Learning to classify quantum states

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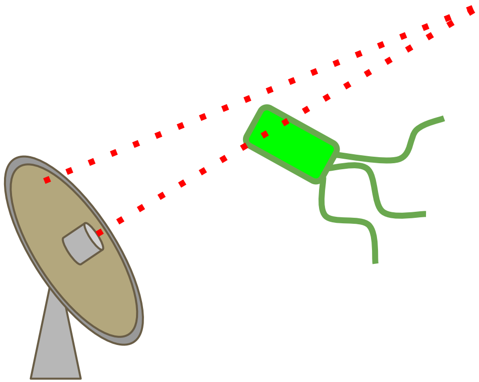
What is quantum state/channel discrimination?

- Discriminating between possible quantum states/channels (unsurprisingly).
- We are presented with an unknown quantum state/channel.
- Our task is to decide which state/channel it is (from a finite set).
- Carry out measurement/interact with probe.
- Quantum metrology is more concerned with parameter estimation.

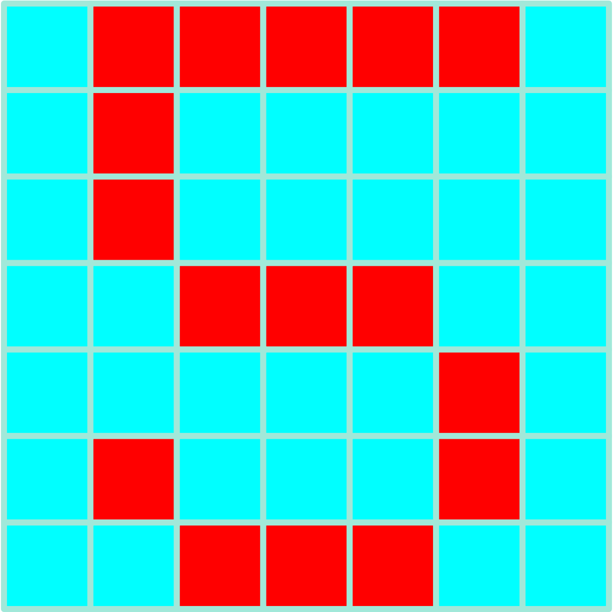
Why carry out quantum state/channel discrimination?

- Many physical experiments can be regarded as state/channel discrimination.
- A process can be modelled as a quantum operation.
- Any probe can be described as a quantum state.
- Deciding which physical process occurs is channel discrimination.
- Example: quantum target detection.
- Example: probing a substance with photons to find transmission.
- Example: quantum reading.



Grouping states/channels into classes

- Suppose we are not interested in which specific state we have.
- Instead, we are interested in a property of that state.
- Example: average photon number.
- For probing a grid of pixels, we might be looking for a global property.
- Example: reading a barcode/number.
- Example: cluster detection.

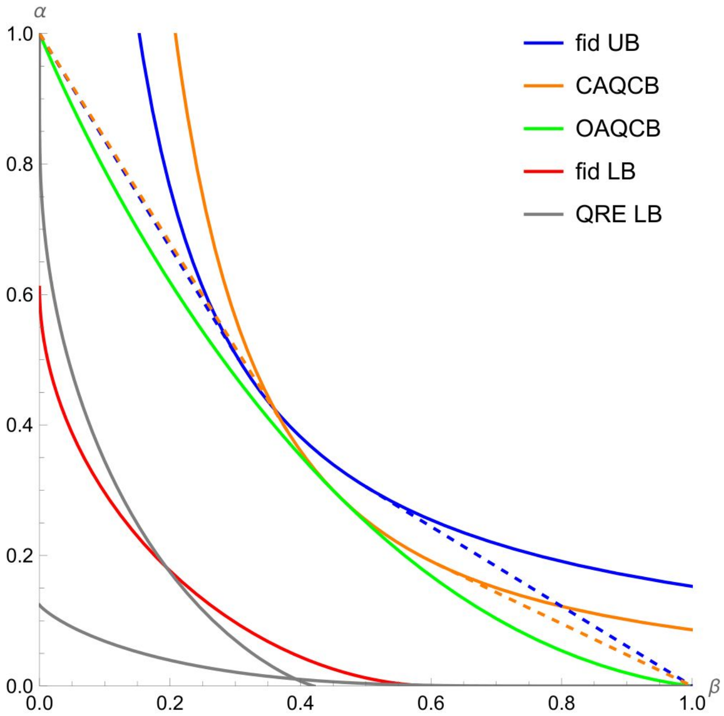


Ultimate bounds on state discrimination

- Better measurement devices perform better measurements!
- We cannot perfectly distinguish between non-orthogonal quantum states.
- There is an ultimate bound on quantum state/channel discrimination.
- Focus on binary hypothesis testing.
- Two cases: symmetric and asymmetric.
- Symmetric: Helstrom bound. Asymmetric: quantum Neyman-Pearson.
- Can be hard to calculate exactly for large states.
- Asymptotic bounds: quantum Stein’s lemma/quantum Hoeffding bound.
- What about CV states/large DV states?

