Geodetic Observatory
Wettzell

Bundesamt für
Kartographie und Geodäsie

High Resolution Inertial Earth Sensing with Large Sagnac Interferometers

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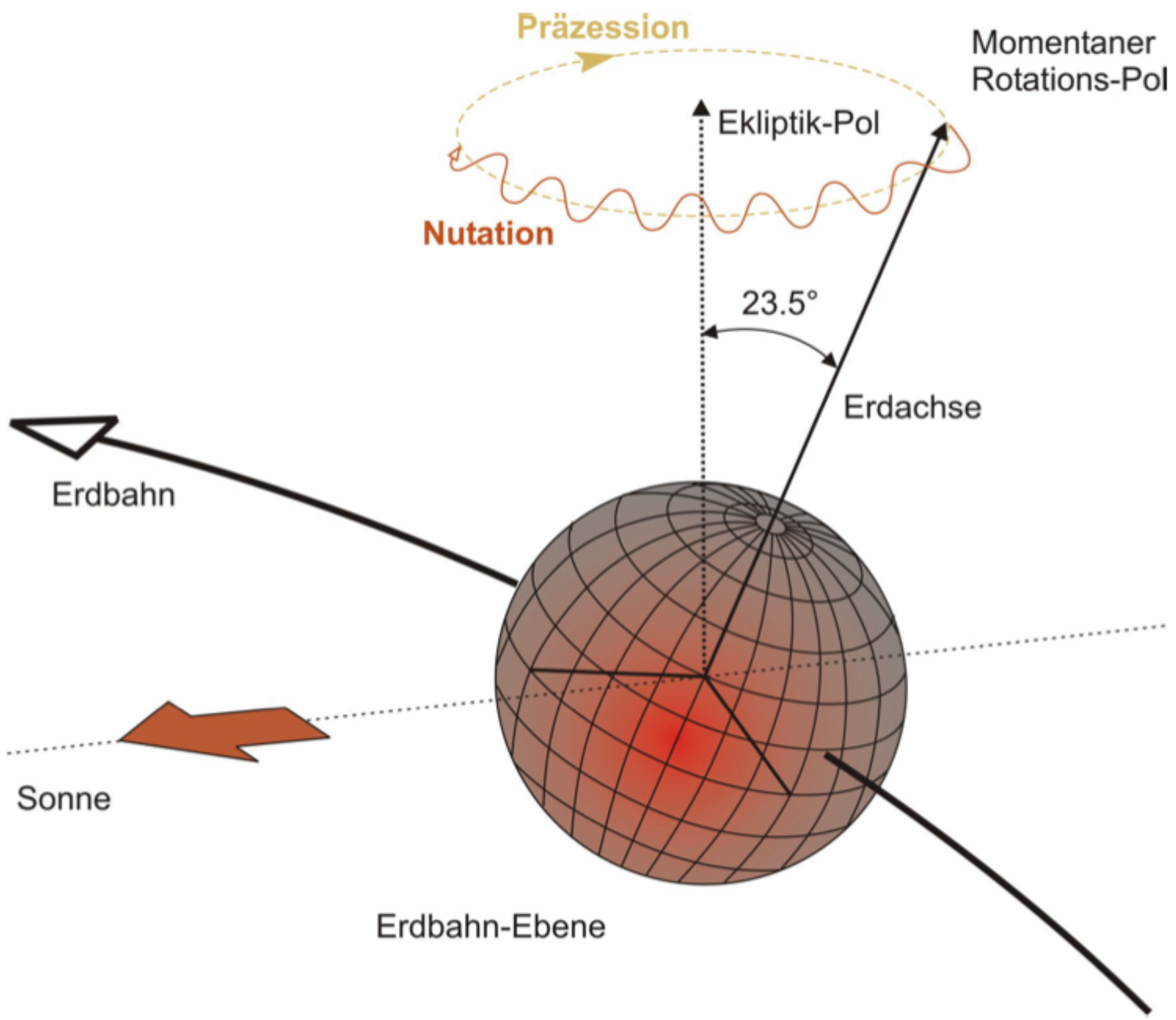
"Requirements for space geodesy" or what is the motivation to build large ring lasers

- Establishment of exact positions and the structure of extra-galactic radio sources (quasars)
- Determination of precise global, regional and local 3D coordinates (navigation, global change)
- Determination of the instantaneous earth rotation axis and the rate of rotation as a function of time. (This allows the transformation between terrestrial and celestial reference frame)
- Determination of the gravity field of the earth and its variation over time (mass transport phenomena)

What did we find on the roadside?

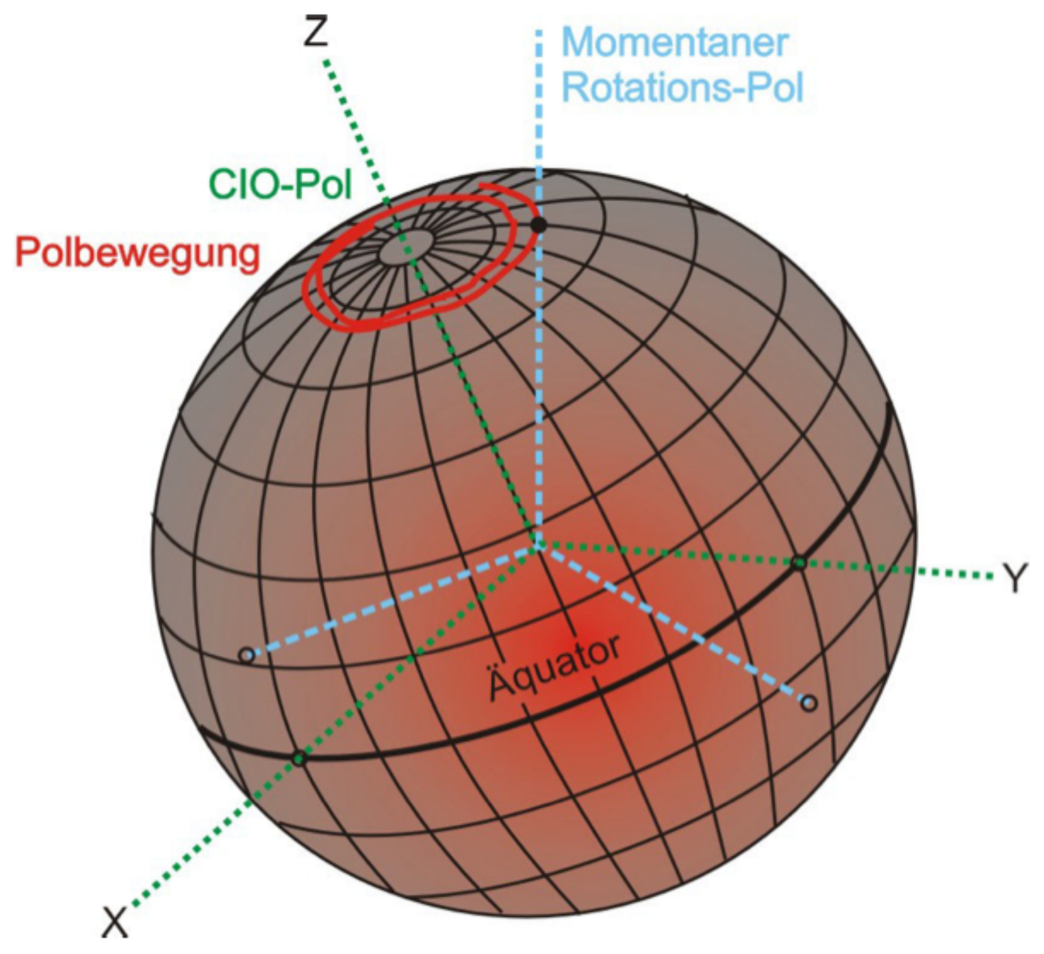
A lot of ground motion and heaps of seismological signals...

Earth rotation shows a complex behavior



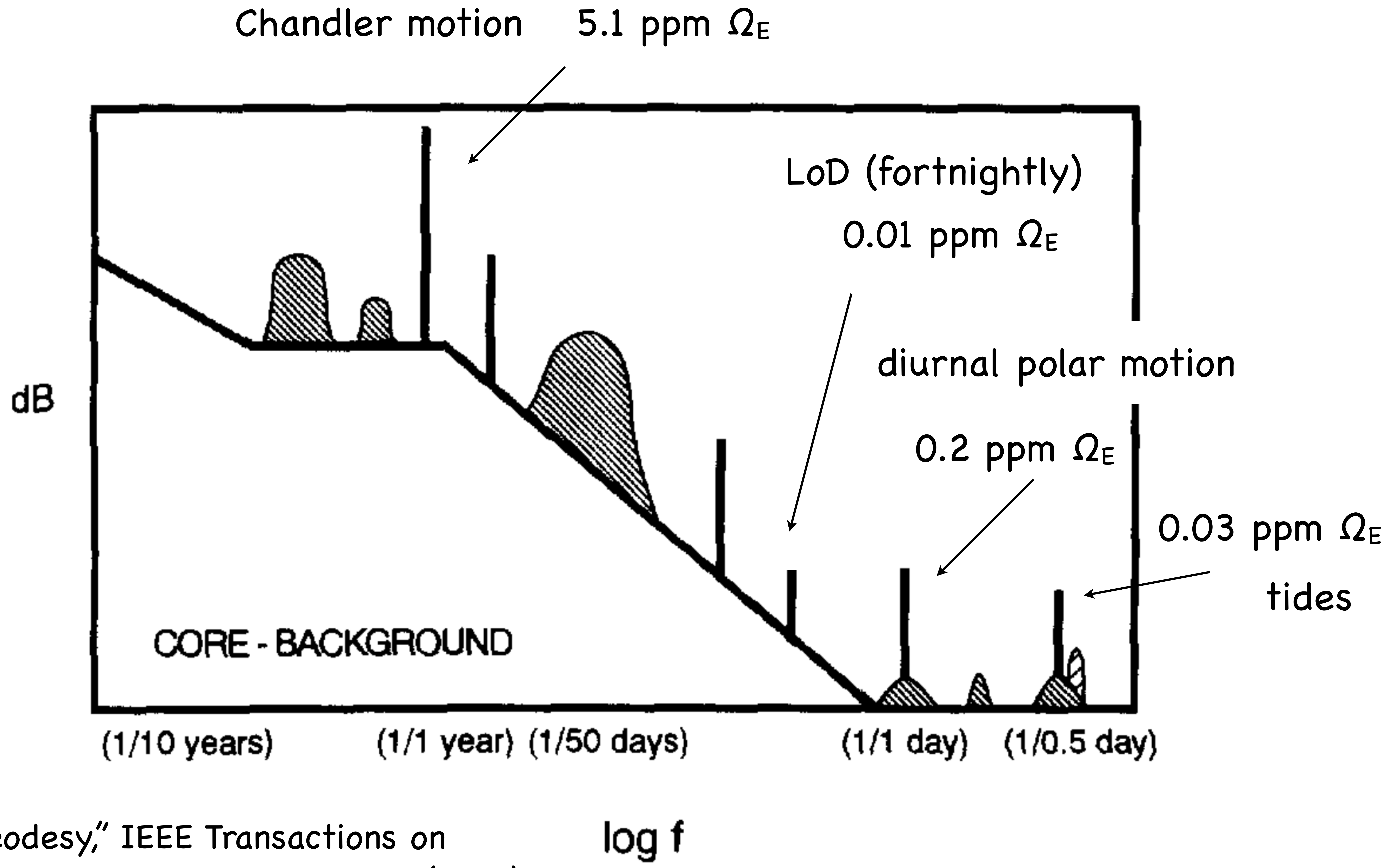
a) the rotation rate of the earth is not constant. Deceleration by dissipation and variation by momentum exchange. Free oscillations excited by ocean, atmosphere

b) gravitational attraction of sun and moon on a near spherical object give rise to precession and nutation



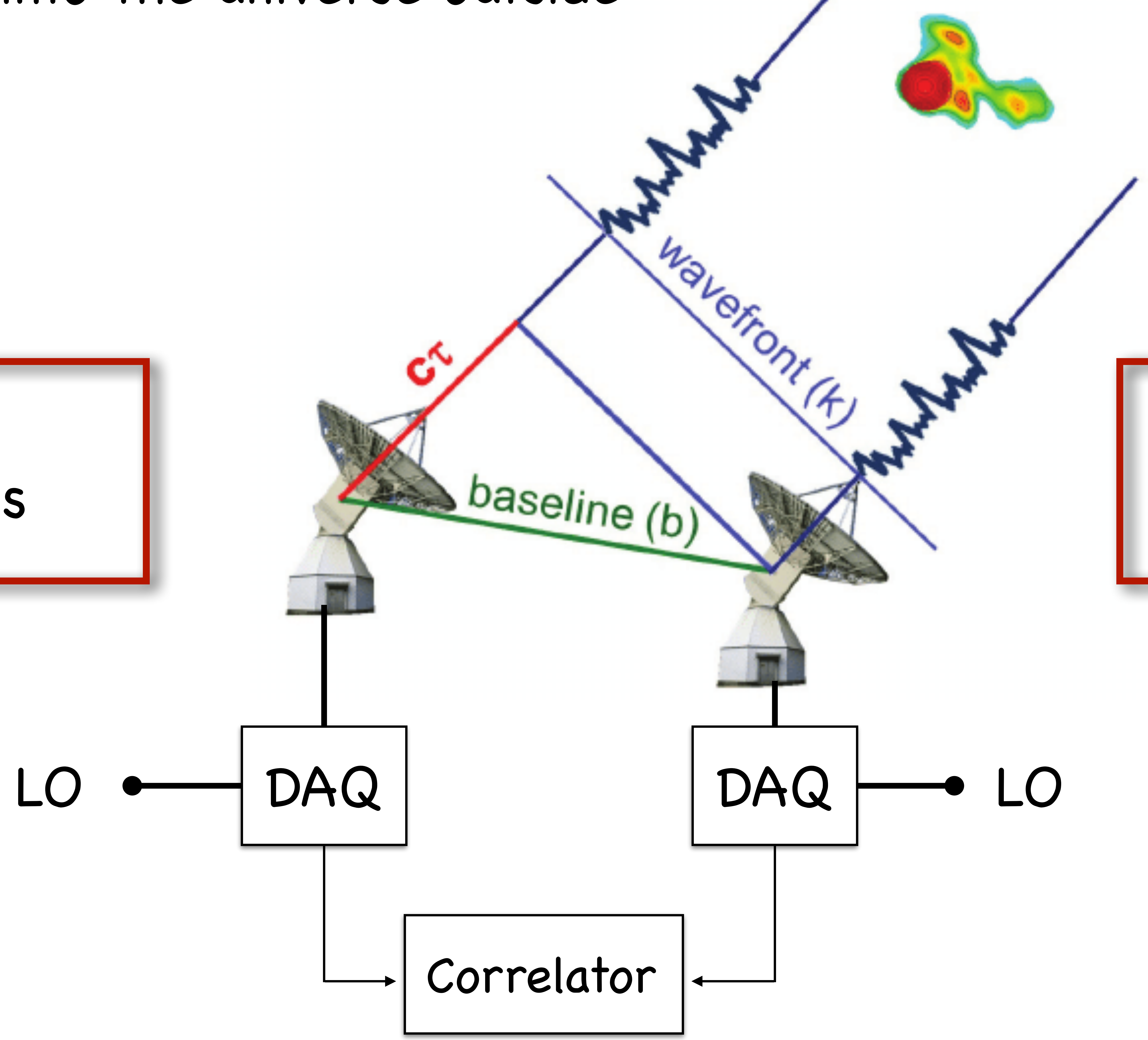
c) mass redistribution on Earth and the fact that the figure axis and the axis of Inertia are not coinciding, give rise to polar motion

What signals do we have to expect:



Bilger et al., "Ring lasers for geodesy," IEEE Transactions on Instrumentation and Measurement, vol. 44, no. 2, pp. 468–470, (1995) Very Long Baseline Interferometry provides a very stable link by looking into the universe outside

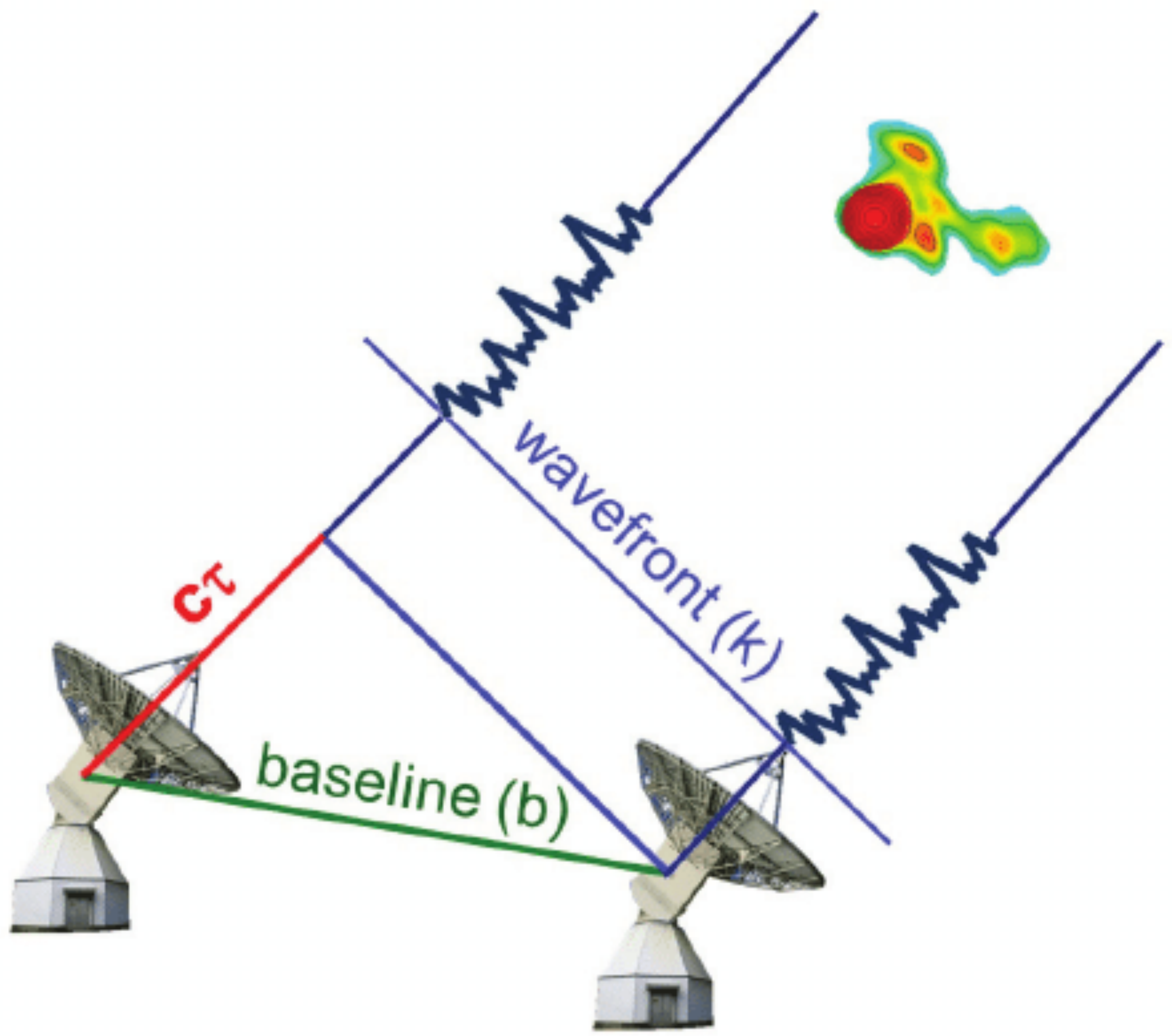
LoD:
50 – 100 μs



Orientation:
10 – 100 μas

There are different concepts for the estimation of the motion of the earth

Star Compass

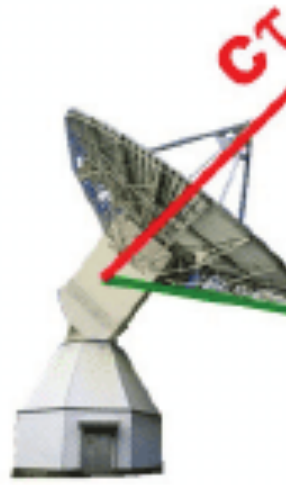


Inertial Compass

The measured rotational motion is not identical, since VLBI is not sensitive to relativistic effects

There are different concepts for the estimation of the motion of the earth

Eos, Vol. 72, No. 49, December 3, 1991

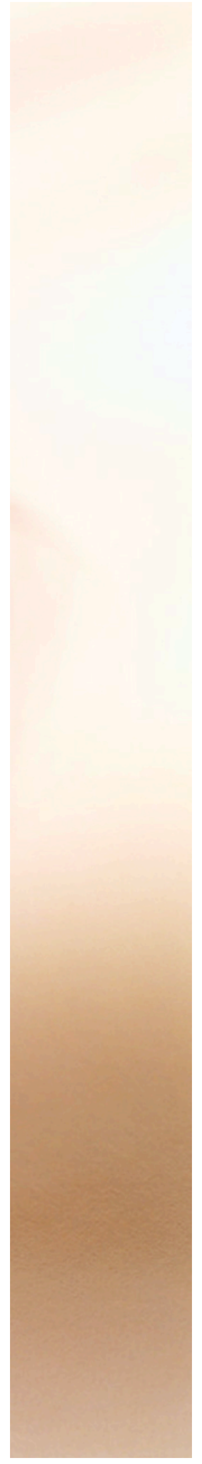


As the World Turns, II

PAGES 550–551

B. Fong Chao

—Come to think of it, no one ever said the Earth's rotation could not be measured with an apparatus in the comfort of a windowless basement.

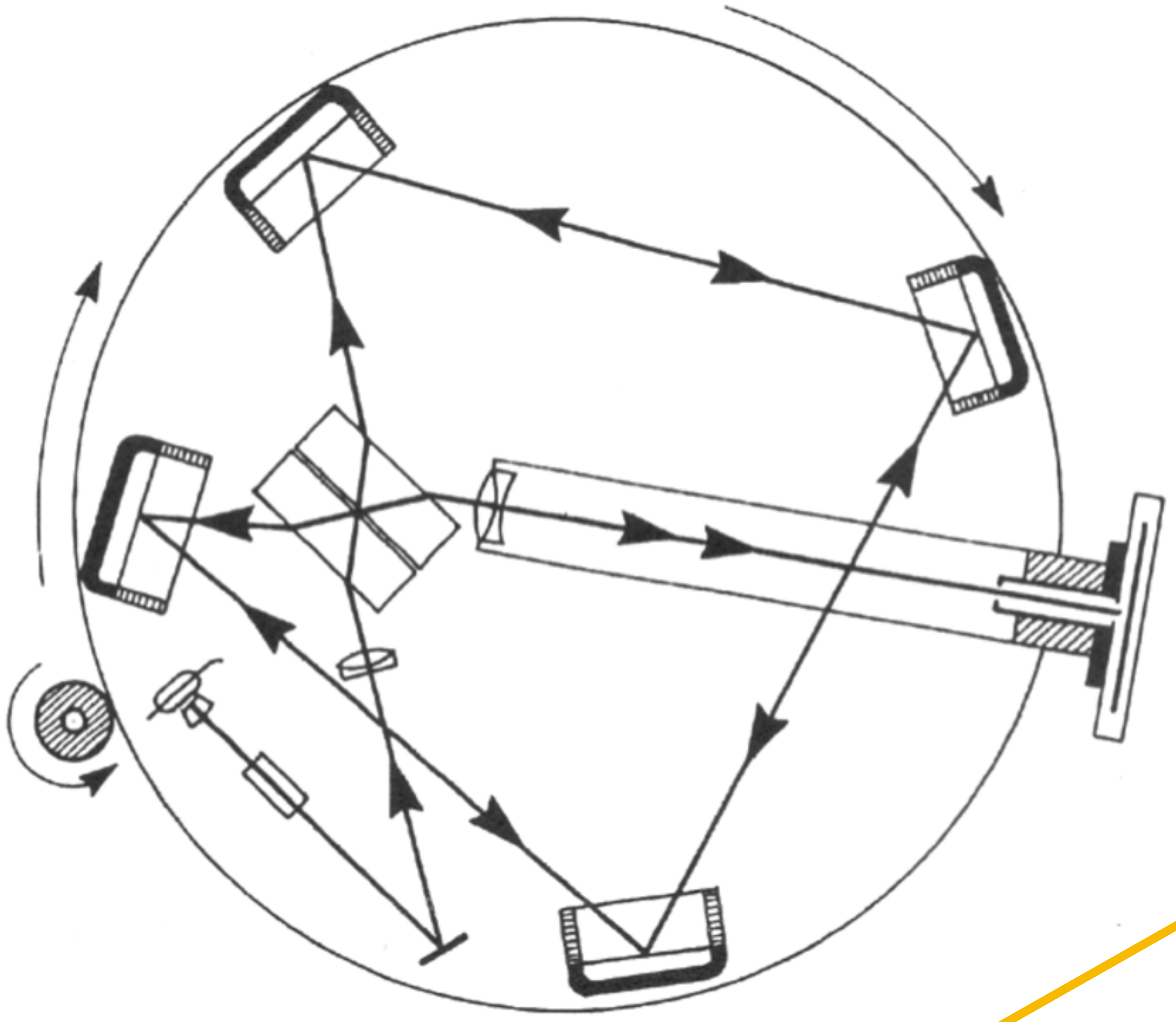


The measured rotational motion is not identical, since VLBI is not sensitive to relativistic effects

Geodetic Observatory Wettzell…

…one of 6 globally distributed fundamental stations

Sagnac interferometer (1913)



GP-B: 2.56 x 10^{-11} rad/s/sqrt(Hz)

(1.35 x 10^{-13} rad/s @ 10h) PRL **106**, 221101 (2011)

PASSIVE

FOG: (large scale factor… but sensitivity, stability insufficient for geodesy)

