

Study of the ISR-FSR interference in radiative return measurements in e^+e^- -annihilation.

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ISR (initial- state radiation)-FSR (final- state radiation) interference

The FSR events

-represent unavoidable background to RRM by large-angle radiative photon.

-in process $e^+(p_2) + e^-(p_1) \rightarrow \gamma(k) + \pi^+(p_+) + \pi^-(p_-)$ (1)

give a non- negligible value in hadronic contribution to the muon anomalous magnetic moment,

- carries very useful information about electromagnetic interaction of hadrons and their structure.

That is why ISR-FSR interference effects at last estimated.

We try to investigate just effects due to ISR-FSR interference in double photon events in

$$e^+(p_2) + e^-(p_1) \rightarrow \gamma(k_1) + \gamma(k_2) + \pi^+(p_+) + \pi^-(p_-). \quad (2)$$

The RRM supposes that we have at least one hard photon radiated from initial state.

We must consider events where there are

- either two ISR photon or
- one is ISR and another is FSR.

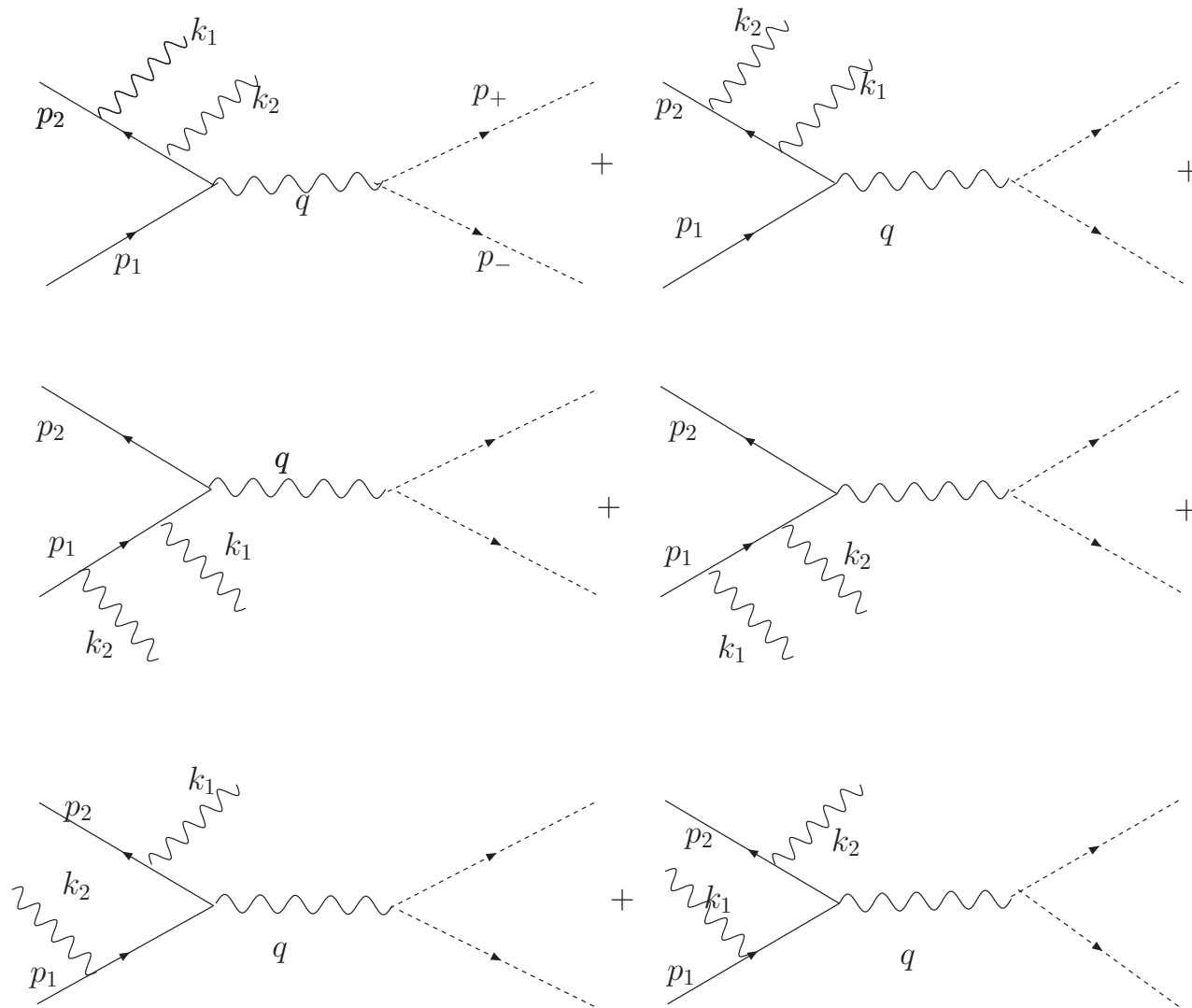


Fig.1 a)

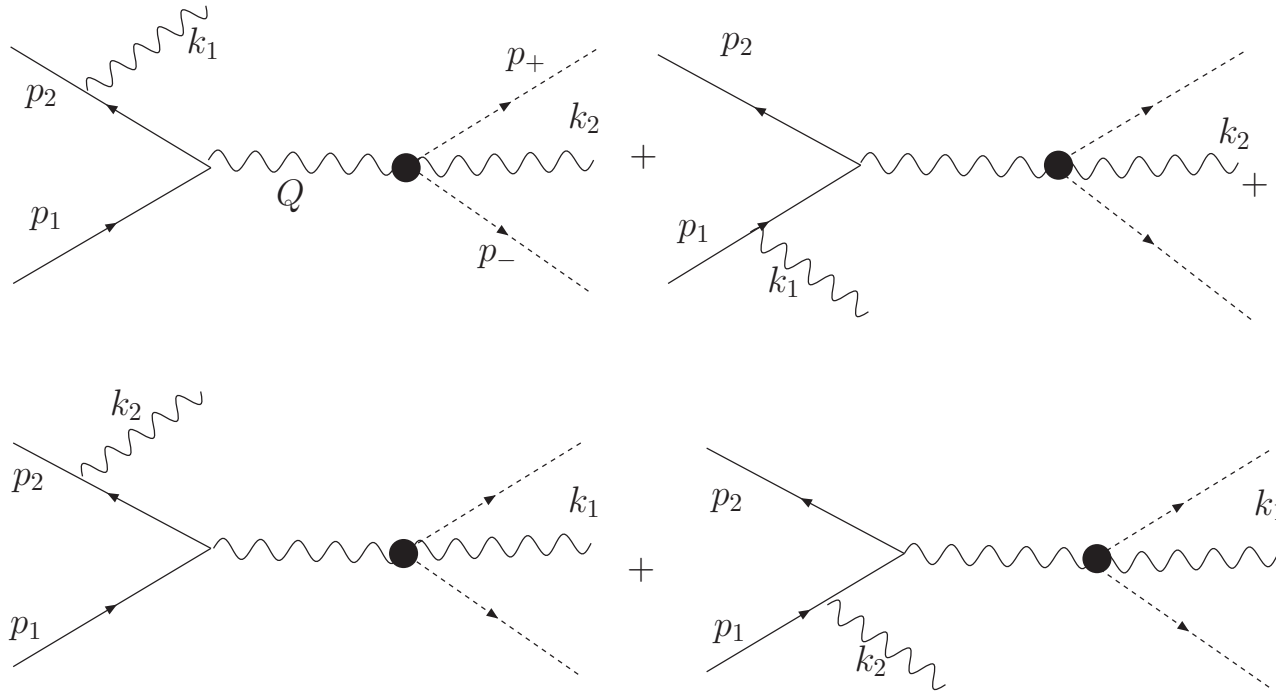


Fig.1 b)

Leptonic blocks can be written by QED rules and are universal.

The hadronic part is model-dependent and changes for different hadronic states and chosen models.

There are two types of interferences effect:

1. the first one is caused by interference of diagrams 1a) and 1b);
2. the second one arises due interference of the two diagrams 1b) with interchanged photon momentum k_1 and k_2 .

