LARGE SCALE SPACE INTERFEROMETRY TO MEASURE GALACTIC GRAVITOMAGNETISM

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INNER SPACE-TIME OF THE MILKY WAY (all included)

$$ds^2 = U(r,z)d\tau^2 - 2N(r,z)rd\varphi d\tau - W(r,z)r^2d\varphi^2 - Q(r,z)(dr^2 + dz^2)$$

The observer is within the galactic mass distribution, including the dark component

Axial symmetry, comoving frame (with the Sun, assumed to be in the galactic plane)

Constraints on *U, N, W, Q*:

- mirror symmetry across the galactic plane: $z = 0 \rightarrow \partial_z U = \partial_z N = \partial_z W = \partial_z Q = 0$

GALACTIC GRAVITO-MAGNETISM

$N/U \rightarrow$ GM vector-potential (in the galactic plane): $\vec{A}$ *= (0, N/U, 0)*
The base is *(dr, rd*φ*, dz)*

$$\bar{B}_g = \bar{\nabla} \times \bar{A} = \frac{N}{U}\left(\frac{\partial_r N}{N} - \frac{\partial_r U}{U}\right)\hat{z}$$

Distance from the Sun to the center of the Milky Way $R_S \cong 2.35 \times 10^{20}$ m

Displacement range $\delta R = 2$ AU $= 2.98 \times 10^{11}$ m $\delta R / R_S \sim 10^{-9}$

GEODETIC MOTION

Looks like the motion of a charge in an electromagnetic field

$$\frac{d^2\bar{x}}{dt^2} \cong -c^2\left(\bar{\nabla}U - 2\frac{\bar{v}}{c}\times\bar{B}_g\right)$$

Gravito-magnetic force on freely falling masses

LIGHT RAYS

$$0 = U(r,z)d\tau^2 - 2N(r,z)rd\varphi d\tau - W(r,z)r^2 d\varphi^2 - Q(r,z)(dr^2 + dz^2)$$

Time of flight along a given path

Antisymmetric

$$\tau = \int \frac{N}{U} r d\varphi \pm \int \frac{1}{U}\sqrt{N^2r^2 + U\left(Wr^2 + Q\left(\left(\frac{dr}{d\varphi}\right)^2 + \left(\frac{dz}{d\varphi}\right)^2\right)\right)}\, d\varphi$$

Symmetric

TIME OF FLIGHT ASYMMETRY FOR LIGHT

Spacely closed path in a plane
(observer's time)

GM Sagnac effect

Time of flight difference between right- and left-handed propagation in the galactic plane

Stokes theorem

$$|\Delta \boldsymbol{t}| = 2\frac{\sqrt{U}}{c}\oint \frac{N}{U} r d\varphi = 2\frac{\sqrt{U}}{c}\oint A_\varphi r d\varphi = 2\frac{\sqrt{U}}{c}\int \bar{B}_g \cdot \hat{u}_n dS$$

VARIOUS COMPONENTS OF $\bar{B}_g$

Very weak field approximation

$$\bar{B}_g \cong \bar{B}_{g\,Kinematics} + \bar{B}_{g\,Milky\ Way} + \bar{B}_{g\,Sun} + \bar{B}_{g\,Earth} + \ldots$$

