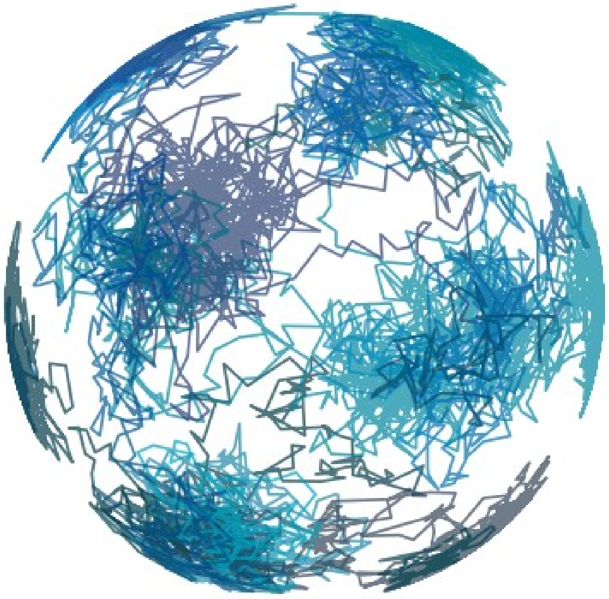
Supersolid phases of bosons in a spherical geometry



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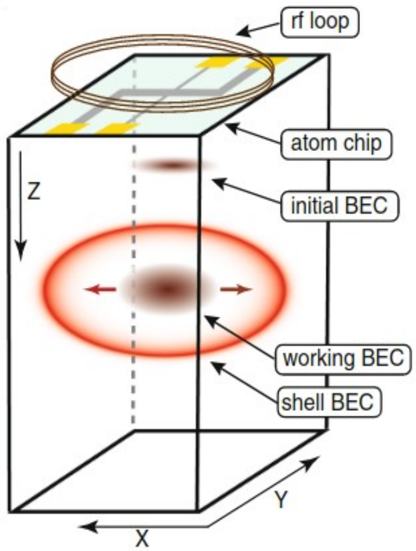
In collaboration with S. Prestipino, G. Pellicane

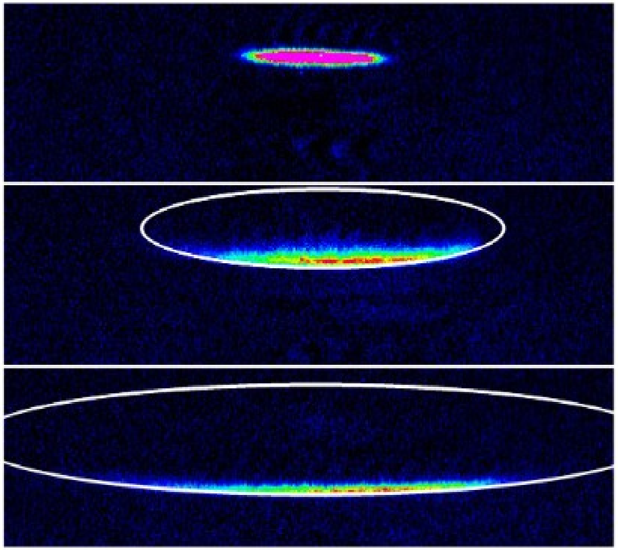
Summary

- Bubble traps
- BEC and superfluidity on the sphere
- Biased PIMC
- **Results**

Bubble traps



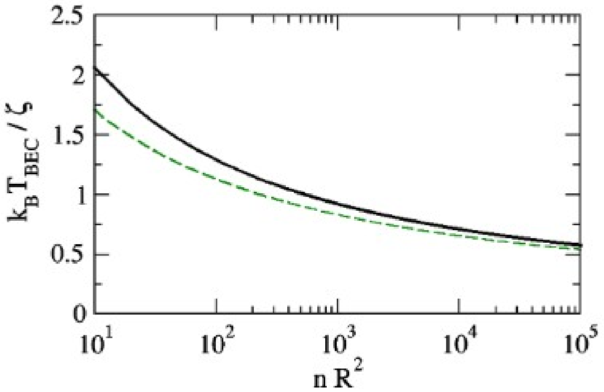
Lundblad et al.
2019



Colombe et al.
2004

Solution: go to space...