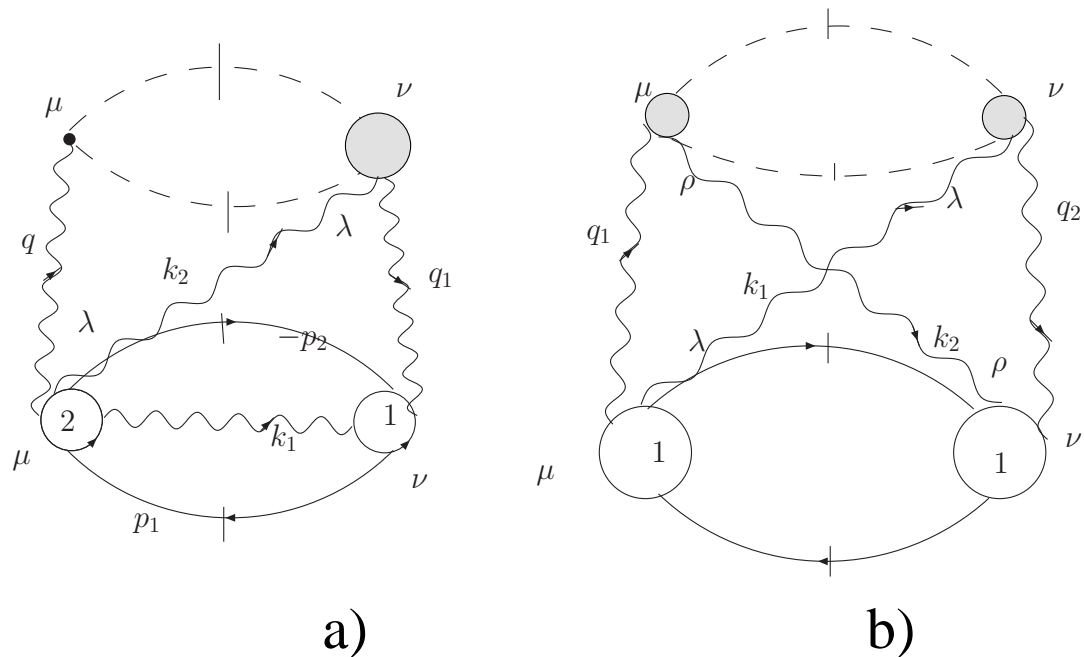


Fig.2.

$$1. \quad M_a = \frac{1}{q^2} \frac{1}{q_1^2} L_{\mu\lambda;\nu} H_{\mu;\lambda\nu} + (1 \leftrightarrow 2), \quad (3)$$

where

$$q = p_1 + p_2 - k_1 - k_2, \quad q_1^2 = (p_1 + p_2 - k_1)^2, \quad q_2^2 = (p_1 + p_2 - k_2)^2.$$

Leptonic tensor $L_{\mu\lambda;\nu}$ depends on four 4-momenta: p_1, p_2, k_1, k_2 and on six invariants.. It satisfies conditions

$$q_\mu L_{\mu\lambda;\nu} = q_{1\nu} L_{\mu\lambda;\nu} = k_{2\lambda} L_{\mu\lambda;\nu} = 0 \quad (4)$$

Hadronic tensor $H_{\mu;\lambda\nu}$ ^[1] depends on three momenta p_+, p_-, k_2 and on three invariants.

$$H_{\mu;\lambda\nu} = K_{\lambda\nu}(p_+, p_-, k_2) J_\mu(p_+, p_-) \quad (5)$$

Tensor $L_{\mu\lambda;\nu}$ is expressed in terms of 36 independent Lorentz structures constructed from 4-vectors

1.S.Dubinsky at al. Eur.Phys.J.C40,41-54(2005).

