Condensed dark matter at galactic scales

With M. H. G. Tytgat and J. Vandecasteele
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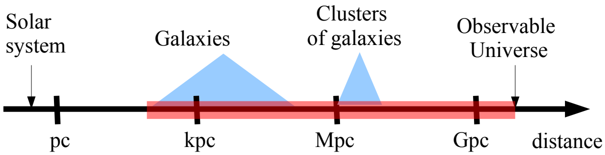
Matter

Dark Matter and visible matter in the Universe



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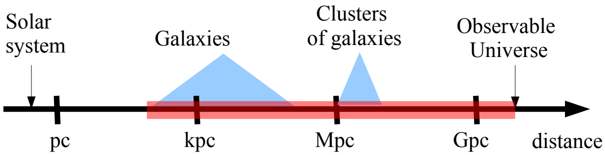

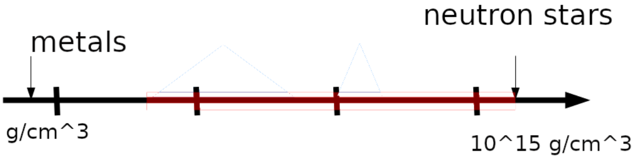

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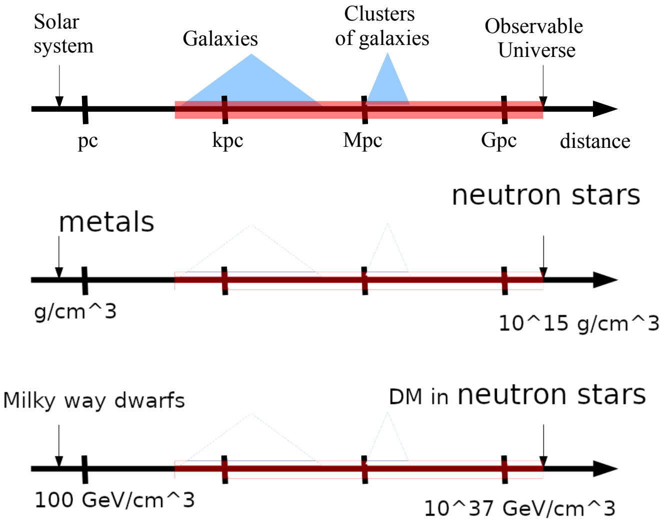


What is new here?

Outline

- Fermion asymmetric DM with yukawa interaction for dark sector. Going beyond non-interacting scenario Domcke & Urbano '14, Randall et al. '16, Gresham &Zurek '18, and Bosonic DM with effective theory for phonons Berezhiani & Khoury '15
- Consistent description of in-medium effect
- Delimiting possible phases of the Yukawa theory
- Generalized 'gap equations' and equation of state for arbitrary mediator masses. Note this regime not encountered in the lab.
- Equation of state and some applications.

Phases in the Yukawa theory

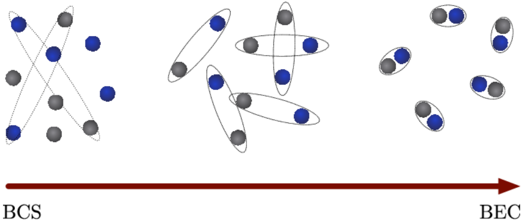
The model

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + \mu\bar{\psi}\gamma^0\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - g\,\bar{\psi}\psi\phi\ .$$

- 4 free parameters: m, m_ϕ, g and the density μ
- Dark particles singlets under SM. The fermion ψ charged under $U(1)_{\rm dark}$ global
- Fermi energy $E_F = \mu \equiv \sqrt{m^2 + k_F^2}$, number density $n = N/V \equiv \langle\bar{\psi}\gamma_0\psi\rangle$

Phases in the Yukawa theory

Scattering in the Yukawa theory



- The scattering length effectively captures the short distance properties of a potential

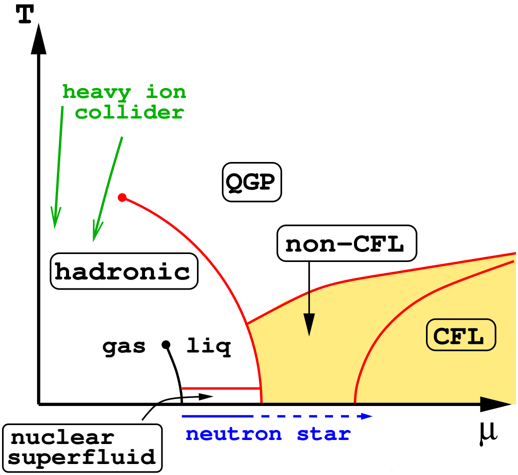
$$\lim_{k \to 0} k \cot \delta_0(k) = -\frac{1}{a} \ .$$

Computable for dilute gases in the non-relativistic limit.

