Nuclear Tapas: the shell model as a cornerstone of nuclear structure
27–28 Apr 2023, Centro Cultural La Corrala, Madrid

Anomalous collective mode from the finite-N triaxial rotor

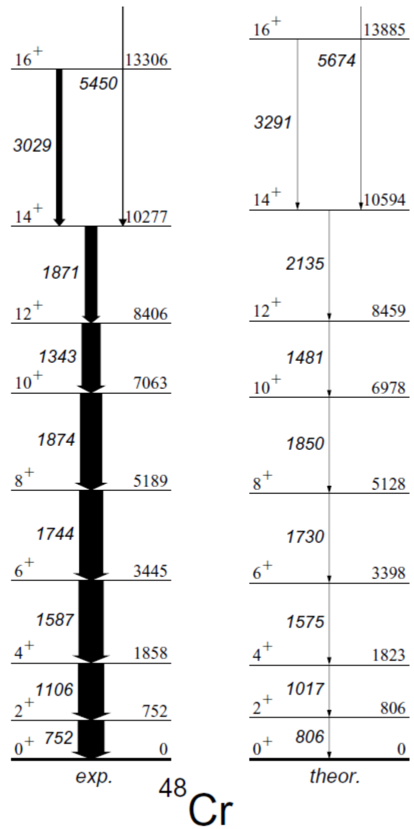
--Some thing that one cannot do with the shell model yet

Chong Qi

Department of Physics, KTH Royal Institute of Technology, Stockholm

An early story that may have not yet ended

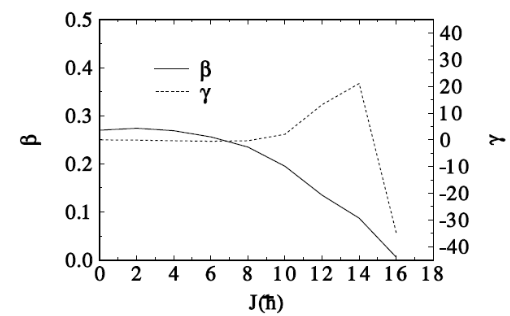
- SM can reproduce the rotational spectra of Cr isotopes perfectly
- But there seems some mismatch in the B(E2) patterns
- Is that something that we should worry about or worth looking into?
- If so, is there any physics one can learn from it



^{48}Cr level scheme; experiment *vs.* theory

Table 5. ^{48}Cr; quadrupole properties of the yrast band

J	$B(E2)_{exp}$	$B(E2)_{th}$	$Q_0(B(E2))$	$Q_0(Q_{spec})$	$Q_0[1f7/2, 2p3/2]$
2	321(41)	228	107	103	104
4	330(100)	312	105	108	104
6	300(80)	311	100	99	103
8	220(60)	285	93	93	102
10	185(40)	201	77	52	98
12	170(25)	146	65	12	80
14	100(16)	115	55	13	50
16	37(6)	60	40	15	40



. Deformation parameters β and γ as a function of J in the CHFB calculation

A. Poves and F. Nowacki: LNP 581, pp. 70–101, 2001.

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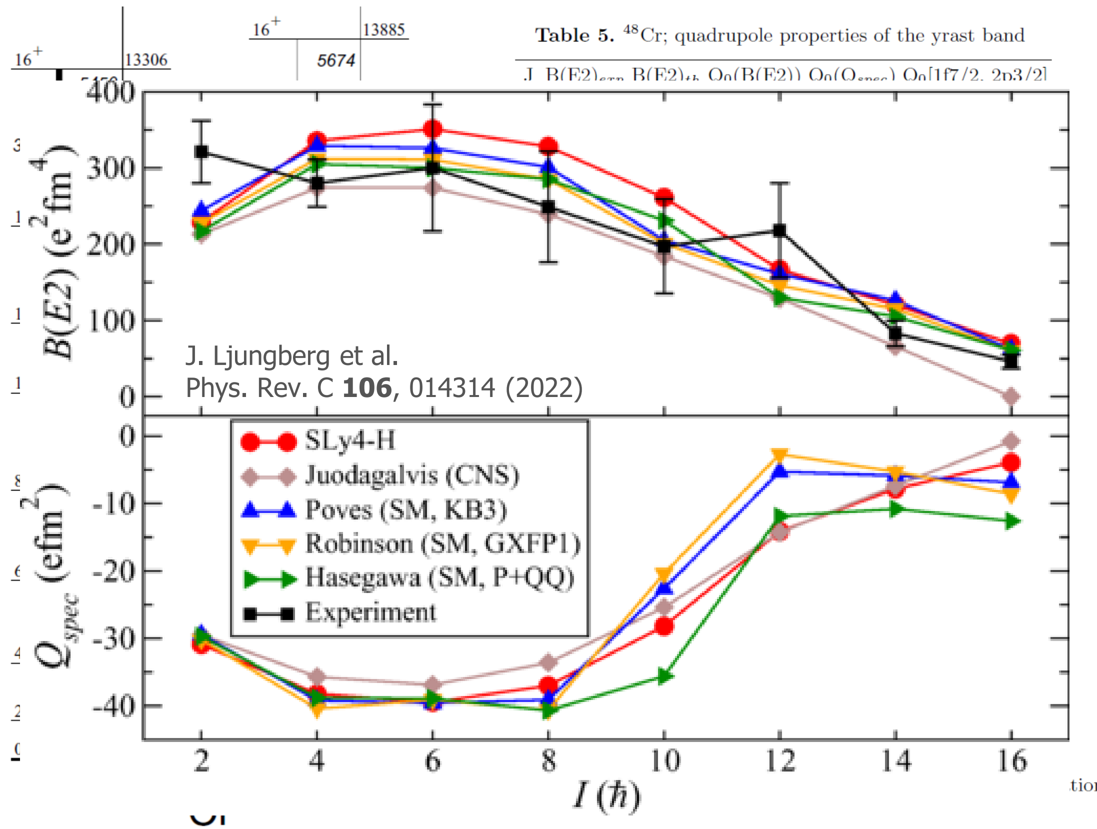


^{48}Cr level scheme; experiment $vs.$ theory

A. Poves and F. Nowacki: LNP 581, pp. 70–101, 2001.

PHYSICAL REVIEW C **70**, 047302 (2004)

Highly anomalous yrast $B(E2)$ values and vibrational collectivity

R. B. Cakirli,[1,2,3] R. F. Casten,[1,3] J. Jolie,[3] and N. Warr[3]
[1]*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520-8124, USA*
[2]*Istanbul University, Department of Physics, Turkey*
[3]*Institut für Kernphysik, Universität zu Köln, Köln, Germany*

(Received 24 June 2004; published 29 October 2004)

It is shown that the existing yrast $B(E2)$ values [especially the $B(E2;4_1^+\rightarrow 2_1^+)/B(E2;2_1^+\rightarrow 0_1^+)$ ratio] in ^{98}Ru are highly anomalous and cannot be plausibly interpreted with existing models. A survey of all even-even nuclei from $40\leqslant Z\leqslant 80$ shows that this phenomenon is rare in collective nuclei. It occurs to a much lesser extent in ^{114}Te, ^{114}Xe, and possibly a few other nuclides. The combination of vibrational-like energies and nonvibrational $B(E2)$ values perhaps points to a different kind of vibrational behavior.

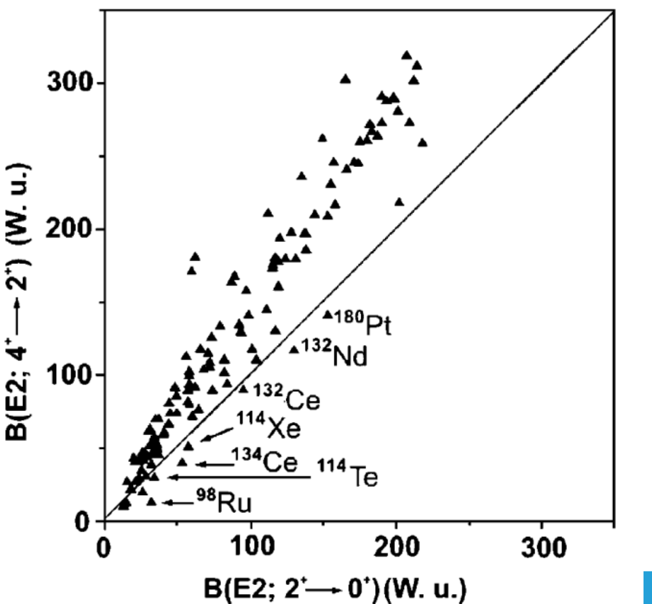
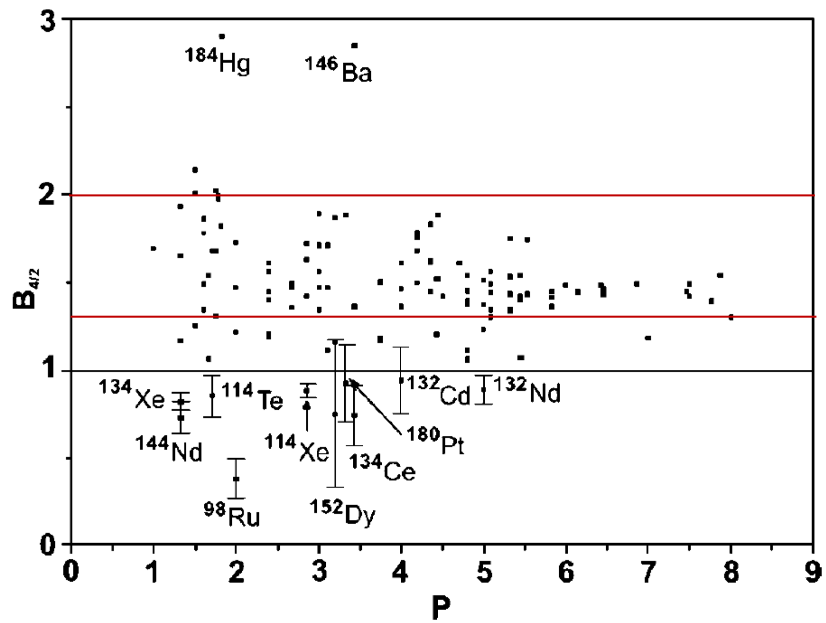


FIG. 2. Experimental yrast $B(E2)$ values for nonmagic nuclei



$$B_{4/2} \equiv B(E2;4_1^+\rightarrow 2_1^+)/B(E2;2_1^+\rightarrow 0_1^+)$$

PHYSICAL REVIEW C **70**, 047302 (2004)

