What could be learned from ring laser gyroscope about the Earth's rotation?

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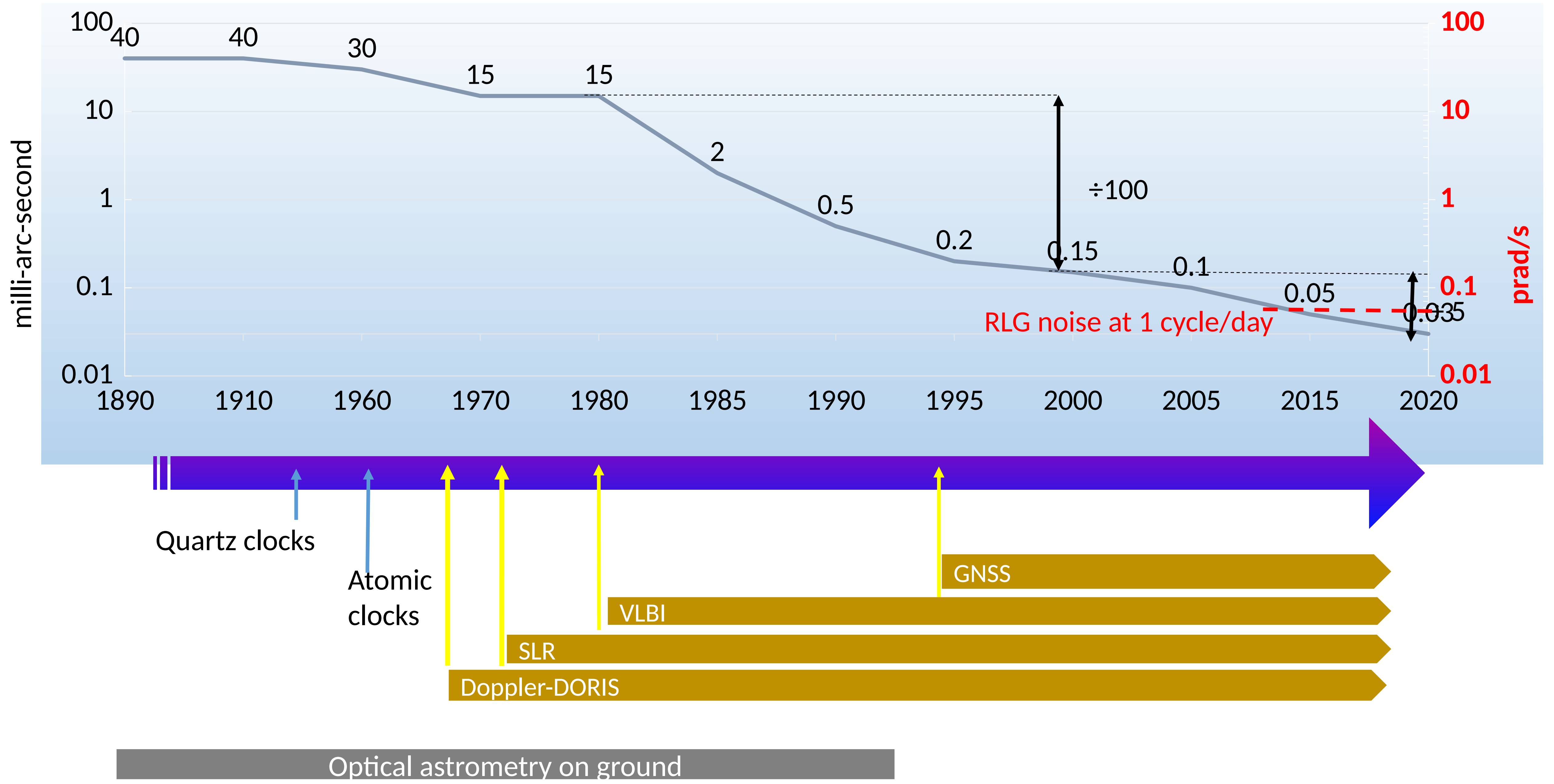
Uncertainty of the Earth Orientation Parameters / equiv. ang. velocity (1890-2020)



Outlines

I. The Earth’s rotation changes from astro-geodesy

II. Reconstruction of Earth angular velocity changes from astro-geodetic Earth Rotation

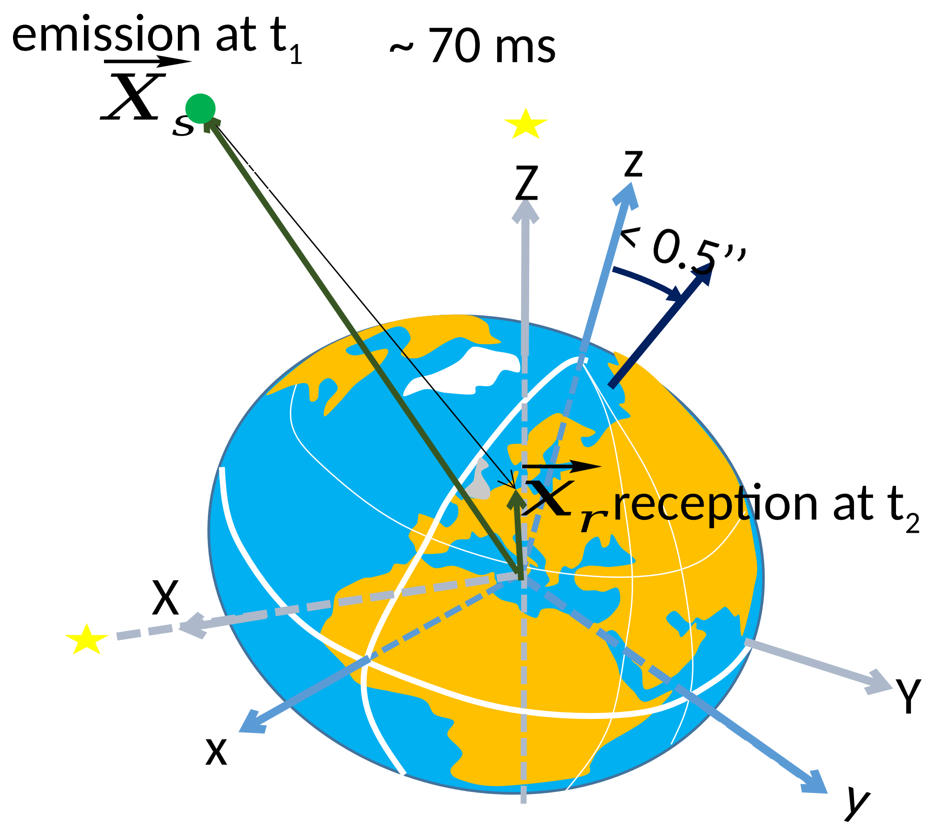
III. How some of the limitations of the astro-geodetic data could be overcome though the Ring Laser Gyro.

Space geodesy observations are primarily "sensitive" to the geometric transformation of rotation of a terrestrial set of observation stations (Terrestrial Reference System, TRS) with respect to a celestial reference system (CRS) and not to the components of the instantaneous rotation vector.

Example of the GNSS:

- : celestial coordinates of the satellite (in GXYZ)
- : terrestrial coordinates of the receiver (in Gxyz)
- Celestial coordinates of the receiver:
  Accumulation of millions of pseudo-distances (light time) per hour in about 100 stations all around the world with slowly varying EOP in

  +....

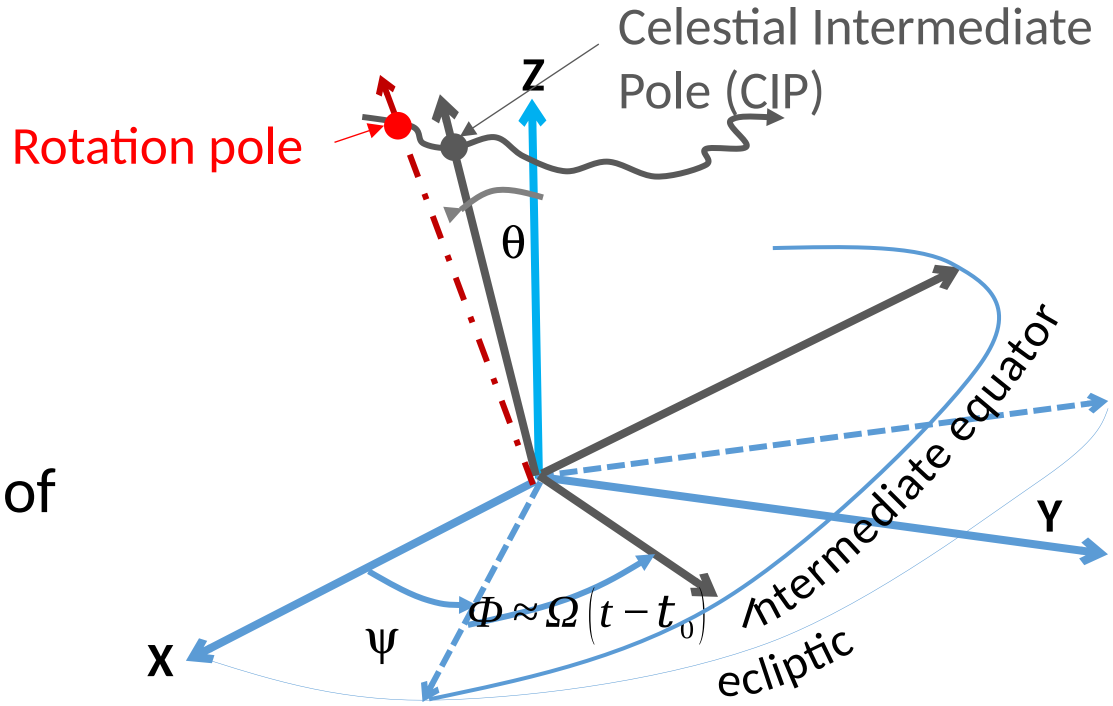


I- Astrogeodesy provides the "Earth orientation" (2)

The **rotation transformation** from a Terrestrial Reference System (TRS) to an Celestial Reference System (CRS) is mostly composed by a modeled (regular) part

with a relative precision of.
Euler angles for describing the modelled part :

and give the motion of the figure axis (of greater mean inertia) caused by the lunisolar moment of force, and define the ***Celestial Intermediate Pole/axis***, close to the axis of rotation (offset < 0.020'').



