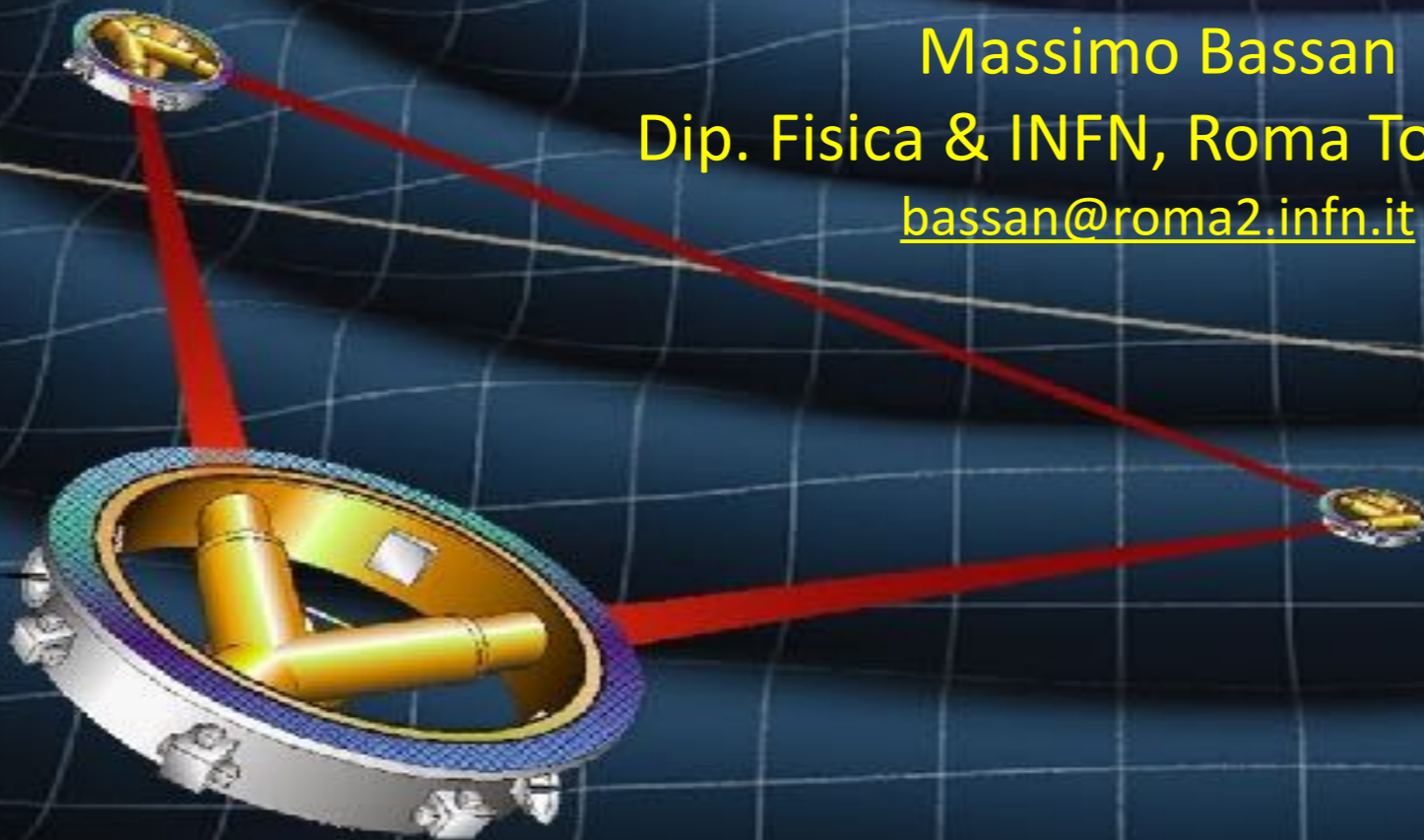


Gravitomagnetism and space missions the case of LISA

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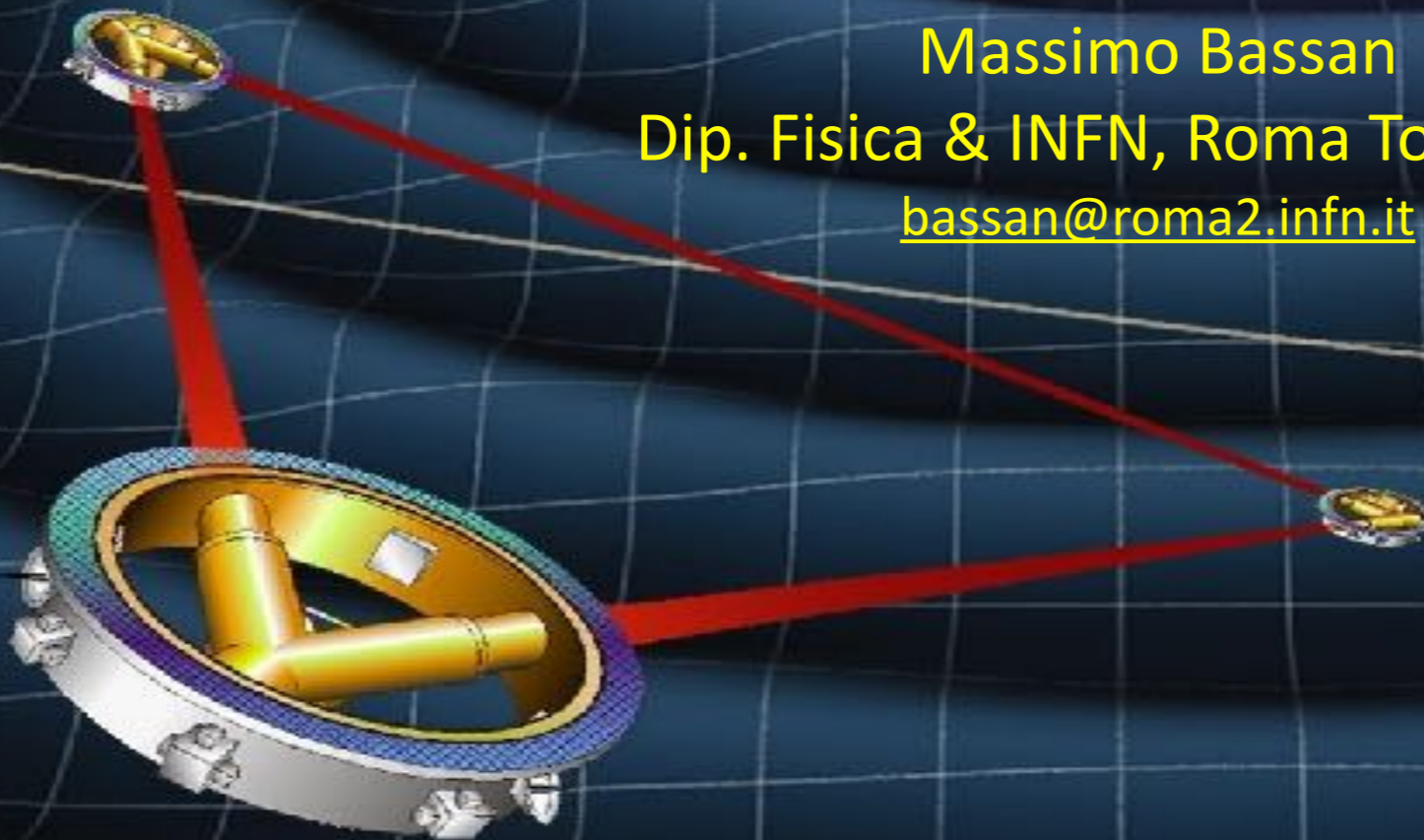
SPOILER:

Gravitomagnetism and space missions bad news from LISA

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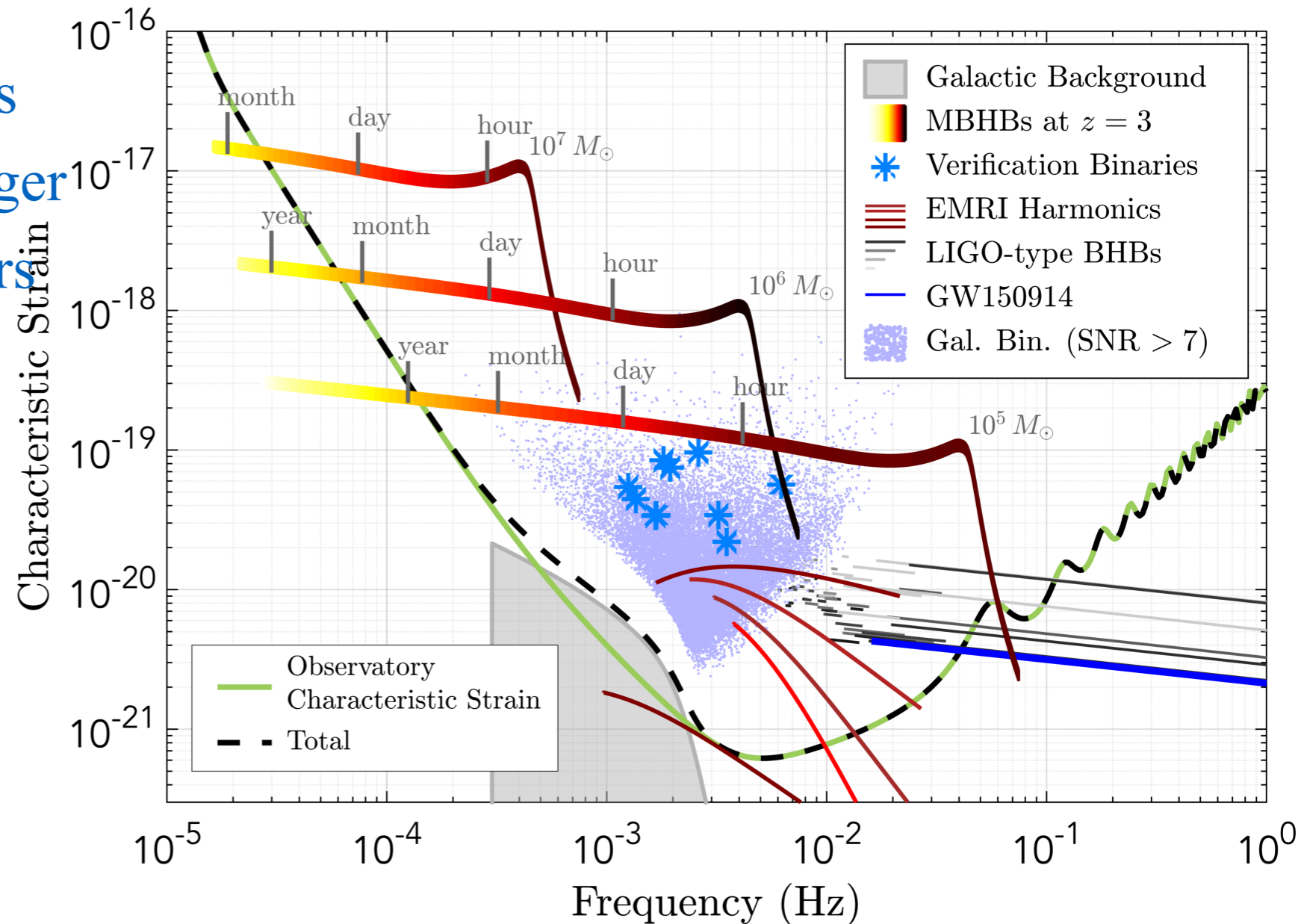
bassan@roma2.infn.it



LISA in two slides

- **LISA Science:**

- - Galactic compact binaries
- - MBHB inspiral and merger
- - EMRI and nuclear clusters
- - Fundamental nature of gravity and black holes
- - Expansion rate and stochastic gw bkgnd
- - Expect the unexpected !

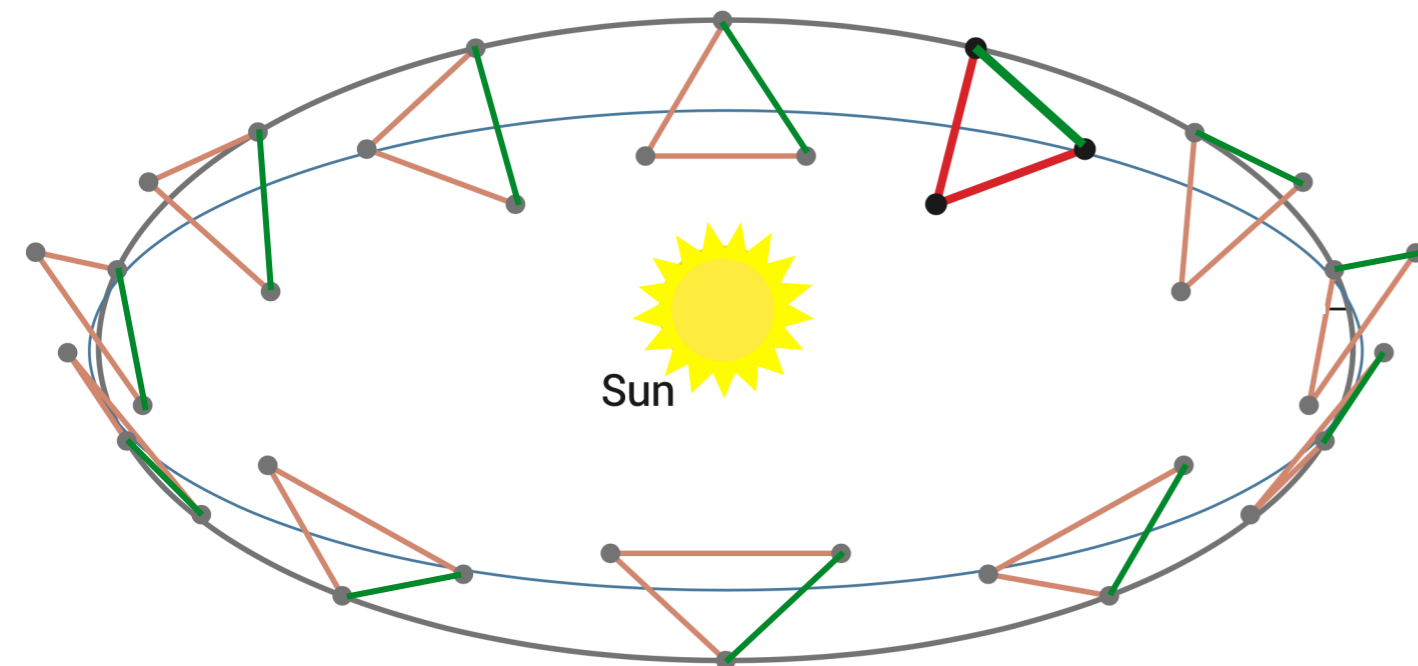
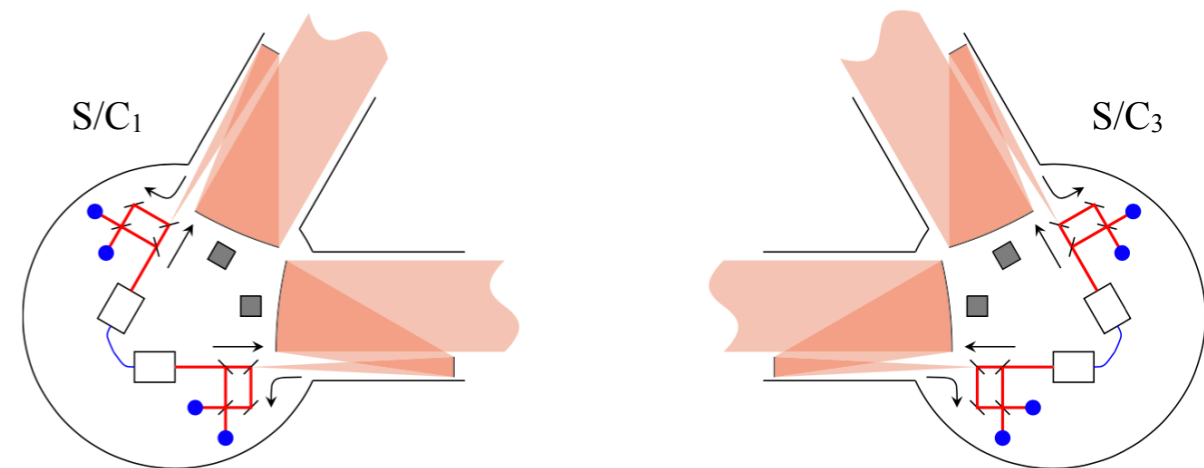
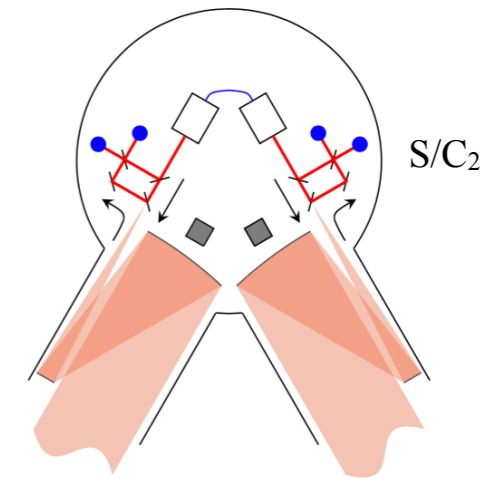