- Anologous to contact interactions in low temperature physics, phases delimitted by dimenionless $k_F a$.

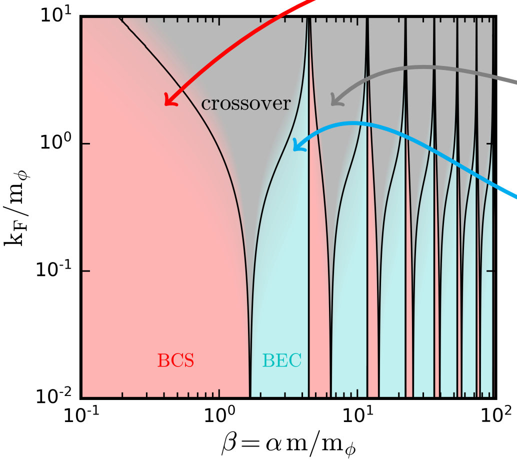
Phases of QCD

Alford, Schmitt, Rajagopal , Schäfer '08



Phases in the Yukawa theory at $T = 0$

Phases in the Yukawa theory RG, M.H.G Tytgat and J. Vandecasteele '22



BCS phase: $(k_F\, a)^{-1} <$ -1

Crossover: -1 $\leq (k_F\, a)^{-1} \leq$ 1

BEC phase: $(k_F\, a)^{-1} >$ 1

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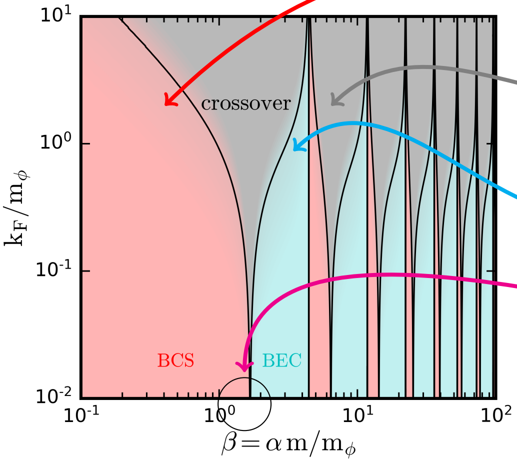
Phases in the Yukawa theory

RG, M.H.G Tytgat and J. Vandecasteele '22



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Unitary gas: $(k_F\, a)^{-1} \rightarrow 0$

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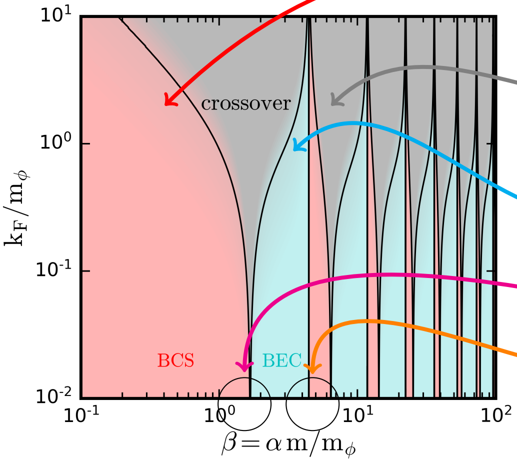

