Crash course on CATS

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D. Mihaylov, VMS et al. Eur.Phys.J.C 78 (2018)

Outline

1. Femtoscopy in a nutshell

2. Basics of CATS
   a. main features
   b. a deeper understanding with few examples

Useful links:

- CATS paper
- Dr. D. Mihaylov PhD thesis
- GitHub CATS repo
- (Old) GitHub tutorial

The correlation function

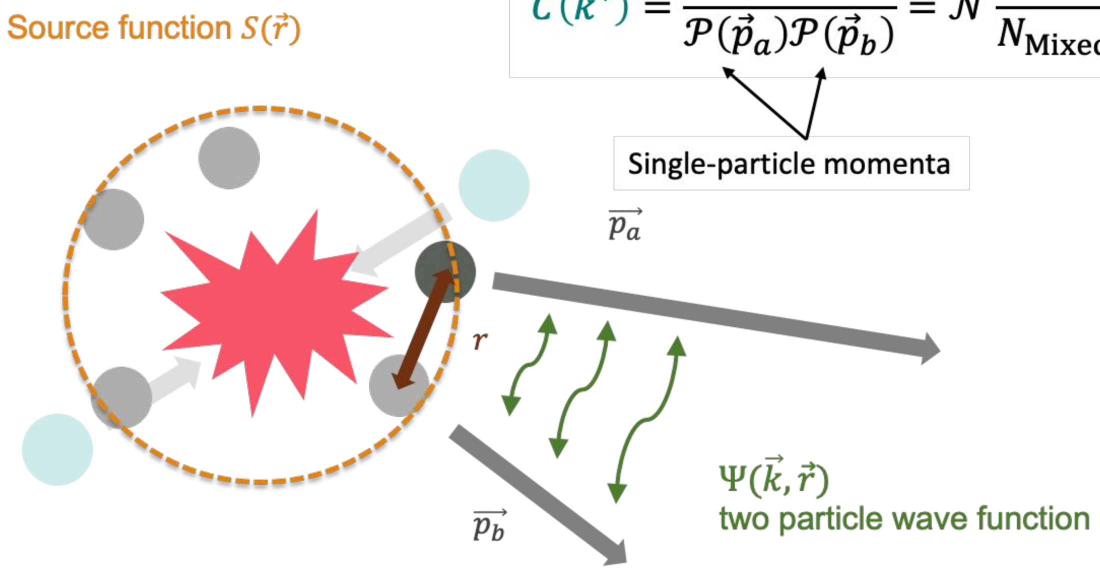
Statistical definition | **Experimental definition** | **Theoretical definition**

$$C(k^*) = \frac{\mathcal{P}(\vec{p}_a, \vec{p}_b)}{\mathcal{P}(\vec{p}_a)\mathcal{P}(\vec{p}_b)} = \mathcal{N}\,\frac{N_{\mathrm{Same}}(k^*)}{N_{\mathrm{Mixed}}(k^*)} = \int S(r)\left|\Psi(\vec{k}^*, \vec{r})\right|^2 \mathrm{d}^3 r \xrightarrow{k^* \to \infty} 1$$

