Systematic treatment of second order NLO QED radiative corrections to exclusive observables

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- **QED version of the factorization theorem in NLO**

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- **Ansatz for the master formula describing exclusive observables in $\mathcal{O}\left(\alpha^2 L^1\right)$**

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- **2 Hard photons**

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- **1 Hard photon $\otimes$ 1-loop Soft + Virtual**

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- **Pairs**

QED Factorization Theorem

The QCD factorization theorem can be adopted for the QED case (omitting for a while pair corrections) *e.g.* **for Bhabha:**

$$\mathrm{d}\sigma = \int_{\bar{z}_1}^1 \mathrm{d}z_1 \int_{\bar{z}_2}^1 \mathrm{d}z_2 \mathcal{D}_{ee}^{\mathrm{str}}(z_1) \mathcal{D}_{ee}^{\mathrm{str}}(z_2) \Big(\mathrm{d}\sigma^{(0)}(z_1, z_2) + \mathrm{d}\bar{\sigma}^{(1)}(z_1, z_2) + \mathcal{O}\left(\alpha^2 L^0\right) \Big)$$

$$\times \int_{\bar{y}_1}^1 \frac{\mathrm{d}y_1}{Y_1} \int_{\bar{y}_2}^1 \frac{\mathrm{d}y_2}{Y_2} \mathcal{D}_{ee}^{\mathrm{frg}}(\frac{y_1}{Y_1}) \mathcal{D}_{ee}^{\mathrm{frg}}(\frac{y_2}{Y_2}),$$

where $\sigma^{(0)}$ is the Born-level cross section,
$\bar{\sigma}^{(1)}$ is the $\overline{\mathrm{MS}}$ subtracted $\mathcal{O}\left(\alpha\right)$ contribution,

$$\mathcal{D}_{ee}^{\mathrm{str,frg}}(z) = \delta(1-z) + \frac{\alpha}{2\pi} d^{(1)}(z, \mu_0, m_e) + \frac{\alpha}{2\pi} L P^{(0)}(z)$$

$$+ \left(\frac{\alpha}{2\pi}\right)^2 \Big(\frac{1}{2} L^2 P^{(0)} \otimes P^{(0)}(z) + L P^{(0)} \otimes d^{(1)}(z, \mu_0, m_e)$$

$$+ L P_{ee}^{(1,\gamma)\mathrm{str,frg}}(z) \Big) + \mathcal{O}\left(\alpha^2 L^0, \alpha^3\right)$$

QED Master Formula Ansatz

Using slicing in the photon energy, we cast the corrected cross section in the form

$$\mathrm{d}\sigma = \mathrm{d}\sigma^{(0)} + \mathrm{d}\sigma^{(1)}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(1)}_{\mathrm{H}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{S+V}} + \mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} + \mathrm{d}\sigma^{(3)LO} + \ldots$$

For many observables we need to know the complete kinematics including hard photon angles, which are integrated over in the QCD-like formula.
Let us decompose the $\mathcal{O}\left(\alpha^2 L^{2,1}\right)$ hard radiation contribution

$$\mathrm{d}\sigma^{(2)NLO}_{\mathrm{H}} = \mathrm{d}\sigma^{(2)}_{\mathrm{HH(coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{HH(s-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(n-coll)}} + \mathrm{d}\sigma^{(2)}_{\mathrm{(S+V)H(coll)}}$$

where slicing in the photon emission angle is applied:
- **"coll" means collinear photon(s) with $\vartheta_\gamma < \theta_0 \ll 1$,**
- **"n-coll" means non-collinear photon with $\vartheta_\gamma > \theta_0$,**
- **"HH(s-coll)" means semi-collinear kinematics,**
  i.e. **one collinear photon and one non-collinear**

Particular NLO contributions (1)

The combined effect of **virtual corrections** and **soft photon emission** ones within the $\mathcal{O}\left(\alpha^2 L^1\right)$ can be obtained by convolution of the structure functions with the kernel cross section according to the general factorization theorem. Here one requires only one non-trivial convolution

$$\frac{\alpha}{2\pi} L \int\limits_{1-\Delta}^{1} \mathrm{d}z \int\limits_{0}^{1} \frac{\mathrm{d}x}{x} P^{(0)}\left(\frac{z}{x}\right) \mathrm{d}\bar{\sigma}^{(1)}(x)$$

This integral can be found for any relevant process as demonstrated in **[A.A., E. Scherbakova, ZhETF Pis'ma 2006]** for the large-angle Bhabha case by getting $\mathrm{d}\sigma_{\mathrm{S+V}}^{(2)NLO}$ in agreement with the complete $\mathcal{O}\left(\alpha^2\right)$ calculation.

Particular NLO contributions (2)

Emission of two hard photons, HH, can be considered in three regions:

1. **non-collinear:** $\theta_{1,2} > \vartheta_0$
   suited for Monte Carlo simulation
2. **semi-collinear:** $\theta_1 > \vartheta_0$ **and** $\theta_2 < \vartheta_0$
   in $\mathcal{O}\left(\alpha^2 L\right)$ **has factorized form** $d\sigma_{\mathrm{H}}^{(1)} \otimes R_{\mathrm{H}}^{\mathrm{ISR,FSR}}(z)$
3. **collinear:** $\theta_{1,2} < \vartheta_0$ **is described by the**
   HH **radiation factor convoluted with the Born**

Emission of one hard photon in $\mathcal{O}\left(\alpha^2 L\right)$ **can be sliced into two domains:**

