Nuclear matrix elements of neutrinoless $\beta\beta$ decay: Where we were and where we are?

Javier Menéndez

University of Barcelona
Institute of Cosmos Sciences

Nuclear Tapas: The shell model as a cornerstone of nuclear structure,
in honor of Alfredo Poves

Madrid, 28th April 2023

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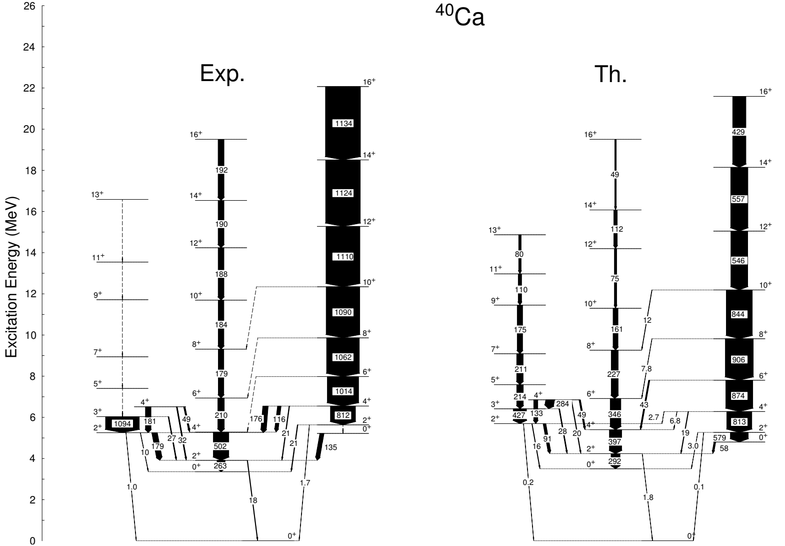
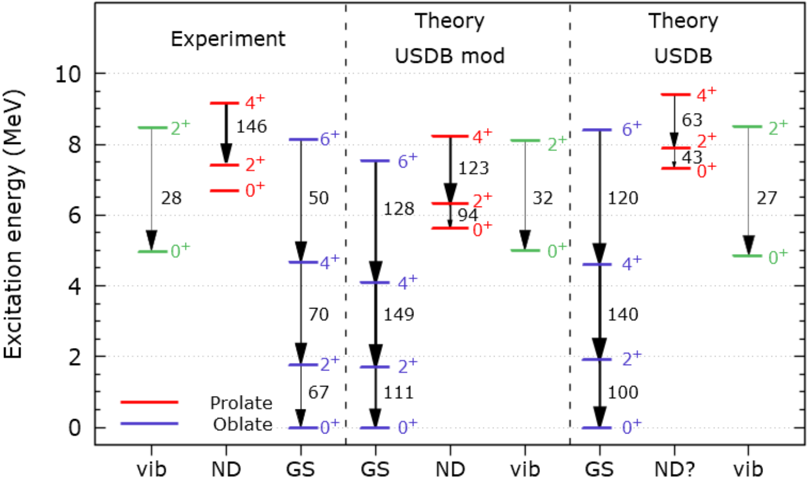
Spherical, deformed, superdeformed bands in ^{40}Ca



Caurier, JM, Nowacki, Poves, PRC 75, 054317 (2007)

Oblate, prolate, superdeformed? bands in ^{28}Si



Frycz, JM, et al. in preparation

Creation of matter in nuclei: $0\nu\beta\beta$ decay

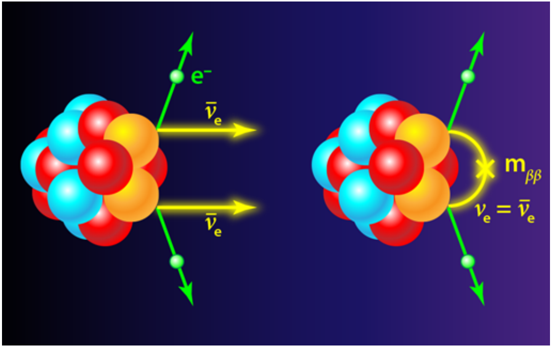
Lepton number is conserved in all processes observed:

single β decay,
$\beta\beta$ decay with neutrino emission...