Analytical bounds for non-asymptotic asymmetric state discrimination

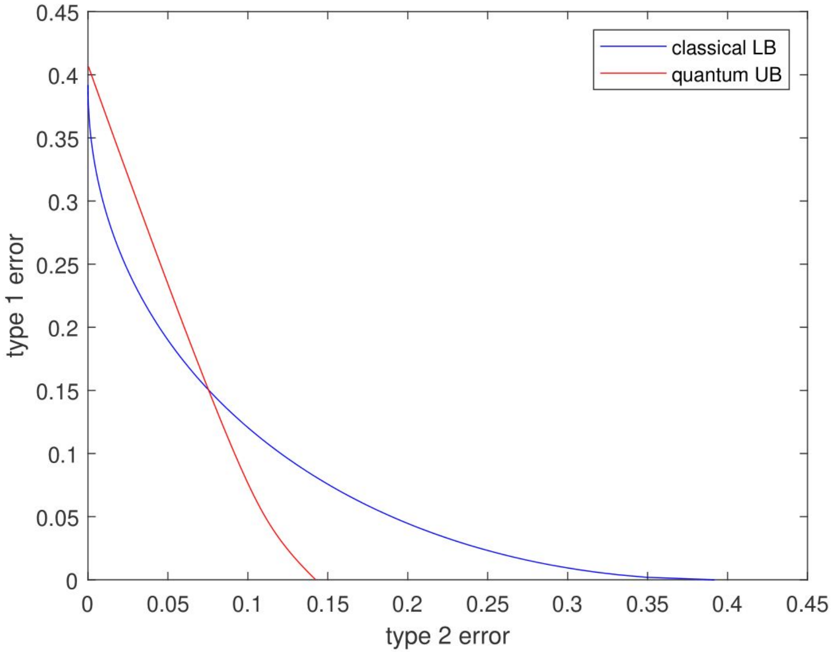
- Upper and lower bounds based on the fidelity, the quantum Chernoff bound, and the quantum relative entropy.
- Lower bound based on fidelity is exact for pure states.
- Optimal upper bound based on the quantum Chernoff bound is asymptotically tight.



J. L. Pereira, L. Banchi, and S. Pirandola. Preprint at https://arxiv.org/abs/2207.10699 (2022).

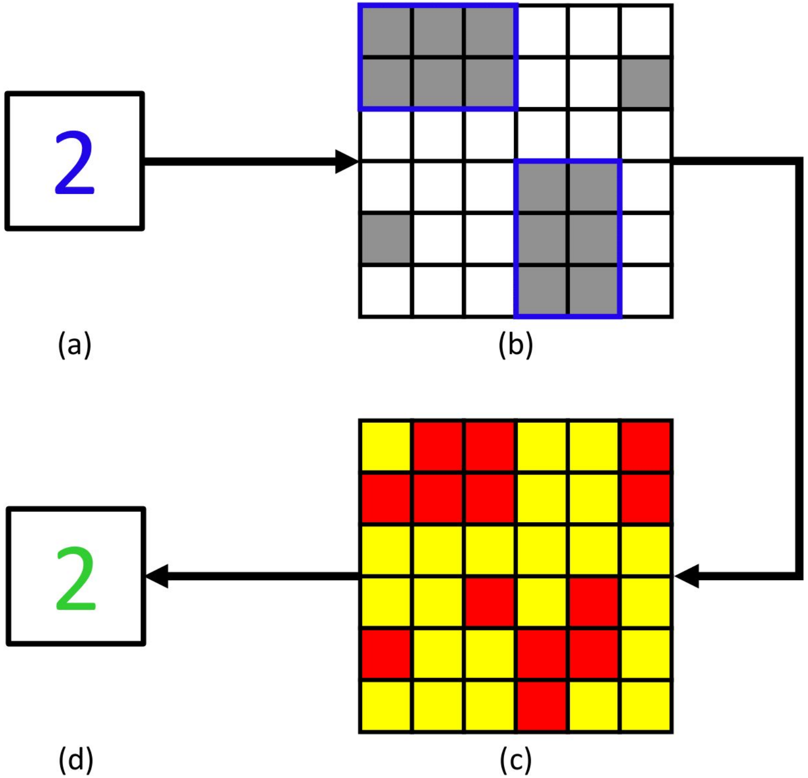
Demonstrating quantum advantage in imaging

- Quantum advantage means better performance than every classical protocol.
- Often compare lower bounds to upper bounds.
- Classical imaging protocols use classical probe states.
- Maximise over all classical probes.
- TMSVs show quantum advantage for probing lossy channels.