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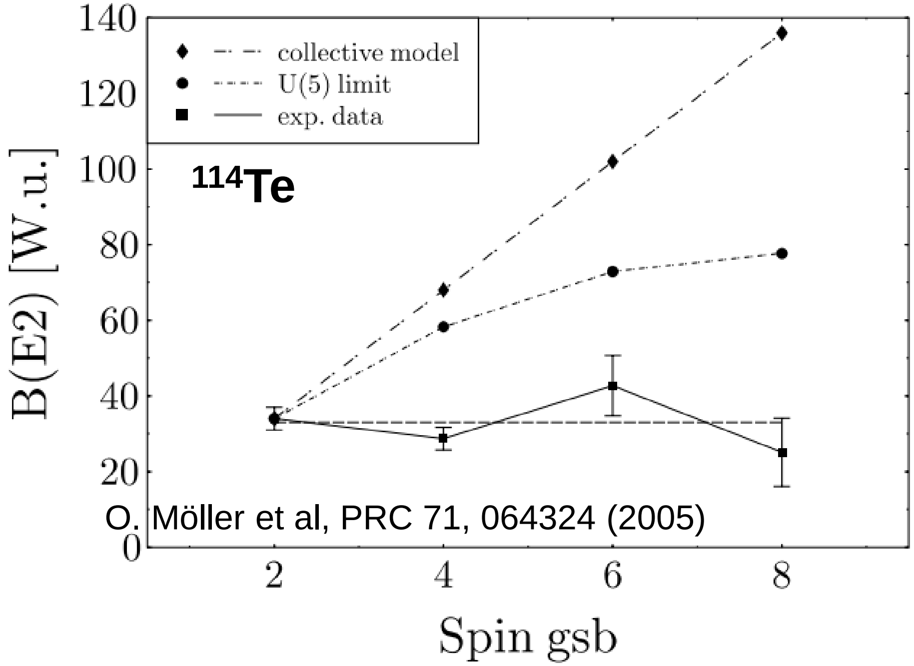
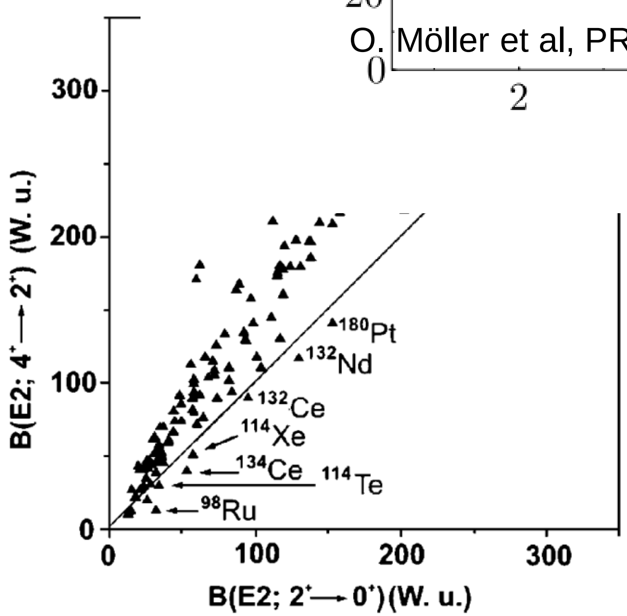


O. Möller et al, PRC 71, 064324 (2005)



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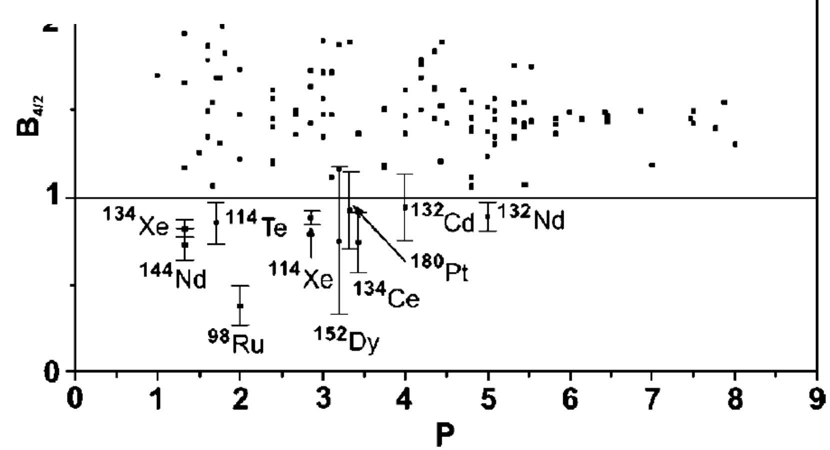

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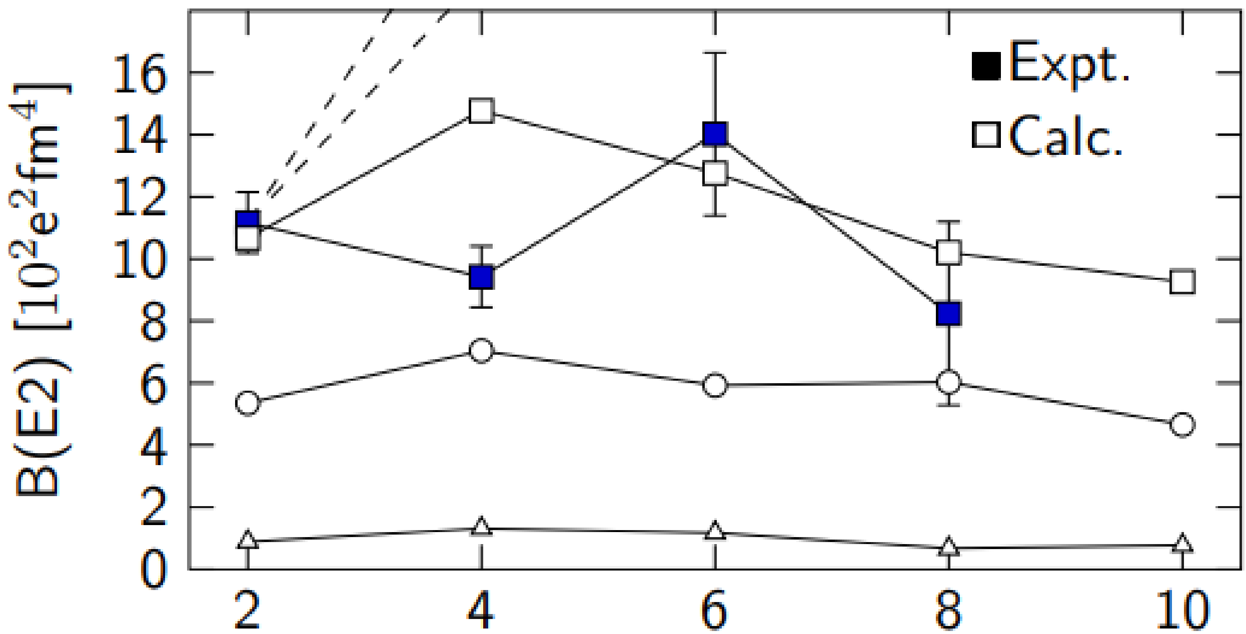
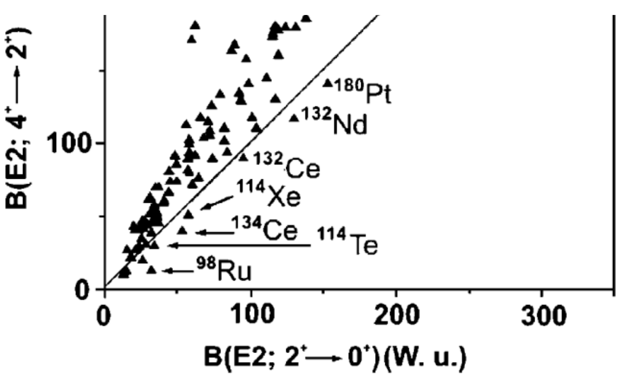
PHYSICAL REVIEW C 70, 047302 (2004)



CQ. Phys. Rev. C 94, 034310 (2016)



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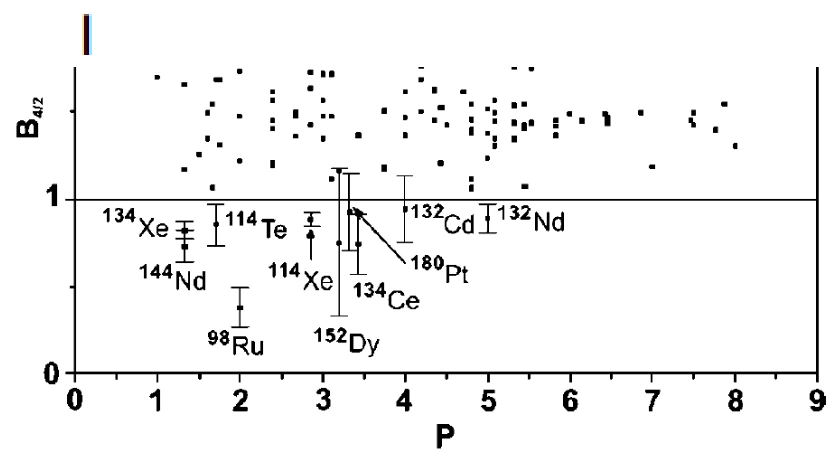
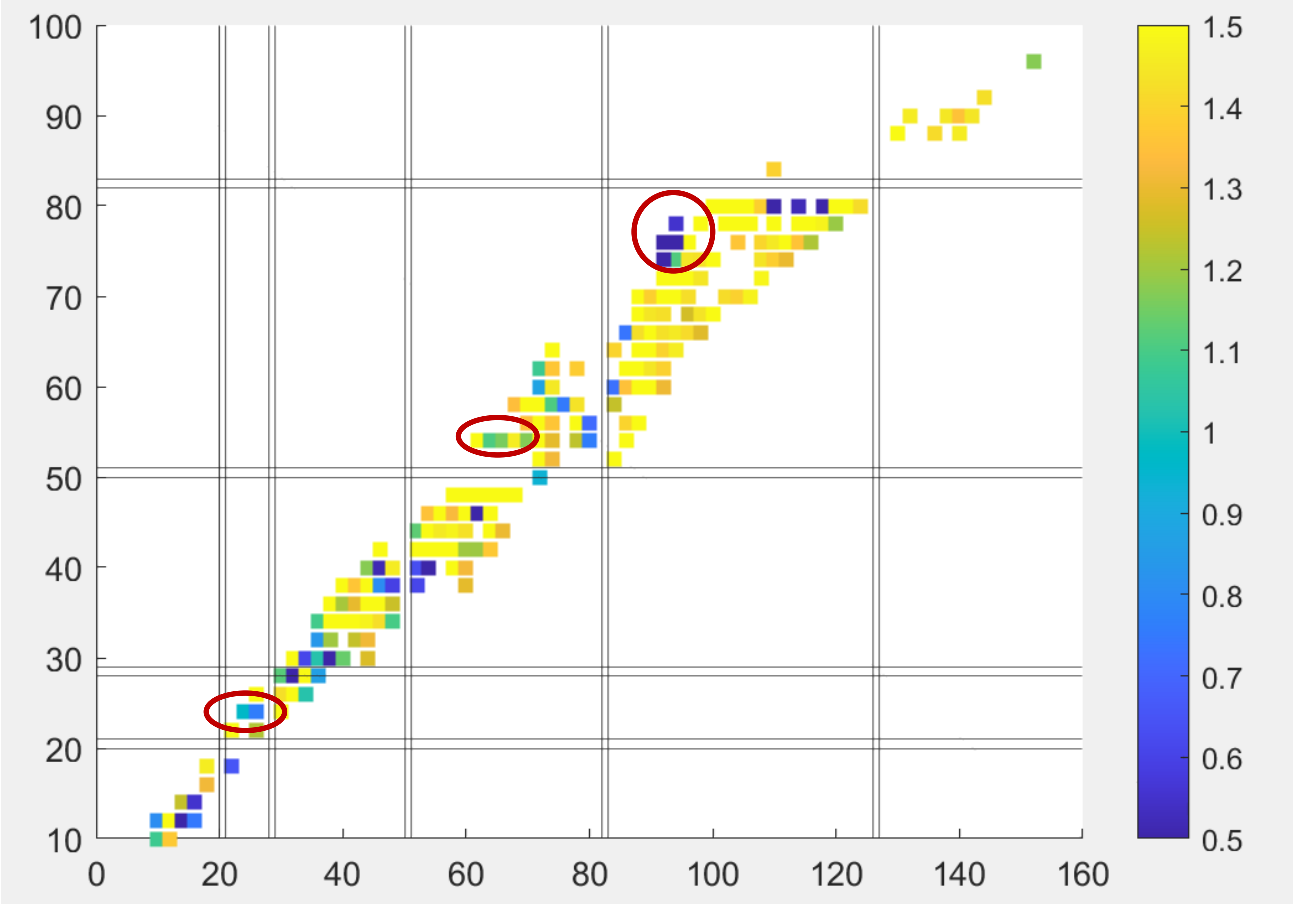