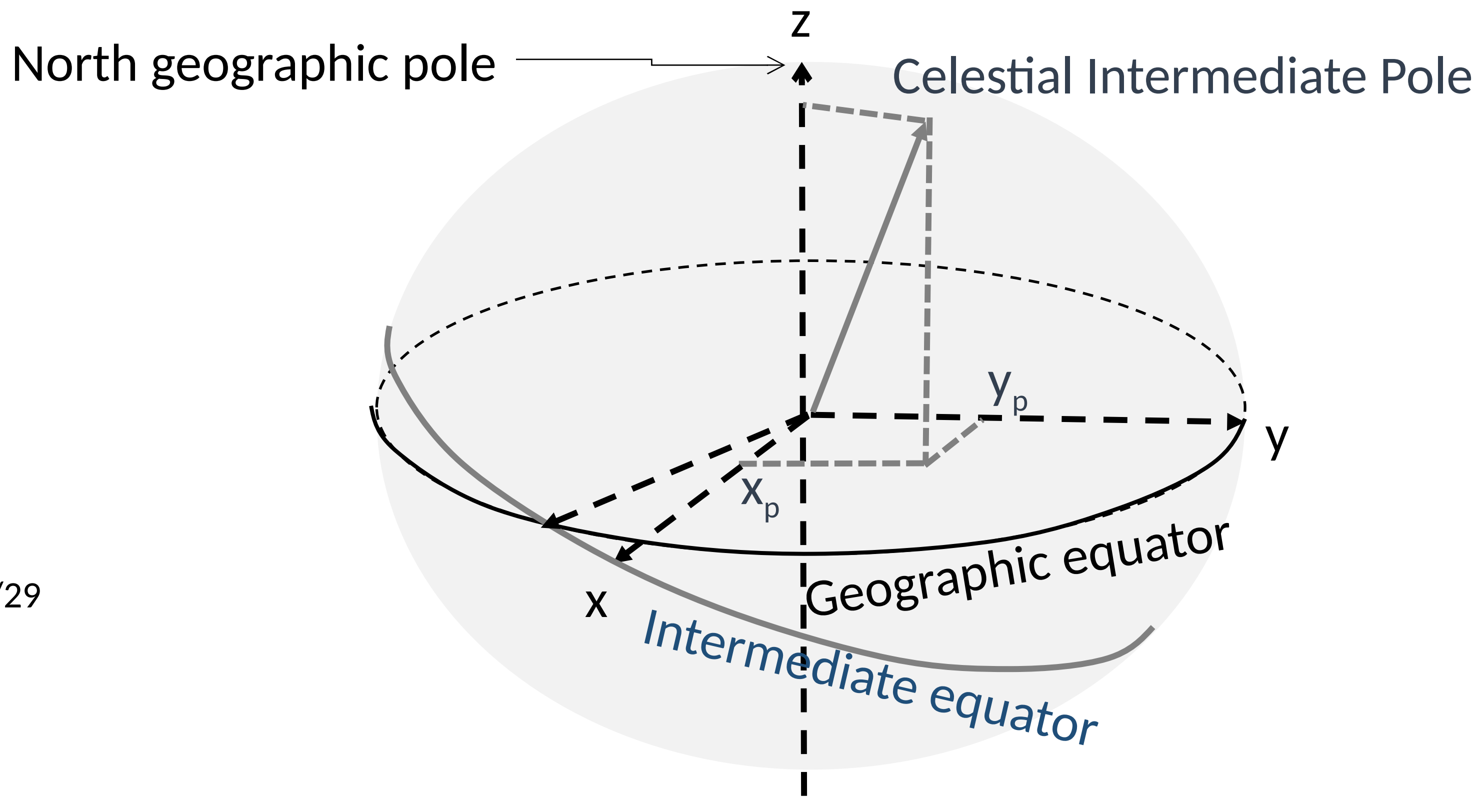
I- Astrogeodesy provides the "Earth orientation" (3)

- Astro-geodesy allows to determine the irregular hardly predictable perturbations to the regular modelled part of M, described by small rotations of angles < 1" :

- Angular corrections rad ~ 0,2'' corresponding to the Earth Orientation Parameters with a time resolution > 1 day
  1. : terrestrial pole coordinates of the CIP < 0.300''
  2. : Earth's rotation time irregularity < 0.02 s
  3. : correction on the Celestial Coordinates of the CIP (precession-nutation) < 0.001''

  In addition **Lenght Of Day offset** (t in days) and **Pole rates** , are determined from satellite techniques

At the first order the angles in are the direction cosines or pole coordinates of the ***Celestial Intermediate Pole (***CIP) in the TRS.



I- Expression of the rotation transformation in terms of quaternions

The matrix formalism is commonly used but does not allow to provide a simple analytical expression of in function of the EOP, in contrast to quaternion that reads:

$$\bar{q}_M = \frac{1}{\sqrt{2(1+Z)}} \left| \begin{array}{ll} \cos\frac{\theta'}{2}\left(X\frac{x}{2} - Y\frac{y}{2} + 1 + Z\right) & + \sin\frac{\theta'}{2}\left(X\frac{y}{2} + Y\frac{x}{2}\right) \\ \cos\frac{\theta'}{2}\left(Y + (1+Z)\frac{y}{2}\right) & + \sin\frac{\theta'}{2}\left(-X - (1+Z)\frac{x}{2}\right) \\ \cos\frac{\theta'}{2}\left(-X + (1+Z)\frac{x}{2}\right) & + \sin\frac{\theta'}{2}\left(-Y + (1+Z)\frac{y}{2}\right) \\ \cos\frac{\theta'}{2}\left(-X\frac{y}{2} + Y\frac{x}{2}\right) & + \sin\frac{\theta'}{2}\left(X\frac{x}{2} - Y\frac{y}{2} - (1+Z)\right) \end{array} \right. .$$

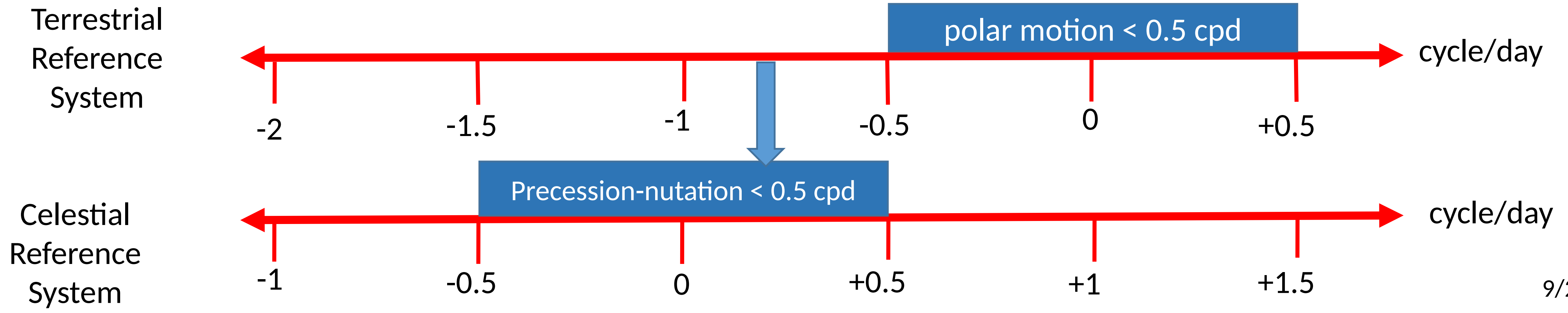
(Bizouard and Cheng, The use of quaternions for describing the Earth’s rotation, J. Geodesy, 2023)

