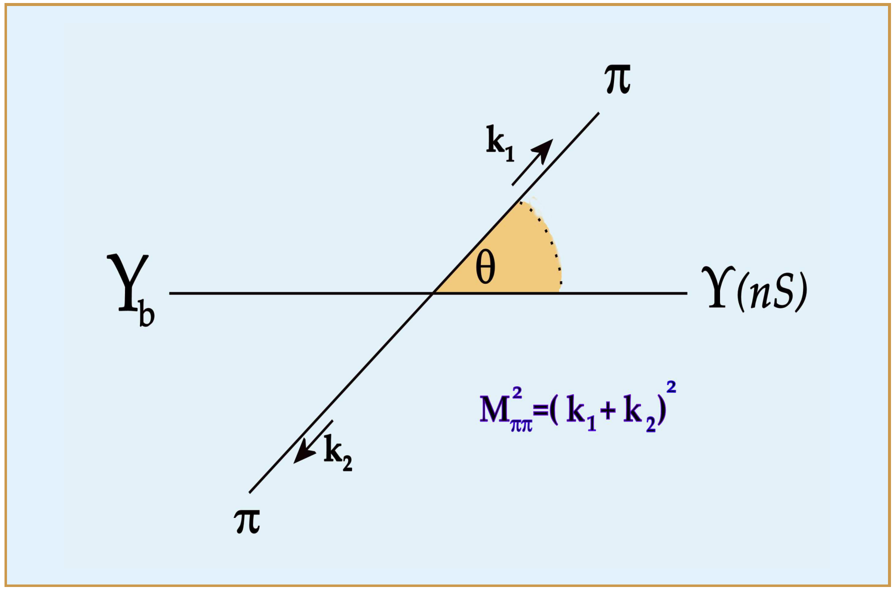

[K. F. Chen *et al*. [Belle Collaboration], Phys. Rev. Lett. **100**, 112001 (2008)]

$$\Gamma(\Upsilon(2S) \to \Upsilon(1S)\pi\pi) \approx 0.0060 \text{ MeV}$$

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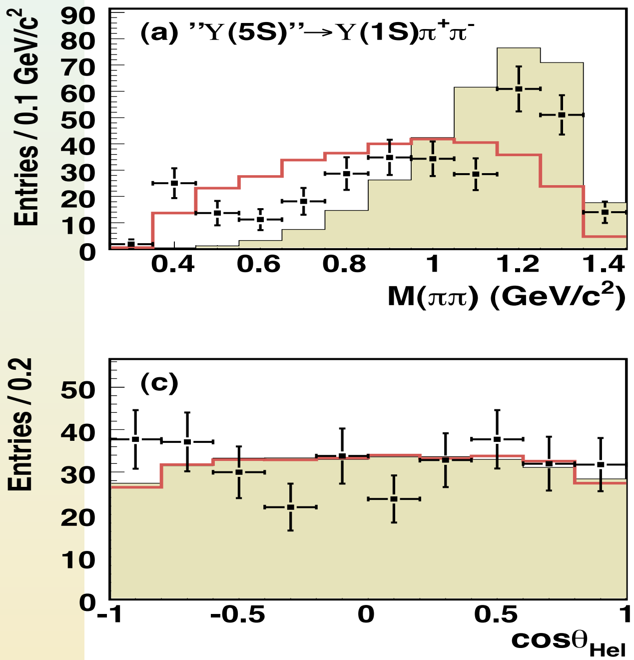

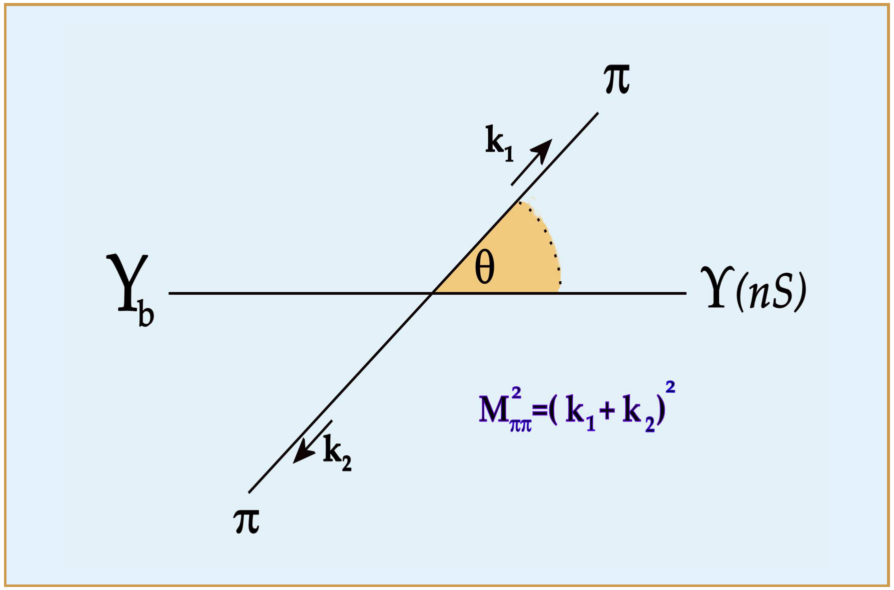

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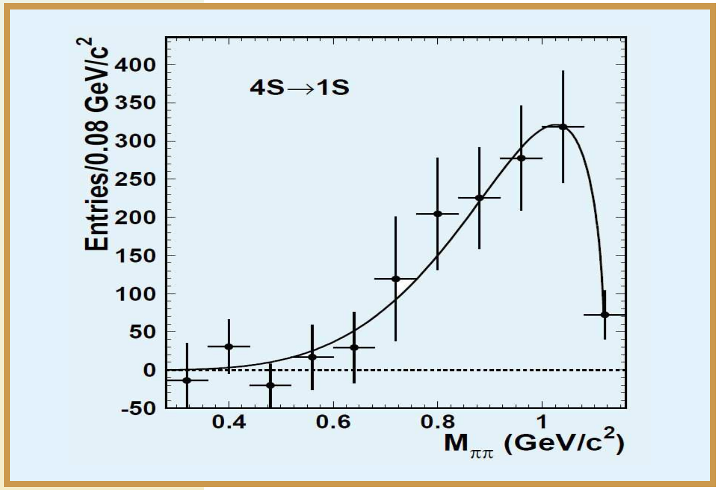
Differs by two orders of magnitude!!

Bottomonia decays

- Typical $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$ decays:



[A. Sokolov *et al*. (2009)]

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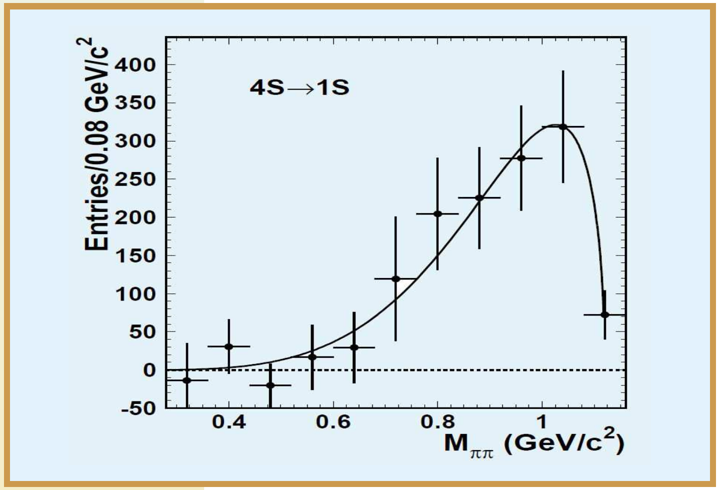

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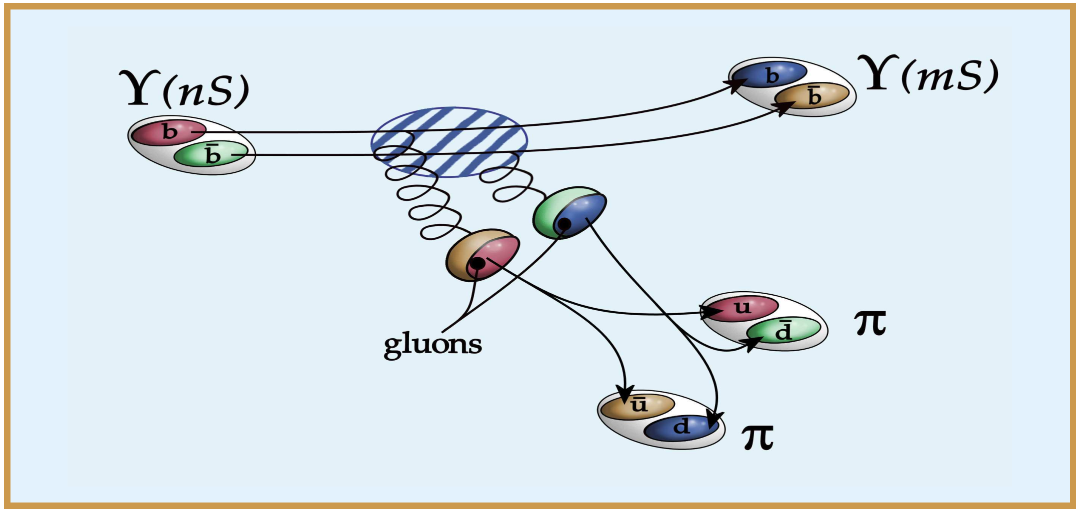

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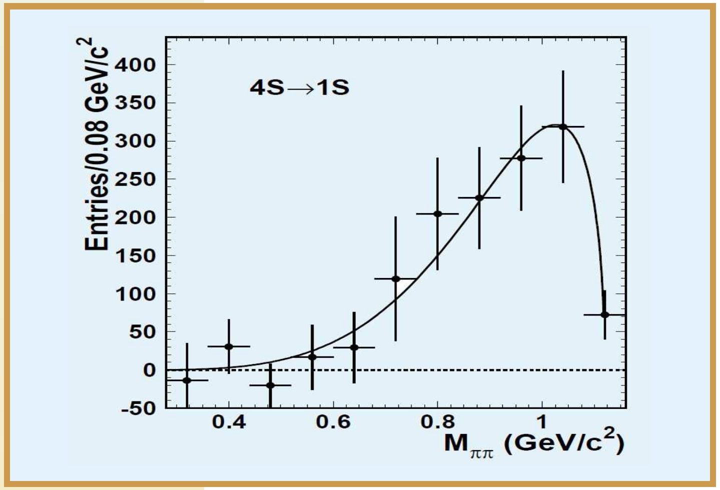