Single-particle momenta

Emitting Source

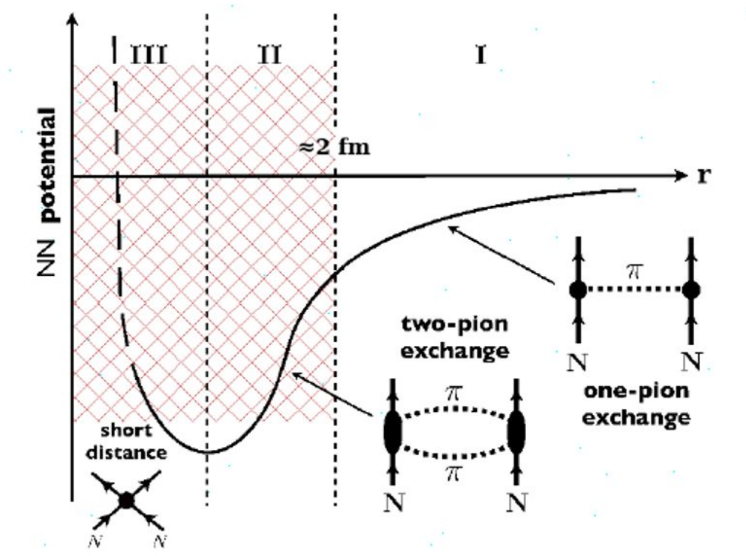
Relative Wavefunction



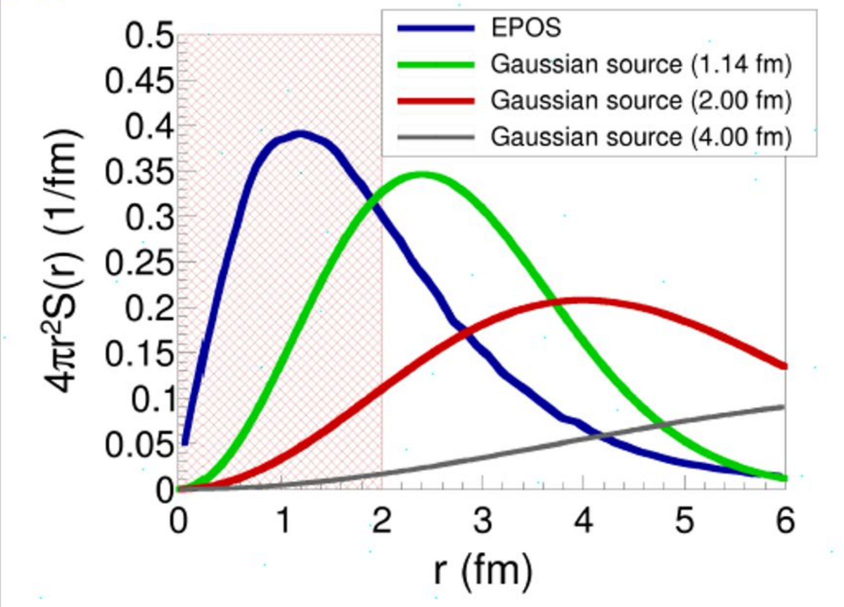
$$C(k^*) \longrightarrow \begin{array}{l} > 1 \text{ attraction} \\ = 1 \text{ no interaction} \\ < 1 \text{ repulsion} \end{array}$$

Length scales in femtoscopy

$$C(k^*) = \int S(r)\left|\Psi(\vec{k}^*,\vec{r})\right|^2 \mathrm{d}^3 r \xrightarrow{k^*\to\infty} 1$$



Based on Holt, Kaiser and Weise, Prog.Part.Nucl.Phys. 73 (2013) 35-83



A-A collisions: R_G ~ 4 fm
p-A collisions: R_G ~ 2 fm
p-p collisions: R_G ~ 1 fm

Our goal

$$C(k) = \frac{\mathcal{P}(\vec{p}_a, \vec{p}_b)}{\mathcal{P}(\vec{p}_a)\mathcal{P}(\vec{p}_b)} = \int \underbrace{S(\vec{r})}_{\text{source}} \underbrace{|\Psi(\vec{r}, k)|^2}_{\text{2-particle wave function}} \mathrm{d}\vec{r} \xrightarrow{k\to\infty} 1$$

Final output

input

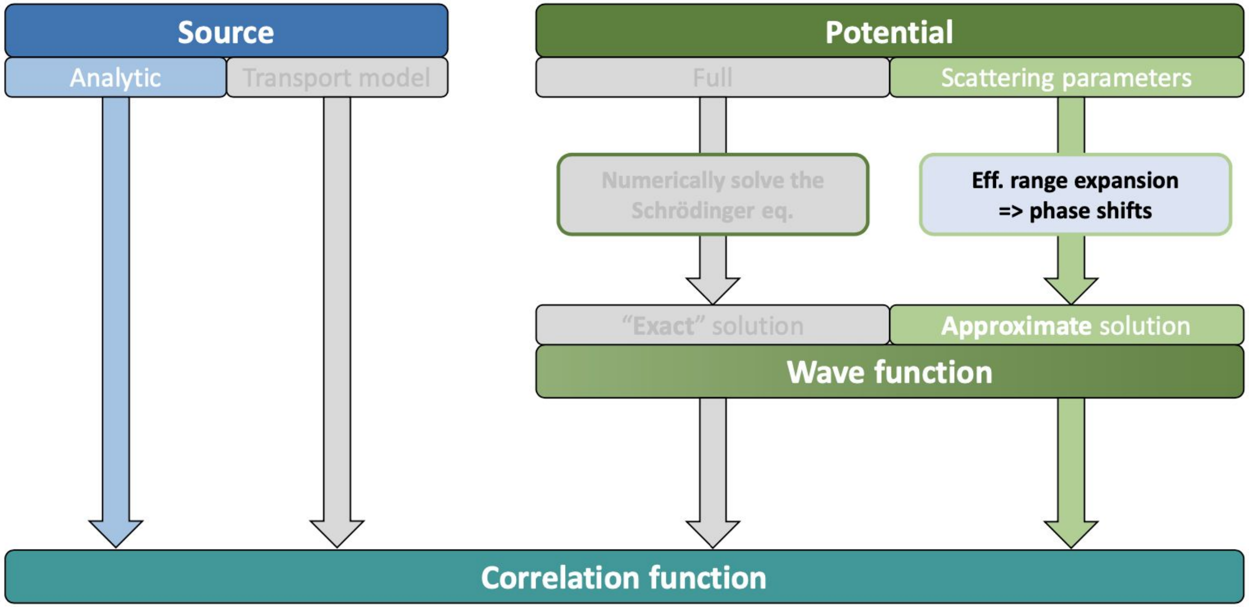
eval. for any local potential V(r)