$\Delta\Omega \leq 10 \times 10^{-8}$ rad/s/sqrt(Hz) (Optics LETTERS **38**, 1092–1094 (2013))

externally injected stabilized laser beams:
(concept shown, backscatter the same as in RLGs)

$\Delta\Omega \leq 1 \times 10^{-9}$ rad/s/sqrt(Hz) (Optics LETTERS **44**, 2732-2735 (2019))

ACTIVE

Large Ring Laser: operational

1.2 x 10^{-11} rad/s/sqrt(Hz) (PRL **107**, 173904 (2011))

(3 x 10^{-13} rad/s @ 10h)

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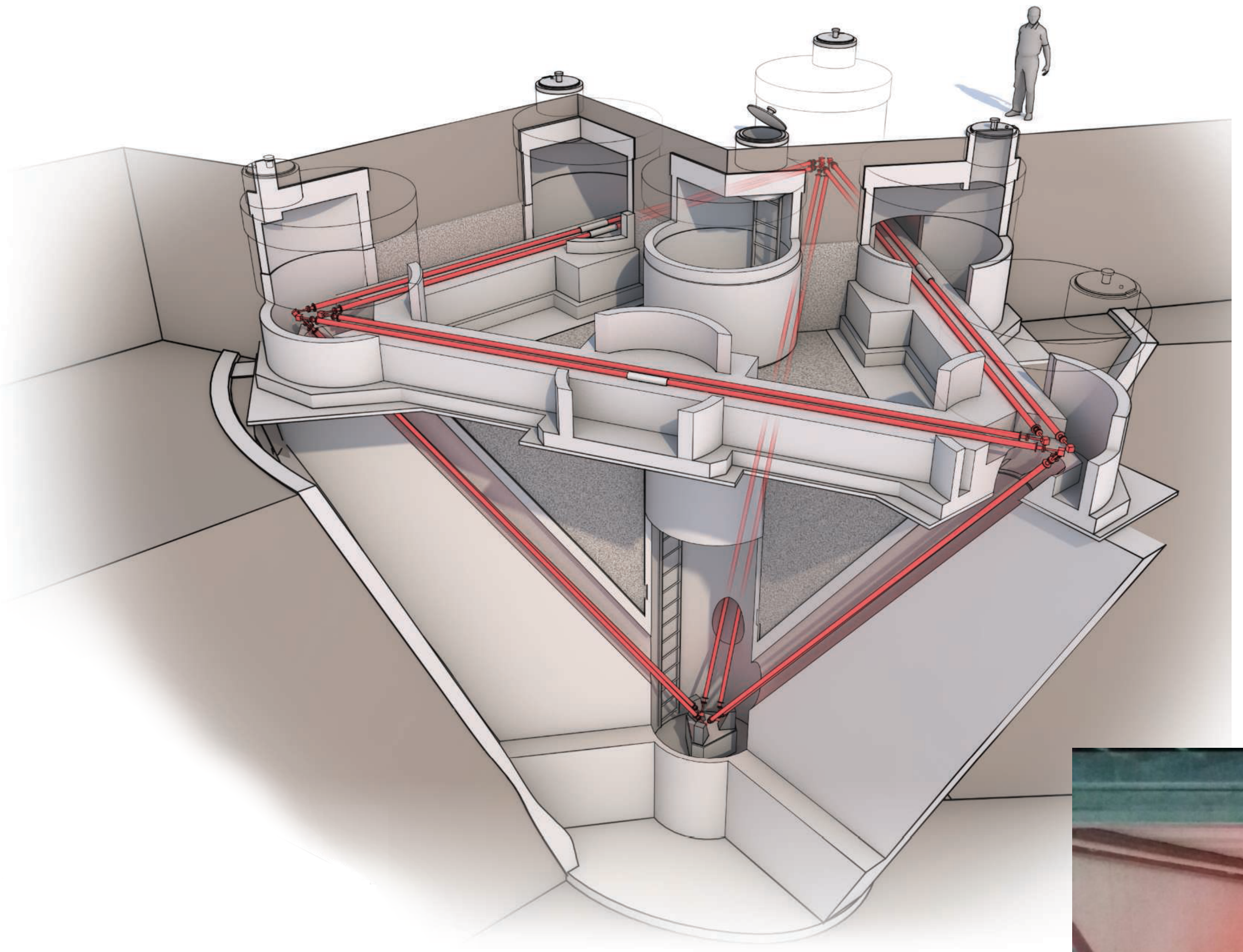
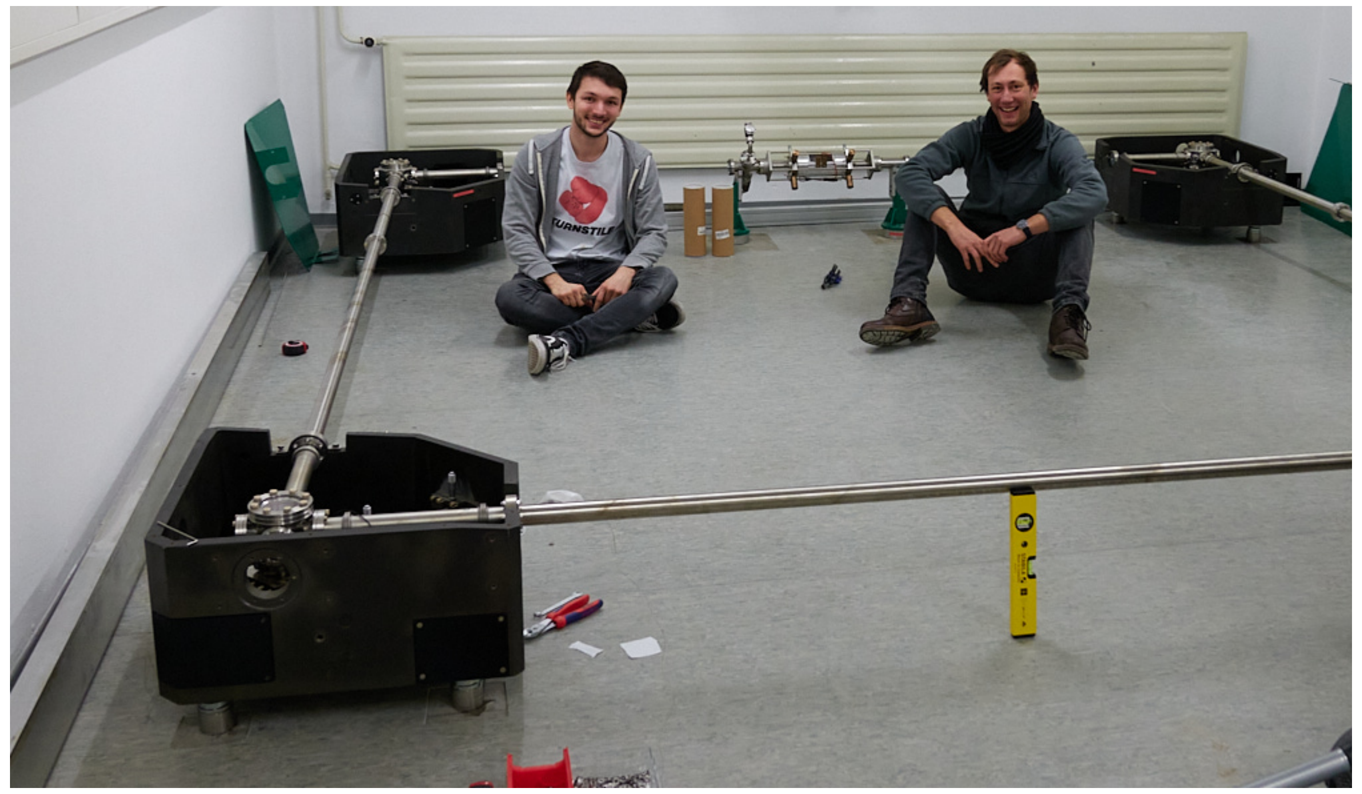
atom interferometry: (short-term)

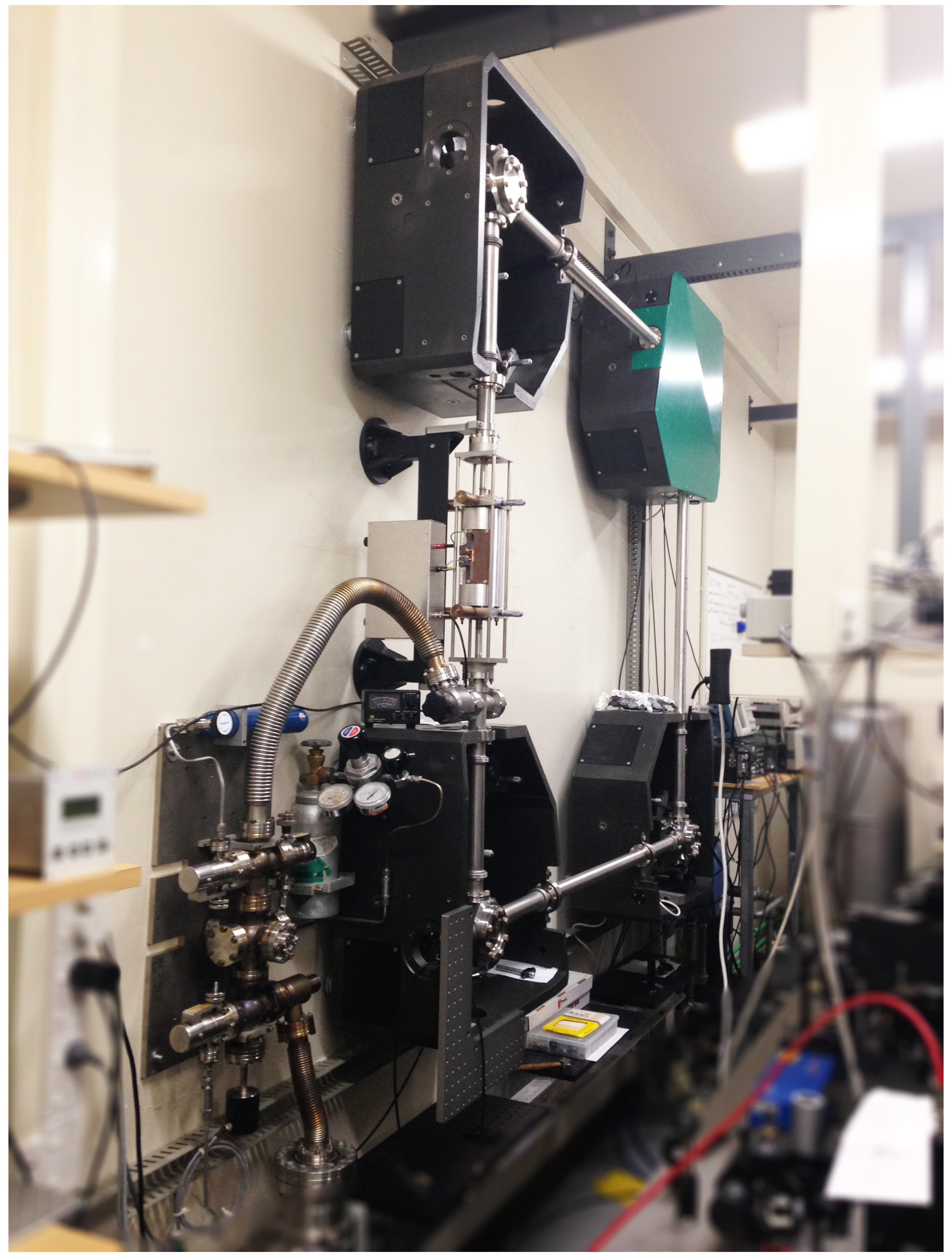
6 x 10^{-10} rad/s/sqrt(Hz) (Class. Quant. Grav. **17**, 2385–2398 (2000))

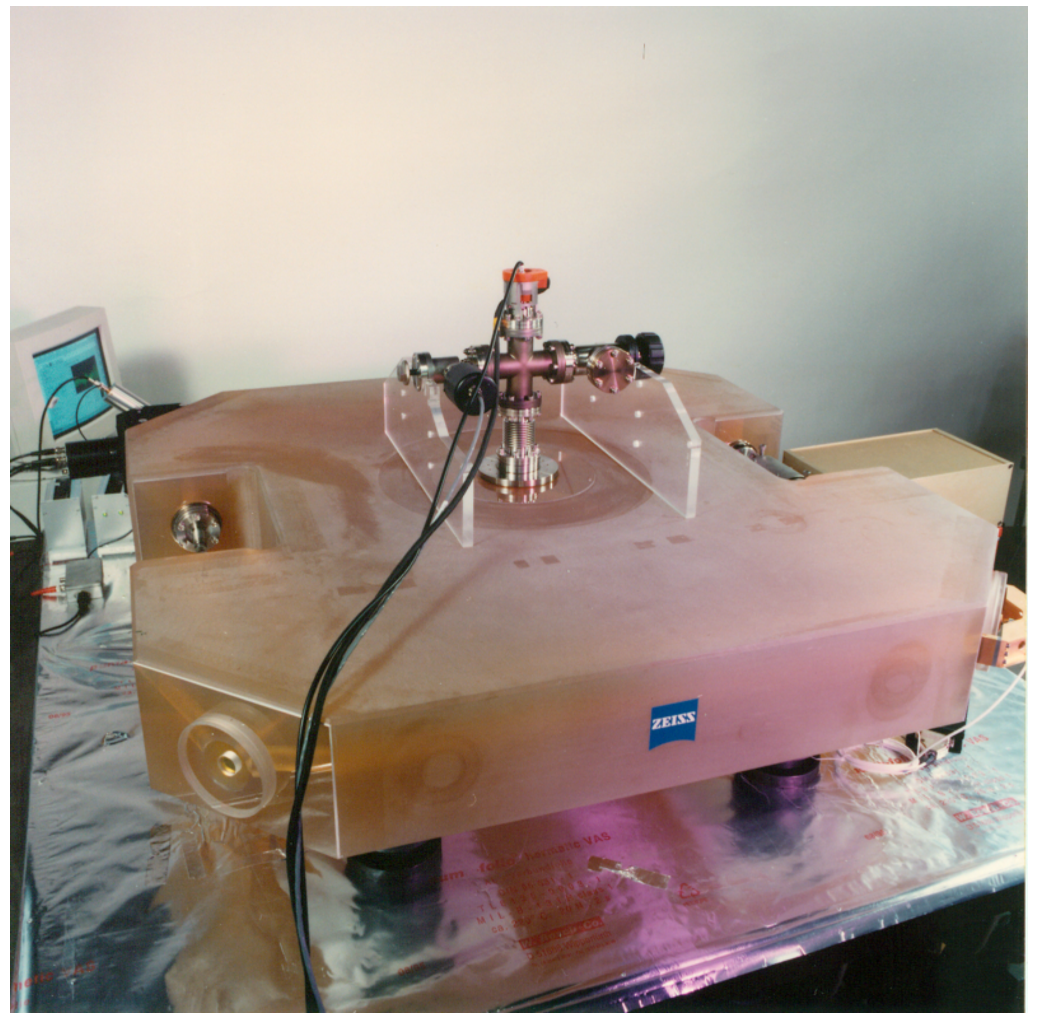
Josephson effect: delicate + small,

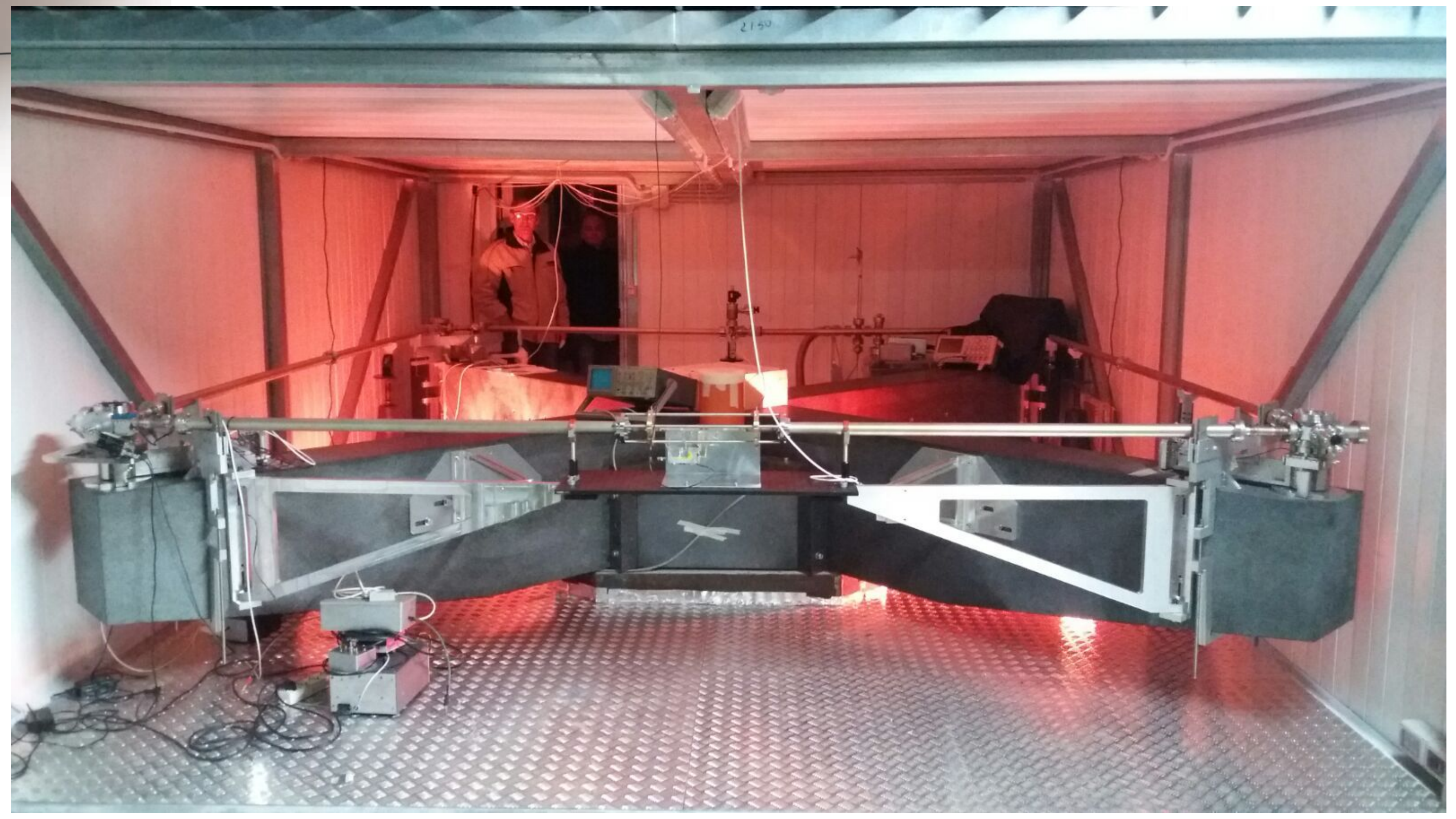
8 x 10^{-9} rad/s/sqrt(Hz) (Rep. Prog. Phys. **75**, 016401, (2012))



