Topological sectors with defects

Florence Theory Group Day

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Based on:

2301.07035, LG

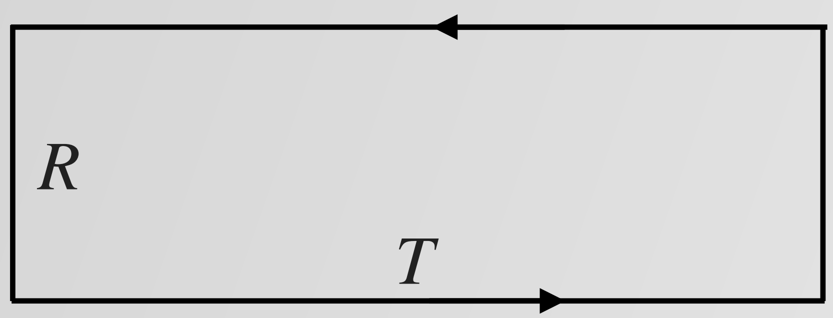
JHEP 04 (2022) 171 with Penati S., Yaakov I.

Why defects?

- Everywhere in nature
  - Boundaries, interfaces, impurities, entropy
  - Extended operators operators in QFT language

- Detect phase transition *[Wilson 74]*

$$W = Tr_R P e^{i\oint dx^\mu A_\mu}$$



  - Area law —> confinement
  - Perimeter law —> Coulomb phase

- Modern application: generalized symmetry *[Gaiotto, Kapustin, Seiberg, Willett 14]*

Conformal lines

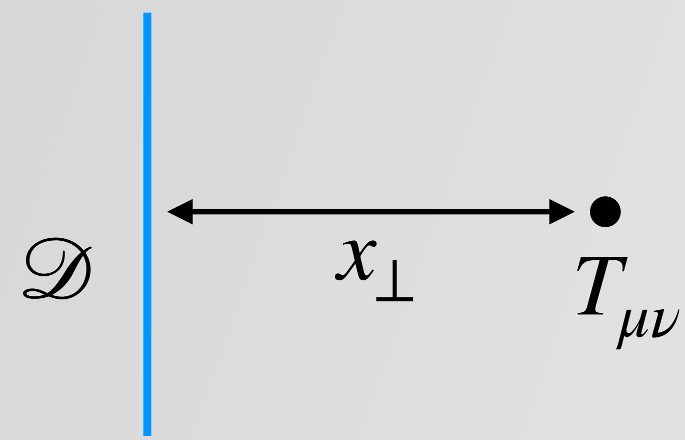
- Conformal line defects: magnetic impurities, Wilson lines…

[Billò, Gonçalves, Lauria, Meineri 2016]

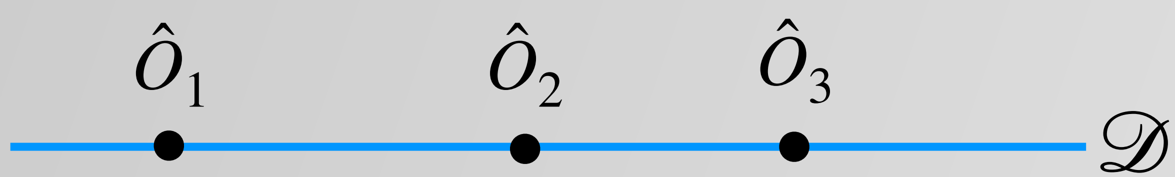
$$SO(d+1,1) \rightarrow SL(2,\mathbb{R}) \times SO(d-1)$$

- Non-trivial one point functions

$$\langle T_{\mu\nu} \rangle_{\mathscr{D}} \equiv \langle T_{\mu\nu}\, \mathscr{D} \rangle = \frac{h_T}{|x_\perp|^3} H_{\mu\nu}$$



- Defect CFT:



- Broken translations —> displacement & bremsstrahlung

$$\partial_\mu T^{\mu i} = \delta(\mathscr{D}) \hat{D}^i \qquad \langle \hat{D}^i(x_\parallel) \hat{D}^j(0) \rangle_{\mathscr{D}} = \frac{12\, B}{|x_\parallel|^4} \delta^{ij}$$

Why supersymmetry?

