Doppler Gyroscopes: Frequency vs Phase Estimation

- John Howell (Chapman, HUJI)
- Merav Kahn (HUJI)
- Ziv Roi (HUJI)
- Einav Grynszpan (HUJI)
- Prof. Nadav Katz (HUJI)
- Stefania Residori (Nice)
- Umberto Bortolozzo (Nice)

- Ziv Cohen, JH, Merav Kahn, Einav Grynszpan, Dvir Jacobovic

Positions available!

- Superoscillations
- Radar/Lidar
- Entangled Photons
- Radiative Cooling
- AI
- Precision Measurements
- Compressive Sensing

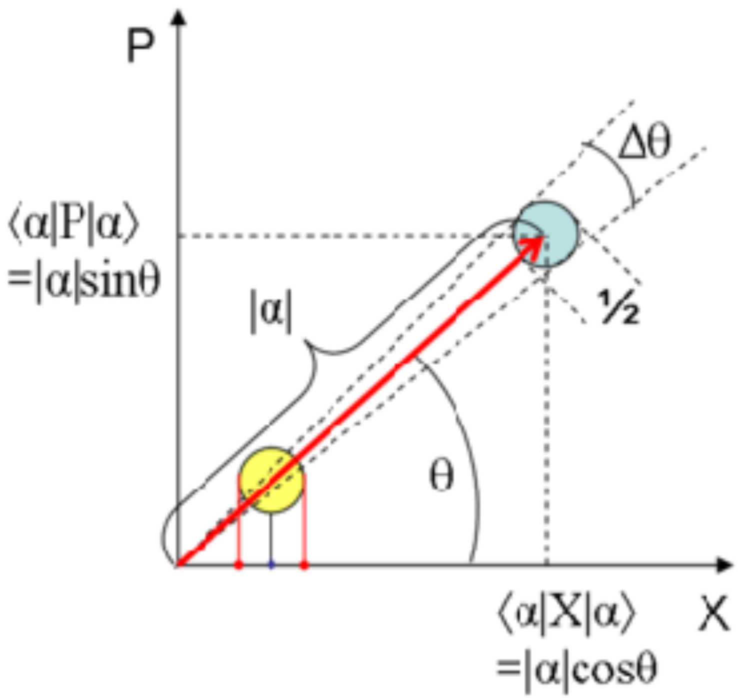
Outline

- Phase estimation standard quantum limit. Preview: we beat it by orders of magnitude
- Frequency vs phase estimation
- Gyroscope fundamentals (Doppler?)
- Experiment
- Results

Standard Quantum Limit: Phase Estimation

Multiple ways to arrive at SQL (coherent states)

- Fisher information for independent measurements
- Central limit theorem
- Phase space quadratures



Simple ad hoc description
Field uncertainty: ½
Distance from origin: $|\alpha|$

$$\alpha\, \Delta\theta = \frac{1}{2}$$

$$\Delta\theta = \frac{1}{2\sqrt{N}}$$

Quantum Metrology Beyond SQL

[1] Carlton M Caves, "Quantum-mechanical noise in an interferometer," Physical Review D **23**, 1693 (1981).

[2] MJ Holland and K Burnett, "Interferometric detection of optical phase shifts at the heisenberg limit," Physical review letters **71**, 1355 (1993).

[3] Agedi N. Boto, Pieter Kok, Daniel S. Abrams, Samuel L. Braunstein, Colin P. Williams, and Jonathan P. Dowling, "Quantum interferometric optical lithography: Exploiting entanglement to beat the diffraction limit," Phys. Rev. Lett. **85**, 2733–2736 (2000).

[4] Nicolas Treps, Nicolai Grosse, Warwick P Bowen, Claude Fabre, Hans-A Bachor, and Ping Koy Lam, "A quantum laser pointer," Science **301**, 940–943 (2003).

[5] Brendon L Higgins, Dominic W Berry, Stephen D Bartlett, Howard M Wiseman, and Geoff J Pryde, "Entanglement-free heisenberg-limited phase estimation," Nature **450**, 393–396 (2007).

[6] Hugo Cable and Gabriel A. Durkin, "Parameter estimation with entangled photons produced by parametric down-conversion," Phys. Rev. Lett. **105**, 013603 (2010).

[7] Vittorio Giovannetti, Seth Lloyd, and Lorenzo Maccone, "Advances in quantum metrology," Nature photonics **5**, 222–229 (2011).

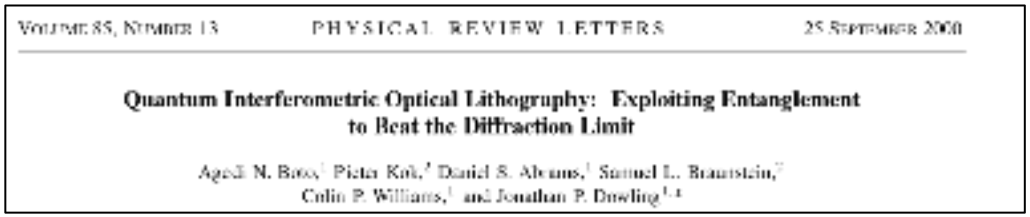
[8] Xiao-Ye Xu, Yaron Kedem, Kai Sun, Lev Vaidman, Chuan-Feng Li, and Guang-Can Guo, "Phase estimation with weak measurement using a white light source," Physical review letters **111**, 033604 (2013).

[9] Junaid Aasi, J Abadie, BP Abbott, Richard Abbott, TD Abbott, MR Abernathy, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, *et al.*, "Enhanced sensitivity of the ligo gravitational wave detector by using squeezed states of light," Nature Photonics **7**, 613–619 (2013).

[10] GJ Pryde, S Slussarenko, MM Weston, HM Chrzanowski, LK Shalm, VB Verma, and SW Nam, "Unconditional shot-noise-limit violation in photonic quantum metrology," in *Conference on Lasers and Electro-Optics/Pacific Rim* (Optical Society of America, 2018) pp. Th4J–1.

[11] Emanuele Polino, Mauro Valeri, Nicolò Spagnolo, and Fabio Sciarrino, "Photonic quantum metrology," AVS Quantum Science **2**, 024703 (2020).

Quantum Metrology
Beyond SQL: NOON States


Volume 85, Number 13 PHYSICAL REVIEW LETTERS 25 September 2000

Quantum Interferometric Optical Lithography: Exploiting Entanglement to Beat the Diffraction Limit

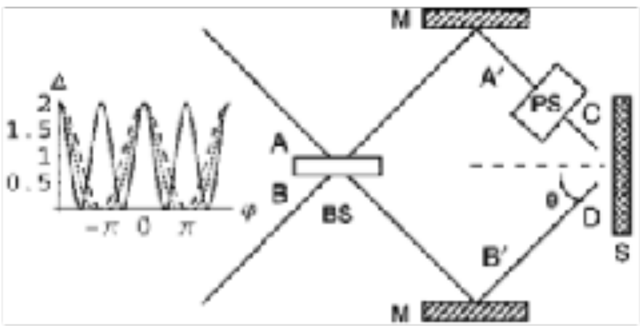
Agedi N. Boto,[1] Pieter Kok,[2] Daniel S. Abrams,[1] Samuel L. Braunstein,[2] Colin P. Williams,[1] and Jonathan P. Dowling[1,*]

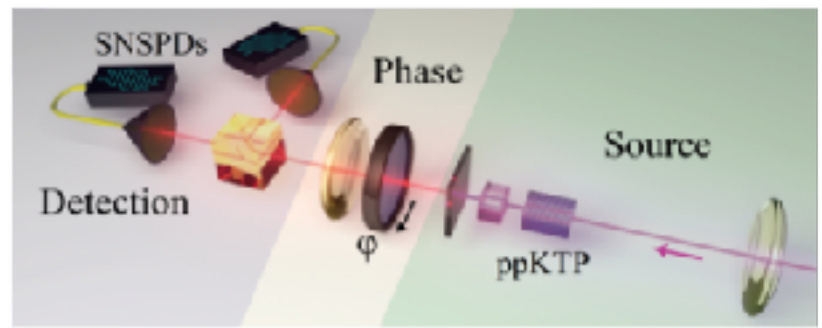

LETTERS

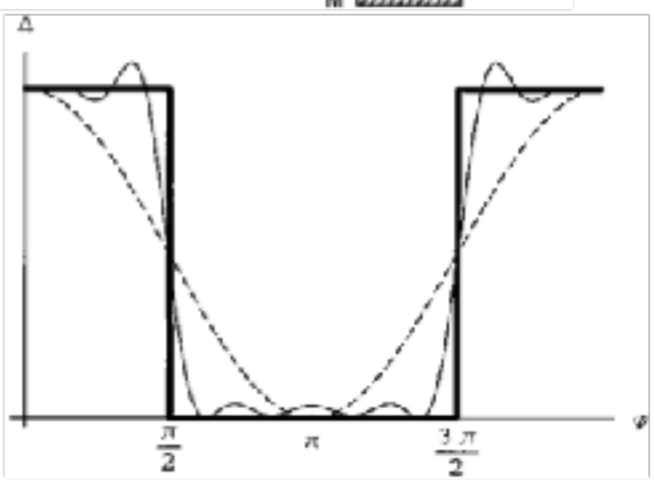
nature photonics

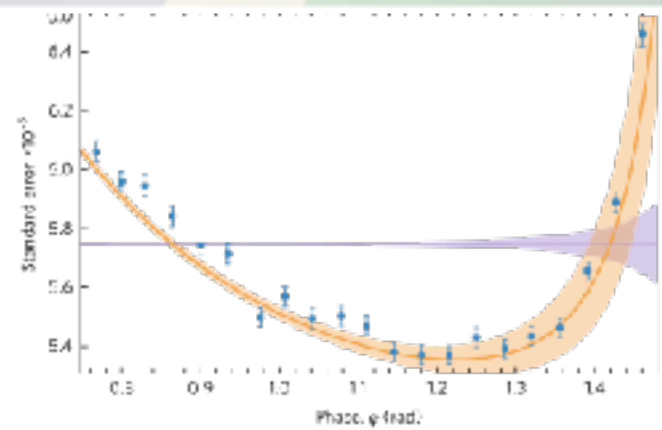
Unconditional violation of the shot-noise limit in photonic quantum metrology