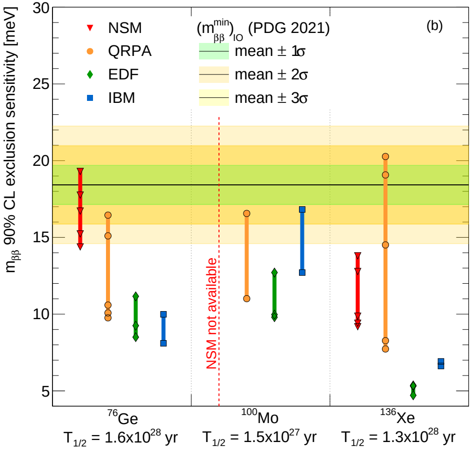
Uncharged massive particles like Majorana neutrinos (ν) allow lepton number violation:

neutrinoless $\beta\beta$ decay
two matter particles (electrons) created

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. in press, arXiv:2202.01787



Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani
Phys. Rev. C 104 L042501 (2021)

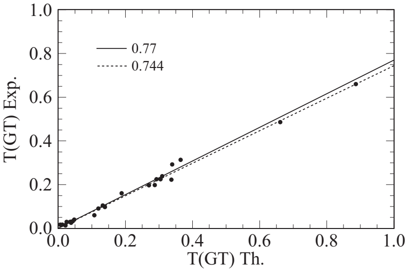
Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty in $m_{\beta\beta}$ prevents to foresee if next-generation experiments will fully cover parameter space of “inverted” neutrino mass hierarchy

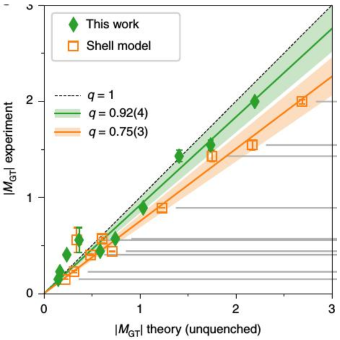
Uncertainty needs to be reduced!

β–decay Gamow-Teller transitions: "quenching"

β decays (e^- capture): phenomenology vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F| \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}}\, |I\rangle\,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Standard shell model needs $\boldsymbol{\sigma}_i \tau$ "quenching"

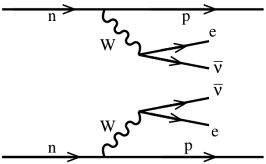
Ab initio calculations including meson-exchange currents and additional nuclear correlations do not need any "quenching"

Two-neutrino $\beta\beta$ decay, 2νECEC

$2\nu\beta\beta$ decay same initial, final states , similar operator ($\sigma\tau$) as $0\nu\beta\beta$
Comparison of predicted $2\nu\beta\beta$ decay vs data

Shell model reproduce $2\nu\beta\beta$ data including "quenching"

Prediction previous to ^{48}Ca measurement!
Caurier, Poves, Zuker
PLB 252 13(1990)



$$M^{2\nu\beta\beta} = \sum_k \frac{\langle 0_f^+ | \sum_n \sigma_n \tau_n^- \ |1_k^+\rangle \langle 1_k^+ | \sum_m \sigma_m \tau_m^- \ |0_i^+\rangle}{E_k - (M_i + M_f)/2}$$

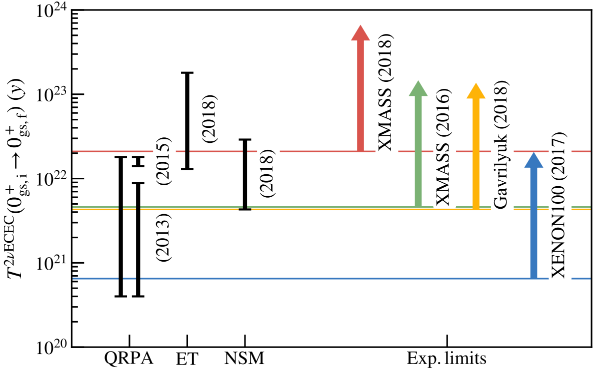
Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.047	kb3
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.048	kb3g
^{48}Ca → ^{48}Ti	0.047 ± 0.003	0.74	0.065	gxpf1
^{76}Ge → ^{76}Se	0.140 ± 0.005	0.60	0.116	gcn28:50
^{76}Ge → ^{76}Se	0.140 ± 0.005	0.60	0.120	jun45
^{82}Se → ^{82}Kr	0.098 ± 0.004	0.60	0.126	gcn28:50
^{82}Se → ^{82}Kr	0.098 ± 0.004	0.60	0.124	jun45
^{128}Te → ^{128}Xe	0.049 ± 0.006	0.57	0.059	gcn50:82
^{130}Te → ^{130}Xe	0.034 ± 0.003	0.57	0.043	gcn50:82
^{136}Xe → ^{136}Ba	0.019 ± 0.002	0.45	0.025	gcn50:82

Caurier, Nowacki, Poves, PLB 711 62 (2012)