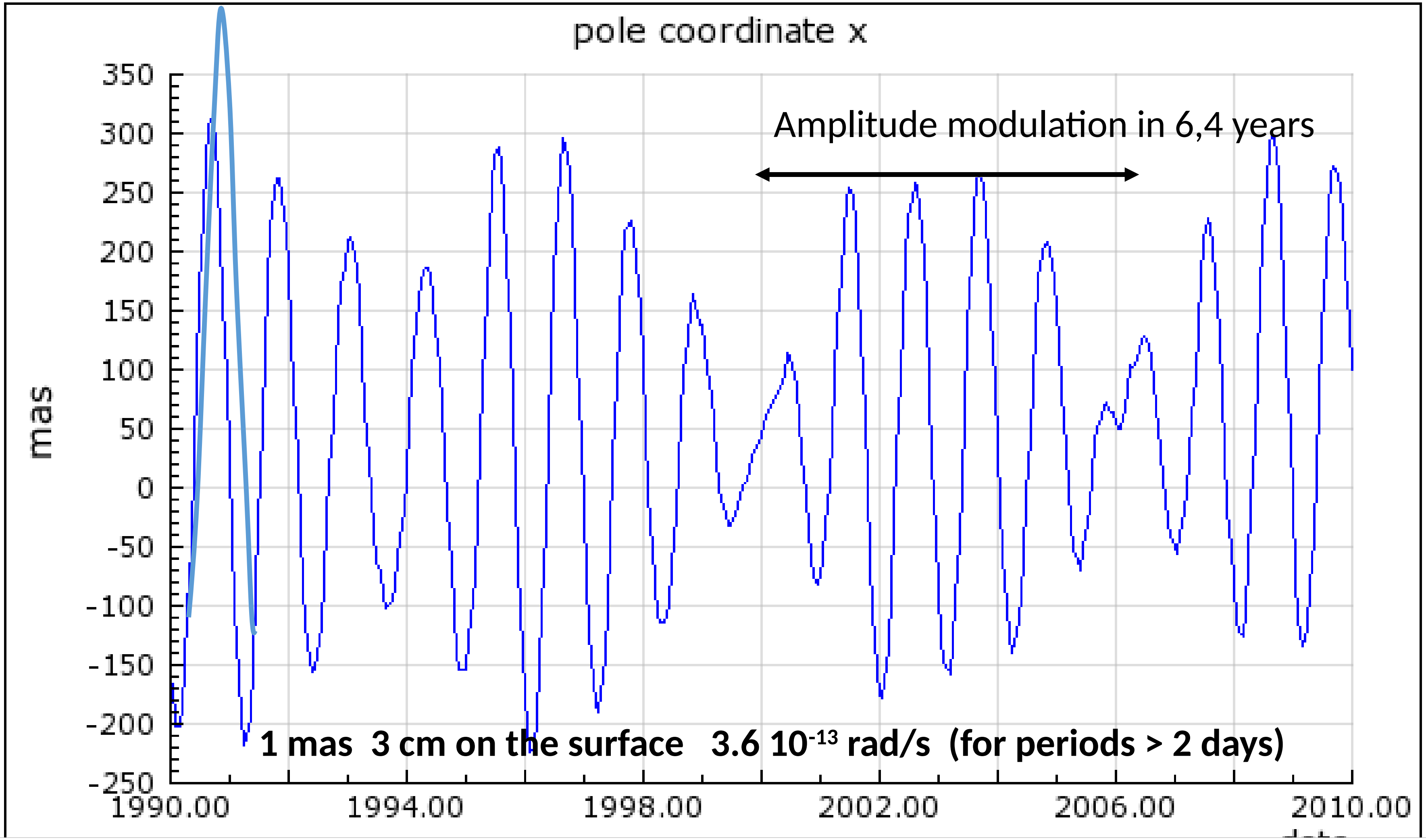
I- Current daily EOP from astro-geodesy : one day time resolution

Current processing: the residual angles or EOP in

are estimated (simultaneously) once per day. So, the highest detectable frequency is **0.5 cycle/day**.

- Satellite techniques (GNSS, Satellite Laser Ranging, …) ✉ with a 1 day time resolution
- Very Long Baseline Interferometry ✉, with a time resolution > 5/7 days (R1/R4 sessions), downgraded UT1 with a time resolution of 1 day (intensive sessions)
- Combination/interpolation in order to obtain homogeneous time EOP series with daily time resolution consistent with international terrestrial and celestial reference frame (C04, Bulletin A,…)
- The frequency band around 0.5-1 cpd is poorly estimated.





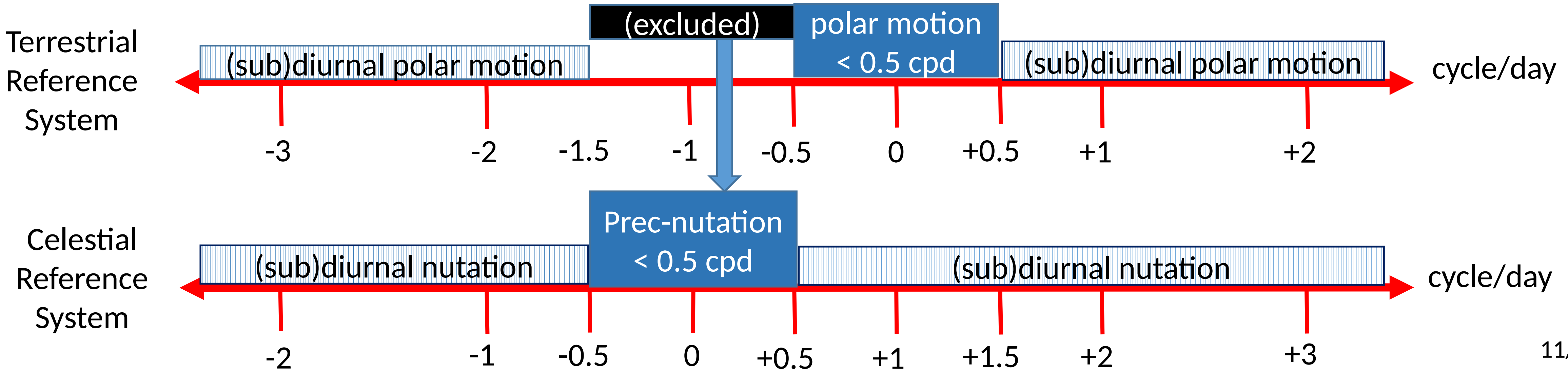
Mostly composed of 433 d (Chandler) and annual terms. Order of magnitude ~ 200 mas **7 10^{-11}** rad/s for the instantaneous pole. Precision obtained by astro-geodesy: 0.05 mas 2 **10^{-14} rad/s**

I- (Sub)diurnal EOP from astro-geodesy

At (sub)diurnal time scale EOP in

are correlated, for they correspond to overlapped frequency bands in the CRS and in the TRS (striped). So, one has to fit either (or .

(Sub)diurnal EOP is dominated by the diurnal and semi-diurnal harmonic effects of ocean tides. These harmonics can be fitted accordingly from VLBI/GNSS observations. On the other hand Intensive VLBI campaign and GNSS observation allows to fit hourly values of .



II-Kinematical relations linking rotation transformation and angular velocity vector

- In term of Euler angles

- In term of matrix

- In term of quaternion:

II- Terrestrial oscillations of the Earth instantaneous rotation vector

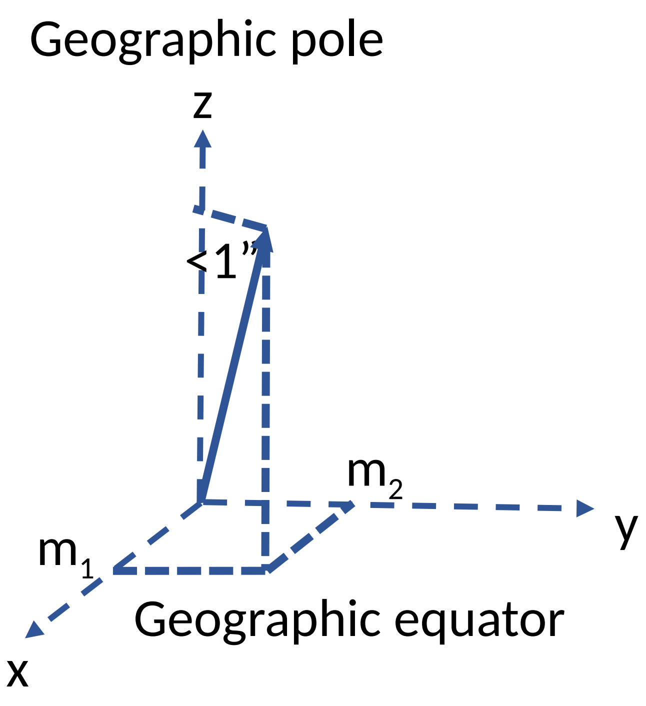
: instantaneous angular rotation vector of the ITRF with respect to the ICRF