- Sensitivity at 20 μ Hz - 1 Hz

LISA in two slides

LISA Technology:

- 3 Spacecrafts in heliocentric orbit $L = 2.5$ Gm
- 6 Test Masses in free-fall along geodesics, shielded by S/Cs
- Interferometry \Rightarrow phase meters w/ $10 \text{ pm}/\sqrt{\text{Hz}}$ noise
- TDI for frequency noise reduction
- 2 independent IFOs + Sagnac veto



The *smart* orbits of the 3 LISA S/Cs

The eccentric anomaly is defined by: $\psi - e \sin \psi = \Omega t$;

Define 3 anomalies, w/ $\pi/3$ dephasing :

$$\psi_k - e \sin \psi_k = \Omega t - \frac{2\pi(k-1)}{3} \quad (k = 1, 2, 3)$$

The 3 orbits are then described as:

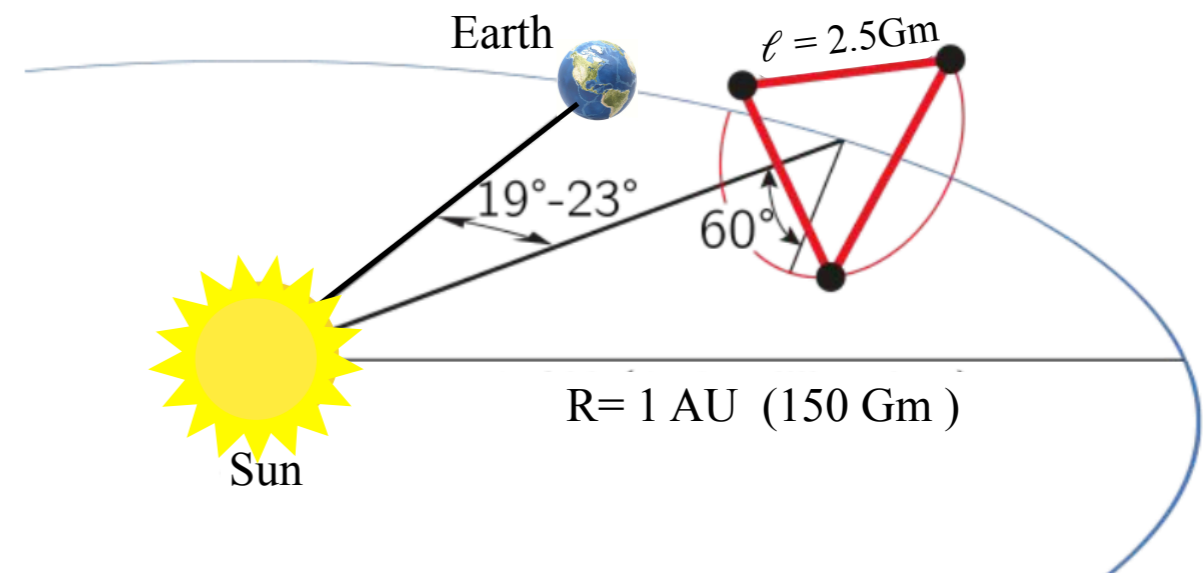
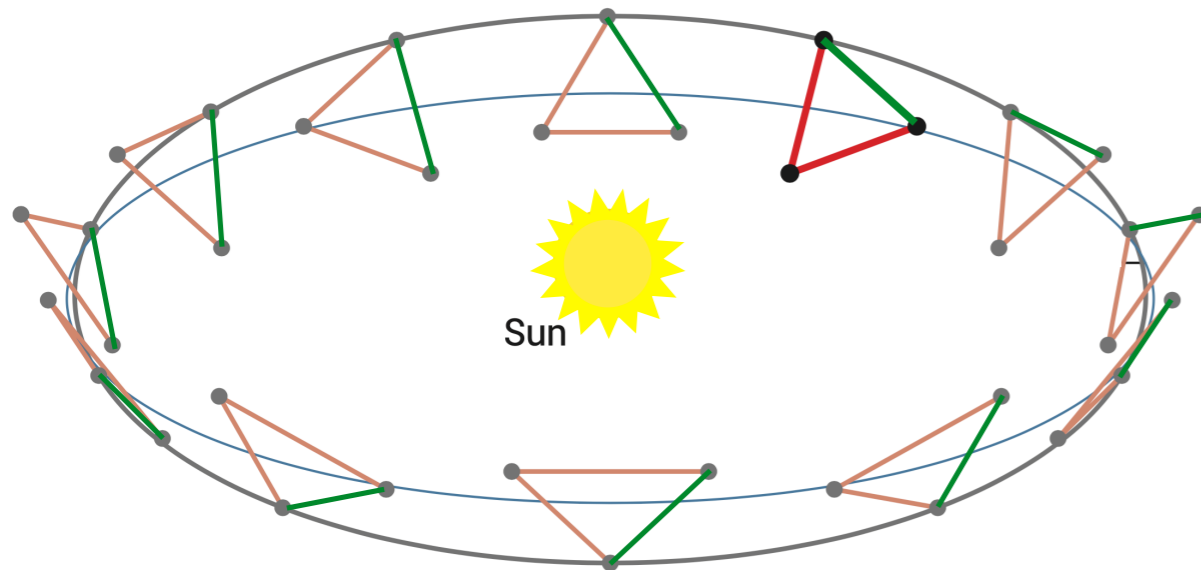
$$X_k = [R(\cos \psi_k - e) \cos i] \cos \phi_k - [R\sqrt{1-e^2} \sin \psi_k] \sin \phi_k$$

$$Y_k = [R(\cos \psi_k - e) \cos i] \sin \phi_k + [R\sqrt{1-e^2} \sin \psi_k] \cos \phi_k$$