$$\tilde{p}_{1\mu} = p_{1\mu} - \frac{p_1 q}{q^2} q_\mu; \quad \tilde{p}_{2\mu} = p_{2\mu} - \frac{p_2 q}{q^2} q_\mu; \quad \tilde{k}_{1\mu} = k_{1\mu} - \frac{k_1 q}{q^2} q_\mu; \quad \tilde{k}_{2\mu} = k_{2\mu} - \frac{k_2 q}{q^2} q_\mu; \quad (6a)$$

$$\tilde{p}_{1\nu} = p_{1\nu} - \frac{p_1 q_1}{q_1^2} q_{1\nu}; \quad \tilde{p}_{2\nu} = p_{2\nu} - \frac{p_2 q_2}{q_2^2} q_{2\nu}; \quad \tilde{k}_{1\nu} = k_{1\nu} - \frac{k_1 q_1}{q_1^2} q_{1\nu}; \quad \tilde{k}_{2\nu} = k_{2\nu} - \frac{k_2 q_1}{q_1^2} q_{1\nu}; \quad (6b)$$

$$- p_{1\lambda} + \frac{k_2 p_1}{k_2 p_2} p_{2\lambda}; \quad - k_{1\lambda} + \frac{k_1 k_2}{k_2 p_1} p_{1\lambda}; \quad k_{2\lambda}, \quad (7)$$

and the second rank tensors

$$\tilde{g}_{\lambda\mu} = g_{\lambda\mu} - \frac{k_{2\mu} q_\lambda}{k_2 q}; \quad \tilde{g}_{\lambda\nu} = g_{\lambda\nu} - \frac{k_{2\nu} q_{1\lambda}}{k_2 q_1}; \quad \tilde{g}_{\nu\mu} = g_{\nu\mu} - \frac{q_{1\mu} q_\nu}{q_1 q}; \quad (8)$$

One test for obtained result. The sets (6a), (6b), (7), (8) satisfied (4).

The second test is the calculation of collinear limit

$$\vec{k}_1 \parallel \vec{p}_1, \quad \omega_1 = (1-x)E, \quad t_1 = -2k_1 p_1$$

Contributions terms t_1^{-1} , $m^2 t_1^{-2}$ proportional $-\frac{1}{x(1-x)} \left(\frac{1+x^2}{t_1} - \frac{2xm^2}{t_1^2} \right)$

$$\int L_{\mu\lambda;\nu}(p_1, p_2, k_2, k_1, \vec{k}_1 \parallel \vec{p}_1) \frac{d^3 k_1}{(2\pi)^3 2\omega_1} = \frac{\alpha}{2\pi} \left(\frac{1+x^2}{1-x} L_0 - \frac{2x}{1-x} \right) L_{\mu\lambda;\nu}(xp_1, p_2, k_2), \quad L_0 = \ln \frac{E^2 \theta_0^2}{m^2}.$$

Six structures in the contraction of leptonic and hadronic tensors

$$F_\pi(q^2) f_1^*(p_+, p_-, k_2), \quad F_\pi(q^2) f_2^*(p_+, p_-, k_2), \quad F_\pi(q^2) f_3^*(p_+, p_-, k_2), \\ F_\pi(q^2) f_1^*(p_+, p_-, k_1), \quad F_\pi(q^2) f_2^*(p_+, p_-, k_1), \quad F_\pi(q^2) f_3^*(p_+, p_-, k_1)$$

Coefficient at $F_\pi(q^2) f_1^*(p_+, p_-, k_2)$ for illustration

$$4 \frac{(p2pm - p2pp)}{t1 t2}$$

$$\left(\frac{1}{v1 v2} (q1^2 (v1^3 + 3 (2 s + t1 + t2) v1 v2 + v2^3 + s (t2 v1 + t1 v2) + (t1 - t2) (v1^2 - v2^2))) + \right. \\ \left. s (2 k2q (2 k2q + q1^2)) (v1 + v2) + \right. \\ \left. 2 k2q (q1^2 ((v1 - v2)^2) + (t1^2 + t2^2) (v1 + v2) + s (v1^2 + v2^2)) \right) + \\ -q1^2 \left(\frac{(t1 ((s + t2) v1 + s t1 + v2^2) - v1^2 (t2 + v1) + q1^2 (t1 + v1) v2)}{v1 \Delta m^2} + \right. \\ \left. \frac{(t2 (v1^2 + t1 v2 + s (t2 + v2)) + q1^2 v1 (t2 + v2) + -v2^2 (t1 + v2))}{v2 \Sigma m^2} \right) \Bigg) +$$