=

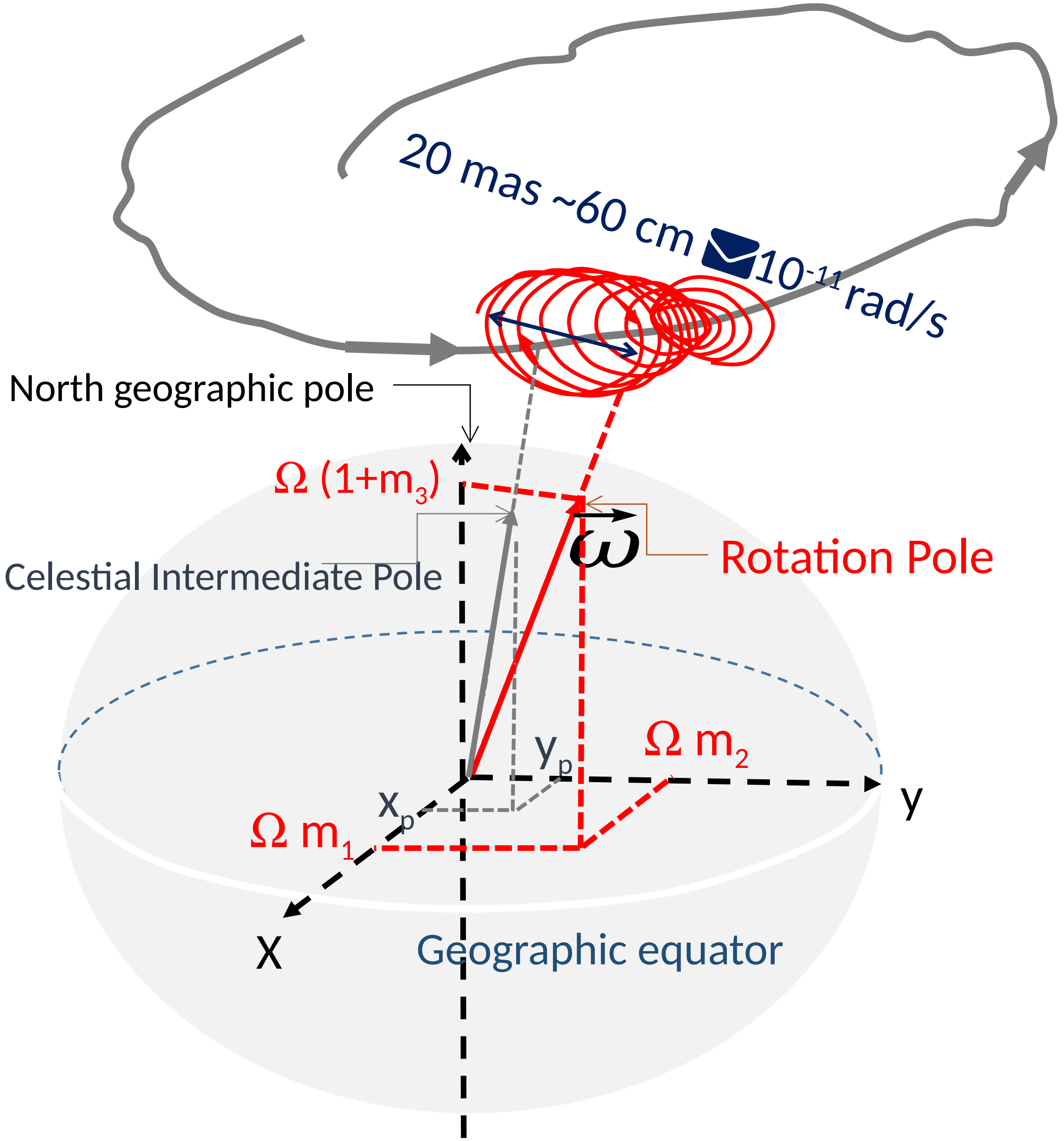
= 7.292 115 10^{-5} rad/s: mean angular velocity

Variations being mostly :

- geophysical for spectral components > 2 days
- tidal (luni-solar) for spectral components < 2 days



II- Celestial Intermediate axis (astrogeodesy) versus instantaneous



From kinematic relations, equatorial coordinates of the *instantaneous rotation pole* and the ones of the *Celestial Intermediate Pole* are linked by the relation:

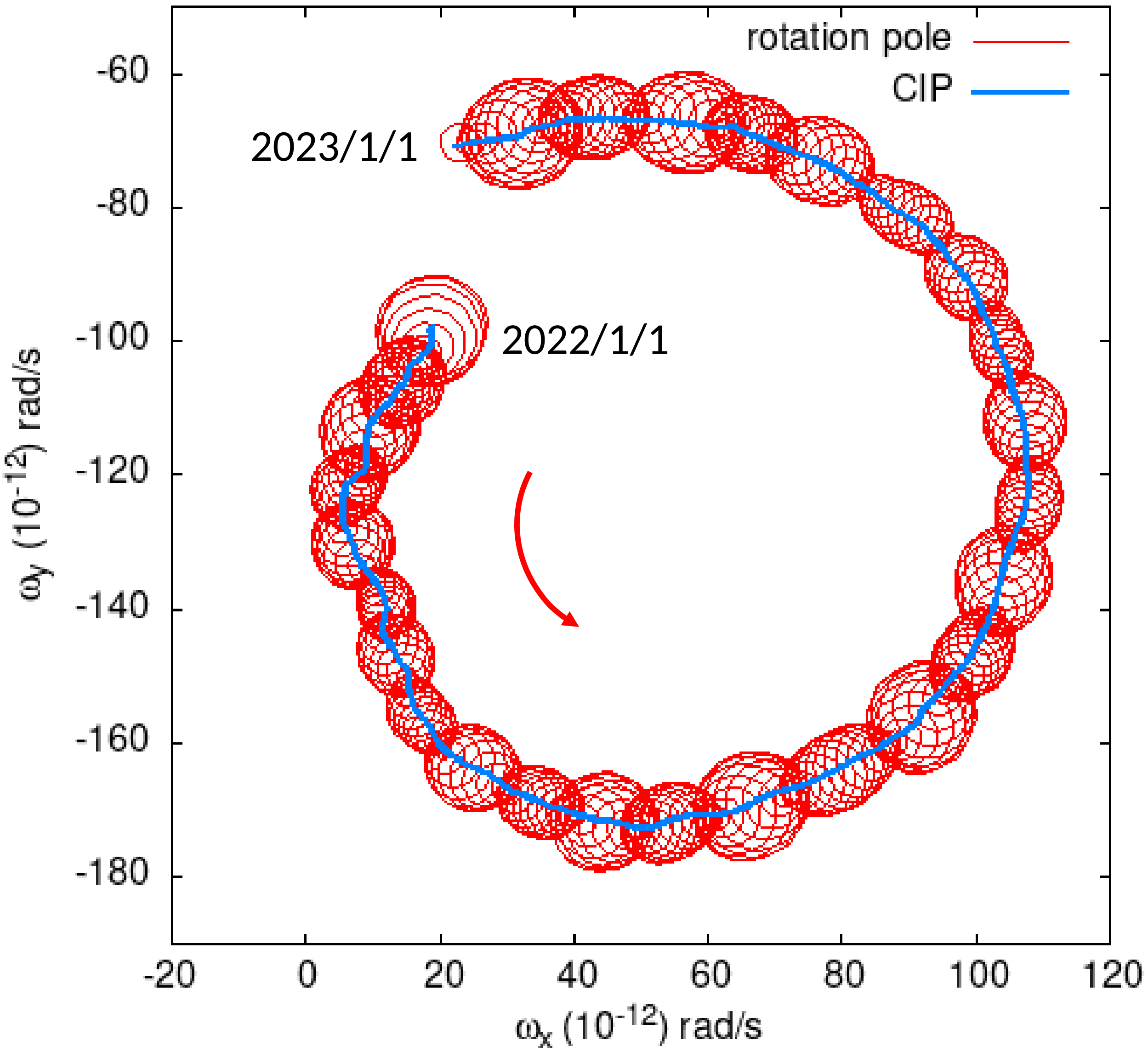
FT

- For periods 2 days
-
-

So conversion of IERS pole coordinate in is only valid for long term frequency

II-Angular velocity vector: equatorial oscillation 2022-2023

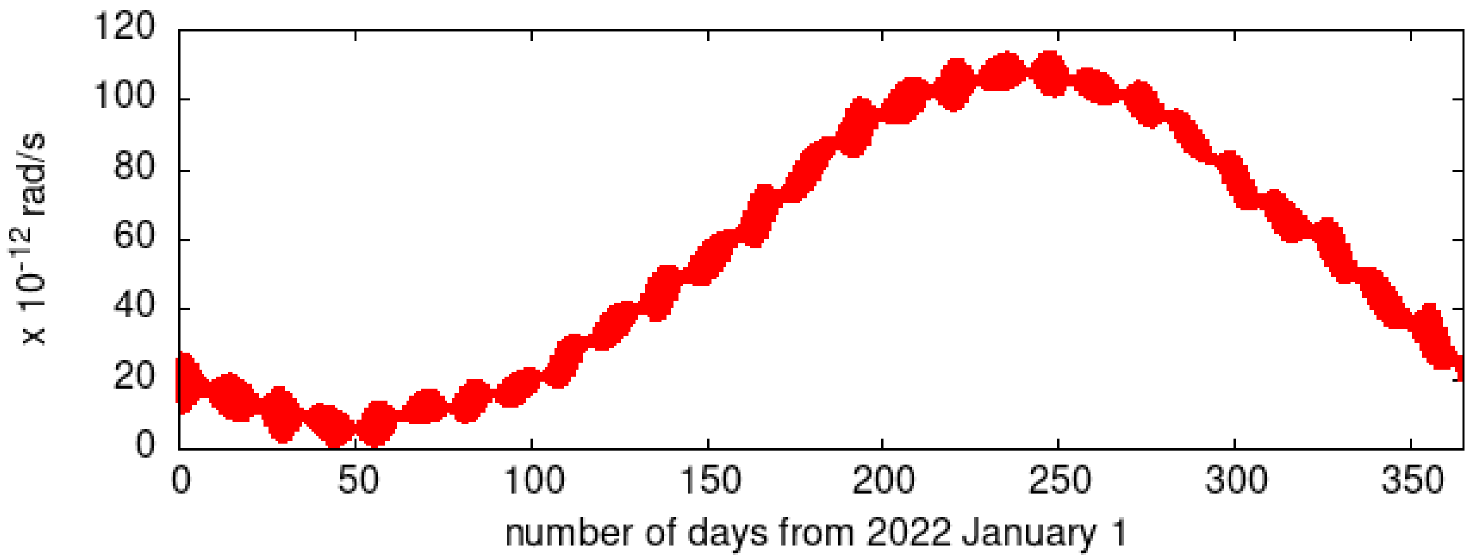
To the long term polar motion > 2 days rad/s) are superimposed retrograde quasi-diurnal oscillations caused by the equatorial luni-solar torque on the Earth bulge rad/s) and ocean tides rad/s).