- Supersymmetric QFTs as theoretical laboratory
  - Confinement in 4d
  - Duality web 3d
  - AdS/CFT
  - Superconformal defects

- 3d $\mathcal{N} = 4$ multiplets
  - Vector multiplet: $\mathbb{V} = (A_\mu \,, \Phi_{\dot{a}\dot{b}} \,, \lambda_{\alpha,a\dot{a}} \,, D_{ab})$
  - Matter multiplet: $\mathbb{H} = (q_a \,, \tilde{q}_a \,, \psi_{\dot{a}} \,, \tilde{\psi}_{\dot{a}})$

BPS line defects

- BPS Wilson loops

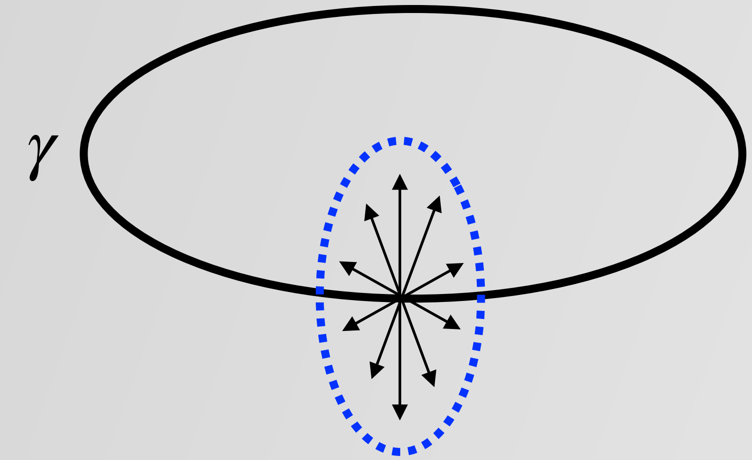
$$W = \mathrm{Tr}_R\, Pexp\left(-i\int d\varphi\left(A_\varphi + i\Phi_{\dot{1}\dot{2}}\right)\right)$$

- Vortex loops: disorder operator

[Kapustin, Willett, Yaakov 2012]
[Drukker, Okuda, Passerini 2012]

$$F = 2\pi q \delta_\gamma$$

$$D^{ab} = iq\, M_V^{ab} \star (\delta_\gamma \wedge dl)$$



- Can we find supersymmetric configurations *with defects and local operators?*

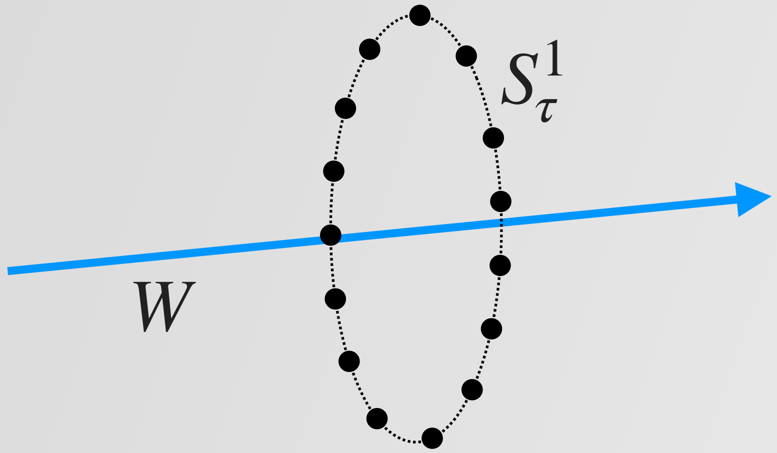
Adding local operators

- 3d $\mathcal{N} \geq 4$ theories admit protected local operator

  [Chester, Lee, Pufu, Yacoby 14]

- $\mathcal{Q}$-invariance and twisted translation $\hat{P}_\tau = P_\tau + R = \{\mathcal{Q}, \tilde{Q}\}$

- Line operator plus topological sector *[LG 23]*



✓ Wilson loop + Coulomb branch operators

$$\Phi(\tau) = v^{\dot{a}}(\tau) v^{\dot{b}}(\tau) \Phi_{\dot{a}\dot{b}}\,, \qquad v^{\dot{a}}(\tau) = \left(e^{\frac{i}{2}\tau},\, -e^{-\frac{i}{2}\tau}\right)$$

✓ Vortex loop + Higgs branch operators

$$Q(\tau) = u^a(\tau) q_a(\tau)\,, \qquad \tilde{Q}(\tau) = u^a(\tau)\tilde{q}_a(\tau) \qquad u^a(\tau) = \left(\cos\frac{\tau}{2},\, \sin\frac{\tau}{2}\right)$$