Observer's motion

Visible + Dark

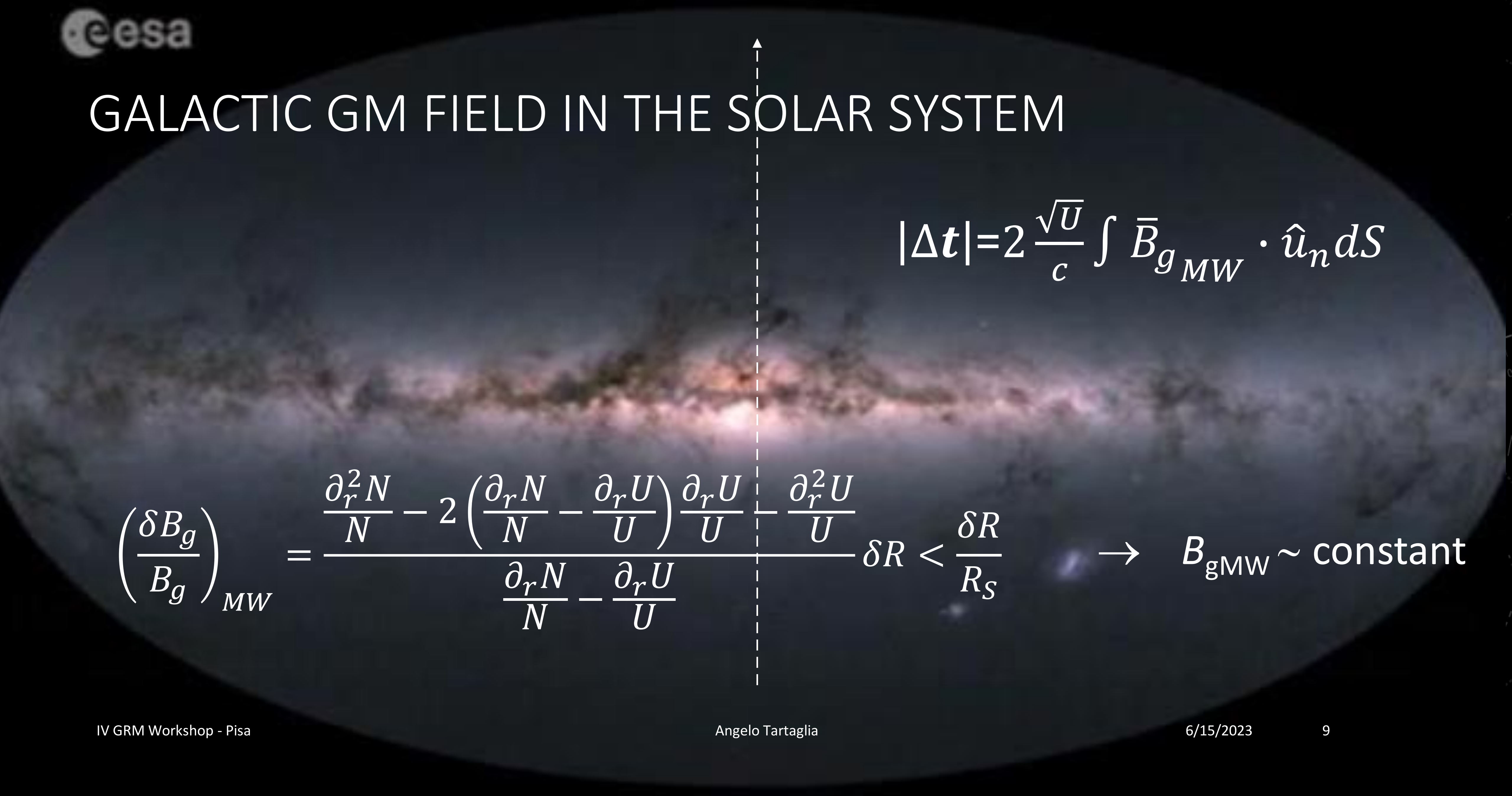
KINEMATIC TERMS: OBSERVER COROTATING WITH THE SUN

$$N' \cong -2\left(N + W\frac{\Omega r}{c}\right) + \ldots$$

$$U' \cong U - 2N\frac{r\Omega}{c} - W\frac{r^2\Omega^2}{c^2} + \ldots$$

These are due to the rotation of the observer

$$A' = -2\frac{N}{U} - 2\left(2\frac{N^2}{U} + W\right)\frac{r\Omega}{cU} - 2\left(3W + 4\frac{N^2}{U}\right)N\frac{r^2\Omega^2}{c^2U^2} + O\left(\frac{r^3\Omega^3}{c^3}\right)$$

