

#florence Theory-group day , 22/02/2023

WHEN QUANTUM TALKS CLASSICAL

FROM QUBITS TO BLACK-HOLES

Caterina Foti



Nicola Pranzini



Paola Verrucchi



Alessandro Cuccoli



Alessandro Coppo

Pietro Liuzzo



the formal tools



- generalized coherent states

dictionary

- large- N limit

translation

- parametric representation

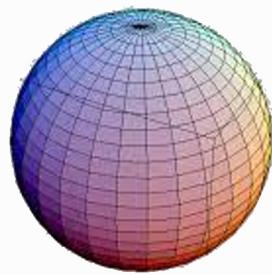
dialogue

dictionary

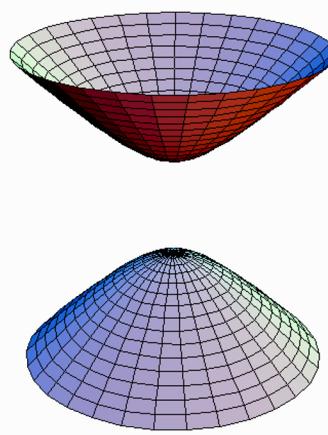
$$|\alpha| \in \mathbb{N} \leftrightarrow \alpha \in \mathcal{M}$$

dictionary

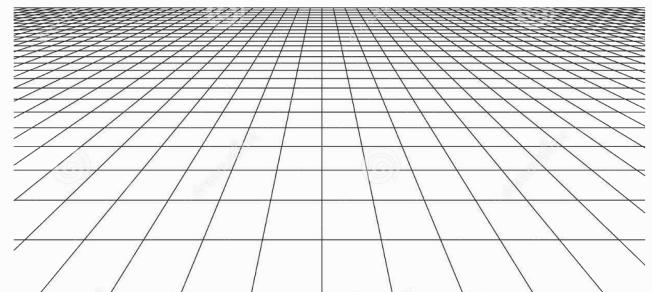
$$|\alpha\rangle \in \mathcal{H} \leftrightarrow \alpha \in \mathcal{H}$$



$\text{su}(2)$



$\text{su}(1,1)$



h_4

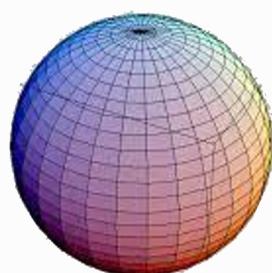
GENERALIZED COHERENT STATES (GCS)

$\mathcal{H}, \mathfrak{g}, |\zeta\rangle$

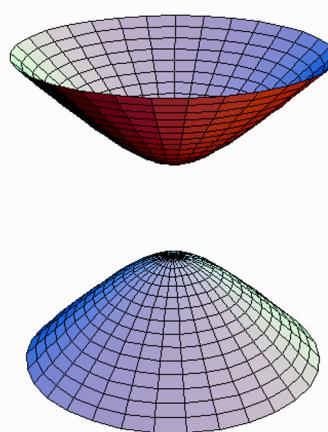
$$|\zeta\rangle \in \mathcal{H} \leftrightarrow \zeta \in \mathcal{M}$$

$$\int_{\mathcal{M}} d\mu(\hat{\zeta}) |\zeta \times \zeta| = \hat{L}_{\mathcal{M}}, \quad d\mu(\hat{\zeta}) \text{ invariant}, \quad \langle \zeta | \zeta' \rangle \neq \delta(\zeta - \zeta')$$

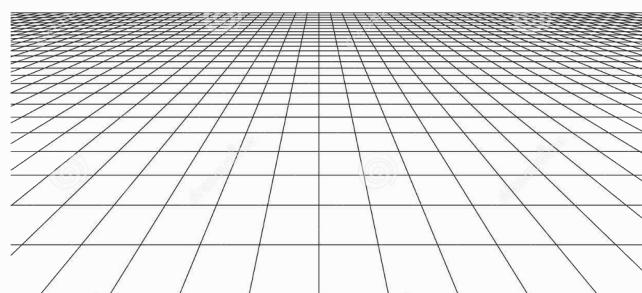
\mathcal{M} is symplectic and depends on \mathfrak{g}



$\mathfrak{su}(2)$



$\mathfrak{su}(1,1)$



\mathbb{H}_4

translation

" CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS
EQUIVALENT TO SOME $N \rightarrow \infty$ LIMIT OF A GIVEN
QUANTUM THEORY "