$$Z_k = R(\cos \psi_k - e) \sin i$$

where $R = 1 \text{ AU}$; $\tan i \simeq \alpha \equiv \frac{L}{2R}$

The *smart* orbits of the 3 LISA S/Cs



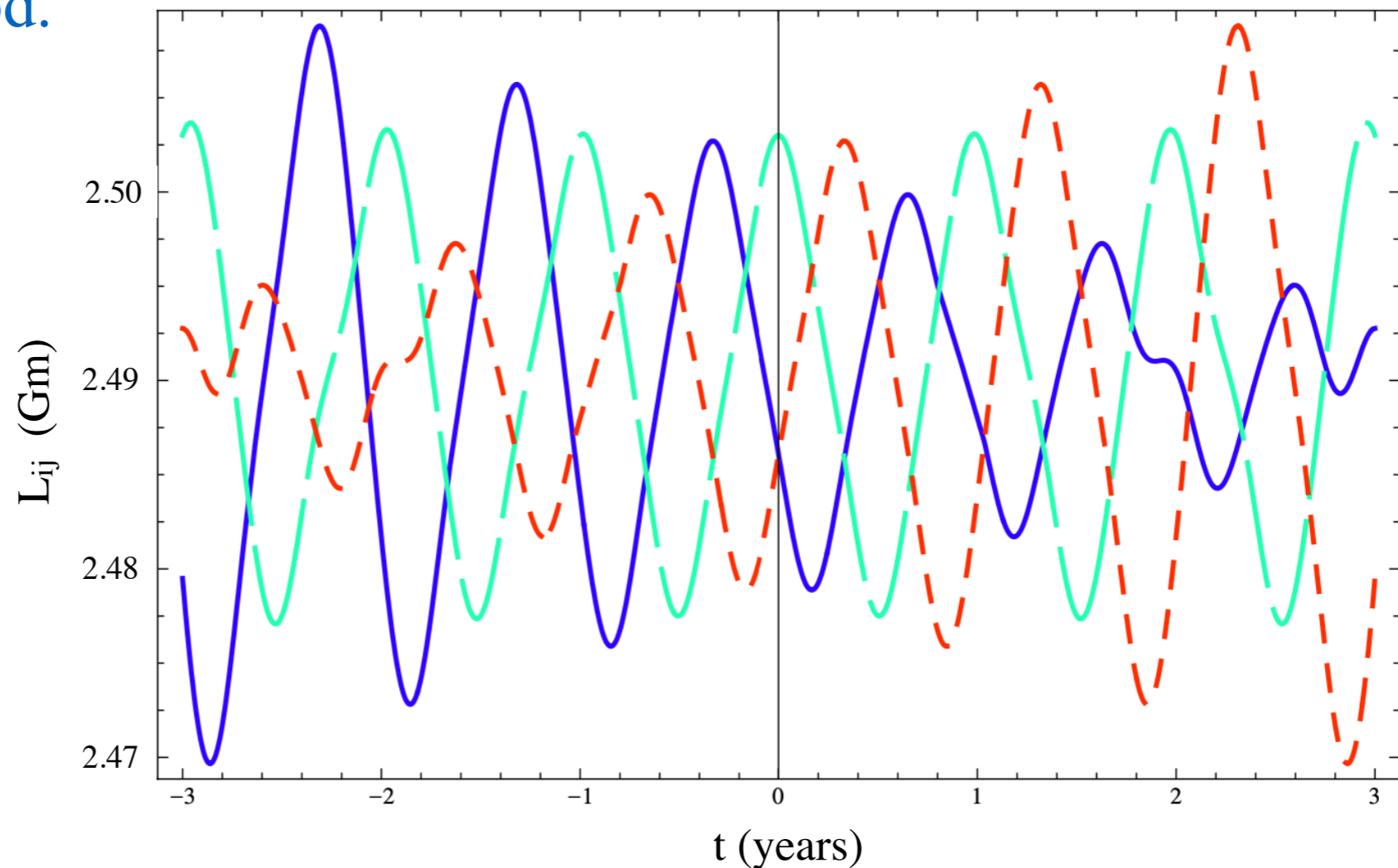
Each S/C rotates with 1 yr period around the Sun

AND around the constellation Center of Mass.

Constellation plane inclined by $\beta = \pi/3$ w.r.t. the ecliptic plane

A perfect triangle formation... almost !

Keplerian dynamics + perturbations produce changes in arm length of $\sim 1\%$ over the 1 yr period.

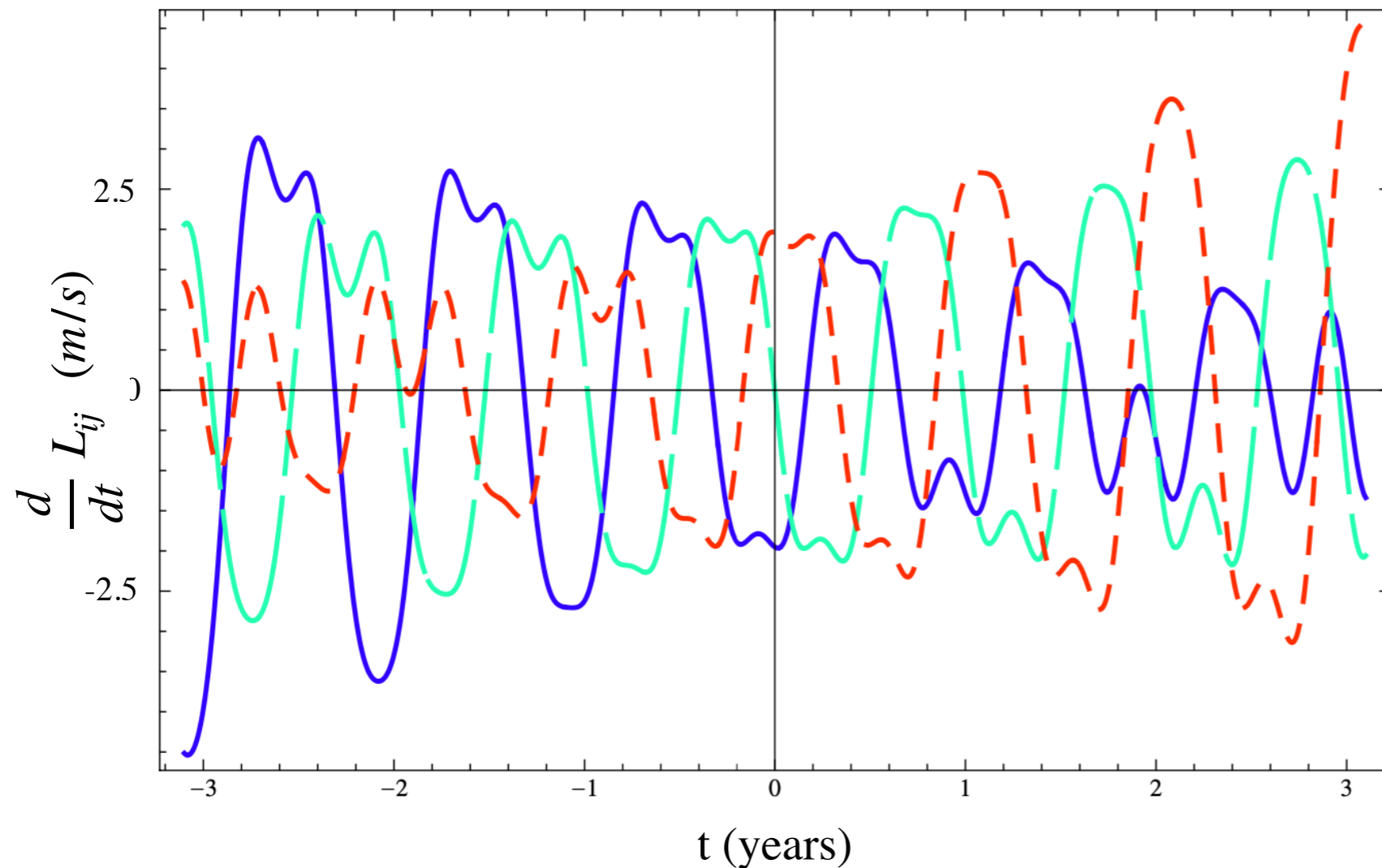


1% of L is some $2 \cdot 10^4$ km !

laser frequency noise $\delta\phi = \frac{\Delta L}{c} \delta f_L$ would kill you !