$B_{4/2} \equiv B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$

$B_{4/2}$ systematics

No theoretical models including SM and GCM can reproduce those anomalous $B_{4/2}$ patterns in open shell nuclei



^{98}Ru, ^{180}Pt seem to be ruled out

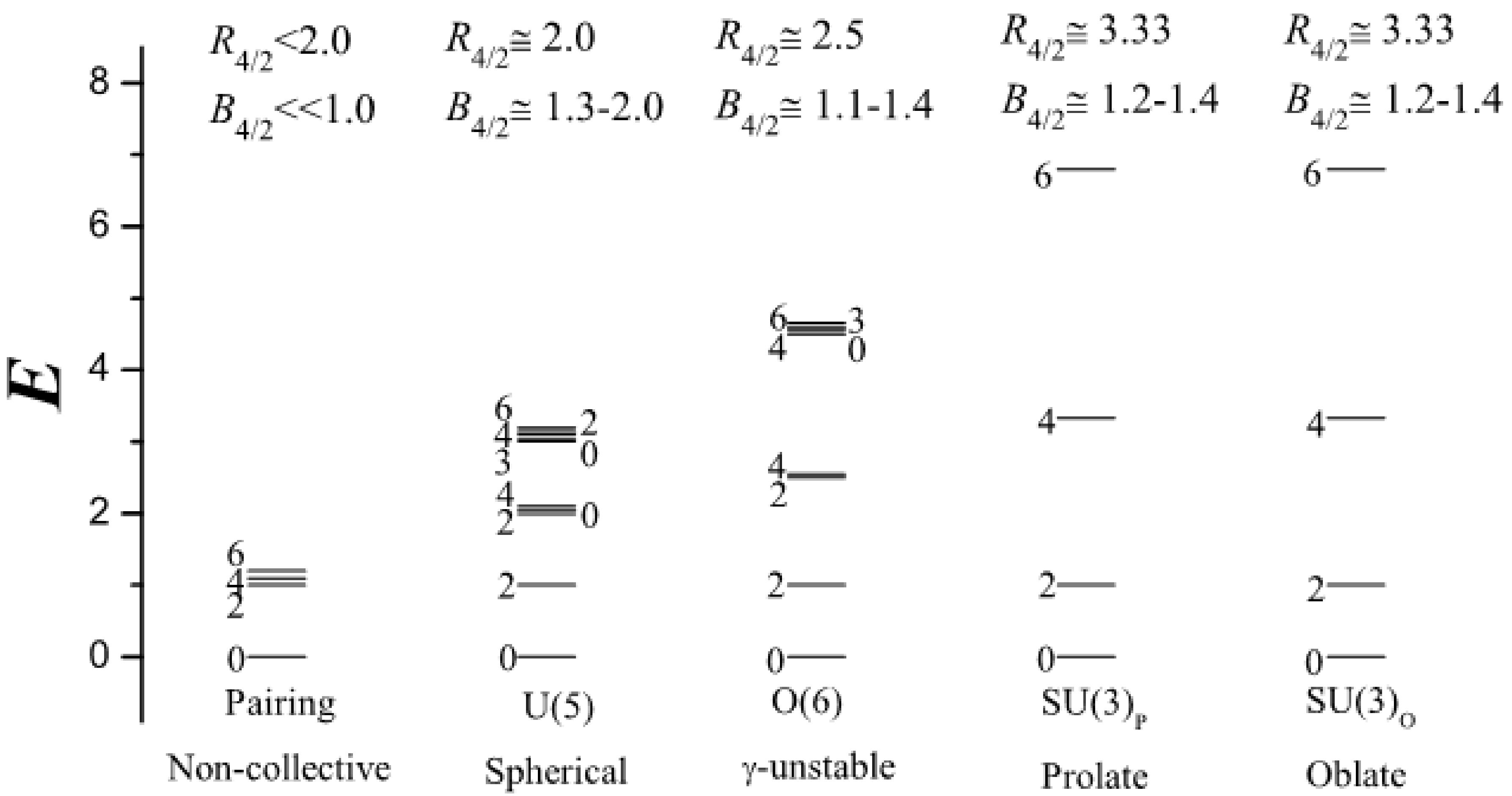
https://www.nndc.bnl.gov/nudat3/ + some recent papers

Outline

- Introduction
- Collective modes in the algebraic model
- Algebraic realization of the **finite-N**, **triaxial** rotor mode within the interacting boson model
- Application in N=90-100 nuclei
- Summary and future works

In collaboration with Yu Zhang, **Ying-Wen He, Daniel Karlsson,** Feng Pan, and J. P. Draayer, Phys. Lett. B 834, 137443 (2022)

Collective modes in the algebraic model



$$B_{4/2} \equiv B(E2;4_1^+ \rightarrow 2_1^+)/B(E2;2_1^+ \rightarrow 0_1^+)$$

$$R_{4/2} \equiv E(4_1^+)/E(2_1^+)$$

Collective modes in the algebraic model

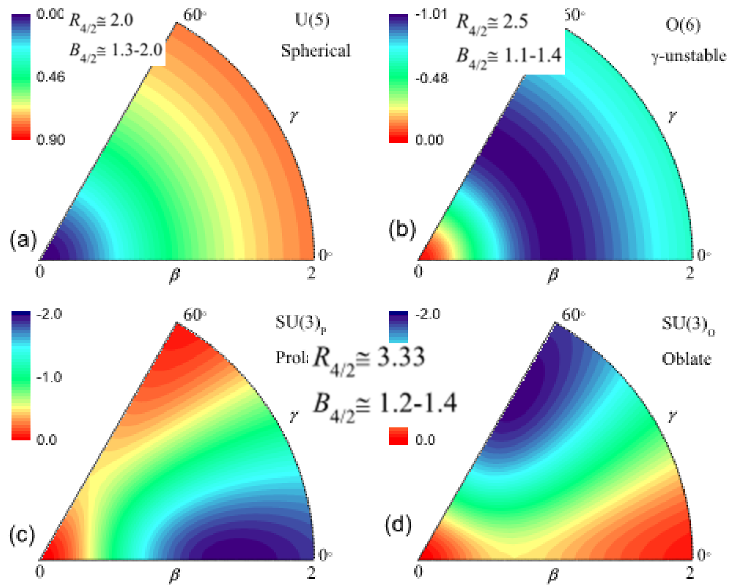


FIG. 1: Contour plots generated from the potential function (6) with the nonzero parameters taken as $\varepsilon = 1.0$ for U(5), $\kappa = -1.0$ for O(6), $(\kappa = -1.0,\ \chi = -\sqrt{7}/2)$ for $SU(3)_P$ and $(\kappa = -1.0,\ \chi = \sqrt{7}/2)$ for $SU(3)_O$.

- **PES in the classical limit**

$$V(\beta,\gamma) = \frac{1}{N}\langle \beta,\gamma,N|\hat{H}_{CQ}|\beta,\gamma,N\rangle|_{N\to\infty}$$
$$= \varepsilon\frac{\beta^2}{(1+\beta^2)} + \delta\frac{6\beta^2}{(1+\beta^2)}$$
$$+ \kappa\frac{4\beta^2}{(1+\beta^2)^2}\left[1-\sqrt{\frac{2}{7}}\chi\beta\cos(3\gamma)+\frac{1}{14}\chi^2\beta^2\right]$$

- **The consistent-Q Hamiltonian**

D. D. Warner and R. F. Casten, Phys. Rev. C 28, 1798 (1983).

$$\hat{H}_{CQ} = \varepsilon\hat{n}_d + \kappa\frac{1}{N}\hat{Q}^{\chi}\cdot\hat{Q}^{\chi}$$

$$\hat{n}_d = d^\dagger\cdot\tilde{d} \text{ and } \hat{Q}^{\chi}_u = (d^\dagger s + s^\dagger\tilde{d})^{(2)}_u + \chi(d^\dagger\times\tilde{d})^{(2)}_u$$

- **ε/κ , χ determine the deformation**
- $d^\dagger\cdot\tilde{d}$ **generators for U(5),** n_d **number conserved**
- **N total boson number**

- **The Hamiltonian spans the (β,γ) space and incorporate following dynamic symmetries:**

