Flavor Symmetry Breaking Effects
in
The Hidden Local Symmetry Model

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LPNHE Paris 6/7

OUTLINE

- Comments on the HLS Model
- SU(3)/U(3) Flavor Breaking of HLS
  (L_A, L_V, L_{YM} & PS Nonet Symm. Breaking)
- The Anomalous Sector (VPγ & Pγγ couplings with Brk)
- The PS and V Mixing Angles : Physical Fields
- Decay Rate Data Sample & Fit Results
- Older Results (glue content of (η,η'), Box Anomaly)
- Isospin Symmetry Breaking & Loop Effects
- Commenting the Numerical Solution (e+e- vs τ)
- Conclusions

The Hidden Local Symmetry Model

- Vector Mesons are gauge bosons of a
  Hidden Local Symmetry Model (HLS)

M.Bando, T. Kugo & K. Yamawaki Phys. Rep. 164 (1988) 217
M. Harada & K. Yamawaki Phys. Rep. 381 (2003) 1

Expanded form in : M.Benayoun & H.O’Connell PR D 58 (1998) 074006

- Four Lagrangian Pieces : L_A, L_V, L_{YM} , $L_{anomalous}$
- *Physics Realm : pion form factor, VPγ & Pγγ decays*

The Hidden Local Symmetry Model

- Define $\xi_{L/R} = \mathrm{e}^{[\mp i P / f_\pi]}$
- Define covariant derivatives $D_\mu \xi_L \, , D_\mu \xi_R$
- Then $L / R = D_\mu \xi_{L/R} \xi_{L/R}^\dagger$ and $L_{A/V} = -\frac{f_\pi^2}{4} Tr[L \mp R]^2$
- The Full HLS Lagrangian : $L_{HLS} = L_A + a \, L_V + L_{YM}$
- Standard VMD : a=2 , Phenomenology a ~ 2.4

Expanded form: M.Benayoun & H.O'Connell PR D 58 (1998) 074006

The Hidden Local Symmetry Model

- Define $\xi_{L/R} = e^{[\mp i P / f_\pi]}$

PS field matrix

- Define covariant derivatives $D_\mu \xi_L \, , D_\mu \xi_R$
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- The Full HLS Lagrangian : $\boxed{L_{HLS} = L_A + a\, L_V + L_{YM}}$
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Expanded form: M.Benayoun & H.O'Connell PR D 58 (1998) 074006

The Covariant Derivatives

- Covariant derivatives ≠ for left- right-ξ fields :

$$D_\mu \xi_{L/R} = \partial_\mu \xi_{L/R} - igV_\mu \xi_{L/R} + i\xi_{L/R} G_{L/R}$$

- With :

$$G_R = eQA_\mu \quad , \quad G_L = eQA_\mu + \frac{g_2}{\sqrt{2}}\left(W_\mu^+ T_+ + W_\mu^- T_-\right)$$

$T_\pm$ is CKM matrix reduced to V_{us} and V_{ud} terms

SU(3)/U(3) Flavor Symmetry Breaking

- HLS almost useless for phenomenology without flavor symmetry breaking → BKY mechanism

M.Bando, T. Kugo & K. Yamawaki Phys. Rep. 164 (1988) 217

- Modified BKY mechanism preserves Hermiticity

M.Benayoun & H.O’Connell PR D 58 (1998) 074006

Breaking of SU(3) Flavor Symmetry I

Several possible schemes

$$L_{A/V} = -\frac{f_\pi^2}{4} Tr\left[L \mp R\right]^2 \Rightarrow$$

$$L_{A/V} = -\frac{f_\pi^2}{4} Tr\left[(L \mp R) X_{A/V}\right]^2$$

With Breaking matrices $X_{A/V} = Diag(1, 1, z_{A/V})$

$$z_A = \left[\frac{f_K}{f_\pi}\right]^2 \cong 1.5$$

PS fields should be renormalized

Breaking of SU(3) Flavor Symmetry II

The Yang-Mills Piece

$$F_{a,\mu\nu} = \partial_\mu V_{a,\nu} - \partial_\nu V_{a,\mu} - i\left[V_{a,\mu}, V_{a,\nu}\right]$$

$$L_{YM} = -\frac{1}{2} Tr\left[F_{a,\mu\nu} F^{a,\mu\nu}\right] \Rightarrow -\frac{1}{2} Tr\left[F_{a,\mu\nu} X_{YM} F^{a,\mu\nu} X_{YM}\right]$$

Vector Field should be renormalized

$$X_{YM} = Diag(1, 1, z_{YM}) \qquad \Longrightarrow \qquad \boxed{V^{\text{Ren}}_{a,\mu} = X_{YM}^{1/2} V_{a,\mu} X_{YM}^{1/2}}$$

M. Benayoun et al. hep-ph 0711.4482→EPJC

U(3) (Nonet) Symmetry Breaking

- In order to do phenomenology :
- Need of Nonet symmetry Breaking

⟹ **add determinant terms**

$$L = L_A + a\, L_V + L_{YM} + \frac{1}{2}\mu_0^2\eta_0^2 + \frac{1}{2}\lambda\, \partial_\mu \eta_0 \partial^\mu \eta_0$$

G. 't Hooft Phys. Rept. 142 (1986) 357

Pseudoscalar field renormalization:

$$\boxed{P_8^{\mathrm{Ren}} + x P_0^{\mathrm{Ren}} = X_A^{1/2}(P_8 + P_0)X_A^{1/2}}$$

$$\left[x \approx \frac{1}{\sqrt{1+\lambda}} \right]$$

M.Benayoun, L. DelBuono & H.O'Connell EPJ C 17 (2000) 593

Anomalous Sector of the HLS Model

- Anomalous sector of HLS Model :**VPγ/Pγγ decays**

$$L = C\varepsilon^{\mu\nu\rho\sigma} Tr\left[\partial_\mu\left(eQA_\nu + gV_\nu\right) X_{BGP} \partial_\rho\left(eQA_\sigma + gV_\sigma\right) P\right]$$

A. Bramon, A. Grau & G. Pancheri PL B 345 (1995) 263

With $V_{a,\mu} \Rightarrow X_{YM}^{-1/2} V_{a,\mu}^{\text{Ren}} X_{YM}^{-1/2}$ and $X_{BGP} X_{YM}^{2} = 1$

- And

$$P_8 + P_0 \Rightarrow X_A^{-1/2}\left(P_8^{\text{Ren}} + xP_0^{\text{Ren}}\right) X_A^{-1/2}$$

- X_{BGP}/X_{YM} plays only for rad. K* decays

M. Benayoun et al. PR D 59 (1999) 114027

G. Morpurgo PR D 42 (1990) 1497

The PS And V Mixing Angles

- In order to fit VPγ and Pγγ decay modes neutrals call for mixing angles
- The (η,η') mixing angle θ_P **not free** :

$$\tan\theta_P = \sqrt{2}\,\frac{1-z_A}{2+z_A}\,x \quad , \left[\!\left[\theta_8 \simeq 2\theta_P, \quad \theta_0 \simeq 0, \quad z_A = \left[\frac{f_K}{f_\pi}\right]^2 \right]\!\right]$$

MB, LD & HO EPJ C 17 (2000) 593

- The ρ^0, ω, φ mixing by constant angles or :

