# Non perturbative aspects of JT gravity and $T\bar{T}$ deformation

Jacopo Papalini

Galileo Galilei Institute

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Based on:
2106.01375, 2203.09683, 2207.05095, 2209.06222
*in collaboration with L. Griguolo, R. Panerai and D. Seminara*

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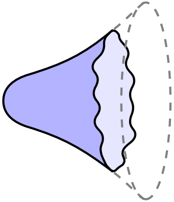

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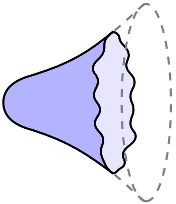

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The path integral runs over all distinct ways of embedding a non-self-intersecting $S^1$ in $\mathrm{EAdS}_2$.

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**Double scaled matrix model**:

$$Z(V, N) = \int \frac{\mathrm{d}H}{\mathrm{vol}(\mathrm{U}(N))} \exp(-N \operatorname{tr} V(H))$$

where one both scales $N \to \infty$ and $V$ to obtain the desidered spectral density $\langle \rho(E) \rangle_0 = e^{S_0} \sinh(2\pi\sqrt{E})/4\pi^2$.

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There are two branches for the deformed Schwarzian spectrum ($t = 4\epsilon^2$)

$$E_{\pm}(t) = \frac{2}{t}\left(1 \mp \sqrt{1 - tE}\right)$$

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The deformed partition function should read [Iliesiu, Kruthoff, Turiaci, Verlinde '20]

$$Z = \int_0^\infty \mathrm{d}E \, \frac{\sinh(2\pi\sqrt{E})}{4\pi^2} \, e^{-\beta E_+(t,E)}$$

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Complexification of the spectrum: the integral is **ill defined!**

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The full result for the disk

$$Z^{\text{disk}} = \frac{\beta}{2\sqrt{t}} \frac{e^{-2\beta/t}}{\beta^2 + \pi^2 t}\, I_2\!\left(\frac{2}{t}\sqrt{\beta^2 + \pi^2 t}\right)$$

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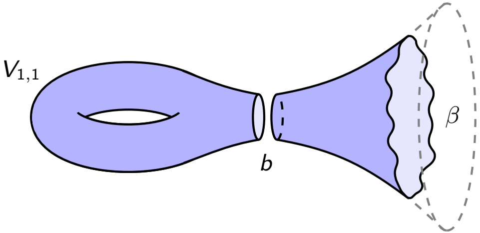
**Example:** $Z_{1,1}$



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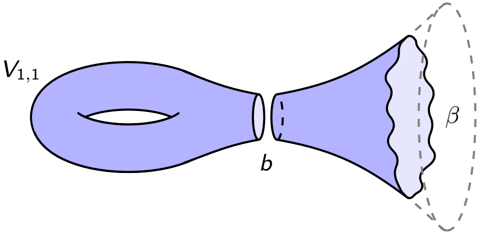


**In general:**

$$Z_{g,n}(\beta_1,\ldots,\beta_n) = \int_0^\infty \mathrm{d}b_1\ b_1 \ldots \int_0^\infty \mathrm{d}b_n\ b_n\ V_{g,n}(b_1,\ldots,b_n) \times Z^{\mathrm{tr}}(\beta_1,b_1)\ldots Z^{\mathrm{tr}}(\beta_n,b_n)\ ,$$

where $V_{g,n}(b_1,\ldots,b_n)$ is the volume of the moduli space of bordered hyperbolic Riemann surfaces.