Two-neutrino ECEC of ^{124}Xe

Predicted 2νECEC half-life:
shell model error bar largely dominated by "quenching" uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective theory (ET) predictions
suggest experimental detection close to XMASS 2018 limit

Two-neutrino ECEC of ^{124}Xe

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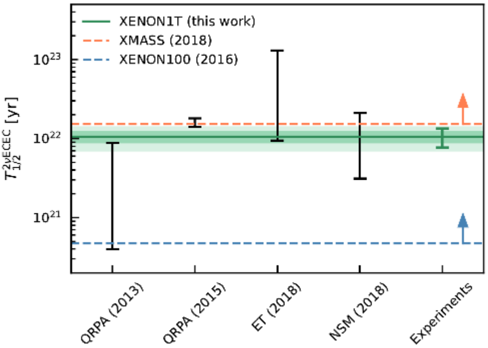


Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

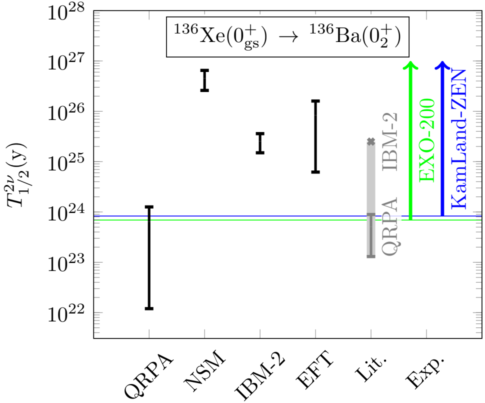
XENON1T
Nature 568 532 (2019)
PRC106, 024328 (2022)

Shell model, QRPA and Effective theory (ET) predictions
good agreement with XENON1T measurement of 2νECEC!

$2\nu\beta\beta$ decay of ^{136}Xe to ^{136}Ba 0_2^+

Current experiments sensitive to two-neutrino $\beta\beta$ of ^{136}Xe to ^{136}Ba 0_2^+

EXO-200, KamLAND-Zen



Nuclear shell model
QRPA, EFT and IBM
very different predictions!

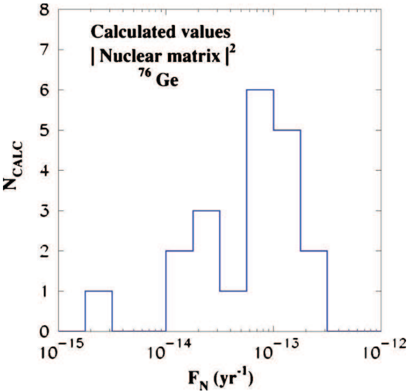
Barea et al.
PRC 91 034304 (2015)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Jokiniemi, Romeo, Brase, Kotila et al.
PLB 838 137689 (2023)

Very good test of theoretical calculations!

$0\nu\beta\beta$ decay nuclear matrix elements: 2003

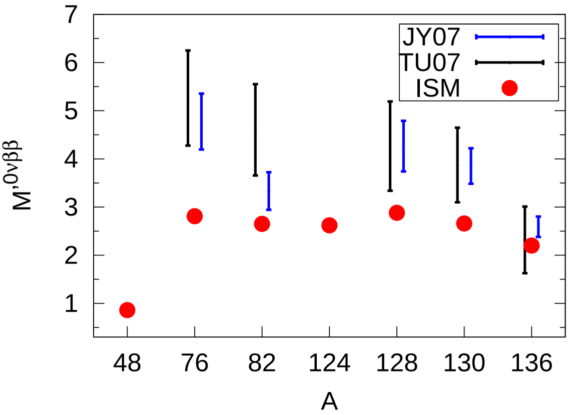


Strong disagreement in calculations of Nuclear Matrix Elements (NMEs)

The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos. The essential problem is that the correct theory of nuclei

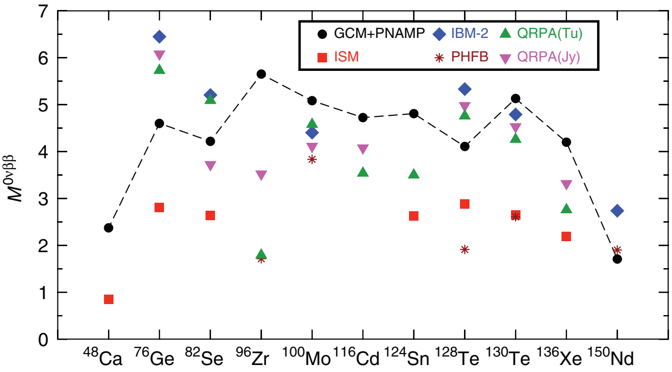
Bahcall, Murayama, Peña-Garay
PRD70 033012 (2004)