Sergei Slussarenko, Morgan M. Weston, Helen M. Chrzanowski, Lynden K. Shalm, Varun B. Verma, Sae Woo Nam and Geoff J. Pryde

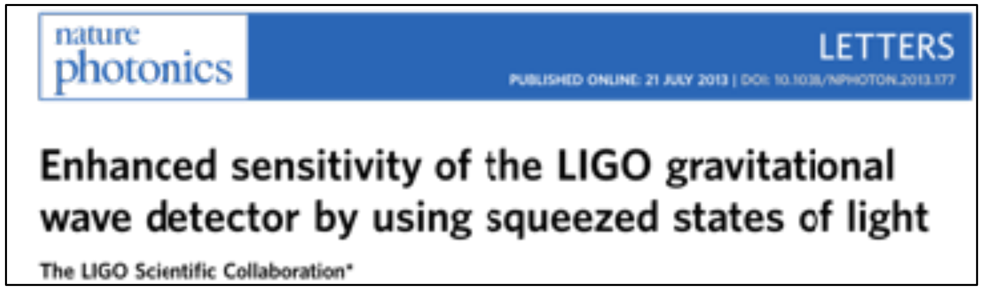








Quantum Metrology
Beyond SQL: Squeezed States

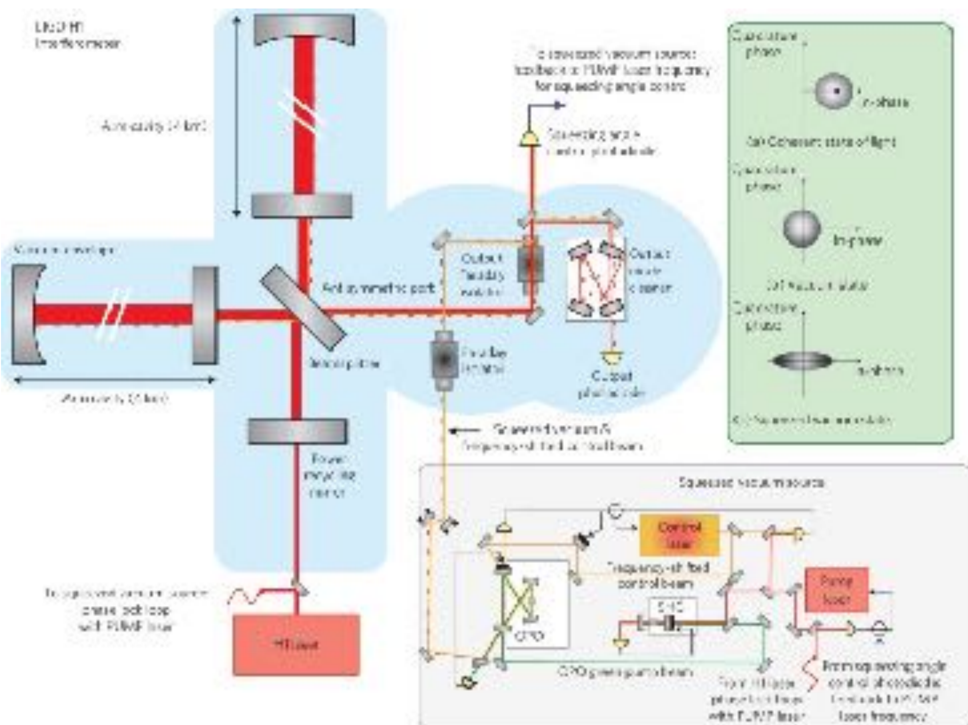

nature photonics

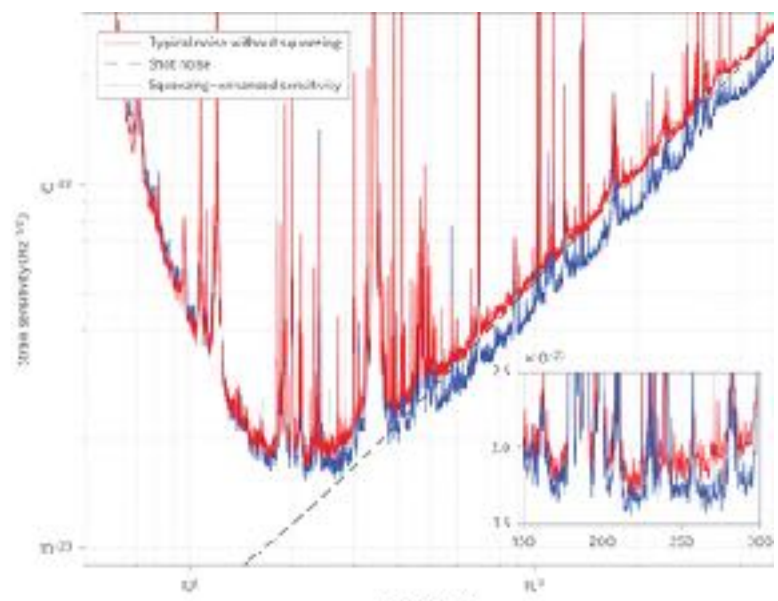
LETTERS

PUBLISHED ONLINE: 21 JULY 2013 | DOI: 10.1038/NPHOTON.2013.177

Enhanced sensitivity of the LIGO gravitational wave detector by using squeezed states of light

The LIGO Scientific Collaboration*

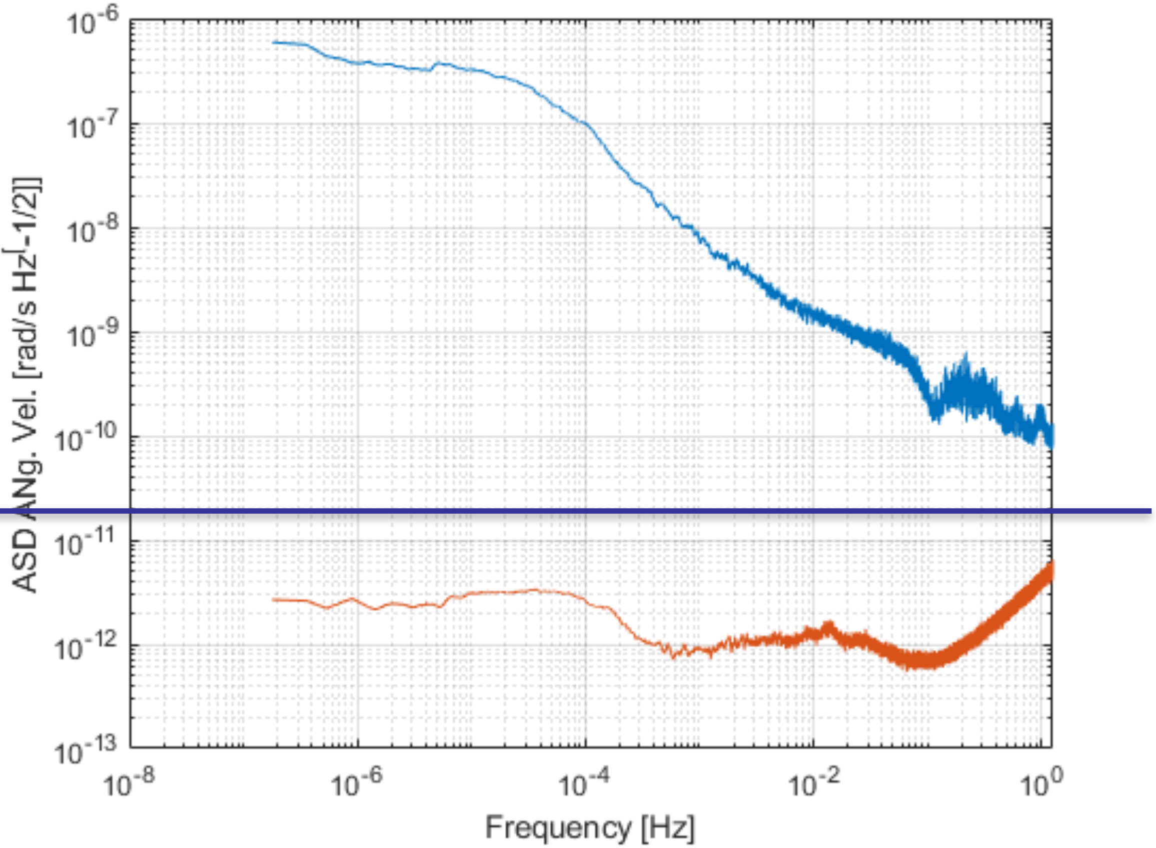




Breakthrough in RLGs

Sub-shot-noise sensitivity in a ring laser gyroscope

Angela D. V. Di Virgilio[1], Francesco Bajardi[2,3], Andrea Basti[1,4], Nicolò Beverini[4], Giorgio Carelli[1,4], Donatella Ciampini[1,4], Giuseppe Di Somma[1,4], Francesco Fuso[1,4], Enrico Maccioni[1,4], Paolo Marsili[1,4], Antonello Ortolan[5], Alberto Porzio[2,6],* and David Vitali[7,8]