1. **Study** the interaction potential between different particle species
   → one needs to be capable of computing the wave function

2. To get reliable results we must find ways to realistically **model** the emission source
   → going beyond the gaussian assumption if required, input from transport models

Lednicky-Lyuboshits workflow

$$C(k) = \frac{\mathcal{P}(\vec{p}_a, \vec{p}_b)}{\mathcal{P}(\vec{p}_a)\mathcal{P}(\vec{p}_b)} = \int \underbrace{S(\vec{r})}_{\text{source}} \underbrace{|\Psi(\vec{r}, k)|^2}_{\text{2-particle wave function}} \mathrm{d}\vec{r} \xrightarrow{k\to\infty} 1 \qquad (1)$$

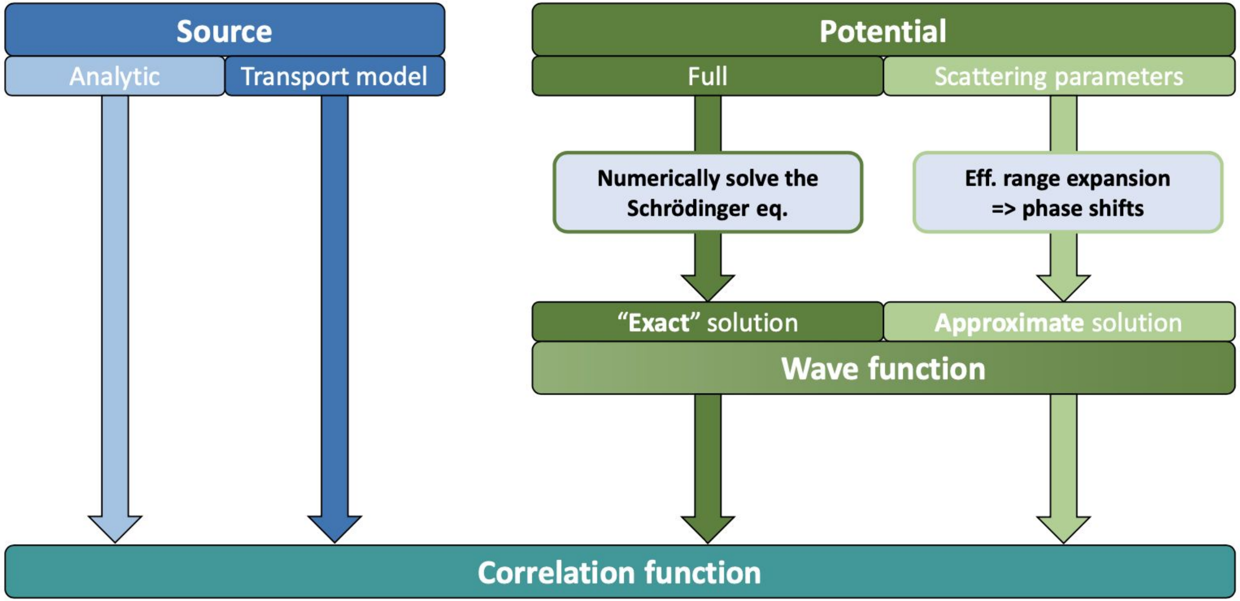


-asymptotic behaviour (far from scatt.centre)
-ERE expansion
-not suited (always) for small sources

R. Lednicky and V. L. Lyuboshits, Sov. J. Nucl. Phys. 35, 770 (1982), [Yad. Fiz.35,1316(1981)]

CATS workflow

$$C(k) = \frac{\mathcal{P}(\vec{p}_a, \vec{p}_b)}{\mathcal{P}(\vec{p}_a)\mathcal{P}(\vec{p}_b)} = \int \underbrace{S(\vec{r})}_{\text{source}} \underbrace{|\Psi(\vec{r},k)|^2}_{\text{2-particle wave function}} \mathrm{d}\vec{r} \xrightarrow{k\to\infty} 1 \qquad (1)$$