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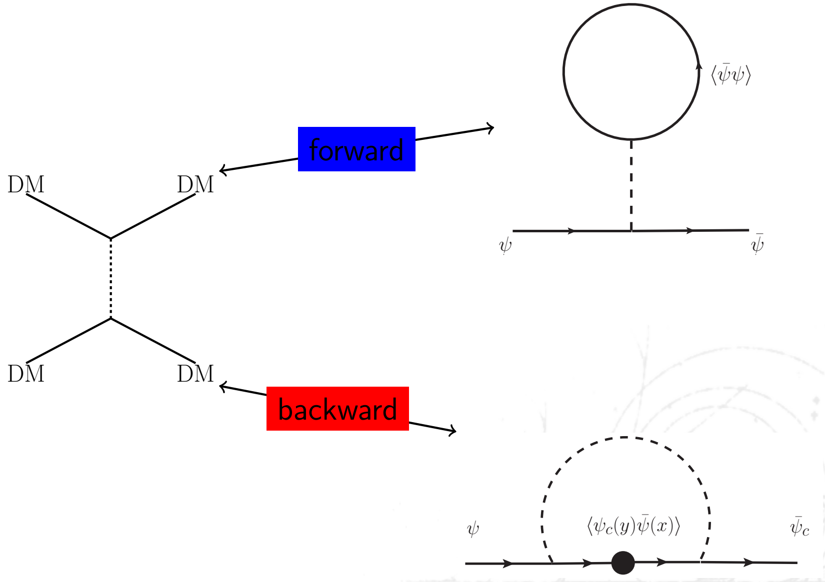
BEC phase: $(k_F\, a)^{-1} > 1$

Unitary gas: $(k_F\, a)^{-1} \to 0$

non-int. gas: $(k_F\, a)^{-1} \to \infty$

The BCS phase

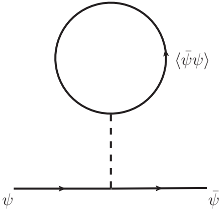
Cooper pairing and in-medium effects



Full forward scattering

Scalar density condensate

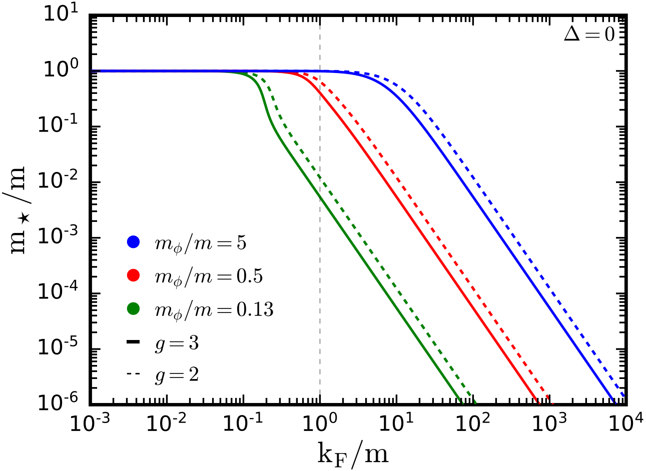
- Tadpole $\neq 0$ when $\mu \neq 0$
- The scalar operator $\bar{\psi}\psi$ has a non-zero mean, $n_s = \langle\bar{\psi}\psi\rangle > 0$ Waleck '74, Gresham et al. '18. $\implies n_s$ sources the scalar field due to its Yukawa interactions with the fermions
  - $\frac{\delta\mathcal{L}}{\delta\phi} = 0 \to m_\phi^2\langle\phi\rangle + g\langle\bar{\psi}\psi\rangle = 0$
  - $m_* = m + g\langle\phi\rangle \to m_* = m - \frac{g^2}{m_\phi^2} n_s(m_*)$



$\implies$ the fermion mass is reduced in the medium! (similar to NJL model of chiral symmetry breaking)

Full forward scattering

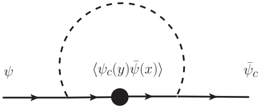
Results for scalar density condensate RG, M.H.G Tytgat and J. Vandecasteele '22



Full backward scattering

Cooper pairing and superfluidity: BCS argument

- Free enrgy for N particles $\Omega_N = E - \mu N$
- Add a particle $\implies$ $\Omega_{N+1} = E_{+1} - \mu\,(N+1)$
- If attractive interactions $\Omega_{N+1} < \Omega_N$
- Formation of many bosonic Cooper pairs which condensate $\sim \langle\psi\psi\rangle$ (Leon Cooper '57). Pairing in ^1S channel.
  - Object that gets a vev $\sim \left\langle \psi_C(y)\bar{\psi}(x)\right\rangle$, a 4×4 quantity