M. Benayoun et al. PR D 59 (1999) 114027

M. Benayoun & H. O'Connell EPJ C 22 (2001) 503

From Ideal To Physical Fields

$$\begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix} = R(s) \begin{pmatrix} \rho_I^0 \\ \omega_I \\ \varphi_I \end{pmatrix}$$

$R(s)$: Real analytic matrix function

+ +(Fit) Subtraction Polynomials

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon)=1$$

$$R(s)=\begin{pmatrix} 1 & \dfrac{\varepsilon_1}{\Pi_{\pi\pi}-\varepsilon_2} & \dfrac{-\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2} \\ \dfrac{-\varepsilon_1}{\Pi_{\pi\pi}-\varepsilon_2} & 1 & \dfrac{-\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2} \\ \dfrac{\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2} & \dfrac{\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2} & 1 \end{pmatrix}$$

$$+O(\varepsilon_i^2)$$

The Decay Data Sample

- Anomalous decay modes VPγ, Pγγ :

$$\rho^0/\omega/\varphi \to \pi^0\gamma/\eta\gamma \,\|\, \varphi \to \eta'\gamma \,\|\, \eta' \to (\rho^0/\omega)\gamma$$

$$K^{*(0,\pm)} \to K^{(0,\pm)}\gamma \quad \|\, \eta/\eta' \to \gamma\gamma \quad \|\, \rho^\pm \to \pi^\pm\gamma$$

- Leptonic modes : $(\rho^0)\,\boxed{\omega\ /\ \varphi\ \to\ e^+e^-}$

$$\boxed{+Modulus\ and\ \ phase\,(\varphi \to \pi^+\pi^-)}$$

$$\left[\left[Modulus\ and\ \ phase\,(\rho^0/\omega \to \pi^+\pi^-) \simeq pion\ FF\right]\right]$$

M.B.,L.D. ,S.E, V.I. & H.OC, PR D 59 (1999) 114027

M.B, H.OC EPJ C22 (2001) 503

The X^2 contributions to fits

Data Set (#data points)	Full Data Fit	No τ data	No Spacelike Data
Decays (18+1)	11.13	11.52	11.48
New Timelike (127+1)	128.1	122.0	125.8
Old Timelike (82+1)	59.1	54.7	55.2
Spacelike (59+2)	65.7	55.2	**89.8/(59)**
τ ALEPH (33)	23.9	**42.3/(33)**	20.8
τ CLEO (25+1)	26.1	**26.2/(25)**	29.7
X^2/dof Probability	313.8/331 74%	257.7/274 75%	238.8/272 93%

Decay Modes (Fit Br/ Fit PDG)

Decay Mode	FIT/PDG	Remark
$\rho^0 \to \pi^0\gamma$	0.86 ±0.15	**1 σ**
$\rho^\pm \to \pi^\pm\gamma$	1.12±0.11	**1 σ**
$\rho^0 \to \eta\gamma$	1.04±0.11	**0.5 σ**
$K^{*\pm} \to K^\pm\gamma$	1.00±0.14	**0.0 σ**
$K^{*0} \to K^0\gamma$	0.98±0.09	**0.0 σ**
$\omega \to \pi^0\gamma$	0.93±0.03	***
$\omega \to \eta\gamma$	1.35±0.11	***
$\phi \to \pi^0\gamma$	0.99±0.08	**0.0 σ**
$\phi \to \eta\gamma$	0.99±0.03	**0.0 σ**

Decay Mode	FIT/PDG	Remark
$\eta' \to \rho^0\gamma$	1.13±0.04	**3.0 σ (Box)**
$\eta' \to \omega\gamma$	1.04±0.11	**0.5 σ**
$\phi \to \eta'\gamma$	0.97±0.12	**0.0 σ**
$\eta \to \gamma\gamma$	0.90±0.02	** f_π not fit
$\eta' \to \gamma\gamma$	0.99±0.07	**0.0 σ**
$\omega \to e^+e^-$	1.00±0.02	**0.0 σ**
$\phi \to e^+e^-$	1.00±0.02	**0.0 σ**
$\phi \to \pi^+\pi^-$	0.98±0.29	**0.0 σ**
Phase [$\phi \to \pi^+\pi^-$]	0.79±0.15	**1.5 σ**

Two New Results

Process	FIT	PDG
$\rho^0 \rightarrow e^+e^-$ [x10^5]	5.56±0.06	4.70±0.08
$\omega \rightarrow \pi^+\pi^-$ (%)	1.13±0.08	1.70±0.27
$\rho^0 \rightarrow \pi^+\pi^-$ [MeV]	144.5±0.6	149.4±1.0

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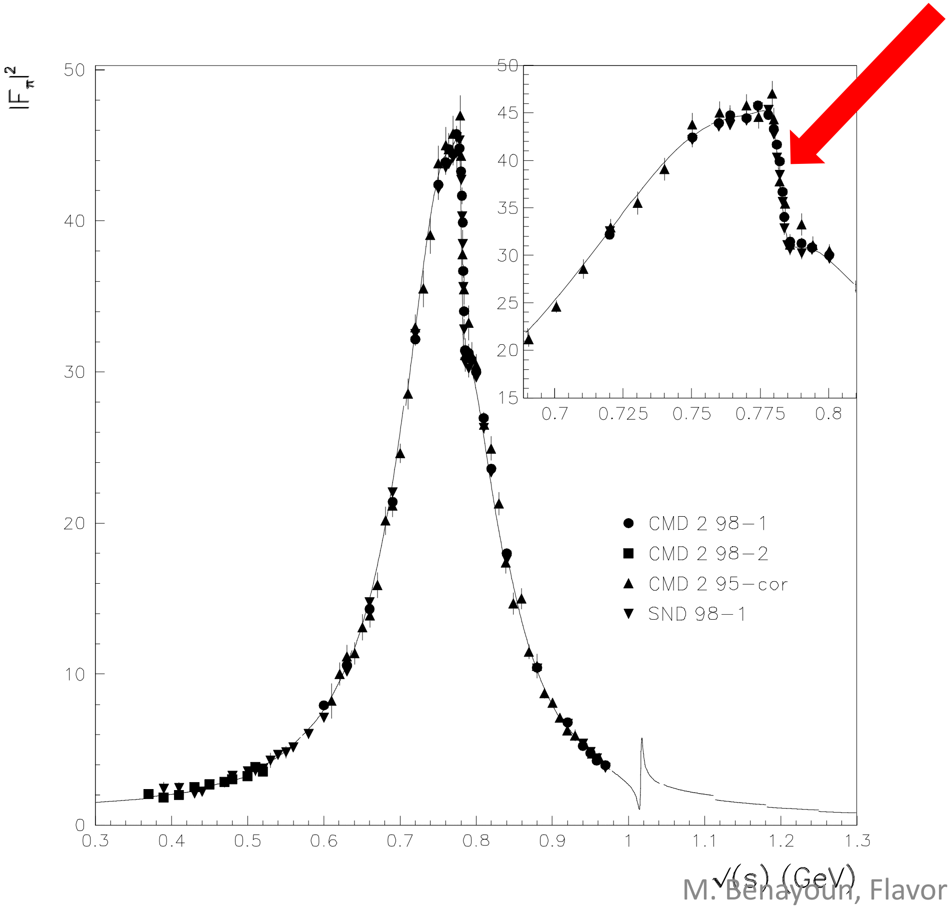
$\rho^0 \rightarrow \pi^+\pi^-$ [MeV]	144.5±0.6	149.4±1.0