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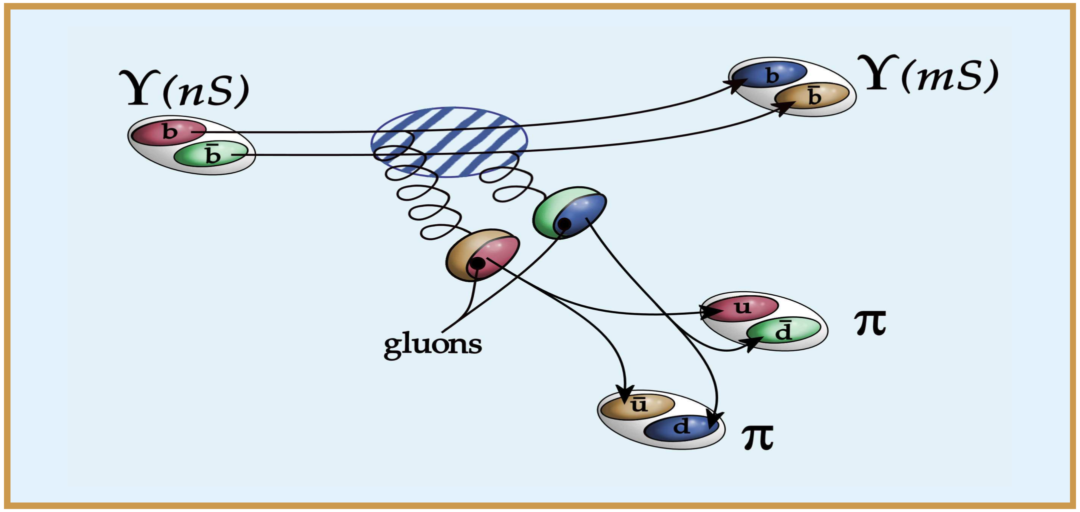

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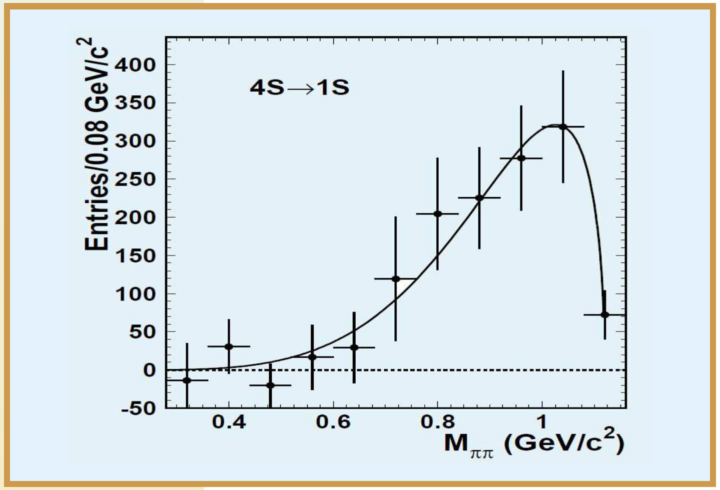

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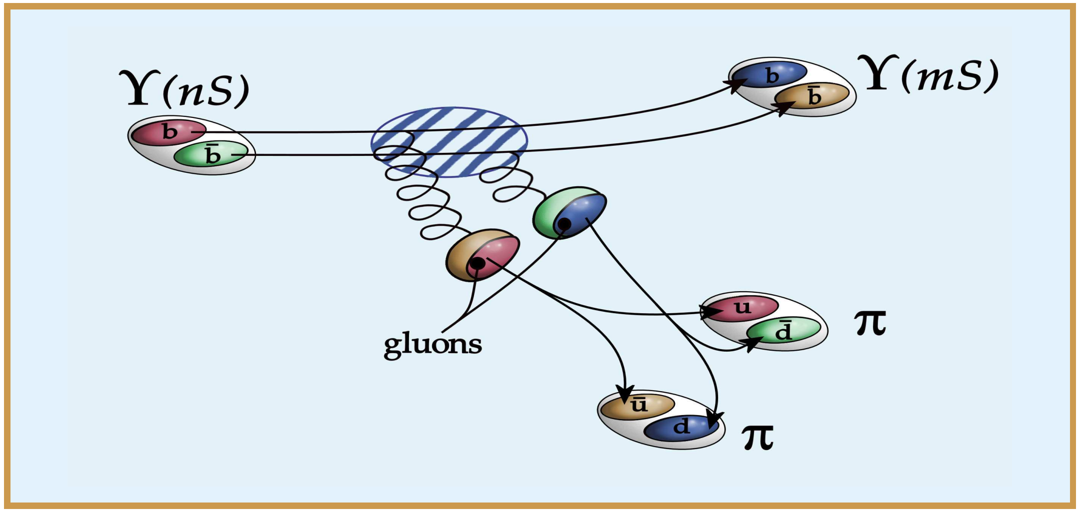


- Zweig forbidden process $\Longrightarrow$ Small cross sections
- Up to now good description for bottomonia
- Fails for $\Upsilon(5S)$
  $\Rightarrow$ Observed state might be 1^{--} tetraquark

Tetraquark Explanation of the Belle anomaly

- Dynamical model to calculate
  $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)PP')$

$$PP' = \pi^+\pi^-, K^+K^-, \eta\pi^0$$

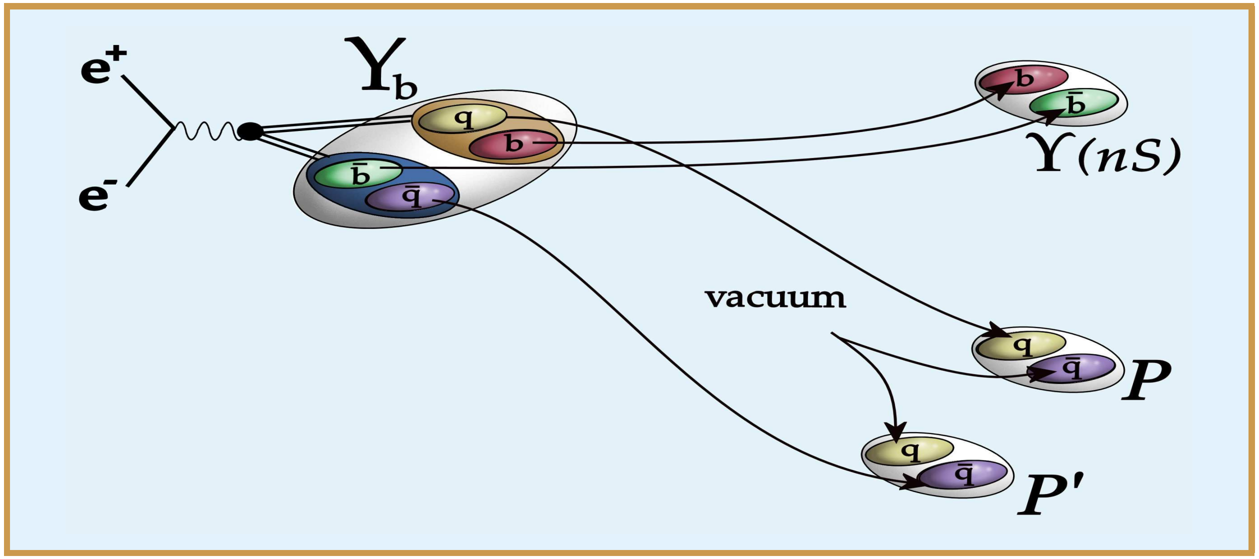
- Fit to the $\Upsilon(1S)\pi^+\pi^-$ Belle spectra

$\Rightarrow$ Testable predictions for $\Upsilon(1S)(K^+K^-, \eta\pi^0)$

[A. A., C. Hambrock and J. Aslam, PRL 104:162001 (2010)]
[A. A., C. Hambrock and S. Mishima, PRL 106:092002 (2011)]

Continuum contribution

- Zweig allowed tetraquark continuum[Brown et al. (1975)] :



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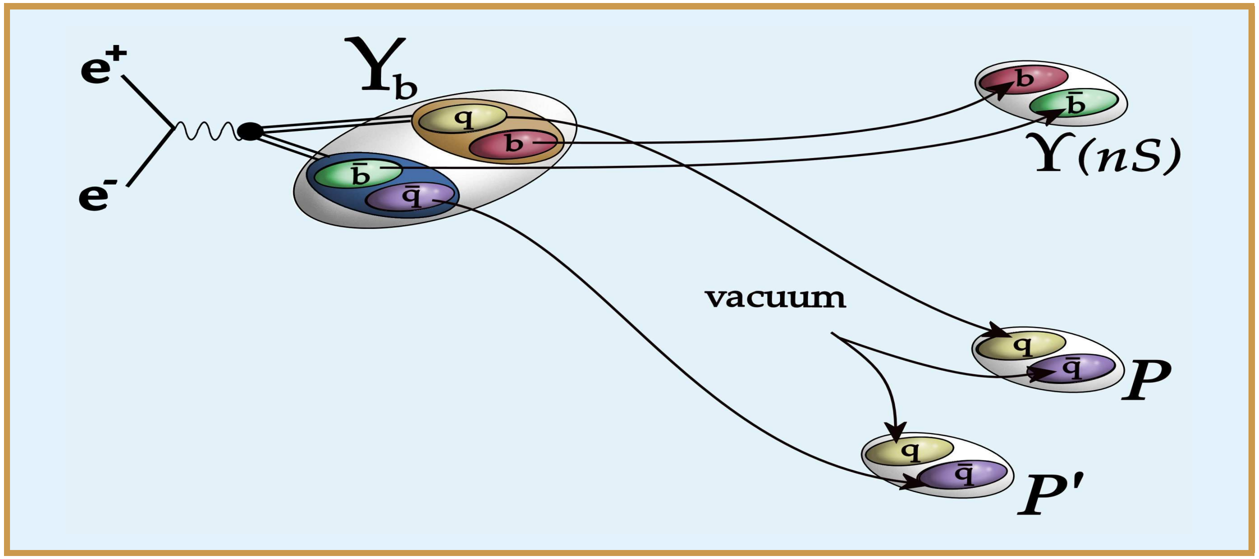
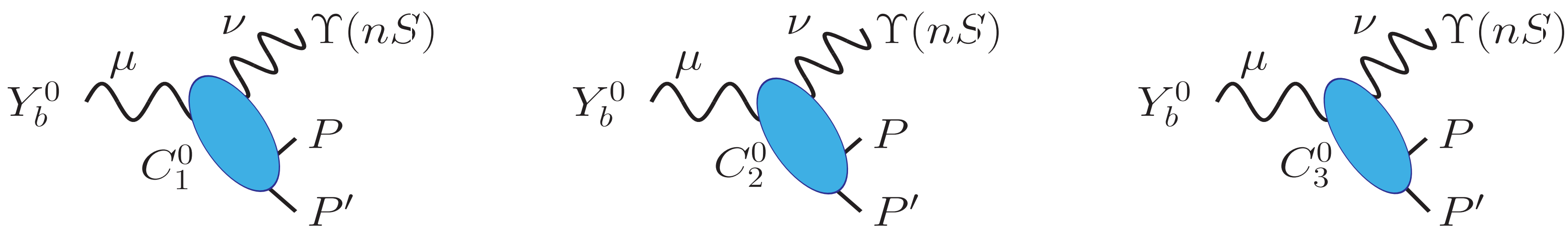