BEC and superfluidity on the sphere



Bose-Einstein Condensation on the Surface of a Sphere

A. Tononi and L. Salasnich
Phys. Rev. Lett. **123**, 160403 – Published 17 October 2019

Thermodynamics in expanding shell-shaped Bose-Einstein condensates

Brendan Rhyno, Nathan Lundblad, David C. Aveline, Courtney Lannert, and Smitha Vishveshwara
Phys. Rev. A **104**, 063310 – Published 13 December 2021

Only for no interaction or weak hard-core interactions...

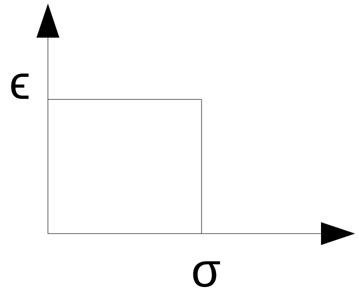
Physical model

$$\mathscr{H} = \sum_{i=1}^{N} \left(\frac{\hat{\mathbf{p}}_i^2}{2m} + u_{\text{ext}}(\hat{\mathbf{r}}_i) \right) + \sum_{i<j} v_{int}(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j)$$

- Spinless bosons of mass *m*
- Canonical ensemble
- Finite system (no PBC)
- Finite temperature

- Soft-core interaction

$$v(r) = \epsilon\theta(r - \sigma)$$

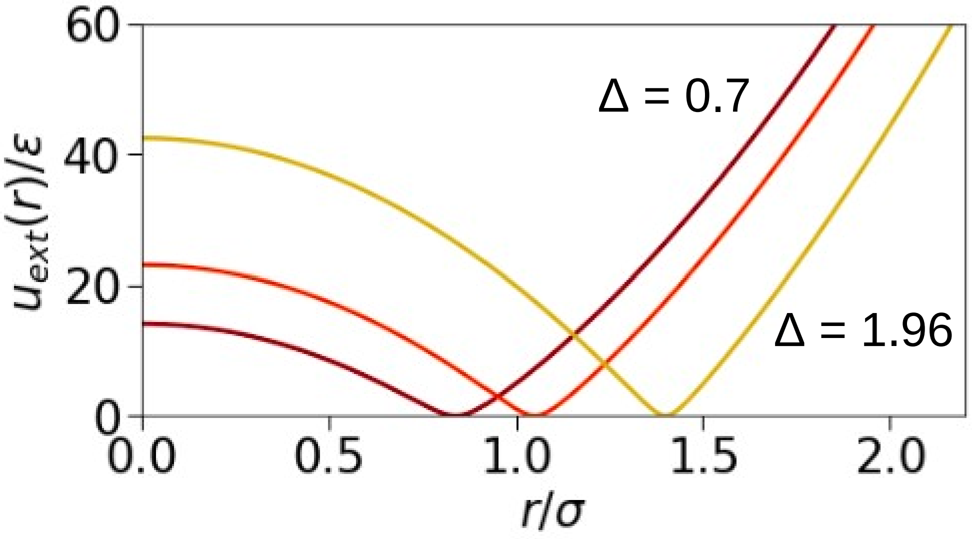


- Dipole-dipole interaction

$$v(\mathbf{r}) = v_{hard}(r) + \frac{\mu_0 d_m^2}{4\pi} \frac{(1 - 3\cos^2\theta)}{r^3}$$

External potential

$$u_{ext}(r) = u_0 \left(\sqrt{\frac{(r^2 - \Delta)^2}{4\Omega^2} + 1} - 1 \right)$$



Minimum:

$$r = \sqrt{\Delta} \quad \Rightarrow \quad u_{ext}(\sqrt{\Delta}) = 0$$

Small oscillation limit:

$$r = x + \sqrt{\Delta}$$

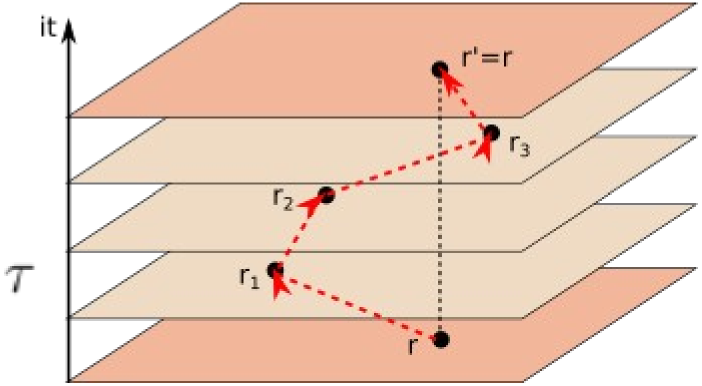
$$u_{ext}(x) \approx \frac{u_0 \Delta}{2\Omega^2} x^2$$