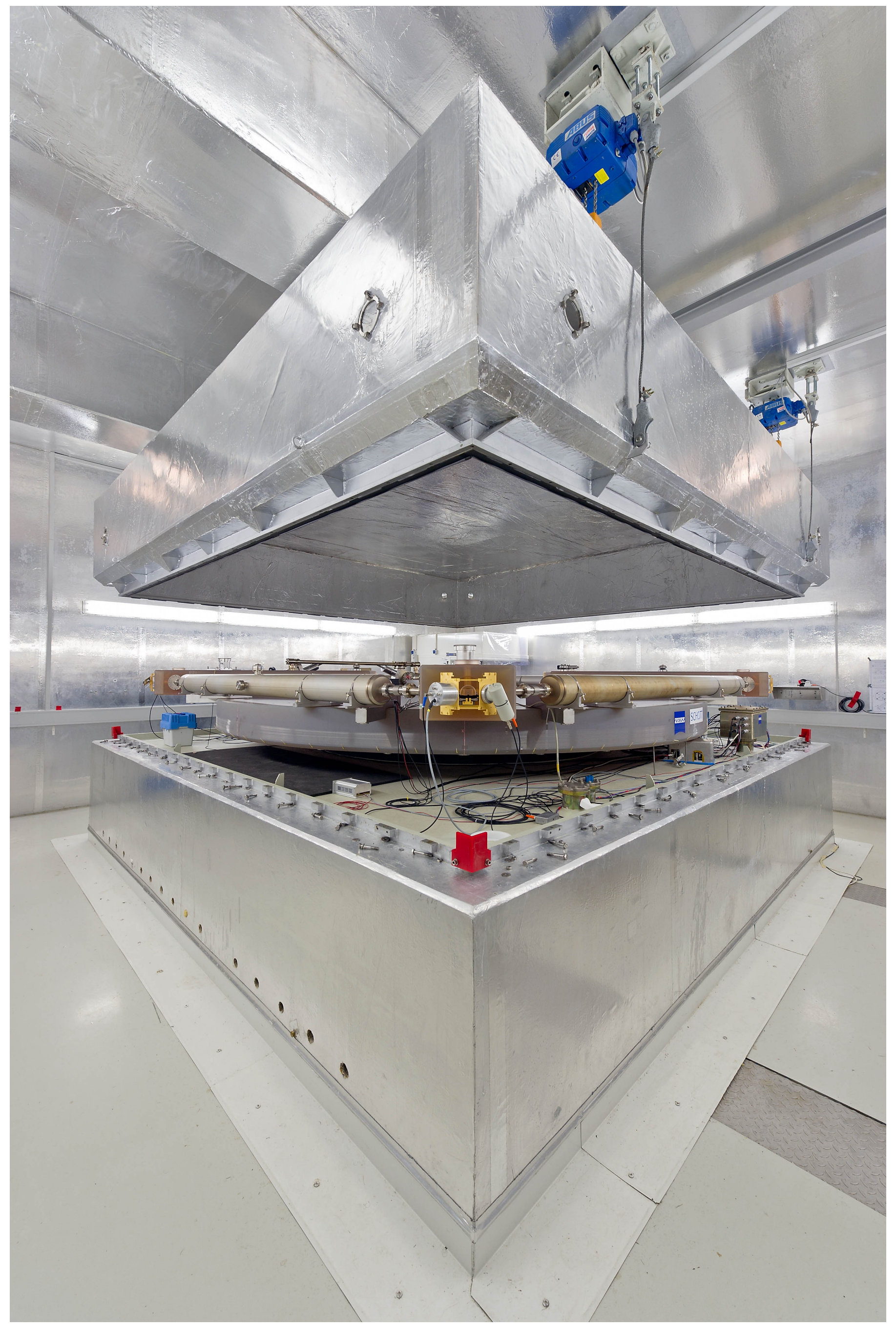


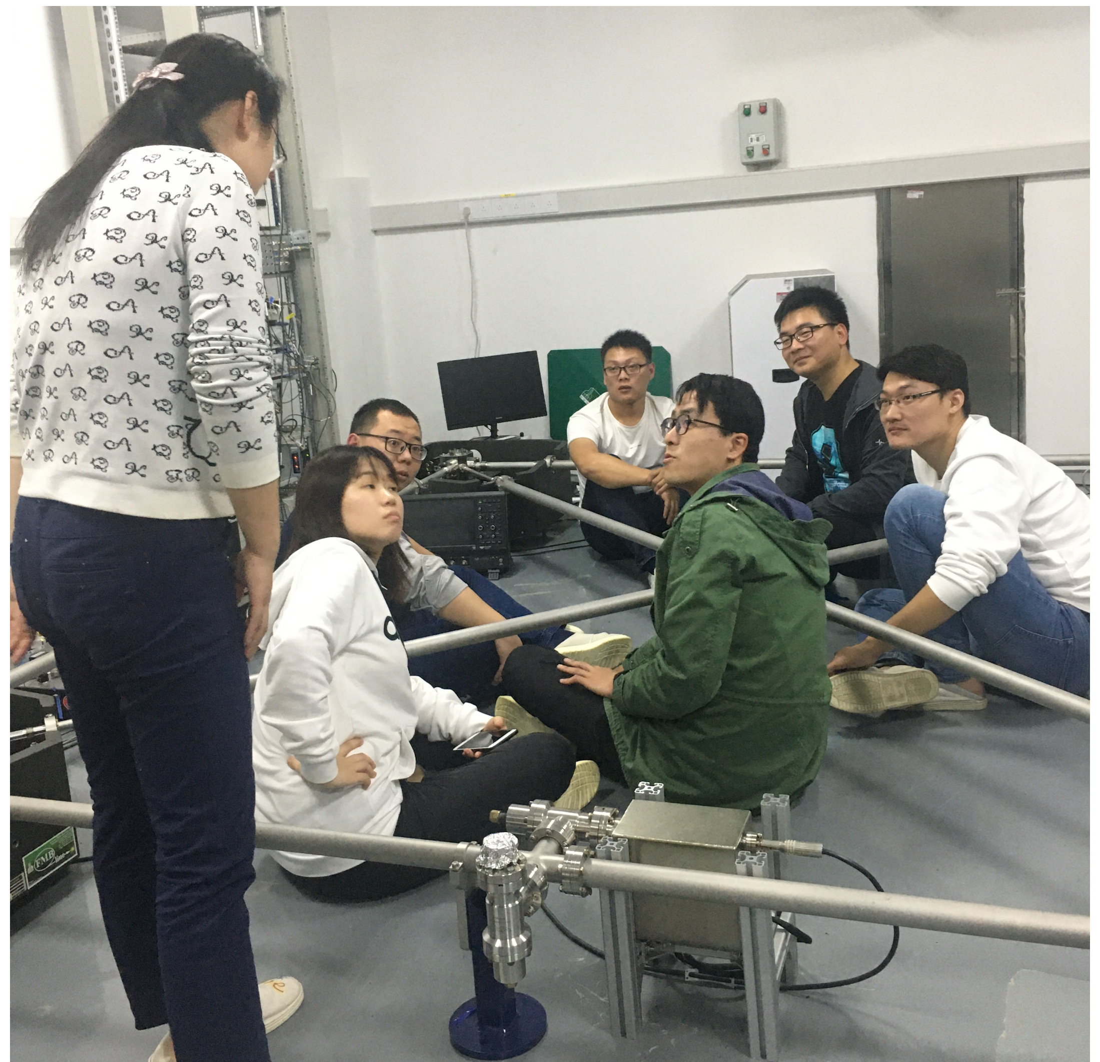


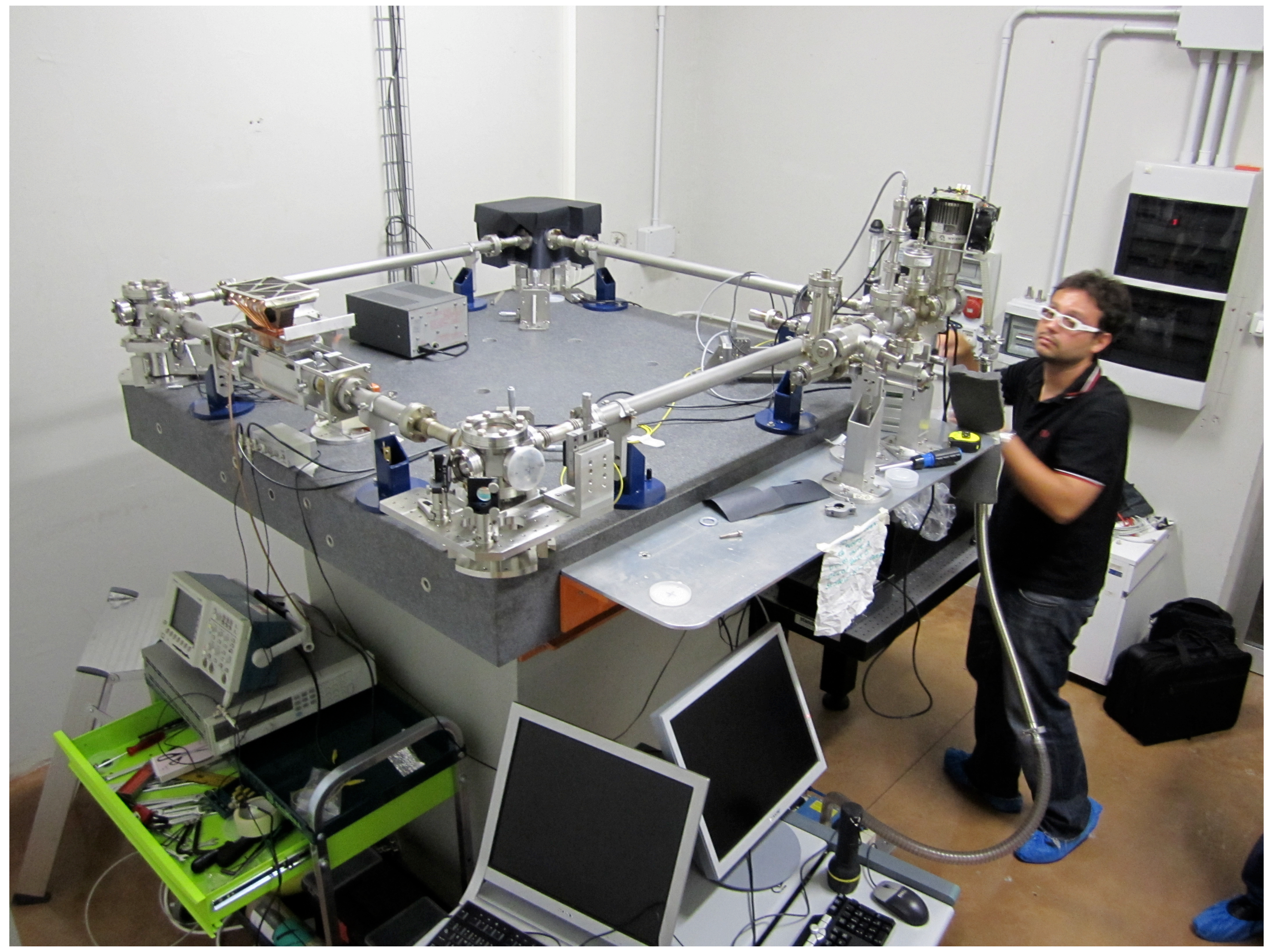




Large Ring Lasers





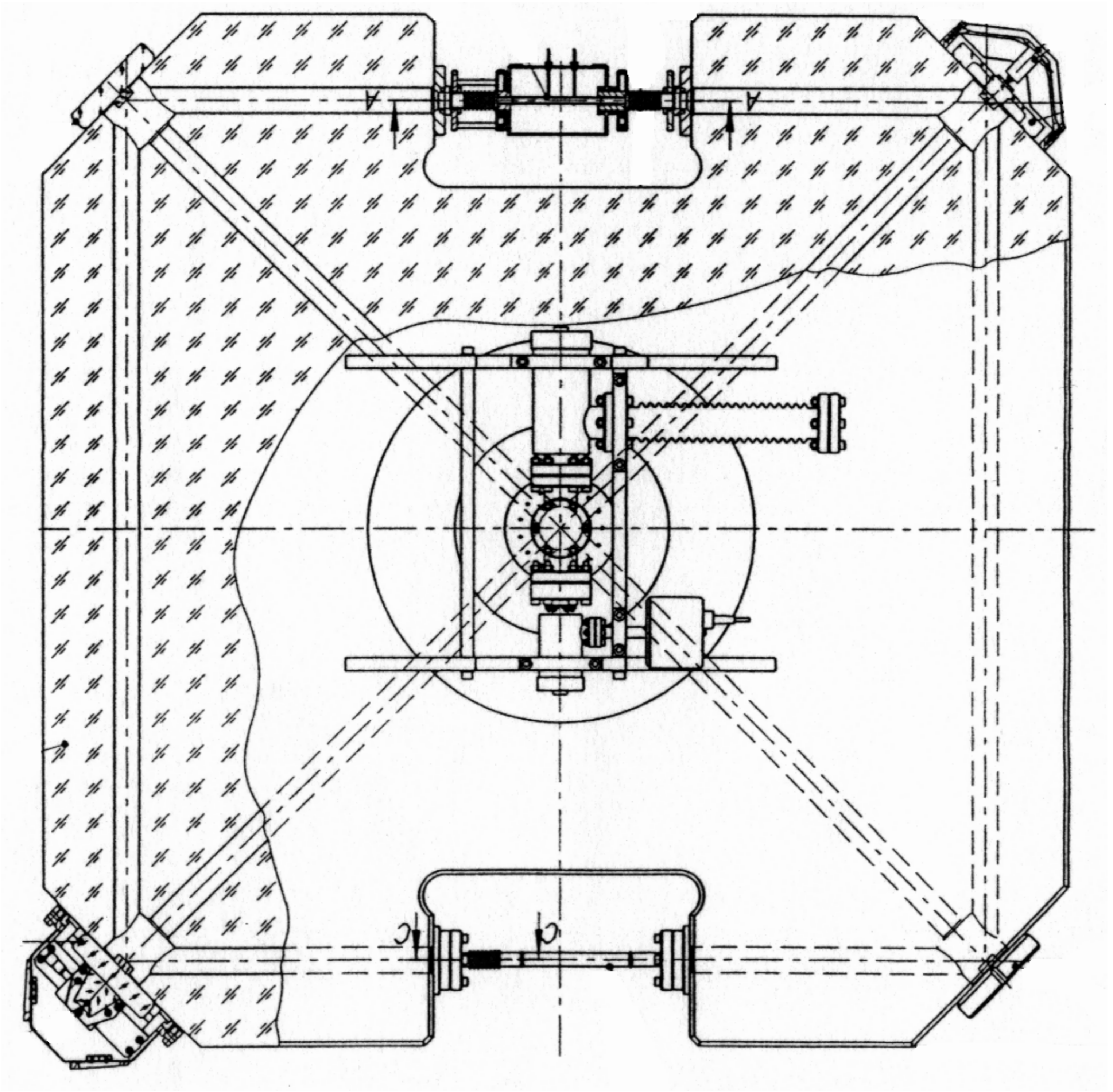


Ring laser essentials

- A ring laser gyroscope is defined by a closed light path around a contour
- It contains a laser gain medium (neutral atom gas) all around the cavity
- The sensitivity is given by the scale factor

$$S = \frac{4A}{\lambda P}$$

- The gyro is entirely insensitive to translations and has a linear transfer function
- The rotation signal is encoded in frequency modulation (beat note)

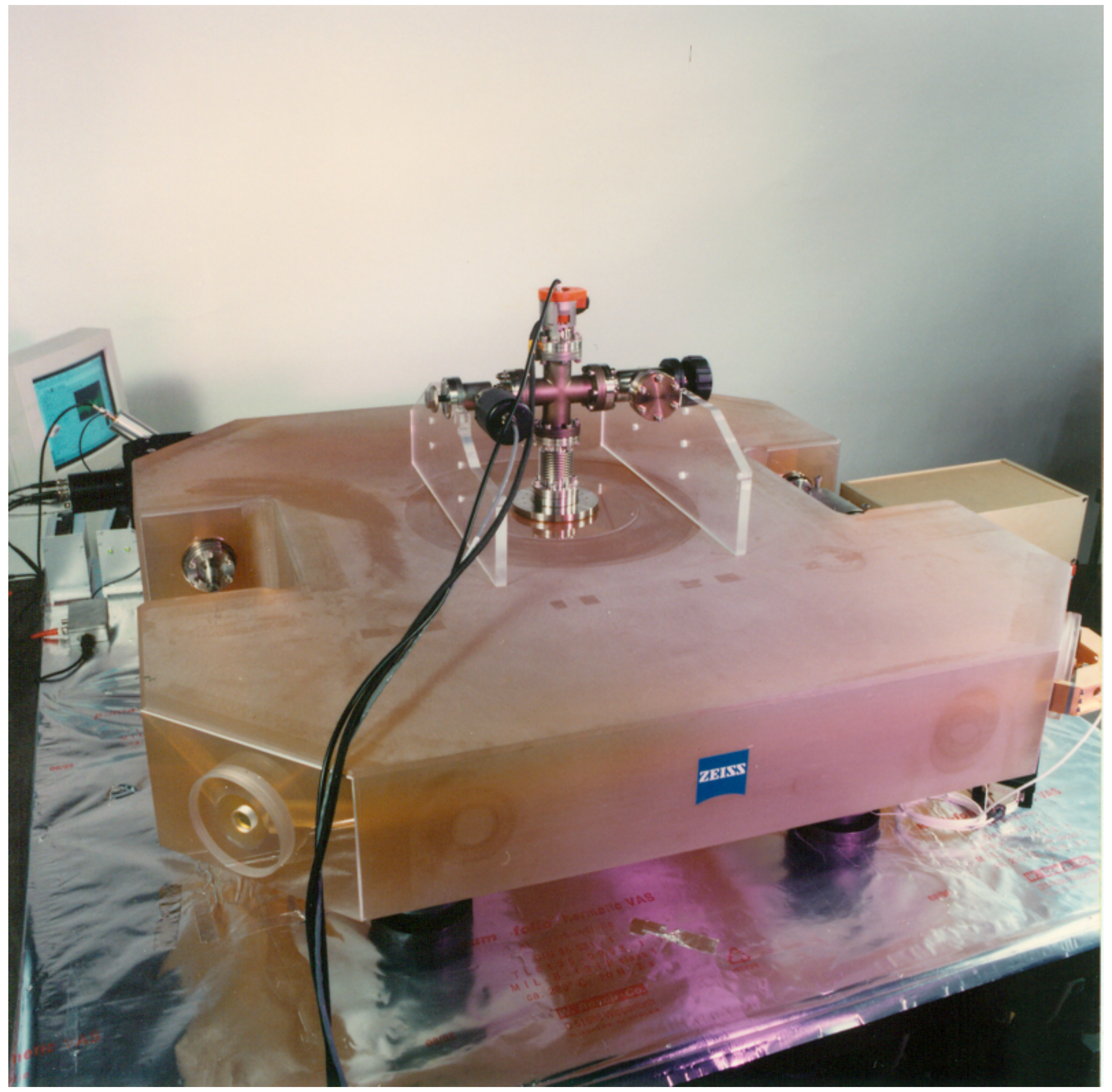


$$\delta f = \frac{4A}{\lambda P}\vec{n}\cdot\vec{\Omega} + f_{nr}$$

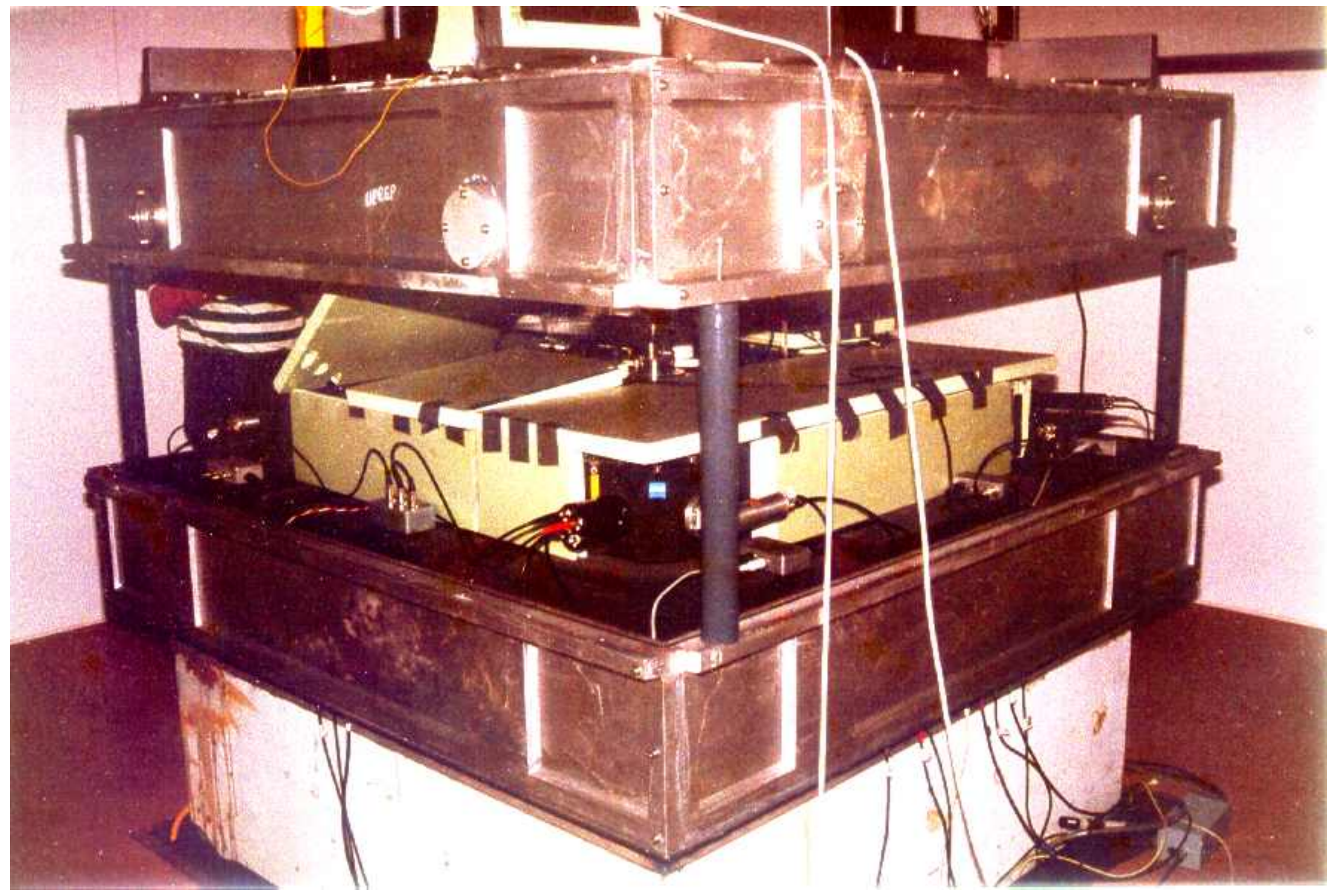
$10^{-9}\Omega_E \approx 0.07$ prad/s

$f_{nr} < 0.3\ \mu\text{Hz}$

Ring laser essentials



C-II



$$\Delta\Omega = \frac{cP}{4AQ}\sqrt{\frac{hf}{P_x t}}$$

sensitive parameters

- The experienced rotation rate is entirely a matter of the cavity
- The enclosed area and the losses determine the ultimately achievable sensor resolution
- We use light and optical interference to probe the rotation sensing ability
- Therefore active and passive systems are entirely equivalent and this includes FOGs as well.
- Things however become tricky when we wish to extract the rotation rate at very high resolution and stability

Sagnac interferometry and the ring laser G

For an active cavity, HeNe is the gain medium of choice: It is a neutral atom gas laser with suitable isotopic shift of ≈ 800 MHz to decouple the 2 beams (no mode competition) and ensures a very narrow line-width

Example G (monolithic)

Perimeter: 16 m

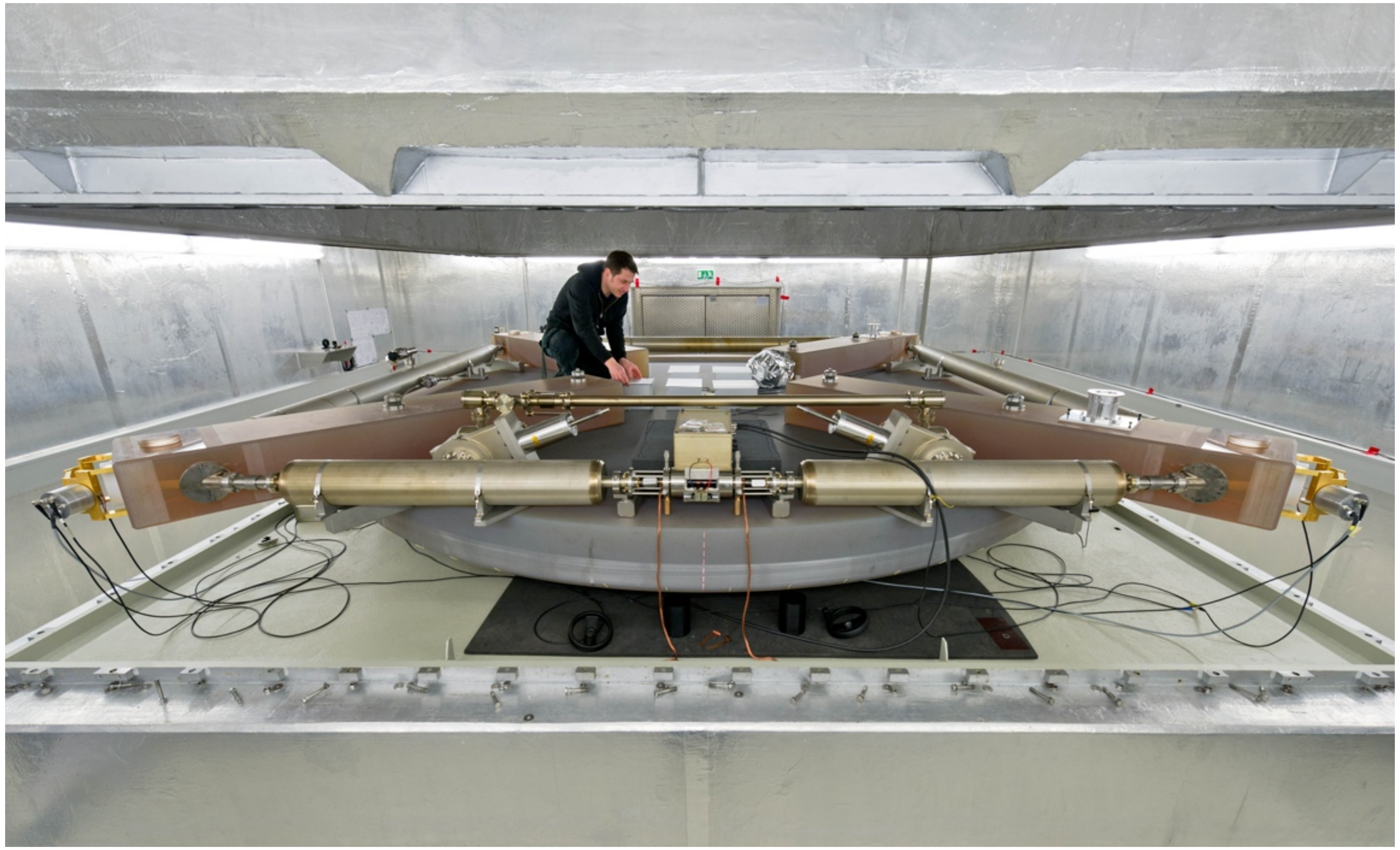
Area: 16 m^2

Losses: ≈ 46 ppm

circ. power: 153 mW

$Q = \omega\tau \approx 5 \times 10^{12}$

$\Delta\Omega \approx 3.17\ 10^{-14}$ rad/s/sqrt(Hz)

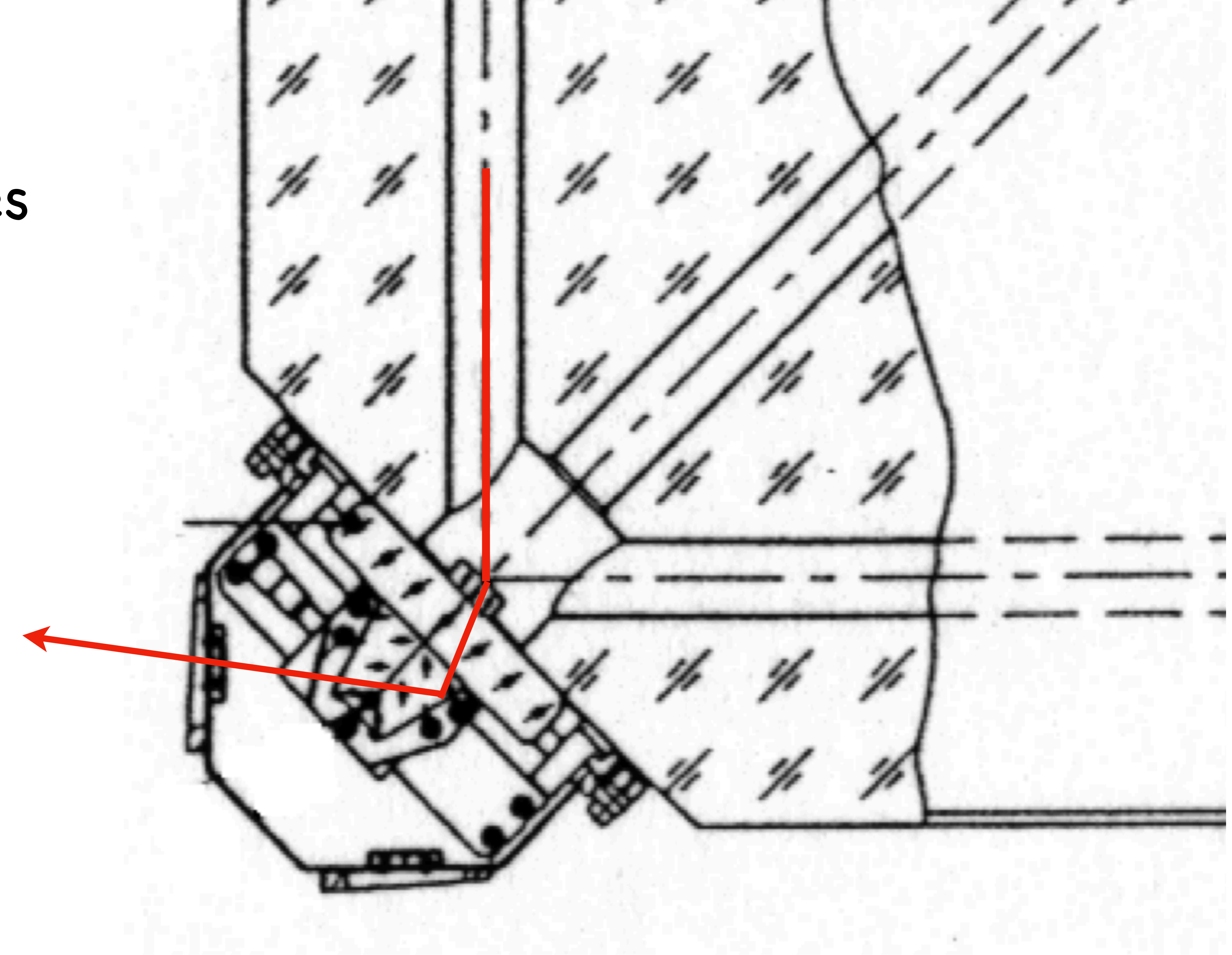


...internal shot noise limit,
but we cannot access that !!!