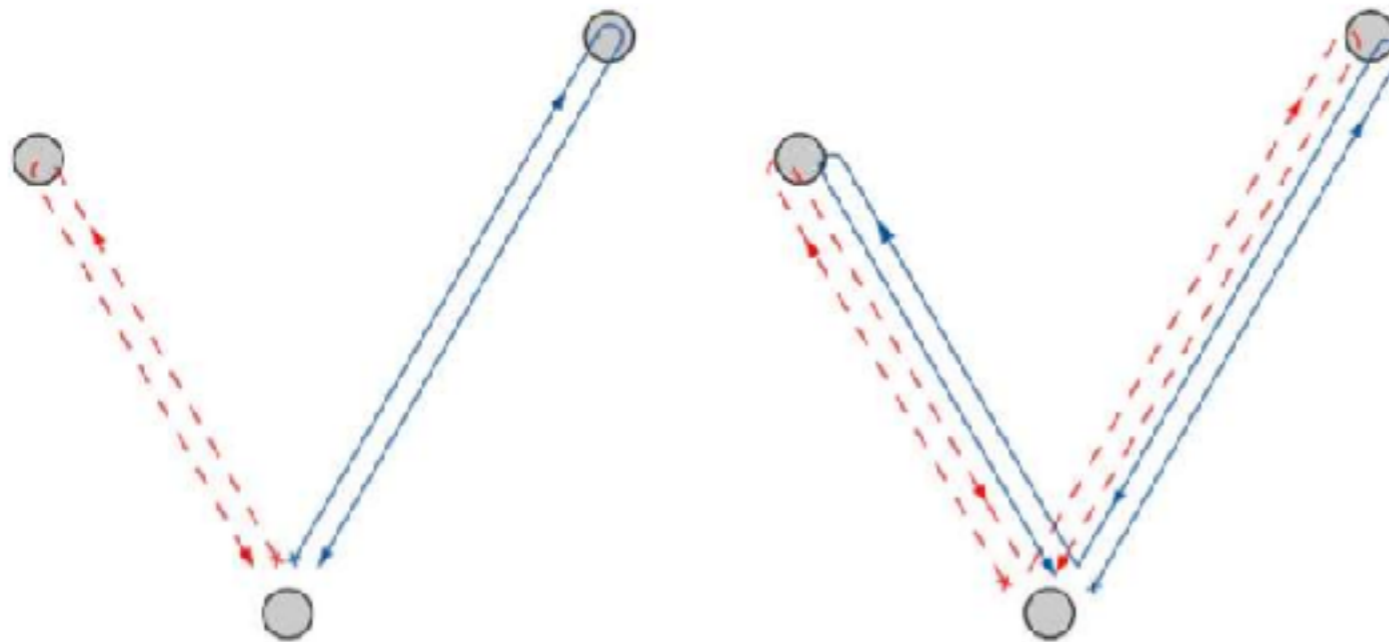
A perfect triangle formation... almost !

Keplerian dynamics + perturbations produce changes in arm length of $\pm 1\%$
 \Rightarrow varying velocity along the line of sight \Rightarrow Doppler signal (undesired)



Can't do traditional interferometry - use TDI

Time Delay Interferometry



Approach for fixed but unequal arms

1st generation TDI

Synthesize equal arm interferometry

2nd generation TDI

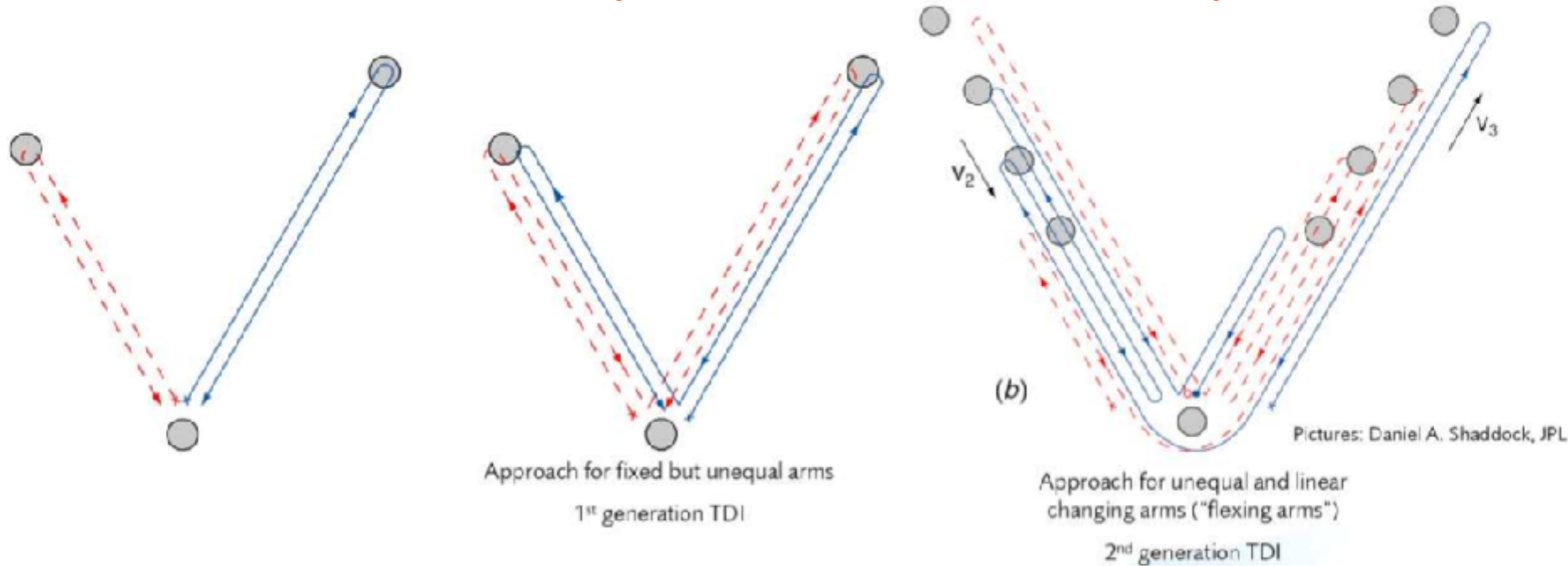
$$\begin{aligned}
 X(t) &= s_1(t) - s_2(t) - [s_1(t - 2\tau_2) - s_2(t - 2\tau_1)] \\
 &= [s_1(t) - s_1(t - 2\tau_2)] - [s_2(t) - s_2(t - 2\tau_1)]
 \end{aligned}$$

$s_i(t)$ = laser phase in arm i

τ_i = one way light travel time down arm i

pictures: D.A. Shaddock - JPL

Time Delay Interferometry



*Compensates for
different arm lengths and different velocities (Doppler)*

Many TDI combinations exist.

Some (Sagnac - like) are insensitive to g.w.

Back to GM...

We have seen that a large detector area is desirable:

$$\Delta\tau = -\frac{2}{c}\sqrt{U} \oint A_{(g)i} dx^i = -\frac{2}{c}\sqrt{U} \int \vec{B}_{(g)} \cdot \hat{u}_n dS \simeq \frac{4G}{c^4} \int \frac{J_{\odot}}{r^3} \cos \eta dS$$

LISA has a respectable area: $S \simeq 2.7 \cdot 10^{18} \text{ m}^2$

and it moves around the Sun, modulating the signal: $\eta = \eta(t)$

“Available” signals: Sun rotation, Galaxy rotation

Let's have a quick look on both

GM signal from the Sun

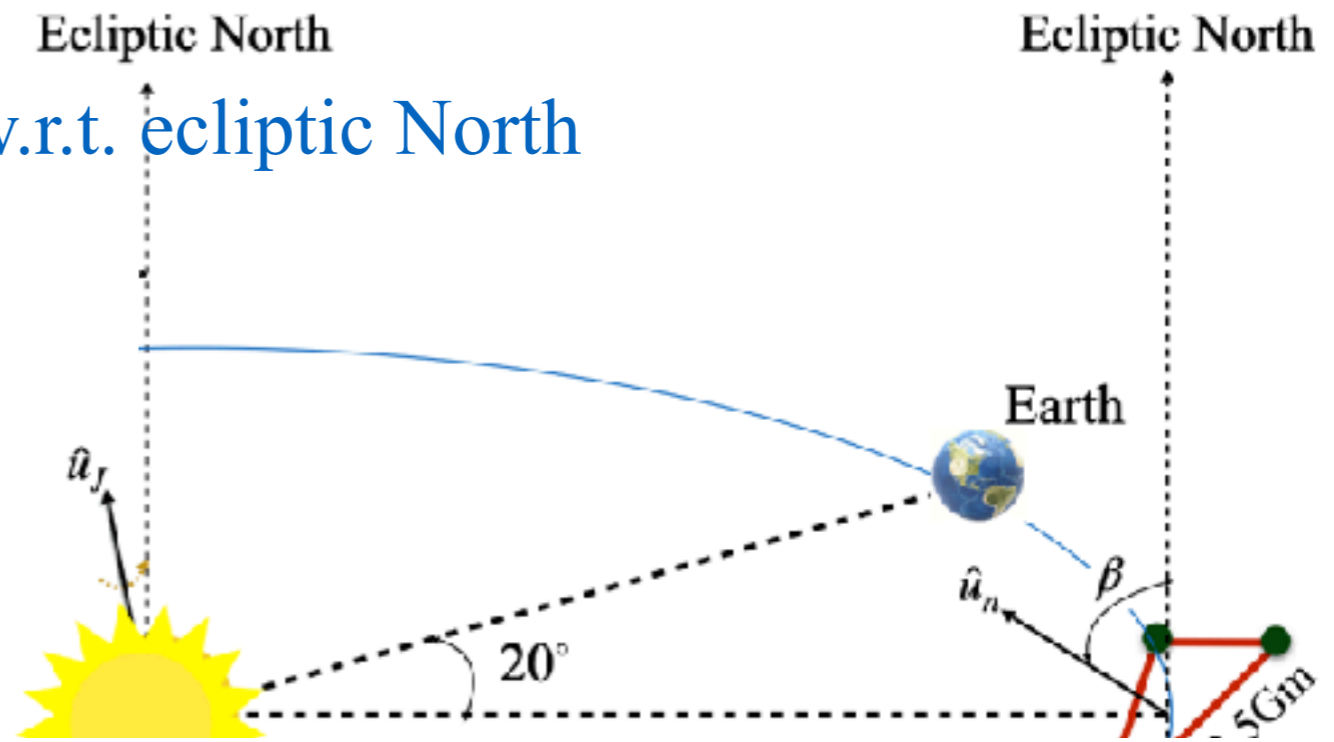
$$J_{\odot} = 1.9 \cdot 10^{41} \text{ kg m}^2 / \text{s}$$