Feynman’s path integral (statistical)

$$\beta = \frac{1}{k_B T}$$

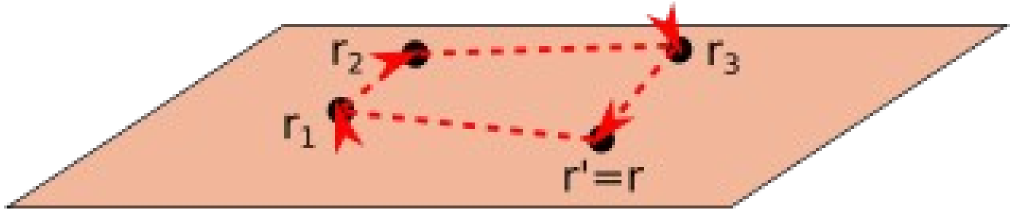
$$\rho(r', r; \beta) = \langle r' | e^{-\mathcal{H}\beta} | r \rangle$$

$$\mathcal{Z} = \mathrm{Tr}\left\{\rho(r', r; \beta)\right\} = \int dr \ \langle r | e^{-\mathcal{H}\beta} | r \rangle$$



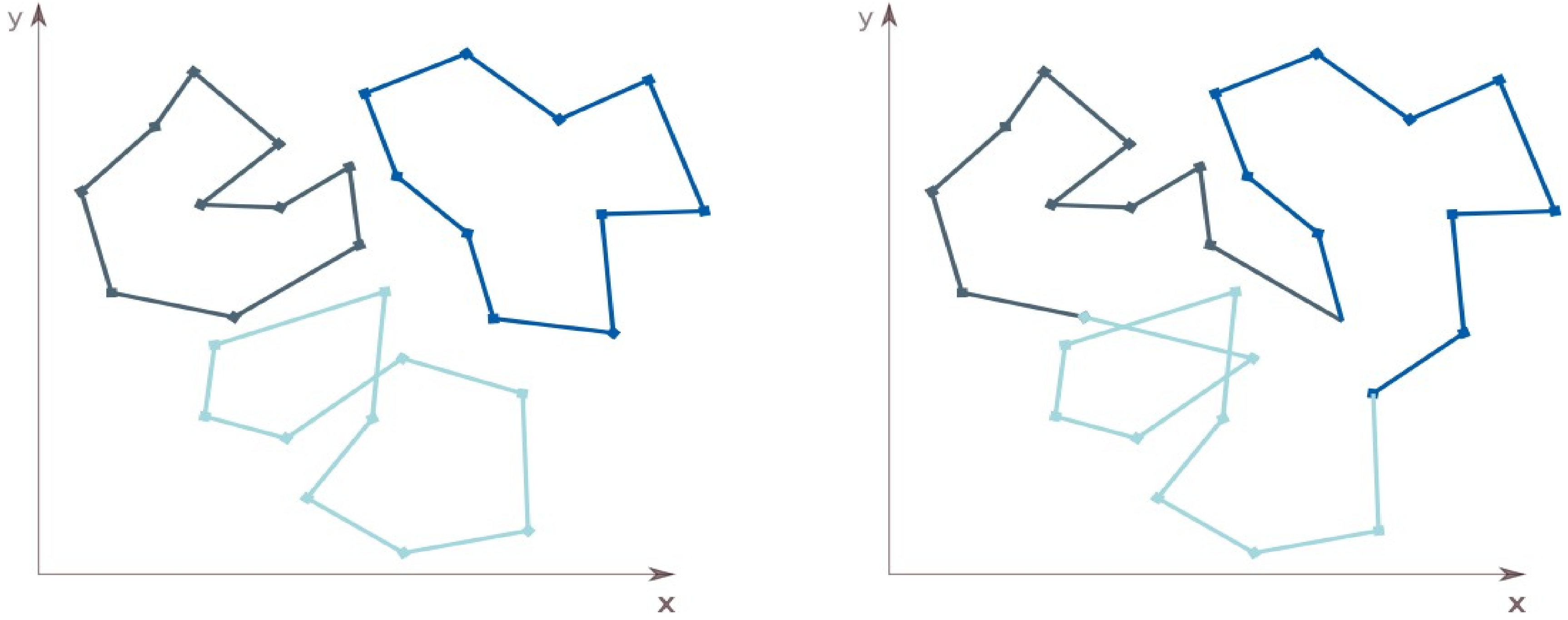
$$= \int \prod_{i=0}^{M-1} dr_i \ \prod_{i=0}^{M-1} \langle r_i | e^{-\mathcal{H}\beta/M} | r_{i+1} \rangle$$