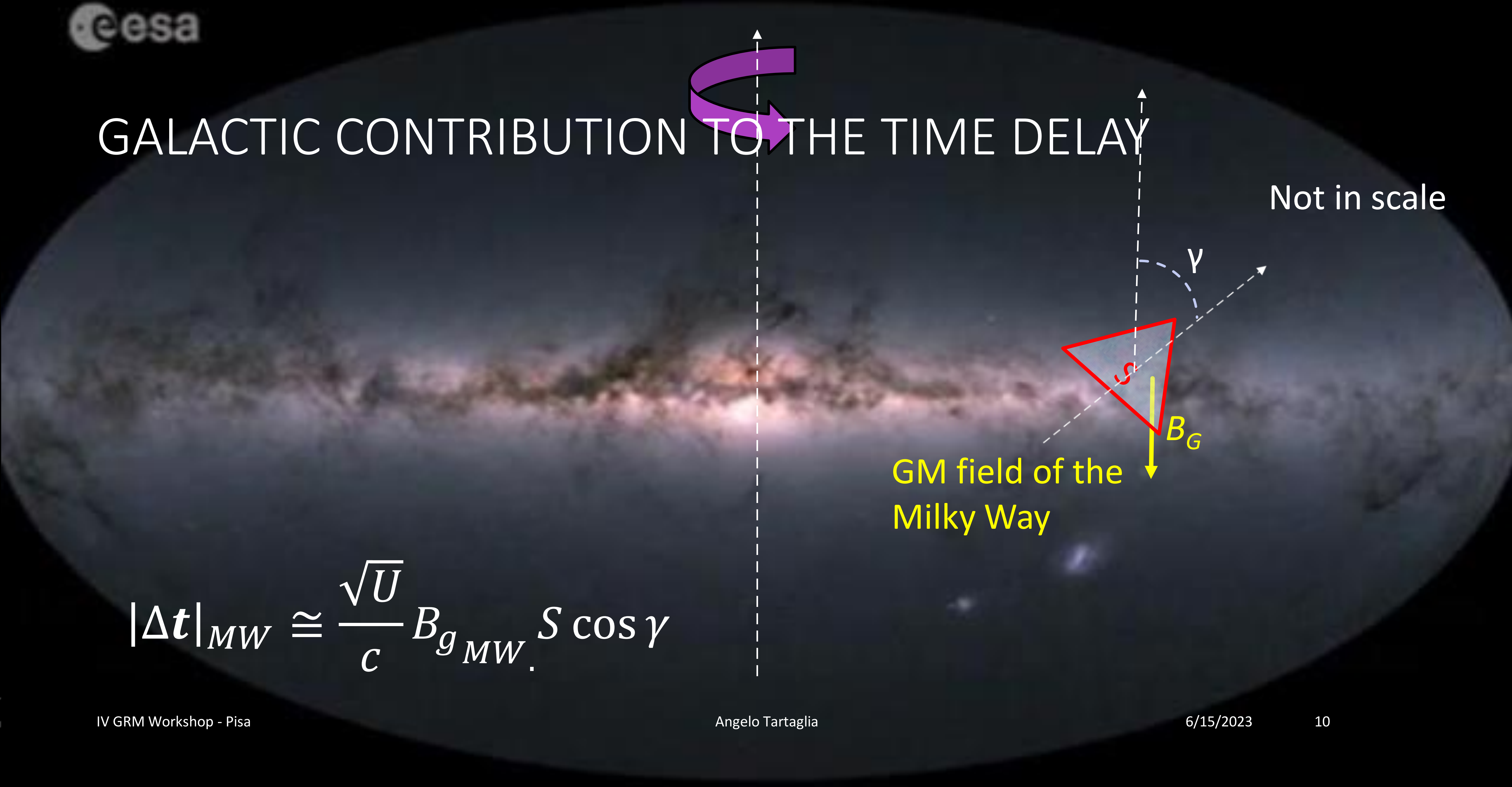


GALACTIC GM FIELD IN THE SOLAR SYSTEM

$$|\Delta \boldsymbol{t}| = 2\frac{\sqrt{U}}{c}\int \bar{B}_{g_{MW}} \cdot \hat{u}_n dS$$

$$\left(\frac{\delta B_g}{B_g}\right)_{MW} = \frac{\frac{\partial_r^2 N}{N} - 2\left(\frac{\partial_r N}{N} - \frac{\partial_r U}{U}\right)\frac{\partial_r U}{U} - \frac{\partial_r^2 U}{U}}{\frac{\partial_r N}{N} - \frac{\partial_r U}{U}}\delta R < \frac{\delta R}{R_S} \quad \rightarrow \quad B_{gMW} \sim \text{constant}$$

GALACTIC CONTRIBUTION TO THE TIME DELAY



$$|\Delta \boldsymbol{t}|_{MW} \cong \frac{\sqrt{U}}{c} B_{g_{MW}} S \cos\gamma$$