arXiv:2301.01386v1 [quant-ph] 3 Jan 2023

August 15, 2013 / Vol. 38, No. 16 / OPTICS LETTERS 3107

Precision Doppler measurements with steep dispersion

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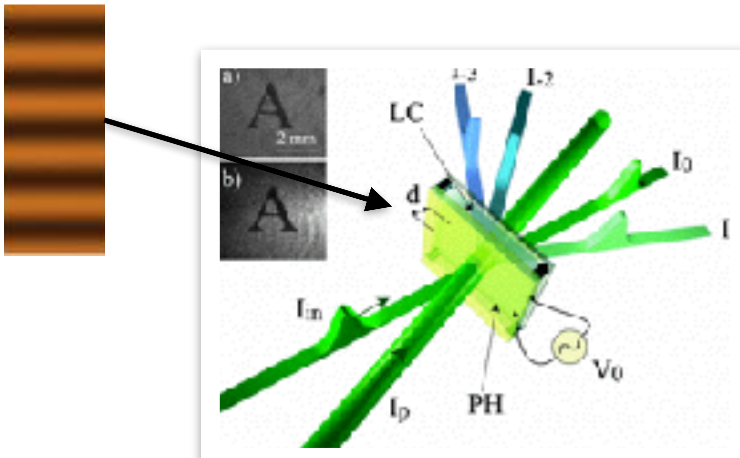
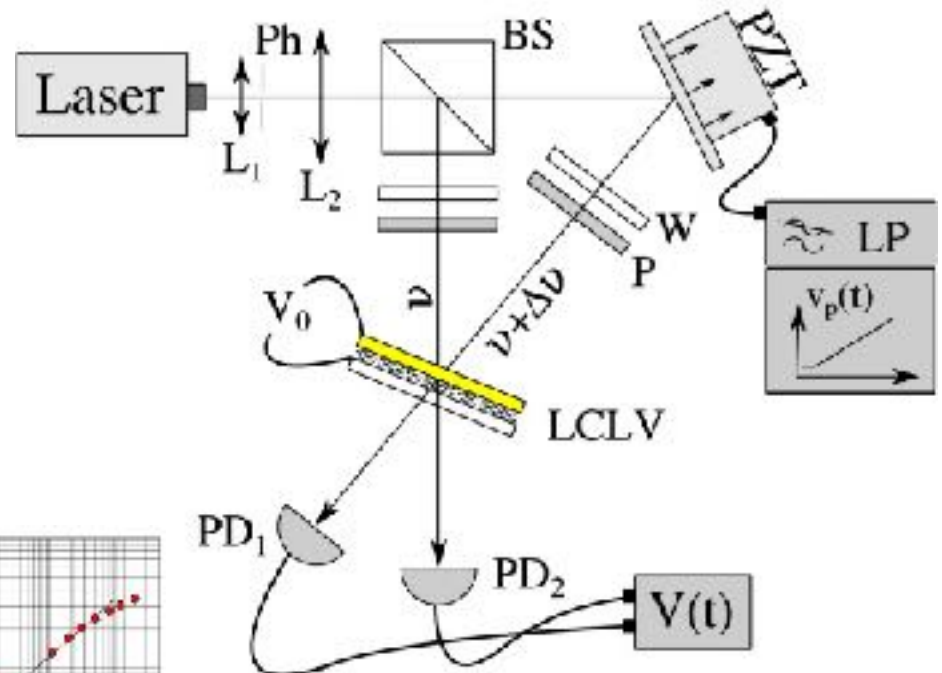
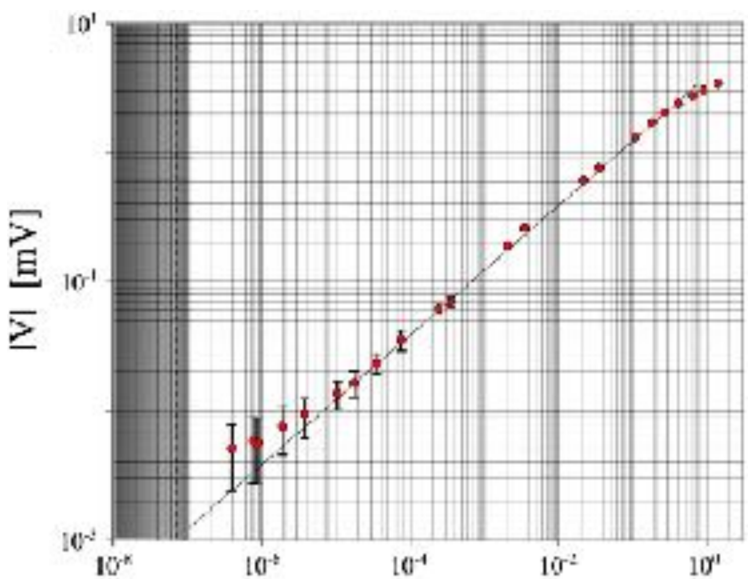