3 diff. terms



$\propto$ $g_{\mu\nu}$ $\qquad$ $\propto$ $g_{\mu\nu}(\cos^2\theta - \frac{1}{3})$ $\qquad$ $\propto$ $g_{\mu\nu}(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu})$

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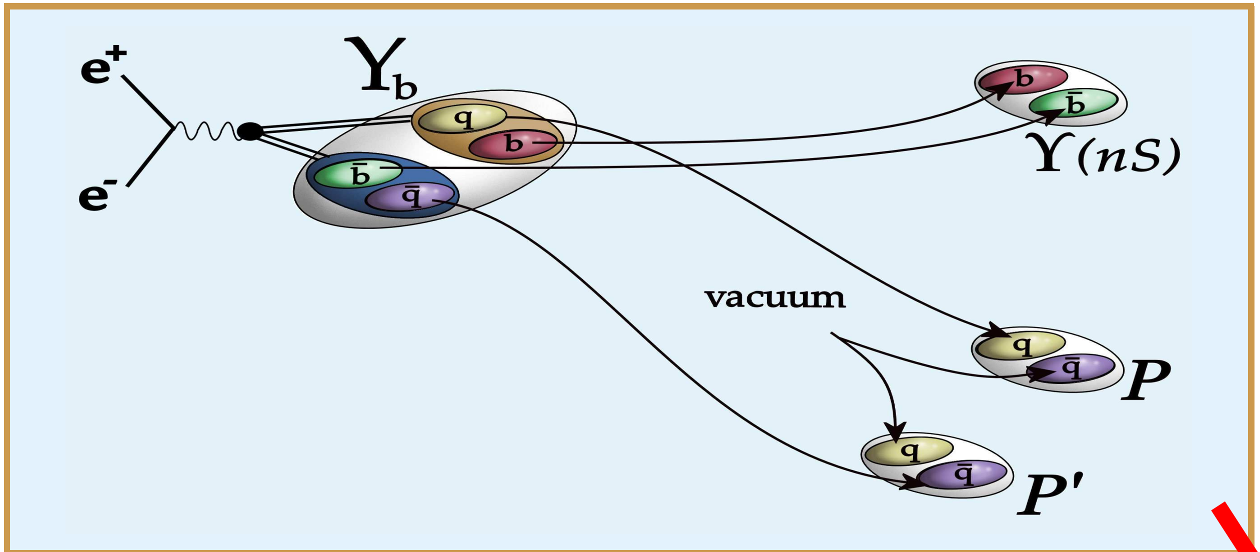
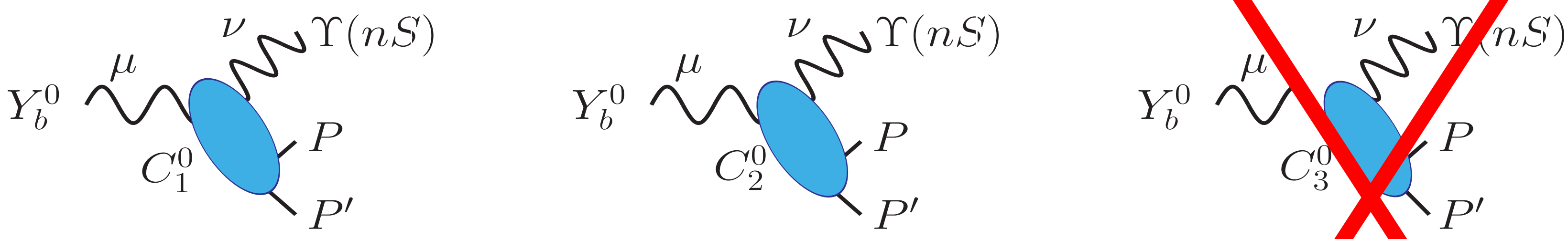


HQ spin interaction $\propto \frac{1}{m_b}$

3 diff. terms



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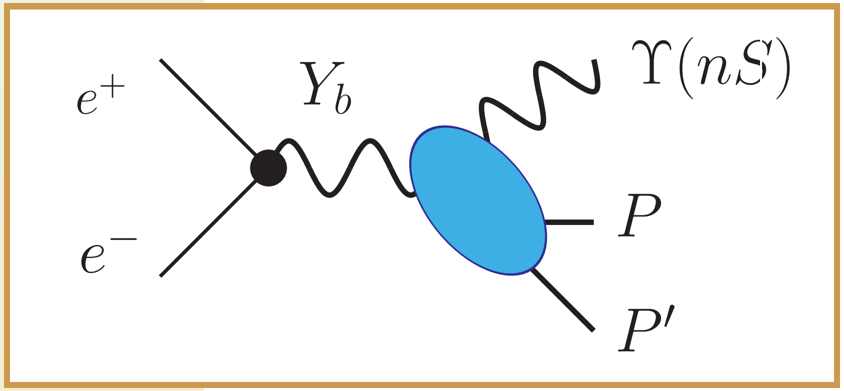
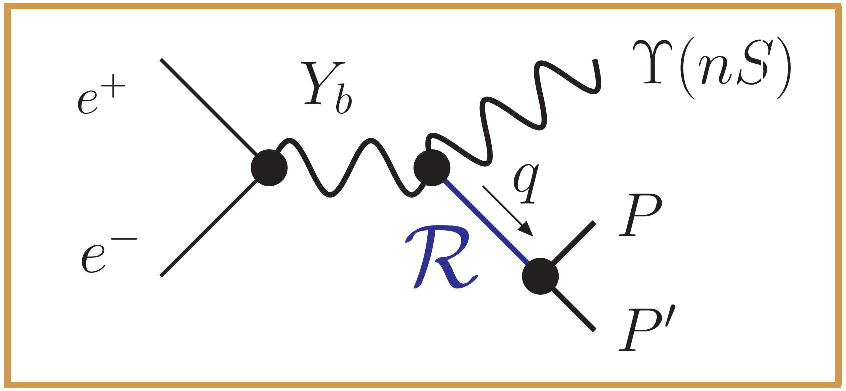
And the resonant contributions

Continuum | Resonance



+



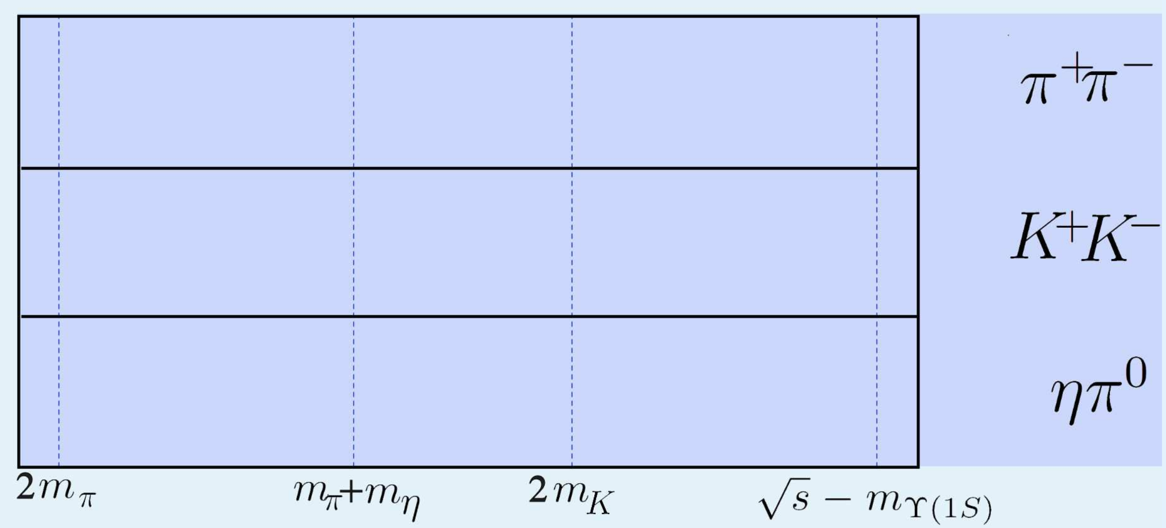
- Breit-Wigner shape for resonance:

$$\frac{1}{(q^2 - M^2) + iM\Gamma}$$

$q^2 \equiv M^2_{PP'} \Rightarrow$ Resonances show in $M_{PP'}$ spectrum

Resonances in $M_{PP'}$ spectrum

- Resonance $\mathcal{R}$ contributions for each channel:



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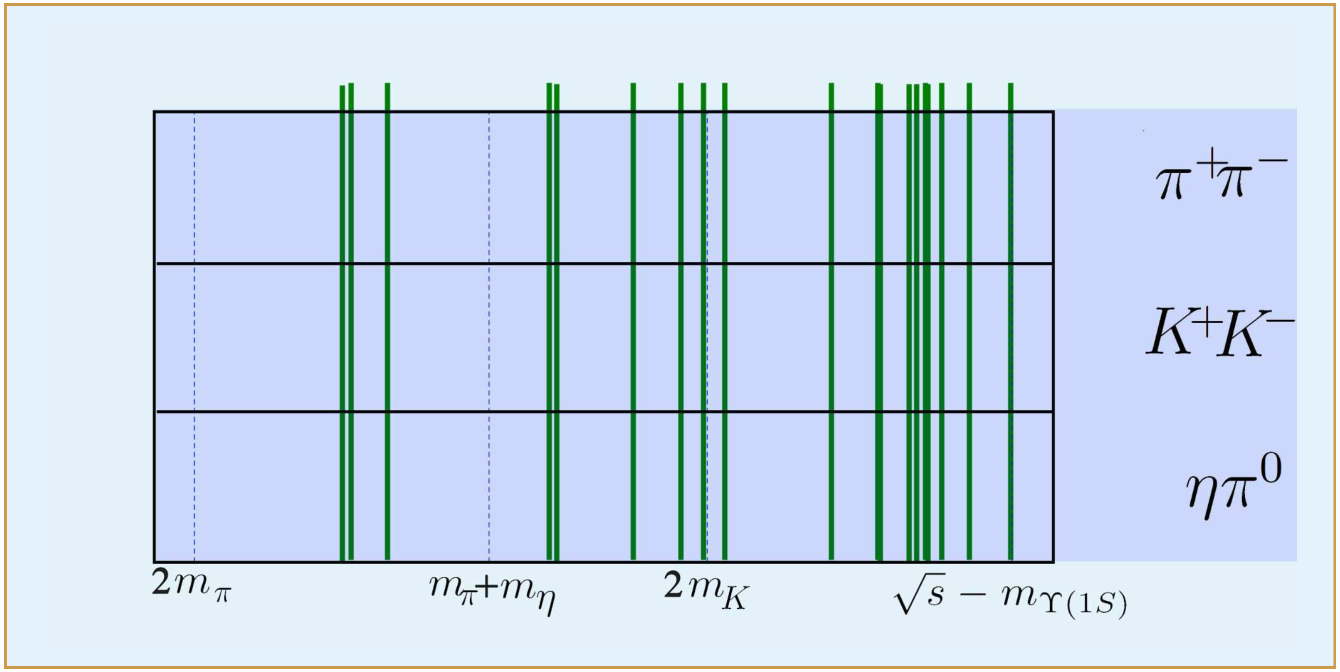

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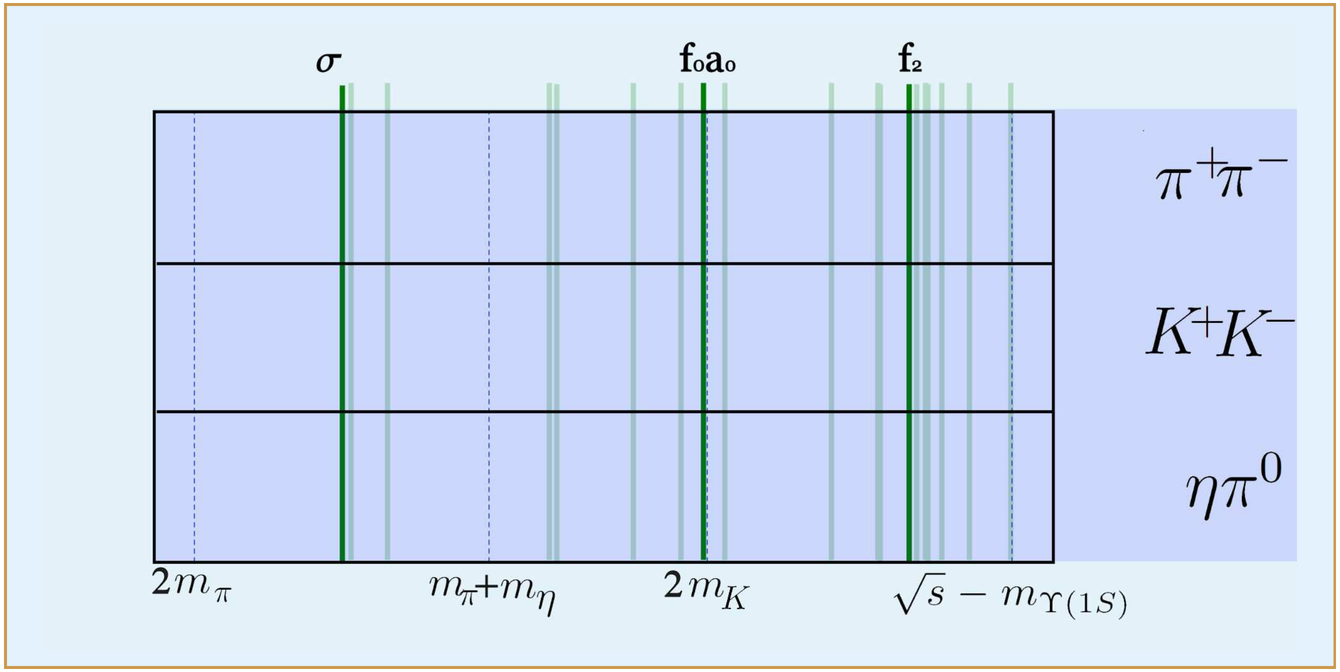

  - Only 0^{++} and 2^{++} allowed

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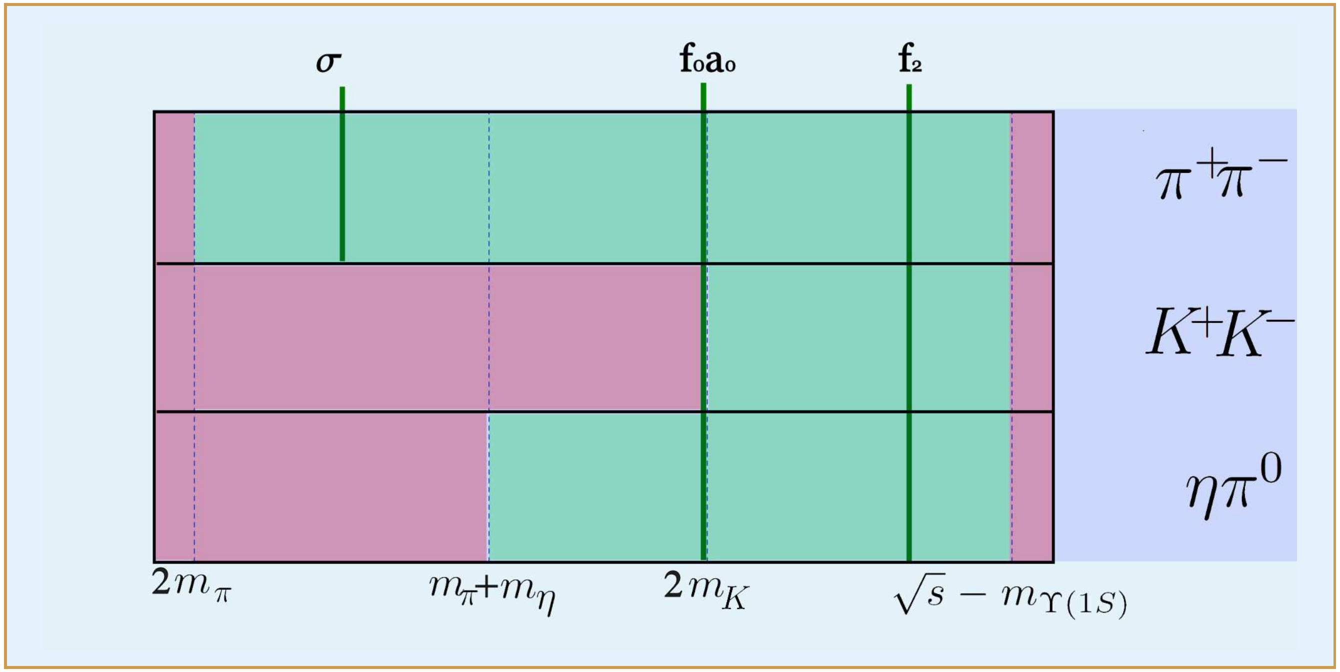

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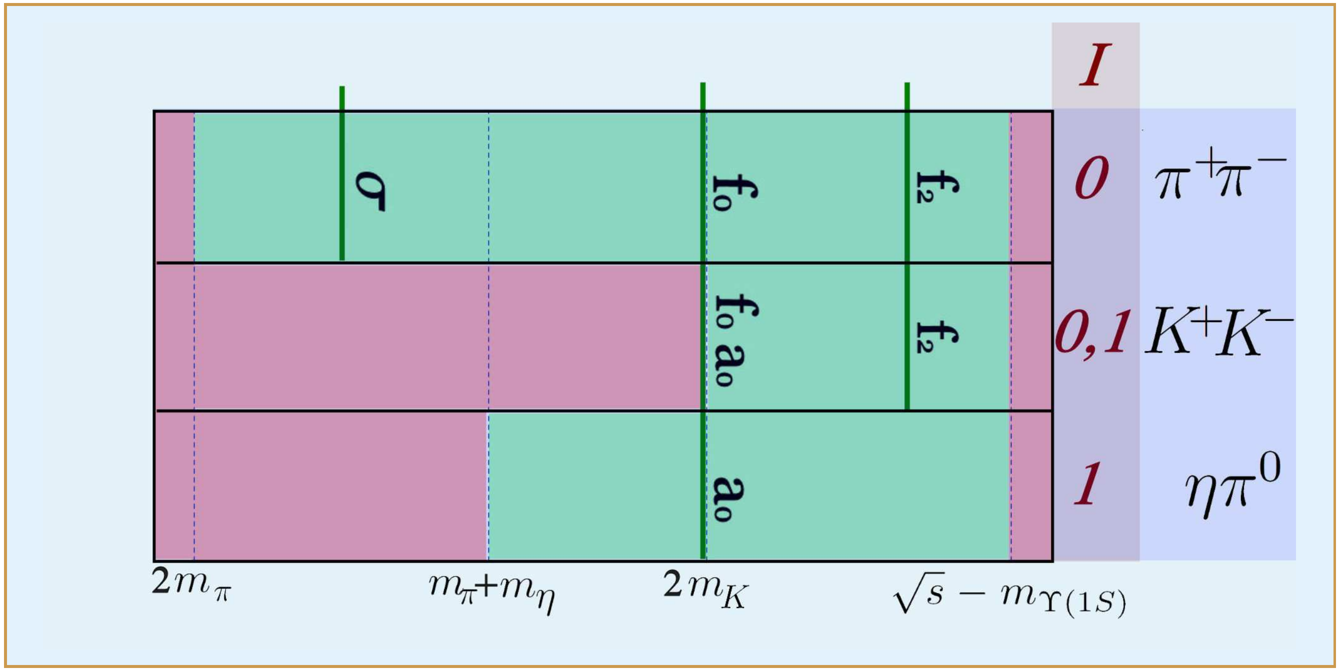