Numerically solve SE

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Solving the Schrödinger equation (I)

$$\Psi_k(\vec{r}) = R_k(k,r)Y(\theta) = \sum_{l=0}^{\infty} R_{k,l}(r)Y_l(\theta) = \sum_{l=0}^{\infty} i^l(2l+1)\frac{u_{k,l}(r)}{r}P_l(\cos\theta)$$

Separation of variables

Expansion in partial waves

Legendre polynomials

With $u_{k,l}(r) = r \cdot R_{k,l}(r)$

$$\frac{d^2 u_{k,l}}{dr^2} = \left[2mV(r) + \frac{l(l+1)}{r^2} - k^2\right]u_{k,l}$$ radial SE

D. Mihaylov, VMS et al. Eur.Phys.J.C 78 (2018)

Solving the Schrödinger equation (II)

- we need to compute the RSE for all partial waves (PW)

- low-energy (low k* pairs)
  → interaction dominated by s-wave (residual effects of p-, d- waves)
  → rest of PW follow free particle solution

$$\Psi_k(\vec{r}) = e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta),$$

spherical Bessel functions

- in PW *l* there is no interaction → free wave (see next discussion with channels)

- symmetrization/anti-symmetrization wf included in CATS

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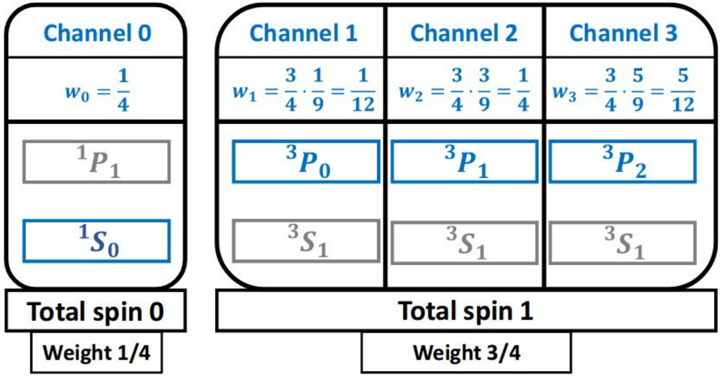
Interaction channels

- Interaction depends on spin/isospin configurations
  - NN interaction (S=0,1)
    - I=1: pp, nn, 1/√2 (pn+np)
    - I=0: 1/√2 (pn-np) → for S=1 we have the deuteron
- Each configuration contributes to C(k*) with weights determined by degeneracy.
  → p-Λ has S=0,1 configurations with weights ¼ and ¾.

Example for p-p
The shaded PWs are Pauli-blocked for identical particles
4 int. channels



$$|\Psi_k(r)|^2 = \sum_{\text{channel}} w_{\text{channel}} \sum_l |\Psi_{k,l}(r)|^2.$$

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Understanding the correlation with CATS

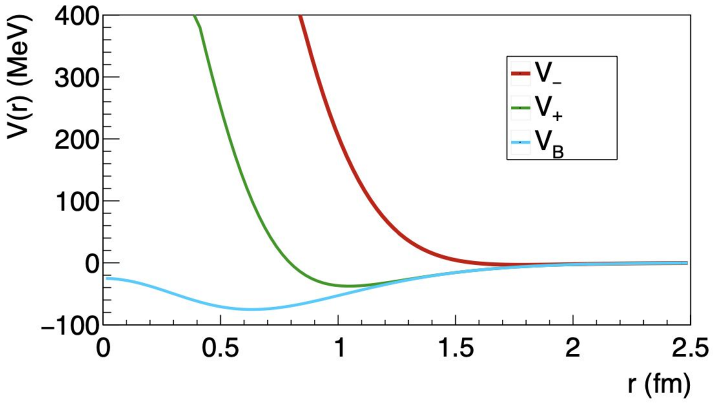
- Toy double-gaussian potential (single channel)

$$V(r) = V_1 \exp(-r^2/\mu_1^2) + V_2 \exp(-r^2/\mu_2^2).$$

- CF as average value of the $|\Psi|^2$ probed by the source acting as pdf

$$C_{\text{th}}(k) = \langle |\Psi|^2 \rangle = \int S(r) \left| \Psi(\vec{k}, \vec{r}) \right|^2 d^3r.$$

- any depletion/enhancement in wf is transferred to CF, source determines which region in *r* to probe



For more details: VMS et al.Ann.Rev.Nucl.Part.Sci. 71 (2021)

Probing different interactions with CATS

1. shifted asymptotic solution determined by δ_0
   → eff. described by scatt. pars