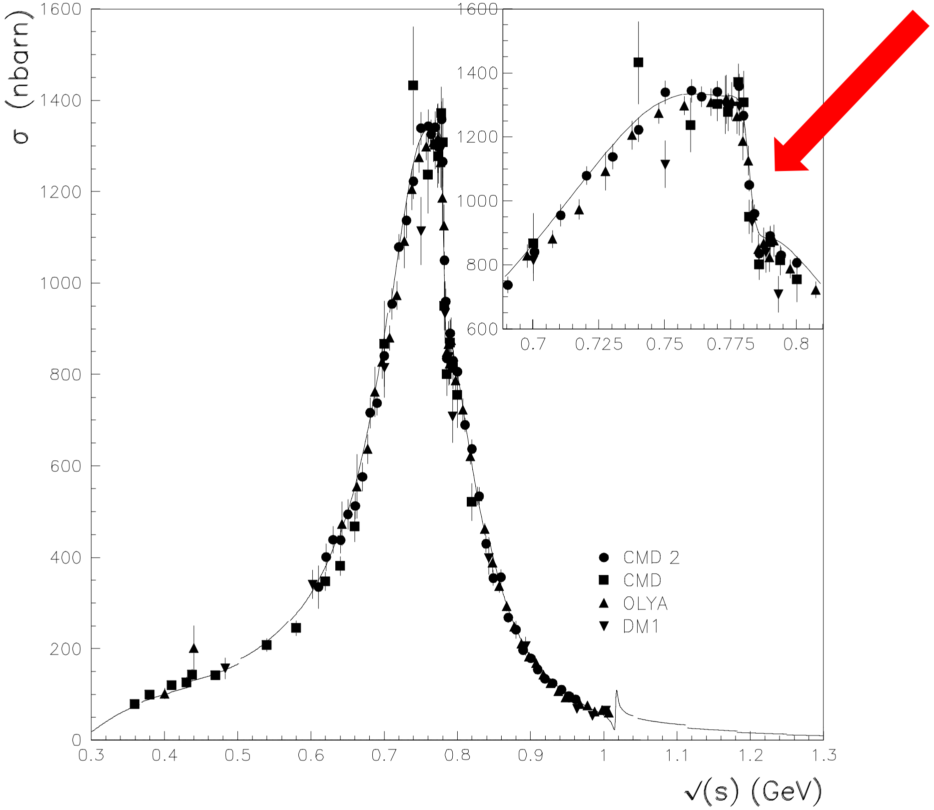
May change the global fit to ω branching ratios

Decays ρ/ω to ππ and e+e-

FF’s New Data



Cross Sections Old Data

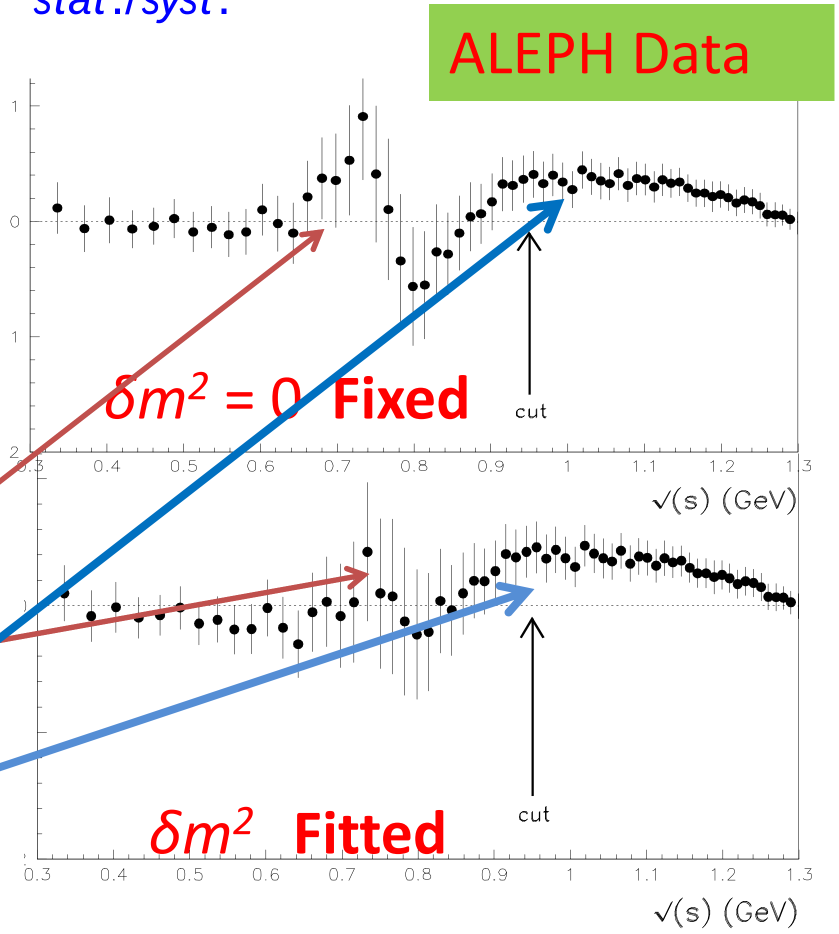


The ρ^0 - ρ$^\pm$ Mass Difference

$$M_{\rho^0} - M_{\rho^\pm} \simeq 1.35 \pm 0.15_{def.} \pm 0.53_{stat./syst.}\, MeV$$

The ρ^0-ρ$^\pm$ Mass Difference :
1/ Only visible in ALEPH data
2/ ~1 .2 σ from ChPT

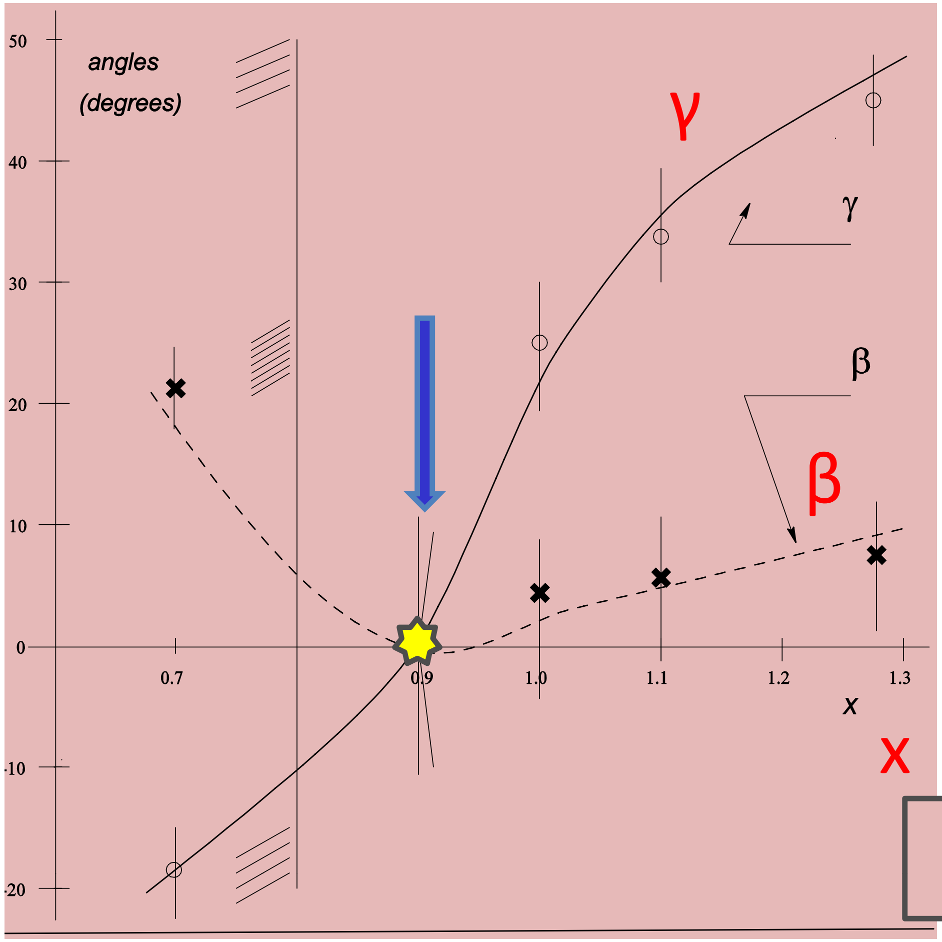
J. Bijnens & P. Gosdzinsky PL B 388 (1996) 203