SOLAR CONTRIBUTION

The GM field of the Sun is dipolar

Angular momentum of the Sun (geometric units)

GM vector potential

$$A_\varphi = 2\frac{J_\odot}{r^2} sin^2\theta$$

Colatitude of the observer from the rotation axis of the Sun

$$\bar{B}_{g_{Sun}} = -\frac{2G}{c^3 r^3} J_\odot \left[2\cos\theta \hat{r} + \sin\theta \hat{l}_\theta\right]$$

$$J_\odot = 1.9 \times 10^{41}\ kg\ m^2/s$$

TIME OF FLIGHT ASYMMETRY DUE TO THE SUN

$$U = 1 - 2\frac{GM_\odot}{c^2 r} \cong 1$$

$$|\Delta \boldsymbol{t}|_\odot \cong \frac{2}{c}\int \bar{B}_{g_\odot} \cdot \hat{u}_n dS \cong \frac{4G}{c^4}\int \frac{J_\odot}{r^3}\cos\eta \, dS$$

Along the orbit of the Earth $\bar{B}_{g_{Sun}} \cong -\frac{2G}{c^3 {r_E}^3} J_\odot = -2.8 \times 10^{-28}\, m^{-1}$

REQUIREMENTS FOR A REAL EXPERIMENT

Extremely weak signal $\rightarrow$ Contured area as wide as possible

Important noise and spurious signals $\rightarrow$ modulation of the looked for effect in order to make it recognizable