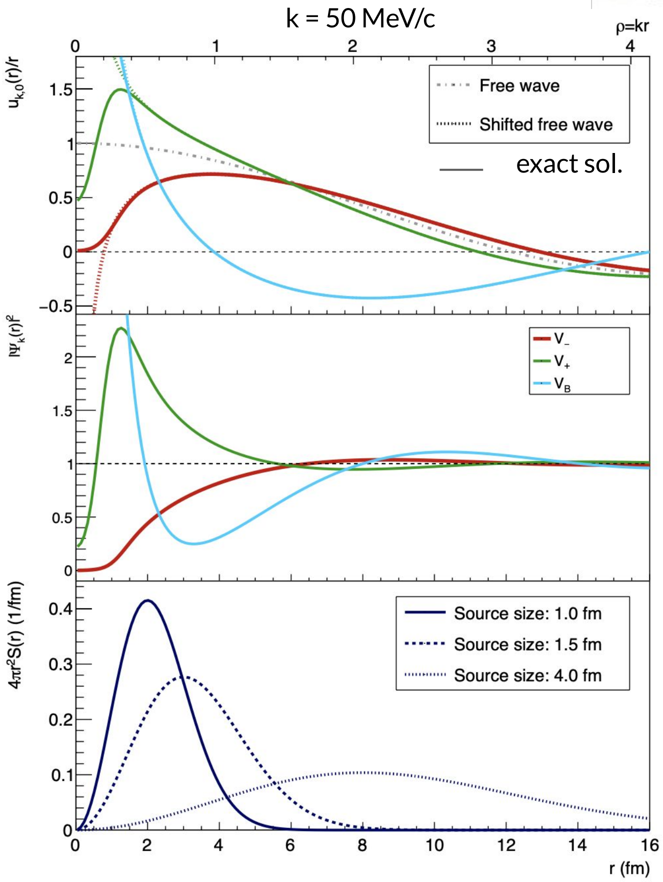
$$k\cot(\delta(k)) \overset{k\to 0}{\approx} \frac{1}{f_0} + \frac{1}{2}d_0k^2 + \mathcal{O}(k^4).$$

   → overlaps with true solution for r > 1-2 fm
   → basics of Lednický model relating CF to f_0,d_0
2. $|\Psi|^2$ behaviour tells us about the interaction
   → pull/push wf of attr./rep
   → interplay for bound-state + localized at low r

for BS

$$|f_0| \geq 2d_0.$$

Potential	f_0 (fm)	d_0 (fm)
V_-	-0.8	0.6
V_+	1.1	5.0
V_B	-4.7	1.3



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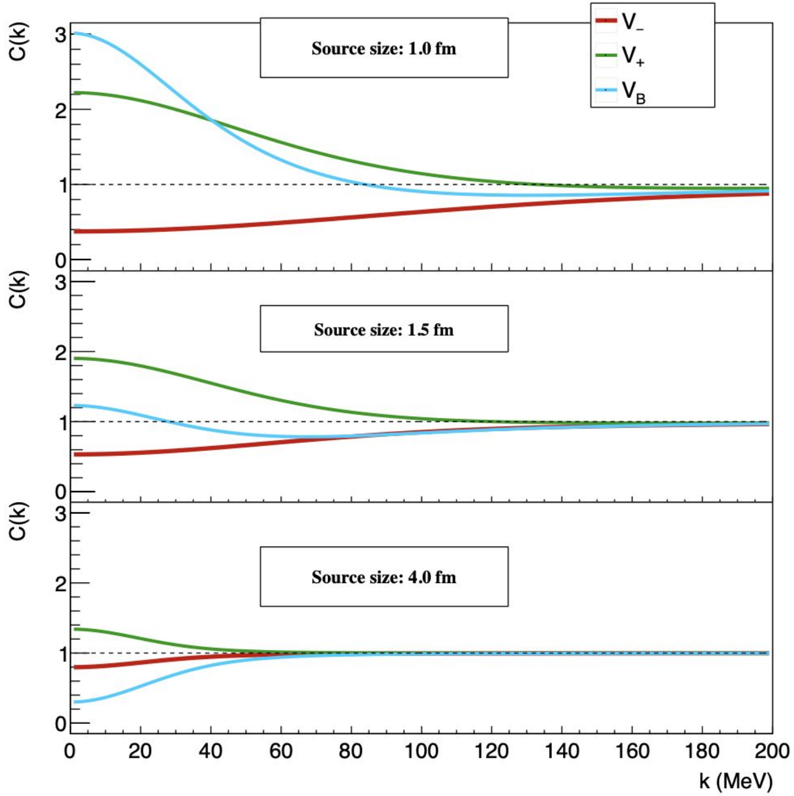
Understanding correlations with CATS