We can only access the light leakage through a mirror, which is small in order to make the losses low

This provides the single mode sensor resolution, where P_x now is 28 nW and $\Delta\Omega$ comes to

8.9 10 x 10^{-11} rad/s/sqrt(Hz)



Further complications are a limited quantum efficiency η_D of the detector as well as the contrast in the interferogram (astigmatism)

More issues follow from the intrinsic electronic noise sources:

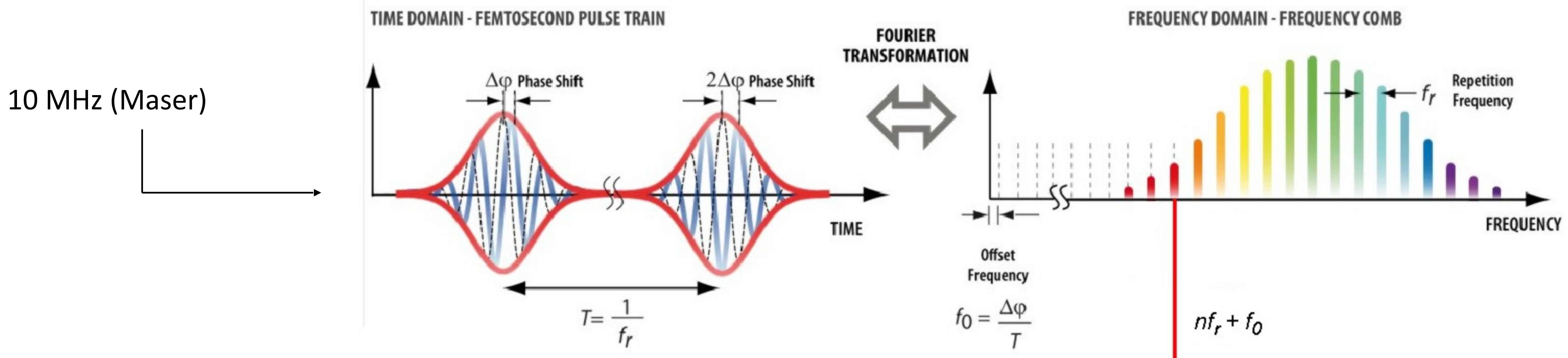
$$\eta_D \approx 0.7 \qquad K \approx 0.8$$

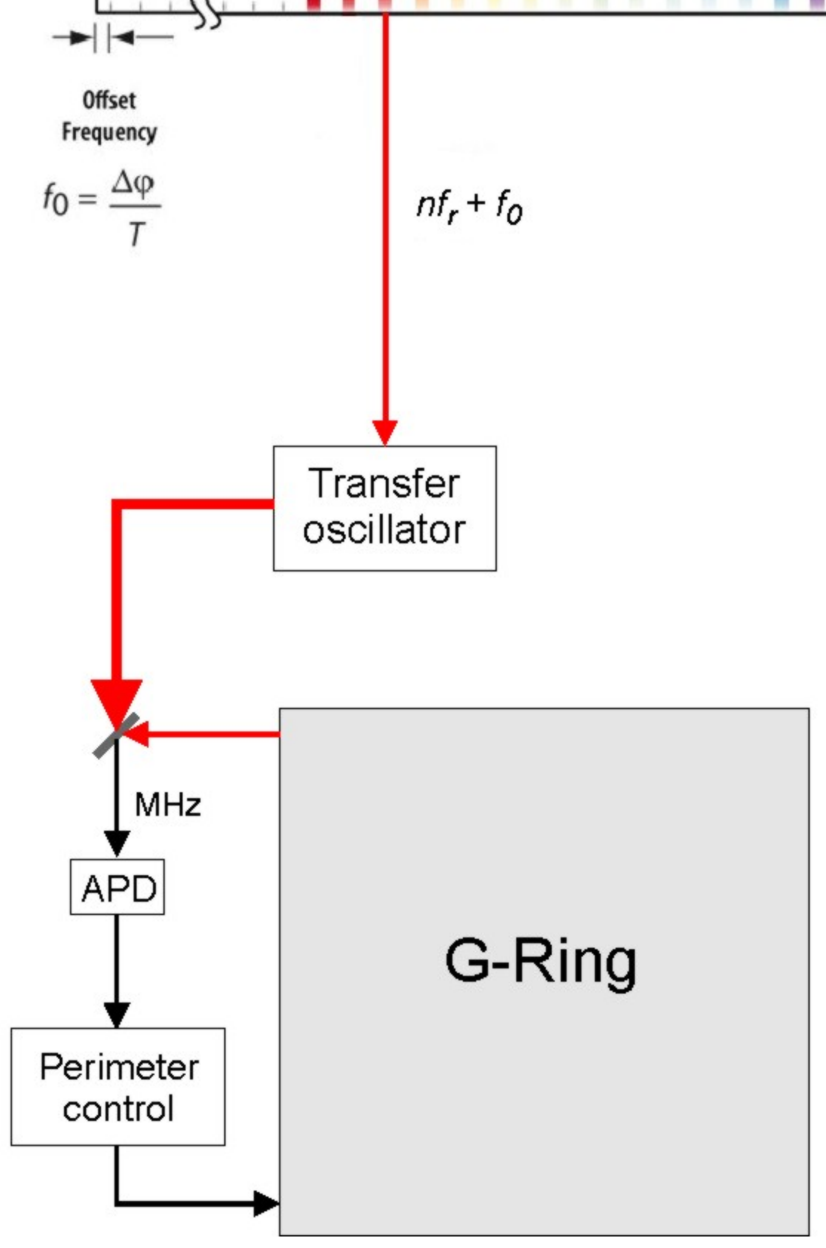
detector, digitizer, frequency estimator, coating and substrate noise (fluctuation, dissipation theorem)

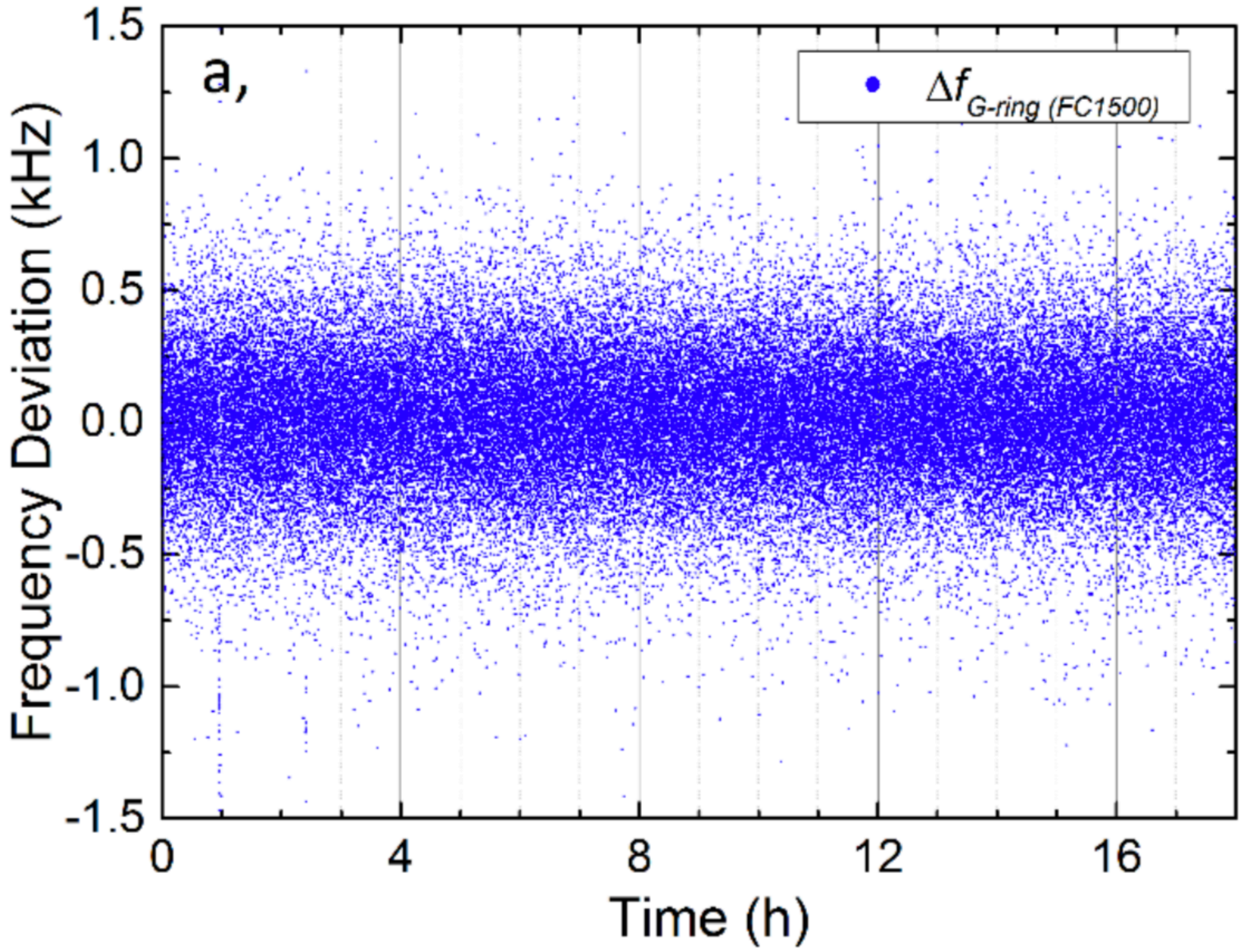
$$\Delta\Omega = \frac{cP}{4AQ}\sqrt{\frac{hf}{P_x \eta_D K t}} \qquad K = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

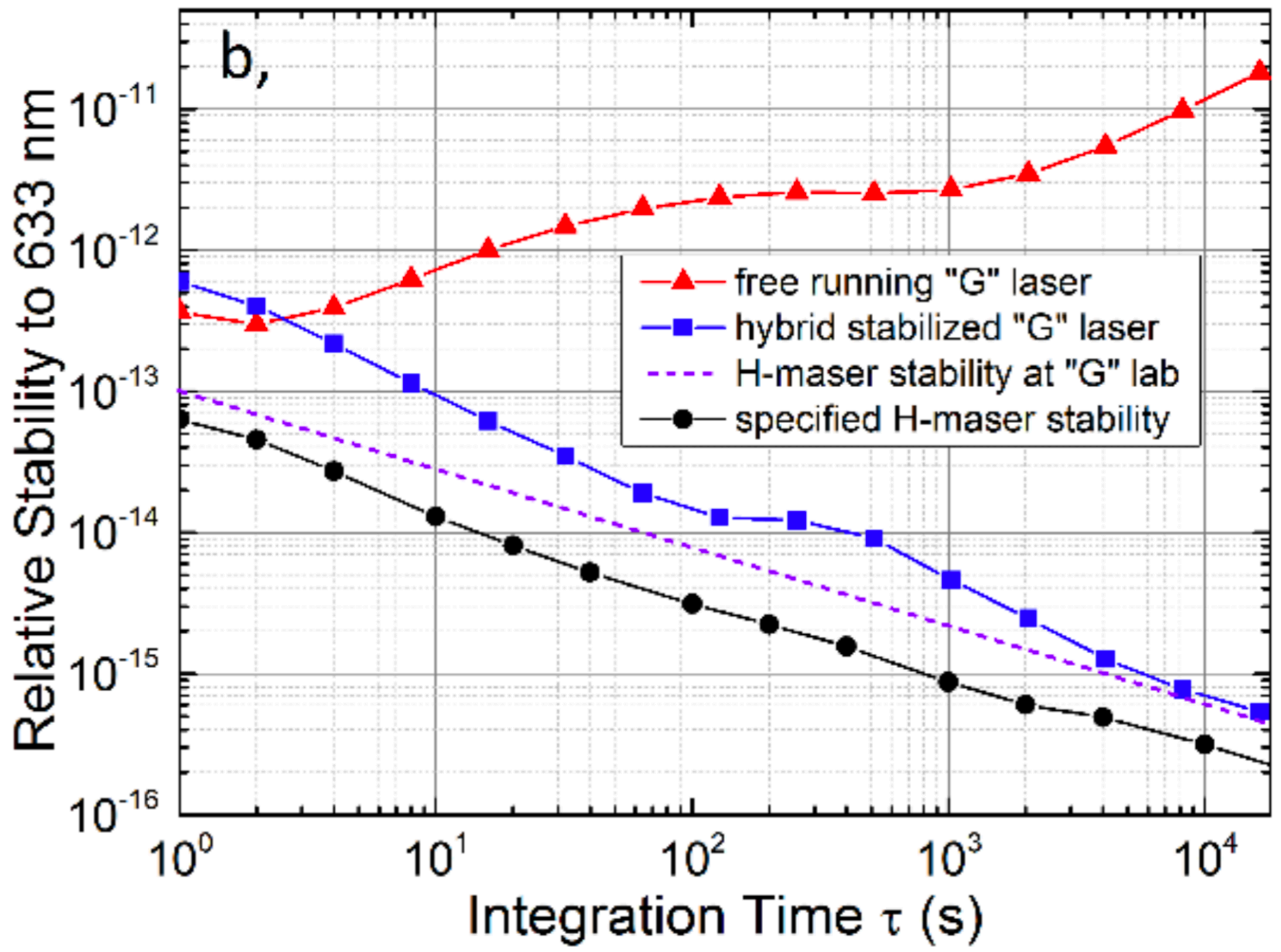
All this only addresses sensitivity. An entirely different story is sensor stability and accuracy.

HeNe ring lasers are perfect spectroscopic systems, because of a very narrow linewidth







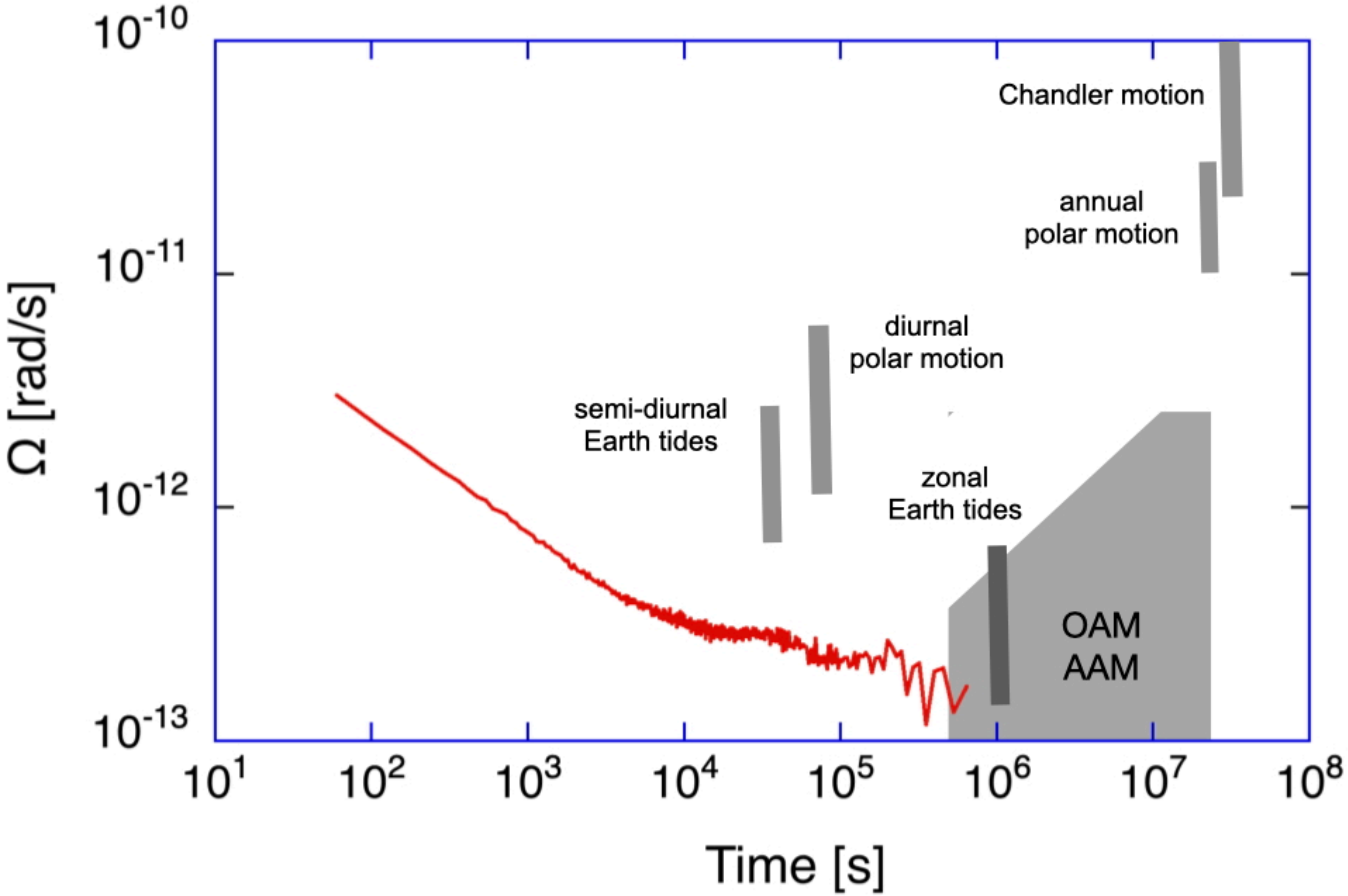


Shawlow - Townes Limit of G is: ≈ 6 μHz

The mechanical limit of the cavity is ≈ 1 kHz

Most of that noise is in "common mode" for Sagnac

Geodetic signals that we get from the G ring laser



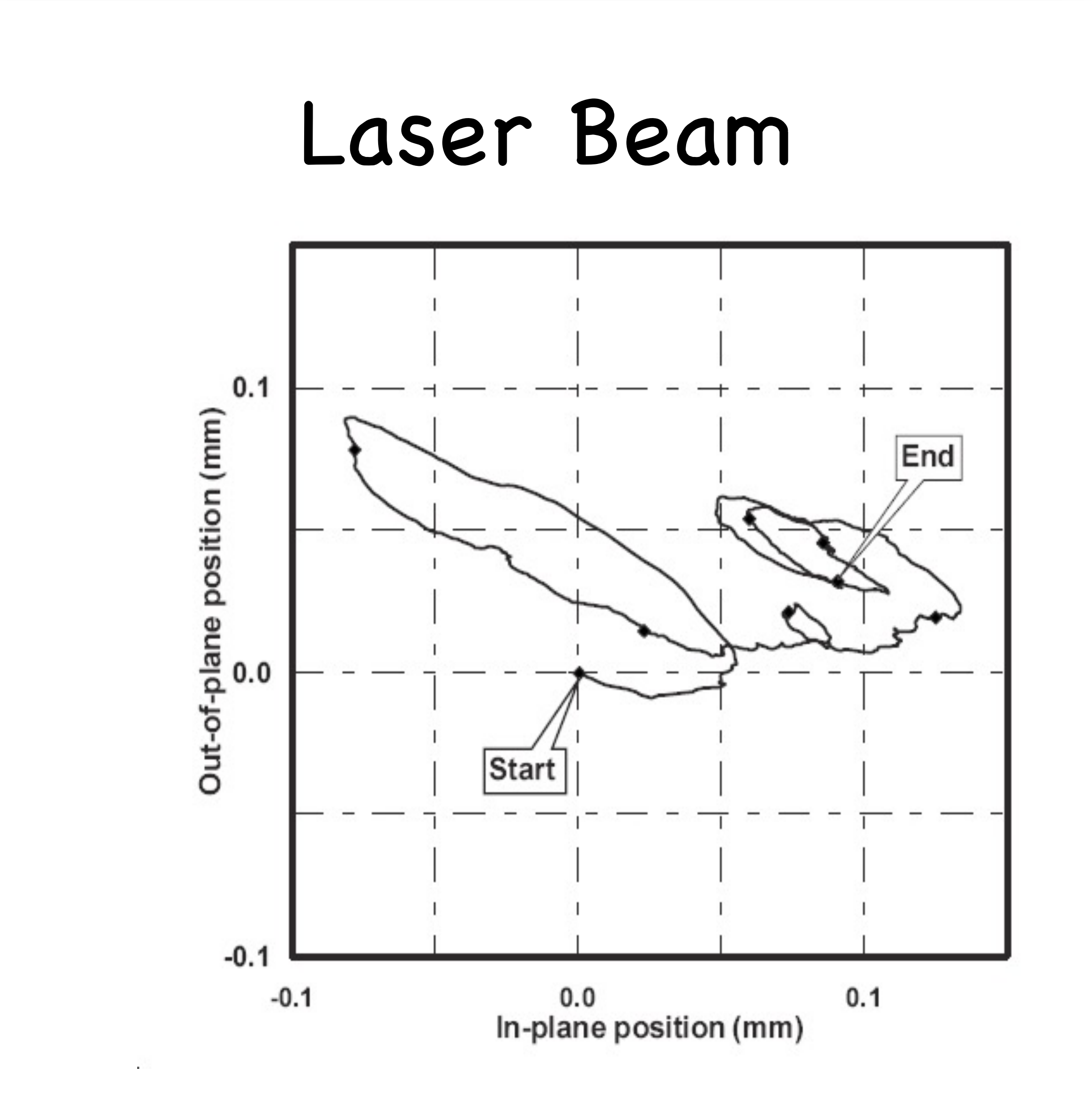
Heterolithic concept: UG-1/2 RLG with up to 834 m^2 of area

University of Chicago Photographic Archive [apf1-04511r]

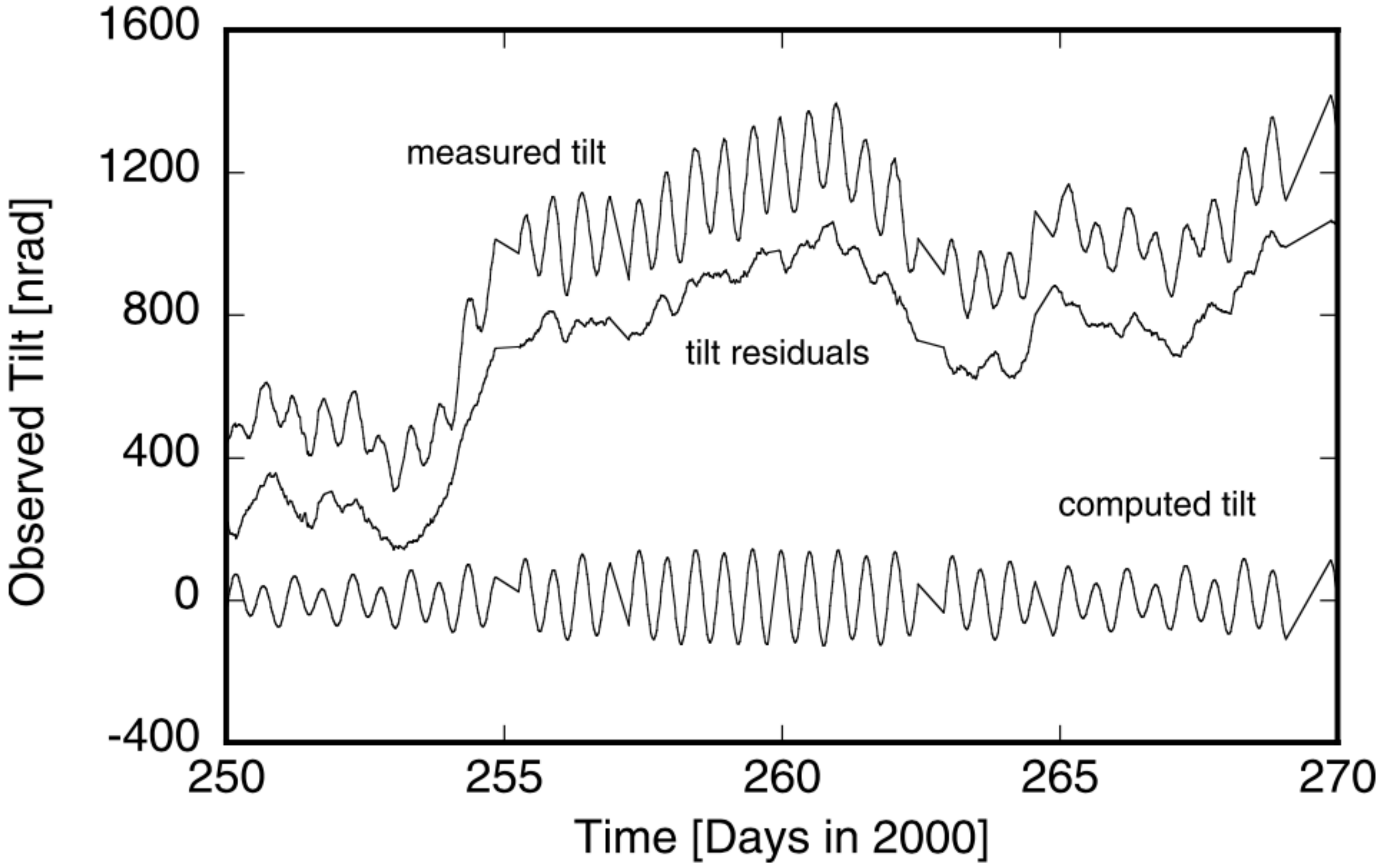
UG-2 is a stainless steel rectangular ring laser structure and about 8 orders of magnitude more sensitive than the Michelson - Gale installation

Heterolithic concept: UG-1/2 RLG with up to 834 m^2 of area

Terrain and sensor deformation caused by local tilt generate beam wander, scale factor variation and a change in sensor orientation