ρ Peak Region

High mass Vector mesons

Older Results, Same Model I



Glue inside (η,η’) system?

β≈0 no Glue inside η

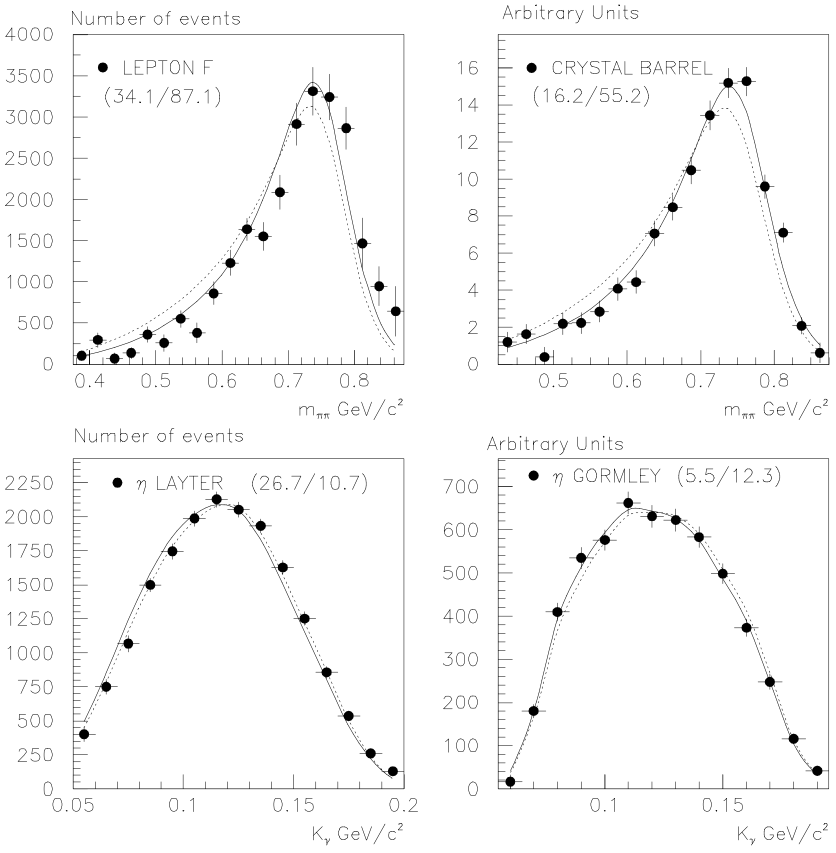
γ≈0 no Glue inside η

Nonet Symm. Breaking x≈0.9

↔No Glue inside ηη’ system

M. Benayoun et al. PR D 59 (1999) 114027
& « Phi to Jpsi » Novosibirsk 1999

Older Results, Same Model II



NOT A FIT

Box Anomaly in
ηη’→ππγ spectra
at expected magnitude

M. Benayoun et al EPJ C 31 (2003) 525

Isospin Breaking & Loops Effects I

- Two Isospin Breaking mechanisms

  Direct Breaking and Breaking in Loops

- **Direct Breaking** : $L_{A/V} \approx Tr\left[A_{A/V} X_{A/V}\right]^2$

with

$$X_{A/V} = Diag\left[1+\varepsilon^u_{A/V}, 1+\varepsilon^u_{A/V}, z_{A/V}\right] \quad \left(\varepsilon^u_{A/V}, \varepsilon^u_{A/V} \neq 0\right)$$

M. Hashimoto PR D 54 (1996) 5611

⇒ **small effects on masses and couplings :**

$$m^2_{\rho^0} = m^2_{\omega} \quad , \quad m^2_{\rho^0} = m^2_{\rho^\pm} + O\left(\varepsilon_V^2\right) \quad ,\ldots$$

Isospin Breaking & Loops Effects II

- **Loop Effects producing mixing among ideal fields**

⇨ **Ideal fields no longer mass eigenstates**

Loop Effects : NOT Neccessarily Isospin Breaking

No Brk : (ω,φ)//Isospin Brk : (ρ,ω) & (ρ,φ)

- **Physical fields :: ideal field combinations which diagonalize the s-dependent Mass Matrix**

M. Benayoun et al. hep-ph 0711.4482→EPJC

Isospin Breaking & Loops Effects III

$$\begin{array}{cc} & \begin{array}{ccc} \rho_I & \omega_I & \phi_I \end{array} \\ M^2(s)= & \begin{pmatrix} m^2+\Pi_{\pi\pi}(s)+\varepsilon_2(s) & \boxed{\varepsilon_1(s)} & \boxed{-\mu\varepsilon_1(s)} \\ \boxed{\varepsilon_1(s)} & m^2+\varepsilon_2(s) & \boxed{-\mu\varepsilon_2(s)} \\ \boxed{-\mu\varepsilon_1(s)} & \boxed{-\mu\varepsilon_2(s)} & z_V m^2+\mu^2\varepsilon_2(s) \end{pmatrix} \end{array}$$

Color Code==

Loop Effects & **Isospin Brk**

Loop Effects

$\varepsilon_1(s)$ and $\varepsilon_2(s)$ real below φ mass

→ **s-dependent mixing angles**

The Mass Matrix Eigen System

- **Expect :** $\boxed{\left(m^2, \Pi_{\pi\pi}(s)\right) \gg \varepsilon_2(s) \gg \varepsilon_1(s)}$
- Then solve for the eigensystem **perturbatively** :

$$M_0^2(s) = \begin{pmatrix} m^2 + \Pi_{\pi\pi}(s) & 0 & 0 \\ 0 & m^2 + \varepsilon_2(s) & 0 \\ 0 & 0 & z_V m^2 + \mu^2 \varepsilon_2(s) \end{pmatrix}$$

and :

$$\delta M^2(s) = \begin{pmatrix} \varepsilon_2(s) & \varepsilon_1(s) & -\mu\varepsilon_1(s) \\ \varepsilon_1(s) & 0 & -\mu\varepsilon_2(s) \\ -\mu\varepsilon_1(s) & -\mu\varepsilon_2(s) & 0 \end{pmatrix}$$

Attn : s=0 delicate !