$$\tau = \beta / M$$



Bose-Einstein statistics

$$\rho_B(R, R'; \beta) = \frac{1}{N!} \sum_{\mathcal{P}} \rho(R, \mathcal{P} R'; \beta)$$



Monte Carlo sampling

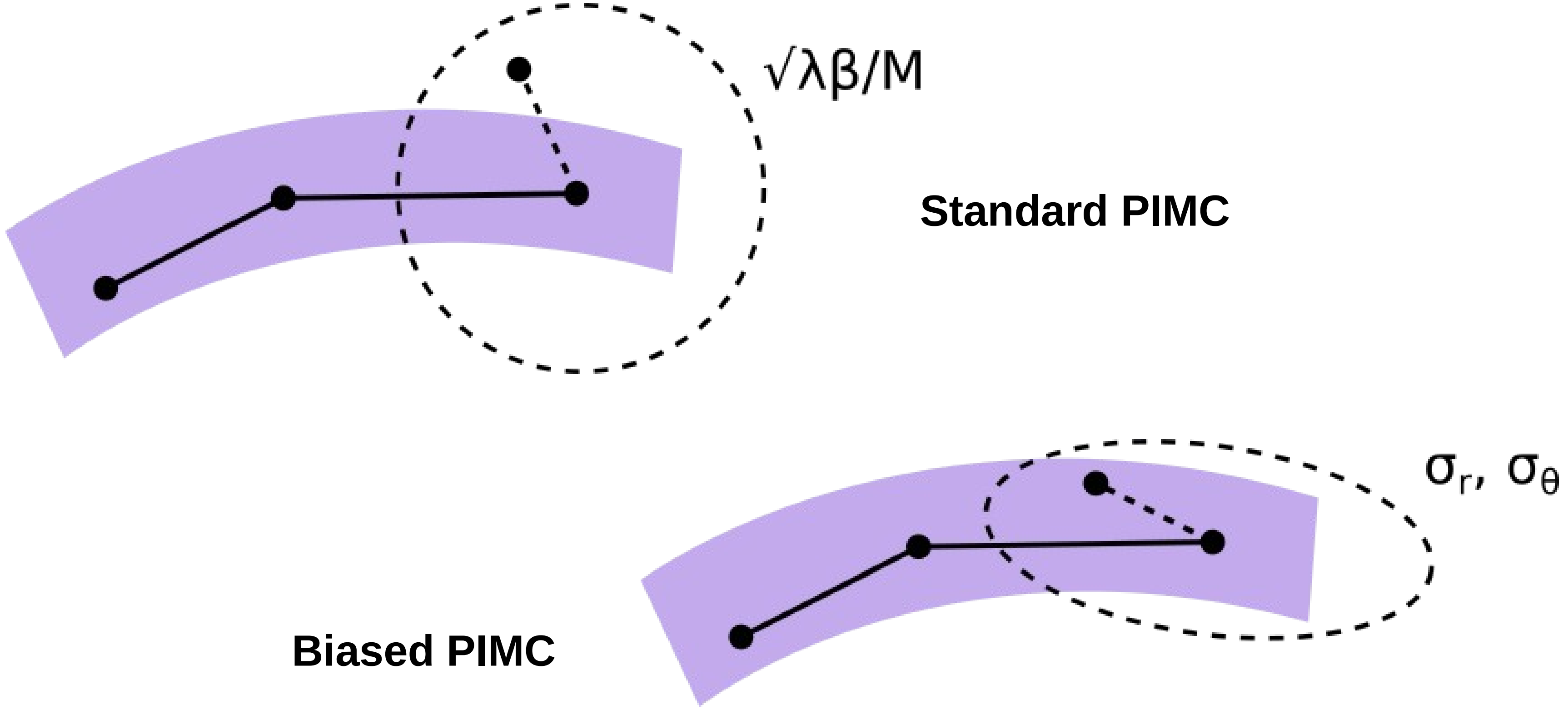
$$\mathcal{H} = \mathcal{T} + \mathcal{V}$$