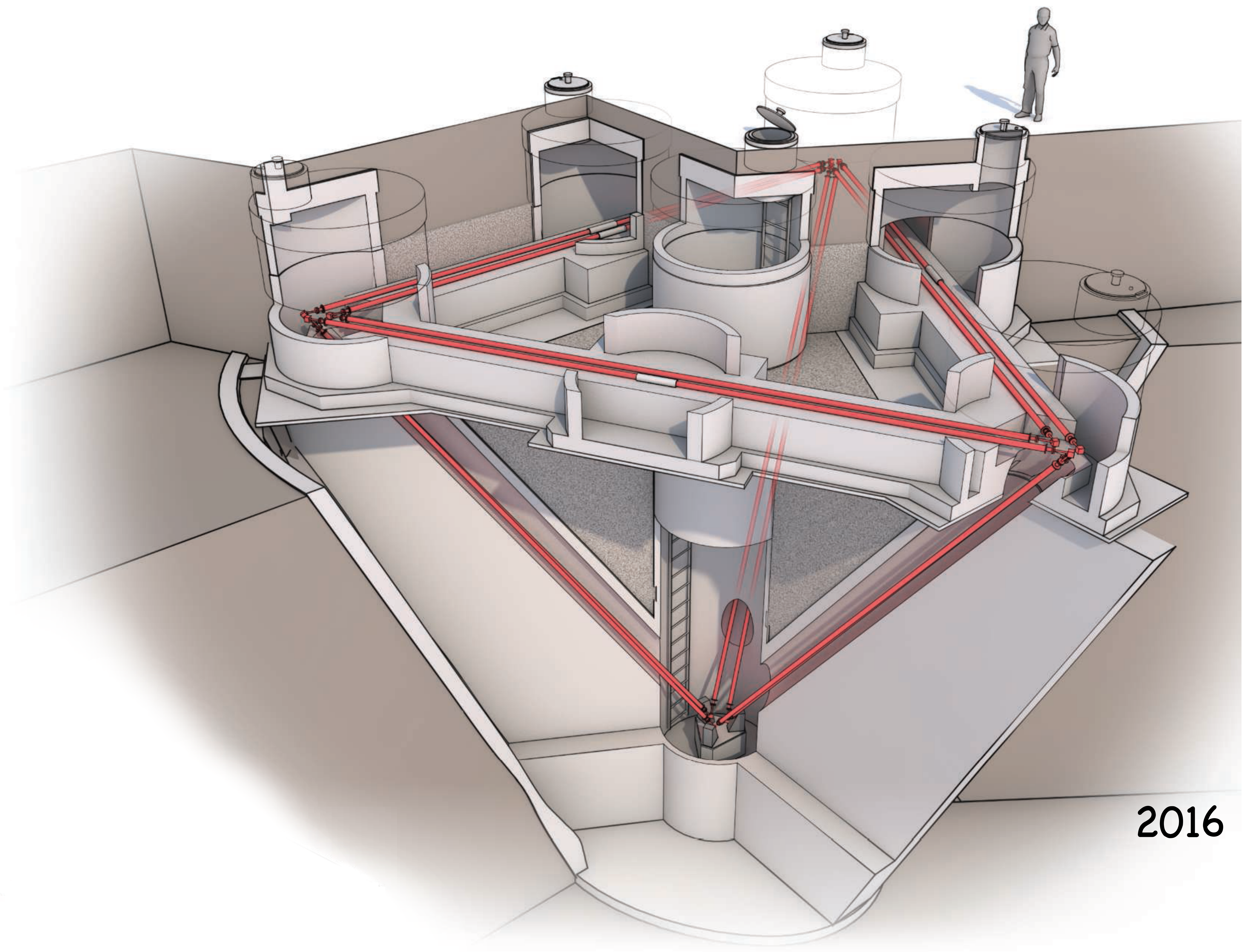


University of Chicago Photographic Archive [apf1-04509r]



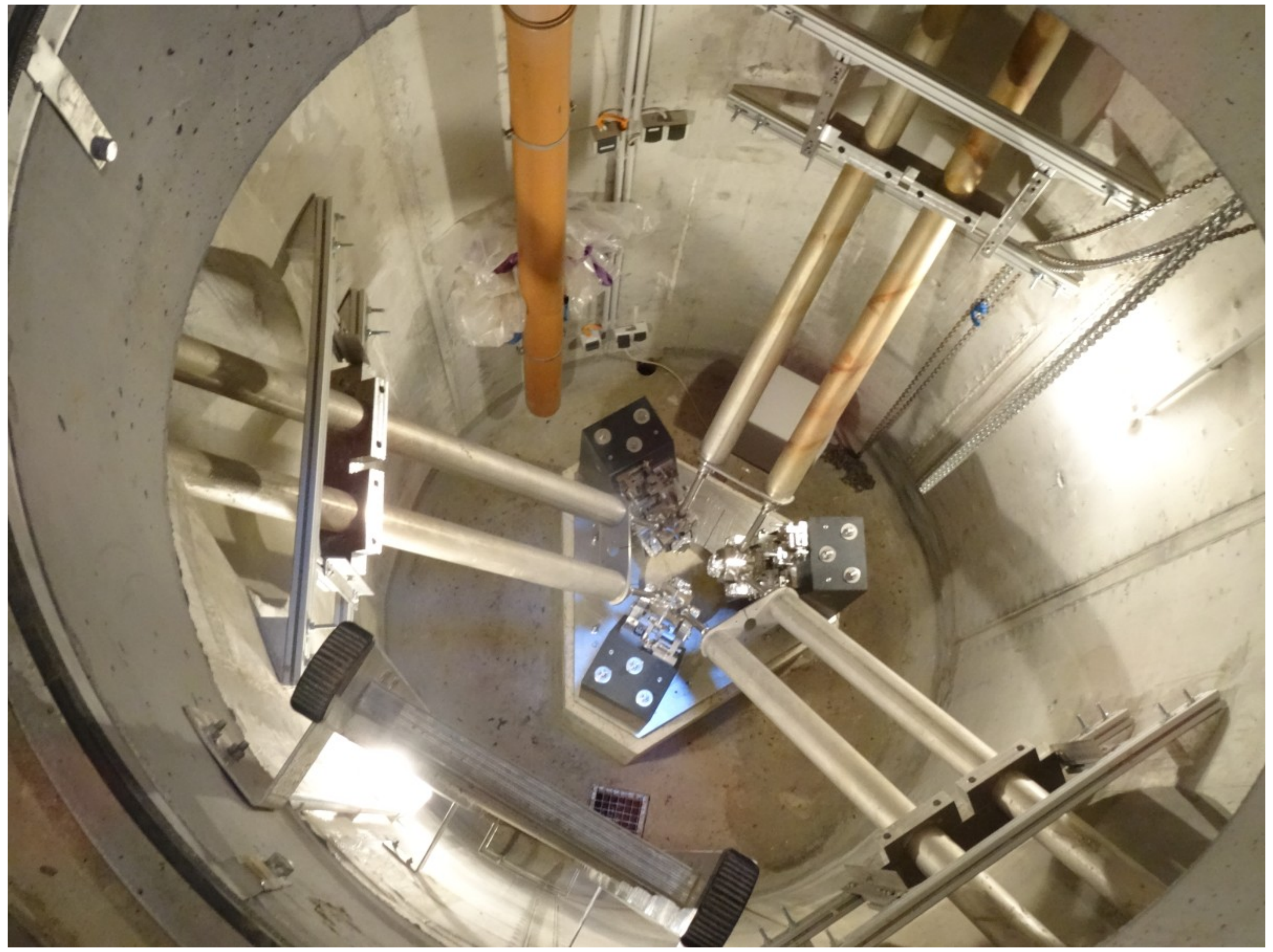
The advantages from upscaling are lost by the lack of stability

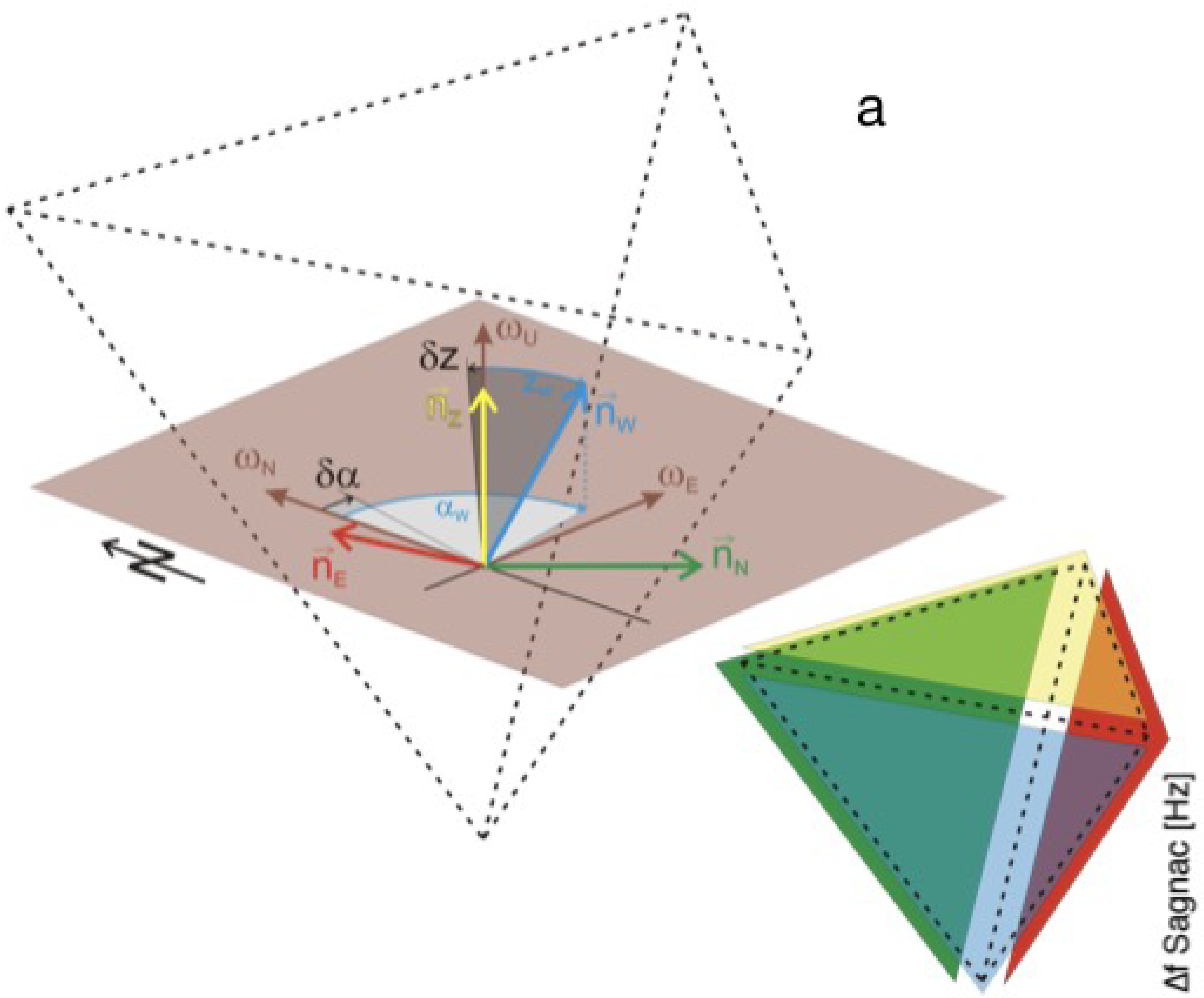
ROMY - A 4 component ring laser structure

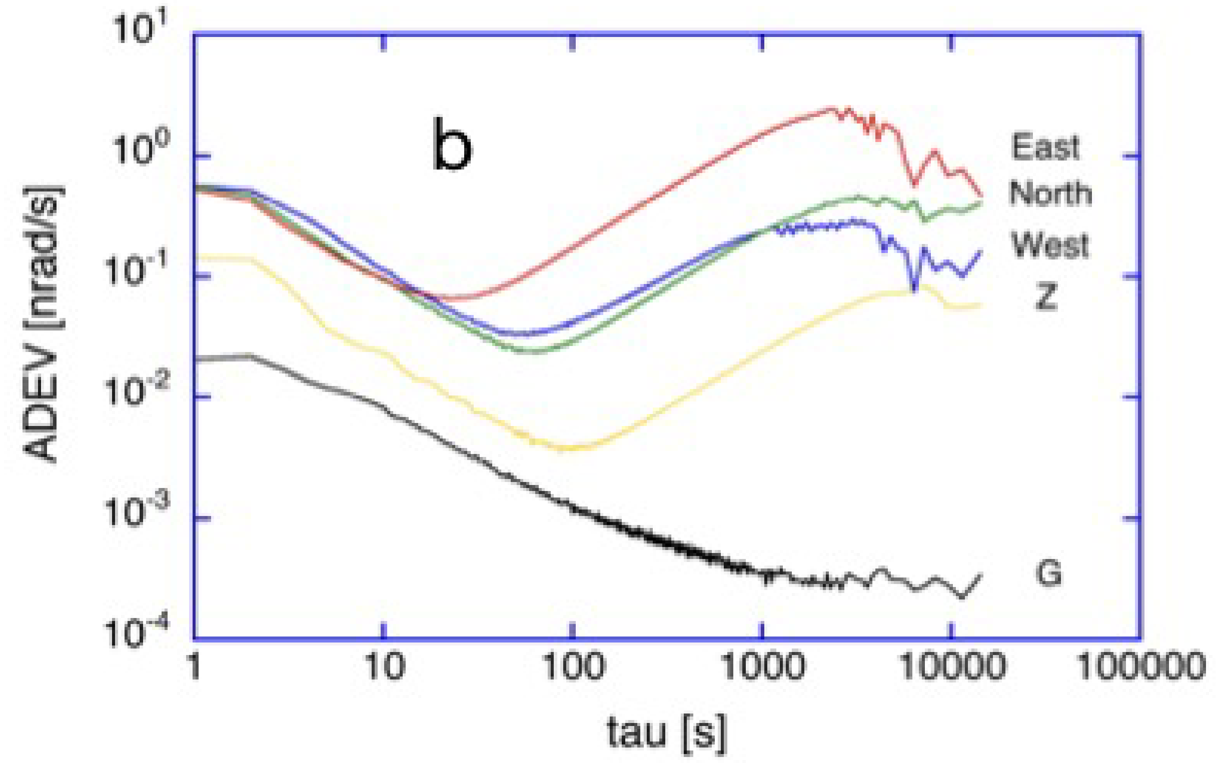


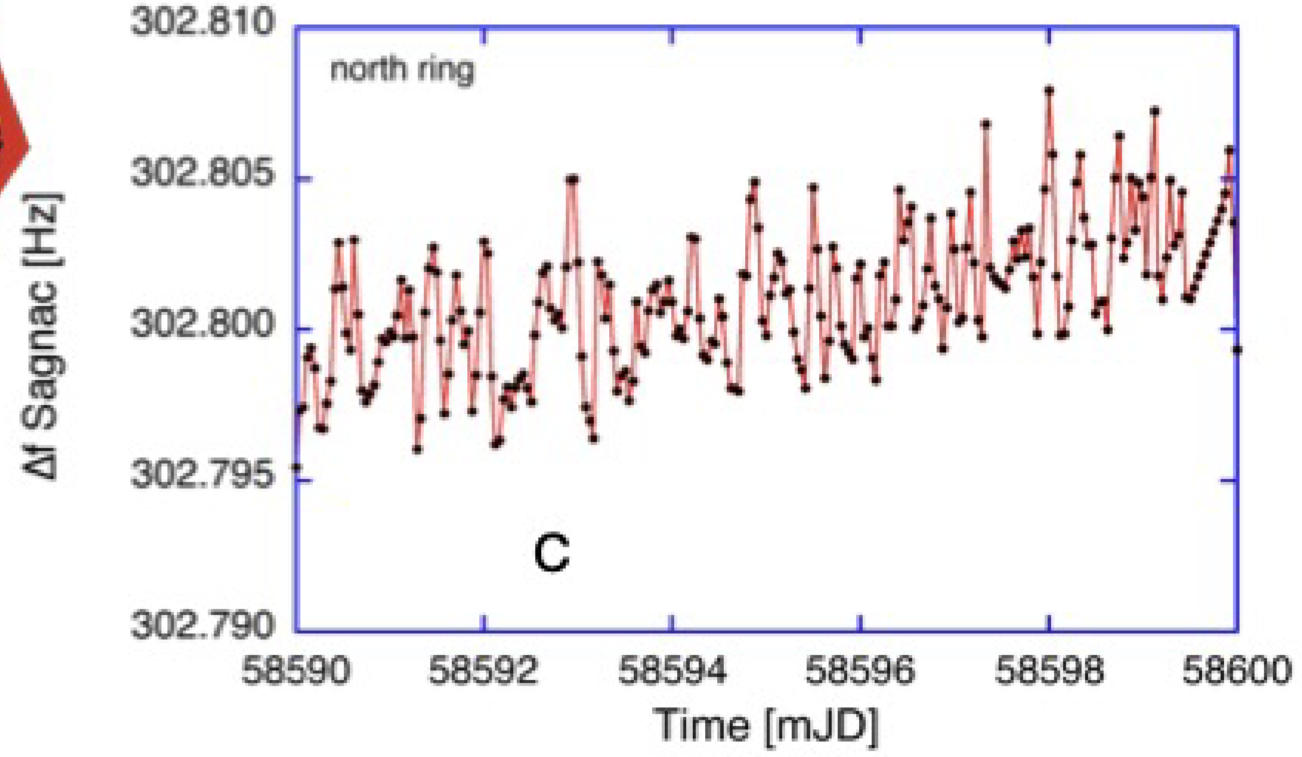
2016

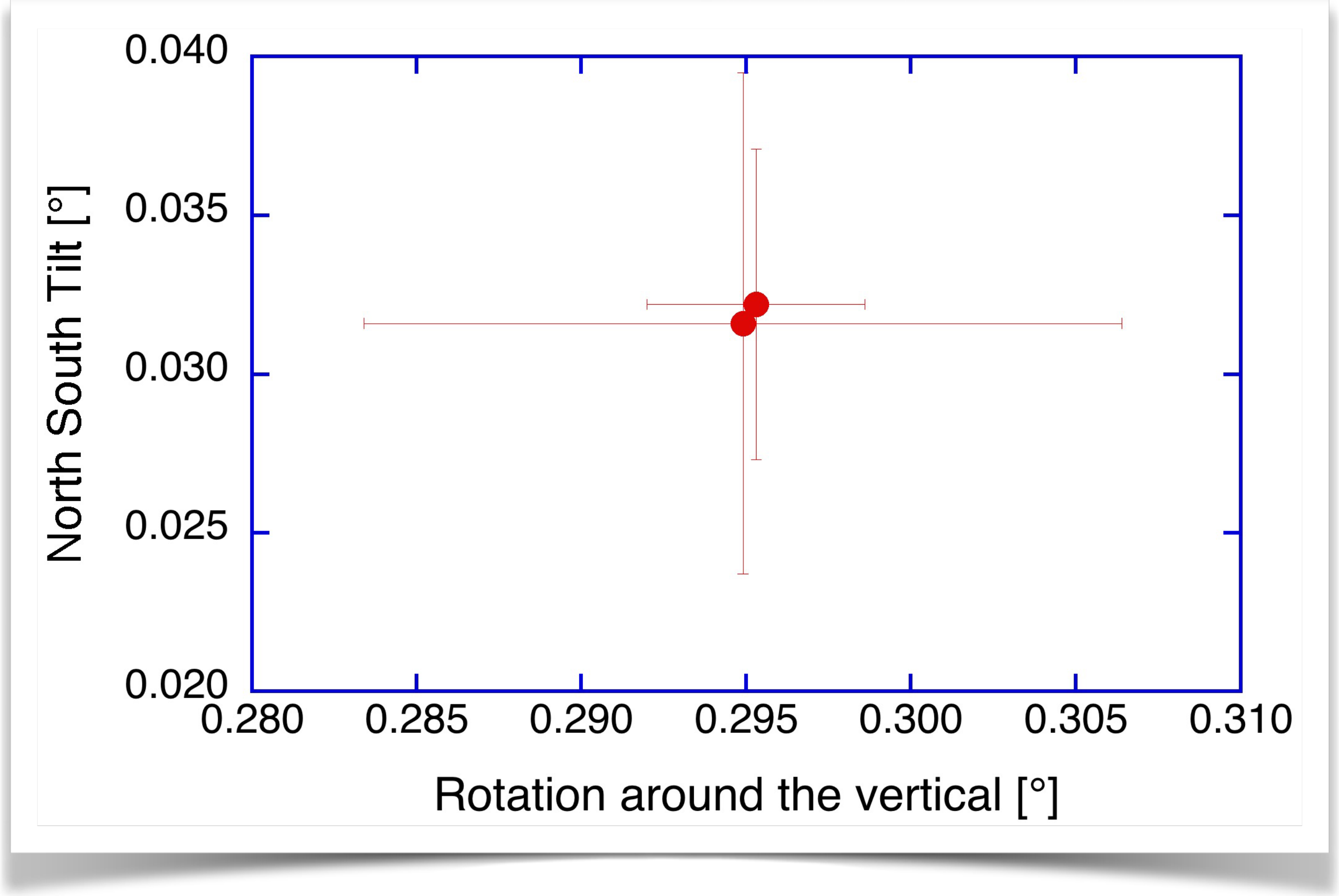
Hand, E. (2017): Lord of the rings; Science; Vol. 356; Issue 6335; pp. 236-238; doi: 10.1126/science.356.6335.236





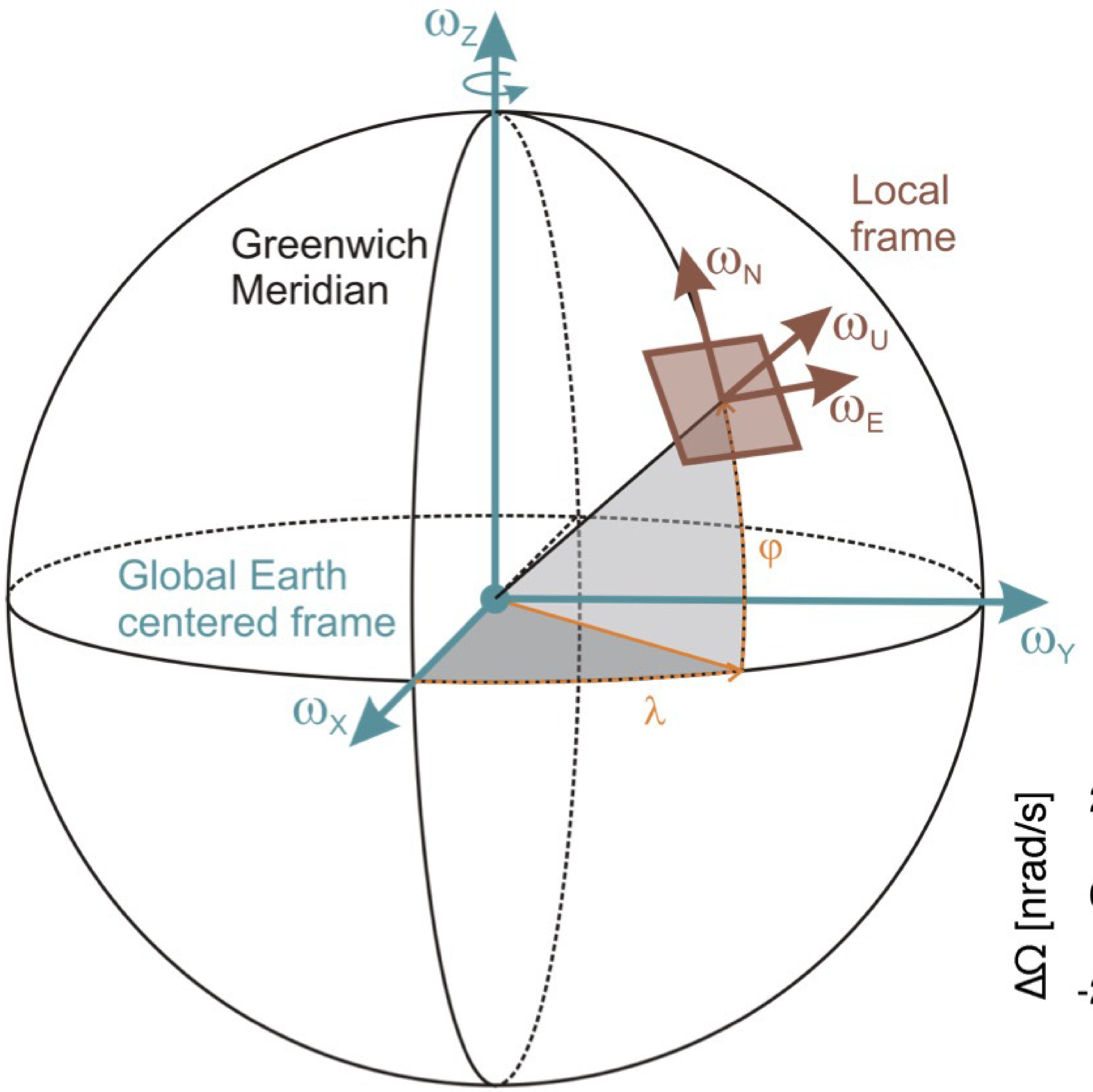


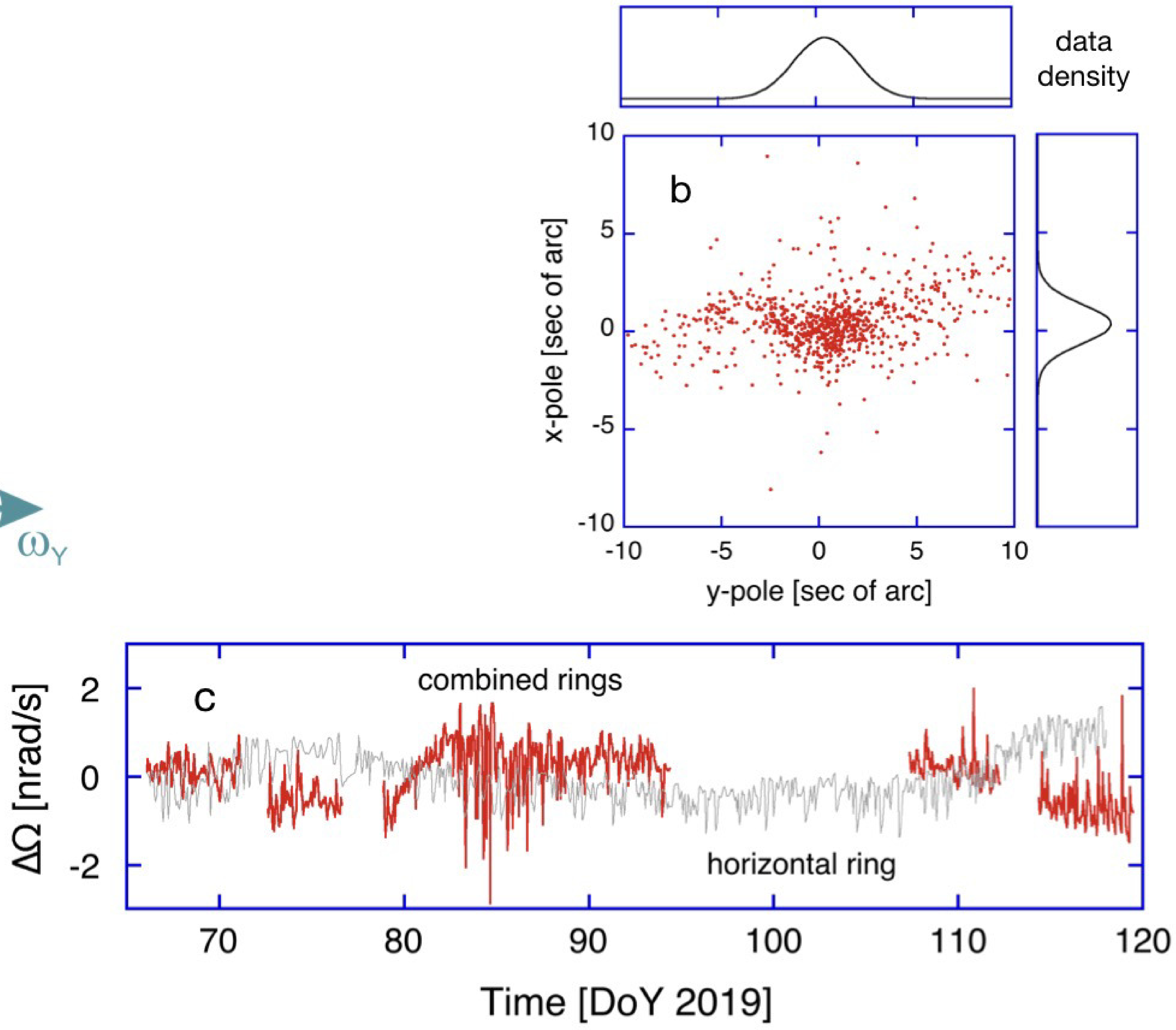




- Top: Reduced stability from the stainless steel structure expressed by Allan Deviation
- Bottom: The trend shows a drift from building setting in the early days of ROMY
- Left: Misalignment of ROMY with respect to North and the local horizontal