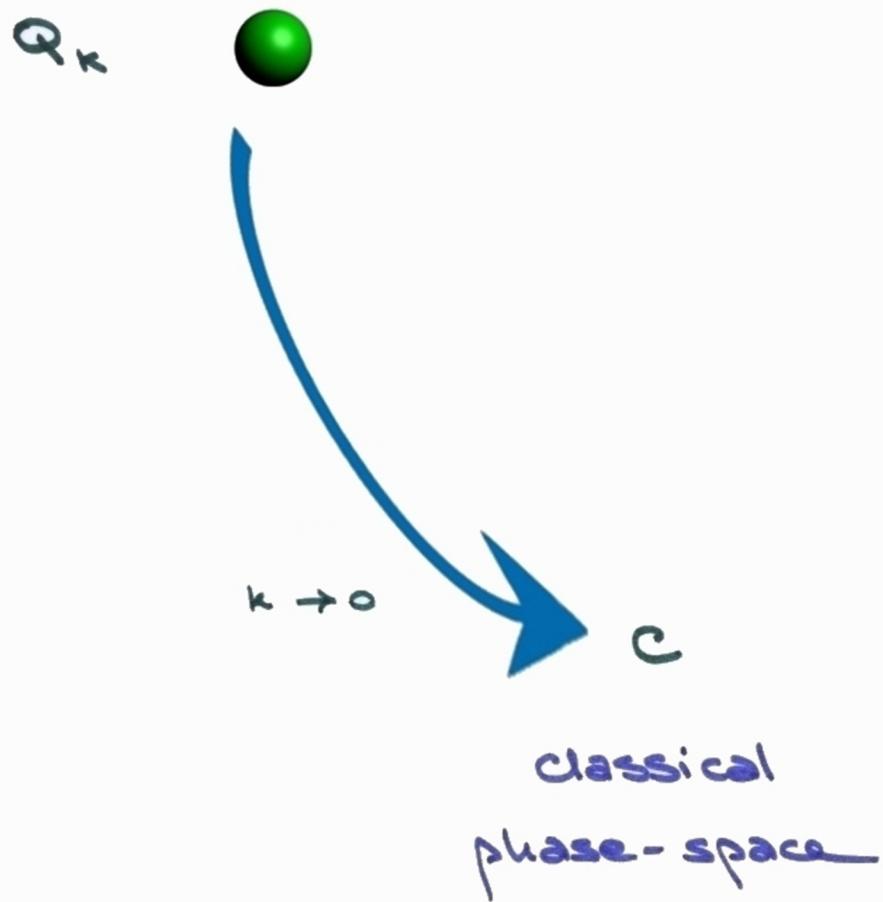
L.G.YAFFE Rev.Mod.Phys. 54, 407 ('82)

translation

" CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME $N \rightarrow \infty$ LIMIT OF A GIVEN QUANTUM THEORY "



L.G.YAFFE Rev.Mod.Phys. 54, 407 ('82)



translation

" CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME $N \rightarrow \infty$ LIMIT OF A GIVEN QUANTUM THEORY "

?

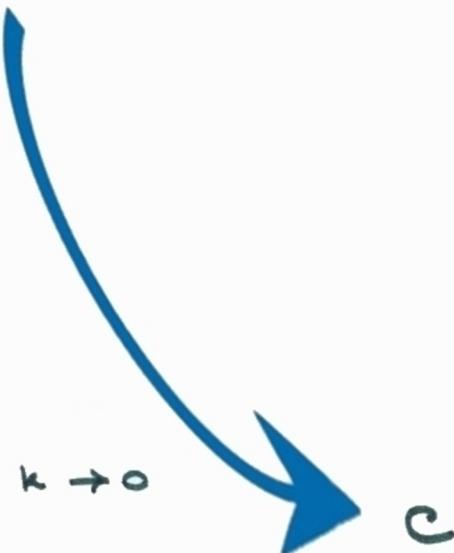
L.G.YAFFE Rev.Mod.Phys. 54, 407 ('82)