$0\nu\beta\beta$ decay nuclear matrix elements: 2009



$0\nu\beta\beta$ decay nuclear matrix elements: 2010

Finally, spread $\sim$ factor 2 in the different calculations of the NMEs

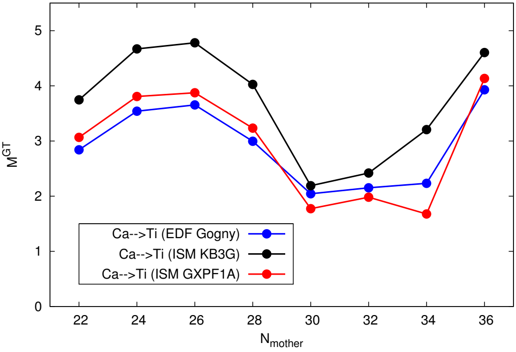


Rodríguez, Martínez-Pinedo PRL105 252503 (2010)

$0\nu\beta\beta$ decay without correlations

Non-realistic spherical (uncorrelated) mother and daughter nuclei:

- Shell model (SM): zero seniority, neutron and proton $J = 0$ pairs
- Energy density functional (EDF): only spherical contributions



In contrast to full (correlated) calculation SM and EDF NMEs agree!

NME scale set by pairing interaction

JM, Rodríguez, Martínez-Pinedo, Poves PRC90 024311(2014)

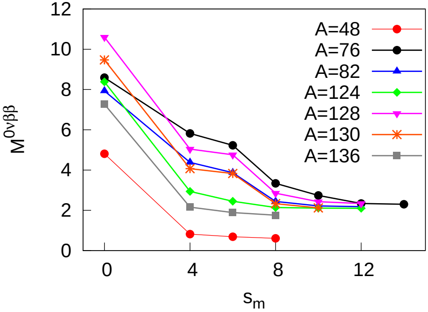
NME follows generalized seniority model:

$$M_{GT}^{0\nu\beta\beta} \simeq \alpha_\pi \alpha_\nu \sqrt{N_\pi + 1}\sqrt{\Omega_\pi - N_\pi}\sqrt{N_\nu}\sqrt{\Omega_\nu - N_\nu + 1},$$ Barea, Iachello PRC79 044301(2009)

Pairing correlations and $0\nu\beta\beta$ decay

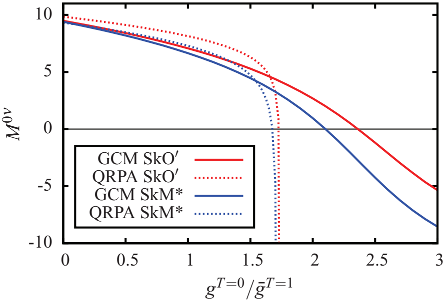
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing,
but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities



Caurier et al. PRL100 052503 (2008)

Addition of isoscalar pairing reduces matrix element value



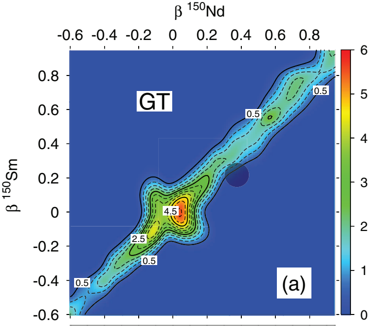
Hinohara, Engel PRC90 031301 (2014)

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j\tau_i\tau_j$ operator

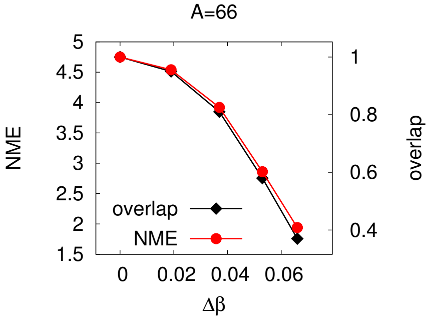
Deformation and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay is disfavoured by quadrupole correlations

$0\nu\beta\beta$ decay very suppressed when nuclei have different structure



Rodríguez, Martínez-Pinedo
PRL105 252503 (2010)



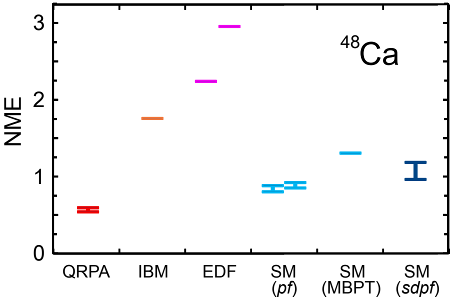
JM, Caurier, Nowacki, Poves
JPCS267 012058 (2011)