# Topological recursion formula(Eynard–Orantin)

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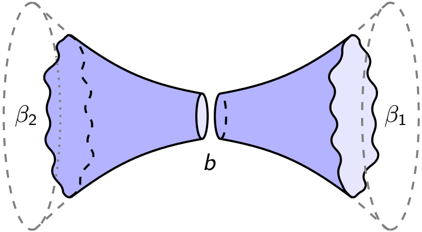


*Our results for general topologies satisfy the Eynard–Orantin recursion relations for a double scaled matrix model!*

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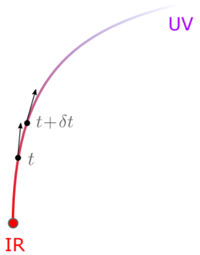

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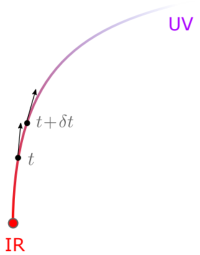


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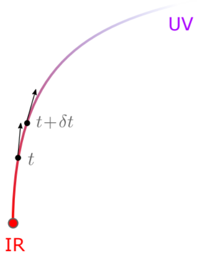


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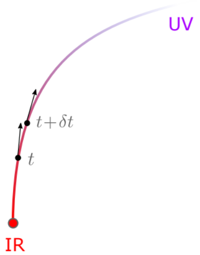


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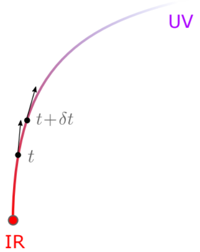


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- equivalent to coupling the theory to JT gravity [Dubovski, Gorbenko, Hernandez-Chifflet; Tolley; ...]

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- For any $\tau \neq 0$: the partition function diverges for $g < 2$ !
- The Hamiltonian is pathological at $c_2(R) \to \frac{N^3}{\tau}$, as $H \to \pm\infty$

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*What about the fate of these non perturbative terms at large $N$?*

- **Weak phase**:
  For $\alpha < \pi^2$ only the 0-instanton sector is not suppressed and yields the following free energy

$$F_0(\alpha, 0) = \frac{3}{4} + \frac{\alpha}{24} - \frac{1}{2}\log\alpha \qquad F_1(\alpha, 0) = -\frac{\alpha}{24} \qquad F_n(\alpha, 0) = 0 \;\; n \geq 2$$

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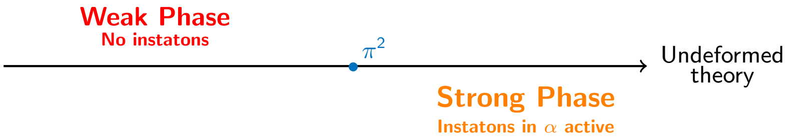
  In fact the ratio

$$\log\frac{Z_{(1,0,\ldots,0)}(\alpha)}{Z_{(0,0,\ldots,0)}(\alpha)} \sim -\frac{2\pi^2 N}{\alpha}\gamma(\alpha/\pi^2)\,, \qquad \gamma(z) = \sqrt{1-z} - \frac{z}{2}\log\frac{1+\sqrt{1-z}}{1-\sqrt{1-z}}\,.$$

vanishes for $\alpha = \pi^2$ and the one-instanton sector is no longer suppressed.

# The $T\bar{T}$ phase diagram at large $N$: a new tricritical point



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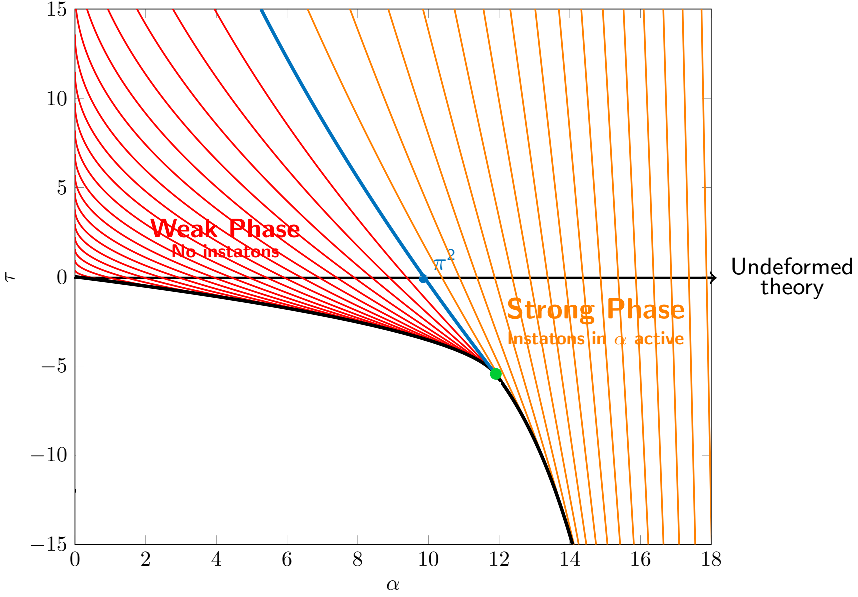