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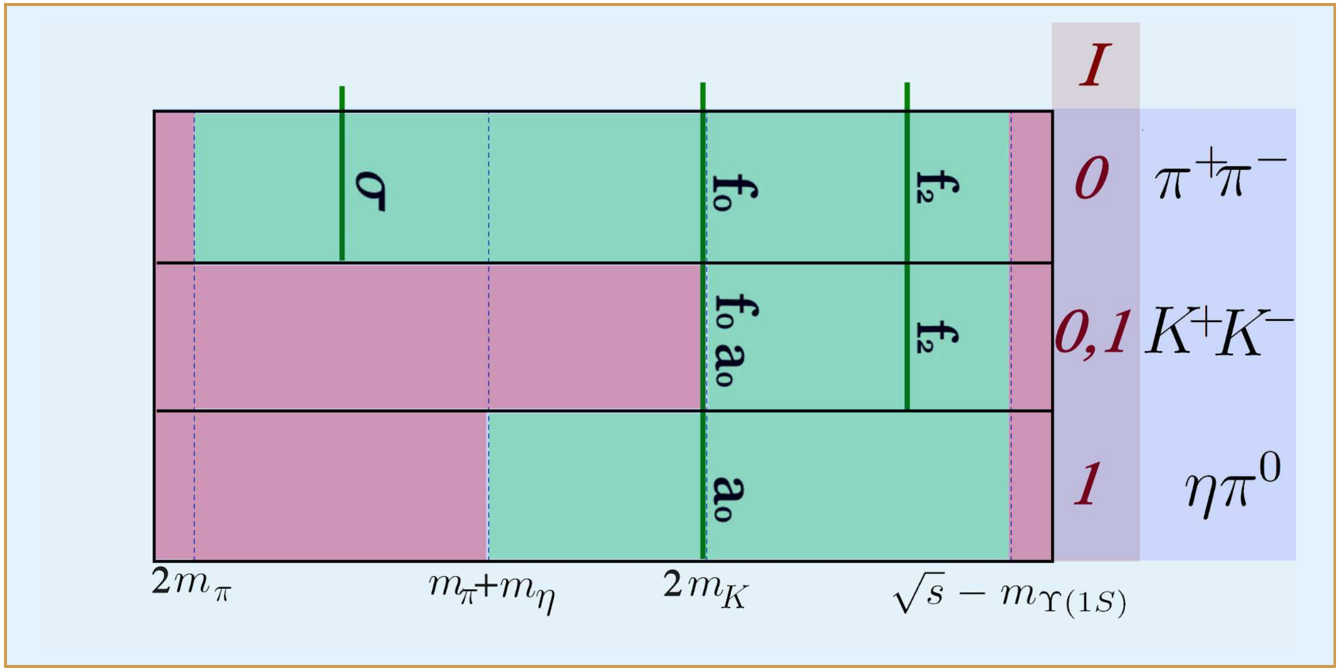
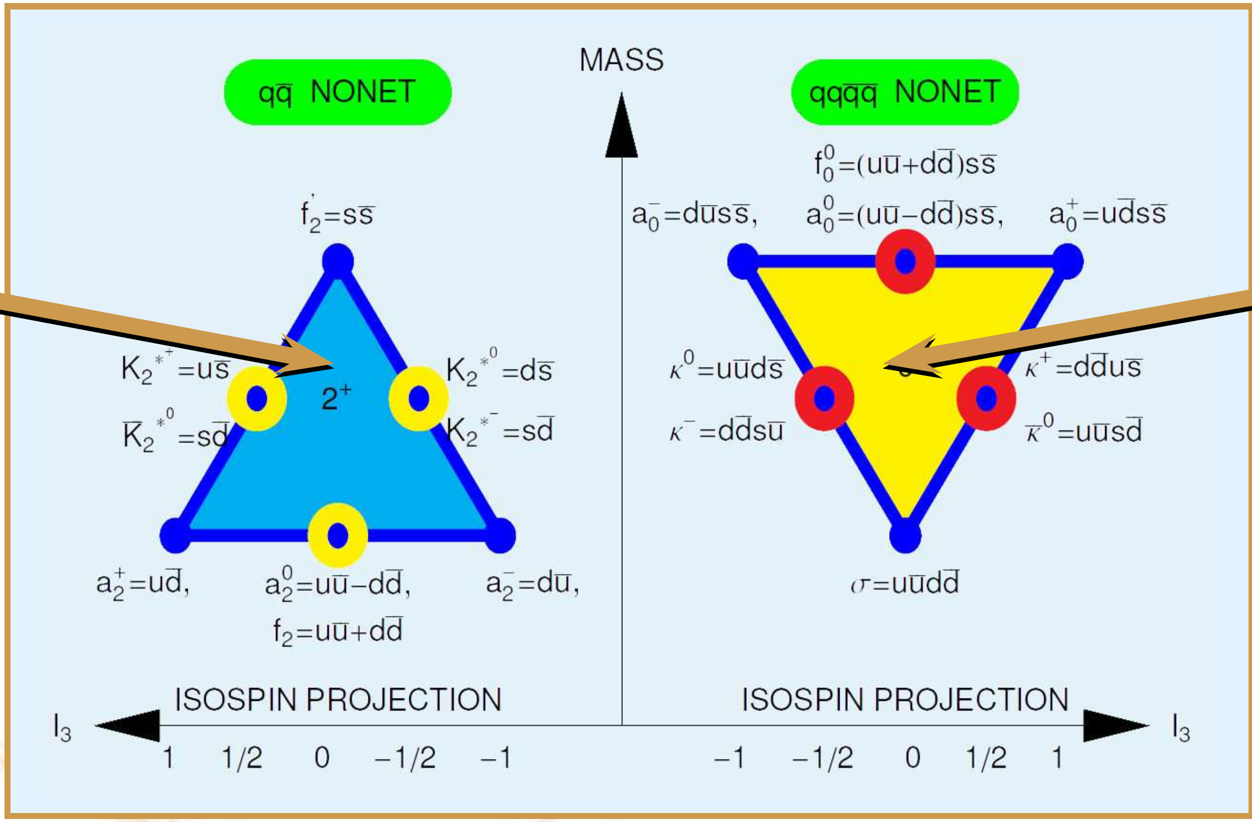


- Only $0^{++}$and 2^{++} allowed
- Kinematical constraints
- Final state isospin
- Threshold effects for $f_0, a_0, \sigma \Rightarrow$ Flatté formalism[Flatte (1976)]

Evidence for light tetraquarks

9 mesons



9 tetraquarks

σ, κ, f_0, a_0

- Masses for light resonances in constituent model
  $\Rightarrow$ Flavor nonets are arranged as triangles

Nature of f_0, a_0, σ

- Light tetraquark $SU(3)_F$ nonet [t'Hooft et al., (2008)] :

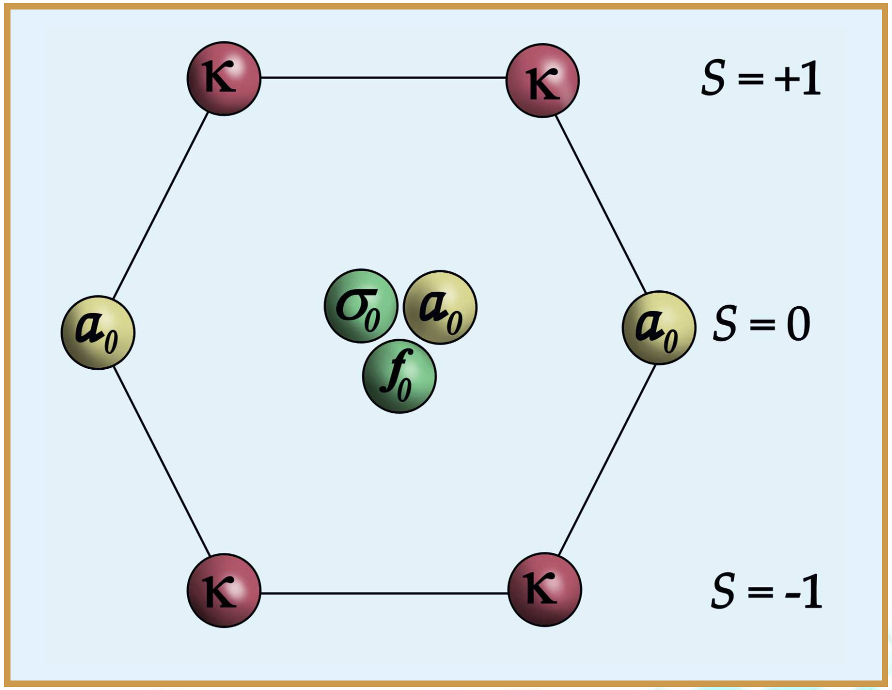
$$\sigma^{[0]} = [ud][\bar{u}\bar{d}]$$

$$\kappa = [su][\bar{u}\bar{d}];\ [sd][\bar{u}\bar{d}]$$

(+conjugate doublet)

$$f_0^{[0]} = \frac{[su][\bar{s}\bar{u}]+[sd][\bar{s}\bar{d}]}{\sqrt{2}}$$

$$a_0 = [su][\bar{s}\bar{d}];\quad [sd][\bar{s}\bar{u}];\quad \frac{[su][\bar{s}\bar{u}]-[sd][\bar{s}\bar{d}]}{\sqrt{2}}$$