k21

$$\left(\frac{4}{t1 t2 v1 v2} (4 k2q^2 s v1 + s^3 (v1 - v2) + (t1^2 + t2^2) (t2 v1 - t1 v2) + (t1 - t2) t2 (v1^2 - v2^2) + \right. \\ \left. (t1 + t2) (2 t1 + t2) v1 v2 + v1^2 (t2 v1 - t1 v2) + t2 v2^2 (v1 + v2) + \right. \\ \left. s q1^2 (4 t2 v1 + 3 (-t1 + v1) v2) + \right. \\ \left. s (2 t1 t2 (-v1 + v2) + v1 (v1^2 - v1 v2 + v2^2) + -t2 (v1^2 - 3 v1 v2 - v2^2)) + \right. \\ \left. 2 k2q (-s t2 v1 + (2 s + t2) v1^2 + (s + t1)^2 v2 + t2 (s + t2) v2 + -q1^2 v1 v2 + t2 v2^2) \right) + \\ - \frac{4 q1^2}{t1 t2} \left(\frac{(s + t2 + v2) (t1 q2^2 + q1^2 v1 - 2 t1 v1)}{v1 \Delta m^2} - \right. \\ \left. \frac{((s + t1)^2 v2 + s t2 (s + t1 + v1) + -t2 (s + v1) (t2 + v2) - t1 t2 v2 + v2^2 (t1 + v2))}{v2 \Sigma m^2} \right) \Bigg) +$$

k11

$$\left(\frac{1}{t1 t2 v1 v2} 4 \right. \\ \left. (4 k2q^2 s v1 - 2 k2q (q1^2 (s (v1 - 2 v2) + v1 v2) + - (s + t2) (v1^2 + v2^2) + - (t1^2 + t2^2) v2) + \right. \\ \left. q1^2 (s t2 v1 + -v1^2 (s + t2 - v2) + 3 (s + t1) v1 v2 + v2^2 (s + t2 - v1 + v2))) \right) + \\ - \frac{4 q1^2}{t1 t2} \left(\frac{(s + t2 + v2) (-v1^2 + t1 v2)}{v1 \Delta m^2} + \frac{((s + v1 + v2 - 2 v2) (t2^2 + v2^2) + q1^2 (t2 + v1) v2)}{v2 \Sigma m^2} \right) \Bigg) + \\ \left(\frac{8 (p2pm - p2pp)}{t1 t2} \right.$$

$$\left(- \left(\frac{1}{t1 t2 v1 v2} \right) \right. \\ \left. (+2 k2q (2 t1 t2 (t1 v1 + t2 v2) + q1^2 (t2 (-3 t1 + t2) v1 + t1 (t1 - 3 t2) v2) - \right. \\ \left. 2 (t1 + t2)^2 v1 v2) + -t1 t2 (t1^2 + t2^2) (v1 + v2) + \right. \\ \left. (t1 + t2) (3 t1^2 - 4 t1 t2 + 3 t2^2) v1 v2 + \right. \\ \left. s (t1 t2 (t1 + t2) (v1 + v2) + -2 t1 t2 (v1 + v2)^2 + 3 (t1 + t2)^2 v1 v2) \right) + \\ \frac{1}{t1 v1 \Delta m^2} (t1^2 (s t1 - t1^2 + s t2 - t2^2) + t1^2 (3 t1 + t2) (-v1 + v2) + \\ q1^2 (t1 t2 (v1 + v2) + v1 (-2 t1 v1 + t2 v2)) + t1 (-s + t1 + 3 t2) v1 v2 + \\ t1^2 (- (v1 + v2) (s + v1 + v2) - (v1 - v2)^2)) + \frac{1}{t2 v2 \Sigma m^2} (t2^2 ((s - t1) t1 + (s - t2) t2) +$$