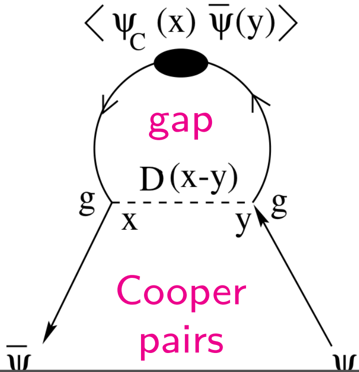
Full backward scattering

Qualitative physics

Yukawa theory when $m_\phi \gg m$: $4-$fermion interaction Schmitt '14

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \gamma^0\mu - m)\psi + G_\phi \bar{\psi}\bar{\psi}\psi\psi$$

$$\approx \begin{pmatrix}\bar{\psi} & \bar{\psi}_C\end{pmatrix} \begin{pmatrix} k\!\!\!/ + \mu\gamma^0 - m & \left\langle \psi\bar{\psi}_C \right\rangle \times G_\phi \\ \left\langle \psi_C\bar{\psi} \right\rangle \times G_\phi & k\!\!\!/ - \mu\gamma^0 - m \end{pmatrix} \begin{pmatrix}\psi \\ \psi_C\end{pmatrix}$$



The consistent set of gap equation

Gap structure and dispersion RG, M.H.G Tytgat and J. Vandecasteele '22

Δ has fermionic indices, respects Fermi statistics

$$\Delta_{\alpha\beta} \equiv \left\langle \psi_{C,\alpha}(x)\, \bar{\psi}_\beta(y) \right\rangle$$

Ansatz for the Yukawa theory Pisarski and Rischke '99

$$\Delta = \Delta_1 \gamma_5 + \Delta_2 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{k}} \gamma_0 \gamma_5 + \Delta_3 \gamma_0 \gamma_5$$

$$\mathcal{L} = \begin{pmatrix} \bar{\psi} & \bar{\psi}_C \end{pmatrix} \underbrace{\begin{pmatrix} \not{k} + \mu\gamma^0 - m & \Delta(k) \\ \Delta(k) & \not{k} - \mu\gamma^0 - m \end{pmatrix}}_{\text{inverse propagator}} \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$$

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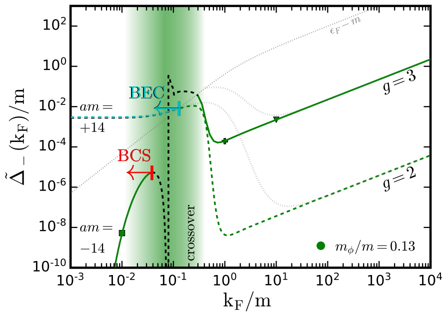
In BCS, $\Delta \ll \mu$

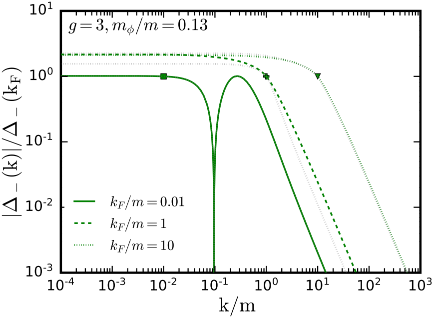
$$\epsilon_{\pm}^2 \approx (\omega \pm \mu)^2 + \left(\Delta_1 \pm \left(\frac{k}{\omega} \Delta_2 + \frac{m}{\omega} \Delta_3 \right) \right)^2$$

Like standard BCS theory but non-trivial momentum dependence

The consistent set of gap equation

Solution to gap equations: moderately light mediators RG, M.H.G Tytgat and J. Vandecasteele '22





Applications

Bullet cluster constraints

Effective range formalism

- s-wave scatterings are best understood in terms of scattering length a, effective range r_e H. Bethe '49, Tulin et al. '13, Chu et al. '20

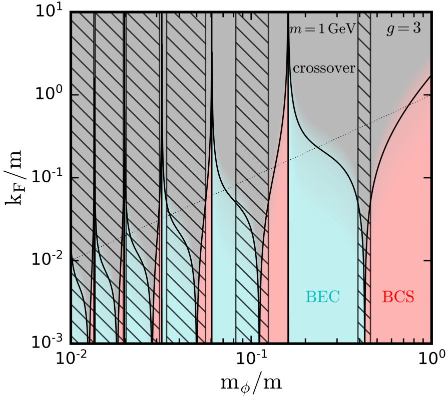
$$\mathcal{M}^{l=0} = \frac{1}{k\left(\cot\delta - i\right)}\,,\ \ k\cot\delta\big|_{\text{s-wave}} \approx \frac{-1}{a} + \frac{r_e}{2}k^2$$