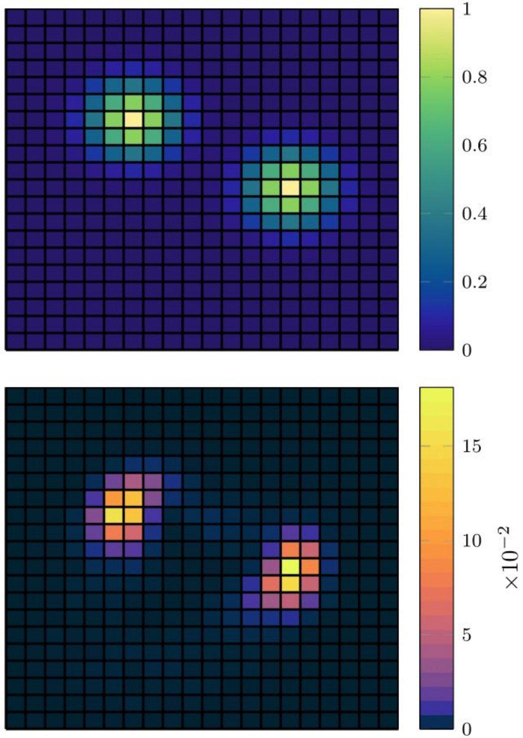


Is it robust enough for pattern recognition?

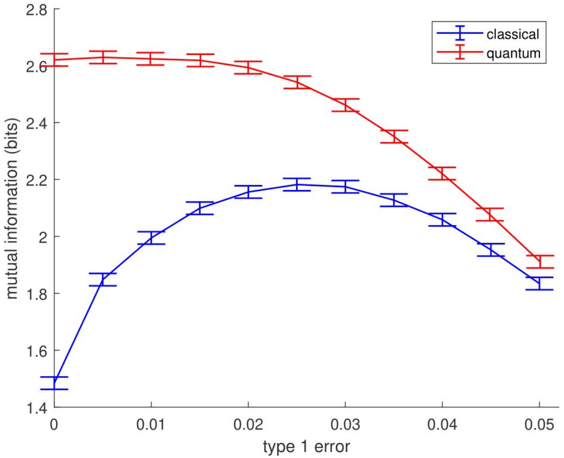
- Quantum advantage may be lost by data processing.
- Machine learning is hard to study analytically.
- Two forms of pattern recognition: supervised and unsupervised.
- Example: Quantum-enhanced barcode decoding and pattern recognition.
- What about unsupervised?

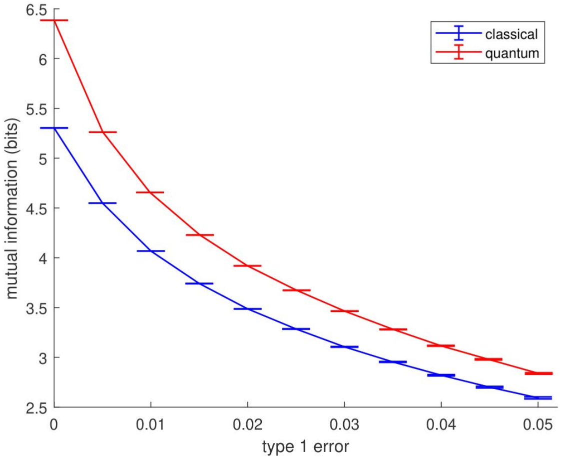


Quantum-enhanced cluster detection in physical images



- Can study numerically.
- We find that we can achieve quantum advantage for cluster detection.





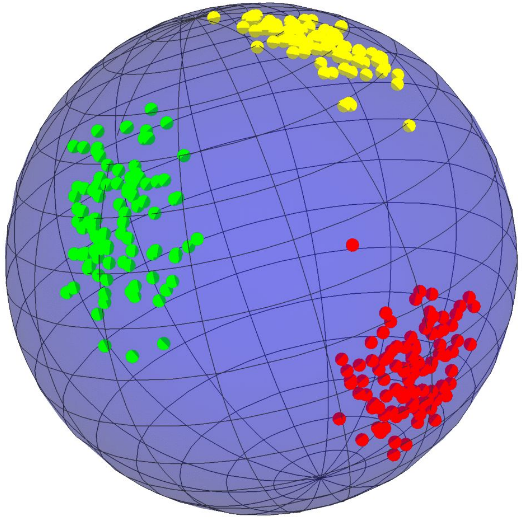
J. L. Pereira, L. Banchi, and S. Pirandola. Preprint at https://arxiv.org/abs/2208.05522 (2022).

Learning a quantum measurement

- We previously considered applying machine learning to a quantum measurement result.
- What about learning a measurement to directly decide the class?
- Suppose we have a limited training set: how well can we generalise?
- Example: quantum phase recognition.
- Now suppose we start with classical data.
- Can consider different embeddings, with different approximation errors.
- What is a “good” embedding?

Generalization in Quantum Machine Learning: a Quantum Information Perspective

- Can bound the generalisation error with a quantity that depends on the embedding.
- Scales with $T^{-½}$, where T is the number of training samples.
- Cannot simultaneously minimise the generalisation error and the approximation error.
- A good embedding has a small intra-class distance and large inter-class distance.



L. Banchi, J. L. Pereira, and S. Pirandola. PRX Quantum 2, 040321 (2021).

Conclusions

- Many physical processes can be modelled as quantum channels.
- Can find ultimate bounds on optimal state/channel discrimination.
- Quantum probes can have quantum advantage.
- This advantage can be robust enough to survive machine learning.
- Measurements learned through machine learning can generalise.
- If we can choose the embedding, we want a good embedding.
- Current work: out of distribution generalisation.