$$\begin{aligned}
& \left((t_1 - t_2) t_2 (v_1 + v_2) + (t_1 - t_2) v_1 v_2 - 2 t_2 v_2^2 \right) q_1^2 + \\
& \quad 2 t_2 \left(t_2 (t_1 + t_2) v_1 + t_2^2 (v_1 - v_2) + (2 t_1 + t_2) v_1 v_2 - t_2 (v_1^2 + v_2^2) \right) \Big) + \\
& k_{11} \\
& \left(- \frac{4}{t_1 t_2^2 v_1 v_2} \right. \\
& \quad \left(-t_2 \left(t_1 t_2 (-v_1 + v_2) + 2 t_2^2 (v_1 + v_2) + (t_1 - 2 t_2) v_1^2 + (t_1 + t_2) v_1 v_2 + \right. \right. \\
& \quad \left. \left. (2 t_1 - t_2) v_2^2 + v_1 (v_1 - v_2) v_2 - 2 s \left(2 t_1 v_1 + t_1 v_2 - t_2 v_2 + v_1 v_2 - 2 v_2^2 \right) \right) + \right. \\
& \quad 6 t_1 (s + t_1) v_1 v_2 + 4 k_2 q^2 t_2 (v_1 - v_2) + \\
& \quad 2 k_2 q \left(2 q_1^2 (-2 t_2 v_1 + t_1 v_2) + (t_1 - 2 t_2) t_2 v_1 + t_2 (-2 s + t_2) v_2 + \right. \\
& \quad \left. -4 (t_1 + t_2) v_1 v_2 \right) \Big) + \\
& \quad \left. \frac{4 (t_1 - v_2) \left(2 t_1 (s + v_2) - 2 t_2 q_1^2 + 4 (s + t_1 + t_2) v_1 + t_1 (t_2 - v_1) + (t_2 + v_1) v_2 \right)}{t_1 t_2 v_1 \Delta m^2} + \right. \\
& \quad \frac{1}{t_1 t_2^2 v_2 \Sigma m^2} 4 \left((t_1 - 2 t_2) t_2^3 + 2 t_1 t_2 (t_2 v_1 + t_1 v_2) + \right. \\
& \quad t_2^2 (4 t_2 v_1 - 4 t_1 v_2 - 9 t_2 v_2) + t_2 (-2 t_2 v_1^2 + t_1 (v_1^2 - 4 v_2^2)) + \\
& \quad 2 \left((t_1 + t_2)^2 + t_2^2 \right) v_1 v_2 + t_2 v_1^2 v_2 + 8 t_2 v_2 (t_1 v_1 - t_2 v_2) + \\
& \quad \left. \left. 2 s \left(t_2 (-3 t_2 + v_1 - 2 v_2) v_2 + t_1 \left(2 t_2^2 + t_2 v_2 + v_1 v_2 \right) \right) \right) \Big) + \right. \\
& k_{21} \\
& \left(\frac{1}{t_1^2 t_2^2 v_1 v_2} 4 \left(t_2 \left((t_1 - 2 t_2) (t_1 + t_2)^2 - t_1 t_2 (4 t_1 + t_2) \right) v_1 + \right. \right. \\
& \quad -2 t_1 t_2 (t_1 + t_2) v_1^2 + t_1 \left((t_1 + t_2)^2 (2 t_1 + t_2) + t_1^2 t_2 \right) v_2 - 4 t_1^3 v_1 v_2 + \\
& \quad 2 (t_1 - t_2) t_2^2 v_1 v_2 + 2 t_1 t_2 (t_1 + t_2) v_2^2 + 5 t_1 t_2 v_1 (v_1 - v_2) v_2 + \\
& \quad 4 v_1 v_2 (t_1^2 v_1 - t_2^2 v_2) + \\
& \quad 2 s \left(2 (t_1 + t_2)^2 (-t_2 v_1 + t_1 v_2) + -t_1 t_2^2 (v_1 + v_2) + t_1 t_2 (v_1 + v_2)^2 - \right. \\