$$B_g \simeq -\frac{2G}{c^3 R^3} J_{\odot} = -2.8 \cdot 10^{-28} \text{ m}^{-1}$$

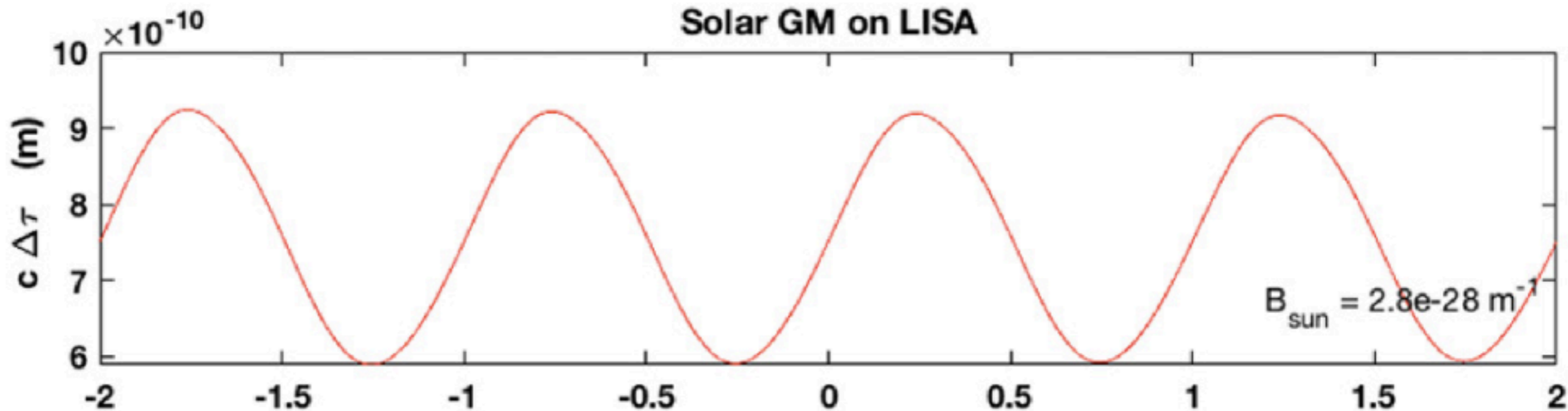
Sun rotation axis is inclined by $\chi \simeq 7^\circ$ w.r.t. ecliptic North

modulation: $\beta - \chi \leq \eta \leq \beta + \chi$

$$0.39 \leq \cos \eta \leq 0.60$$



Solar GM on LISA

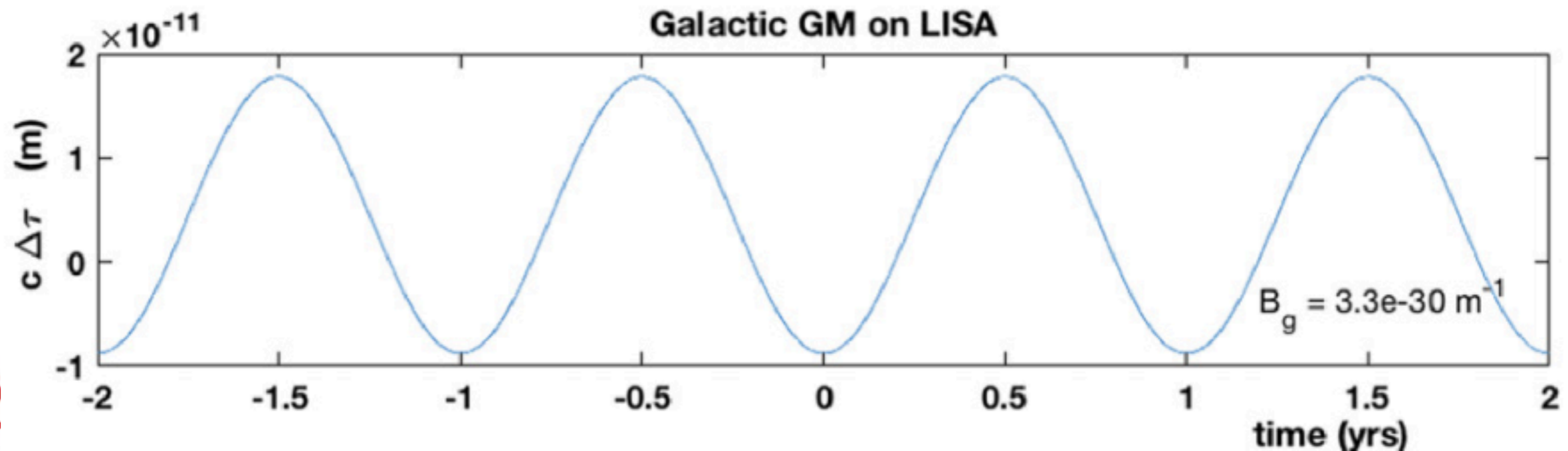
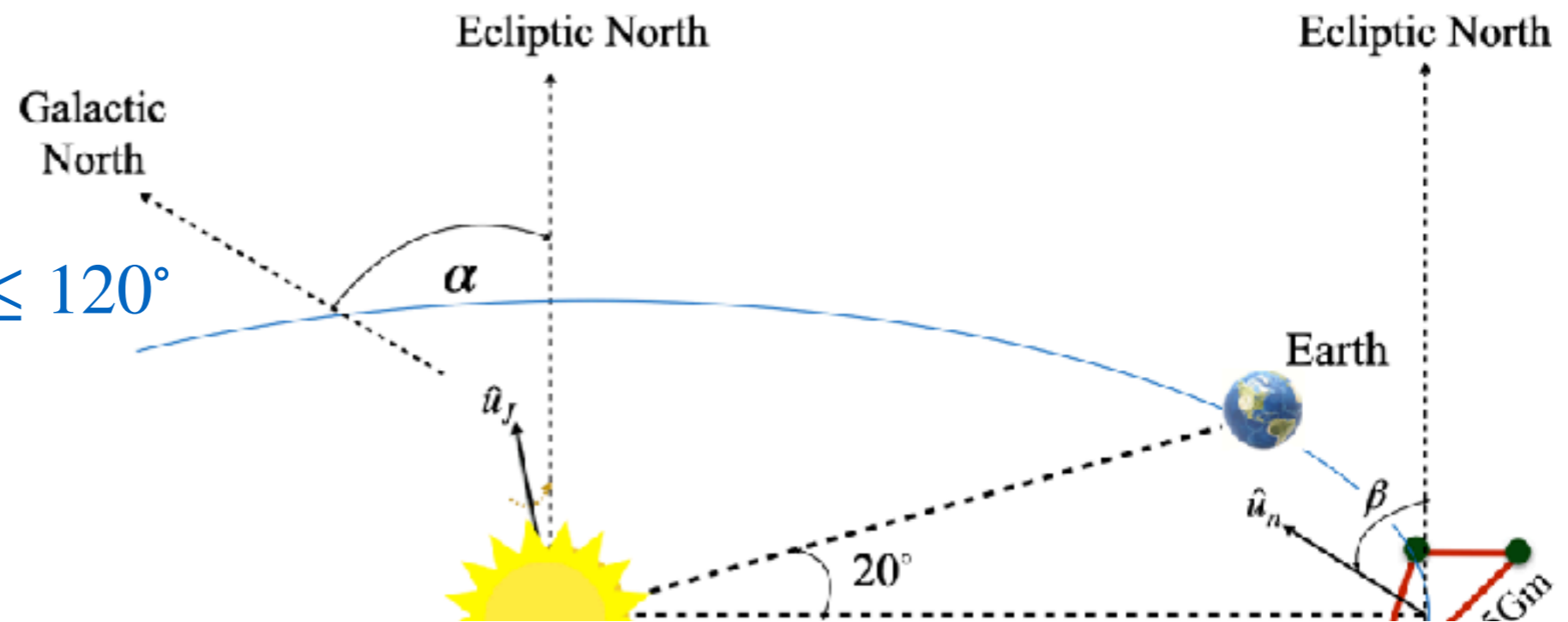