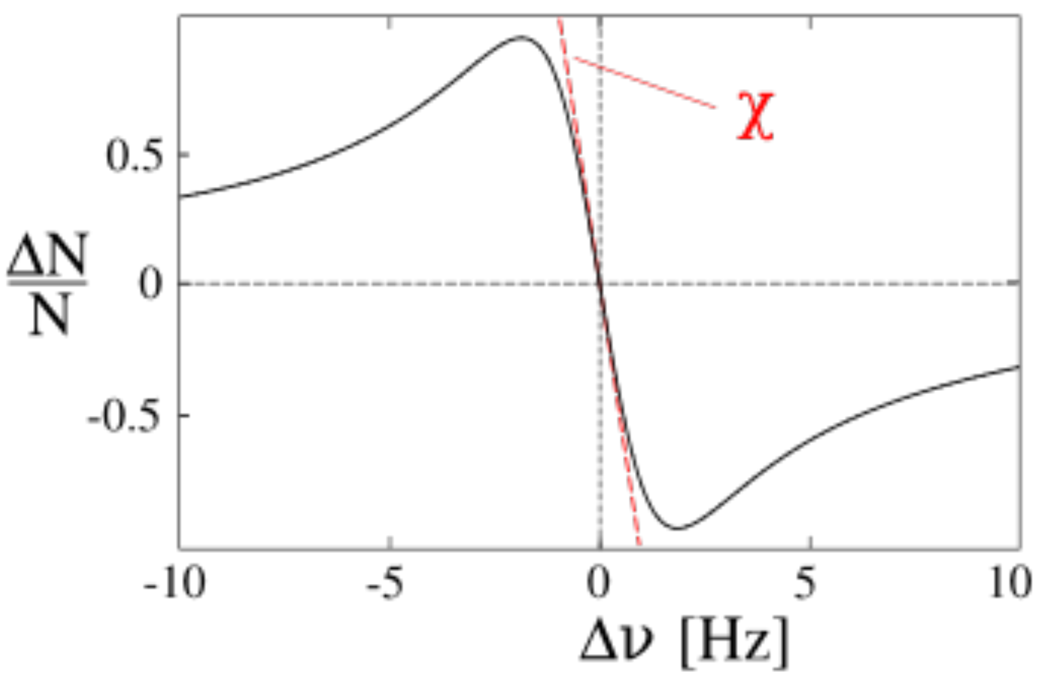


Frequency Estimation
Liquid Crystal Light Valve

Shot noise limit

$$\Delta f = \frac{1}{|\chi|\sqrt{N}}$$

Gain Curve



Frequency vs Phase

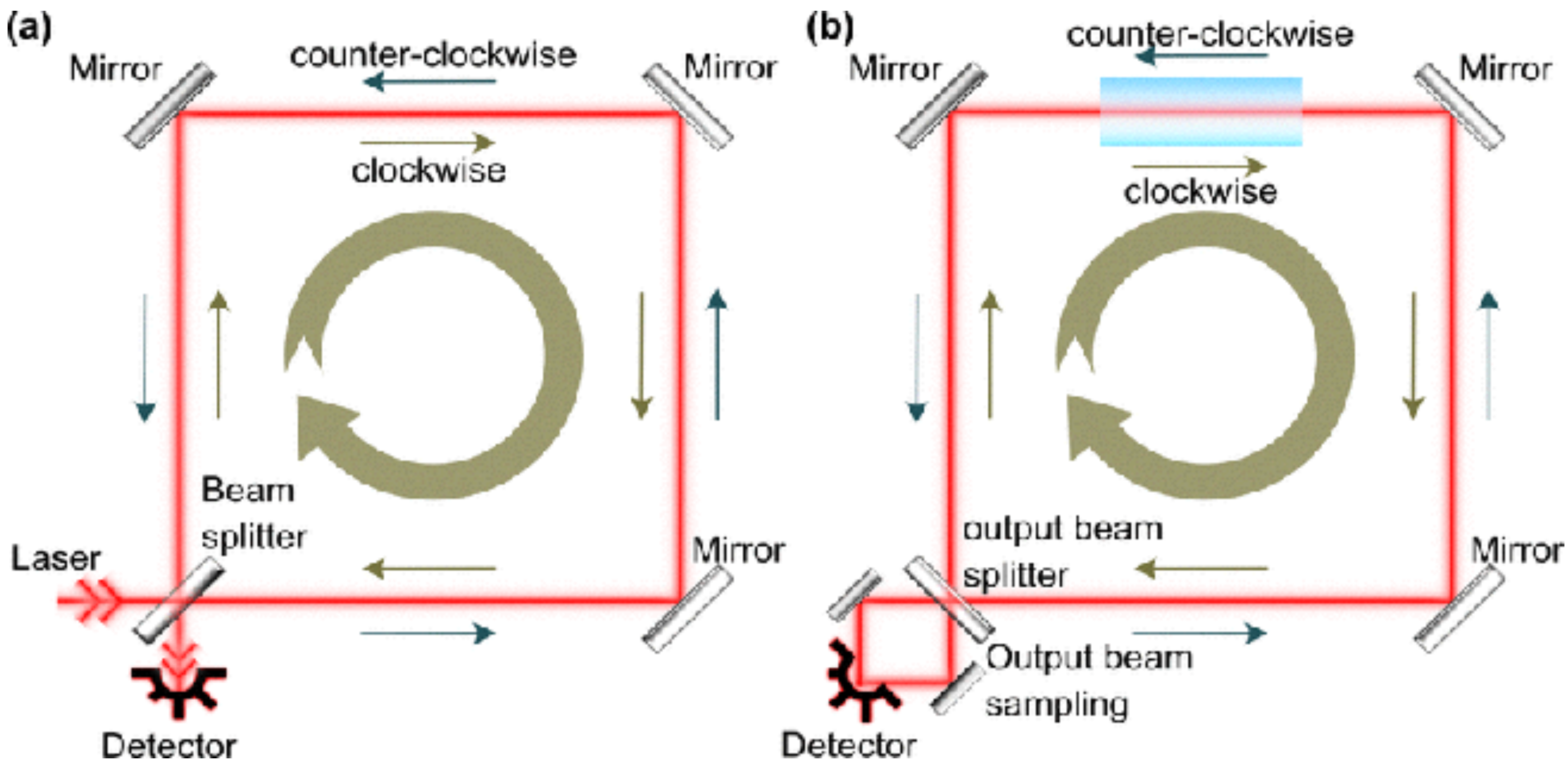
$$\Delta f = \frac{1}{|\chi|\sqrt{N}}$$

$$\Delta\theta = \frac{1}{2\sqrt{N}}$$

Shift theorem in Fourier transforms:
Relating phase gradients and frequency offsets