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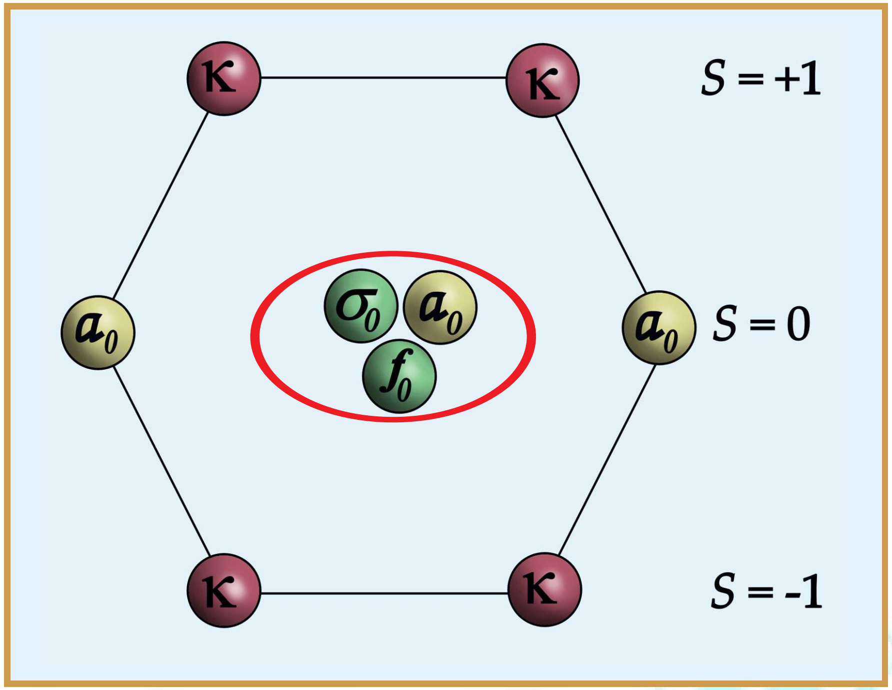
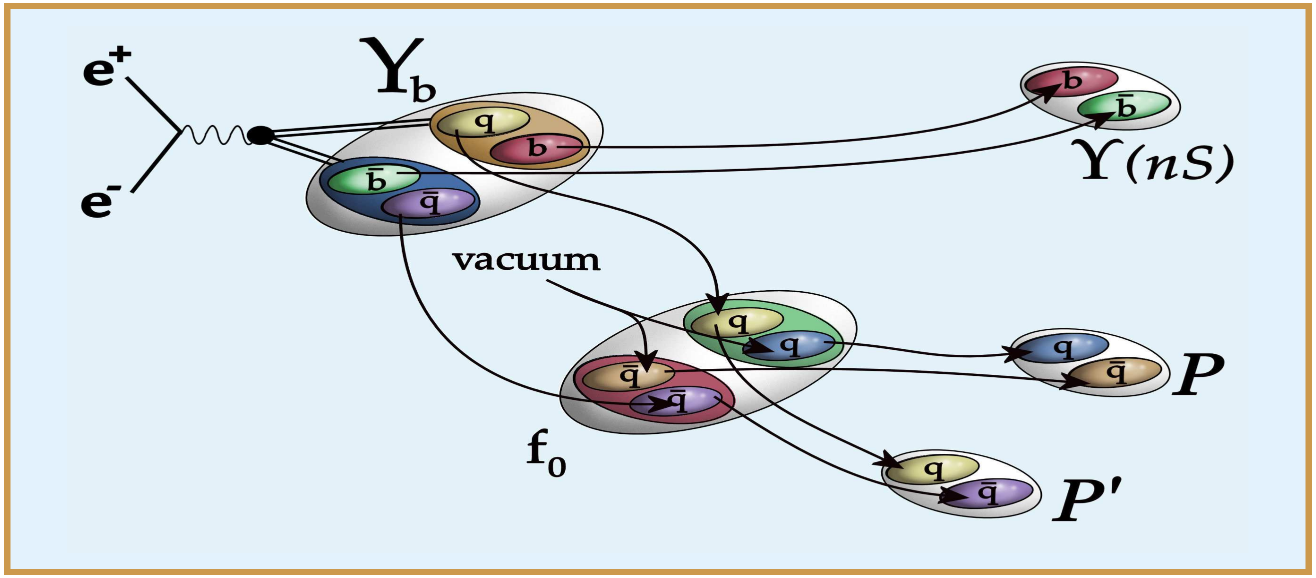


$SU(3)_F$ limit $\Rightarrow$ Identical couplings

S-wave resonance contribution

Zweig allowed light tetraquark resonance contributions:

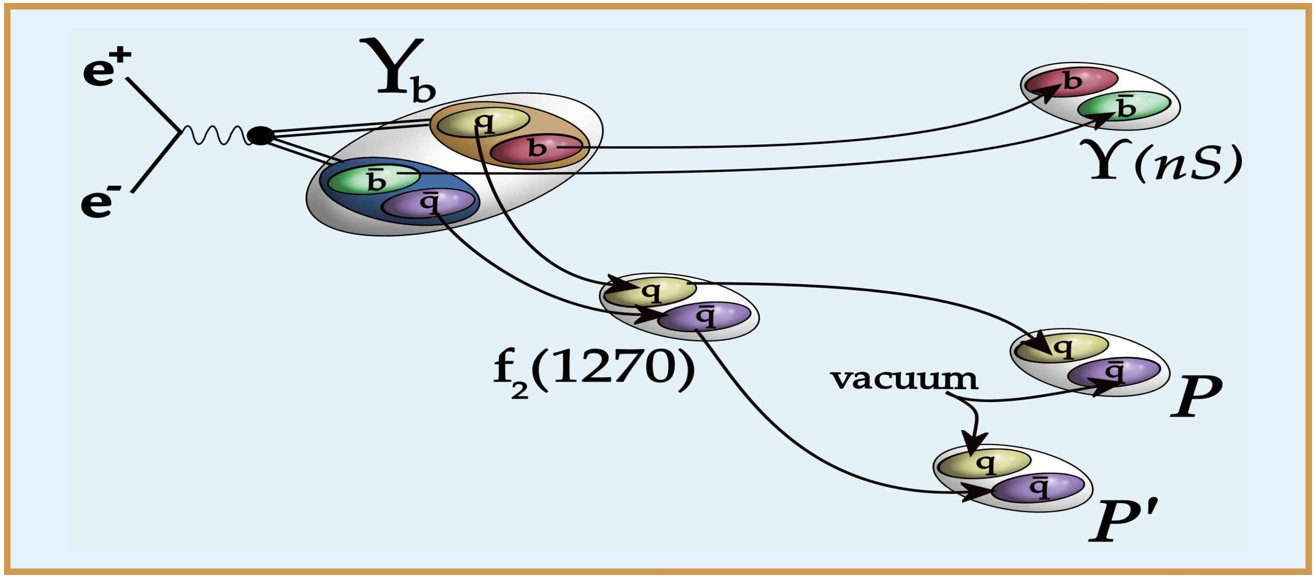


- Effective Lagrangian for 0^{++} tetraquarks:

$$\mathcal{L} = g_{SPP'}(\partial_\mu P)(\partial^\mu P')\, S + g_{Y_b^I \Upsilon(nS) S}\, Y_{b\mu}^I \Upsilon^\mu S$$

D-wave resonance contribution

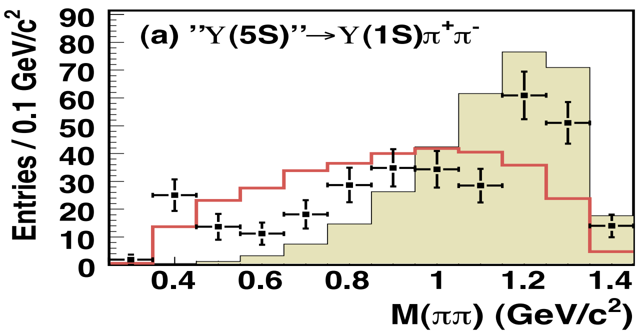
Zweig allowed $f_2(1270)$ contribution:

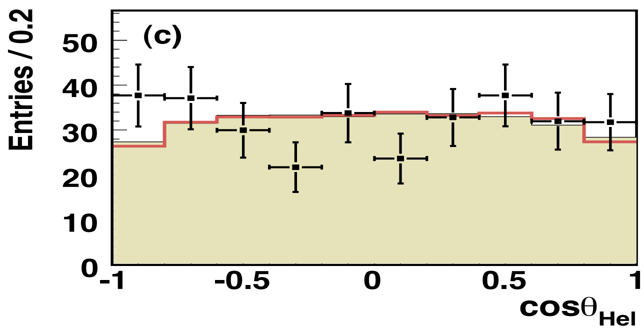


- $f_2(1270)$ effective Lagrangian:

$$\mathcal{L} = 2 g_{f_2 P P'} (\partial_\mu P)(\partial_\nu P') f_2^{\mu\nu} + g_{Y_b^I \Upsilon(nS) f_2} Y_{b\mu}^I \Upsilon_\nu f_2^{\mu\nu}$$

Fit to $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-)$





- Fit to normalized cross section:

$$\widetilde{\sigma}_{\pi^+\pi^-} \equiv \sigma_{\Upsilon(1S)\pi^+\pi^-} / \sigma^{\text{Belle}}_{\Upsilon(1S)\pi^+\pi^-} \quad \text{with} \quad \sigma^{\text{Belle}}_{\Upsilon(1S)\pi^+\pi^-} = 1.61 \pm 0.16 \text{ pb}$$

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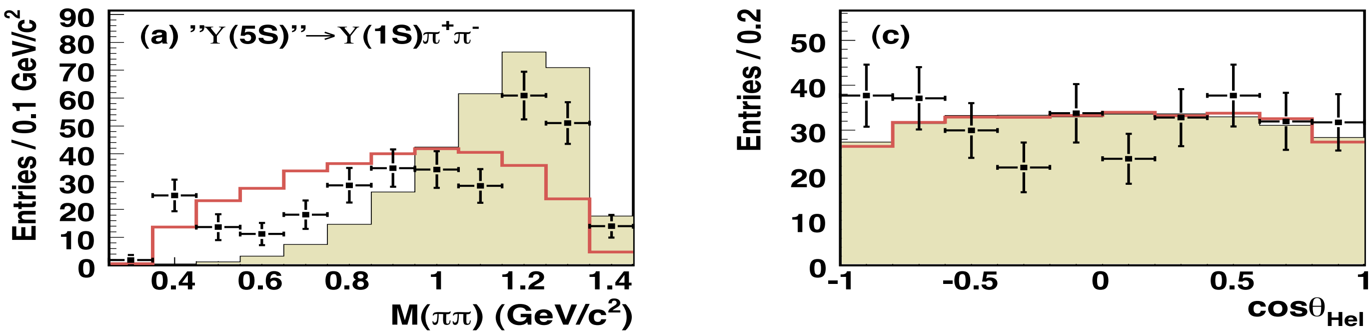

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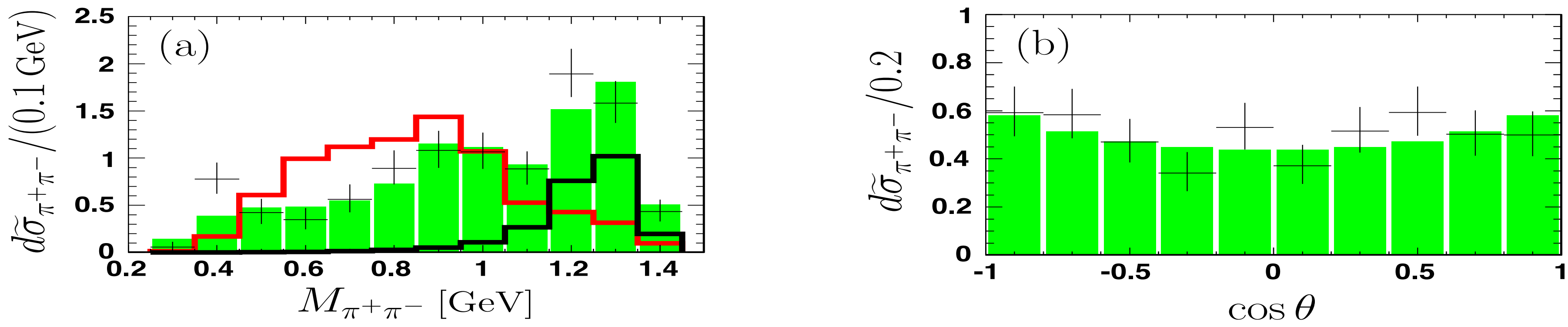
- Fit features:
  - ROOT $\mathcal{O}(5000)$ fits - checked with Mathematica
  - Simultaneous fit
  - Binning taken into account
  - Different Flatté couplings BES, CB and KLOE