GM signal from Galaxy rotation

$B_{(g)gal} \sim 10^{-30} m^{-1}$ with ample uncertainty

Galactic North is inclined by $\alpha \simeq \pi/3$ w.r.t. ecliptic North

modulation: $0 \leq \eta \leq 120^\circ$



Can LISA see these signals ?

- Compare the expected GM signal with expected LISA noise:

- timing noise: $6 \times \frac{\delta L_{shot}}{c}$
2 beams, 3 links
 $\sim 15 \text{ pm}/\sqrt{\text{Hz}} @ 2 \text{ mHz}$

would allow a noise limit $B_g \sim 10^{-31} \text{ m}^{-1}$
with 1000 s integration time

BUT: nobody knows the noise @ 30 nHz (i.e. 1 yr⁻¹)

BUT BUT: Classic Sagnac is orders of magnitude larger

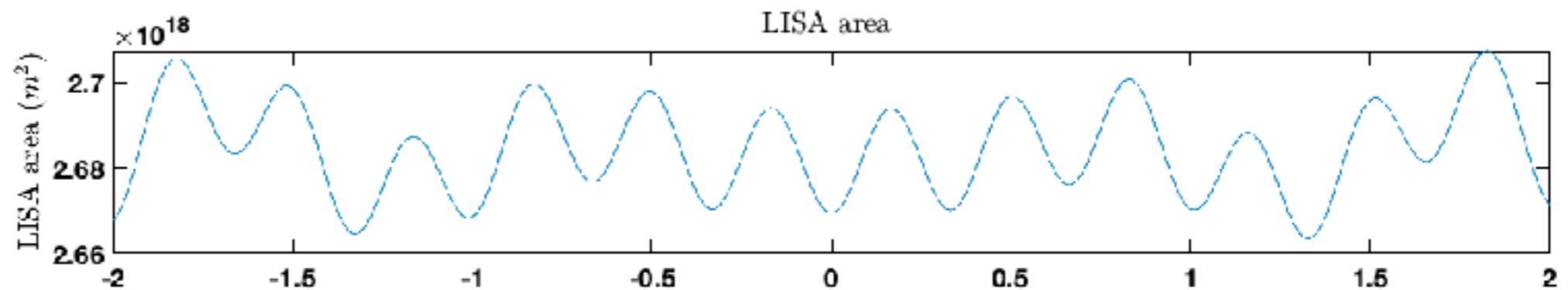
Sagnac effect in LISA

BUT: Classic Sagnac $\Delta\tau_{Sagnac} = \frac{4S}{c^2} \vec{\omega} \cdot \hat{u}_n$ is orders of magnitude larg

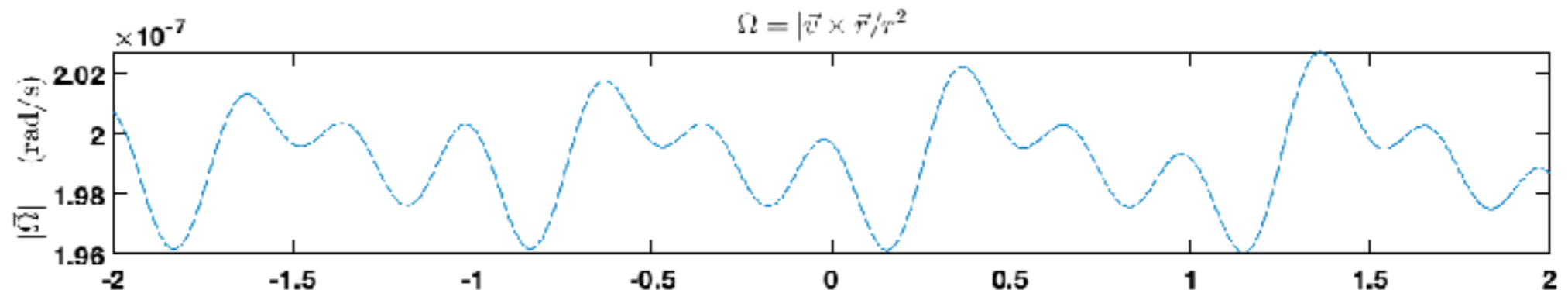
Hard to model: $S(t)$ and $\omega(t)$ because of flexing.

- what is the rotation angular frequency for a non rigid body ?