$$\mathscr{F}\left\{g(t-\tau)\right\} = Ge^{-i\omega\tau}$$

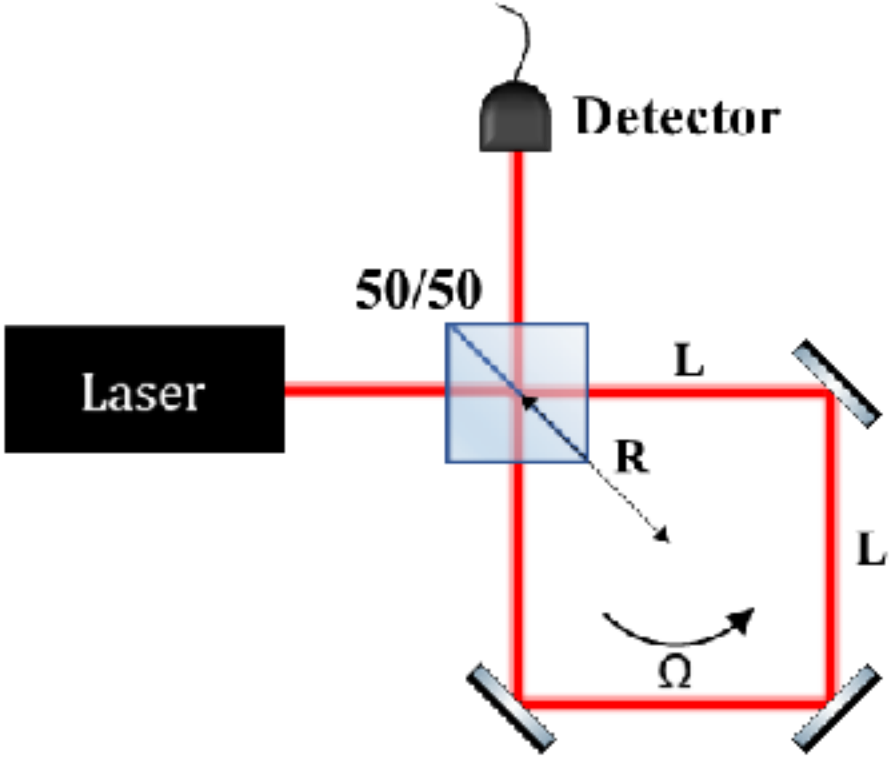
Active vs Passive Gyroscopes



Sagnac Effect

$$\Delta\theta = \frac{8\pi A\Omega}{\lambda c}$$

CLOSED System!
Light folds back onto itself

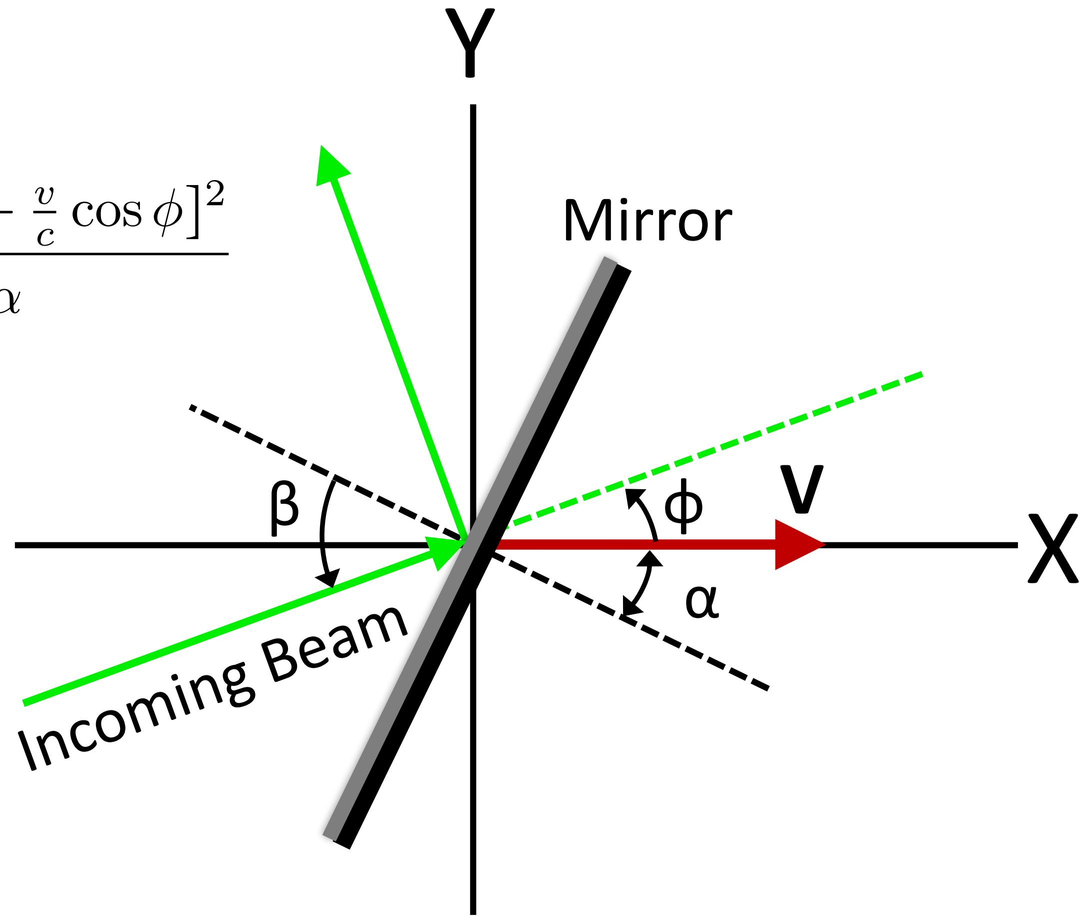


Verifying the Ashworth-Davies Doppler Shift

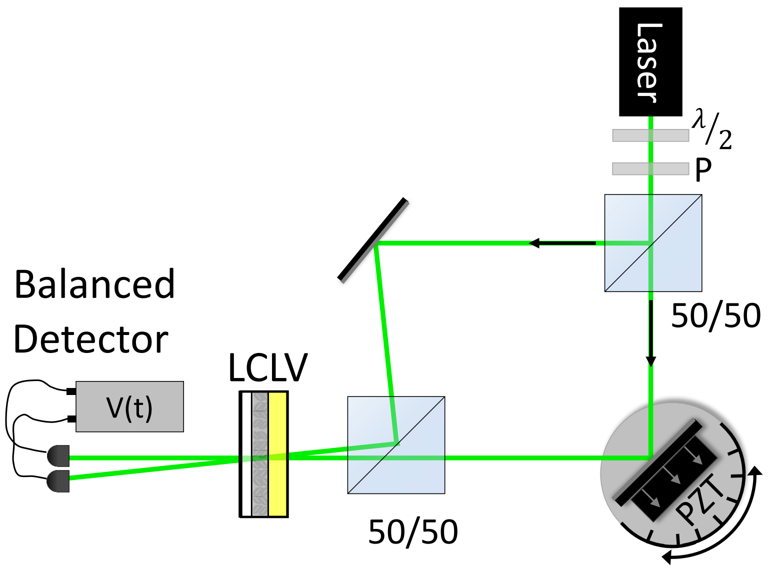
$$f_f = f_i \frac{[\tan\alpha + \frac{v}{c}\sin\phi]^2 + [1 - \frac{v}{c}\cos\phi]^2}{1 - \frac{v^2}{c^2} + \tan^2\alpha}$$

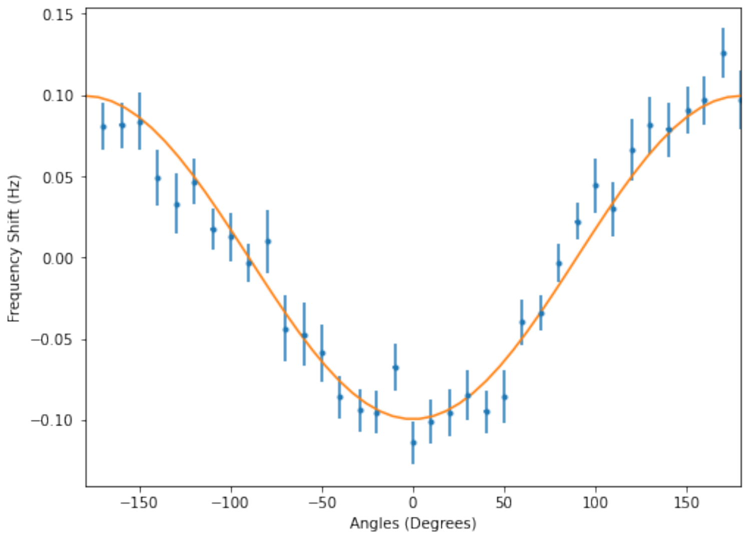
$$v \ll c$$

$$\Delta f = -\frac{2v}{\lambda}\cos\beta\cos\alpha$$



Verifying the Ashworth-Davies Doppler Shift





Arguments against Doppler shift in Sagnac effect

Physics – Uspekhi 43 (12) 1229 – 1252 (2000)

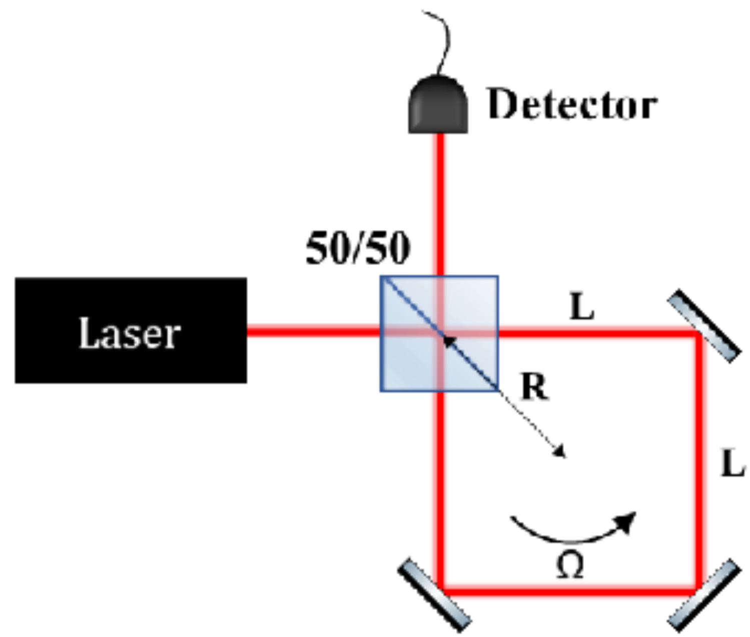
METHODOLOGICAL NOTES

PACS numbers: 01.65.+g, 03.30.+p, 07.60.Ly, 42.57.Bg

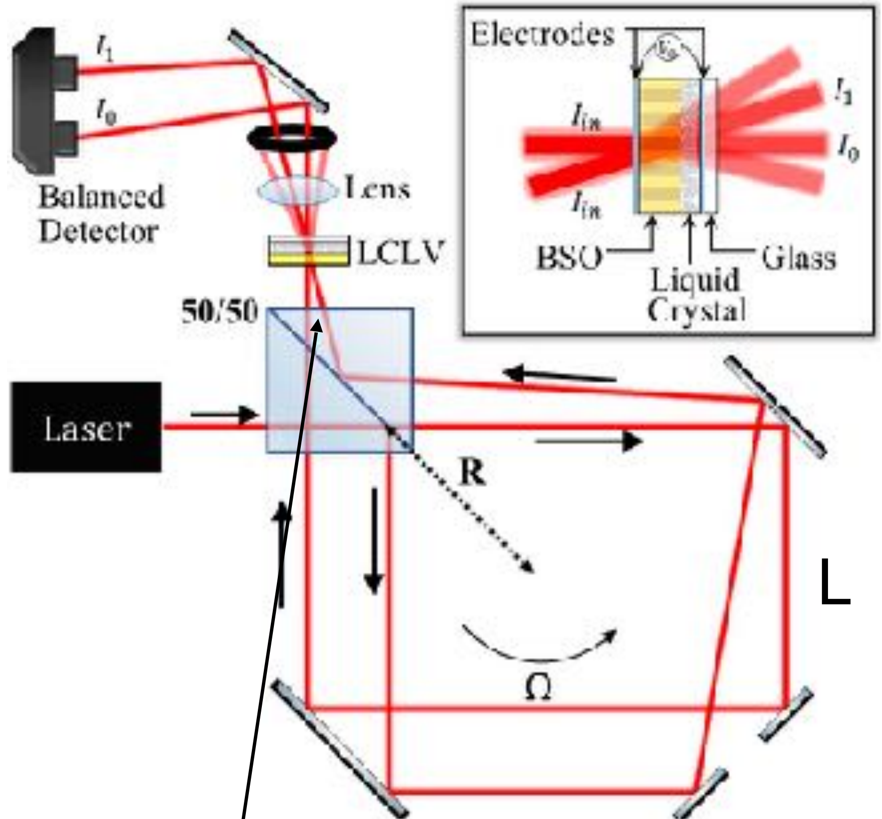
The Sagnac effect: correct and incorrect explanations

G B Malykin

1. Emitter and receiver the same for closed loop (beamsplitter)
2. In material medium of index n, Doppler predicts $2n^2$-fold larger signal