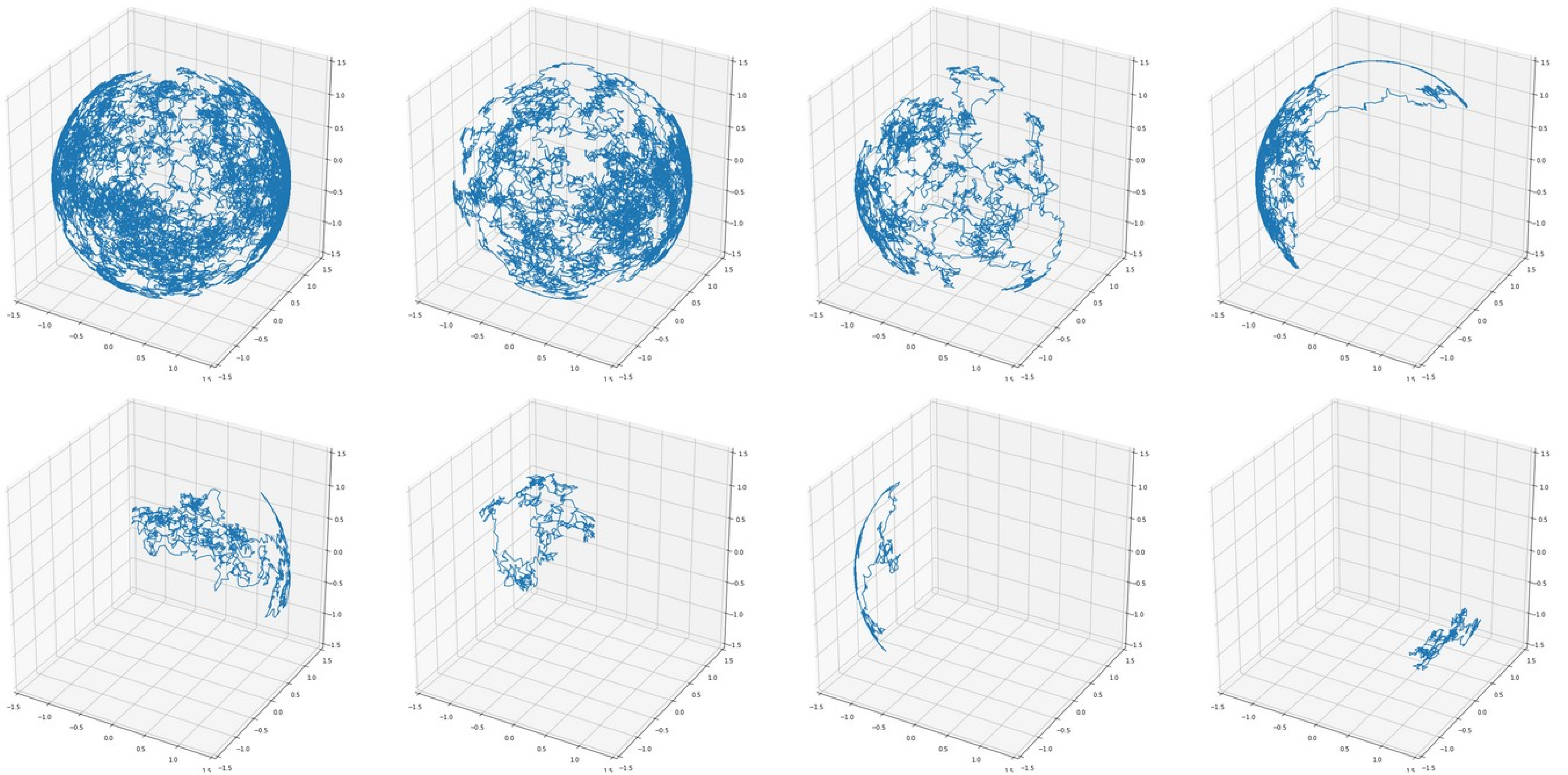
$$e^{-\tau\mathcal{H}} = e^{-\tau(\mathcal{T}+\mathcal{V})} = e^{-\tau\mathcal{T}} e^{-\tau\mathcal{V}} e^{O(\tau^2)}$$