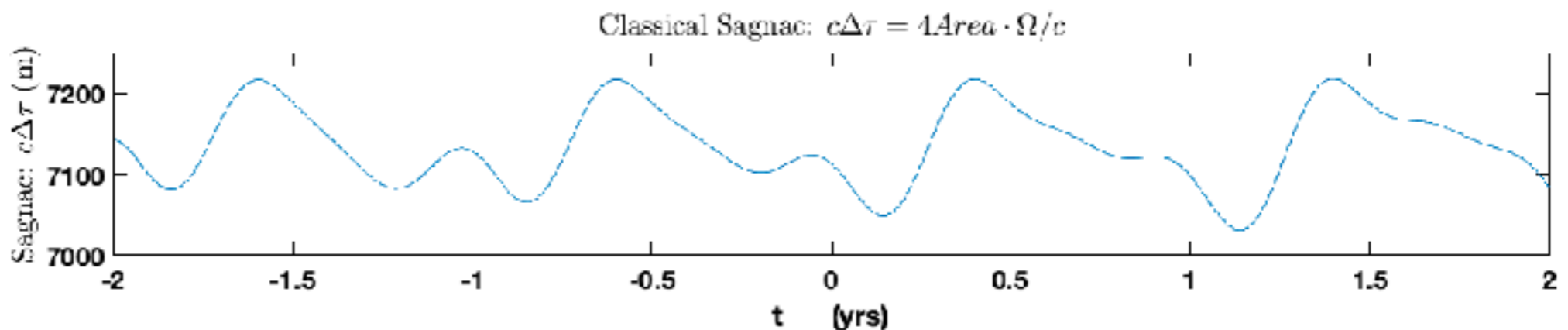
Area



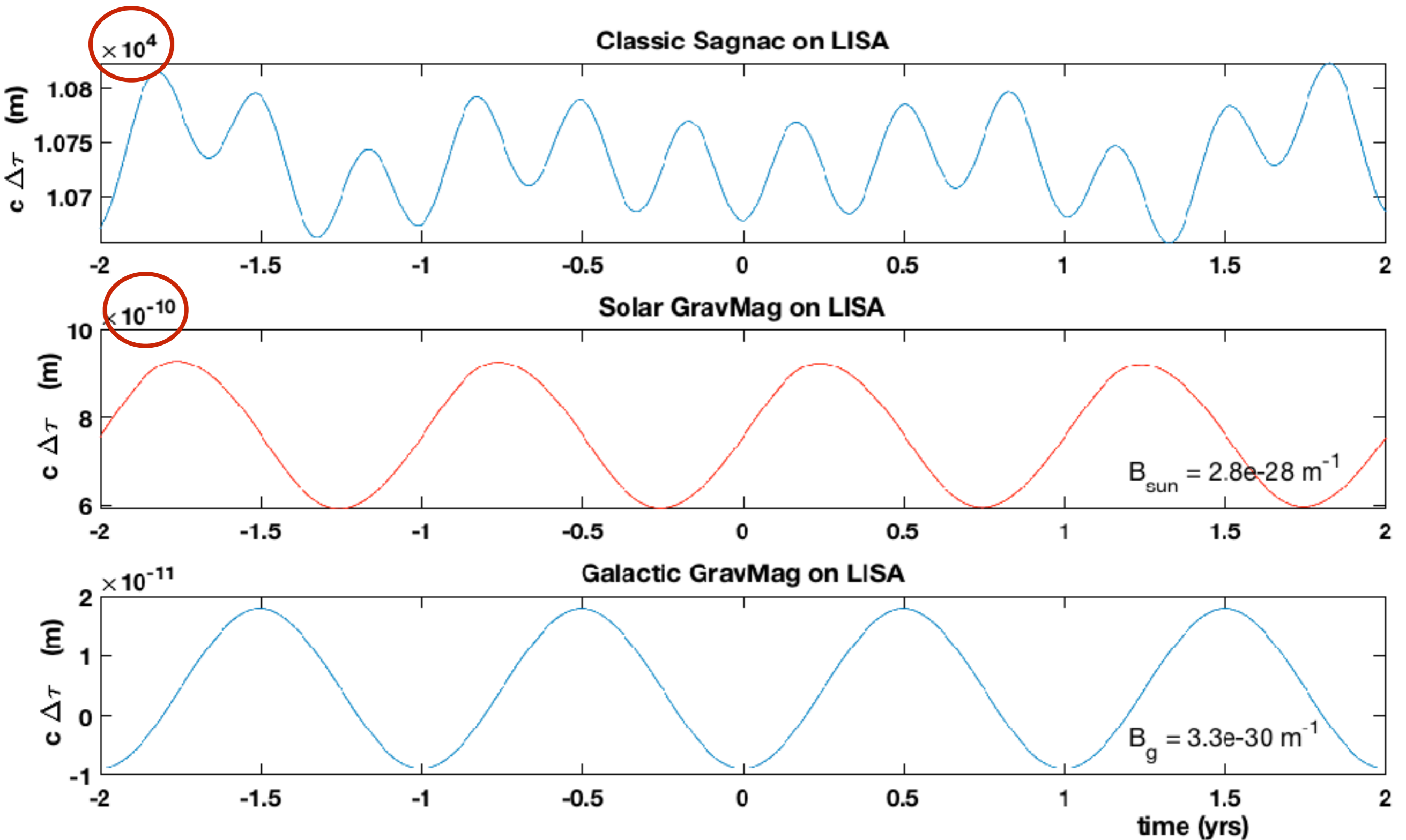
Ω



Sagnac



Comparing the signals



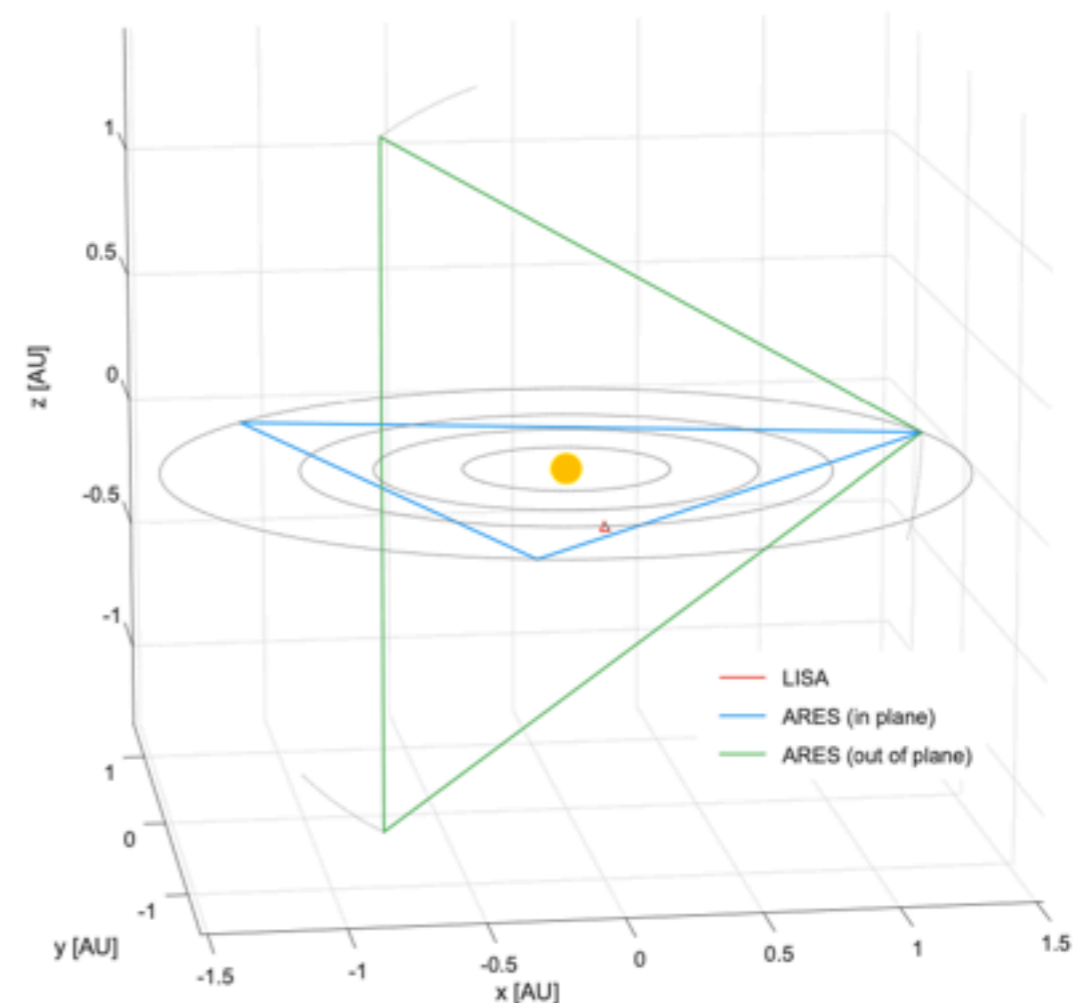
Concluding:

GM signal are present in LISA, and of measurable amplitude.

There is however a much larger Sagnac signal superimposed

The two effects appear to be indistinguishable. But, never despair:

- We might come up with super-smart TDI capable of discriminating
- In the future: non-planar constellation could be more suited to disentangle GM effects from Sagnac.



XXV SIGRAV Conference on General Relativity and Gravitation

Sep 4 – 8, 2023
SISSA (Miramare campus)
Europe/Rome timezone

<https://indico.sissa.it/event/96/>

Overview

Committees

Timetable

Registration

Call for Abstracts

Contribution List

Book of Abstracts

How to reach us

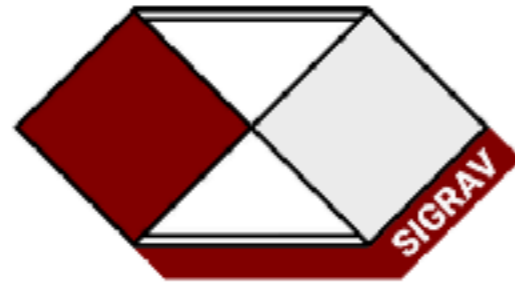
Accommodation

Connected events

Free Circulation of
Scientists and Code of
Conduct

Contact

✉ sigrav-xxv@sissa.it



**Società Italiana di Relatività Generale e
Fisica della Gravitazione**

The Italian Society of General Relativity and Gravitation announces the XXV SIGRAV Conference, to be held in Trieste, Italy from September 4th to the 8th, 2023. The conference will be hosted in the Miramare SISSA Building near the sea.



The conference aims to discuss aspects of Classical and Quantum Gravity, including General Relativity tests, Cosmology, Gravity experiments and Gravitational Waves from the experimental, theoretical and data analysis points of view.