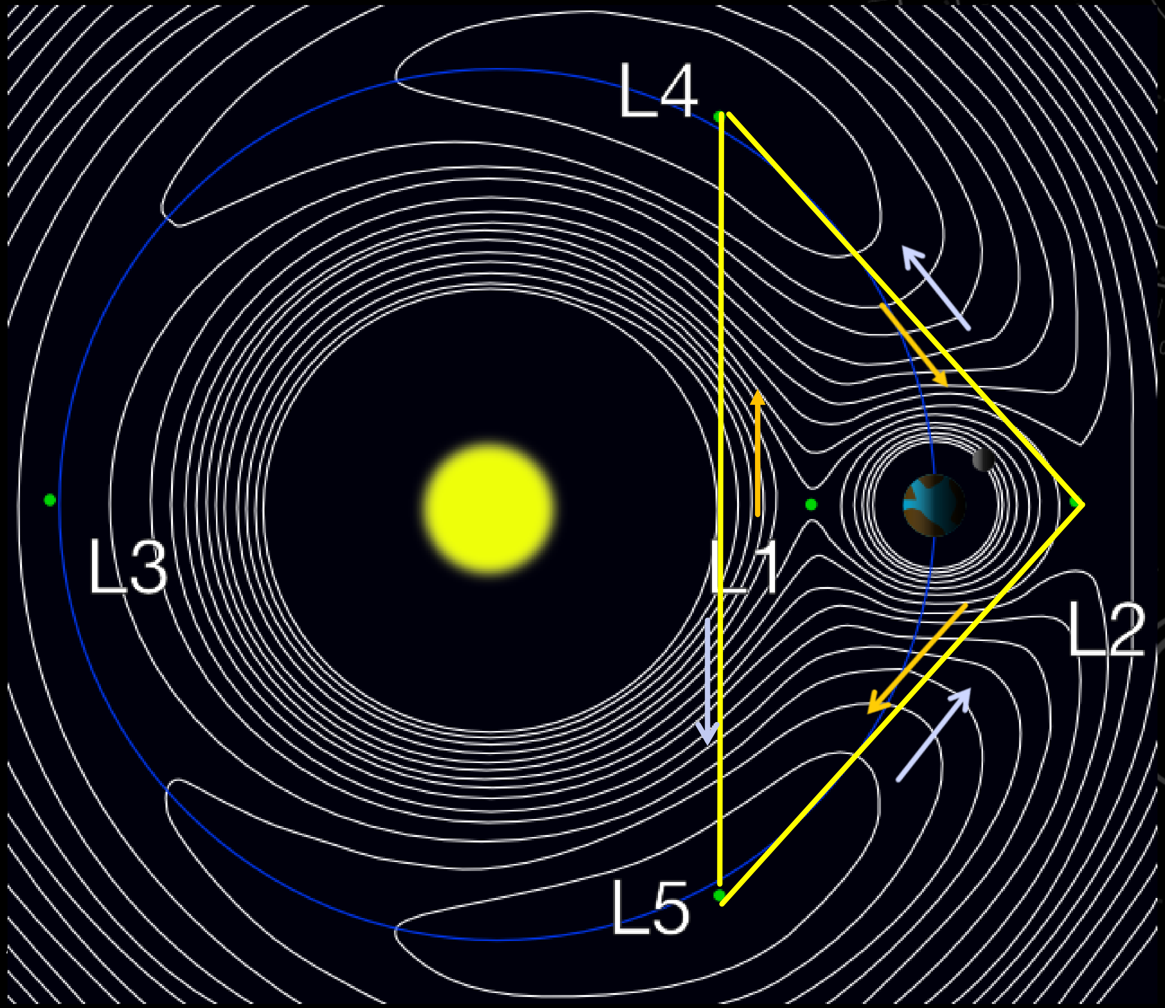
Various possible principle experiments

A SUN-EARTH LAGRANGEAN TRIANGLE

Locate emitters/transponders in the Lagrange points of the sun-earth pair

The system "rigidly" rotates together with the earth

$S_{245} \cong 9.9\times10^{21}\ \mathrm{m}^2$ $\alpha \sim 62.6°$

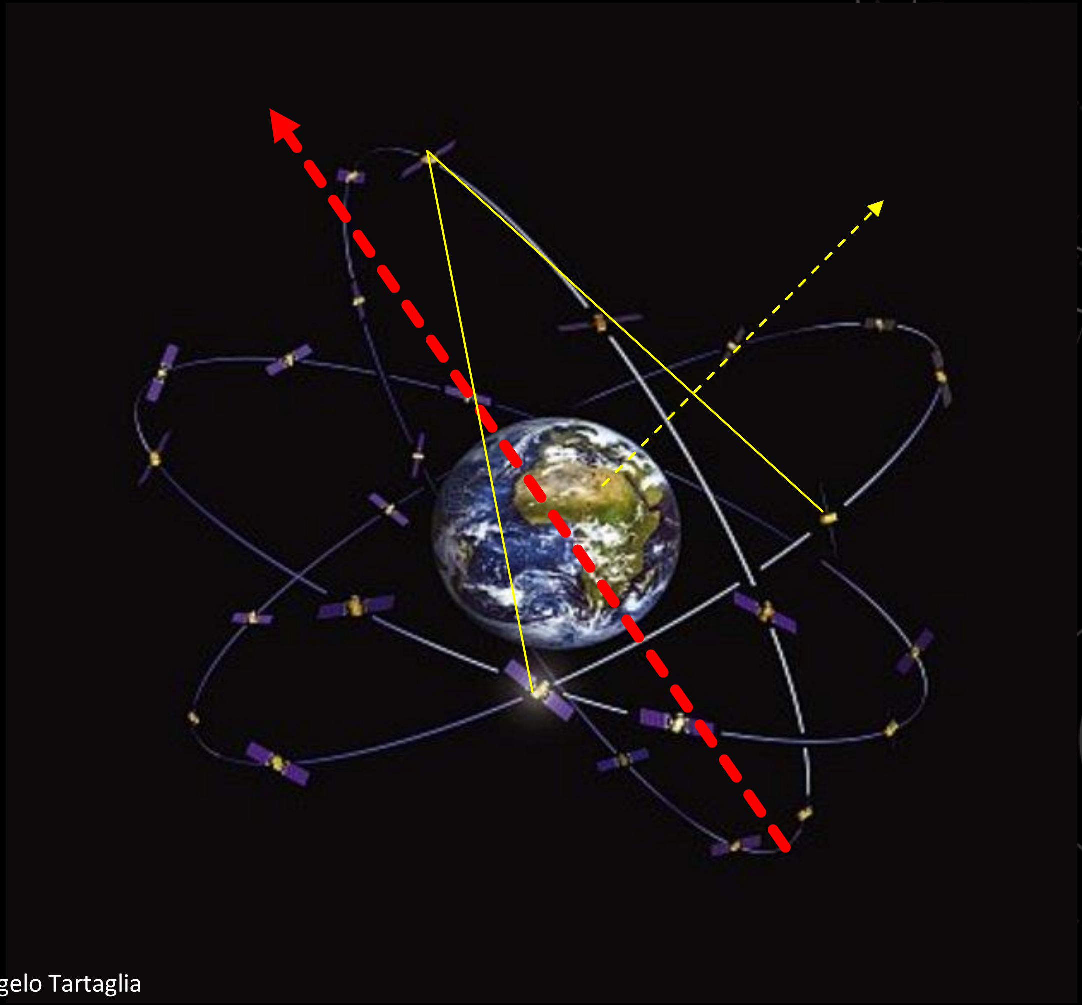


LAGRANGIAN POINTS REFERENCE FRAME

- L4 and L5: ~150 million km ahead and behind the earth. Stable.
- L1 and L2 : ~1.5 million km along the sun-earth line in opposite directions from the planet. (Weakly) unstable.
- L3: ~300 million km along the sun-earth line behind the sun. Not visible from earth; weakly unstable.