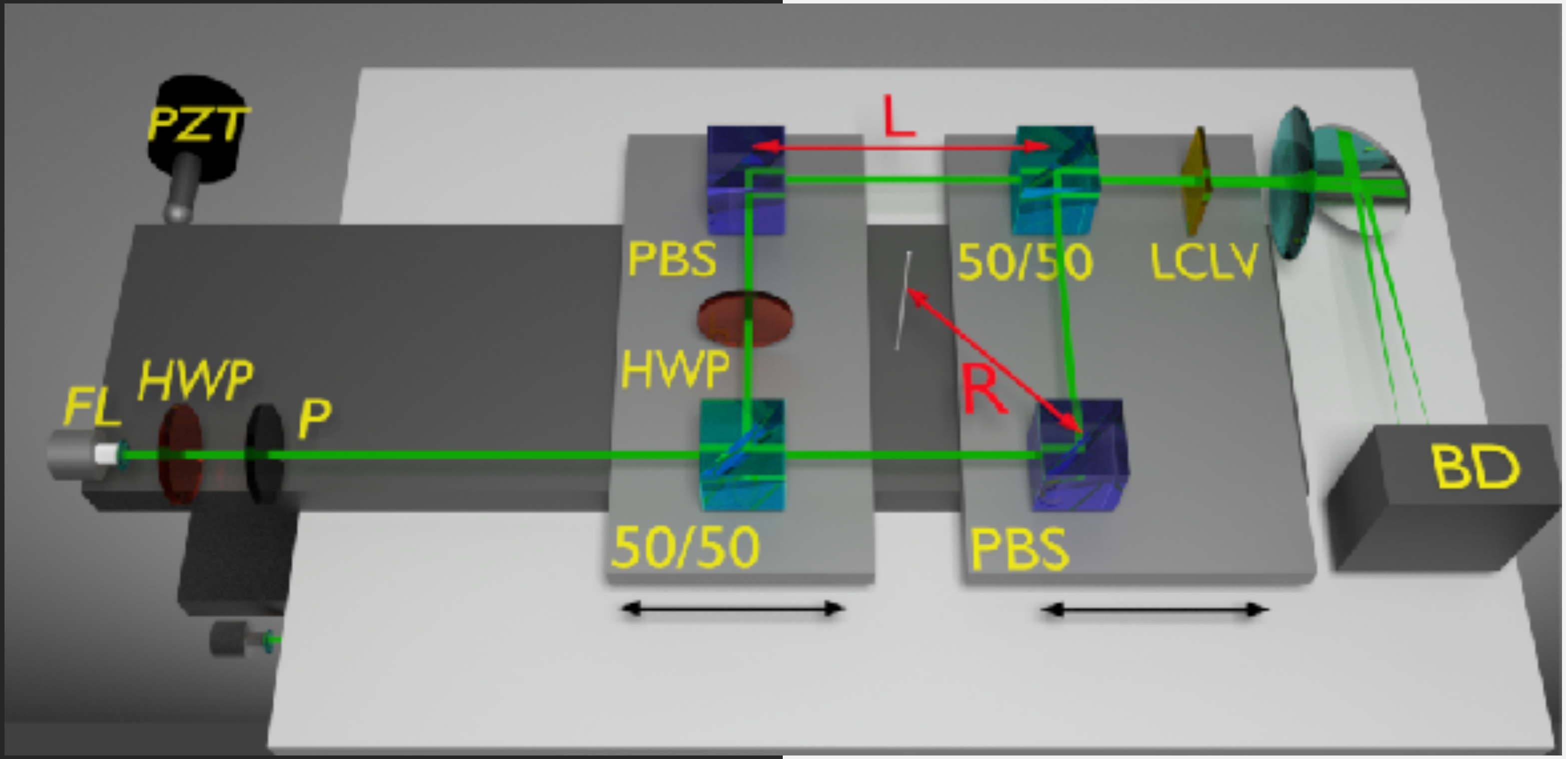


Our Broken Symmetry System



$$\Delta f = \frac{L\Omega\varepsilon}{2\lambda}$$

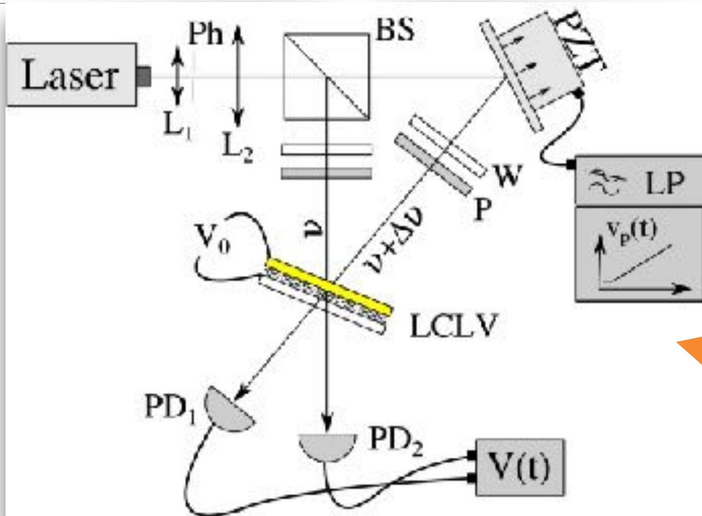
Doppler shift prediction for square interferometer