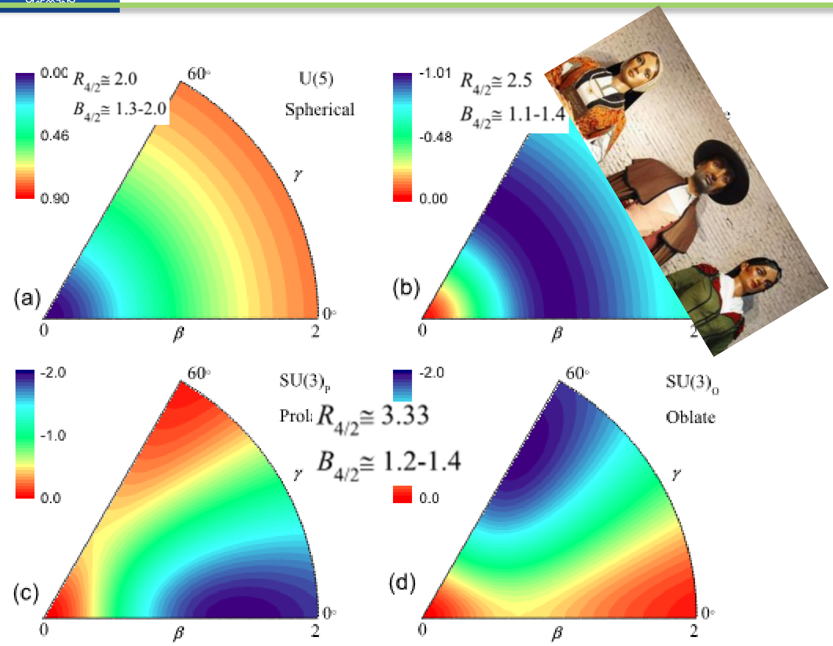
U(5) when $\varepsilon > 0$ and $\kappa = 0$;

O(6) when $\varepsilon = 0,\ \kappa < 0$ and $\chi = 0$;

SU(3) when $\varepsilon = 0,\ \kappa < 0$ and $\chi = \pm\sqrt{7}/2$.

- **No triaxial minimum or rotor mode**

Collective modes in the algebraic model



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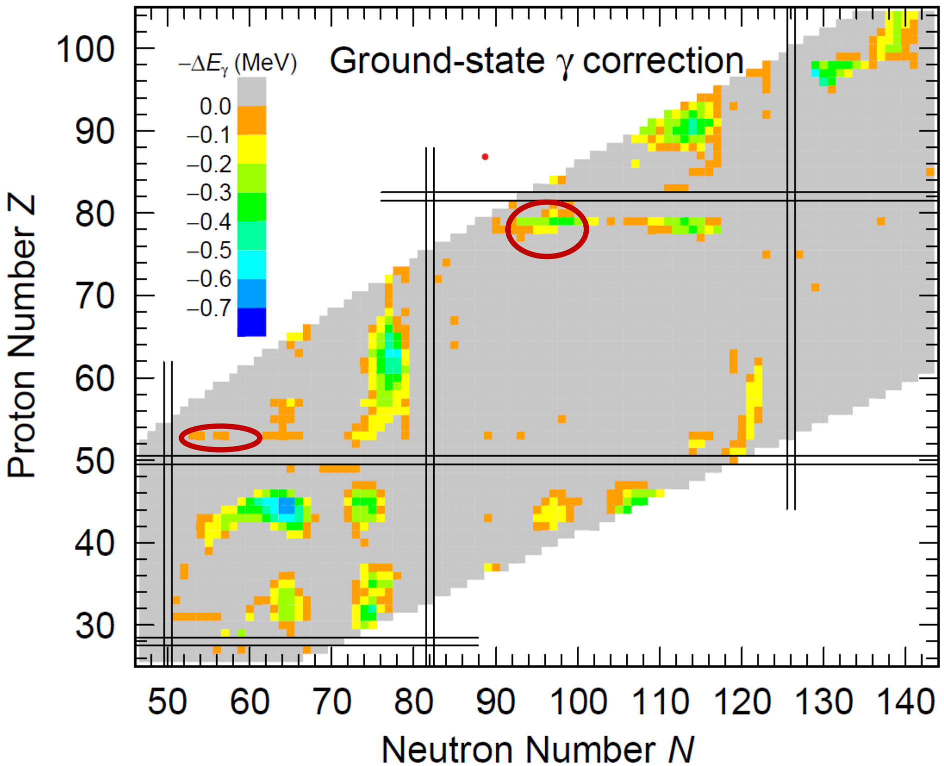
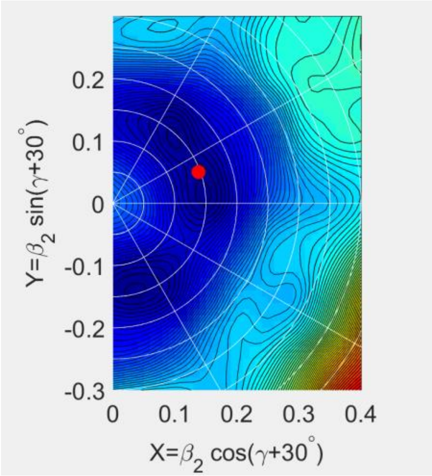
$$\begin{aligned} V(\beta,\gamma) &= \frac{1}{N}\langle \beta,\gamma,N|\hat{H}_{CQ}|\beta,\gamma,N\rangle|_{N\to\infty} \\ &= \varepsilon \frac{\beta^2}{(1+\beta^2)} + \delta \frac{6\beta^2}{(1+\beta^2)} \\ &+ \kappa \frac{4\beta^2}{(1+\beta^2)^2}\left[1 - \sqrt{\frac{2}{7}}\chi\beta\cos(3\gamma) + \frac{1}{14}\chi^2\beta^2\right] \end{aligned}$$

- **No triaxial minimum or rotor mode**

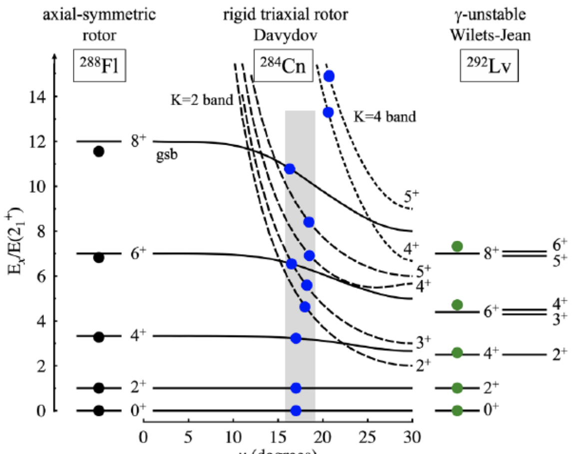
Ground state triaxial deformation



P. Möller, et al., At. Data Nucl. Data Tables 94 (2008) 758



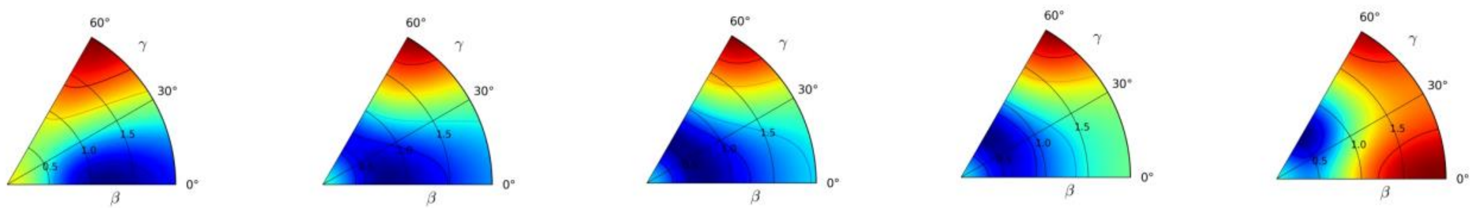
PES from deformed Woods-Saxon for ^{172}Pt



J. Luis Egido and Andrea Jungclaus Phys. Rev. Lett. **126**, 192501 (2021)

Triaxiality in CQ Hamiltonian with higher order terms

$$H = \xi \hat{n}_d - (1-\xi)\left[\frac{(\hat{Q}\cdot\hat{Q})}{N} + k_3 \frac{(\hat{Q}\times\hat{Q}\times\hat{Q})^{(0)}}{N^2}\right] .$$



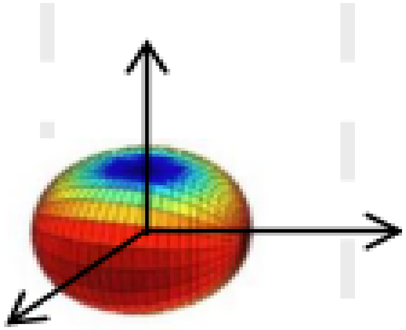
$k_3 = 0.0$ $k_3 = 1.5$ $k_3 = 1.77$ $k_3 = 2.0$ $k_3 = 3.0$

FIG. 12: (Color online) Potential energy surfaces for the CCQH for $\chi = -\sqrt{7}/2$ and $\xi = 0.5$, with different values of k_3 indicated in the figure. The observed minima range from an axially deformed prolate minimum for $k_3 = 0.0$ and $k_3 = 1.5$ to an oblate one for $k_3 = 2.0$ and $k_3 = 3.0$, passing through the triaxial region. The triaxial minimum when $k_3 = 1.77$ is not apparent from this figure, because it is very shallow (see next figure).

L. Fortunato, C. E. Alonso, J. M. Arias, J. E. García-Ramos, and
A. Vitturi, Phys. Rev. C **84**, 014326 (2011)

The SU(3) theory for the rotor mode

In body-fixed principle axis system



$$H_{\rm rot} = A_1 L_1^2 + A_2 L_2^2 + A_3 L_3^2 \quad \text{(1) Geometric}$$

$$H_{\rm rot} = a L^2 + b X_3^c + c X_4^c, \quad \text{(2) Algebraic}$$

$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

$$k_0 \beta_s = k\sqrt{\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu+3)}$$

$$X_3^c = \sum_{\alpha\beta} L_\alpha Q^c_{\alpha\beta} L_\beta = \lambda_1 L_1^2 + \lambda_2 L_2^2 + \lambda_3 L_3^2,$$

$$X_4^c = \sum_{\alpha\beta\gamma} L_\alpha Q^c_{\alpha\beta} Q^c_{\beta\gamma} L_\gamma = \lambda_1^2 L_1^2 + \lambda_2^2 L_2^2 + \lambda_3^2 L_3^2,$$

$$\langle \hat{Q} \rangle \propto \sqrt{\langle \hat{C}_2 \rangle} = \sqrt{\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu}$$

Y. Leschber and J. P. Draayer, Phys. Lett. B **190**, 1 (1987).
O. Castaños, J. P. Draayer, and Y. Leschber, Z. Phys. A **329**, 33 (1988).