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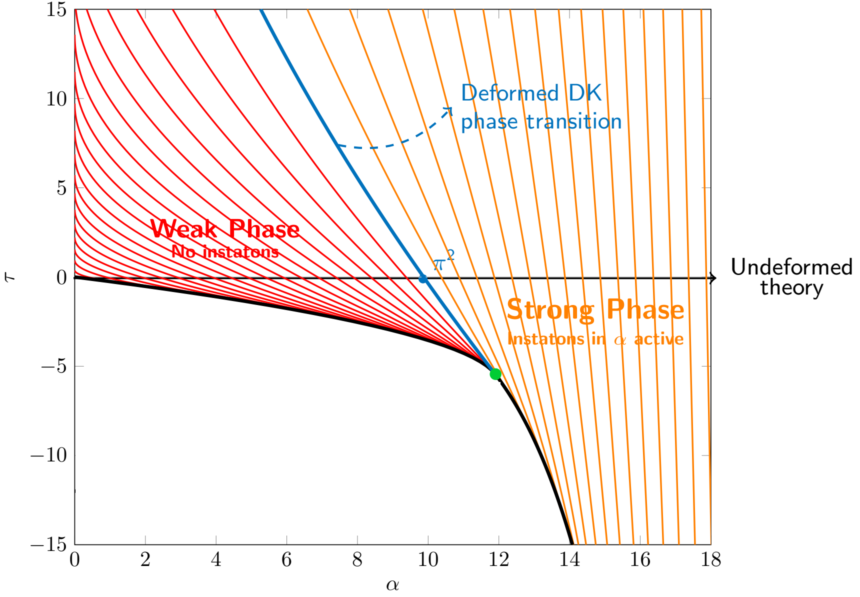

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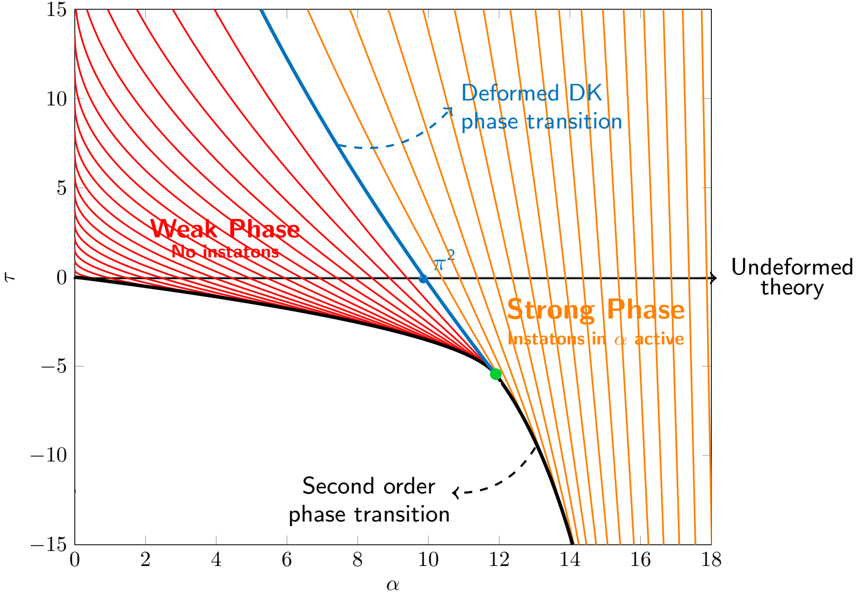