A cohomological Ward identity

- Conserved current multiplet $\mathbb{J} = \left(J_{ab}, K_{\dot{a}\dot{b}}, j_\mu, \chi_{a\dot{a}} \right)$, $\partial_\mu j^\mu = 0$
  - Topological operator $J = u^a u^b J_{ab}$
  - Mass deformations $S_{mass}[m] = \int \mathbb{V}_{back.}[m]\, \mathbb{J}$
- $J(\tau)$ integrated correlators from mass derivatives *[Agmon, Chester, Pufu 20]*

$$\frac{1}{(4\pi r)^n} \frac{1}{Z} \frac{d^n Z(m)}{d^n m}\bigg|_{m=0} = \langle \int d\tau_1\, J(\tau_1) \dots \int d\tau_n\, J(\tau_n) \rangle$$

- The following *cohomological Ward identity* holds *[LG, Penati, Yaakov 2021]* *[Bomans, Pufu 2021]*

$$\frac{\partial}{\partial m} S_{mass}[m] = 4\pi r^2 \oint_{S^1_\tau} J(\tau) + \{\mathcal{Q}, \dots\}$$

Defect generating functional

- Line defects preserve the cohomological supercharge

- Vortex loop + Higgs branch sector

$$\frac{1}{(4\pi r)^n}\frac{1}{\langle V\rangle}\frac{d^n\langle V\rangle(m)}{d^n m}\bigg|_{m=0} = \langle \int d\tau_1 J(\tau_1)\ldots \int d\varphi_n J(\tau_n) \rangle_V$$

- Wilson loop + Coulomb branch sector

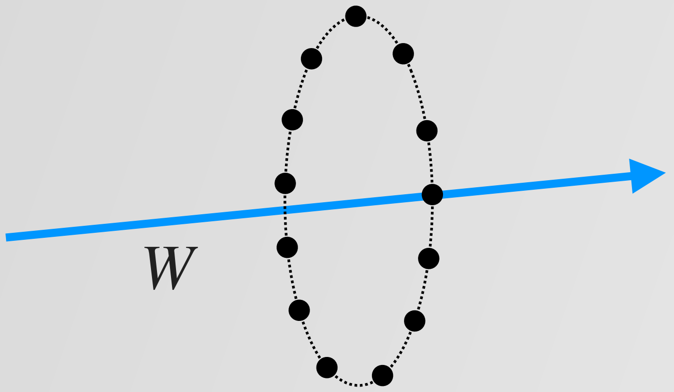
$$\frac{1}{(4\pi i r^2)^n}\frac{1}{\langle W\rangle}\frac{\partial^n}{\partial\zeta^n}\langle W\rangle(\zeta) = \langle \int_{S^1_{\tau_1}} d\tau_1\, \Phi(\tau_1)\cdots \int_{S^1_{\tau_n}} d\tau_n\, \Phi(\tau_n)\rangle_W$$

Example: ABJM

- $\mathcal{O}(\tau)$ is part of the stress tensor multiplet

$$\mathcal{C}(\tau) = e^{\frac{i}{2}\tau} C_1 - e^{-\frac{i}{2}\tau} C_3 \,, \qquad \bar{\mathcal{C}}(\tau) = e^{\frac{i}{2}\tau} \bar{C}^1 - e^{-\frac{i}{2}\tau} \bar{C}^3 \,,$$

$$\mathcal{O}(\tau) = \mathrm{Tr}\left(\bar{\mathcal{C}}(\tau)\mathcal{C}(\tau)\right)$$



- Give mass to C_1 and C_3 —>Ward identity

$$\frac{1}{(4\pi r)^n}\frac{1}{\langle W\rangle}\frac{d^n\langle W\rangle(m)}{d^n m}\bigg|_{m=0} = \langle \int d\tau_1 \mathcal{O}(\tau_1)\ldots \int d\tau_n \mathcal{O}(\tau_n) \rangle_W$$

- Localization —> l.h.s. is a tractable matrix model (e.g. $k \gg 1$)

$$\langle \mathcal{O}\rangle_W \propto h_T \propto B \propto \frac{2\pi N_1 N_2}{k(N_1+N_2)} - \frac{\left(\pi^3 N_1 N_2 \left(N_1 N_2 - 3\right)\right)}{3k^3\left(N_1+N_2\right)} + O(k^{-4})$$

$$\langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle_W = \frac{N_1 N_2}{64\pi^2 r^2} - \frac{N_1 N_2\left(N_1^2 + N_2^2 - 14\right)}{384 k^2 r^2}$$

Discussion

- New supersymmetric system with local and line operators
- Agreement with previous results for B

[Lewkowycz, Maldacena 2014]
[Bianchi, Griguolo, Mauri, Penati, Preti, Seminara 2017]
[Bianchi, Griguolo, Mauri, Penati, Seminara 2018]

- Formula for 1 point function of stress tensor for $\frac{1}{3}$-BPS lines
- ABJM at strong coupling
- Bootstrap?
- Application in holography?

Thank you!