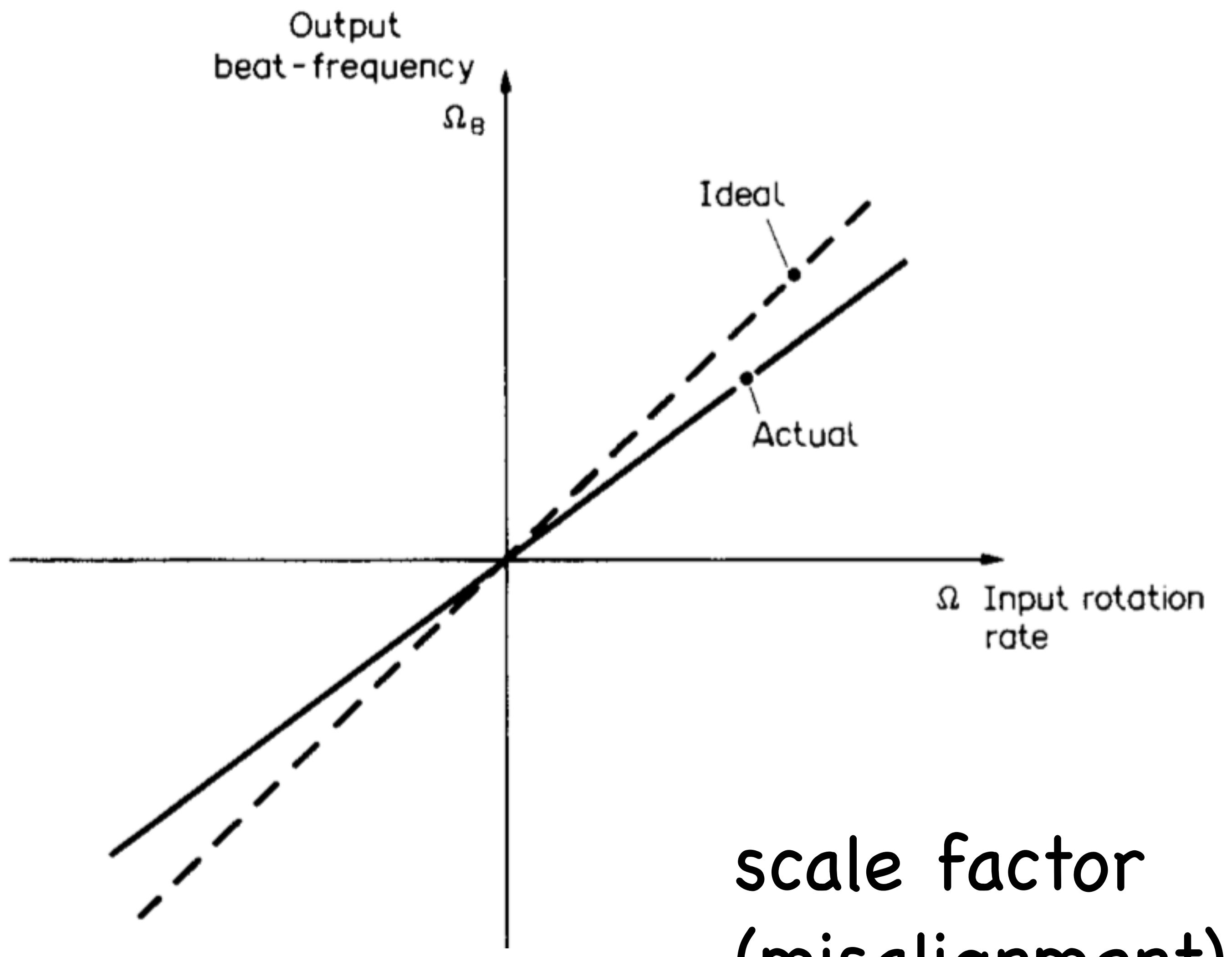
Polar motion derived by the ROMY array...



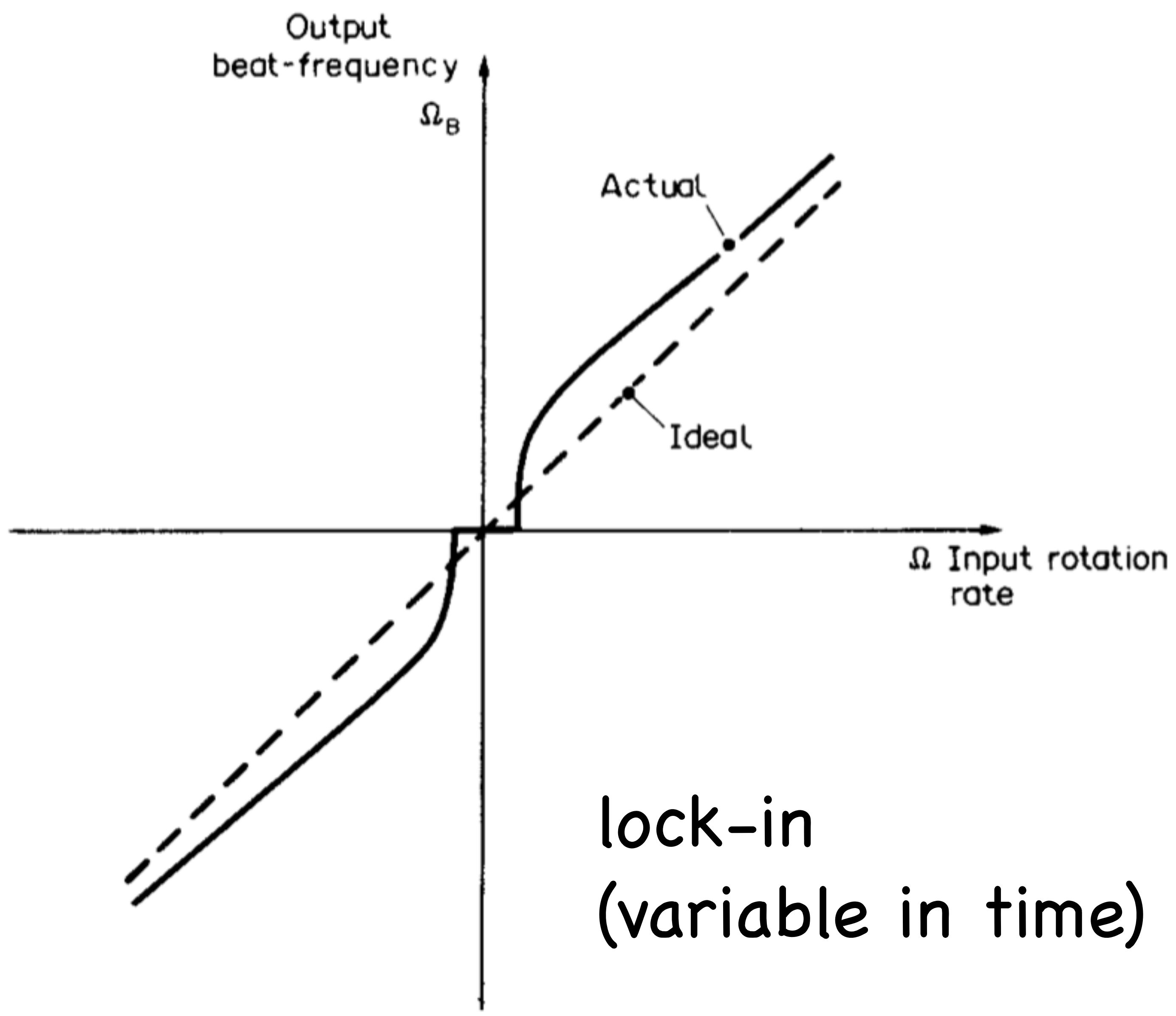


...and the instantaneous rotation rate of the Earth (ΔUT1)

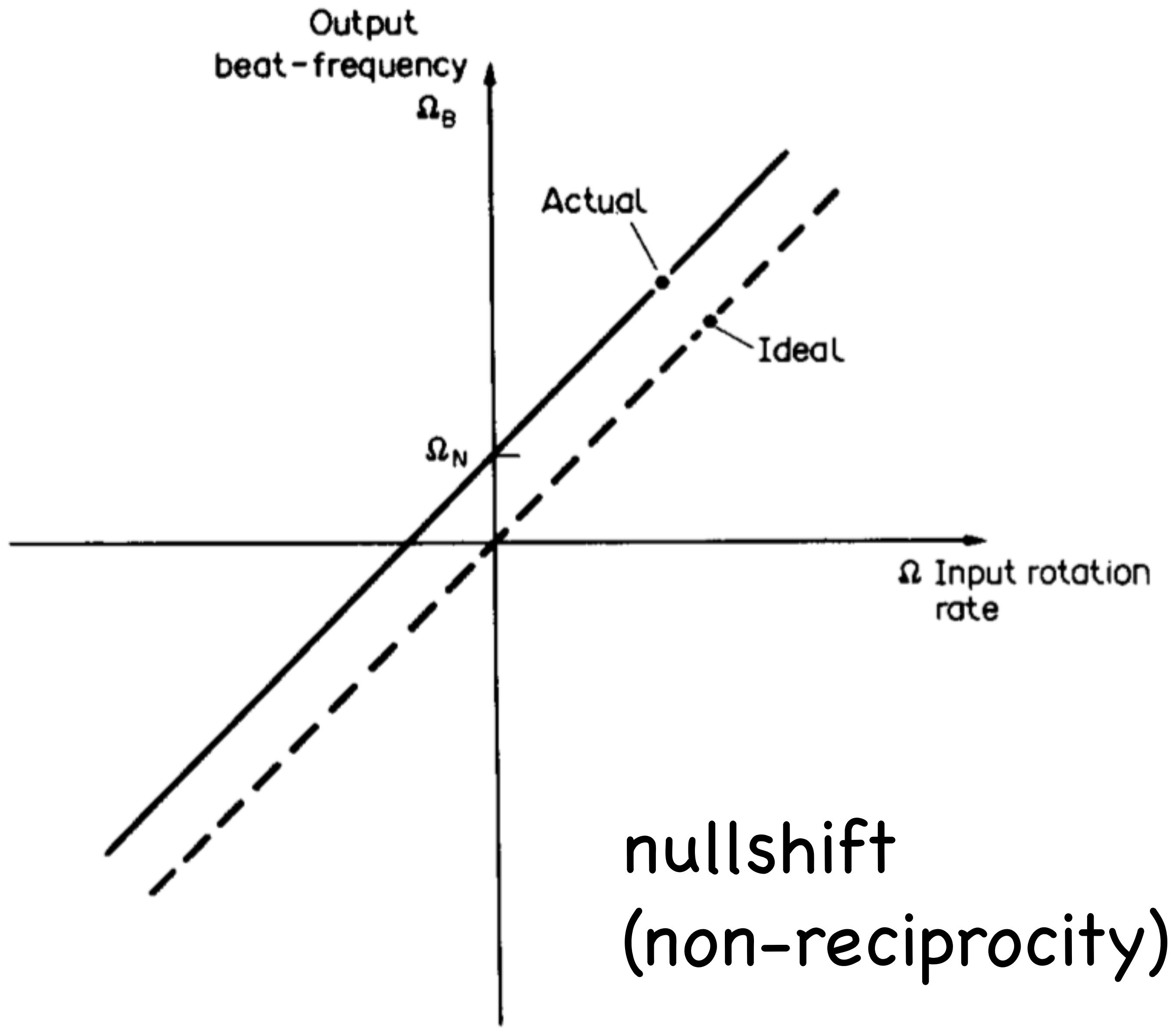
The literature usually presents precision!
however, we need accuracy and stability and that is much more difficult



scale factor
(misalignment)



lock-in
(variable in time)



nullshift
(non-reciprocity)

The classical error sources

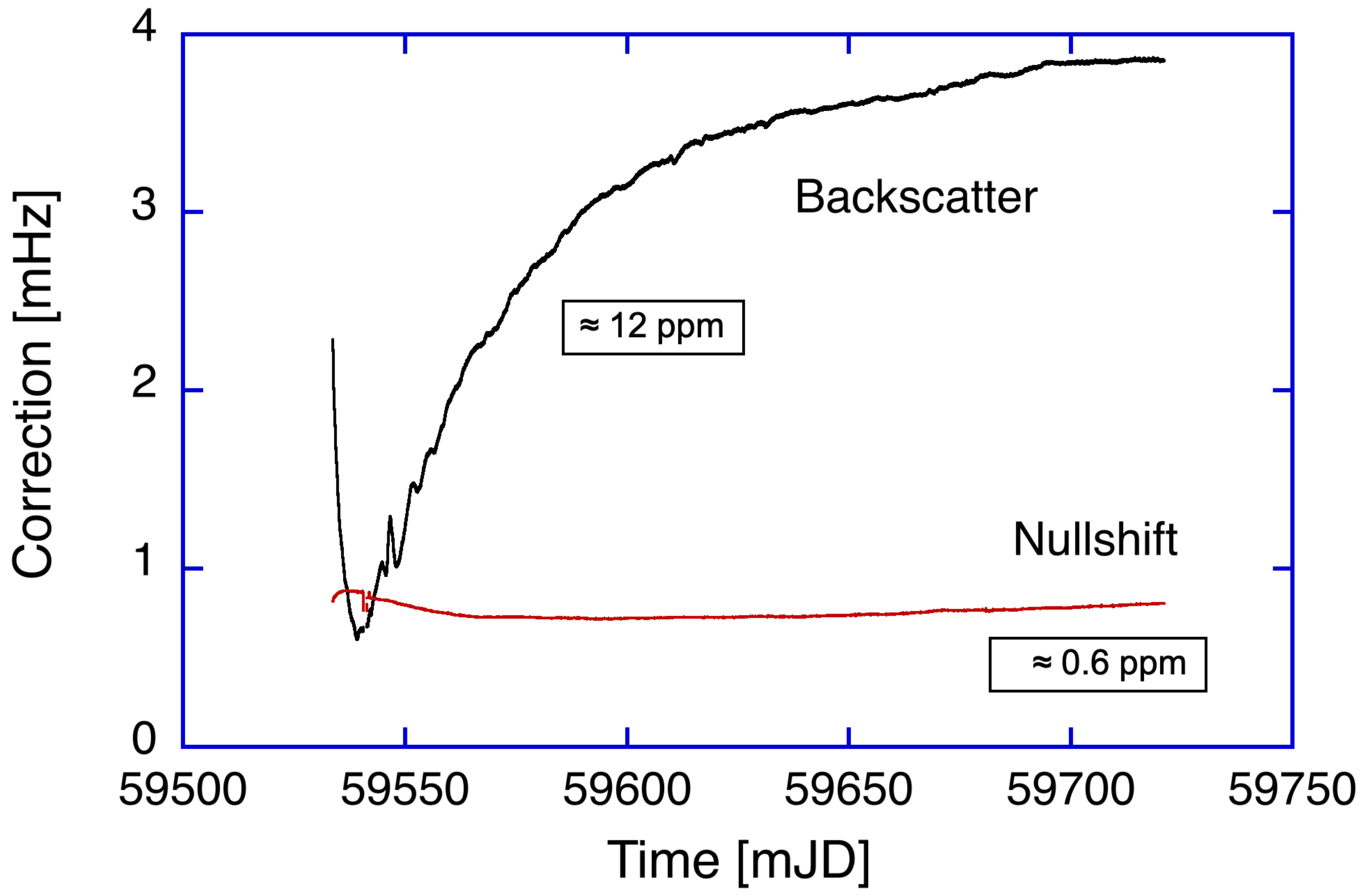
G Random Walk Estimate ≈ 1.76 nrad/h
Hi-end nav. Gyro:≈ 0.001°/h ≈ 17.5 μrad/h

Error contributions in the G ring laser gyro

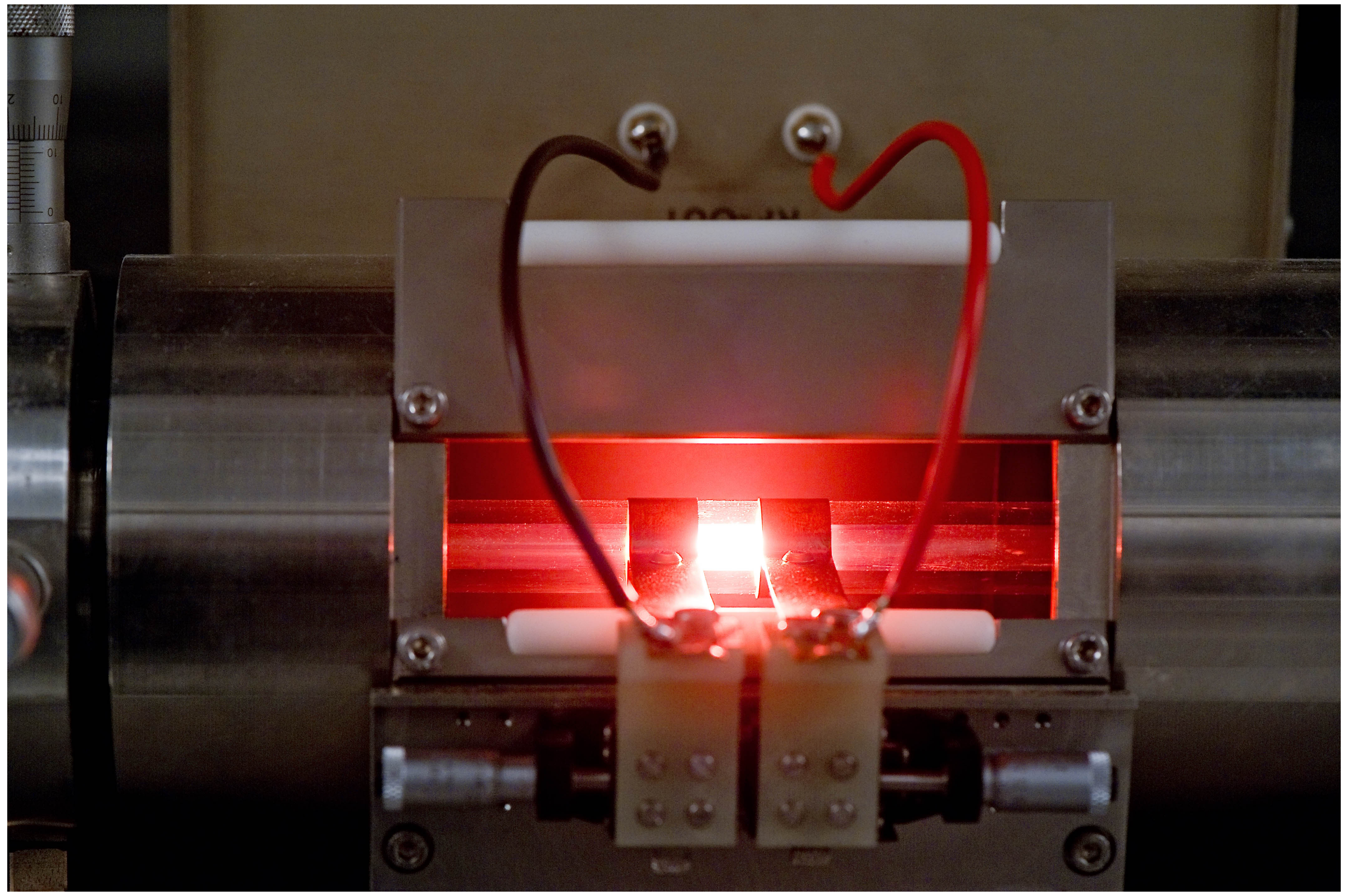
Scale Factor

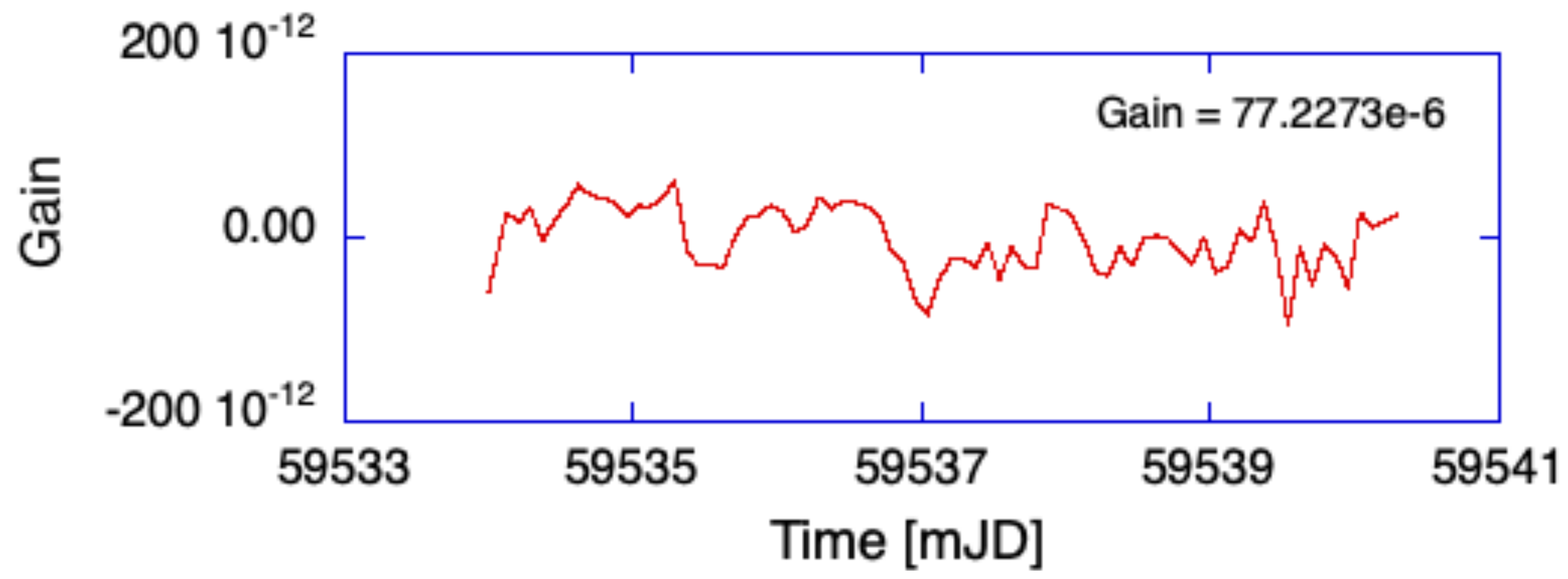
Effect	Value for G	Error (ppb)
Goos Hänchen Displacement	1 + 8.78e-8	1.6
Refractive Index	1 - 6.616e-7	1.6
Dispersion from Mirrors and Plasma	1 - 2.527e-7	0.2
Beam Abberation	1 + 4.11e-8	0.3