gl



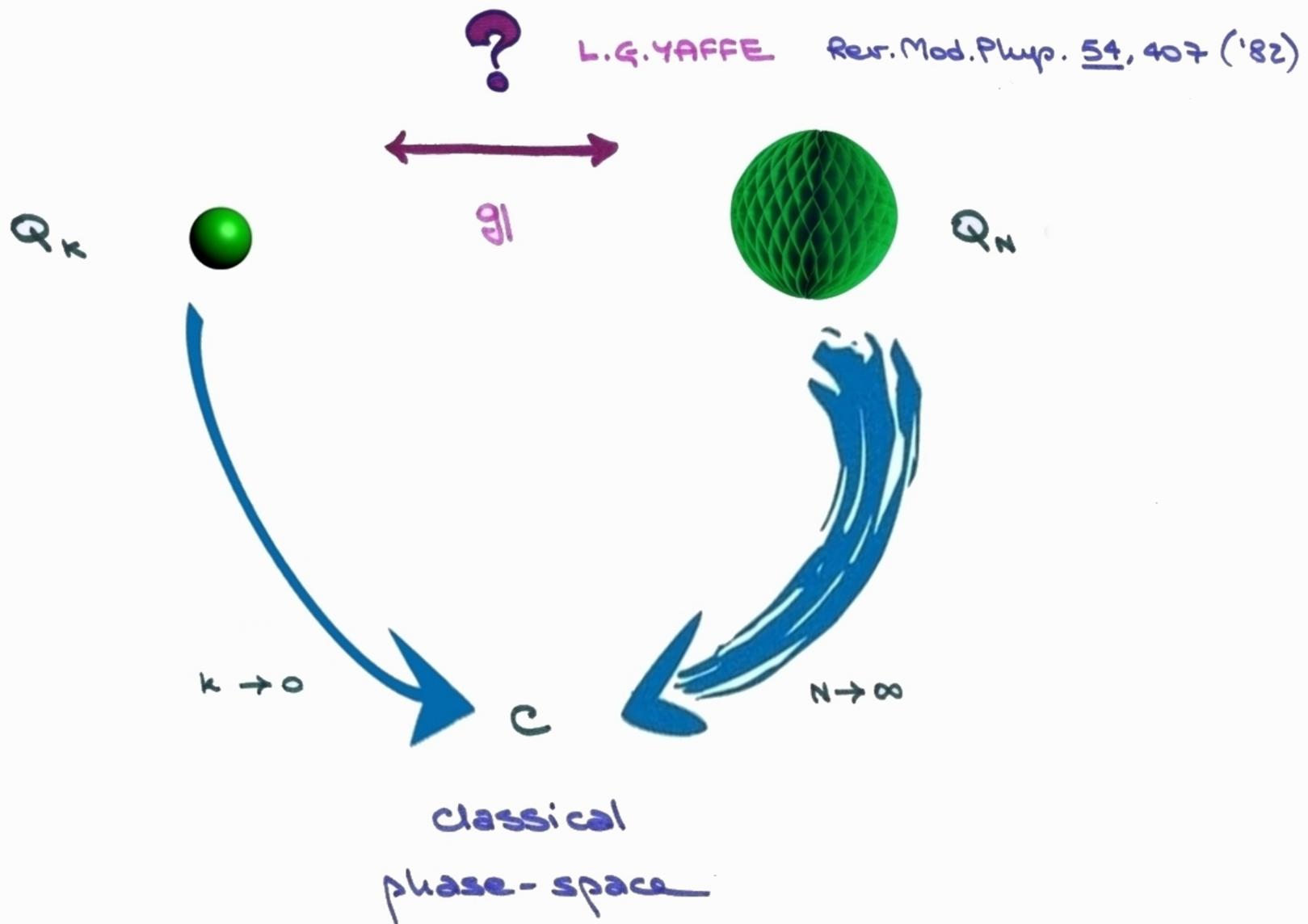
Q_N



classical
phase-space

THE LARGE- N LIMIT

"CAN ONE FIND A CLASSICAL SYSTEM WHOSE DYNAMICS IS EQUIVALENT TO SOME $N \rightarrow \infty$ LIMIT OF A GIVEN QUANTUM THEORY?"