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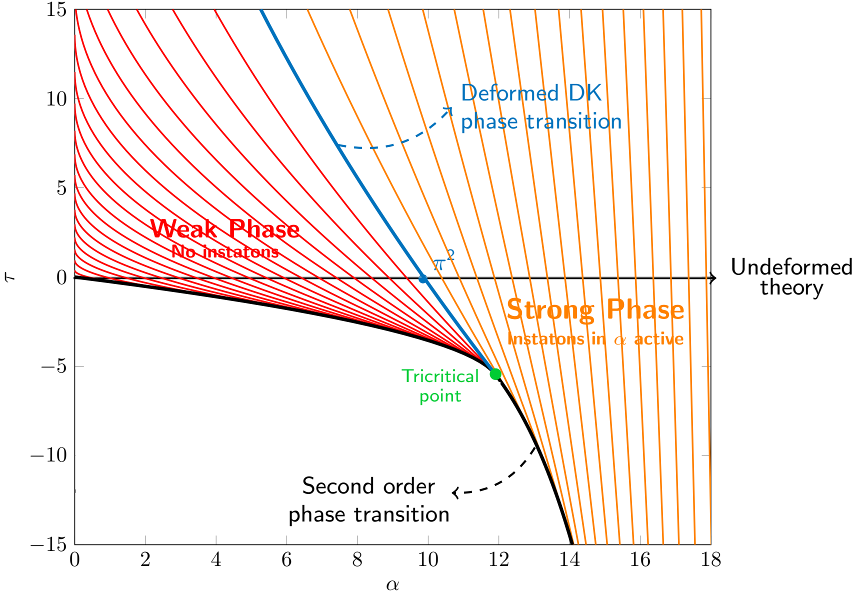

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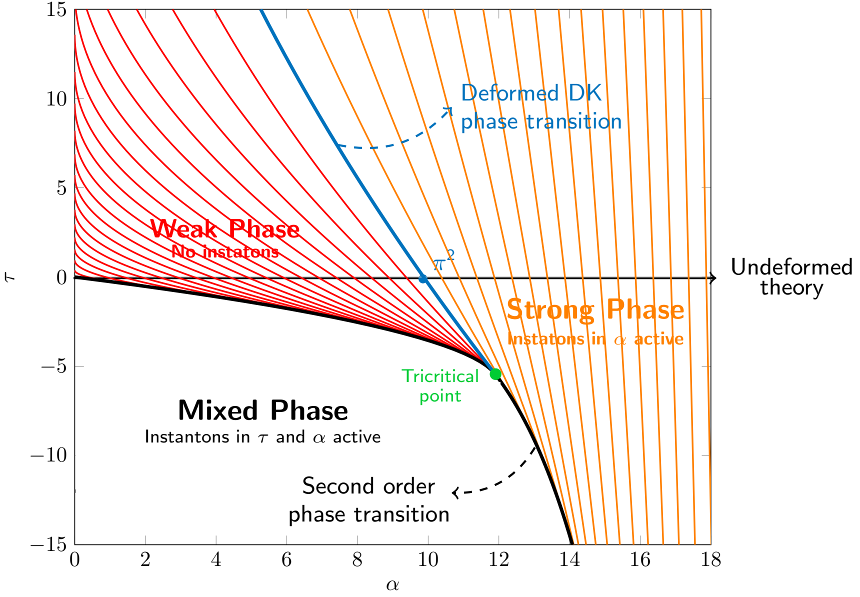

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# The end

Thank you!