A triaxial rotor Hamiltonian

$$\hat{H}_{\mathrm{SU}(3)} = a\hat{C}_2 + b\hat{C}_2^2 + c\hat{C}_3 + dX_3 + eX_4 + f\hat{L}^2 + g\hat{L}^4 + h\hat{C}_2\hat{L}^2$$

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

$$\hat{C}_2[\mathrm{SU}(3)] = 2\hat{Q}\cdot\hat{Q} + \tfrac{3}{4}\hat{L}^2,$$

$$\hat{C}_3[\mathrm{SU}(3)] = -\tfrac{4\sqrt{35}}{9}(\hat{Q}\times\hat{Q}\times\hat{Q})_0^{(0)} - \tfrac{\sqrt{15}}{2}(\hat{L}\times\hat{Q}\times\hat{L})_0^{(0)}$$

$$\hat{H}_{\mathrm{S}} = \tfrac{a_1}{N}\hat{C}_2[\mathrm{SU}(3)] + \tfrac{a_2}{N^3}\hat{C}_2[\mathrm{SU}(3)]^2 + \tfrac{a_3}{N^2}\hat{C}_3[\mathrm{SU}(3)],$$

$$\hat{H}_{\mathrm{D}} = t_1\hat{L}^2 + t_2(\hat{L}\times\hat{Q}\times\hat{L})^{(0)} + t_3(\hat{L}\times\hat{Q})^{(1)}\cdot(\hat{L}\times\hat{Q})^{(1)}$$

$$X_3^b = \sum_{\alpha\beta} L_\alpha^b Q_{\alpha\beta}^b L_\beta^b = \frac{\sqrt{30}}{6}(L^b \times Q^b \times L^b)^{(0)}$$

$$X_4^b = \sum_{\alpha\beta\gamma} L_\alpha Q_{\alpha\beta}^c Q_{\beta\gamma}^c L_\gamma$$

$$= \frac{-5\sqrt{3}}{18}[(L^b \times Q^b)^{(1)} \times (L^b \times Q^b)^{(1)}]^{(0)},$$

$$\langle \hat{C}_2[\mathrm{SU}(3)]\rangle = \lambda^2 + \mu^2 + 3\lambda + 3\mu + \lambda\mu,$$

$$\langle \hat{C}_3[\mathrm{SU}(3)]\rangle = \tfrac{1}{9}(\lambda-\mu)(2\lambda+\mu+3)(\lambda+2\mu+3)\,.$$

$$\gamma_{\mathrm{S}} = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

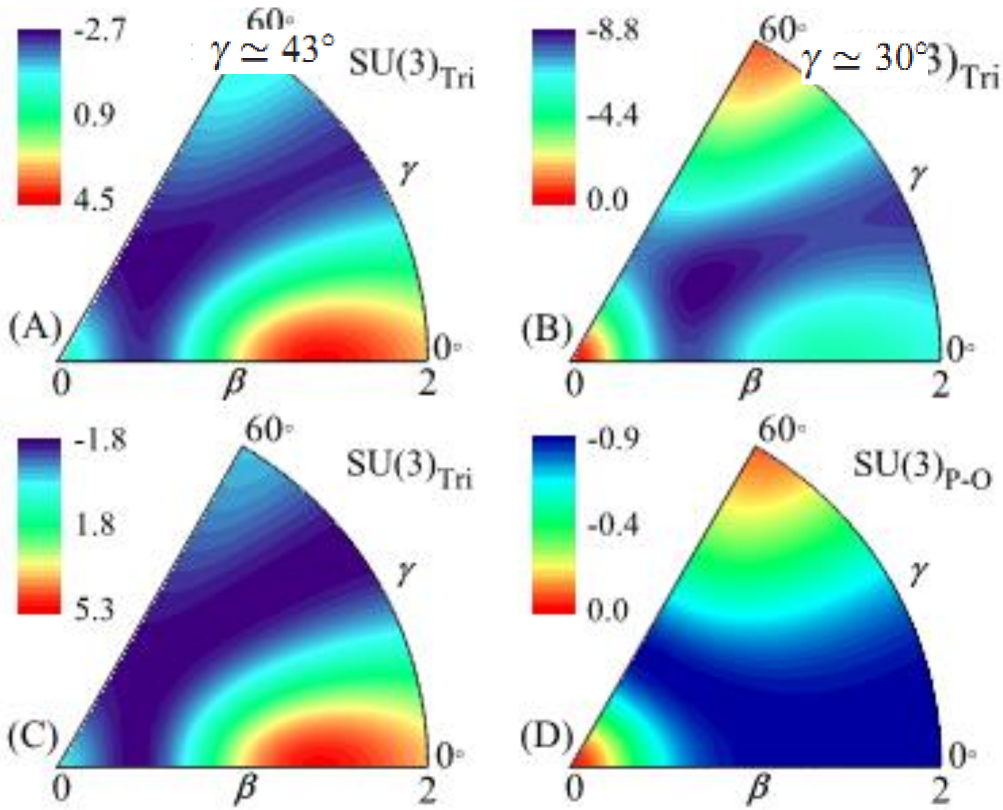
$$k_0\beta_{\mathrm{S}} = \sqrt{\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu+3},$$

Y. F. Smirnov, N. A. Smirnova, and P. Van Isacker, Phys. Rev. C 61, 041302(R) (2000).

Y. Zhang, F. Pan, L. R. Dai, and J. P. Draayer, Phys. Rev. C **90**, 044310 (2014).

PES from the static Hamiltonian

$$\hat{H}_{\rm S} = \frac{a_1}{N}\hat{C}_2[{\rm SU}(3)] + \frac{a_2}{N^3}\hat{C}_2[{\rm SU}(3)]^2 + \frac{a_3}{N^2}\hat{C}_3[{\rm SU}(3)]$$



$$\begin{aligned}\hat{H}_{\rm S} &= \frac{a_1}{N}\hat{C}_2[{\rm SU}(3)] + \frac{a_2}{N^3}\hat{C}_2[{\rm SU}(3)]^2 + \frac{a_3}{N^2}\hat{C}_3[{\rm SU}(3)], \\ &\propto b_2[QQ]^0 + b_3[QQQ]^0 + b_4([QQ]^0)^2 \quad |b_2| \gg |b_3| \gg |b_4|\end{aligned}$$

$$\hat{C}_2[{\rm SU}(3)] = 2\hat{Q}\cdot\hat{Q} + \tfrac{3}{4}\hat{L}^2, \tag{6}$$

$$\hat{C}_3[{\rm SU}(3)] = -\tfrac{4\sqrt{35}}{9}(\hat{Q}\times\hat{Q}\times\hat{Q})_0^{(0)} - \tfrac{\sqrt{15}}{2}(\hat{L}\times\hat{Q}\times\hat{L})_0^{(0)}. \tag{7}$$

FIG. 4: Contour plots generated from (14) with $a_1 : a_2 : a_3 =$ (A) $-\frac{27+10N}{3N} : 1 : 1$; (B) $-7.5 : 1 : 5$; (C) $-\frac{24+8N}{3N} : 1 : 0$; (D) $-\frac{2N+3}{3N} : 0 : 1$.

A consistent Q Hamiltonian with triaxiality

$$\hat{H} = \hat{H}_{\mathrm{CQ}} + \hat{H}_{\mathrm{Tri}}$$

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \tfrac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi}$$

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

$$\hat{H}_{\mathrm{S}} = \tfrac{a_1}{N} \hat{C}_2[\mathrm{SU}(3)] + \tfrac{a_2}{N^3} \hat{C}_2[\mathrm{SU}(3)]^2 + \tfrac{a_3}{N^2} \hat{C}_3[\mathrm{SU}(3)],$$

$$\hat{H}_{\mathrm{D}} = t_1 \hat{L}^2 + t_2 (\hat{L} \times \hat{Q} \times \hat{L})^{(0)} + t_3 (\hat{L} \times \hat{Q})^{(1)} \cdot (\hat{L} \times \hat{Q})^{(1)}$$

$$\hat{C}_2[\mathrm{SU}(3)] = 2\hat{Q} \cdot \hat{Q} + \tfrac{3}{4} \hat{L}^2,$$

$$\hat{C}_3[\mathrm{SU}(3)] = -\tfrac{4\sqrt{35}}{9} (\hat{Q} \times \hat{Q} \times \hat{Q})^{(0)}_0 - \tfrac{\sqrt{15}}{2} (\hat{L} \times \hat{Q} \times \hat{L})^{(0)}_0.$$

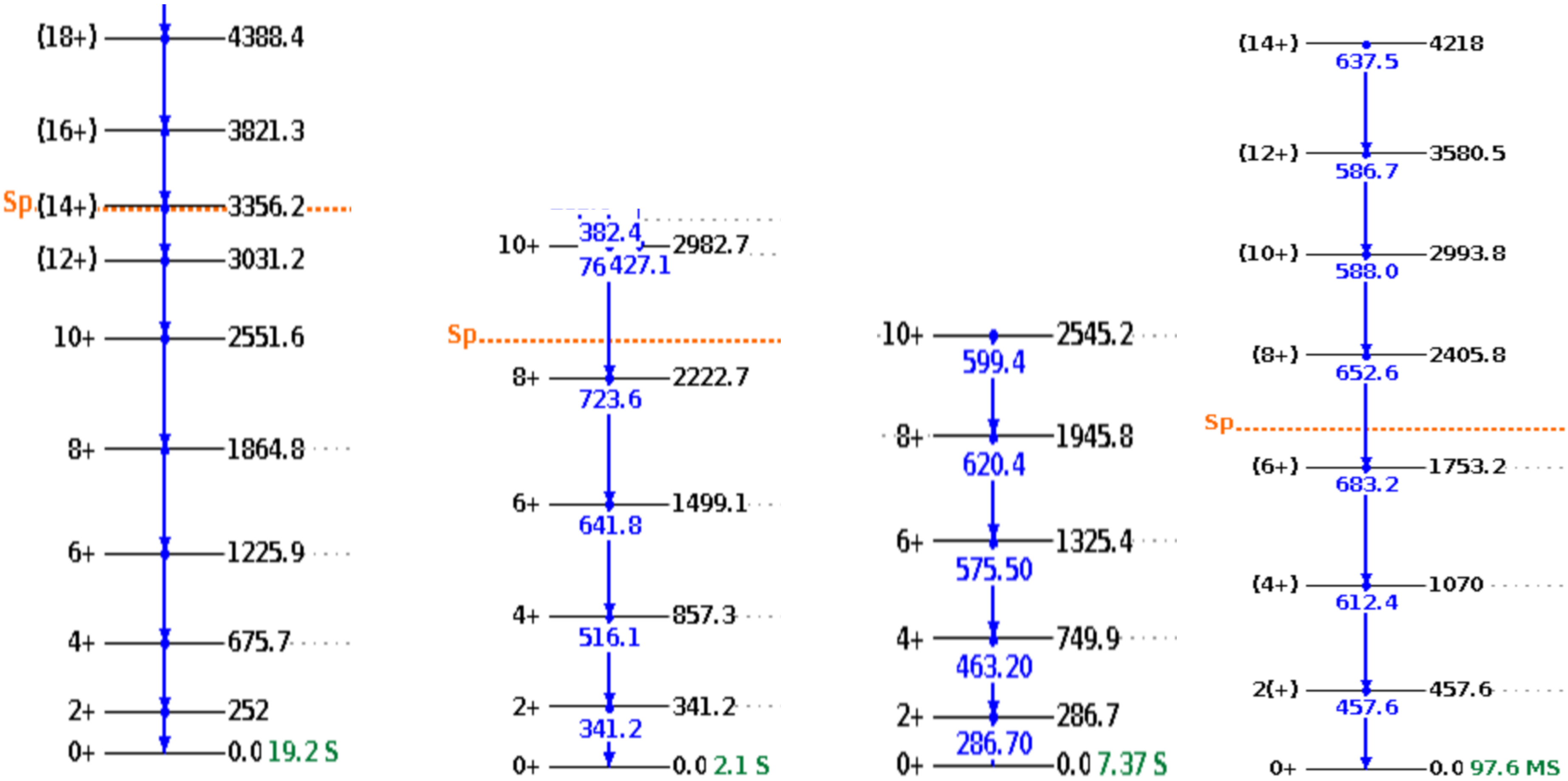
Included: Spherical vibrator, Prolate rotor, Oblate rotor, Gamma-soft rotor, Triaxial rotor