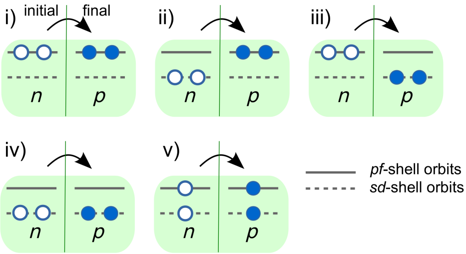
Suppression also observed with QRPA Fang et al. PRC83 034320 (2011)

Shell model configuration space

For ^{48}Ca enlarge configuration space from *pf* to *sdpf* (4 to 7 orbitals) increases matrix elements but only moderately 30%

Iwata et al. PRL116 112502 (2016)

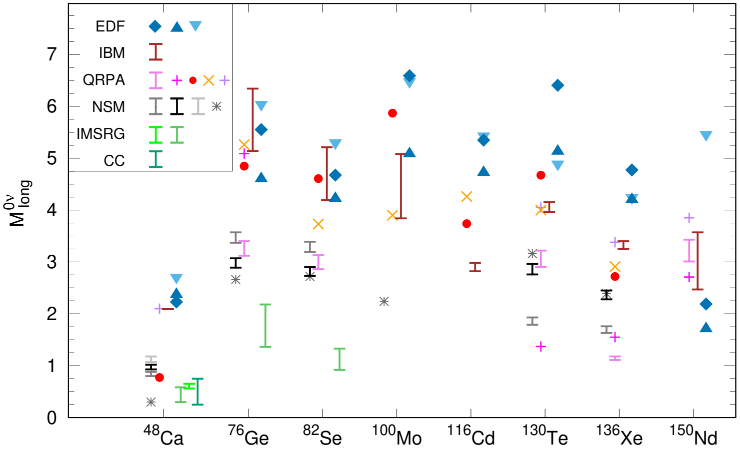




The contributions dominated by pairing (2p-2h) excitations enhance the $\beta\beta$ matrix element,
but the contributions dominated by 1p-1h excitations suppress the $\beta\beta$ matrix element

$0\nu\beta\beta$ decay nuclear matrix elements: now

Large difference in nuclear matrix element calculations: factor ~ 3



Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. in press, arXiv:2202.01787

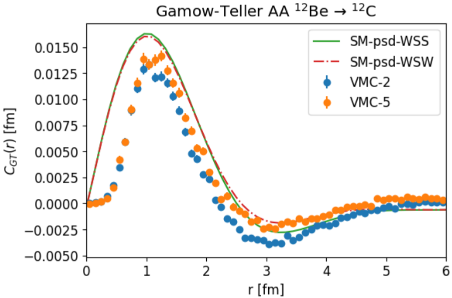
Shell model vs quantum Monte Carlo: correlations

Compare $\beta\beta$ transition densities in
nuclear shell model and quantum Monte Carlo calculations in light nuclei

$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle \, ,$$

$$M_{GT}^{0\nu} = \int_0^\infty dr \, C_{GT}^{0\nu} \, ,$$

Agreement at long distances, missing short-range correlations in shell model



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Similar findings in Wang et al. PLB 798 134974 (2019)

Generalized contact formalism (GCF)

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

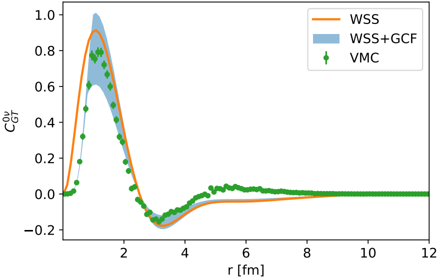
Separation of scales: wf, transition density factorize for two nearby nucleons

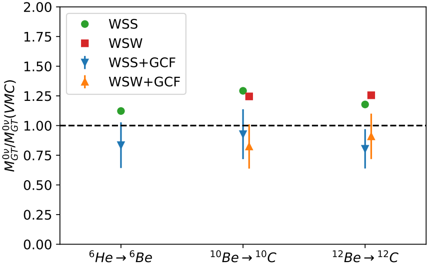
$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi^\alpha(\boldsymbol{r}_{ij}) A^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}), \quad \rho_{GT}(r) \xrightarrow[r\to 0]{} -3|\varphi^0(r)|^2 C^0_{pp,nn}(f,i)$$

with $\varphi(r)$ the solution of the two-nucleon Schrödinger equation

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ is model dependent

Replace shell-model by QMC contact
to improve transition density and nuclear matrix element