$$\rho(r, r'; \tau) = \underbrace{\frac{1}{\sqrt{4\pi\lambda\tau}^{\,d}}\, e^{-\frac{(r-r')^2}{4\lambda\tau}}}_{\text{Sampling}} \underbrace{e^{-\tau V(r)}}_{\text{Acceptance/rejection}}$$

Sampling Acceptance/rejection

Monte Carlo sampling

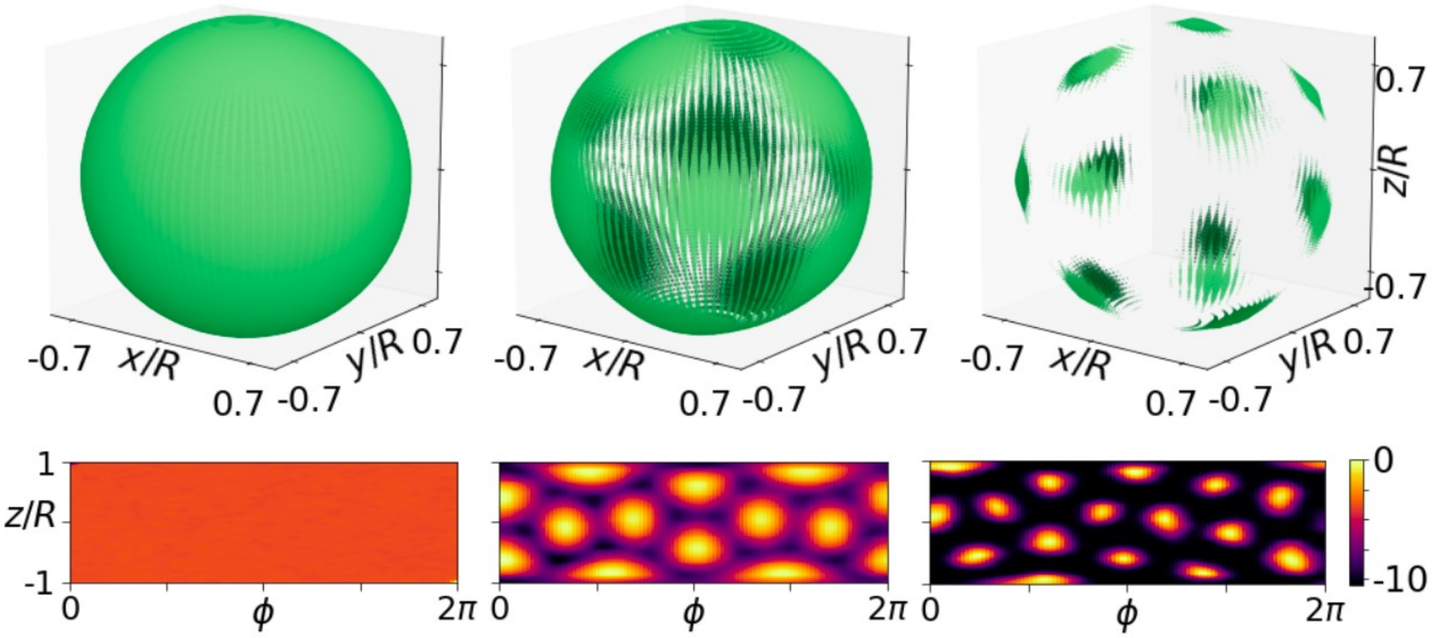




-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5

Results: soft-core interactions