^{172}Pt and ^{168}Os

A 8-boson system with 4 (6) proton holes and 12 (10) neutrons

$$(\lambda, \mu) = (16,0), (12,2), (8,4), (4,6), (0,8)$$
$$(10,0), (6,2), (2,4), (4,0), (0,2)$$

Neutron-deficient nuclei around N=90-100



^{166}W $^{168,170}Os$ ^{172}Pt

https://www.nndc.bnl.gov/nudat3/

Neutron-deficient nuclei around N=90-100

PHYSICAL REVIEW LETTERS **121**, 022502 (2018)

Lifetime Measurements of Excited States in ^{172}Pt and the Variation of Quadrupole Transition Strength with Angular Momentum

B. Cederwall,[1,*] M. Doncel,[2] Ö. Aktas,[1] A. Ertoprak,[1,3] R. Liotta,[1] C. Qi,[1] T. Grahn,[4] D. M Cullen,[5] D. Hodge,[5] M. Giles,[5] S. Stolze,[4] H. Badran,[4] T. Braunroth,[6] T. Calverley,[4] D. M. Cox,[4,†] Y. D. Fang,[7] P. T. Greenlees,[4] J. Hilton,[4] E. Ideguchi,[7] R. Julin,[4] S. Juutinen,[4] M. Kumar Raju,[7] H. Li,[8] H. Liu,[1] S. Matta,[1] V. Modamio,[9] J. Pakarinen,[4] P. Papadakis,[4,‡] J. Partanen,[4] C. M. Petrache,[10] P. Rahkila,[4] P. Ruotsalainen,[4] M. Sandzelius,[4] J. Sarén,[4] C. Scholey,[4] J. Sorri,[4,12,§] P. Subramaniam,[1] M. J. Taylor,[11] J. Uusitalo,[4] and J. J. Valiente-Dobón[12]

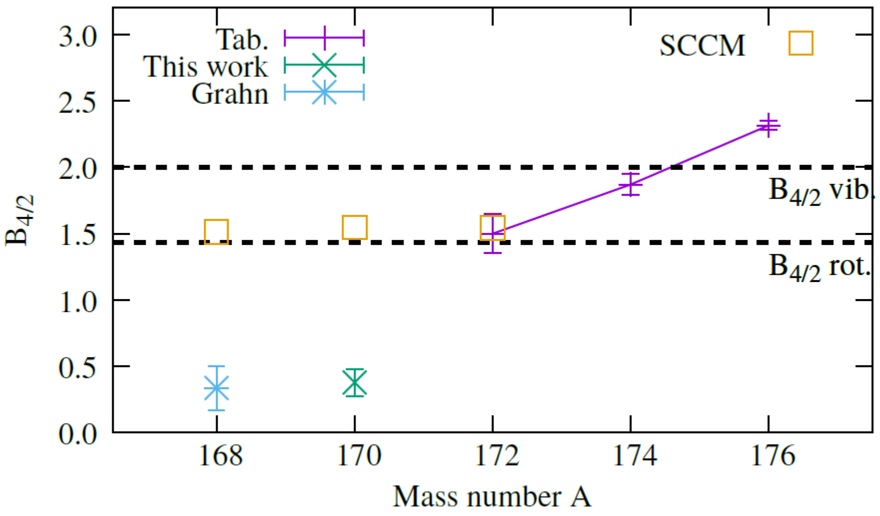
TABLE I. Energies of the $2_1^+ \rightarrow 0_{gs}^+$ and $4_1^+ \rightarrow 2_1^+$ transitions (E_γ), deduced lifetime values (τ) for the 2_1^+ and 4_1^+ states, and corresponding reduced transition probabilities [$B(E2\downarrow)_{exp}$] in Weisskopf units (W.u.).

Transition	E_γ (keV)	τ (ps)	$B(E2\downarrow)_{exp}$ (W.u.)
$2_1^+ \rightarrow 0_{gs}^+$	458	15(3)	49(11)
$4_1^+ \rightarrow 2_1^+$	612	6.2(17)	27(7)

$R_{4/2}$=2.34

$B_{4/2}$=0.55(19)

- **Several ee nuclei in that region including ^{166}W, ^{168,170}Os and ^{172}Pt and a few of their odd-A neighbors show strongly suppressed $B_{4/2}$**
- **172Pt is ”not” far from the Z=82 shell closure**
- **LSSM in $\pi gd_{5/2}d_{3/2}sh$-$\nu hfpi$ space with a large variety of effective interactions cannot reproduce $B_{4/2}$ and B(E2) simultaneously**



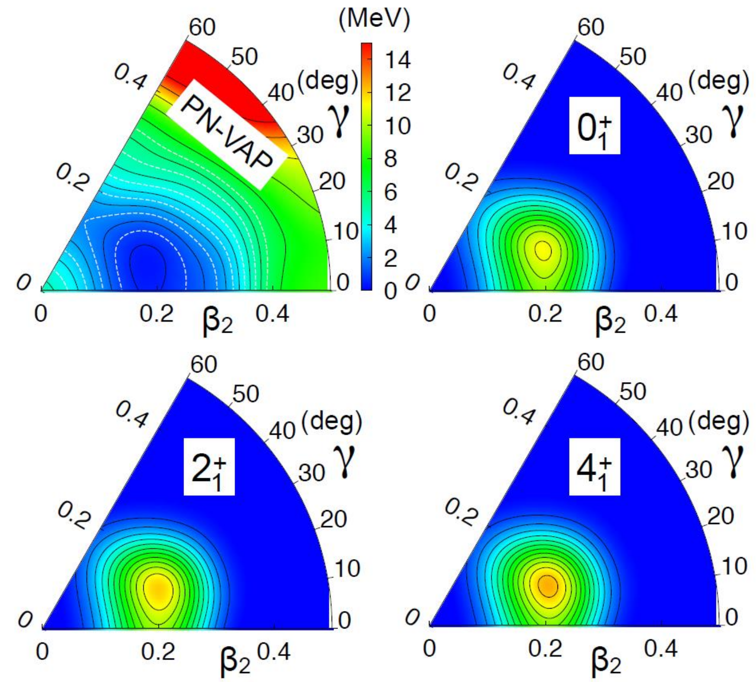


FIG. 7. (Color online) The PN-VAP potential energy surface for ^{170}Os (top left panel) and collective wave functions for the ground state (top right) as well as the yrast 2^+ (bottom left) and 4^+ (bottom

A. Goasduff, J. Ljungvall, T.R. Rodríguez, F.L. Bello Garrote, A. Etile, et al.. $B(E2)$ anomalies in the yrast band of ^{170}Os. Phys.Rev.C, 2019, 100 (3), pp.034302. 10.1103/PhysRevC.100.034302 .

A consistent Q Hamiltonian with triaxiality

$$\hat{H} = \hat{H}_{\mathrm{CQ}} + \hat{H}_{\mathrm{Tri}}$$

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \frac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi}$$

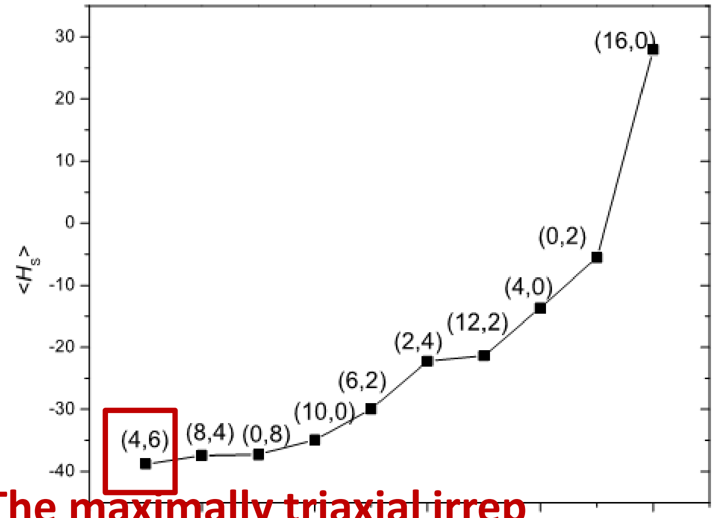
$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

Including: Spherical vibrator, Prolate rotor, Oblate rotor, Gamma-soft rotor, Triaxial rotor

^{172}Pt and ^{168}Os

A 8-boson system with 4 (6) proton holes and 12 (10) neutrons

$$(\lambda,\mu) = (16,0),(12,2),(8,4),(4,6),(0,8)\\ (10,0),(6,2),(2,4),(4,0),(0,2)$$