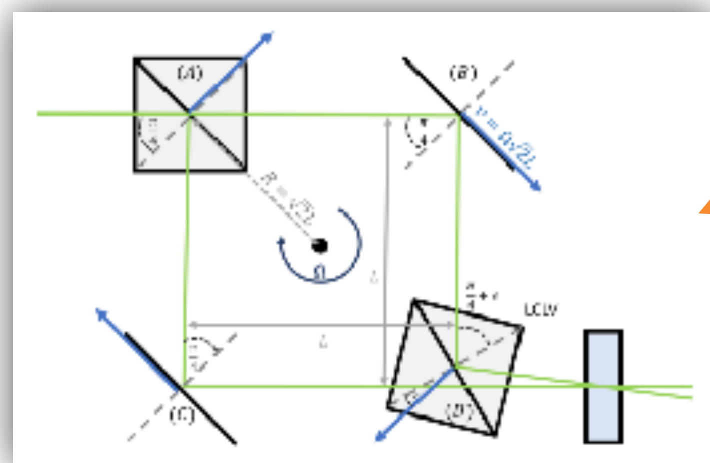


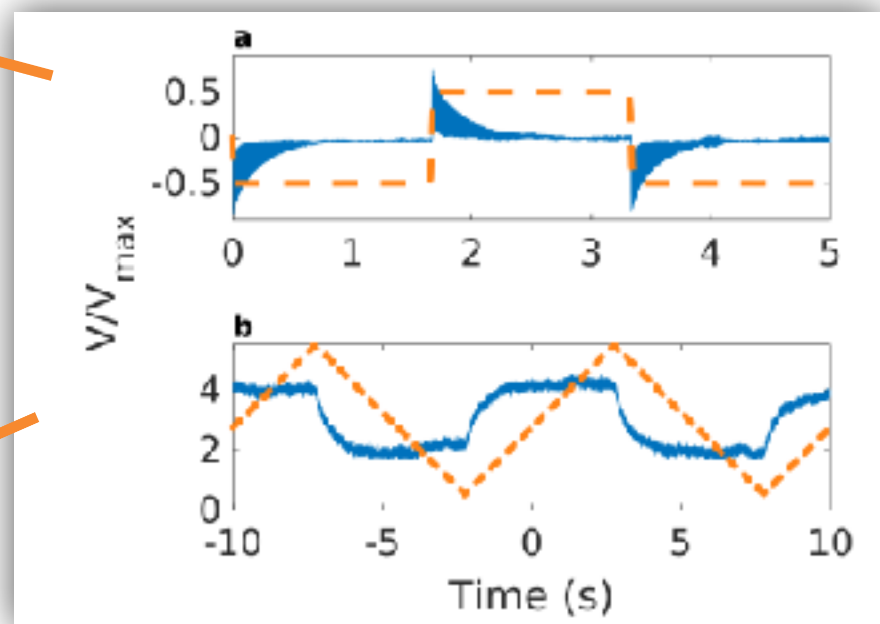
Our Actual Setup: Mach-Zehnder

Phys. Rev. Lett. **129**, 113901

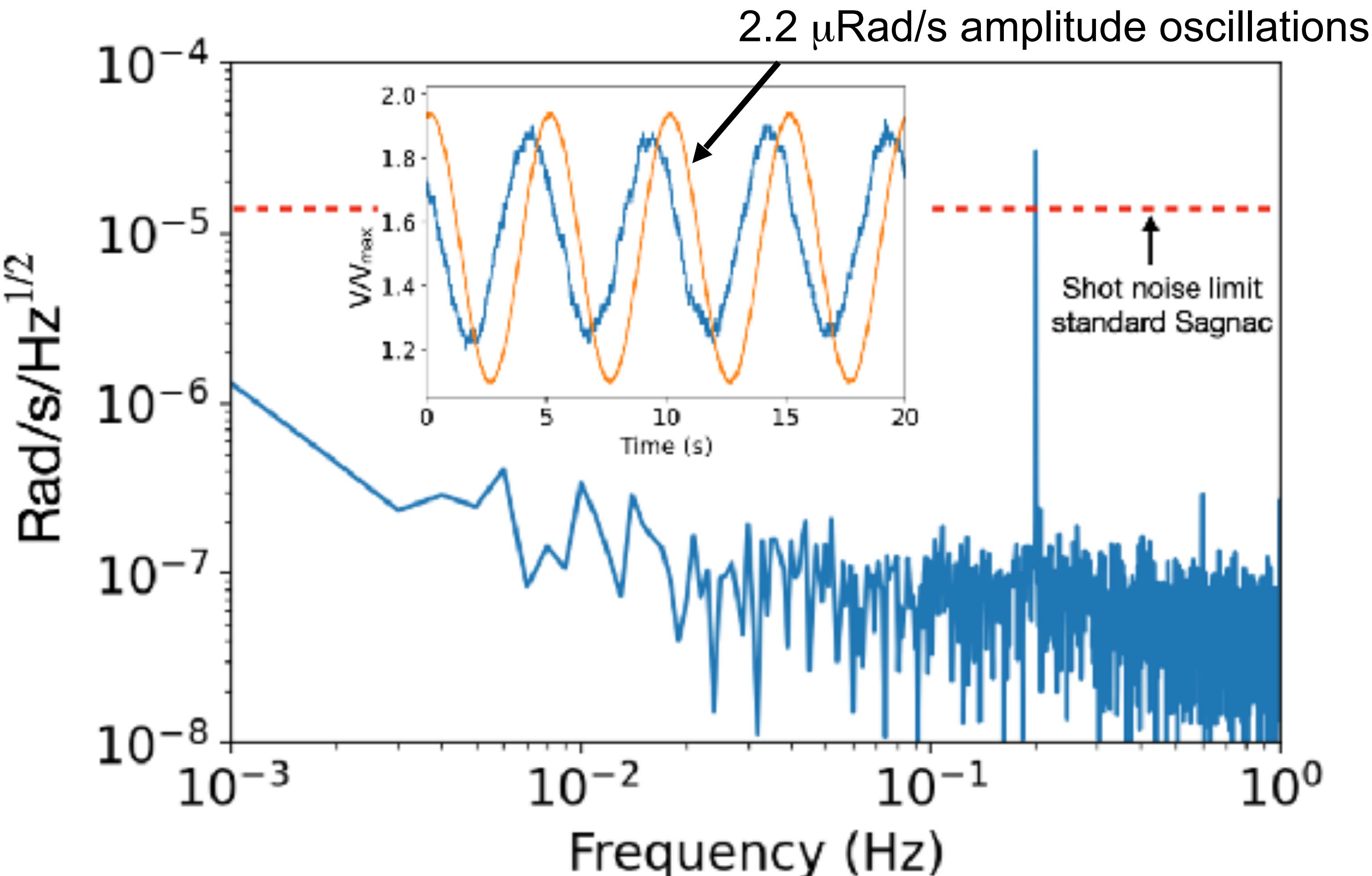
LCLV - Frequency Measurement





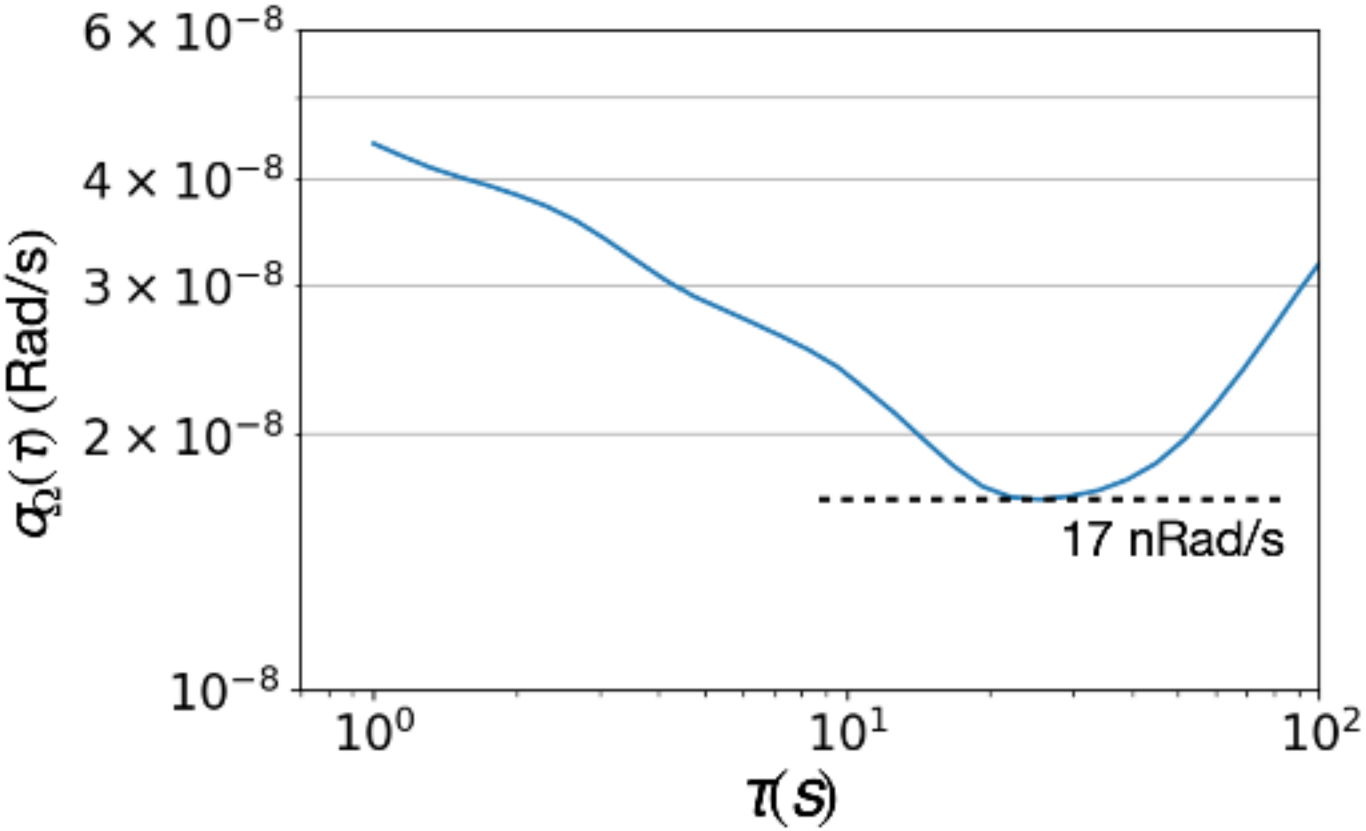


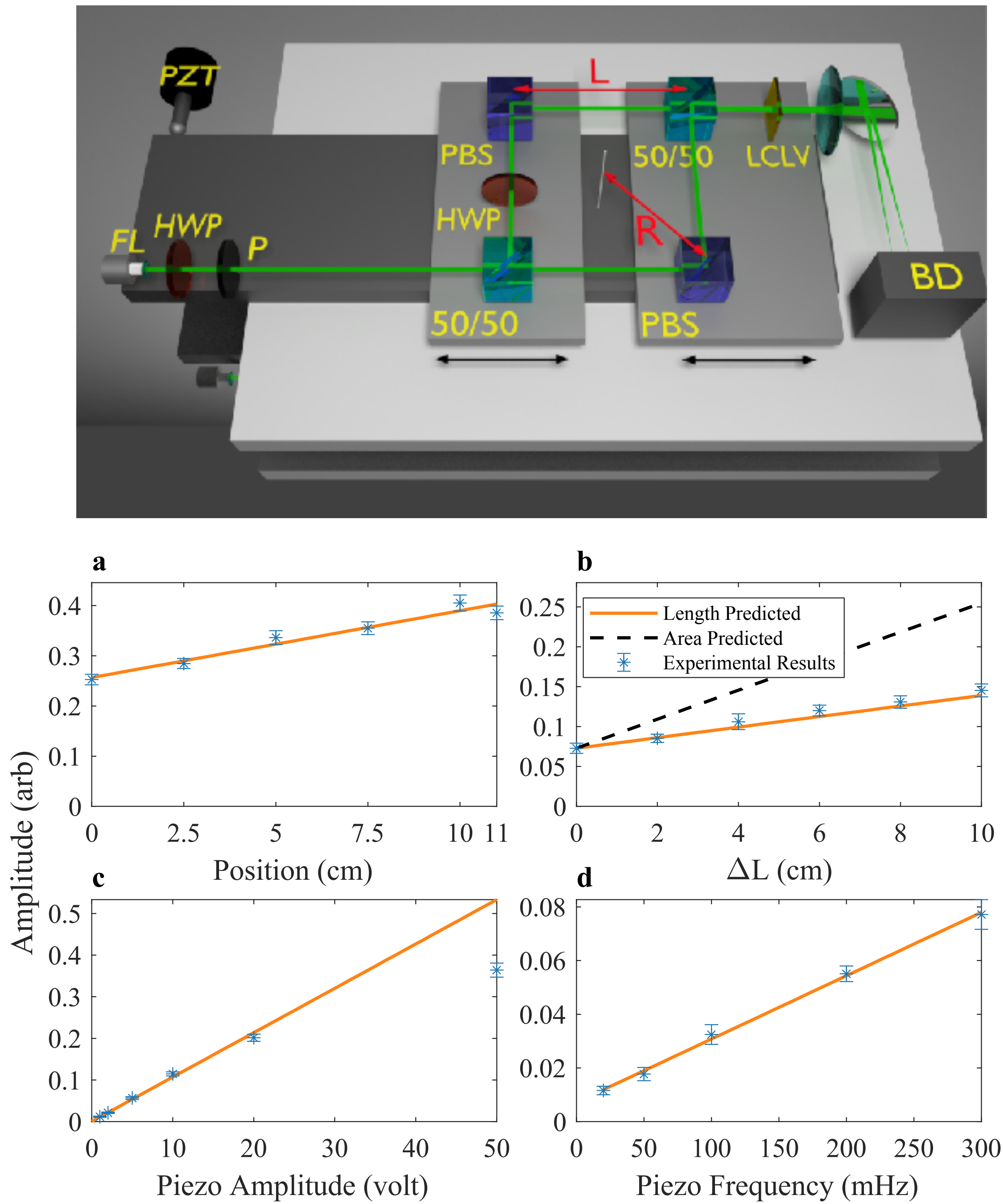
Way Below SQL Phase Estimation



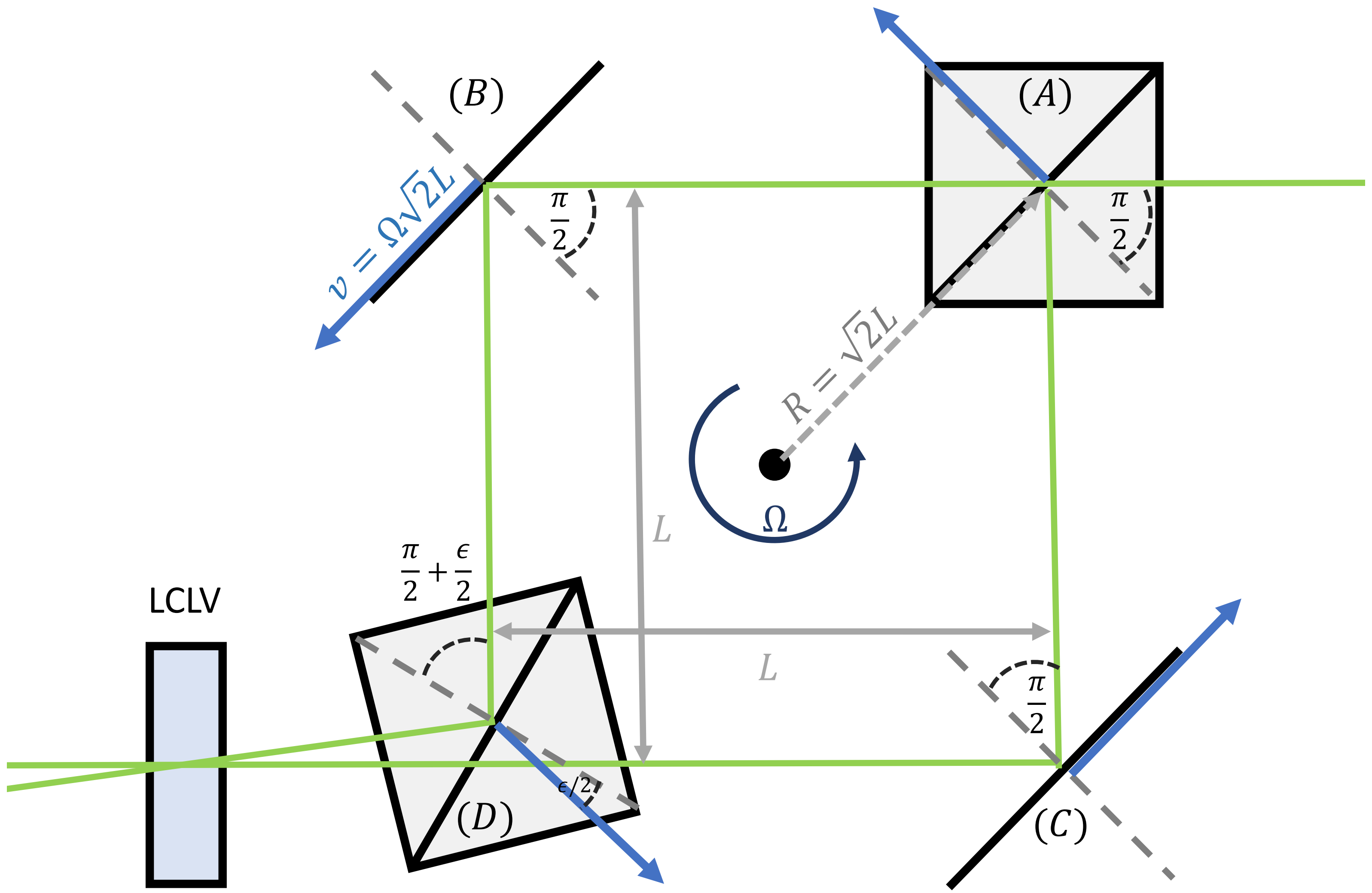
Phys. Rev. Lett. **129**, 113901

Allan Deviation

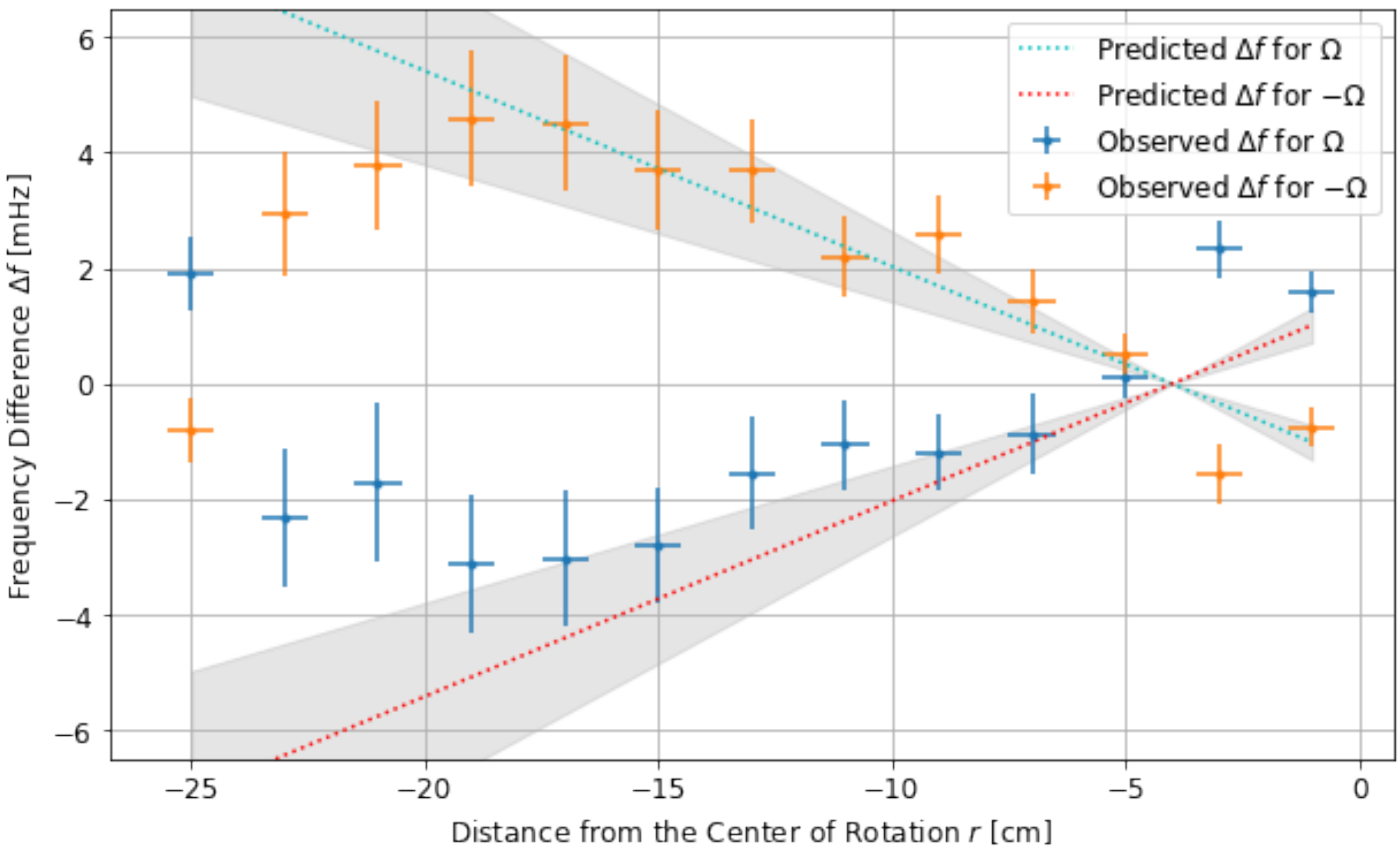



PZT
L
PBS
50/50
LCLV
HWP
R
FL
HWP
P
BD
50/50
PBS
a
b
Amplitude (arb)
Position (cm)
ΔL (cm)
Length Predicted
Area Predicted
Experimental Results
c
d
Piezo Amplitude (volt)
Piezo Frequency (mHz)

Generalized Theory



Theory vs Experiment for large radius rotations



Takeaways

- Beat SQL for phase estimation by orders of magnitude. Theoretically up to 5 orders of magnitude, experimentally by 2.
- Strong evidence Doppler shifts do exist within Sagnac effect.
- Sensitivity linear in length, not area.
- Sensitivity linear dependent on position of interferometer relative to axis of rotation.