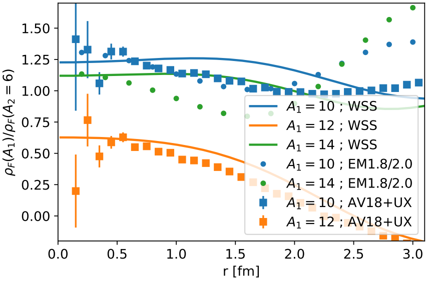


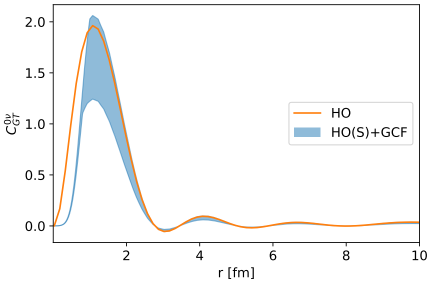


GCF: model independence of ratios

Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

The contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ is model dependent (shell model, quantum Monte Carlo, no-core shell model...)
but for two nuclei the ratio $C^0_{pp,nn}(X)/C^0_{pp,nn}(Y)$ relatively model independent:
combine QMC calculation in light nuclei with two shell model calculations:



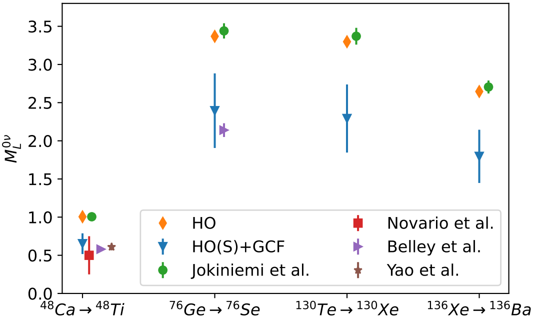


Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Shell model + Generalized contact formalism: NMEs

GCF builds QMC short-range correlations to shell model transitions densities can be extended to heavy nuclei where shell model calculations are possible

Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)



Short-range correlations included by GCF reduce $0\nu\beta\beta$ NMEs moderately

$\sim$ 30% reduction in general consistent with ab initio NMEs in ^{48}Ca, ^{76}Ge

Good agreement in benchmark NMEs in light nuclei with ab initio calculations

Light-neutrino exchange: contact operator

Contact operator suggested to contribute to light-neutrino exchange to absorb cutoff dependence of two-nucleon decay amplitude

Contribution of high-energy neutrinos

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\rm long} + M^{0\nu}_{\rm short}\right)^2 \frac{m_{\beta\beta}^2}{m_e^2},$$

Cirigliano et al. PRL120 202001(2018)

$$M^{0\nu}_{\rm short} \equiv \frac{1.2A^{1/3}\,{\rm fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \mathbb{1} \left[\frac{2}{\pi} \int j_0(qr)\, 2g_\nu^{\rm NN}\, g(p/\Lambda)\, p^2 dp\right] |0_i^+\rangle,$$

$$M^{0\nu}_{\rm GT} \simeq \frac{1.2A^{1/3}\,{\rm fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[\frac{2}{\pi} \int j_0(qr)\, \frac{1}{p^2}\, g_A^2\, f^2(p/\Lambda_A)\, p^2 dp\right] |0_i^+\rangle$$

Unknown value (and sign) of the hadronic coupling $g_\nu^{\rm NN}$!

Lattice QCD calculations can obtain value of $g_\nu^{\rm NN}$

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

or match $nn \to pp + ee$ amplitude calculated with approximate QCD methods

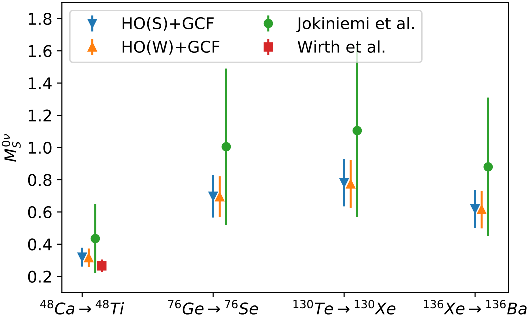
Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

Short-range NME: GCF + shell model

Shell model with short-range correlations from QMC using the GCF give consistent contribution of new term M_S

$\sim$ 25% impact of short-range NME in GCF + shell model obtained with $g_\nu^{\rm NN}$ from AV18 CIB term