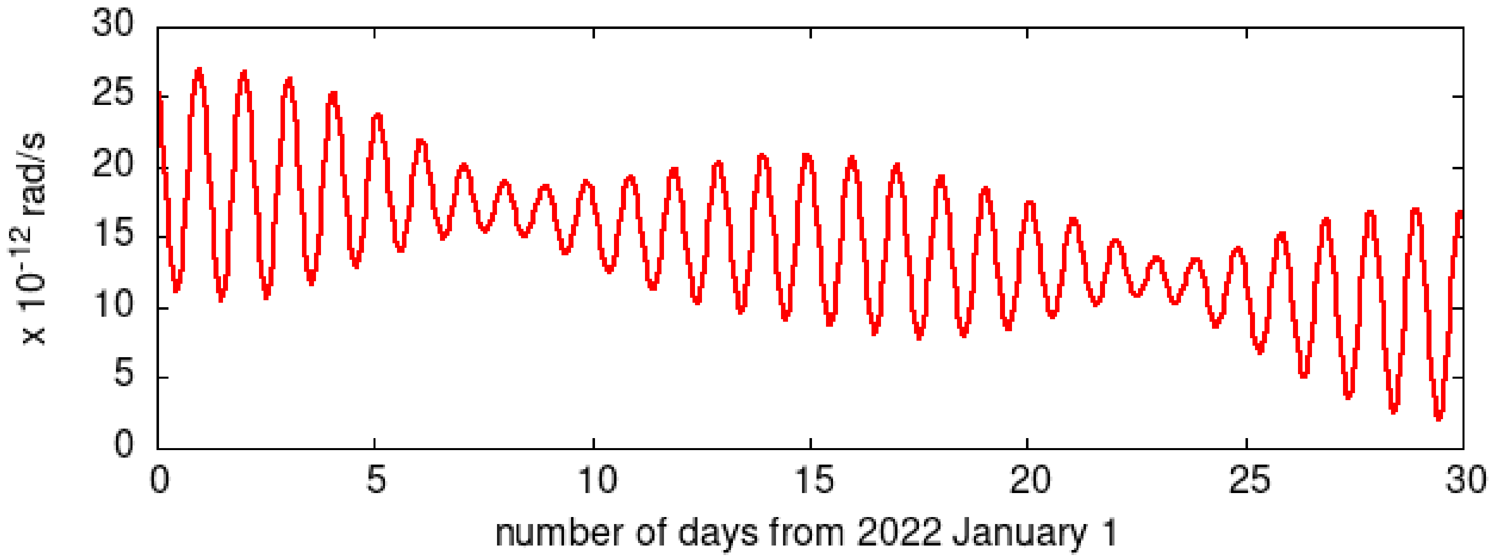


III- Equatorial oscillation:

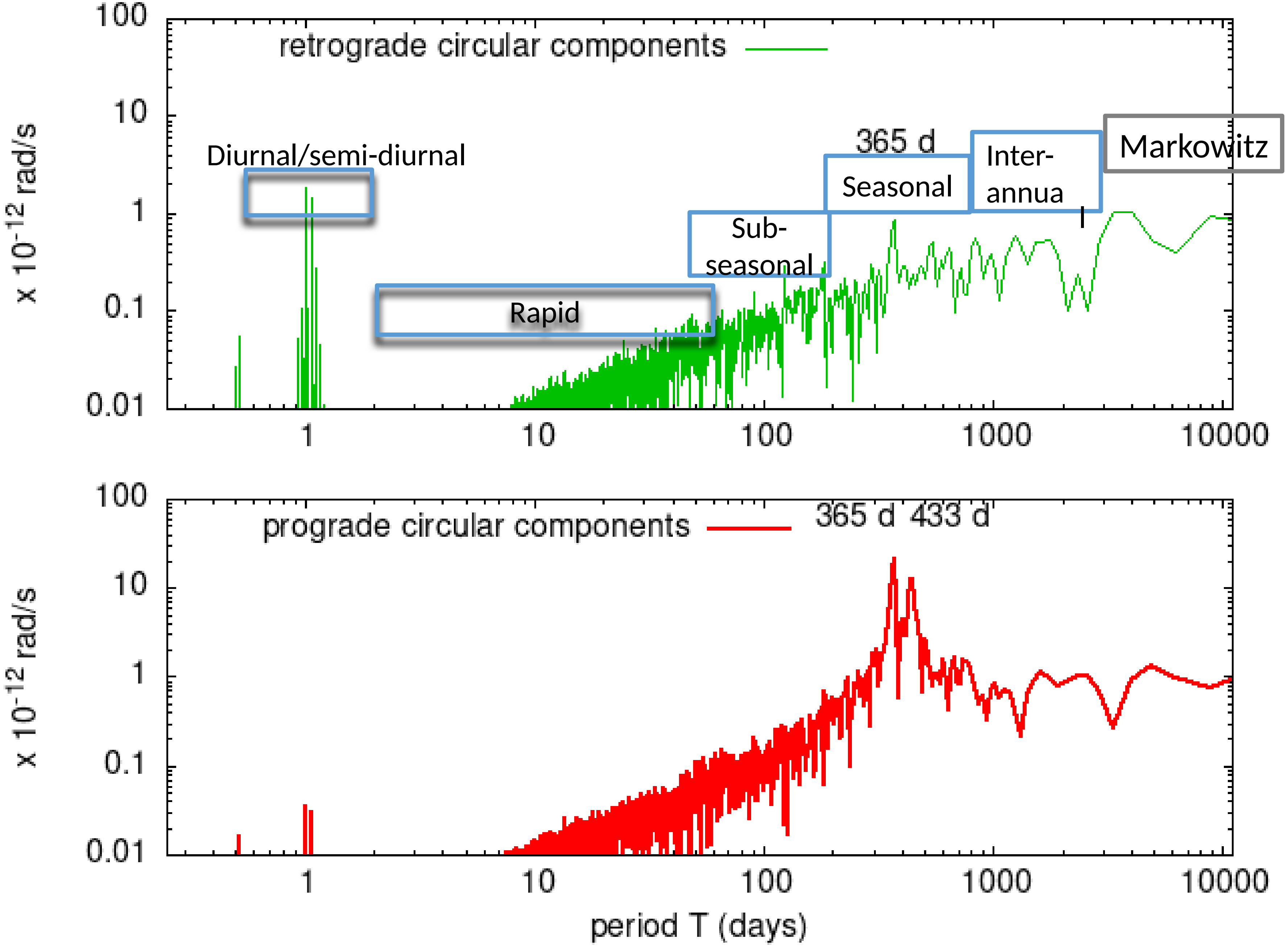
We mainly see the effect of the luni-solar tidal torque on the equatorial bulge.

13.6 d modulation coming from the beating of sidereal K1 oscillation (precession in CRS) and O1 oscillation