Physical Fields versus Ideal Fields

$$\begin{pmatrix} \rho^0 \\ \omega \\ \varphi \end{pmatrix} = R(s) \begin{pmatrix} \rho_I^0 \\ \omega_I \\ \varphi_I \end{pmatrix}$$

$R(s)$: Real analytic matrix function

fulfills Unitarity Condition

$$R(s+i\varepsilon)\tilde{R}(s+i\varepsilon)=1$$

$$R(s)=\begin{pmatrix} 1 & \dfrac{\varepsilon_1}{\Pi_{\pi\pi}-\varepsilon_2} & \dfrac{-\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2} \\ \dfrac{-\varepsilon_1}{\Pi_{\pi\pi}-\varepsilon_2} & 1 & \dfrac{-\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2} \\ \dfrac{\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2} & \dfrac{\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2} & 1 \end{pmatrix}$$

$$+O(\varepsilon_i^2)$$

Check Numerical Solution

- **Correction terms should be small wrt 1 in the physics region of interest**

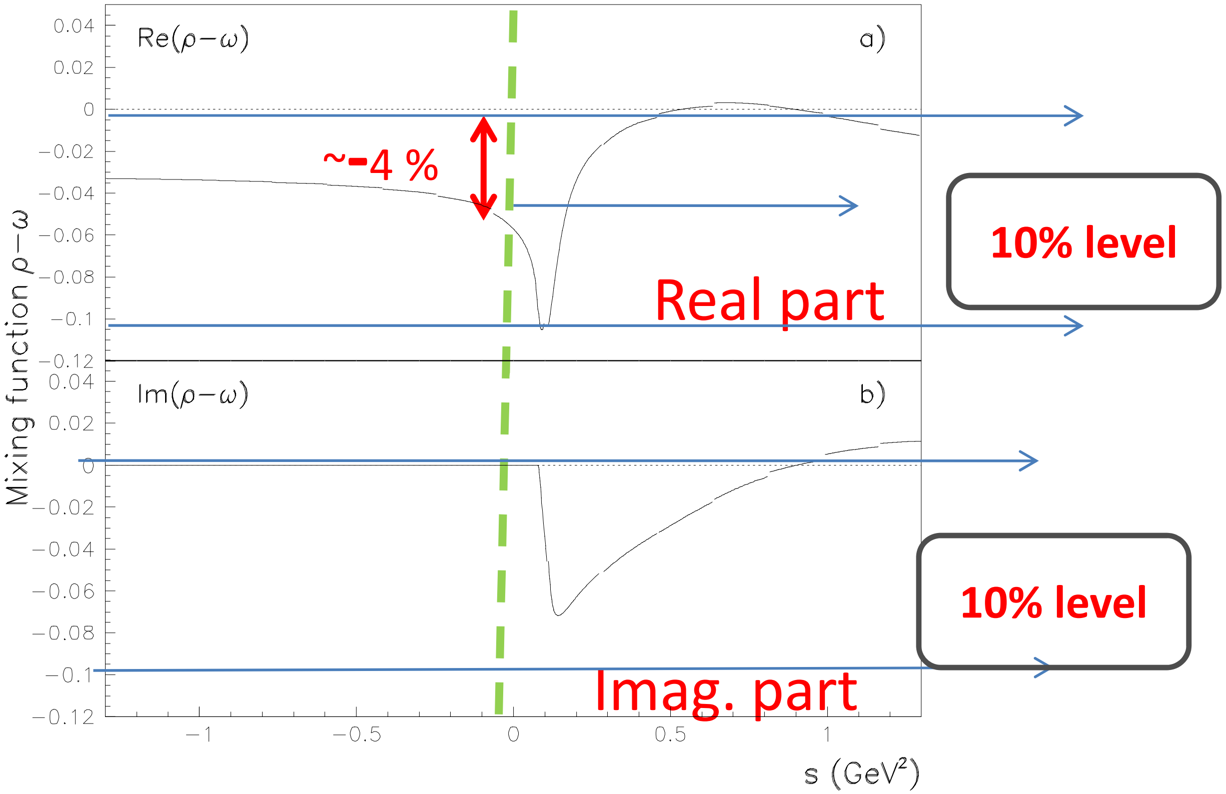
- **Can only been checked with final solution**

M. Benayoun et al. hep-ph 0711.4482→EPJC

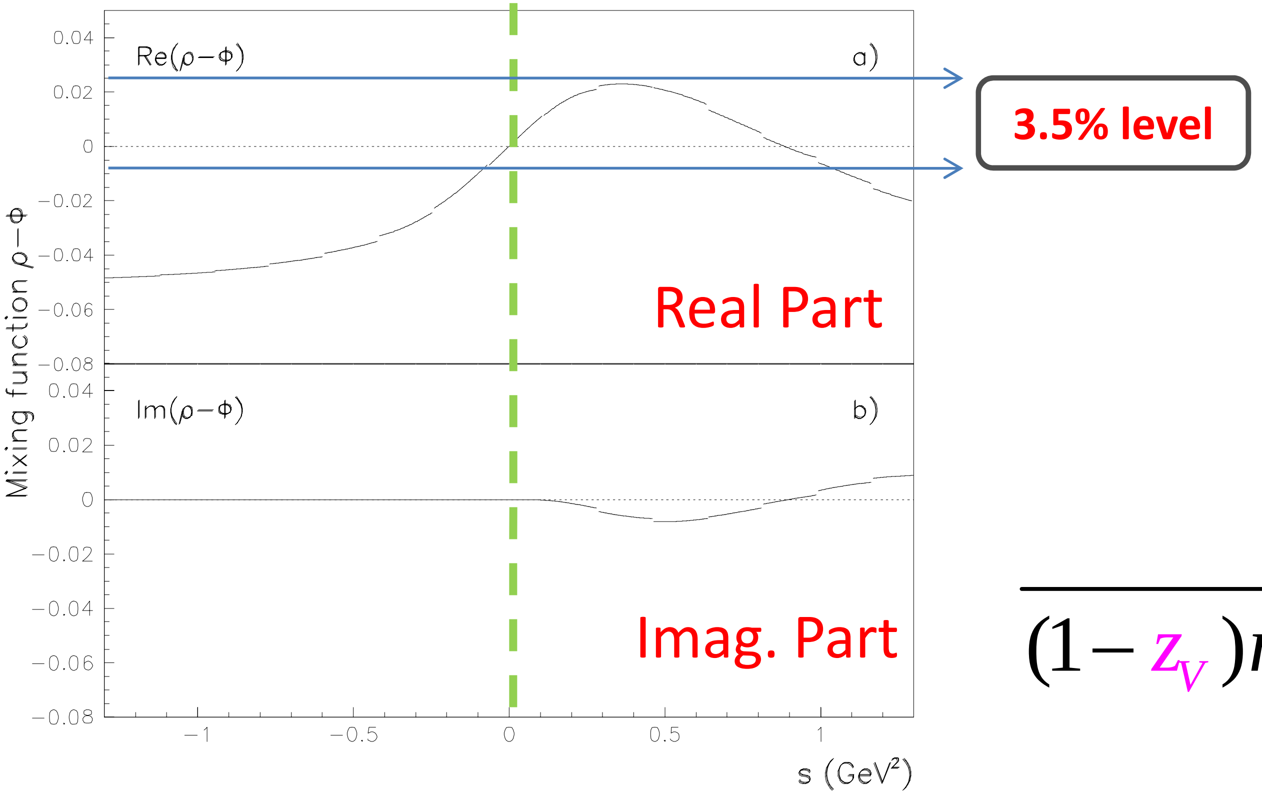
- **Check numerically the mixing « angles »**