$\alpha = 5$ $\alpha = 17.5$ $\alpha = 250$

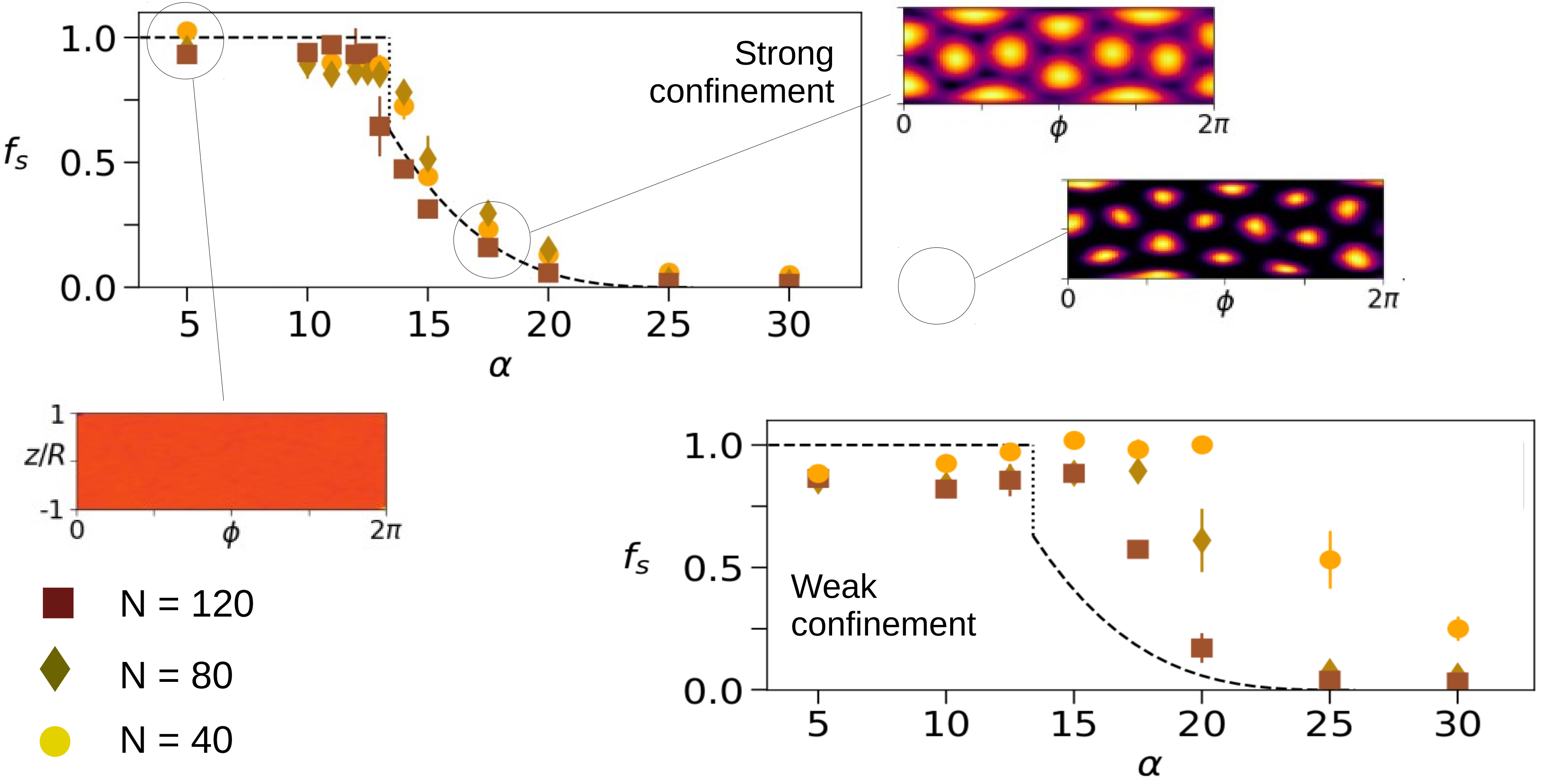


12 clusters 16 clusters

$$\lambda = \hbar^2/(2m)$$

$$\alpha = \rho\sigma^4\epsilon/(2\lambda)$$

Results: soft-core interactions



Future prospects:

- **Including gravity**
- **Dynamics in the strongly-interacting regime**
- **Generalizing to other geometries with curvature**

Thank you!