Laser Dynamics



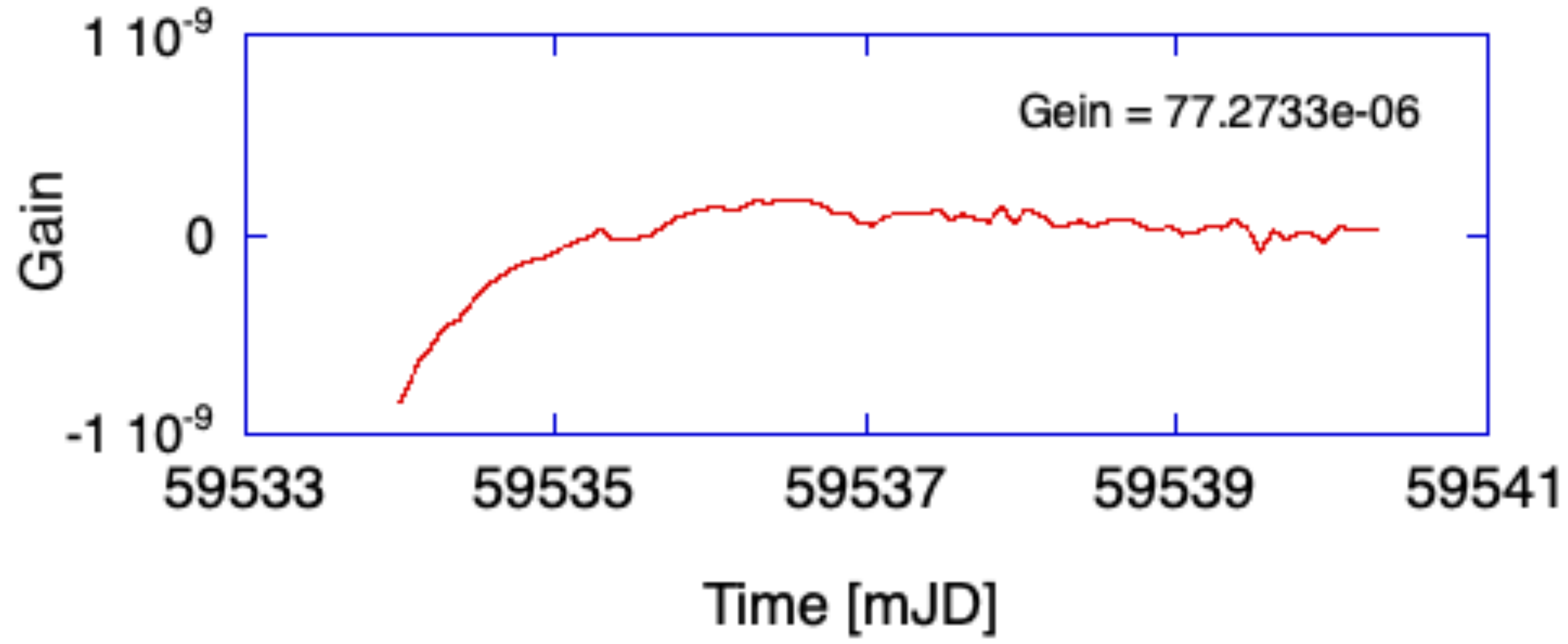
Beam intensities have never been equal (nullshift)



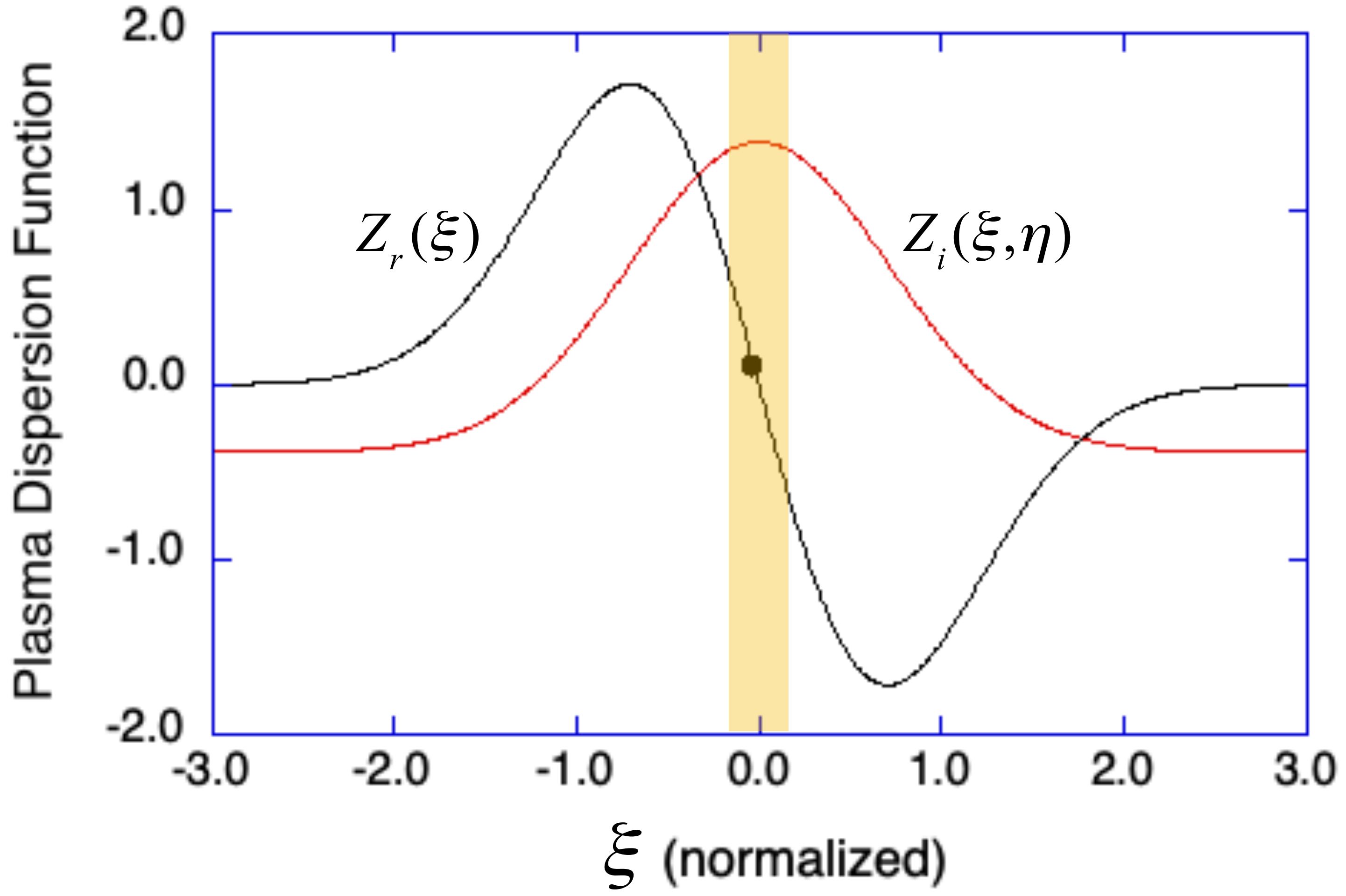


controlled

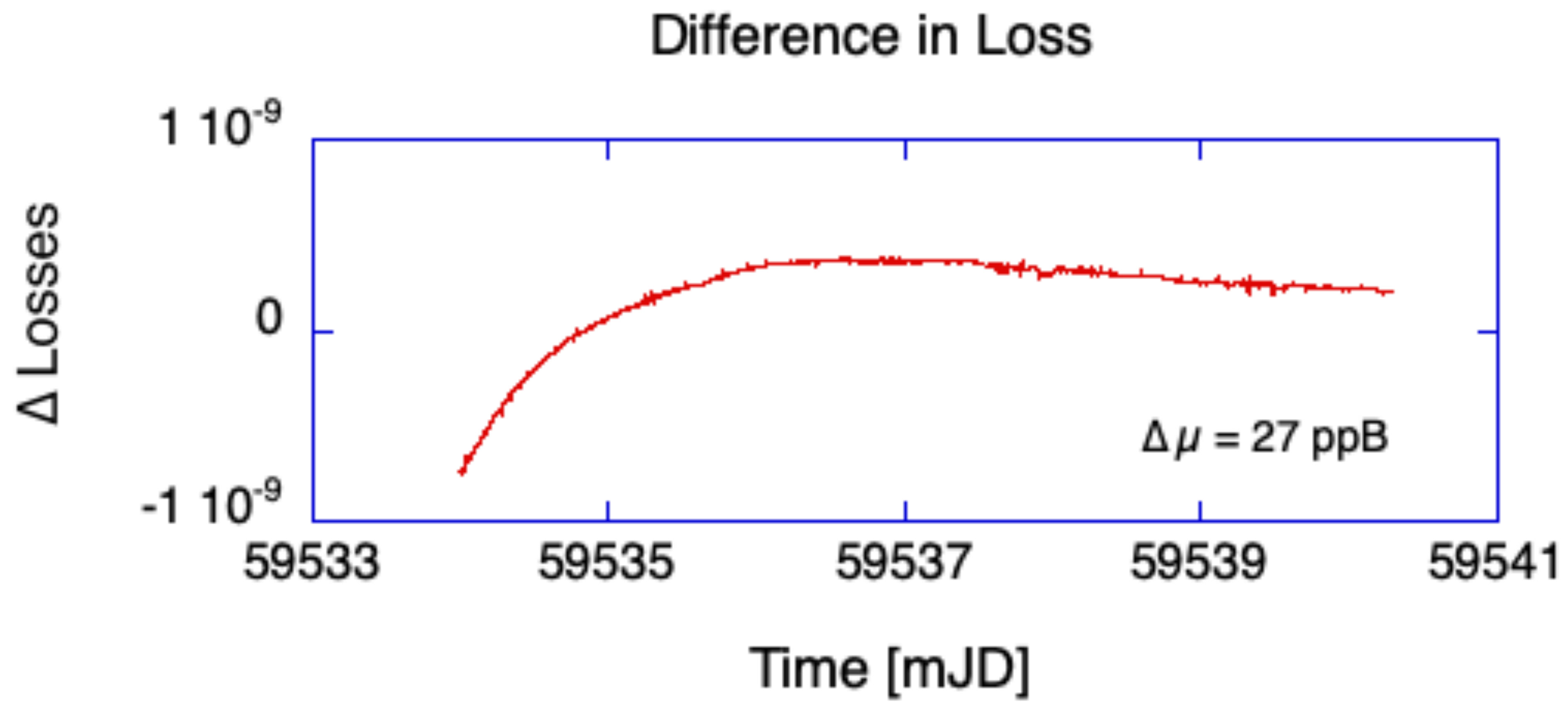
Beam



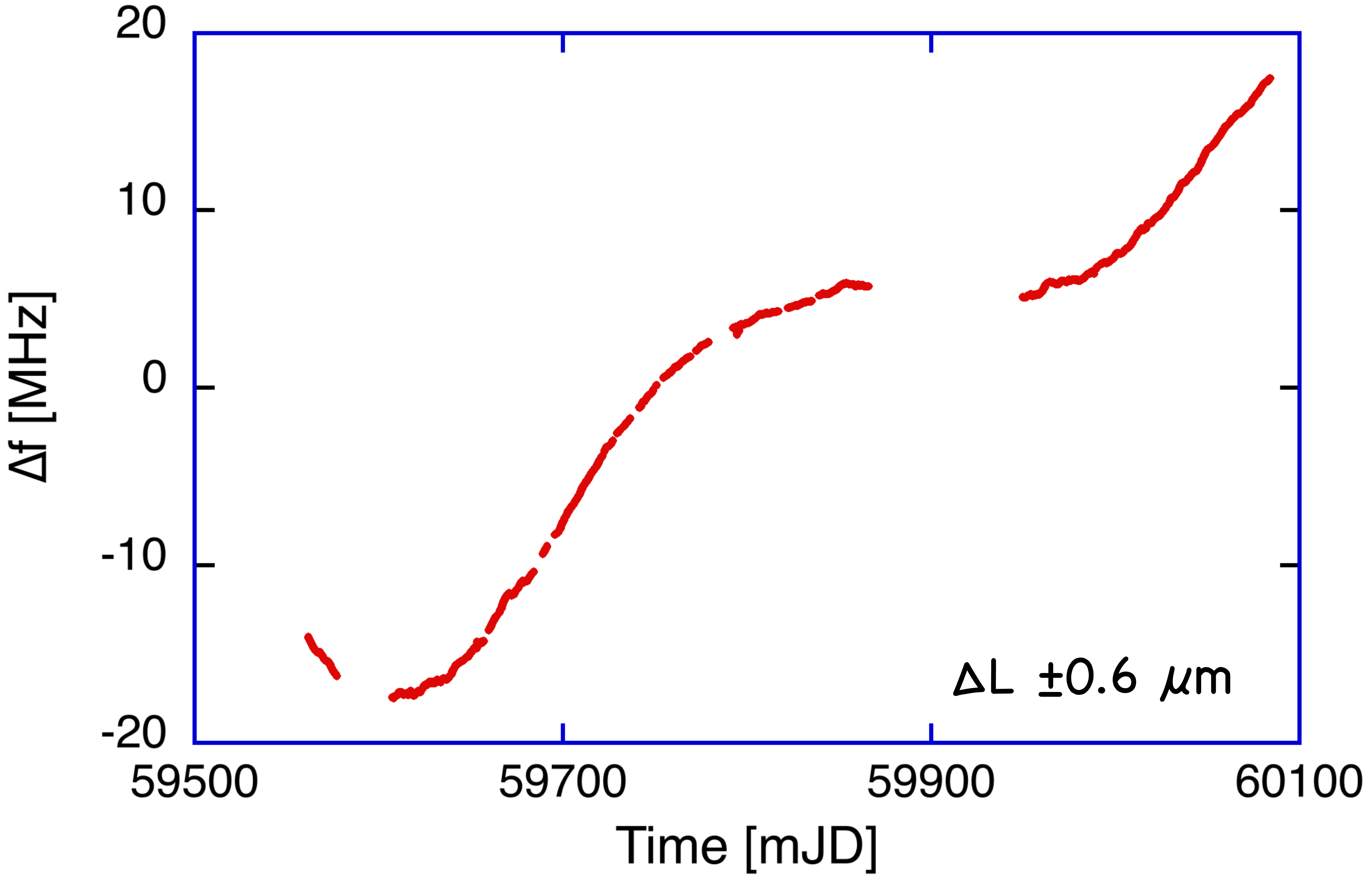
uncontrolled

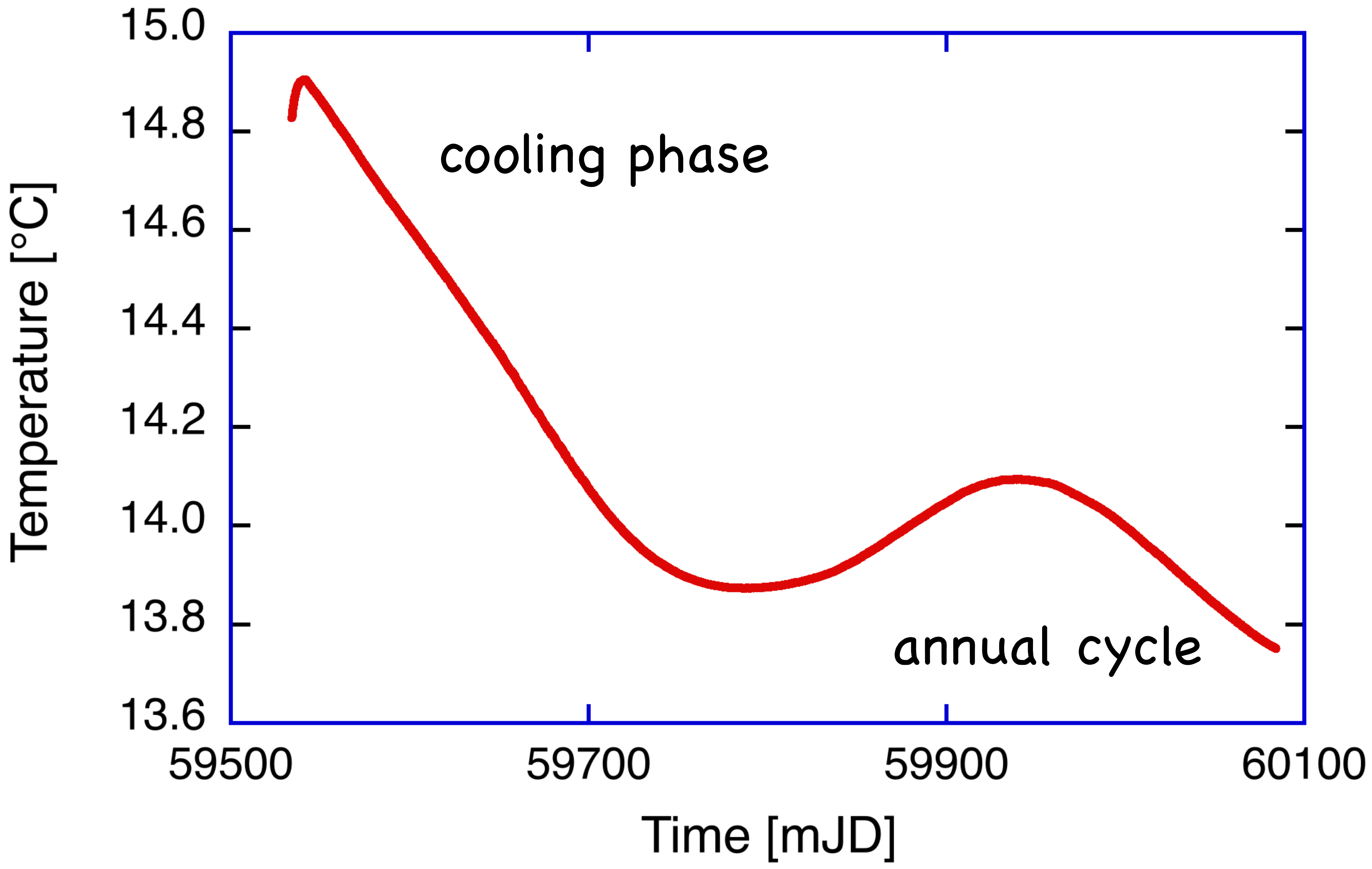


Conditions at the plasma appear to be very conservative and stable (applicable to G)



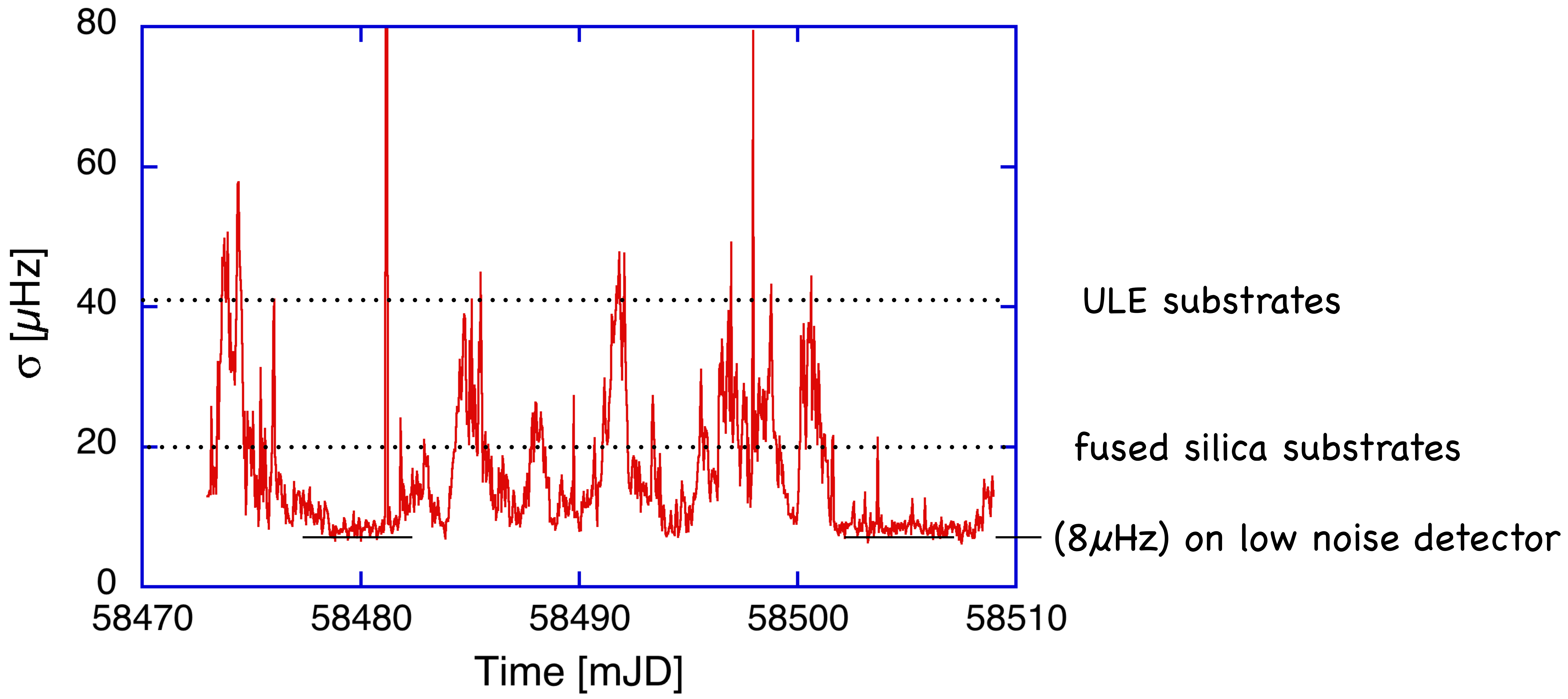
Optical Frequency and temperature in G over more than 500 days





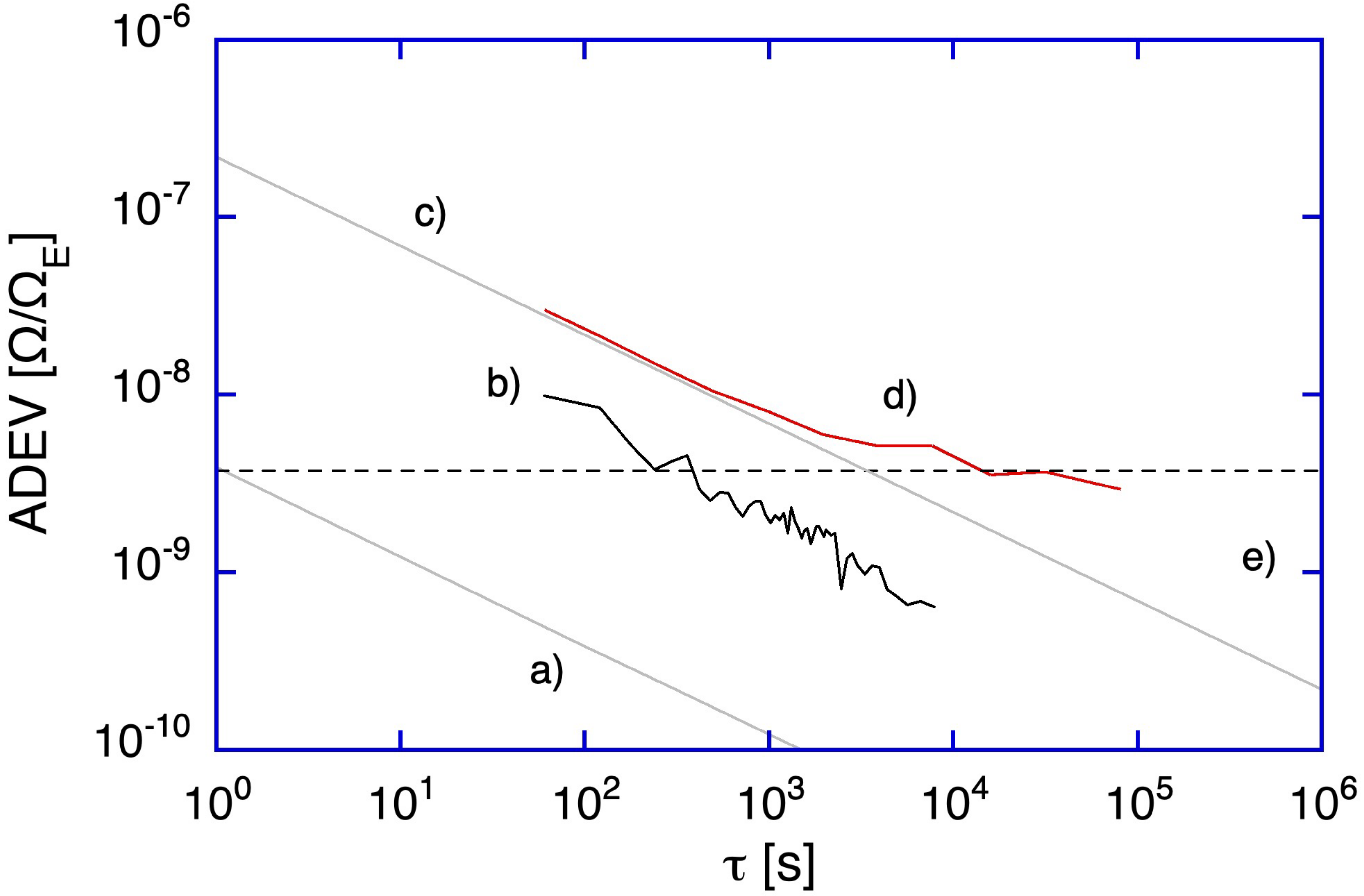
Sensor noise

(mirror substrate, electronics, frequency estimator and finally the earth)



→ Interferometer is not yet limited by micro-seismics under low noise conditions

Noise contributions on G...



a) noise floor from intensity stabilization (electronics)

b) frequency estimator noise

c) quantum noise limit of G

d) observed G performance

e) coating and cavity noise

Sensor noise

(mirror substrate, electronics, frequency estimator and finally the earth)