DANCING SATELLITES

- Two satellites in one orbital plane
- One satellite on a different plane

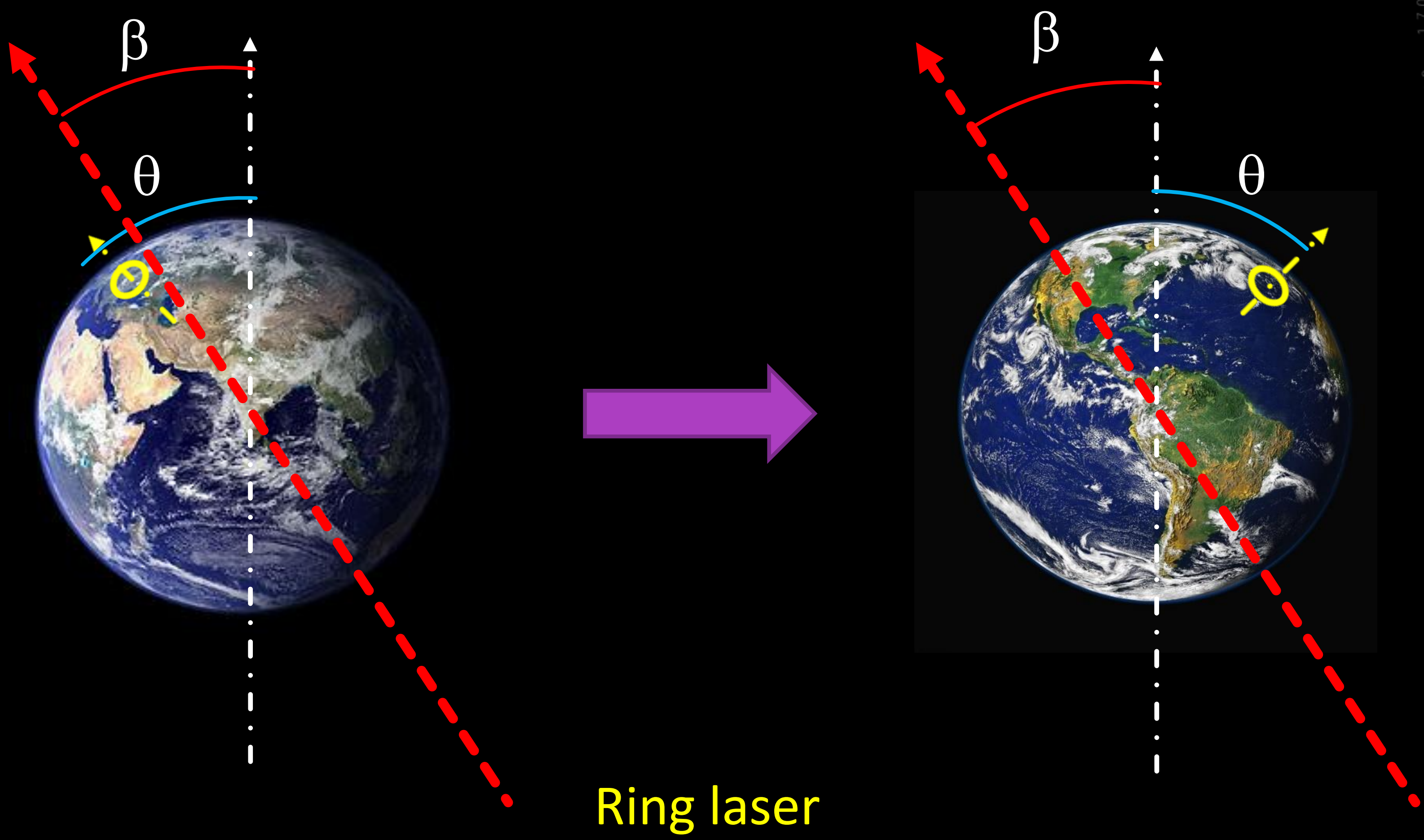


The plane of the triangle oscillates with respect to the axis of the Milky way

14 hours modulation

$\delta t \sim 6\times10^{-18}$ s

MEASUREMENTS ON EARTH?



PERIODIC SIGNAL

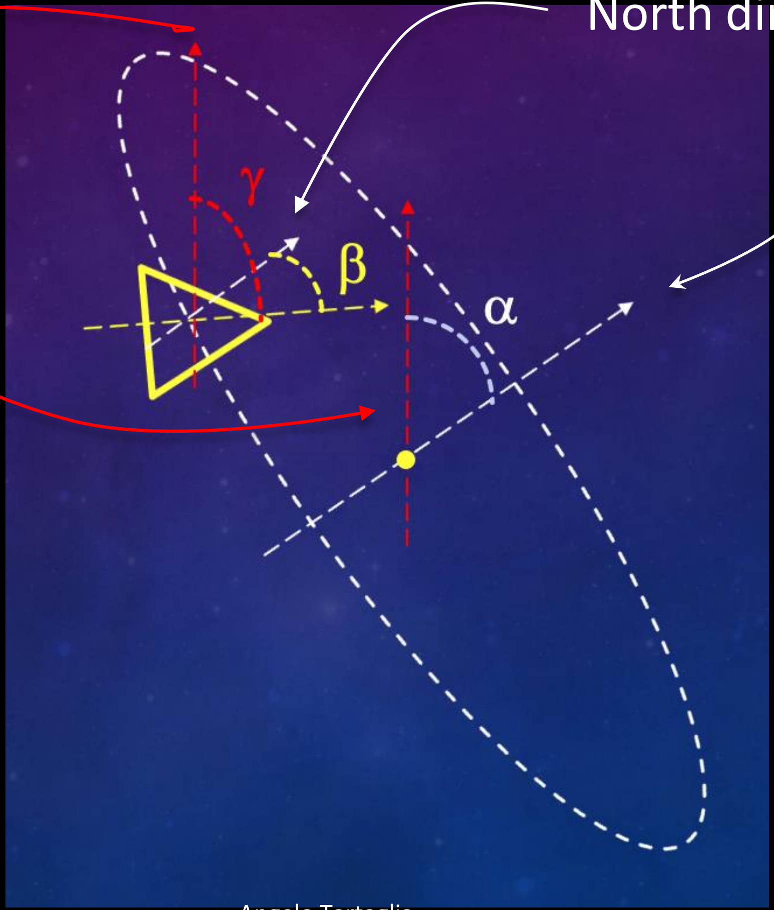
$$\delta\tau_D \cong \frac{2}{c} B_g S \cos\chi$$

Angle χ oscillates in the range $\beta \pm \theta$
in one stellar day.

The time dependence should allow to distinguish the galactic contribution from the constant kinematical Sagnac and terrestrial Lense-Thirring effects

WHAT ABOUT LISA? AN ORBITING TRIANGLE

Direction of the axis of the Milky Way



North direction of the ecliptic

Angle γ changes seasonally during the orbital motion, thus modulating the expected time asymmetry

REFERENCES

- A. Tartaglia, in *General Relativity and Gravitational Physics*, AIP Conference Proceedings, **751** , 136-145, (2004)
- A. Tartaglia et al., *Gen Rel. Grav*., **50**-9, 1-22 (2018)
- A. Tartaglia, *Int. J. Mod. Phys. D*, **27**, 1847012-1-5 (2018)
- A. Tartaglia, M. Bassan, G. Pucacco, V. Ferroni and D. Vetrugno. *Classical and Quantum Gravity*, ***39***, 195010 (2022)