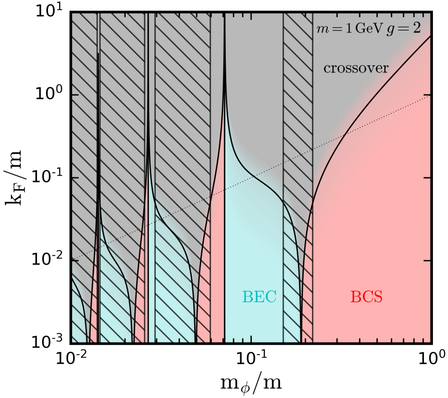
$$\sigma_0 \approx \frac{4\pi a^2}{1 + k^2\left(a^2 - a r_e\right) + \frac{1}{4}a^2 r_e^2 k^4}$$

- Phase shift $\leftrightarrow$ Schrödinger equation(dilute gases, non-relativistiic)
- also gap equation $\leftrightarrow$ Schrödinger equation (large density + relativistic)

Bullet cluster constraints

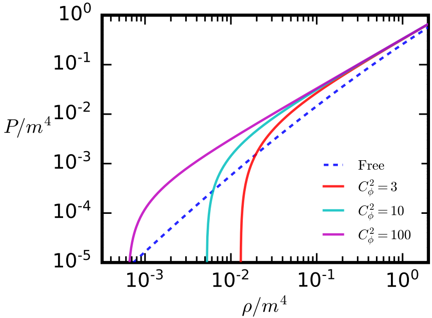
$\sigma/m \approx$ barn/GeV

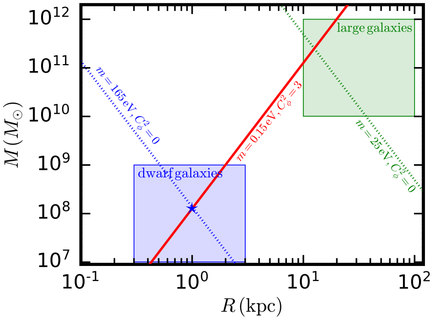




Equation of state

Application to Halos





Conclusions and Outlook

- Emergent phenomena can be realized in dark sectors due to DM−DM interactions. Many interesting phenomena arise with very little ingredients.
- Using scattering length we have delimitted phases of Yukawa theory.
- Very general framework to describe superfluidity, motivated by DM phenomenology. For arbitrary mediator masses all the way from non-relativistic limit to relativistic limit.
- Construct EoS that corectly interpolates between condensate dominated high density regions and low density Maxwellian regimes $\rightarrow$ realistic description of DM halos at dwarf galaxy scales.

Additional information

Scattering length in the non-relativistic limit

- Wave-function in CM frame $\psi = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$
- The s-wave amplitude $f_0 = \frac{e^{2i\delta_0 - 1}}{2ik}$
- At $k \to 0$ the wave function is $\psi = 1 - \frac{a}{r}$
- Scattering length $\lim_{k\to 0} k \cot \delta_0(k) = -\frac{1}{a}$
- Deutron (n-p spin triplet) $a = +7$ fm, whereas n-p spin singlet $a = -20$ fm
- In co-ordinate space Yukawa potential is $\pm\alpha\frac{e^{-m_\phi r}}{r}$
- Solve

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{l,k}}{dr}\right) + \left(k^2 - \frac{l(l+1)}{r^2} - m\,V(r)\right)R_{l,k} = 0\ ,$$

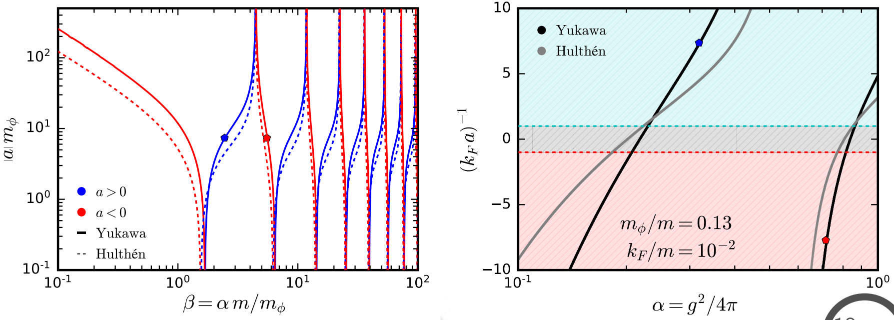
with boundary condition $r R_{l,k} = 0$ at $r = 0$