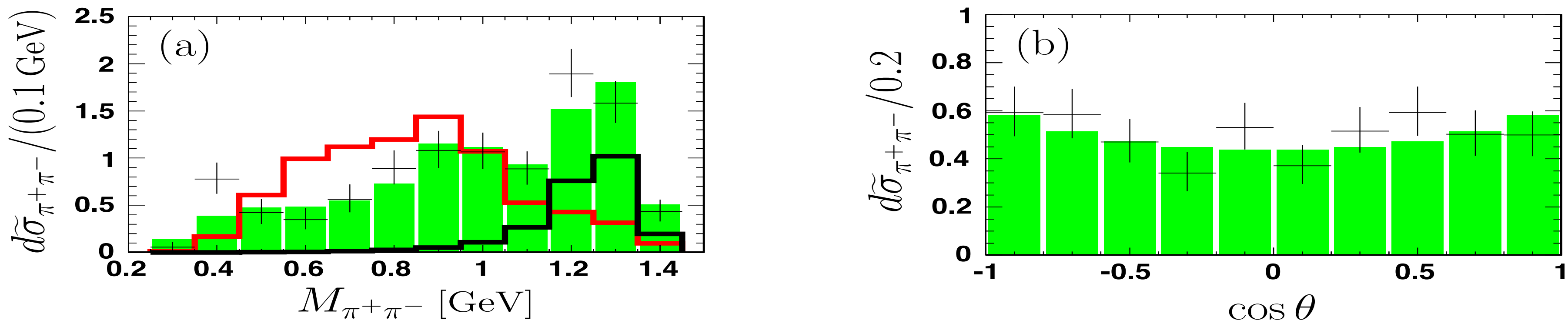
Fit to $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-)$



- Fit results:

	A'	B'	$g'_{Y_b^0\Upsilon(1S)f_0}$	$g'_{Y_b^0\Upsilon(1S)f_2}$	φ_σ	φ_{f_0}	φ_{f_2}
BES, CB	0.000079	-0.00020	0.318	0.439	0.36	-2.76	-0.46

Fit to $\sigma(e^+e^- \to Y_b \to \Upsilon(1S)\pi^+\pi^-)$



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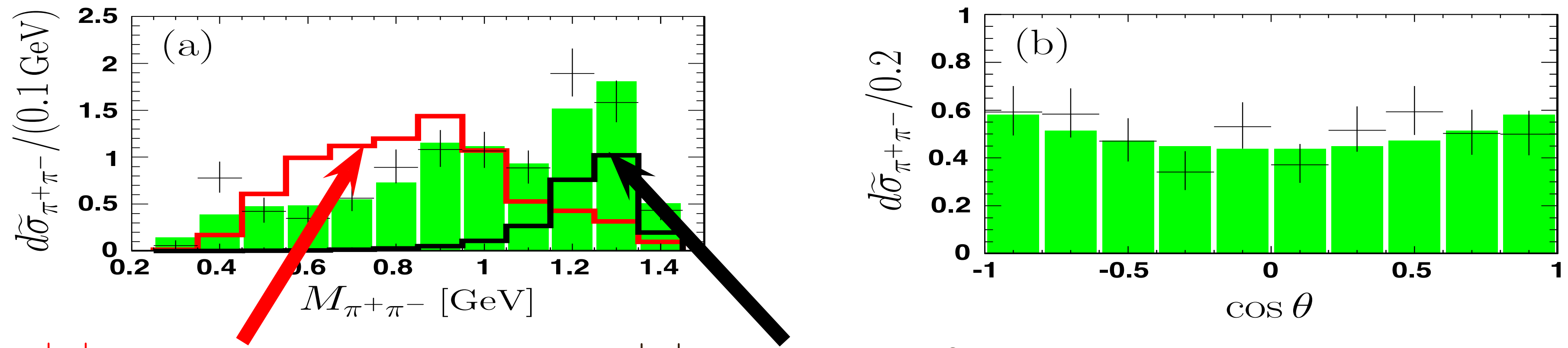

0^{++} **tetraquarks** 2^{++} **meson** f_2

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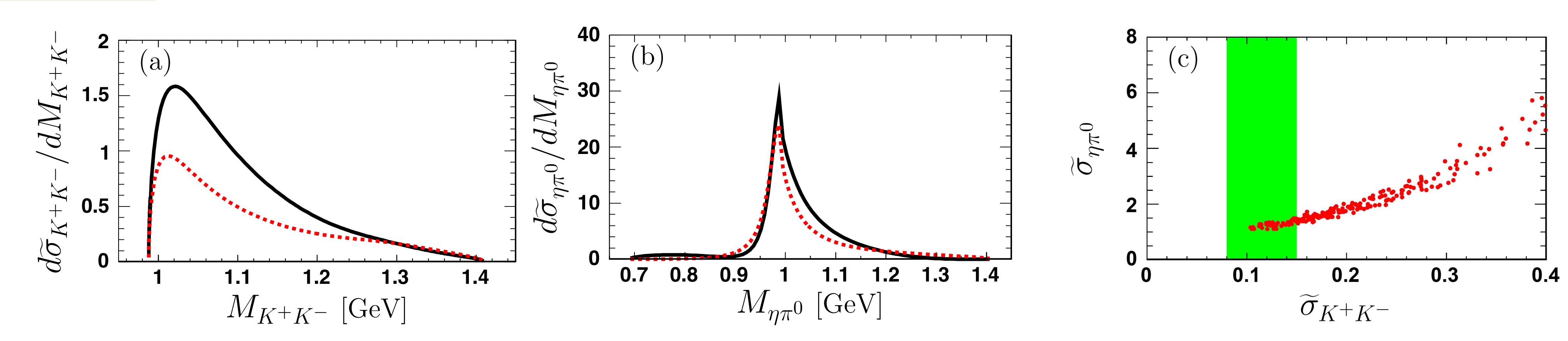
- Stable for BES, CB and KLOE input
- Clear resonance dominance!

Predictions for $\Upsilon(1S)(K^+K^-, \eta\pi^0)$

Fit determines couplings $\Longrightarrow$ predictions for spectra:



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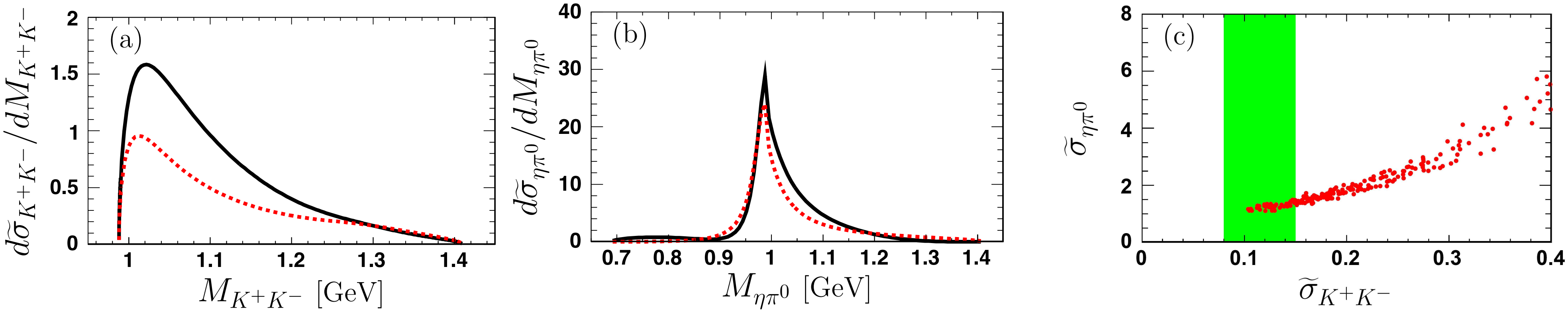

- Agreement with $\tilde{\sigma}_{K^+K^-} = 0.11^{+0.04}_{-0.03}$ (BELLE)

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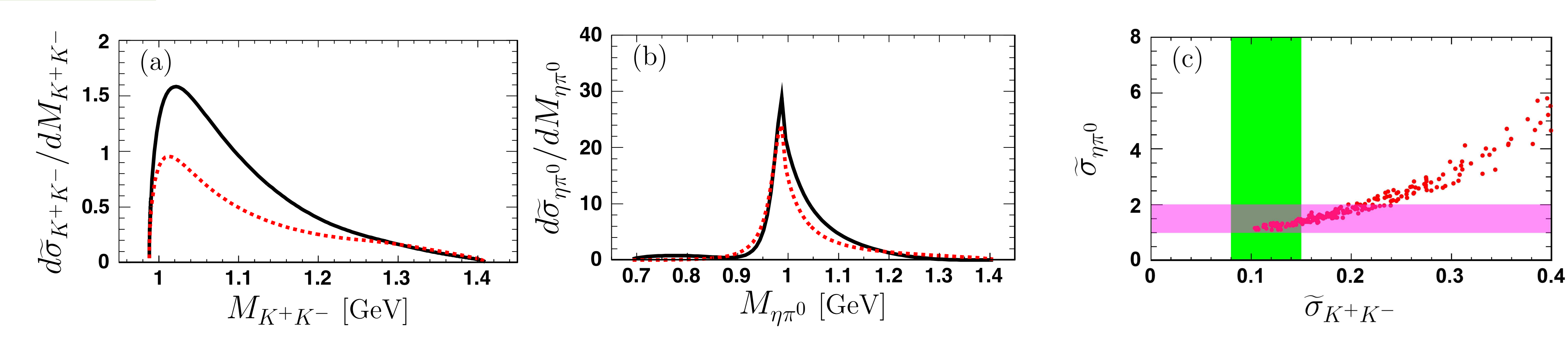