1. signal strength at low k*
   a. decreases as source size increases

2. For presence of BS
   a. flip below unity when measured in large colliding systems
   b. strength of depletion depends on binding energies (e.g. pΩ, DD*)



Additional slides

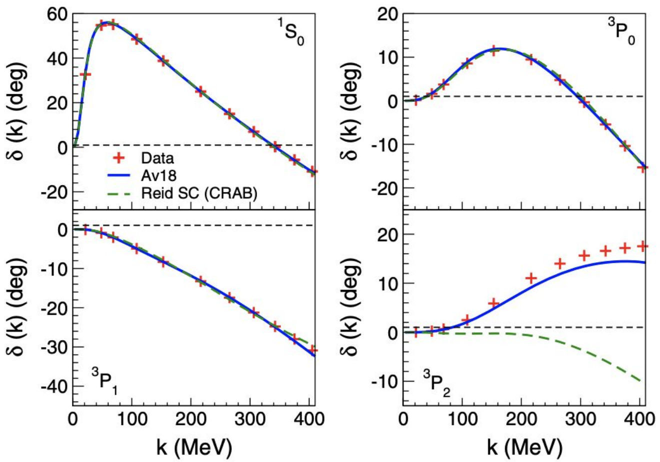
CATS and scattering data

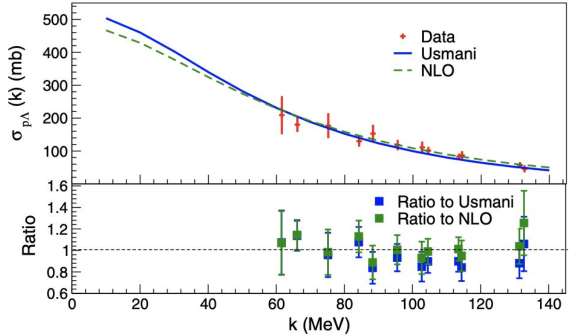
- Extract for any potential phase shifts δ_l

$$\sigma = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2(\delta_l).$$

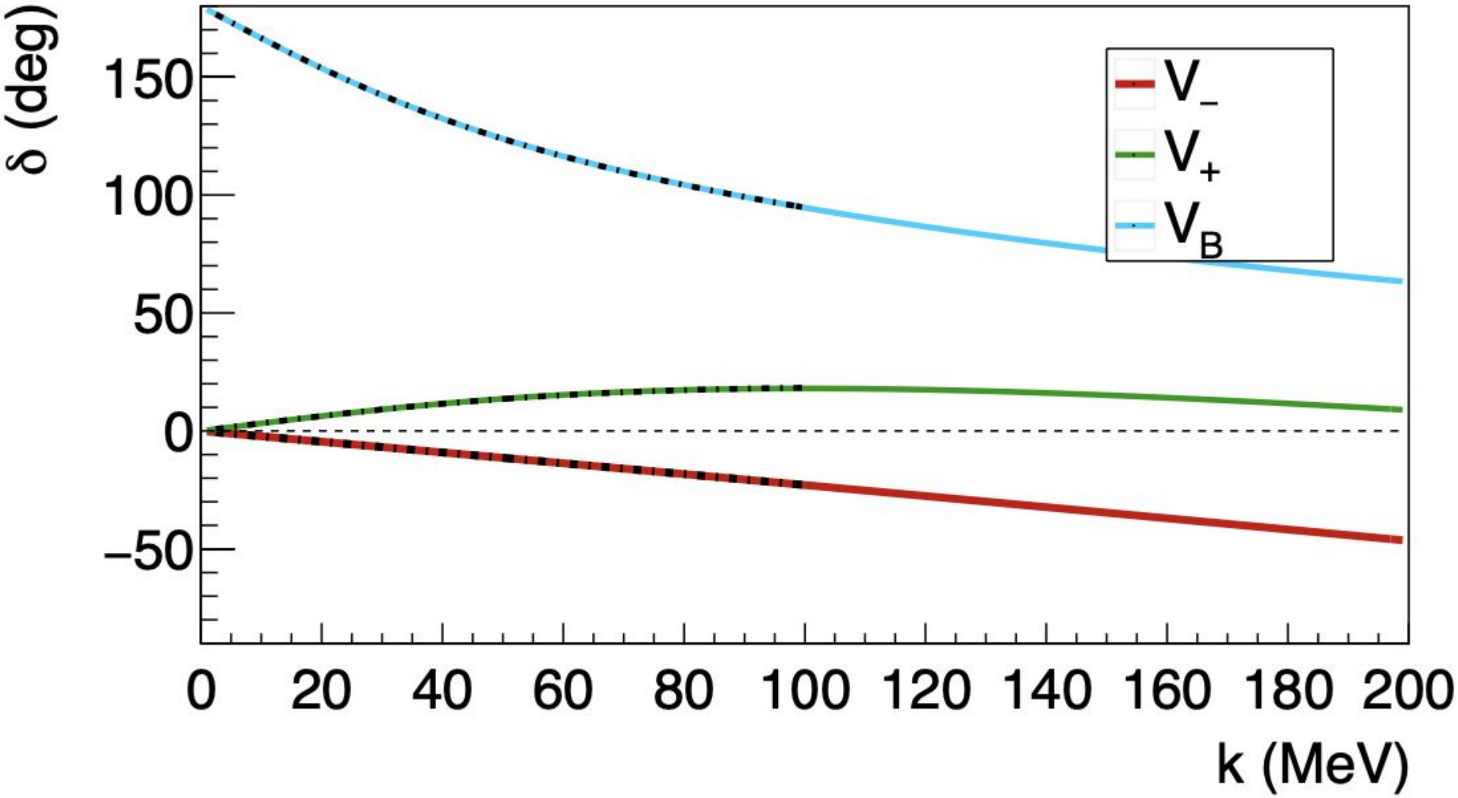
- p-p as benchmark, Argonne V18 pot.
  - perfect agreement with data
- p-Λ cross section data
  - comparison Usmani/NLO
  - discrepancies between models only at low k*
    → femtoscopy!!