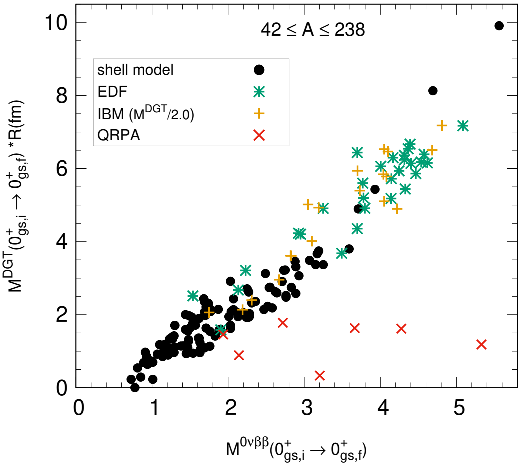
consistent with 43% effect in IM-GCM for ^{48}Ca using synthetic data on $nn \to pp + ee$ decay Wirth et al. PRL127 242502 (2021)



Weiss, Soriano, Lovato, JM, Wiringa, PRC106 065501 (2022)

Correlation of $0\nu\beta\beta$ decay to DGT transitions

Double GT transition to ground state
good linear correlation with $0\nu\beta\beta$ decay NMEs



Double Gamow-Teller correlation with $0\nu\beta\beta$ decay holds across nuclear chart

Shimizu, JM, Yako
PRL120 142502 (2018)

Common to shell model energy-density functionals interacting boson model, disagreement to QRPA

Also correlation in VS-IMSRG (but weaker)

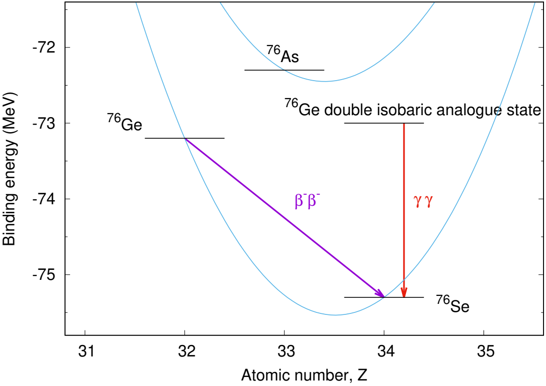
Yao et al. PRC106 014315(2022)

Experiments at RIKEN, INFN, RCNP? access DGT transitions

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays, focused on double-M1 transitions

$$M^{\gamma\gamma}_{M1\,M1} = \sum_k \frac{\langle 0^+_f | \sum_n (g^l_n \boldsymbol{l}_n + g^s_n \boldsymbol{\sigma}_n)^{IV} | 1^+_k(\text{IAS}) \rangle \langle 1^+_k(\text{IAS}) | \sum_m (g^l_m \boldsymbol{l}_m + g^s_m \boldsymbol{\sigma}_m)^{IV} | 0^+_i(\text{DIAS}) \rangle}{E_k - (E_i + E_f)/2}$$



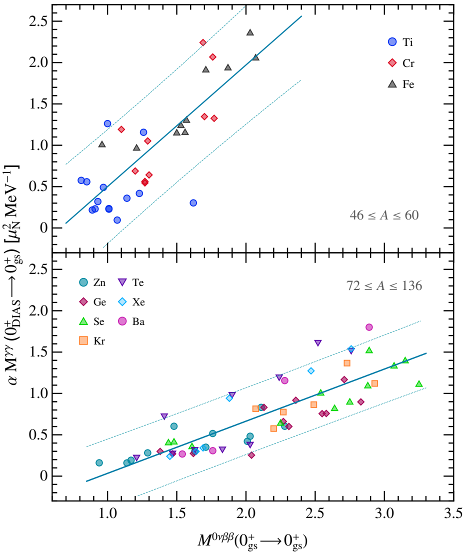
Similar initial and final states but both in same nucleus for electromagnetic transition

M1 and GT operators similar, physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



Good correlation between
$M1M1$ same-energy photons
and shell-model $0\nu\beta\beta$ NMEs
A dependence: energy denom.

$\gamma\gamma$ decays observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015)

Soderstrom et al. Nat.Comm.11 3242('20)

Particle emission, $M1$, $E1$:
$10^{-7} - 10^{-8}$ BR

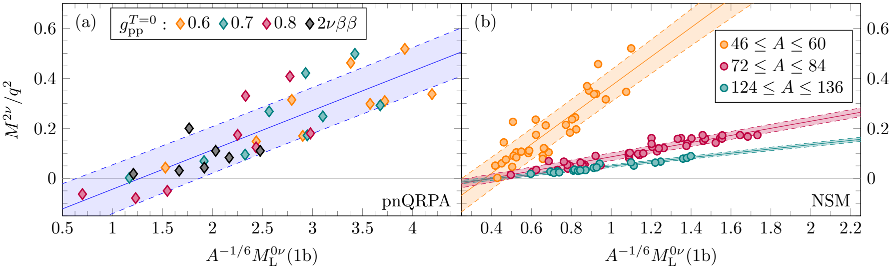
Experimental proposal for ^{48}Ti
by Valiente-Dobón et al.