GCS AND THE LARGE-N LIMIT

conditions: $Q_N \xrightarrow[N \rightarrow \infty]{} \Omega$, in terms of GCS



states $|\psi\rangle$ that survive:

$$\langle \omega | \omega' \rangle \xrightarrow[N \rightarrow \infty]{} \delta(\omega - \omega')$$

$$|\omega\rangle \in \mathcal{H}$$

$$\omega \in \mathcal{M}$$

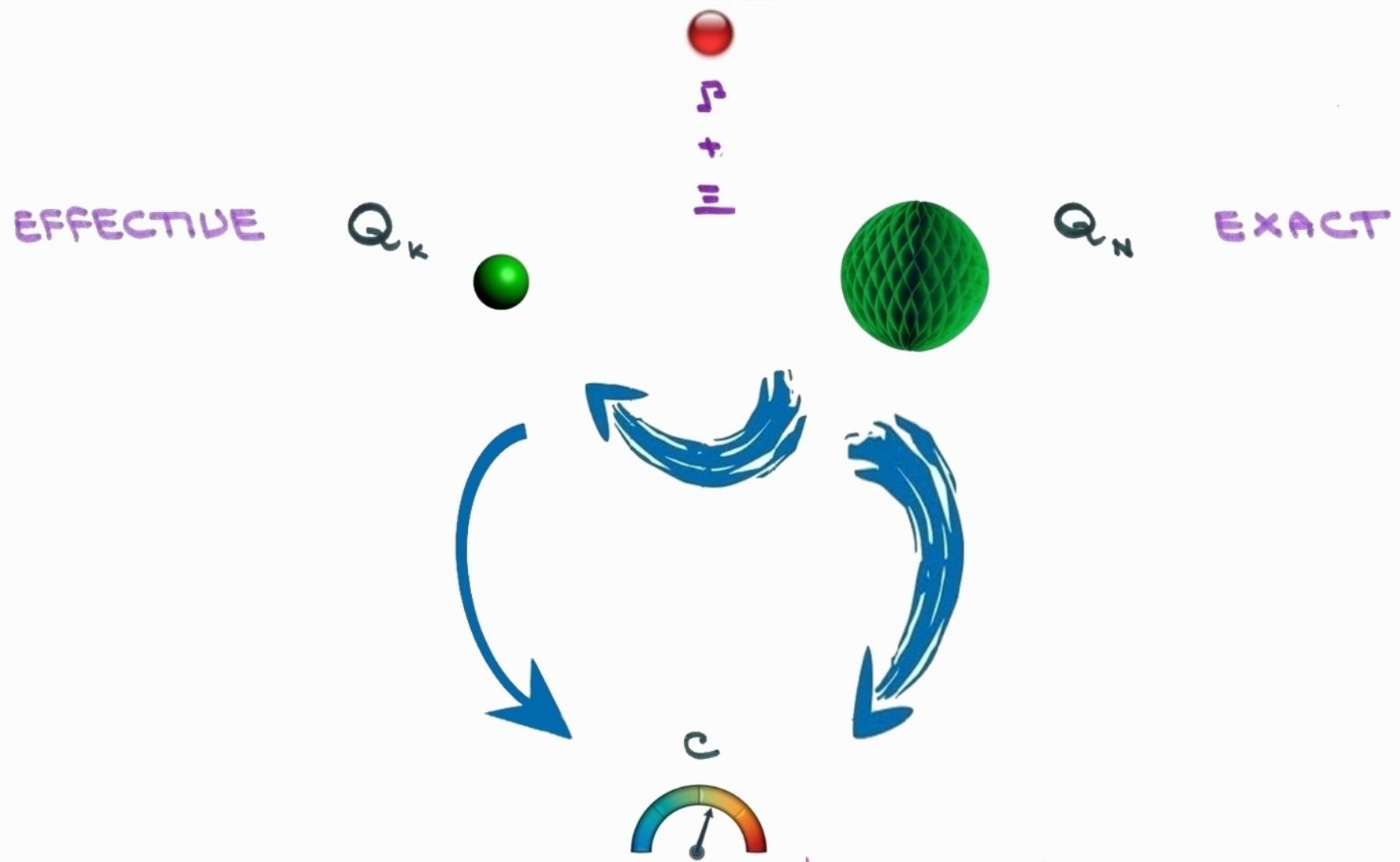
observables \hat{A} that survive:

$$\frac{\langle \omega | \hat{A} | \omega' \rangle}{\langle \omega | \omega' \rangle} \xrightarrow[N \rightarrow \infty]{} \infty$$

$$\langle \omega | \hat{A} | \omega \rangle$$

$$A(\omega)$$

dialogue



GCS AND COMPOSITE SYSTEMS $\Psi = C + F$

A PARAMETRIC REPRESENTATION

$$|\Psi\rangle = \sum_{\gamma} c_{\gamma} |\gamma\rangle \otimes |\gamma\rangle \in \mathcal{H}_{\Psi} = \mathcal{H}_c \otimes \mathcal{H}_f$$