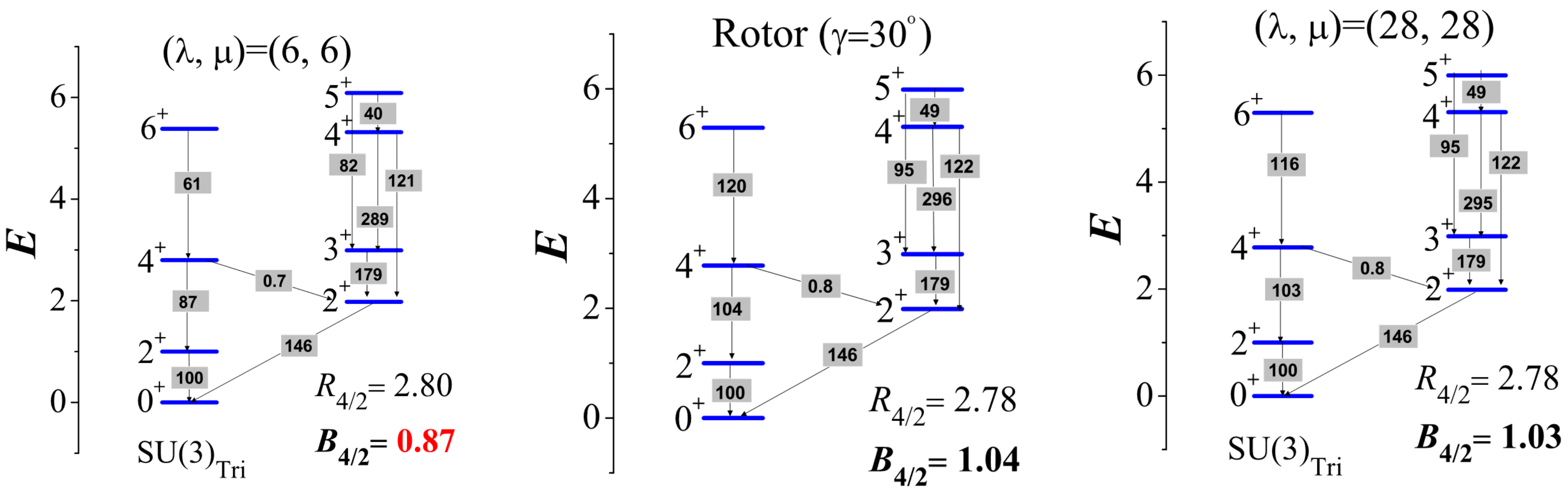


The maximally triaxial irrep for N = 8.

A surprising mode emerging from the SU(3) theory already at step one!!

(in practice the last step)

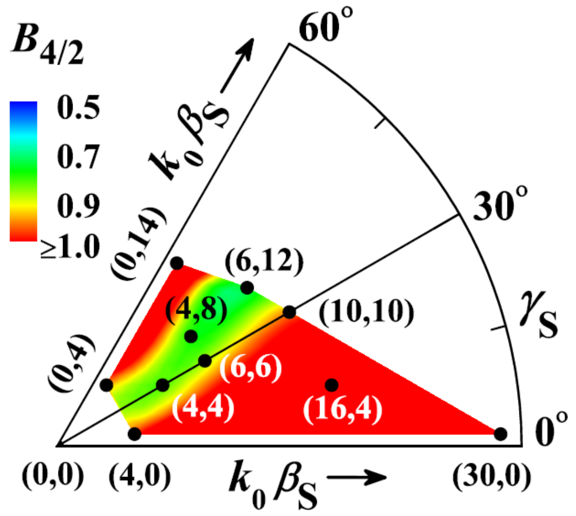
$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$



$t_1 = 3,\ t_2 = 0.553283,\ t_3 = -0.0226757$ ⟵ $A_1 : A_2 : A_3 = 3:1:4$ ⟶ $t_1 = 3,\ t_2 = 0.133551,\ t_3 = -0.00132118$

$$\hat{T}_u^{(E2)}(\mathrm{Rotor}) = q\beta\left[\cos\gamma\, D^{(2)}_{u,0} + \frac{1}{\sqrt{2}}\sin\gamma\left(D^{(2)}_{u,2} + D^{(2)}_{u,-2}\right)\right]$$

Assumption: $\gamma = \gamma_s$



$B_{4/2}$
0.5
0.7
0.9
≥1.0
60°
30°
0°
$k_0\beta_S$
γ_S
(0,14)
(6,12)
(4,8)
(10,10)
(0,4)
(6,6)
(4,4)
(16,4)
(0,0)
(4,0)
(30,0)

A consistent Q Hamiltonian with triaxiality

$$\hat{H} = \hat{H}_{\mathrm{CQ}} + \hat{H}_{\mathrm{Tri}}$$

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \frac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi}$$

$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

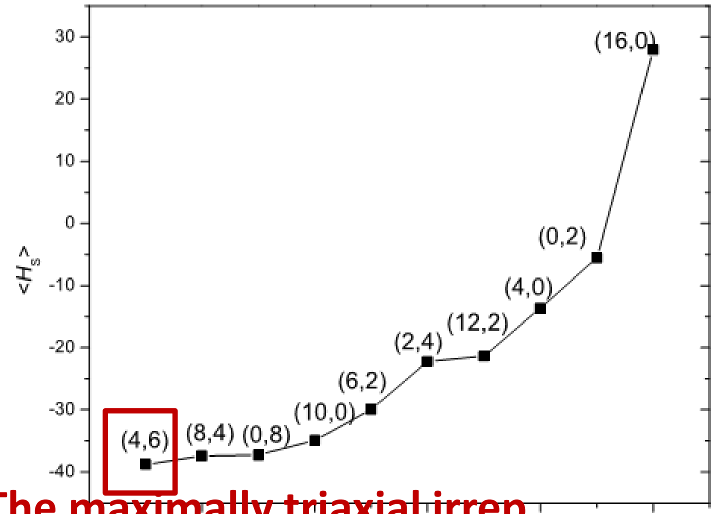
Including: Spherical vibrator, Prolate rotor, Oblate rotor, Gamma-soft rotor, Triaxial rotor

^{172}Pt and ^{168}Os

A 8-boson system with 4 (6) proton holes and 12 (10) neutrons

E(MeV)	^{172}Pt	IBM$_a$	IBM$_b$	^{168}Os	IBM$_a$	IBM$_b$
$E(2_1^+)$	0.458	0.458	0.458	0.341	0.341	0.341
$E(4_1^+)$	1.070	1.107	1.070	0.857	0.872	0.857
$E(6_1^+)$	1.753	1.900	1.742	1.499	1.527	1.504
$E(8_1^+)$	2.405	2.594	2.279	2.222	2.152	2.116
$E(2_2^+)$	-	0.749	0.916	-	0.525	0.686
$E(0_2^+)$	-	0.422	0.913	-	0.287	0.568
Transition	^{172}Pt	IBM$_a$	IBM$_b$	^{168}Os	IBM$_a$	IBM$_b$
$2_1^+ \to 0_1^+$	1.0	1.0	1.0	1.0	1.0	1.0
$4_1^+ \to 2_1^+$	**0.55(19)**	**0.677**	**0.552**	**0.34(18)**	**0.656**	**0.351**
$6_1^+ \to 4_1^+$	-	0.174	0.131	-	0.111	0.121
$2_2^+ \to 0_1^+$	-	0.005	0.201	-	0.001	0.353
$0_2^+ \to 2_1^+$	-	0.331	0.007	-	0.237	0.004

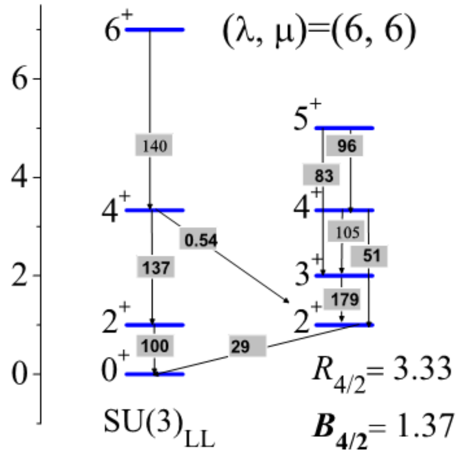
$$(\lambda,\mu) = (16,0),(12,2),(8,4),(4,6),(0,8)$$
$$(10,0),(6,2),(2,4),(4,0),(0,2)$$



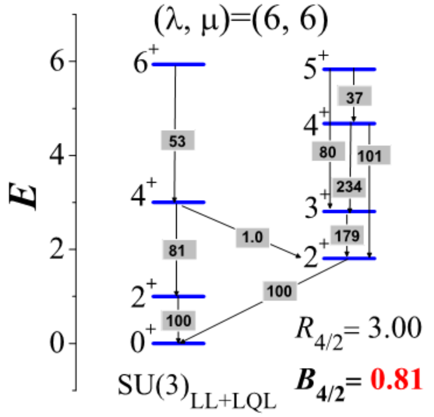
The maximally triaxial irrep for N = 8.

Band mixing within the SU(3) irrep (6,6)