The (ρ, ω) Mixing «Angle»

$$\frac{\varepsilon_1(s)}{\Pi_{\pi\pi}(s)-\varepsilon_2(s)}$$

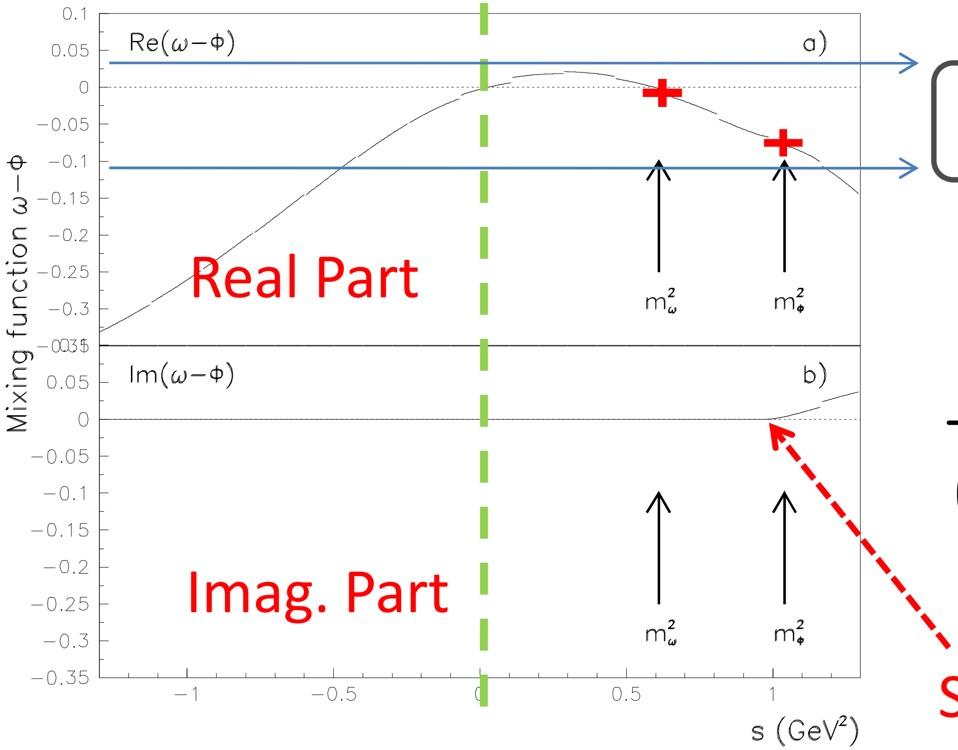


The (ρ, ϕ) Mixing «Angle»



$$\frac{-\mu\varepsilon_1}{(1-z_V)m^2+\Pi_{\pi\pi}-\mu^2\varepsilon_2}$$

The (ω, ϕ) mixing «angle»



15% level

$$\frac{-\mu\varepsilon_2}{(1-z_V)m^2+(1-\mu^2)\varepsilon_2}$$

Start non-zero imaginary part

The Main Guess

- Main Guess ≈ Proof of Principle

18 Decay Modes + Pion FF in e^+e^- annihilation

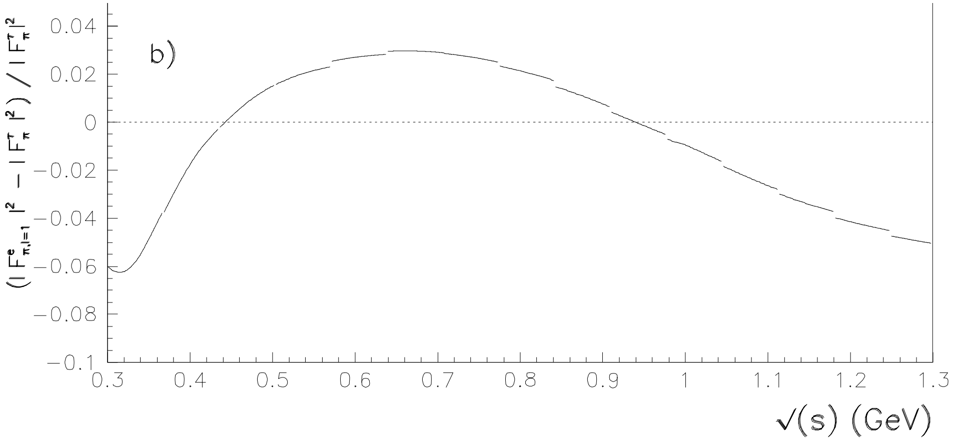
$++\delta m^2$

-

Fully reconstruct

The Pion FF in τ decay

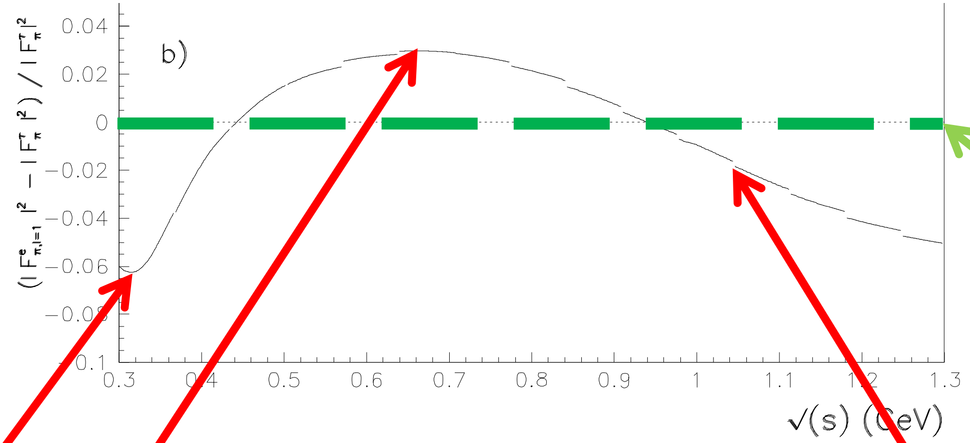
Isospin Symmetry Breaking : ρ^0 VS ρ$^\pm$



$$\frac{\left(| F_{\pi}^{e}{}_{"I=1"}(s) |^2 - | F_{\pi}^{\tau}(s) |^2 \right)}{| F_{\pi}^{\tau}(s) |^2}$$