Parameter	Usmani	χEFT NLO
$a_0^{(s)}$ (fm)	− 2.88	− 2.91
$r_{eff}^{(s)}$ (fm)	2.92	2.78
$a_0^{(t)}$ (fm)	− 1.66	− 1.54
$r_{eff}^{(t)}$ (fm)	3.78	2.72

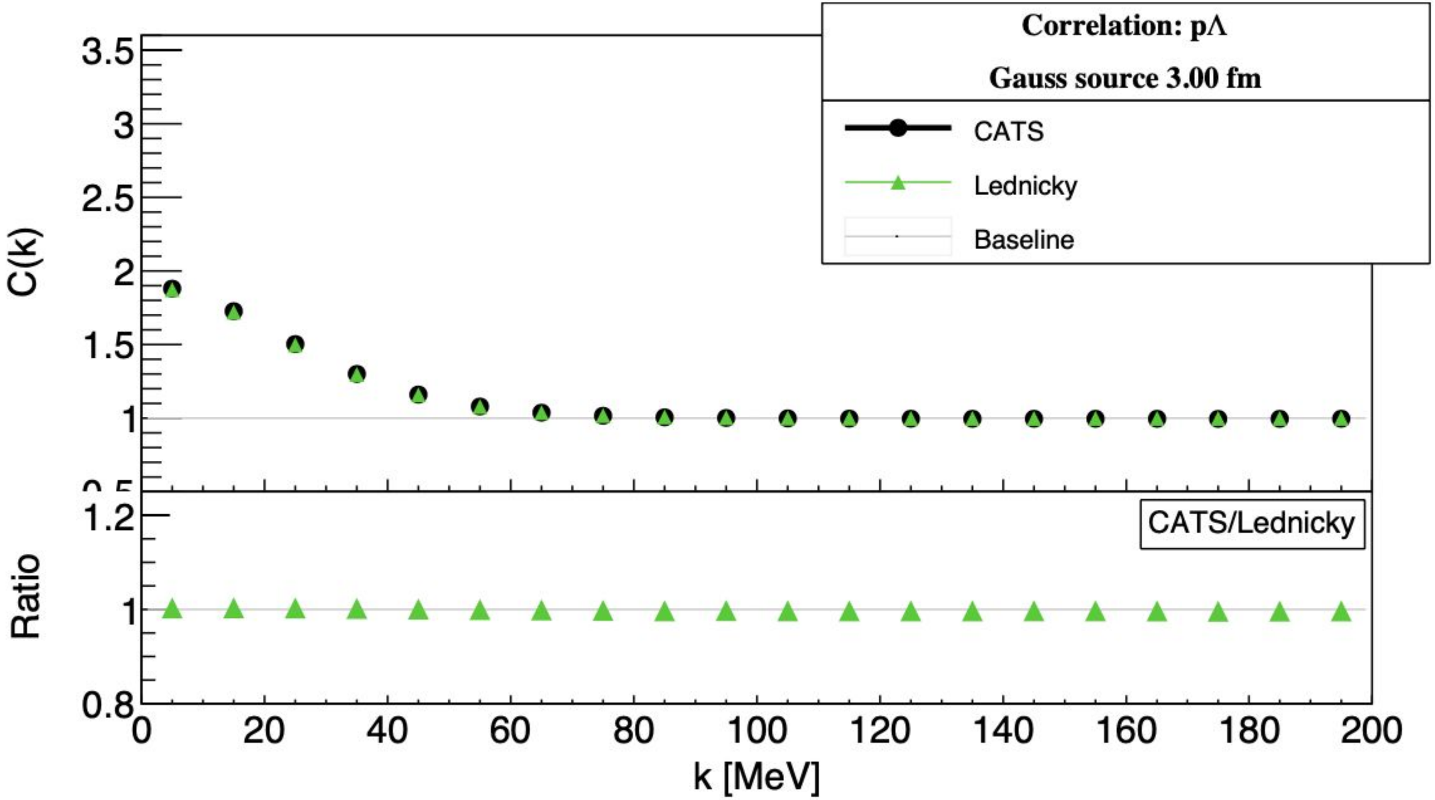