& \quad \left. t_1 (t_1 + t_2) v_1 v_2 + - (t_1 + t_2)^2 v_1 v_2 \right) + 2 s^2 (t_1 + t_2) (-t_2 v_1 + t_1 v_2) + \\
& \quad 2 k_2 q t_1 \left(t_2 (6 s + 7 t_2) v_1 - 2 t_1 (s + t_1 + t_2) v_2 + t_2 (-t_1 + 2 t_2) v_2 + \right. \\
& \quad \left. 4 (t_1 + t_2) v_1 v_2 + t_2 (-v_1^2 + v_2^2) \right) + 4 k_2 q^2 t_1 t_2 (-v_1 + v_2) \Big) + \\
& \quad - \frac{1}{t_1 t_2^2 v_2 \Sigma m^2} 4 \left(2 s^2 (t_1 + t_2) (t_2 + v_2) + t_2^2 (3 t_1^2 + 2 t_1 t_2 + 2 t_2^2) + \right. \\
& \quad t_2 (t_1^2 - 4 t_2^2) v_1 + 2 \left((t_1 + t_2)^3 + t_2^2 (t_1 + 2 t_2) \right) v_2 + \\
& \quad t_2 \left(2 t_1 v_2 (-3 v_1 + v_2) + t_2 (4 v_1^2 + 5 v_2^2) \right) + t_2 v_1 v_2^2 + \\
& \quad 2 s \left(t_1 t_2 (t_1 + 2 t_2) + 2 \left(t_1 (t_1 + 2 t_2) v_2 + t_2^2 (v_1 + v_2) \right) + t_2 v_2 (2 v_1 + v_2) \right) \Big) + \\
& \quad \frac{1}{t_1^2 t_2 v_1 \Delta m^2} 4 \left(2 s^2 (t_1 + t_2) (t_1 + v_1) + t_1^2 t_2 (2 t_1 + t_2) + 8 t_1 t_2 (t_1 + t_2) v_1 + \right. \\
& \quad 2 (t_2^3 v_1 + t_1^3 v_2) + t_1 t_2 (4 t_1 + 3 t_2) v_2 + \\
& \quad v_1 \left(2 t_2^2 v_2 - t_1^2 v_1 + -2 t_1 (t_1 + t_2) (v_1 - v_2) \right) + t_1 v_1^2 v_2 + \\
& \quad 2 s \left(t_1 \left((t_1 + t_2)^2 + t_1 t_2 \right) + \left(t_1^2 + 5 t_1 t_2 + 2 t_2^2 \right) v_1 + t_1 (t_1 + t_2) v_2 - \right. \\
& \quad \left. \left. t_1 v_1^2 + (t_1 + t_2) v_1 v_2 \right) \right) \Big) \Big) m[e]^2 - \\
& \frac{32 (t_1 + t_2)}{t_1^2 t_2^2 v_1 v_2 \Delta m^2 \Sigma m^2} \\
& \left((p_2 p_m - p_2 p_p) \left(t_1 (t_2 - v_1) v_1 (t_2 + v_2) \Delta m^2 + t_2 (t_1 + v_1) (t_1 - v_2) v_2 \Sigma m^2 + \right. \right. \\
& \quad \left. \left. - \Delta m^2 \Sigma m^2 \left(2 t_1 v_1 v_2 + 2 t_2 v_1 v_2 + t_2 v_1 \Delta m^2 + t_1 v_2 \Sigma m^2 \right) \right) + \right. \\
& \quad \left. 2 k_2 q \Delta m^2 \left(k_{11} t_1 (t_2 v_1 + v_2 (v_1 + \Sigma m^2)) + k_{21} v_1 (t_2 (t_1 + \Sigma m^2) + t_1 v_2) \right) \right) m[e]^4;
\end{aligned}$$