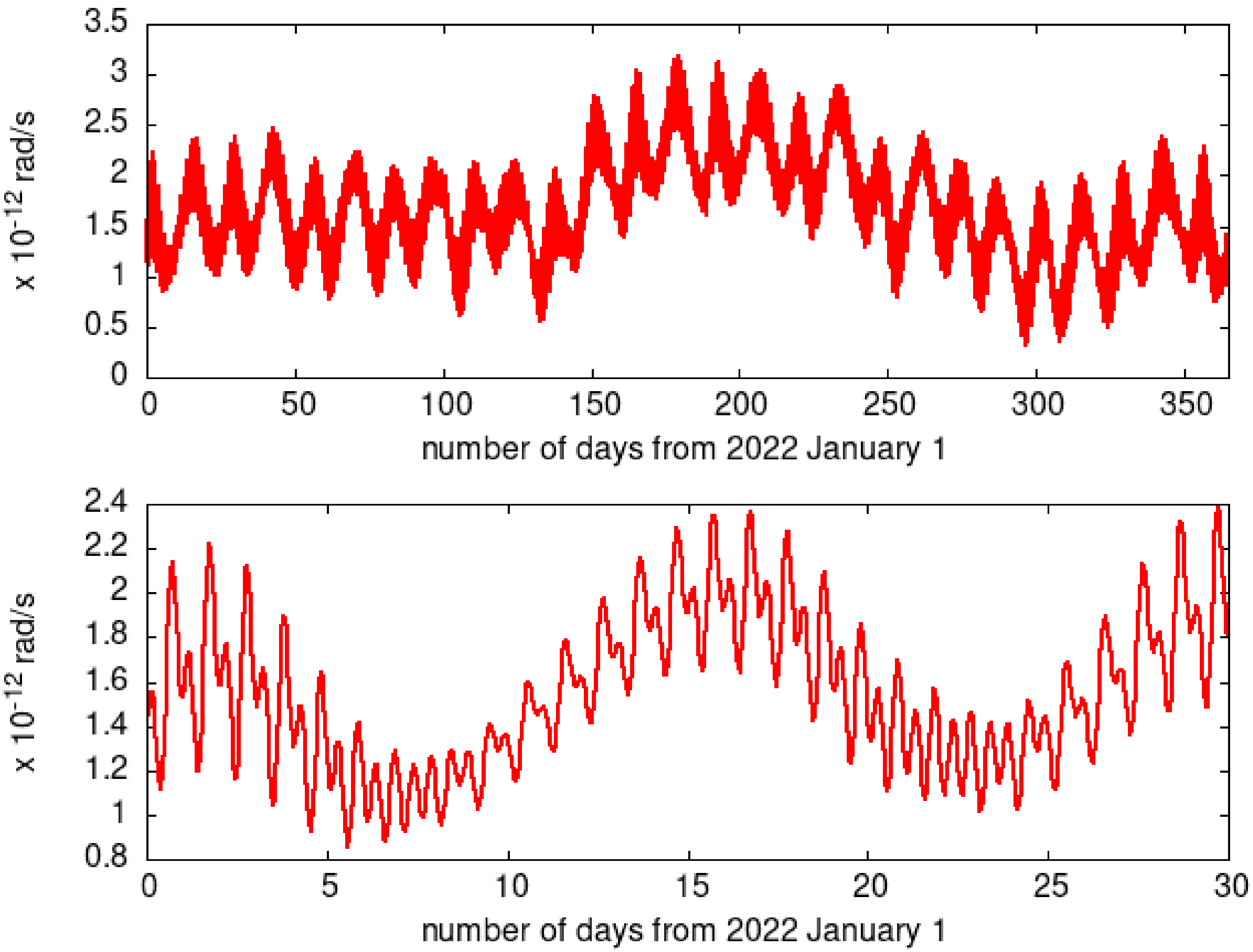
II-Equatorial oscillation of : complex spectrum over the period 1990-2023



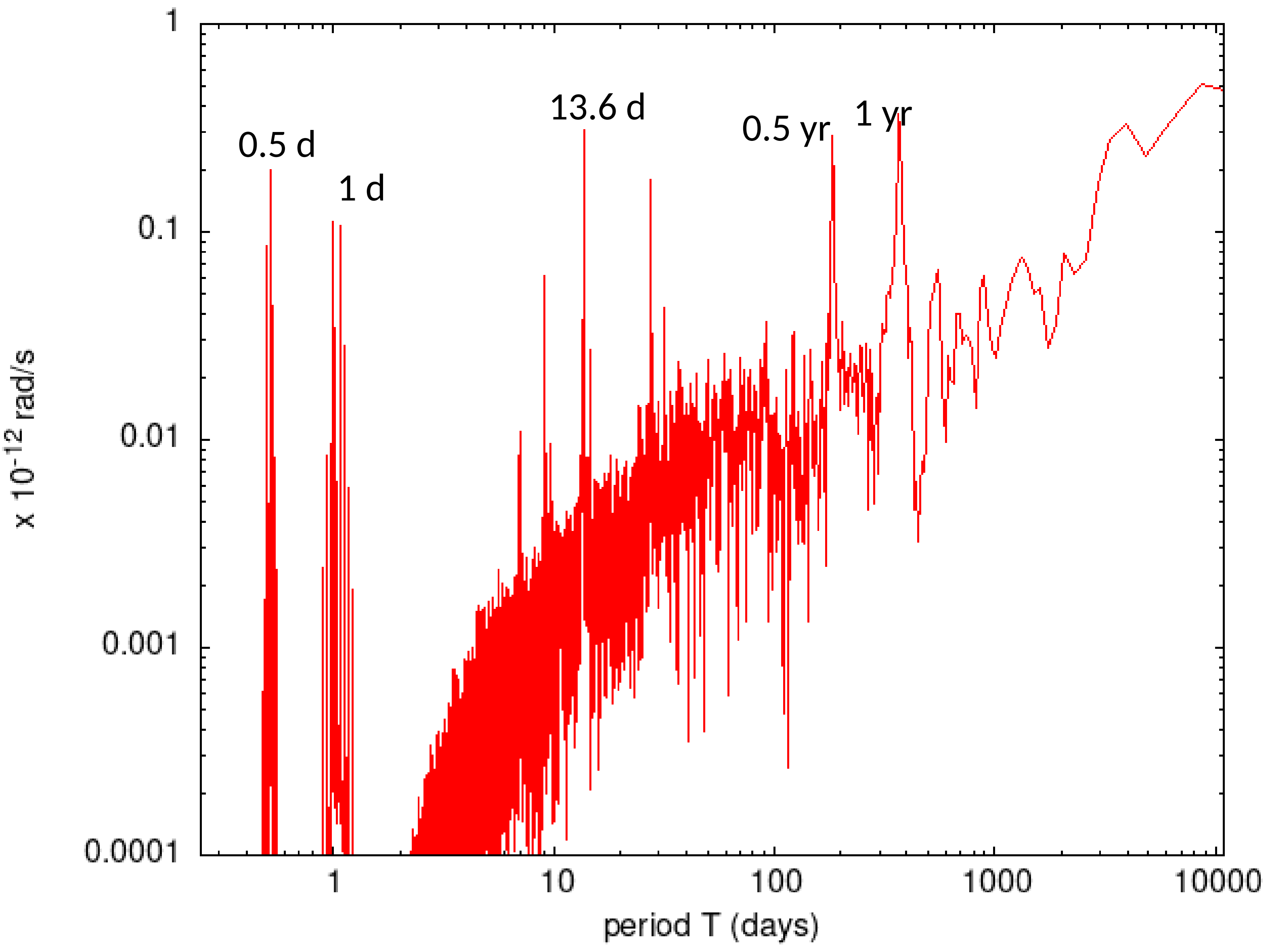
II-Axial variation : $-\Omega \sim 10^{-12}$ rad/s

We see mainly the effects of the:

- Seasonal and sub-seasonal hydro-atmospheric circulation
- Zonal solid Earth tide (> 7 days)
- Ocean tides (~1 day, ~0.5 day)



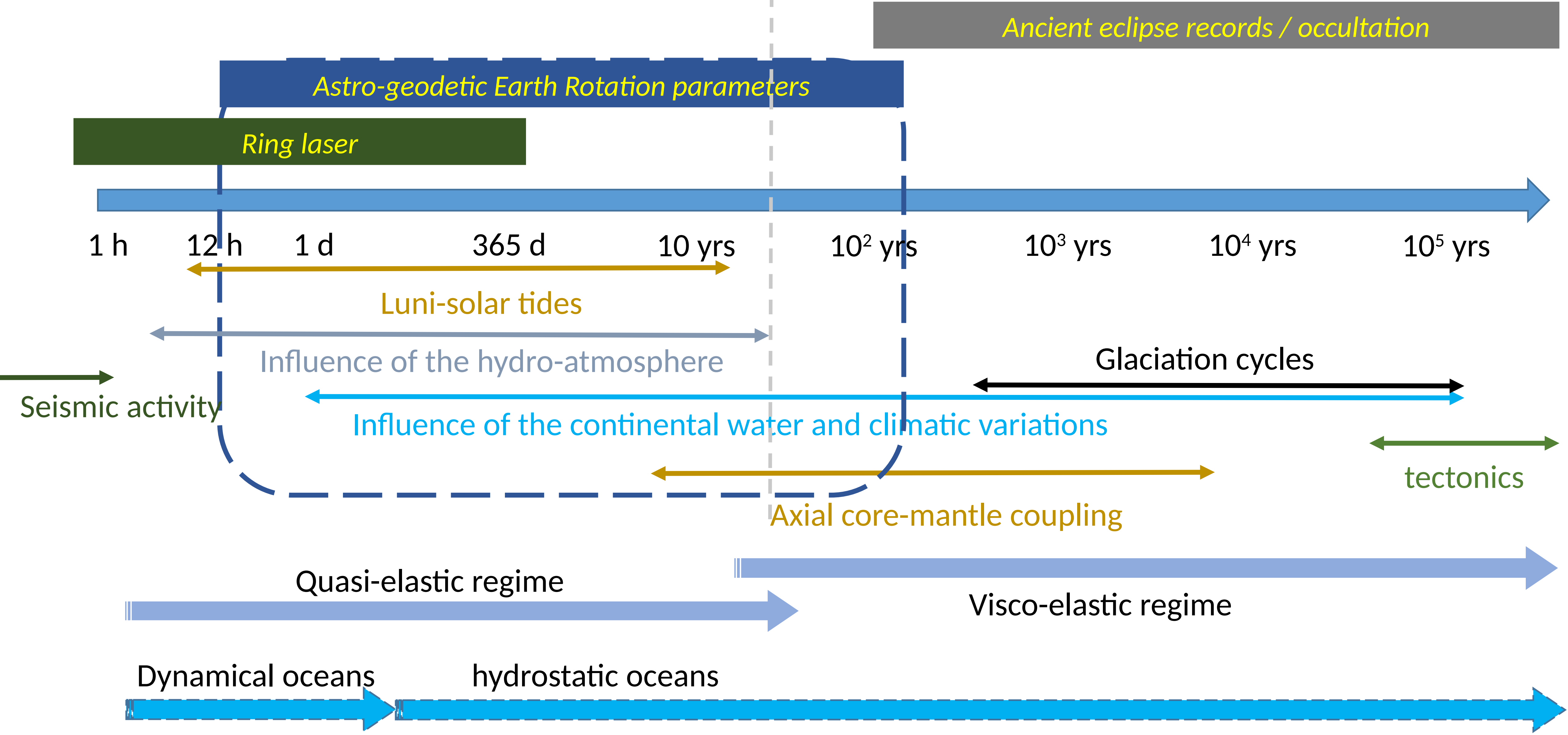
II- Spectrum of the axial change: 1990-2023