GCS AND COMPOSITE SYSTEMS $\Psi = C + F$

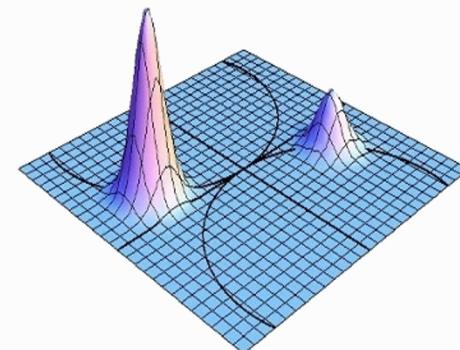
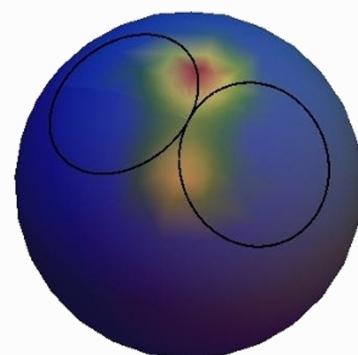
A PARAMETRIC REPRESENTATION

$$\begin{aligned}
 |\Psi\rangle = & \sum_{\lambda} c_{\lambda} |\lambda\rangle \otimes |\psi\rangle \in \mathcal{H}_{\Psi} = \mathcal{H}_e \otimes \mathcal{H}_F \\
 & \underbrace{\qquad\qquad\qquad}_{\int d\mu(\omega) |e\rangle |\chi(\omega)\rangle} \\
 = & \int_{\mathcal{M}} d\mu(\omega) |\chi(\omega)\rangle \otimes |\phi(\omega)\rangle
 \end{aligned}$$

GCS AND COMPOSITE SYSTEMS $\Psi = C + F$

A PARAMETRIC REPRESENTATION

$$\begin{aligned}
 |\Psi\rangle = & \sum_{\Omega} c_{\Omega} |\Omega\rangle \otimes |\chi\rangle \in \mathcal{H}_{\Psi} = \mathcal{H}_e \otimes \mathcal{H}_F \\
 & \underbrace{\qquad\qquad\qquad}_{\int d\Omega(\omega) | \omega \rangle \otimes |\chi\rangle} \\
 = & \int_{\mathcal{M}} d\mu(\omega) |\chi(\omega)\rangle |\omega\rangle \otimes |\phi(\omega)\rangle \\
 & \chi^2(\omega) = \sum_{\Omega} \left| \sum_{\chi} c_{\Omega\chi} \langle \omega | \chi \rangle \right|^2
 \end{aligned}$$



MEASURE-LIKE INTERACTION



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A LETTERS JOURNAL EXPLORING
THE FRONTIERS OF PHYSICS



LETTER

Parametric description of the quantum measurement process

P. Liuzzo-Scorpo^{1,2}, A. Cuccoli^{2,3} and P. Verrucchi^{4,2,3}

Published 3 September 2015 • Copyright © EPLA, 2015

[Europhysics Letters, Volume 111, Number 4](#)

ANY INTERACTION



F



=



 **Quantum**
the open journal for quantum science

Whenever a quantum environment emerges as a classical system, it behaves like a measuring apparatus

Caterina Foti^{1,2}, Teiko Heinosaari³, Sabrina Maniscalco³, and Paola Verrucchi^{4,1,2}

No interaction



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Time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

[Caterina Foti](#)✉, [Alessandro Coppo](#), [Giulio Barni](#), [Alessandro Cuccoli](#) & [Paola Verrucchi](#)

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Alessandro Coppo, Nicola Pranzini, and Paola Verrucchi

Phys. Rev. A **106**, 042208 – Published 13 October 2022



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via black holes*

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Time and classical equations of motion from quantum entanglement via the Page and Wootters mechanism with generalized coherent states

Caterina Foti^{1,2,3}, Alessandro Cocco^{1,2}, Giulio Barni¹, Alessandro Cuccoli^{1,2} & Paola Verrucchi^{1,2,4}

and

PAW # 1, 4, 5

GCS # 23, 25, 40

LARGE-N # 19, 21

PARAMETRIC REPRESENTATION
WITH GCS # 24, 41