Scattering length in the non-relativistic limit continued

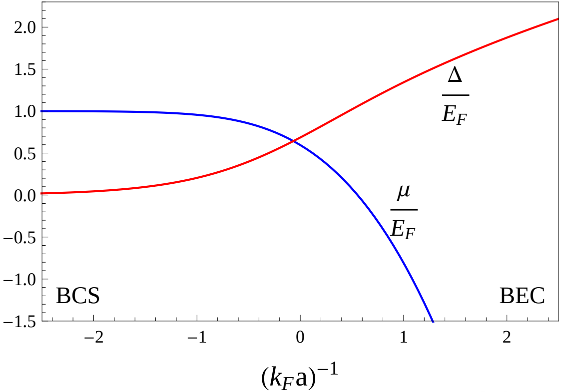
- With further massaging (see Chu, Garcia-Cely, Murayama '19)

$$\frac{d\delta_{l,k}(r)}{dr} = -k\,m\,r^2\,V(r)\,\mathrm{Re}\left[e^{i\delta_{l,k}(r)}h_l^{(1)}(kr)\right]^2 \;,$$

which is to be solved with boundary conditions $\delta_{l,k}(0) = 0$ and $\delta_{l,k} \to \delta_l$ at $r \to \infty$.



BCS-BEC crossover A. Schmitt '15



The consistent set of gap equation

Deriving the gap equations with variational methods

- Ansatz for the Dirac structure of gap Δ. In BCS $\Delta = \Delta_1 \gamma^5$.
- Starting from the action,
  $S = \int_{x,y} \left[\bar{\psi}(x)\, G_0^{-1}(x,y)\, \psi(y) - \frac{1}{2}\phi(x)\, D^{-1}(x,y)\, \phi(y)\right] - g \int_x \bar{\psi}(x)\, \psi(x)\, \phi(x)$
  do Hubbard-Stratonovich transformation to introduce gaps as auxilliary fields
  $1 \propto \int \mathcal{D}\bar{\Delta}\mathcal{D}\Delta \exp\{-\frac{1}{2}(\Delta - \psi_c\bar{\psi})\frac{D}{2}(\bar{\Delta} - \psi\bar{\psi}) - \frac{1}{2}(\bar{\Delta} - \psi\bar{\psi})\frac{D}{2}(\Delta - \psi_c\bar{\psi})\}$

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  do Hubbard-Strantanovich transformation to introduce gaps as auxilliary fields
- $S = \int_{x,y} \bar{\psi}(x) G_0^{-1}(x,y) \psi(y) + \bar{\Delta} D \Delta + \psi \bar{\Delta} \psi + \bar{\psi} \Delta \bar{\psi}$
  $Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\bar{\Delta} \mathcal{D}\Delta \, e^{-S}$
  Use mean-field approximation then compute path integrals

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  $\Omega = -\frac{T}{V} \log Z$

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  Use mean-field approximation then compute path integrals
- Obtain free energy Ω from the partition function
  $\Omega = -\frac{T}{V} \log Z$
- Differentiate Ω w.r.t gaps, set to 0 :
  $\frac{\partial \Omega}{\partial \Delta} = 0$
- Solve by iterative numerical methods. **Very general setup!**

The consistent set of gap equation

Advantages of variational methods

- Introduce auxilliary fields to bring fermion bilinears to quadratic form Stratonovich '57, Hubbard '59
- Captures multi-channel condensation Kleinert '11
- Applicable in all regimes without spurious cut-off Alford '01, Alford et al. '07, Kleinert '11

The consistent set of gap equation

RG, M.H.G Tytgat and J. Vandecasteele '22

$$\Sigma(0) = \frac{-g^2}{m_\phi^2}\sum_\eta \int \frac{d^3k}{(2\pi)^3}\left\{\frac{m_*}{\omega_k}\left(\frac{\omega_k+\eta\mu}{\epsilon_\eta(k)}-1\right)-\eta\frac{k}{\omega_k}\frac{\tilde{\kappa}(k)}{\omega_k}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}\right\},$$