1. **non-collinear:** $\theta_\gamma > \vartheta_0$ **as a product of two factors**
   $d\sigma_{\mathrm{H}}^{(1)} \times \delta_{\mathrm{Soft+Virt}}^{\mathrm{LO}}$
2. **collinear:** $\theta_\gamma < \vartheta_0$ **is described by the collinear**
   NLO H radiation factor (see below)

QED Collinear Radiation Factors in NLO (1)

A. Arbuzov, E. Scherbakova,
Phys. Lett. B 660 (2008) 37 [arXiv:0706.2984]

$$d\sigma[a(p_1) + b(p_2) \to c(q_1) + d(q_2) + \gamma(k \sim (1-z)p_1)]$$
$$= d\hat{\sigma}[a(zp_1) + b(p_2) \to c(q_1) + d(q_2)] \otimes R_{\mathrm{H}}^{\mathrm{ISR}}(z)$$

Emission of collinear photons in FSR and ISR with conditions

$$\vartheta_\gamma < \vartheta_0, \qquad \frac{m}{E} \ll \vartheta_0 \ll 1, \qquad l_0 = \ln\frac{\vartheta_0^2}{4}, \quad \frac{E_\gamma}{E} > \Delta \ll 1$$

In $\mathcal{O}(\alpha)$ the result is well known:

$$R_{\mathrm{H}}^{\mathrm{ISR}}(z) = \frac{\alpha}{2\pi}\left[\frac{1+z^2}{1-z}\left(\ln\frac{E^2}{m^2} - 1 + l_0\right) + 1 - z + \mathcal{O}\left(\frac{m^2}{E^2}\right) + \mathcal{O}\left(\vartheta_0^2\right)\right]$$

QED Collinear Radiation Factors in NLO (2)

Emission of two collinear photons (HH) in the same direction is described by a one-fold integral of results from [A.A. et al., Nucl. Phys. B 483 (1997) 83]:

$$R_{\mathrm{HH}}^{\mathrm{ISR}}(z) = \left(\frac{\alpha}{2\pi}\right)^2 L \bigg\{ (L + 2l_0)\bigg(\frac{1+z^2}{1-z}(2\ln(1-z) - 2\ln\Delta - \ln z)$$

$$+\frac{1+z}{2}\ln z - 1 + z\bigg) + \frac{1+z^2}{1-z}\bigg(\ln^2 z + 2\ln z - 4\ln(1-z) + 4\ln\Delta\bigg)$$

$$+(1-z)\bigg(2\ln(1-z) - 2\ln\Delta - \ln z + 3\bigg) + \frac{1+z}{2}\ln^2 z\bigg\}$$

FSR factor is restored with help of the Gribov-Lipatov relation generalized for the collinear emission case:

$$R_{\mathrm{HH}}^{\mathrm{FSR}}(z) = -zR_{\mathrm{HH}}^{\mathrm{ISR}}\left(\frac{1}{z}\right)\bigg|_{\ln\Delta \to \ln\Delta - \ln z; \; l_0 \to l_0 + 2\ln z}$$

QED Collinear Radiation Factors in NLO (3)

Emission of one collinear hard photon accompanied by one-loop soft and virtual correction (H(S+V)) is received using the NLO QED splitting functions

$$R_{\mathrm{H(S+V)}}^{\mathrm{ISR}}(z) \otimes \mathrm{d}\hat{\sigma}(z) = \delta_{(\mathrm{S+V})}^{(1)} R_{\mathrm{H}}^{\mathrm{ISR}}(z) \otimes \mathrm{d}\sigma^{(0)}(z)$$

$$+ \left(\frac{\alpha}{2\pi}\right)^2 L\left[2\frac{1+z^2}{1-z}\left(\mathrm{Li}_2\left(1-z\right) - \ln(1-z)\ln z\right)\right.$$

$$\left. -(1+z)\ln^2 z + (1-z)\ln z + z\right] \otimes \mathrm{d}\sigma^{(0)}(z),$$

$$\delta_{(\mathrm{S+V})}^{(1)} = \frac{\mathrm{d}\sigma_{\mathrm{Soft}}^{(1)} + \mathrm{d}\sigma_{\mathrm{Virt}}^{(1)}}{\mathrm{d}\sigma^{(0)}},$$

where $\sigma^{(0)}(z)$ is the boosted Born cross section, and $\delta_{(\mathrm{S+V})}^{(1)}$ is the relative $\mathcal{O}\left(\alpha\right)$ Soft + Virtual radiative correction with $E_\gamma^{\mathrm{Soft}} < \Delta E$. The corresponding FSR factor is received again using the Gribov-Lipatov relation.

Pair Corrections

Leptonic and hadronic pair corrections are important for a number of precision observables. **Exclusive** treatment here is of ultimate importance. Monte Carlo has to be used for real or for **hard** pairs, then soft and virtual ones can be treated analytically (semi-analytically for the hadronic case).

Singlet and non-singlet **NLO** pair contributions in $\mathcal{O}\left(\alpha^2 L\right)$ to **inclusive** observables can be described within the QCD-like factorization approach.

But if we have a MC for hard pairs, we can extract analytically the soft+virtual part, so that

$$\mathrm{d}\sigma_{\mathrm{pair}}^{(2)} = \mathrm{d}\sigma_{\mathrm{H\ pair}}^{(2)MC} + \mathrm{d}\sigma^{(0)} \times \delta_{\mathrm{S+V\ pair}}^{(2)}$$

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▸ Negatively weighted events within this approach are possible but not numerous

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- ► $\mathcal{O}\left(\alpha^2 L^0\right)$ **contributions can be put into the same structure**

- ► **MCGPJ can be upgraded**

- ► **MC integrator and generator for Bhabha scattering is under development (upgrade of SAMBHA MC)**