II- Interactive web tool for computing

https://hpiers.obspm.fr/eop-pc/index.php?index=rotation&lang=en

III- Astrogeodetic window: 12h – 170 years



III- Limitations of the astrogeodetic EOP's (1)

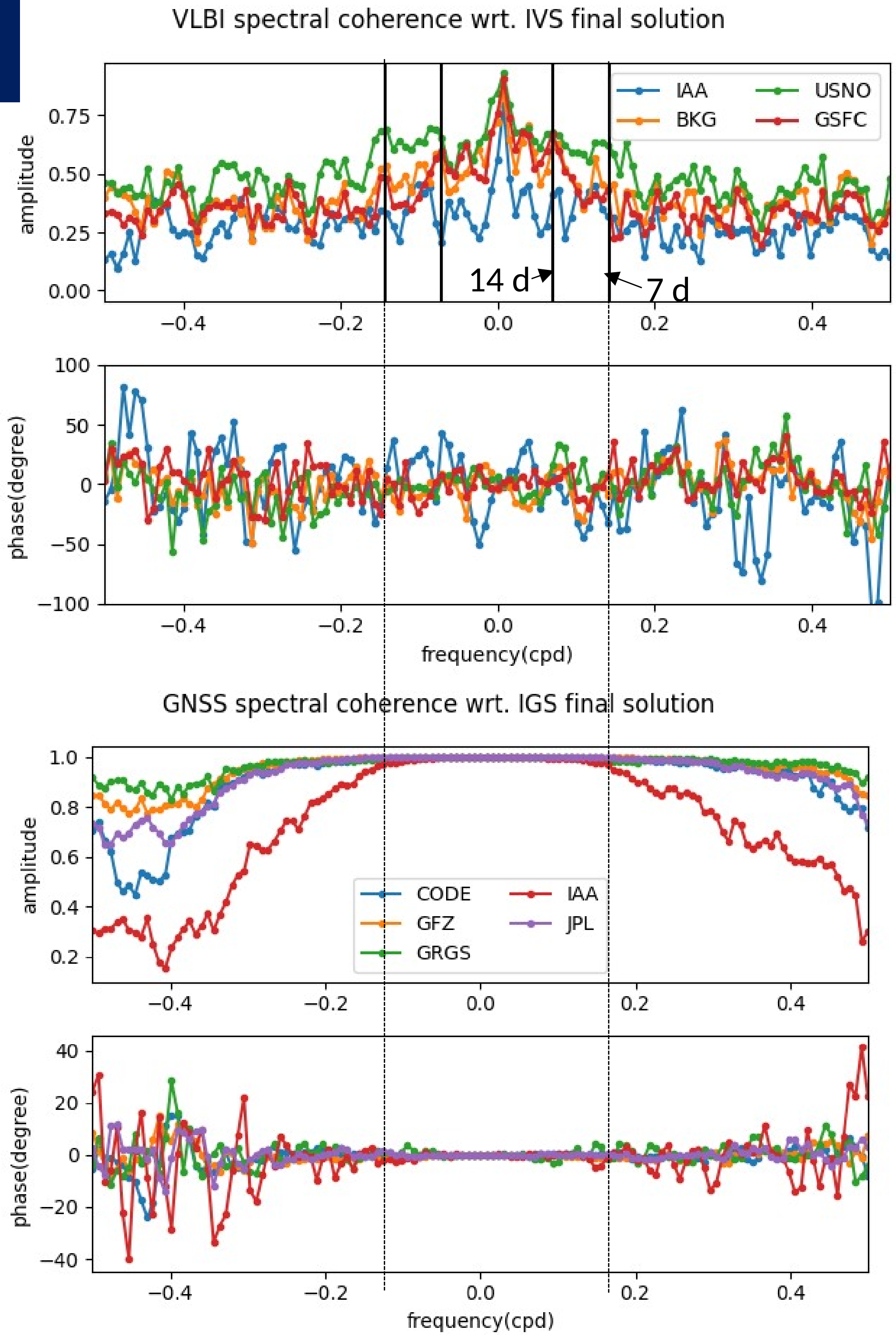
- Current time resolution (1 day), Nyquist frequency of 0.5 cycle/day (cpd).

Actually:

- Inconsistency of high frequency nutation offsets > 0.07 cpd (periods < 14 days) determined by various VLBI analysis centers – see upper right plot

- Same remark for pole coordinates below 7 days

– see bottom right plot



III-Limitations of the astrogeodetic EOP’s

Below one day

- Rare VLBI intensive campaign of 3 weeks
- Experimental hourly GNSS solution with unsolved correlations between orbital elements and EOP
- Fit of diurnal and semidiurnal tidal harmonics

Possibility to improve the present EOP from sub-daily to seasonal time scale

GINGERO

- 2 days precision rad/s or 0.003 mas
- 24 h/12h rad/s or 0.010 mas

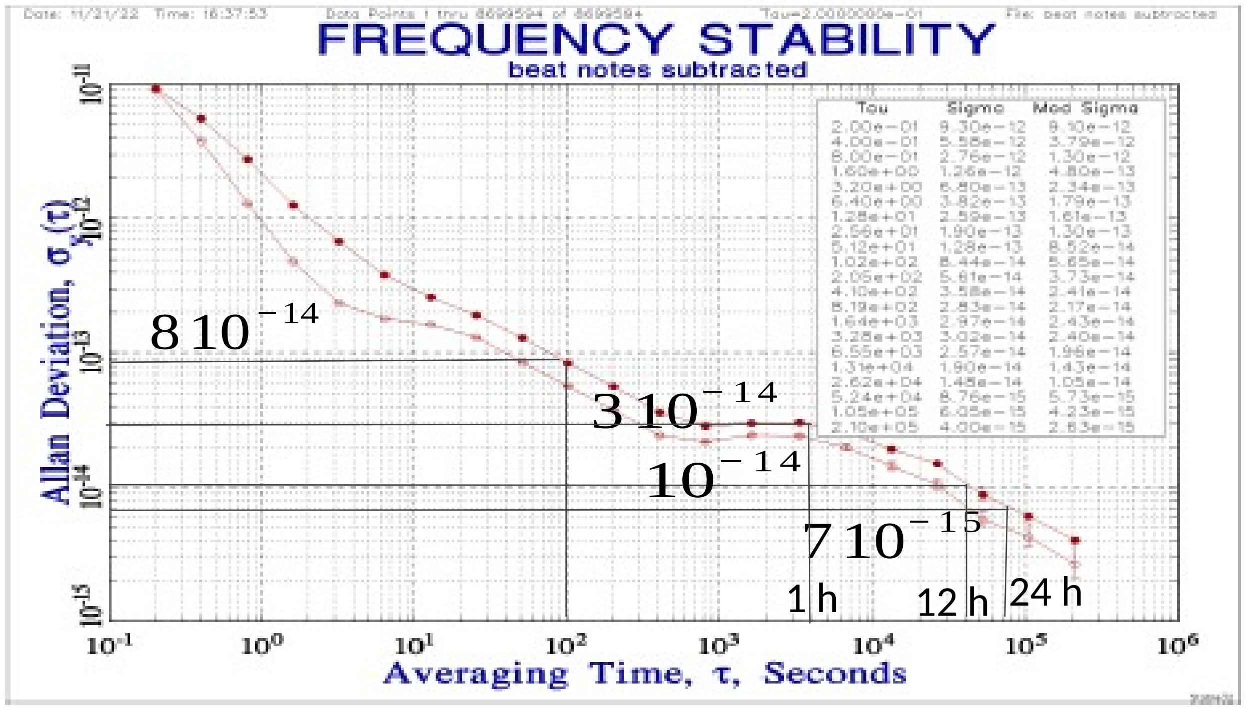


FIG. 5. Overlapped and Modified Allan Deviation of Ω_{Tn} expressed in rad/s. The plot have been obtained by using STABLE32 freely available at: http://www.stable32.com/

Di Virgilio et al, 2023 presented by A. Porzio

III- Possibility to detect transient mass redistribution (1)

- Rotational effect of continuous mass redistributions (hydro-atmospheric circulation) with time scale > 1 day currently monitored by astro-geodesy
- But transient phenomena (< 1 hour) cannot be hardly detected: Earthquakes, geomagnetic jerks in the fluid core,...?
- With a precision of about prad/s after 100 s integration (GINGERO), RLG could fulfill this task.