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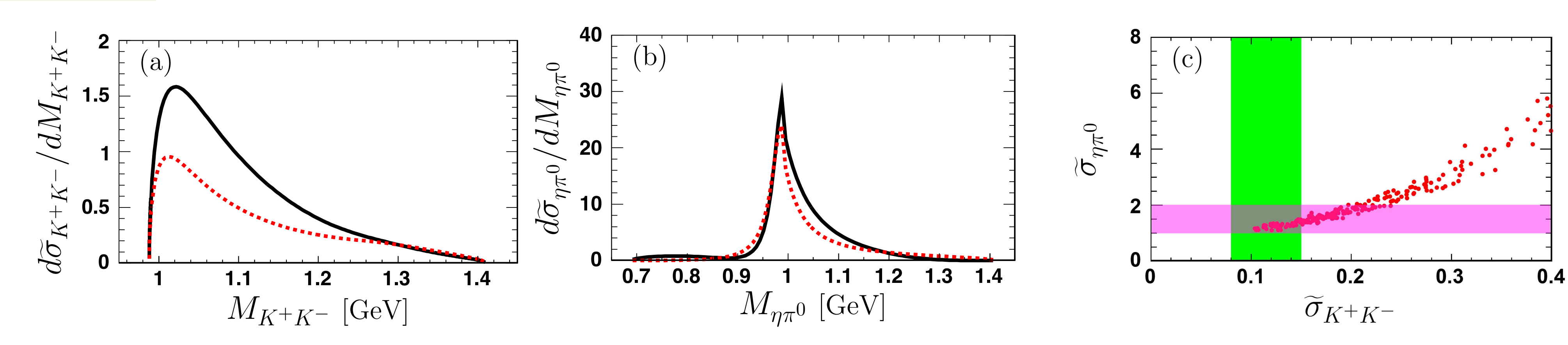

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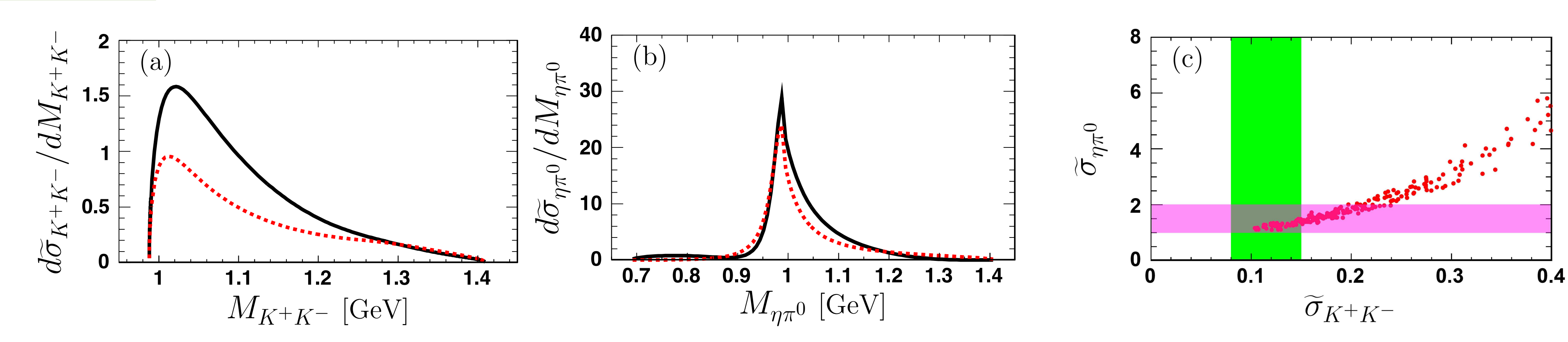

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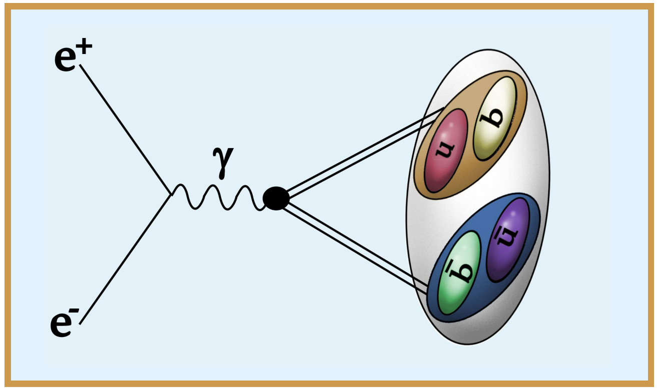

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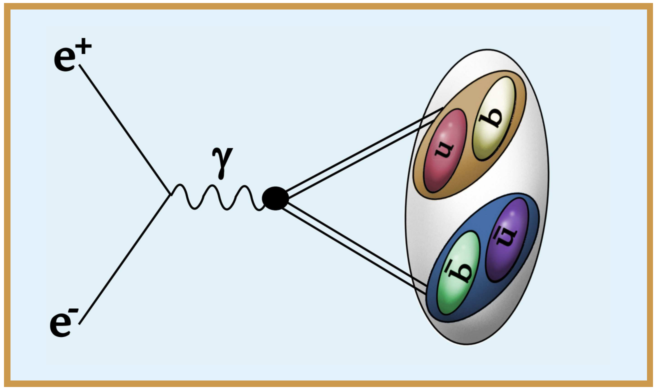

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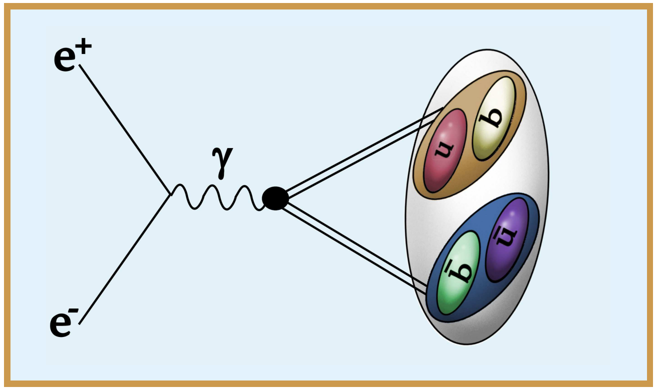

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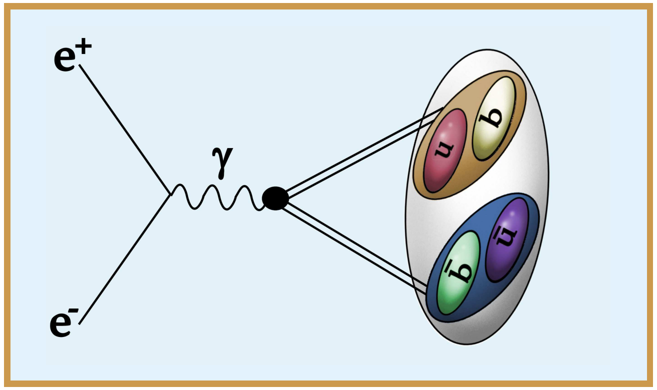

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Distinct for tetraquarks with pointlike diquarks

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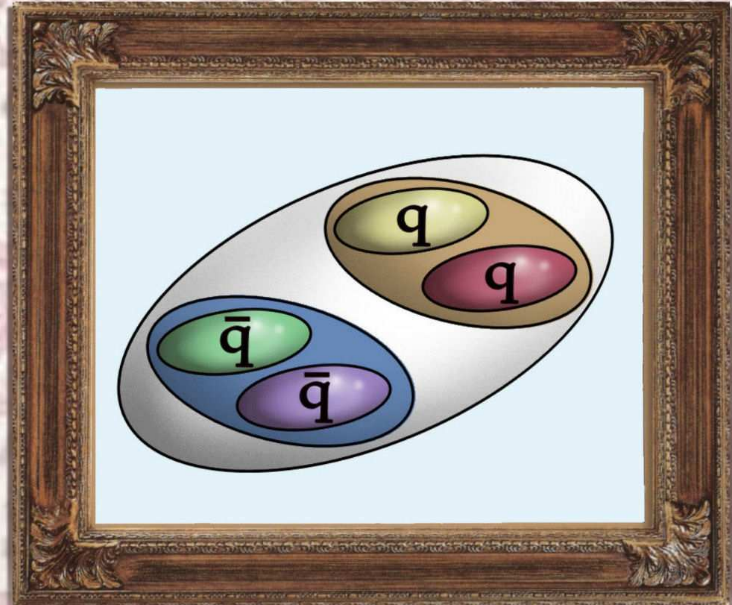
$\Rightarrow$ Forthcoming Belle data, and crucially data from the Super-B factories eagerly awaited!



thank you!

Backup

Light tetraquark interactions

The effective Lagrangian ($i, \ j$ are flavor indices):

$$\mathcal{L} \propto \mathrm{Det}(Q_{LR}) \ , \qquad (Q_{LR})^{ij} = \bar{q}^i_L q^j_R$$

induces tetraquark-meson 6-quark interactions via

$$\mathrm{Tr}(J^{[4q]} J^{2q}) \ , \quad \text{with} \quad J^{[4q]}_{ij} = [\bar{q}\bar{q}]_i [qq]_j \ , \quad J^{2q}_{ij} = \bar{q}_j q_i$$

tetraquark-meson mixing

tetraquark decay

Diquarks: Evidence in lattice QCD

gauge invariant two-density correlators:

$$C_\Gamma(\mathbf{r}_u, \mathbf{r}_d, t) \equiv \langle 0 | J_\Gamma(\mathbf{0}, 2t) J_0^u(\mathbf{r}_u, t) J_0^d(\mathbf{r}_d, t) J_\Gamma^\dagger(\mathbf{0}, 0) | 0 \rangle$$

where $J_0^f(\mathbf{r}, t) =: \bar{f}(\mathbf{r}, t)\gamma_0 f(\mathbf{r}, t) :$, $f = u, d$ and

$$J_\Gamma(x) = \epsilon^{abc}\left[{u^T}_a(x) C\, \Gamma d_b(x) \pm {d^T}_a(x) C\, \Gamma u_b(x) \right] s_c(x)$$

static quark:	s_c	correlator:	$C_{\gamma_5}(r, r_{ud}) \propto e^{-r_{ud}/r_0(r)}$
flavor symmetry:	$+/-$	quark distance:	$r_{ud} = 2r\sin(\theta/2)$
0^+:	$\Gamma = \gamma_5$	angle:	$\theta = \cos^{-1}(\vec{r}_u \cdot \vec{r}_d)$
1^+:	$\Gamma = \gamma_i$	diquark size:	$r_0(r)$