Phase shifts



pΛ correlation: CATS vs Lednicky



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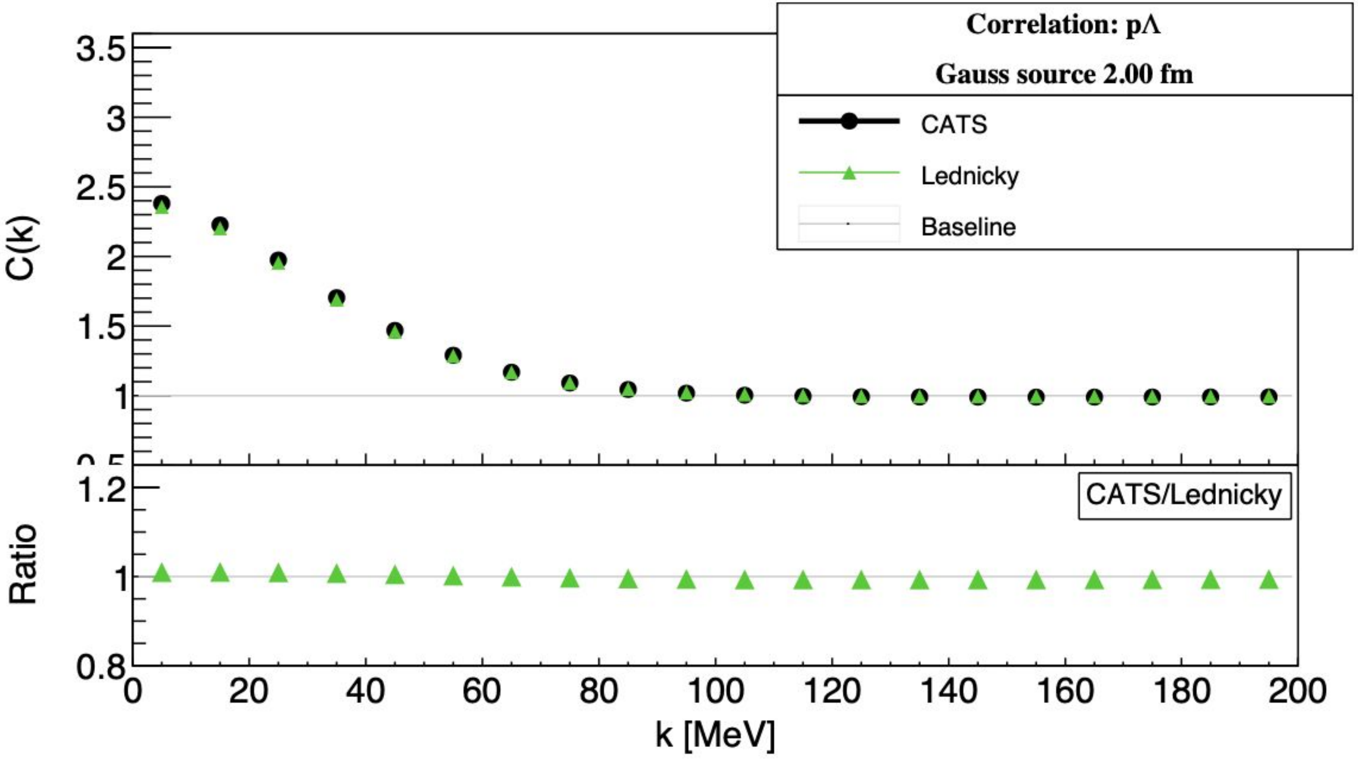

pΛ correlation: CATS vs Lednicky