There

$$\Delta m^2 = (p_1 - k_1 - k_2)^2 - m^2, \quad \Sigma m^2 = (p_2 - k_1 - k_2)^2 - m^2,$$

$$v_{1,2} = -2k_2 p_{1,2}, t_{1,2} = -2k_1 p_{1,2}, k_{1l} = k_1 l, k_{2l} = k_2 l, l = p_+ - p_-, pp^- \rightarrow p_+, pm^- \rightarrow p_-.$$

2. The diagram 2b) corresponds

$$M_b = \frac{1}{q_1} \frac{1}{q_2} L_{\mu\lambda;\nu\rho}(p_1, p_2, k_1, k_2) K_{\mu\rho}(p_+, p_-, k_2) K_{\nu\lambda}(p_+, p_-, k_1) \quad (12)$$

We can write tensor $L_{\mu\lambda;\nu\rho}$ in terms of 4-vectors

$$\tilde{x}_\mu^{(i)} = x_\mu^{(1)} - \frac{x^{(i)} q_1}{q_1^2} q_{1\mu}, \quad \tilde{x}_\nu^{(i)} = x_\nu^{(1)} - \frac{x^{(i)} q_2}{q_2^2} q_{2\nu}, \quad x_\mu^{(i)} = p_{1\mu}, p_{2\mu}, k_{1\mu}, k_{2\mu} \quad (13)$$

and second rank tensor

$$\begin{aligned} \tilde{g}_{\rho\mu} &= g_{\rho\mu} - \frac{k_{2\mu} q_{1\rho}}{k_2 q_1}; & \tilde{g}_{\lambda\mu} &= g_{\lambda\mu} - \frac{k_{1\mu} q_{1\lambda}}{k_1 q_1}; & \tilde{g}_{\rho\nu} &= g_{\rho\nu} - \frac{k_{2\nu} q_{2\rho}}{k_2 q_2}; \\ \tilde{g}_{\lambda\nu} &= g_{\lambda\nu} - \frac{k_{1\nu} q_{2\lambda}}{k_1 q_2}; & \tilde{g}_{\mu\nu} &= g_{\mu\nu} - \frac{q_{2\mu} q_{1\nu}}{q_1 q_2}. \end{aligned} \quad (14)$$

In collinear limit diagram 2b) does not contribute.

We calculate the distribution over the pion invariant mass q^2 .

The reduced phase space can be written as

$$\frac{d^3k_1}{2\omega_1} \frac{d^3k_2}{2\omega_2} \frac{d^3p_+}{2E_+} \frac{d^3p_-}{2E_-} \delta^4(p_1 + p_2 - k_1 - k_2 - p_+ - p_-) =$$

$$\frac{\pi}{32s} dq^2 d\cos\theta d\phi ds_3 ds_4 |\vec{p}_-|^2 \frac{d\cos\theta_- d\phi_-}{|\vec{p}_-|(2E - \omega_1 - \omega_2) + E_-(\omega_1 c_{1-} + \omega_2 c_{2-})} \quad (15)$$

where θ - is angle between \vec{p}_1 and \vec{k}_1 , ϕ angle between planes (\vec{p}_1, \vec{k}_1) and (\vec{k}_1, \vec{k}_2) , θ_-, ϕ_- - polar and azimuthal angles of negative pion with 4- momentum p_- . Invariants s_3, s_4 define the energy of radiated photons

$$\omega_1 = \frac{s - s_3}{2\sqrt{s}}; \quad \omega_2 = \frac{s - s_4}{2\sqrt{s}},$$

and

$$c_{1-} = \frac{(\vec{k}_1 \vec{p}_-)}{|\vec{k}_1| |\vec{p}_-|}, \quad c_{2-} = \frac{(\vec{k}_2 \vec{p}_-)}{|\vec{k}_2| |\vec{p}_-|},$$

The energy of pion can be derived from conservation laws ^[2]

$$q^2 - 2E_-(2E - \omega_1 - \omega_2) - 2|\vec{p}_-|(\omega_1 c_{1-} + \omega_2 c_{2-}) = 0$$

The angular region in (15) covers the whole space, and the integration region over variables s_3, s_4 at fixed value of q^2 was obtained in ^[3].

$$q^2 + \varepsilon s < s_3 < (1 - \varepsilon)s \quad \frac{sq^2}{s_3} < s_4 < s + q^2 - s_3$$

$$q^2(1 + \varepsilon) < s_3 < q^2 + \varepsilon s \quad \frac{sq^2}{s_3} < s_4 < (1 - \varepsilon)s$$

2. V.A.Khose et al. Eur.Phys.J.C25,199-213(2002).

3. F.A.Berends et al. Nucl.Phys,B297(1988)429.

Summary

1. We obtained the analytical expressions for matrix element squared of interference processes (2).
2. The advantages of the used approach in this case are:
 - the applied hadronic tensor $K_{\lambda\nu}$ has general form based on Lorentz and gauge invariance.
 - The obtained results may be used
 - for verification of results obtained with Monte Carlo generators,
 - in calculation of charge-odd asymmetry.
3. For complete numerical result we must take into account the emission of soft photon and loop corrections.