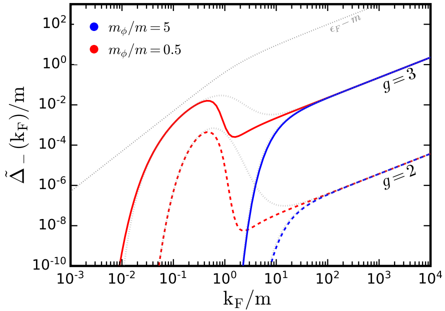
$$\begin{aligned}\tilde{\Delta}_\pm(p) &= \frac{g^2}{32\pi^2}\sum_\eta\int_0^\infty dk\,\frac{k}{p}\left\{\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\mp\eta\right.\\ &\quad \frac{kp}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)\\ &\quad \left.\pm\eta\frac{m_*^2}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)},\end{aligned}$$

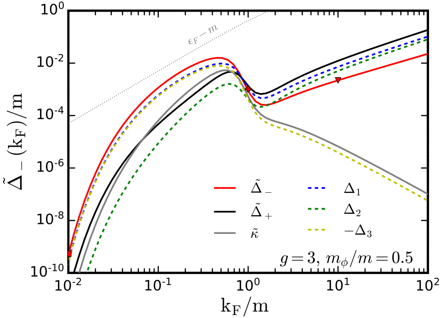
$$\begin{aligned}\tilde{\kappa}(p) &= \frac{g^2}{32\pi^2}\sum_\eta\int_0^\infty dk\,\frac{k}{p}\left\{-\eta\frac{m_*k}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)\right.\\ &\quad \left.-\eta\frac{m_*p}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}.\end{aligned}$$

The consistent set of gap equation

Solution to gap equations: moderately heavy mediators RG, M.H.G Tytgat and J. Vandecasteele '22

$$\Delta = \Delta_1 \gamma_5 + \Delta_2 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{k}} \gamma_0 \gamma_5 + \Delta_3 \gamma_0 \gamma_5$$





Equation of state

Thermodynamic considerations

- Consider first the case when $\Delta = 0$ but $m_\star \ll m$. Free energy Kapusta and Gale '11 also Gresham and Zurek '18

$$\Omega = -2T \int \frac{d^3p}{(2\pi)^3} \left[\log\left(1 + e^{-\beta(\omega_* - \mu)}\right) + \log\left(1 + e^{-\beta(\omega_* + \mu)}\right)\right] + \frac{1}{2} m_\phi^2 \phi_0^2$$

- parameterized by $C_\phi^2 = \frac{4}{3\pi} \alpha \frac{m^2}{m_\phi^2}$, and $m_*/m = 1 - \varphi$, $n = k_F^3/3\pi^2$

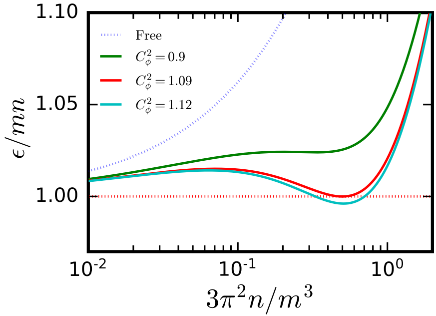
$$P = -\Omega = \frac{m^4}{3\pi^2} \left(-\frac{\varphi^2}{2C_\phi^2} + \int_0^{k_F/m} \frac{x^4}{\sqrt{x^2 + (1-\varphi)^2}} dx \right)$$

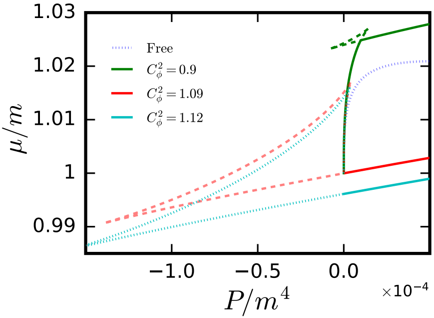
$$\epsilon = \mu n - P = \frac{m^4}{3\pi^2} \left(\frac{\varphi^2}{2C_\phi^2} + 3 \int_0^{k_F/m} x^2 \sqrt{x^2 + (1-\varphi)^2} dx \right) ,$$

Equation of state

With Maxwell construction

$$C_\phi^2 = \frac{4}{3\pi}\alpha\frac{m^2}{m_\phi^2},\; n = k_F^3/3\pi^2$$





Equation of state

as a function of coupling

$$C_\phi^2 = \frac{4}{3\pi}\alpha\frac{m^2}{m_\phi^2},\ n = k_F^3/3\pi^2$$