$$I = 1 \equiv \rho^0 \ part$$

Isospin Symmetry Breaking : ρ^0 VS $\rho^\pm$



0 :: NO IS Brk

Threshold : -6%

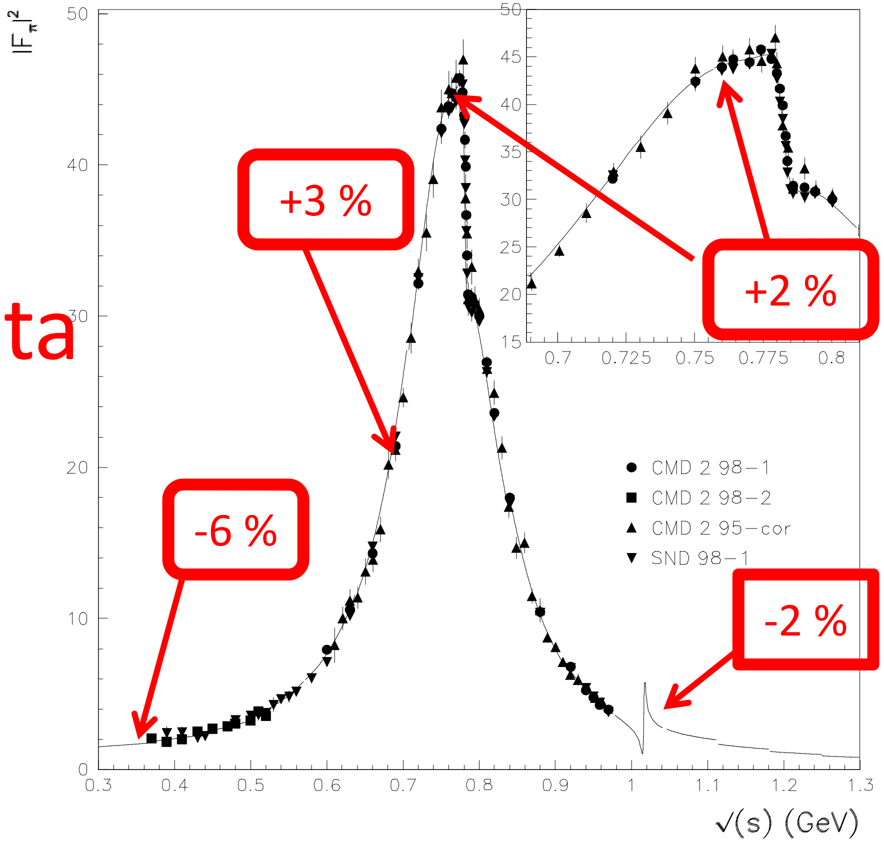
$$\frac{\left(|F_\pi^{e}{}_{"I=1"}(s)|^2 - |F_\pi^\tau(s)|^2\right)}{|F_\pi^\tau(s)|^2}$$

ρ Peak : + 3%

Φ Mass : -2%

Magnitude of Isospin Breaking

FF’s New Data



+3 %

+2 %

-6 %

-2 %

Conclusions

- **The HLS Model suitably broken defines a quite wide framework for low energy physics**
- **It reconciles e^+e^- and τ data along with a good account of VPγ & Pγγ & Ve+e-& V π+π- decays**
- Radiative decays & pion form factor in e^+e^- :

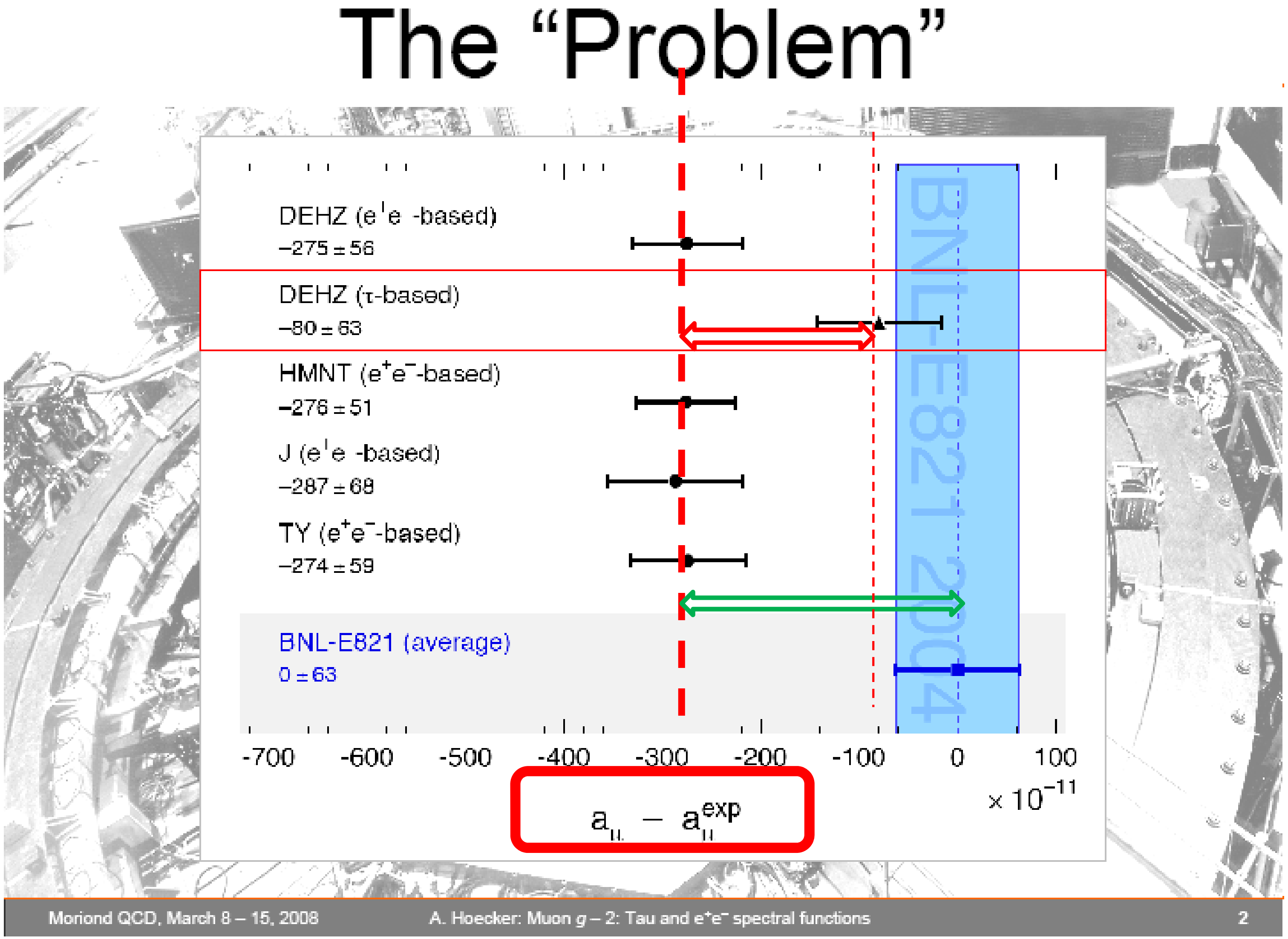
⇨ **predict τ decay data**

predict dipion spectrum in η/η’→π+π-γ

- **Newly identified effect :**

  I=0 (ω_I, ϕ_I) component inside the ρ^0 meson.