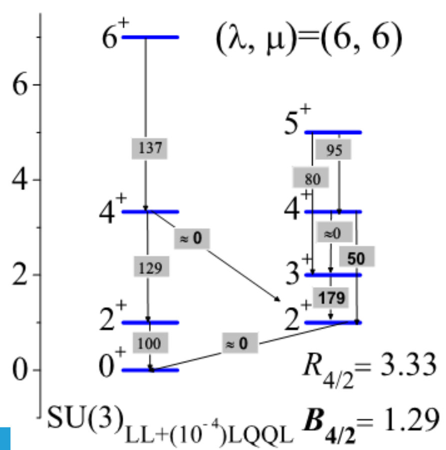
LL



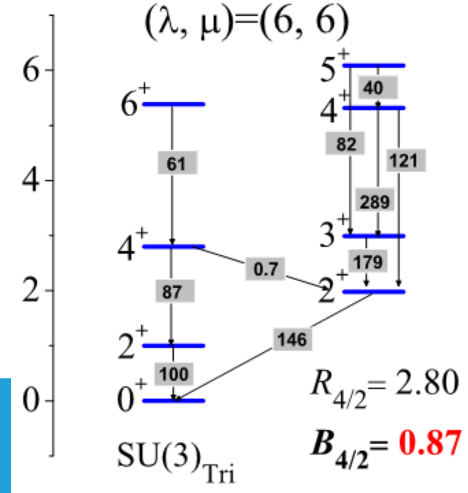
LL+LQL



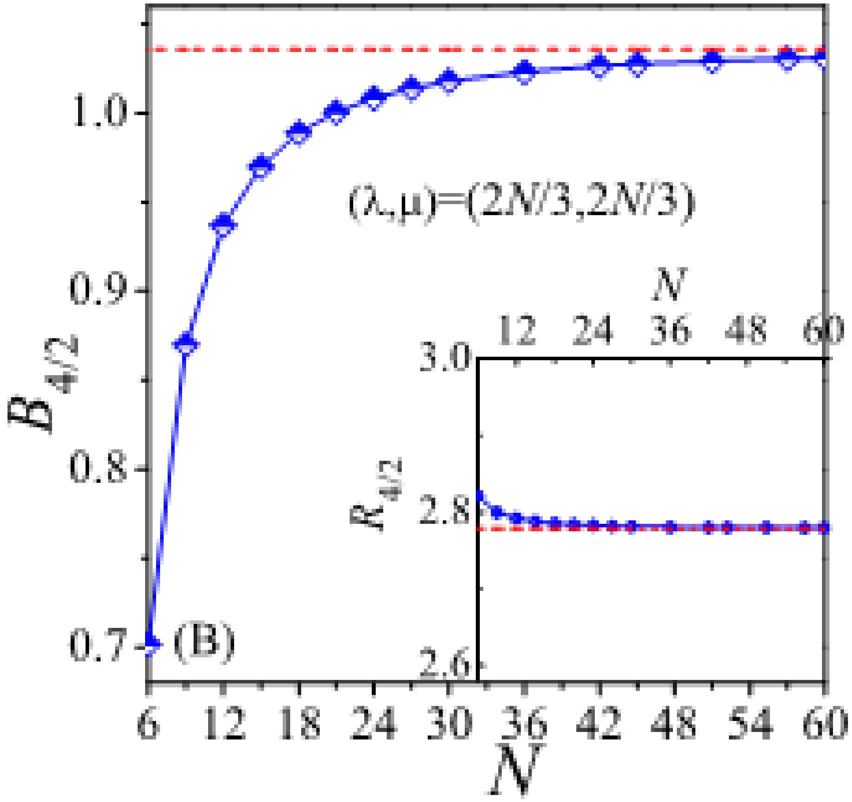
LL+δ LQL



All together



Finite N Effect



of N. All the results are solved from $\hat{H}_{\mathrm{Tri}}$ with the parameters $a_1 : a_2 : a_3 = -\frac{27+10N}{3N} : 1 : 1$ generating $(\lambda_0, \mu_0) = (2N/3, 2N/3)$ and $t_i = g_i(\lambda_0, \mu_0, A_1, A_2, A_3)$. The dashed lines denote those solved directly from the triaxial rotor Hamiltonian (12).

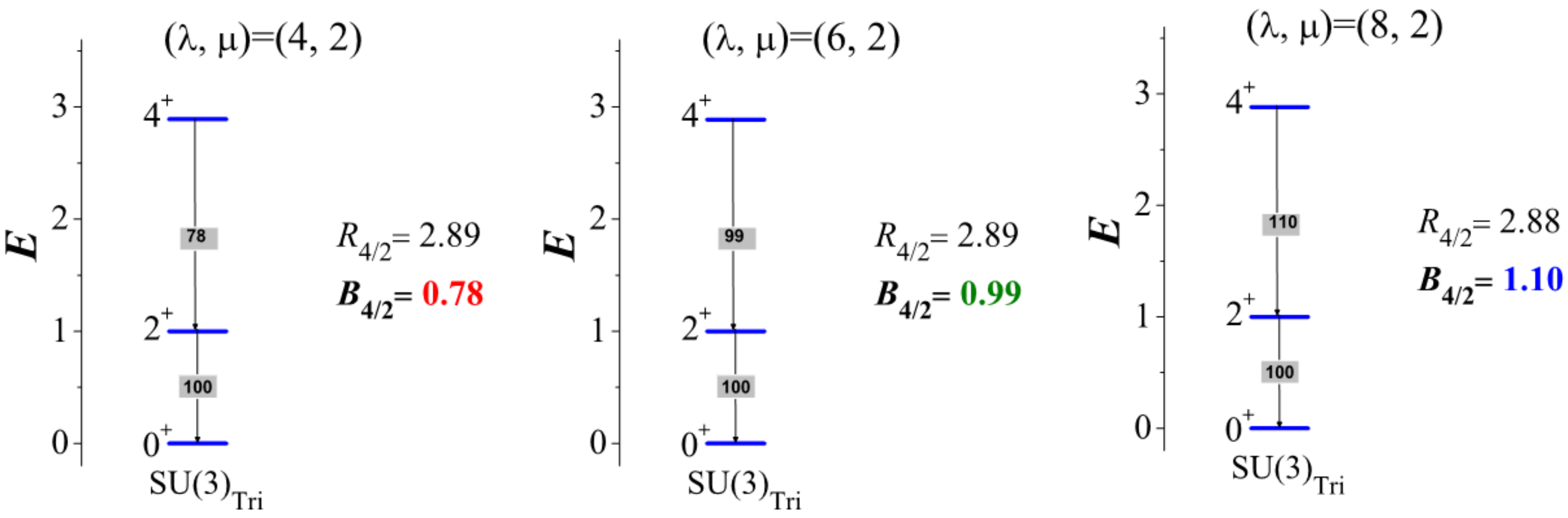
$$\hat{H}_{\mathrm{Tri}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{D}}$$

$$\begin{aligned}(\lambda, \mu) \;=\; & (2N,0),(2N-4,2),\ldots,(0,N) \text{ or } (2,N-1) \\ & (2N-6,0),(2N-10,2),\ldots\end{aligned}$$

$B_{4/2}$ in rotor mode

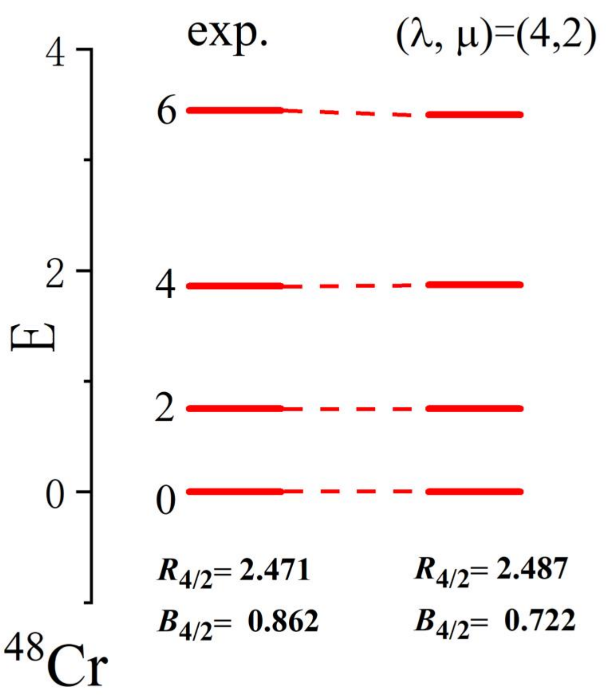
$$\frac{B\left(E2{:}\,4_1^+ \rightarrow 2_1^+\right)}{B\left(E2{:}\,2_1^+ \rightarrow 0_1^+\right)} = \frac{10}{7}\left[\frac{(2N-2)(2N+5)}{(2N)(2N+3)}\right]$$

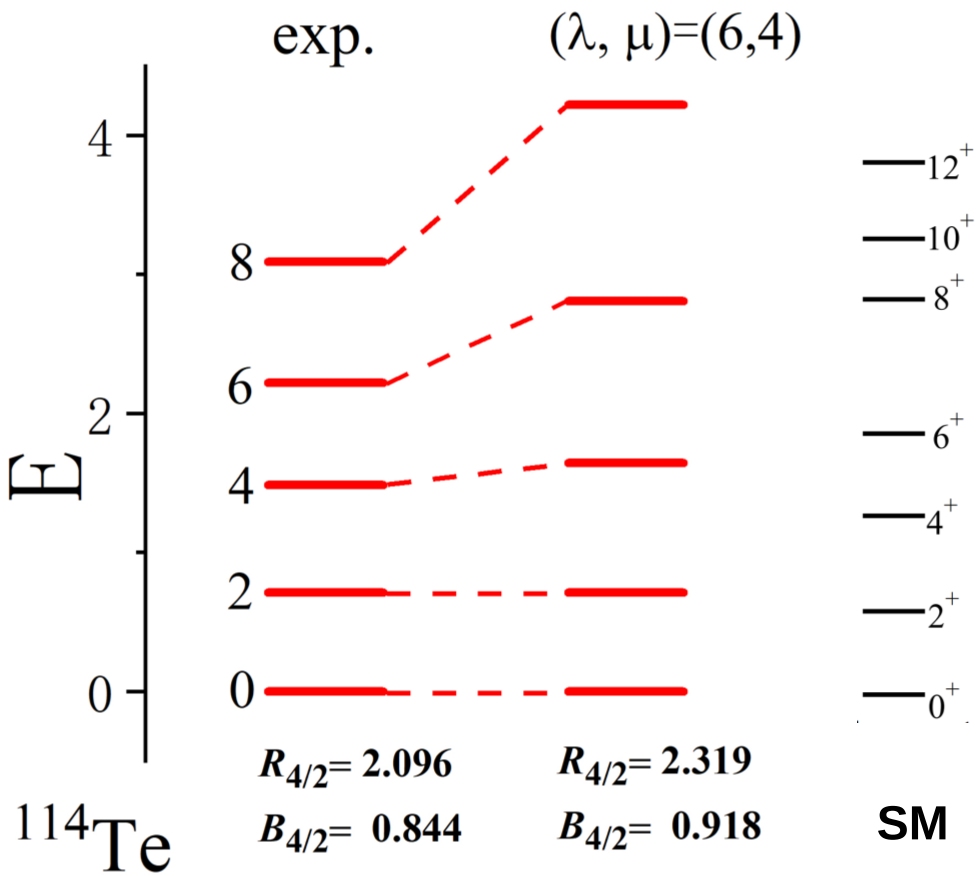
R.F. Casten, Nuclear Structure from a Simple Perspective, Oxford (1990)



(λ, μ)=(4, 2)
E
3
2
1
0
4+
2+
0+
78
100
SU(3)Tri
R4/2= 2.89
B4/2= 0.78
(λ, μ)=(6, 2)
E
3
2
1
0
4+
2+
0+
99
100
SU(3)Tri
R4/2= 2.89
B4/2= 0.99
(λ, μ)=(8, 2)
E
3
2
1
0
4+
2+
0+
110
100
SU(3)Tri
R4/2= 2.88
B4/2= 1.10

^{48}Cr, ^{114}Te


exp.
(λ, μ)=(4,2)
E
4
2
0
6
4
2
0
$R_{4/2}$= 2.471
$B_{4/2}$= 0.862
$R_{4/2}$= 2.487
$B_{4/2}$= 0.722
^{48}Cr


exp.
(λ, μ)=(6,4)
E
4
2
0
8
6
4
2
0
12^+
10^+
8^+
6^+
4^+
2^+
0^+
$R_{4/2}$= 2.096
$B_{4/2}$= 0.844
$R_{4/2}$= 2.319
$B_{4/2}$= 0.918
SM
^{114}Te

Summary

- A collective mode with $B_{4/2}<1.0$ and $R_{42}>2.0$ can naturally appear in the SU(3) theory for the triaxial rotor.
- It is dominated by some triaxial irreps (λ,μ) as well mixture from other configurations.
- It can provide a simple description of the anomalous B(E2) neutron-deficient nuclei
- Systematical calculations on the triaxial and γ-soft nuclei; Generalization to IBM2; Mapping to shell model-like Hamiltonian?