III- Possibility to detect transient mass redistribution (2)

Consider a mass distribution accompanied by an angular momentum variation in TRS. If the mass redistribution occurs with tremendous velocities (winds, oceanic currents, fluid core motion, Earthquake), it causes a relative angular momentum . According to *Euler-Liouville linear equations*, polar motion and angular velocity perturbation are ruled by:

: complex polar motion resonance (Chandler) frequency

with and (angular momentum function)

: equatorial moment of inertia, : dynamical flattening,

, depend on the Earth's rheology (elasticity of the mantle, core-mantle decoupling).

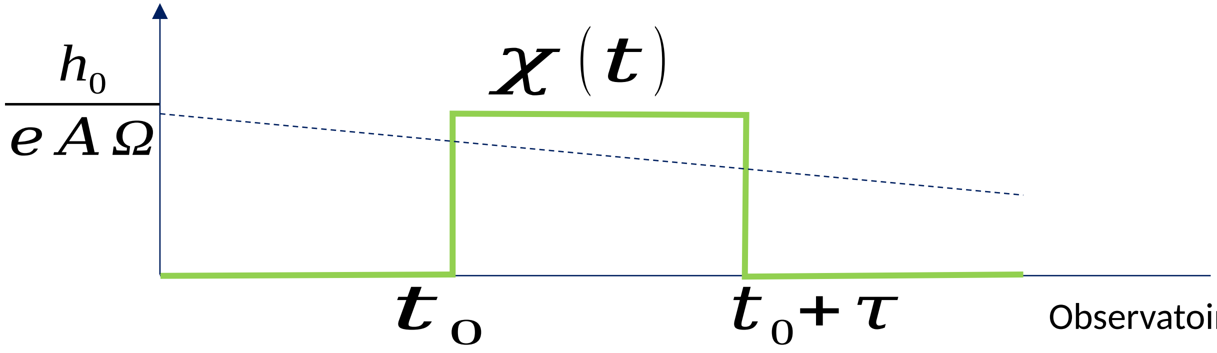
III- Possibility to detect transient mass redistribution (3)

Solution :

For “mega” Earthquakes (Sumatra 2004, Japan 2011, Chili 2011,…) relative angular momentum variations up to during no more than 100 s. This can be modelled as the boxcar function , giving :

rad (0.2 mas)

rad/s just at the level of the present noise level



III- Possibility to detect the Earth’s geodetic precession?

- The Earth’s Precession (~50”/year rad/s) with respect to quasars (kinematic CRS) is measured with an accuracy of 1 microarsecond/year (**rad/s!**) .

- But the dynamical theory of the Earth rotation is not enough precise to isolate in that precession rate the drift of the inertial CRS with respect to the kinematic CRS, namely the geodesic precession (20 mas/year ~ **rad/s**) around the ecliptic pole

- RLG allows to measure the Earth precession with respect to a inertial CRS through the K1 (sidereal) clockwise oscillation in rad/s , which is obtained with a precision of rad/s even less:
(Euler kinematic relation to a constant phase with obliquity of the ecliptic)

Conclusion

- Limitation of the astrogeodetic EOP (low time resolution)
  - ✓ blind zone of the current EOP's between 7 and 1 day
  - ✓ (Sub)hourly resolution of EOP
  - ✓ correlation with orbits (GNSS)
- Could be overcome by RLG:
  - 2 days precision rad/s or 0.003 mas for pole coordinates
  - 24 h/12h rad/s
  - 1h rad/s
  - 100 s rad/s
- Possibility to detect transient effect in the Earth's relative angular momentum caused by
  - ➢ Earthquake
  - ➢ Geomagnetic jerks,…
- What about the geodetic precession through the K1 diurnal oscillation?

II- Sagnac frequency and geodetic variation (1)

- Sagnac angular frequency with the scale factor

  - ✓ unit vector normal to the area
  - ✓ perimeter of the ring cavity, : wavelength

- Neglecting the inherent variations of the RLG, is constant, and we assume that the variation of comes exclusively from rotation/local deformation:
  and

Here is the Earth reference angular velocity vector in the TRS. The reference normal is given by the spherical coordinates in the TRS.

II- Sagnac frequency and geodetic variation (2)

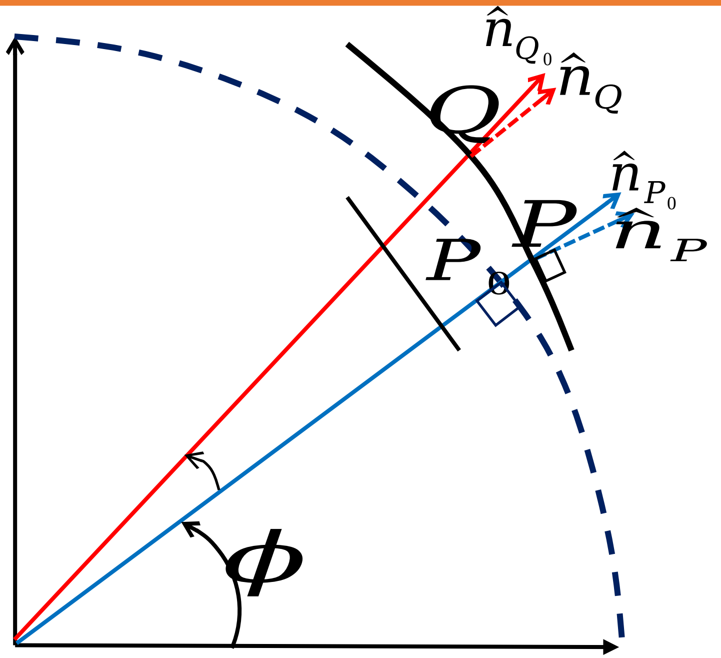
- spherical coordinates of the unit normal vector of the RLG in the TRS:
  : only the latitude variation of the RLG normal impacts

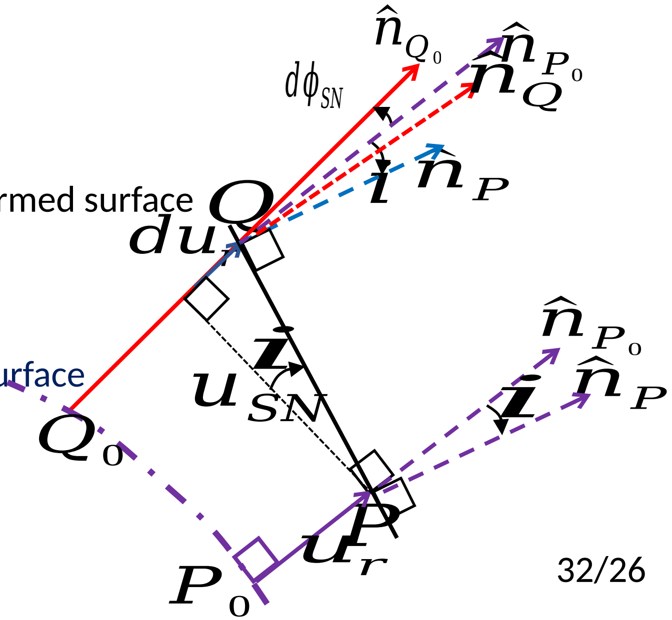
- is composed of a global part , corresponding to the Earth's rotation changes, and possibly a local part associated with the ground deformation :

  with in the TRS

- A point of the undisturbed surface is displaced to the point Q, by composition of the radial displacement   and south-north horizontal displacement . The normal to the crust in is , the normal to the crust in Q is
- The deformed surface is tilted with respect to the initial one, because the geocentric distance of Q is slightly different than the P one. This tilt is evaluated by
  with , , : displacement Love number of degree 2.

  Hence the initial normal is tilted by becoming
- The translation of P to Q introduces a change latitude of the normal ,   slightly compensating the tilt   .
- The total change of latitude of the normal, namely the angle   / is





GINGERO, G-RLG, Romy and Pisa GP2

	GINGERO (Gran-Sasso)	G-RLG (Wetzell)	Romy (Munich)	Wuhan
Latitude	42°25'16''	49.099645°		
Longitude	13°30'59''	12.878005°		
Perimeter	14.4 m	16 m	36 m	6.4 m
Wavelength	633 nm			
Sagnac frequency	280.4 Hz			
	5 700 259.459	6 322 866.227		
Surface	12.990 m^2	16.010 m^2		
Structure	heterolytic	Monolytic (ZERODUR)	4 triangular heterolytic RLG	
Uncertainty on	rad/s			

Comparison of astro-geodesy and RLG for determining Earth's rotation changes

	Modern astro-geodesy	**RLG**
Observed quantities	Light time, Doppler frequency	Sagnac frequency
Astrogeodetic quantities	Rotation transformation between TRF and CRF, deformation	Angular velocity in the TRF, tilt of the normal to the area in the TRF
Precision	50 µas **2.5 10^{-14} rad/s**	**GINGERINO: 4 10^{-14}** rad/s after 3.5 d integration
Time resolution	1 day, exceptionally 1 hour	**up to a few minutes**
Network of station	Very dense	Quasi-inexistent
Long term stability	10 µas over one century	Uncontrolled drift at seasonal scale
Deformation	Displacement in the TRS	Displacement not measured
Separation deformation/rotation	Well achieved	Necessity to add complementary instrument, like tilt-meter
Possibility to monitor vertical	no	For optical table aligned along the vertical