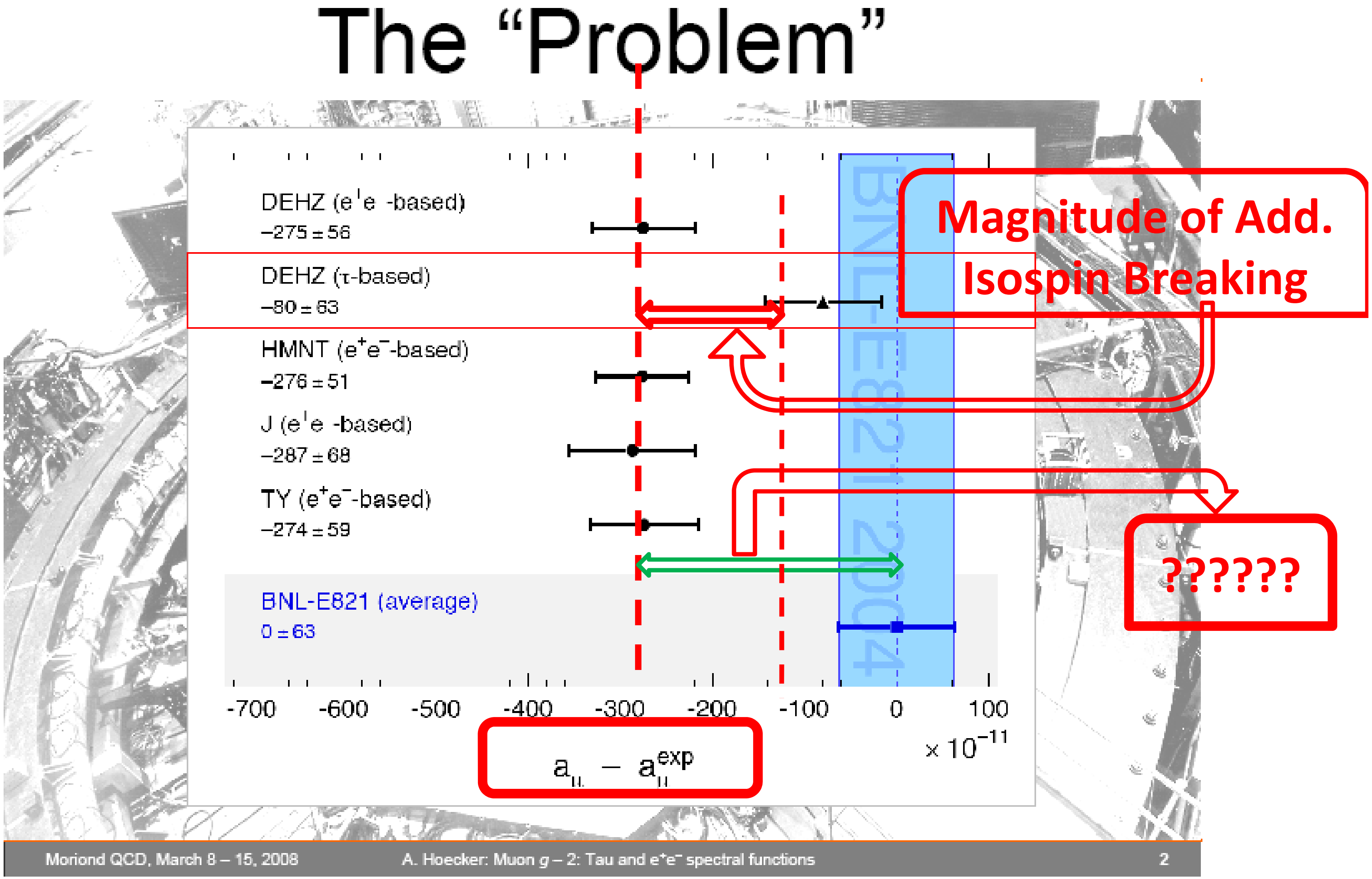
A. Hoecker Moriond QCD 2008



The “Problem”

Moriond QCD, March 8 – 15, 2008 A. Hoecker: Muon $g-2$: Tau and e^+e^- spectral functions 2

A. Hoecker Moriond QCD 2008



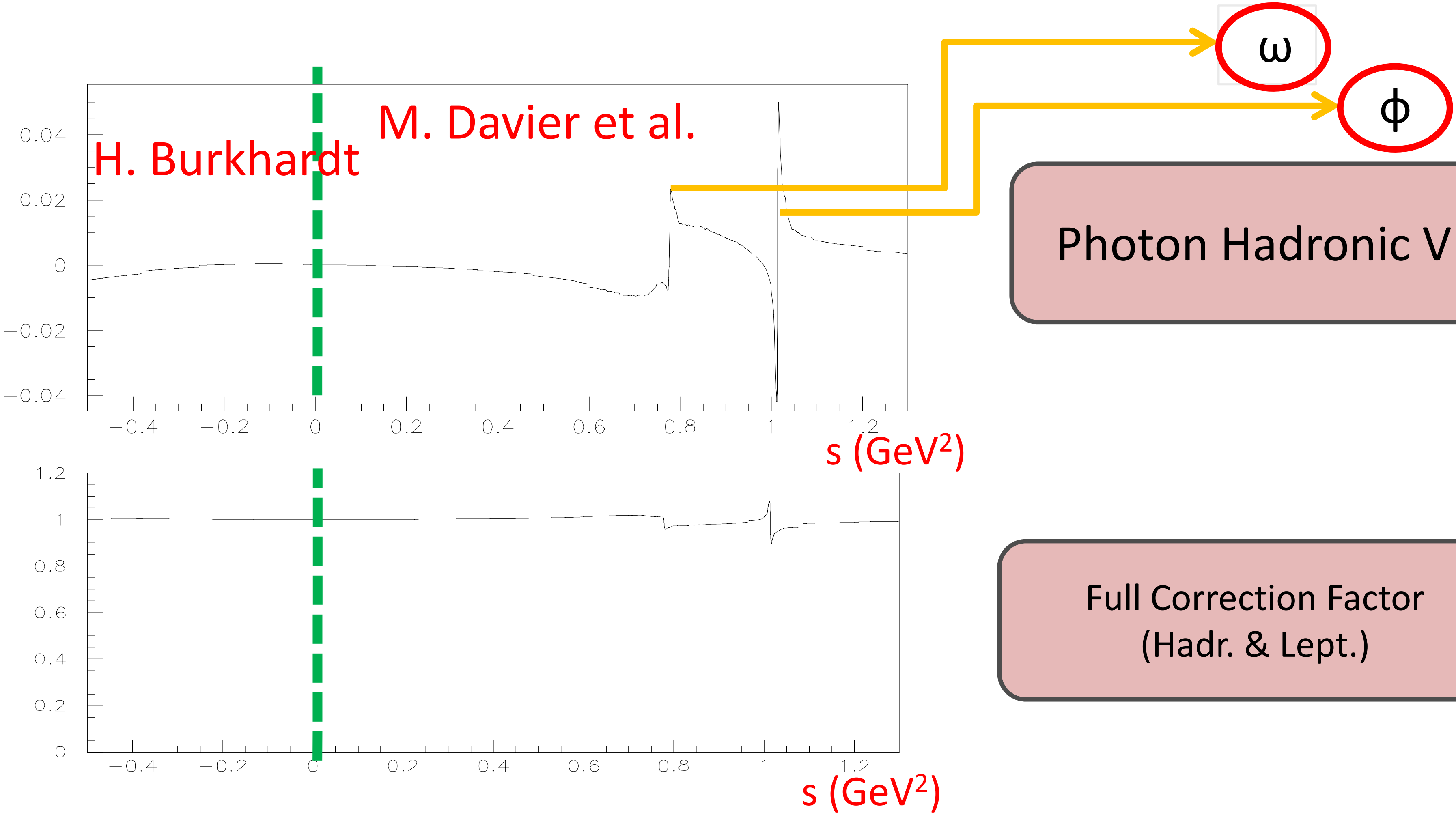
The "Problem"

Magnitude of Add. Isospin Breaking

??????

Moriond QCD, March 8 – 15, 2008 A. Hoecker: Muon g – 2: Tau and e+e− spectral functions 2

Hadronic Photon Vacuum Polarization



Photon Hadronic VP

Full Correction Factor
(Hadr. & Lept.)

The Pion Form Factor