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca Horoi et al. arXiv:2203.10577



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

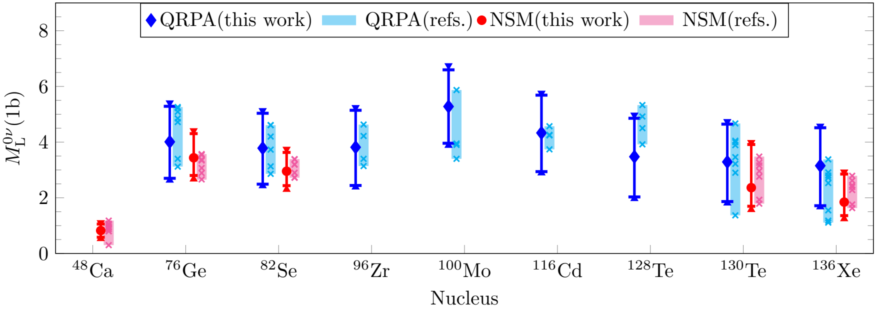
Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs!

$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

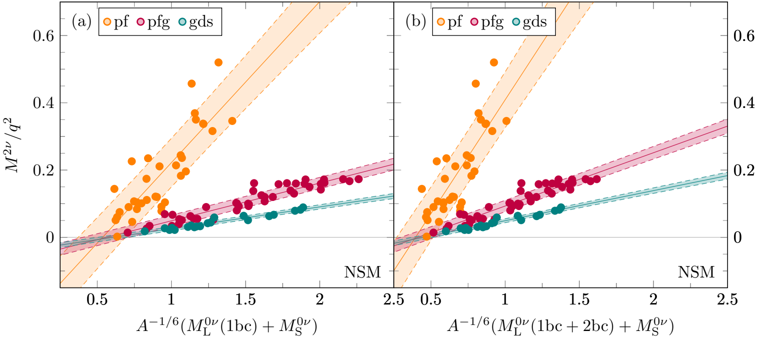
Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: general case

A good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also appears when we include to the calculation of $0\nu\beta\beta$ NMEs
2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents, short-range NME

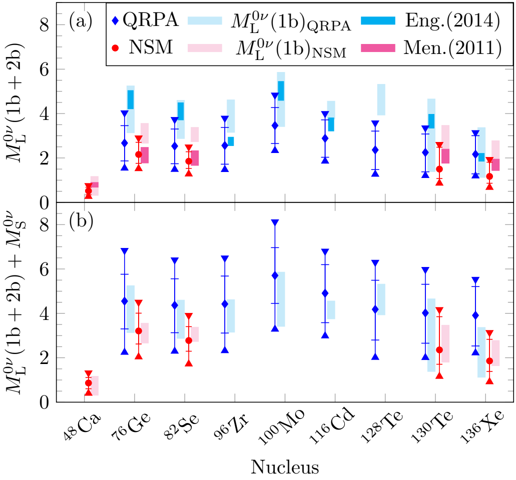
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation, calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108

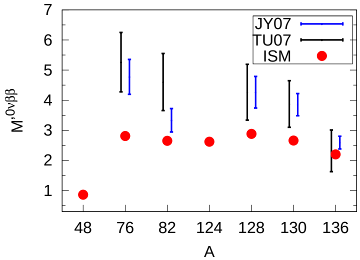


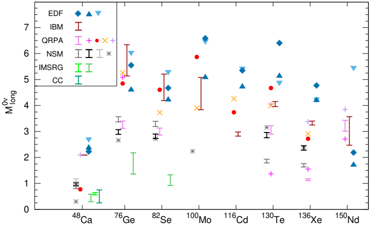
Summary

Calculations of $0\nu\beta\beta$ NMEs
challenge nuclear many-body methods,
searches demand reliable NMEs

Ab initio suggests relatively small NMEs
as first indicated by
shell-model NME calculations
with relevant correlations

Double Gamow-Teller transitions,
electromagnetic *M*1*M*1 decay of DIAS,
$2\nu\beta\beta$ decay NMEs
good correlation with $0\nu\beta\beta$ NMEs:
exploit correlation to obtain $0\nu\beta\beta$ NMEs
with theoretical uncertainties





Thank you very much!