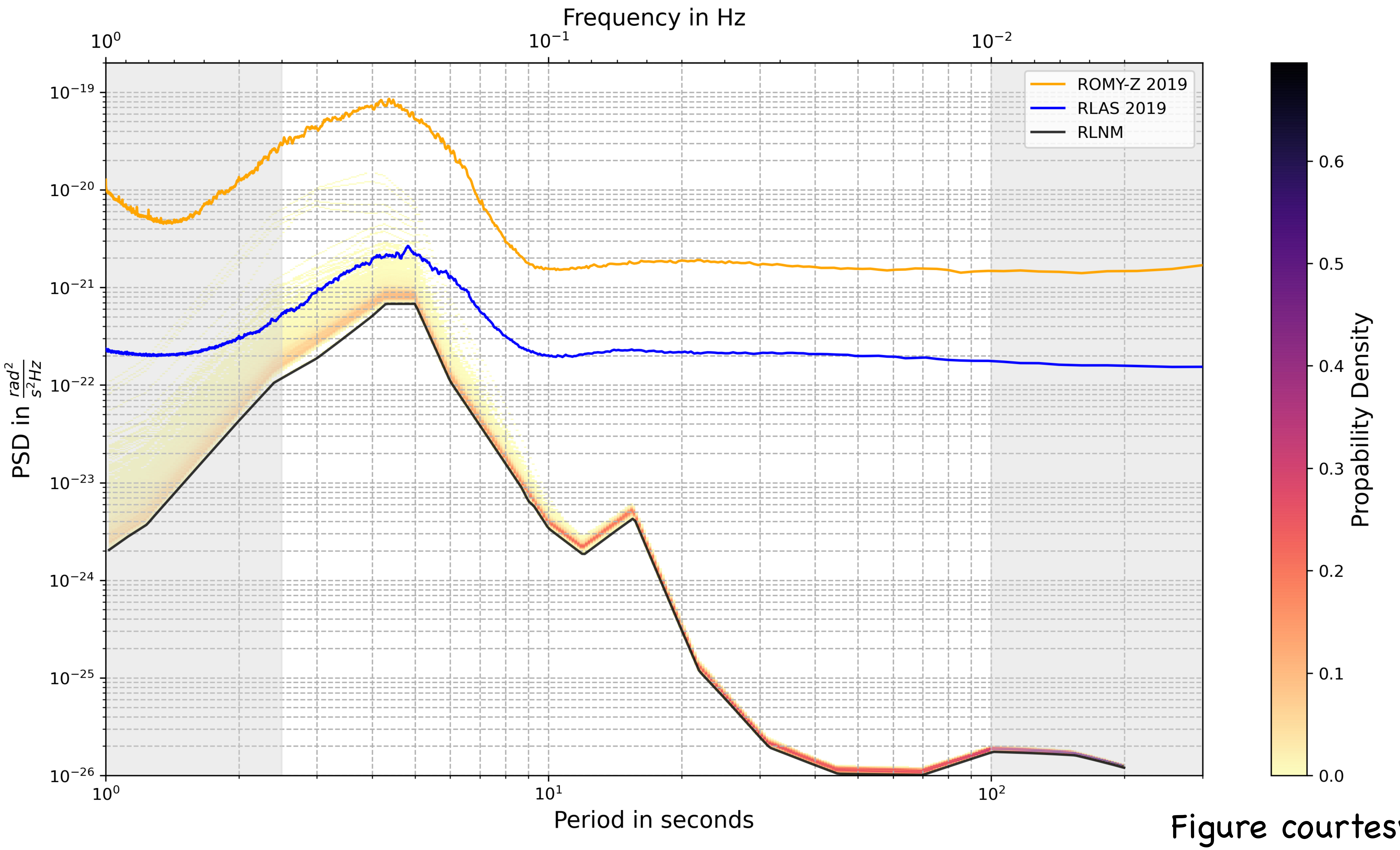
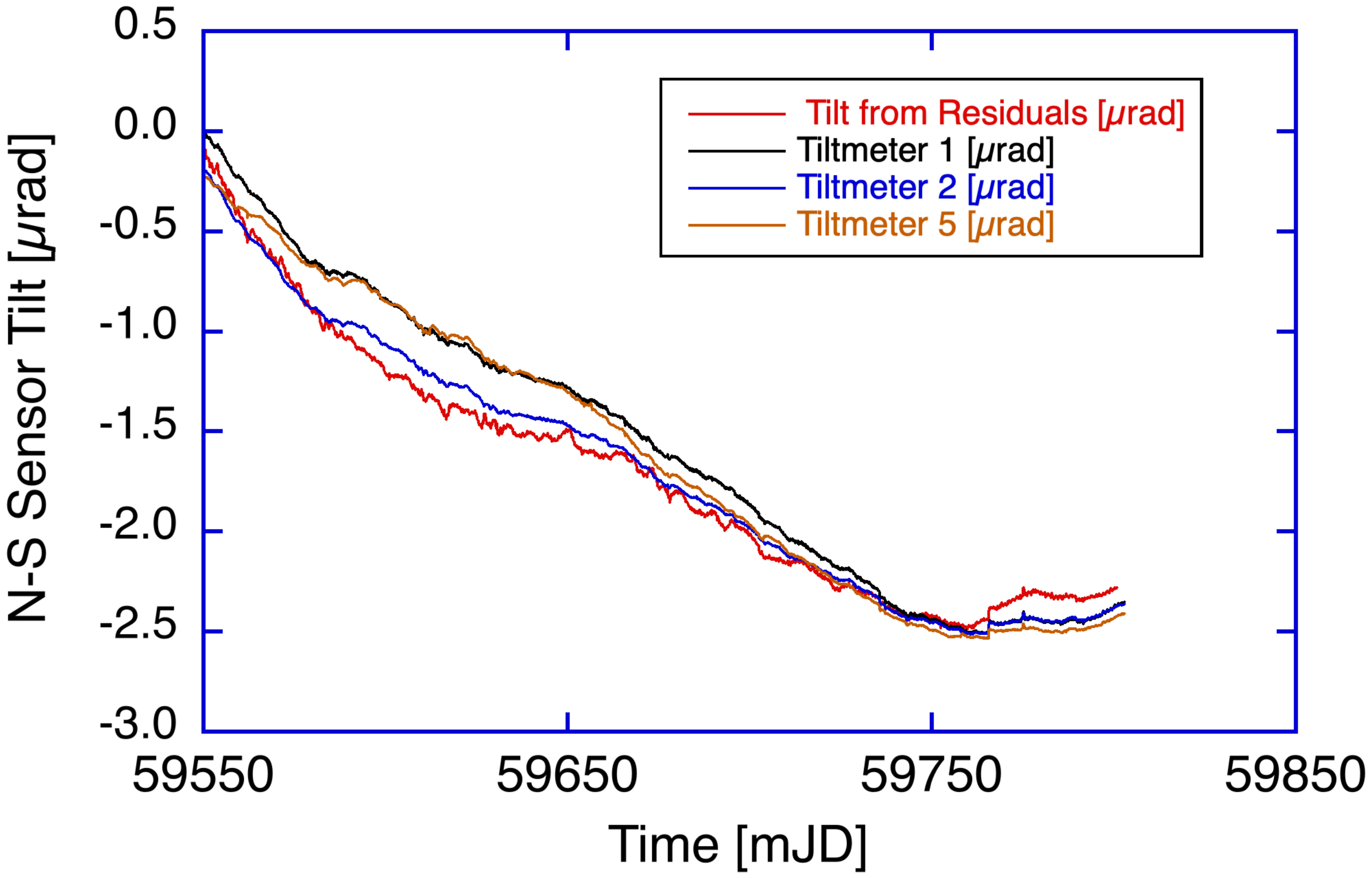


Figure courtesy of A. Brotzer

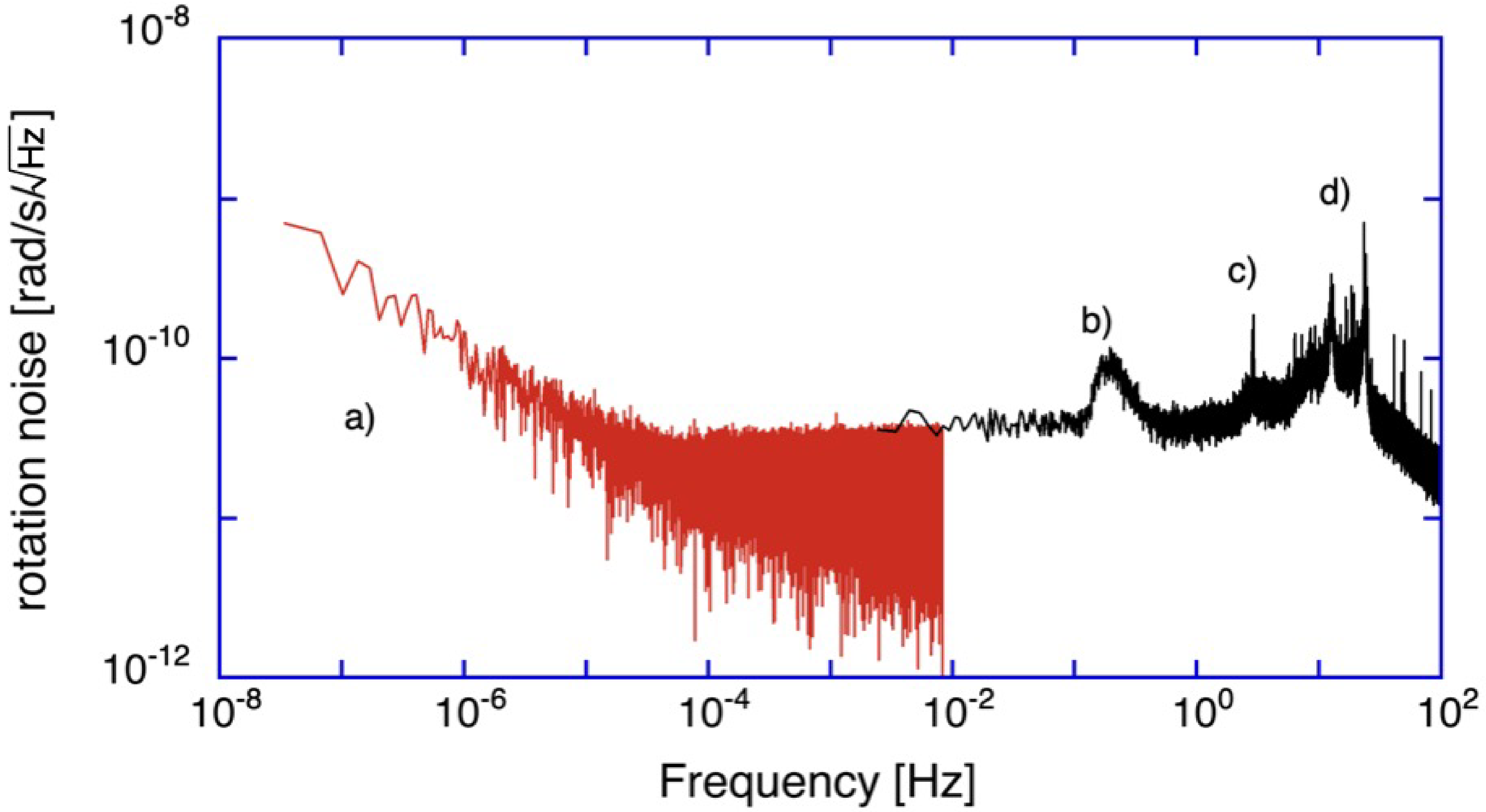
There is a smoking gun in the tiltmeters



- The tiltmeters show significant systematics and their contribution is considerably large
- 3 nrad in tilt ≈ correspond to 1 μHz in Sagnac (note that LoD has about 5 μHz_{pp})
- The apparent drift between the tiltmeters is much larger than the respective deviation in Sagnac
- If we convert the trend of the ring laser residuals to tilt, we get the red curve
- There is a little more "small scale" variability in the red curve compared to the various tiltmeters

Power spectral density over nearly 10 decades

1 year of data   3 days of data

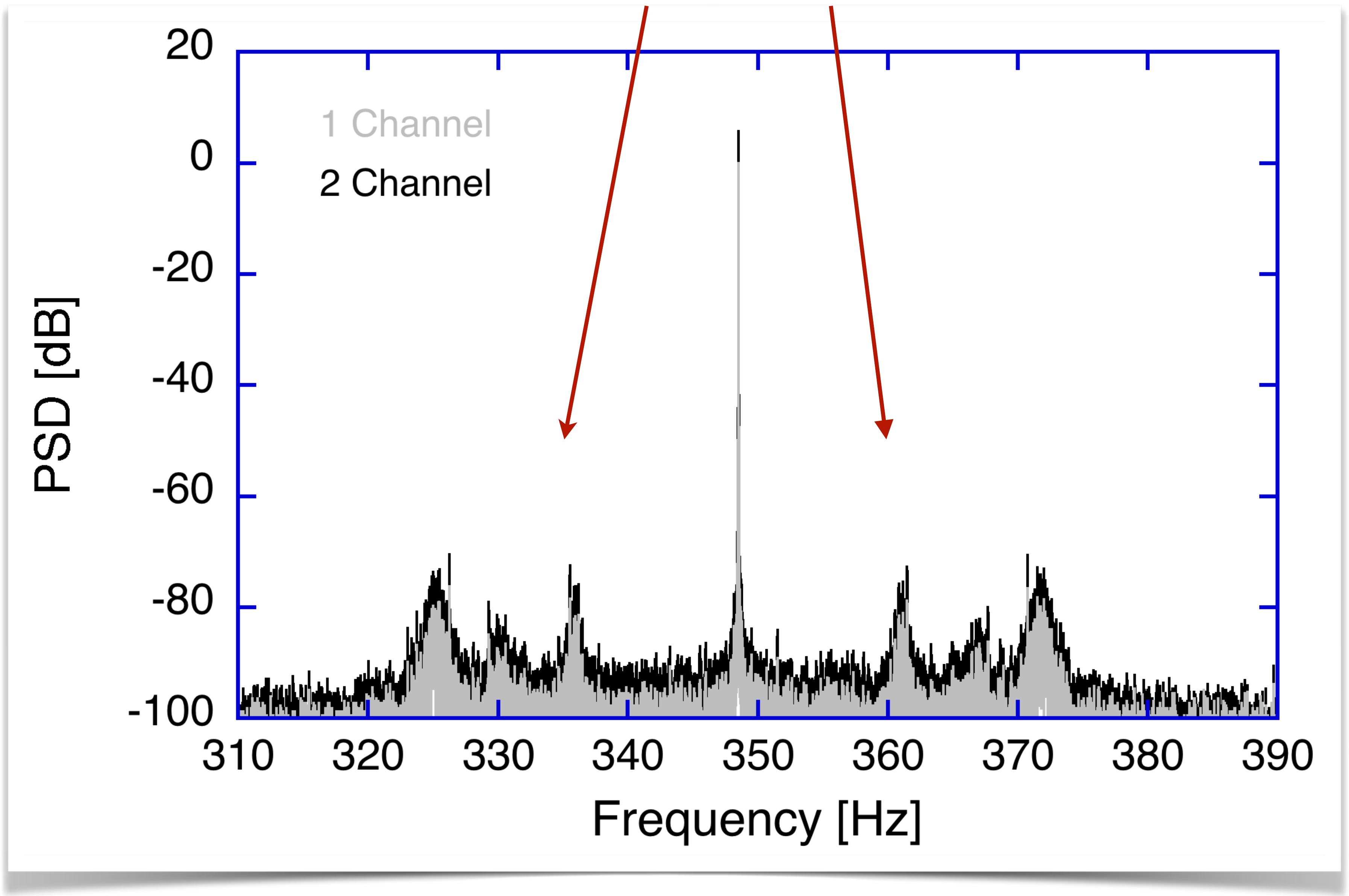


a) 1/f noise boundary

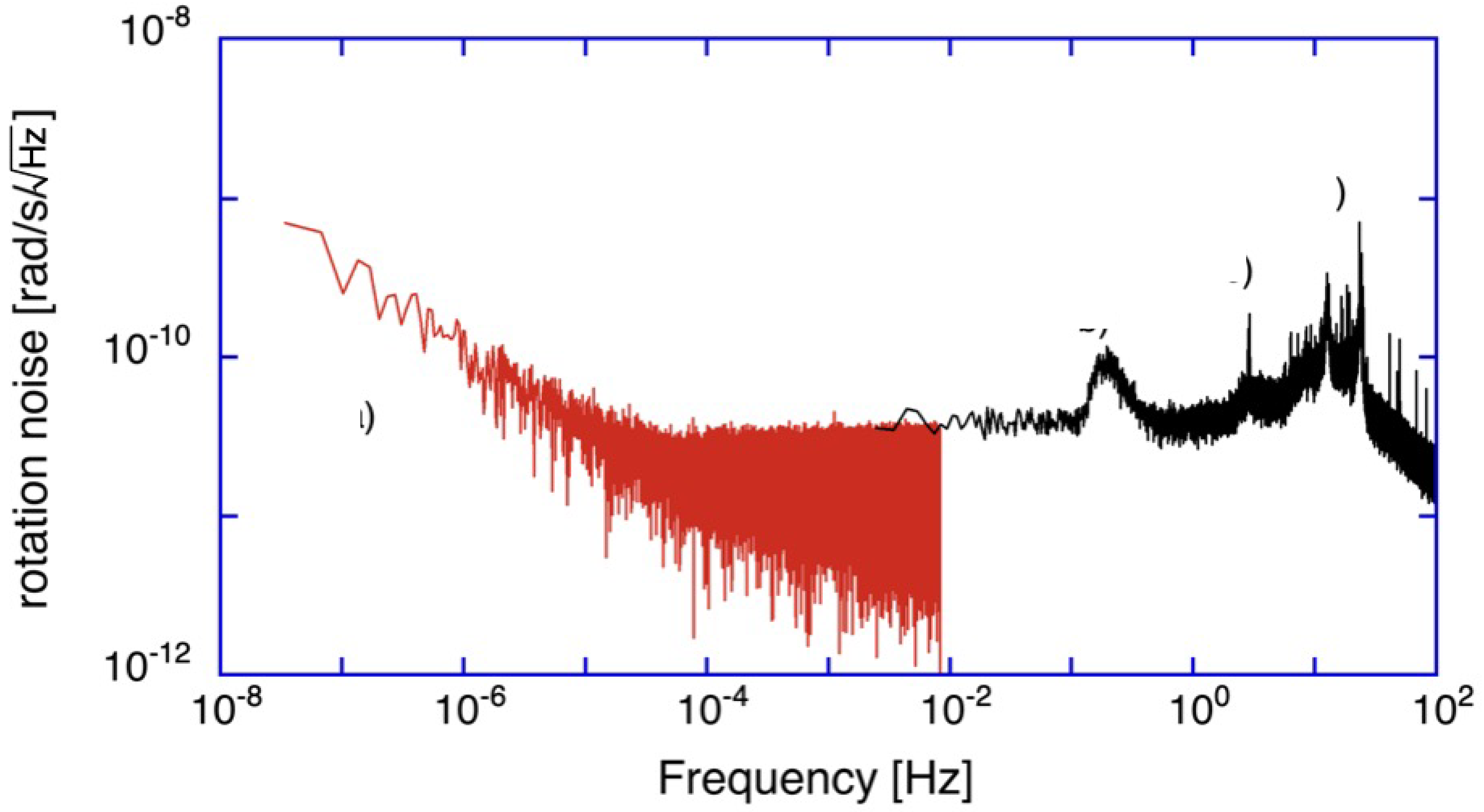
b) secondary microseismic band

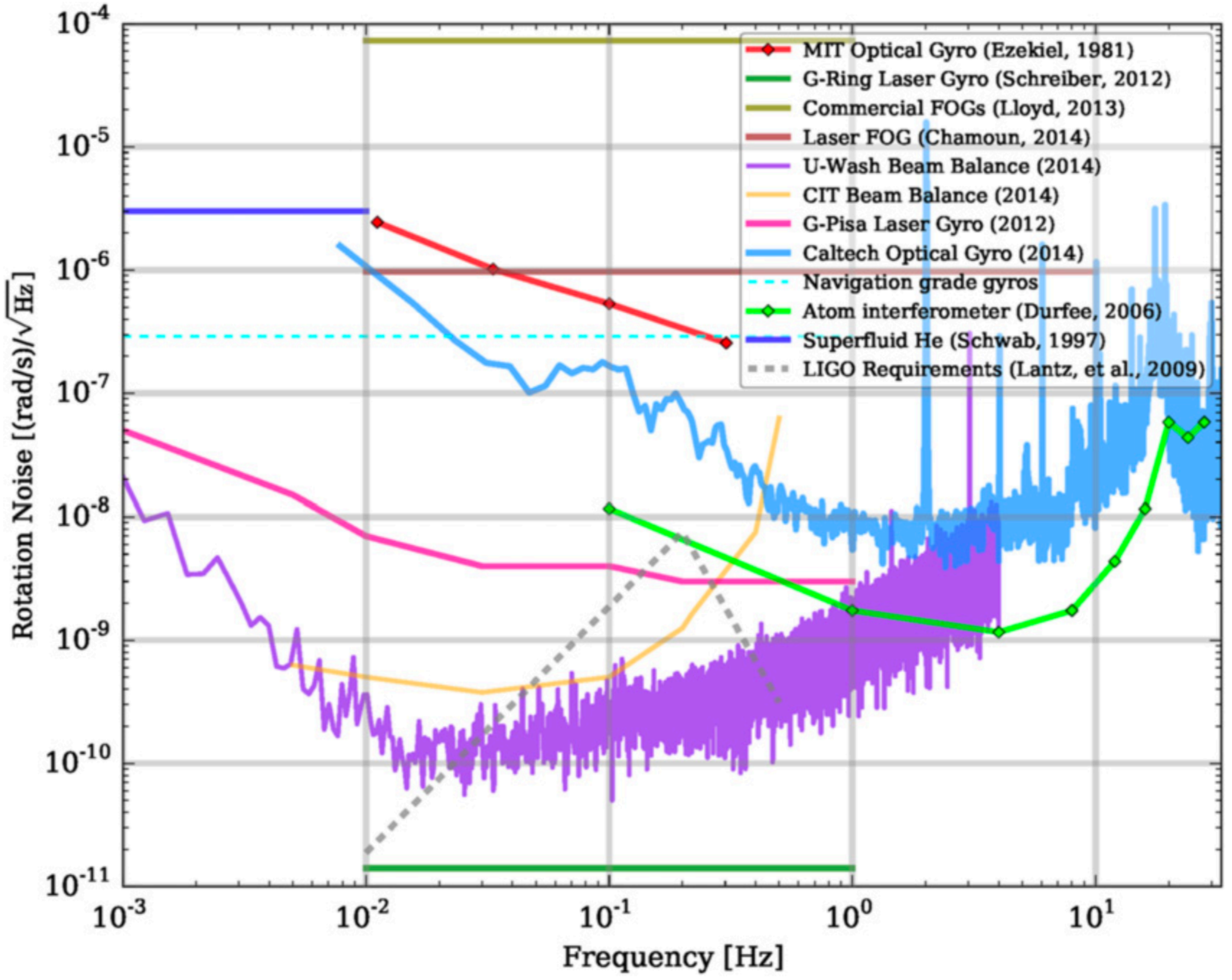
c) intensity feedback loop

d) plasma generated modulation (subharmonics from mains)



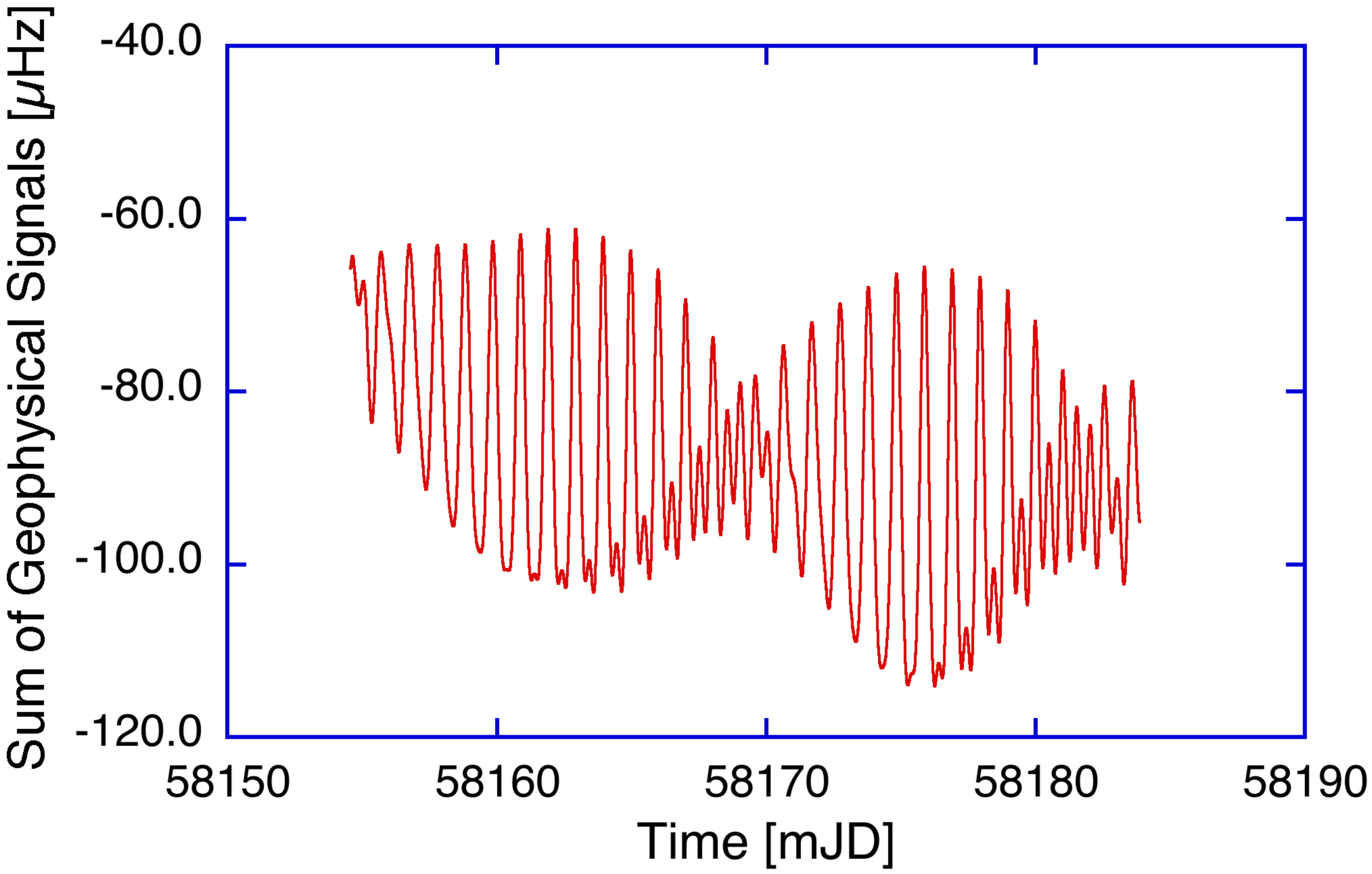
World map of rotation sensing



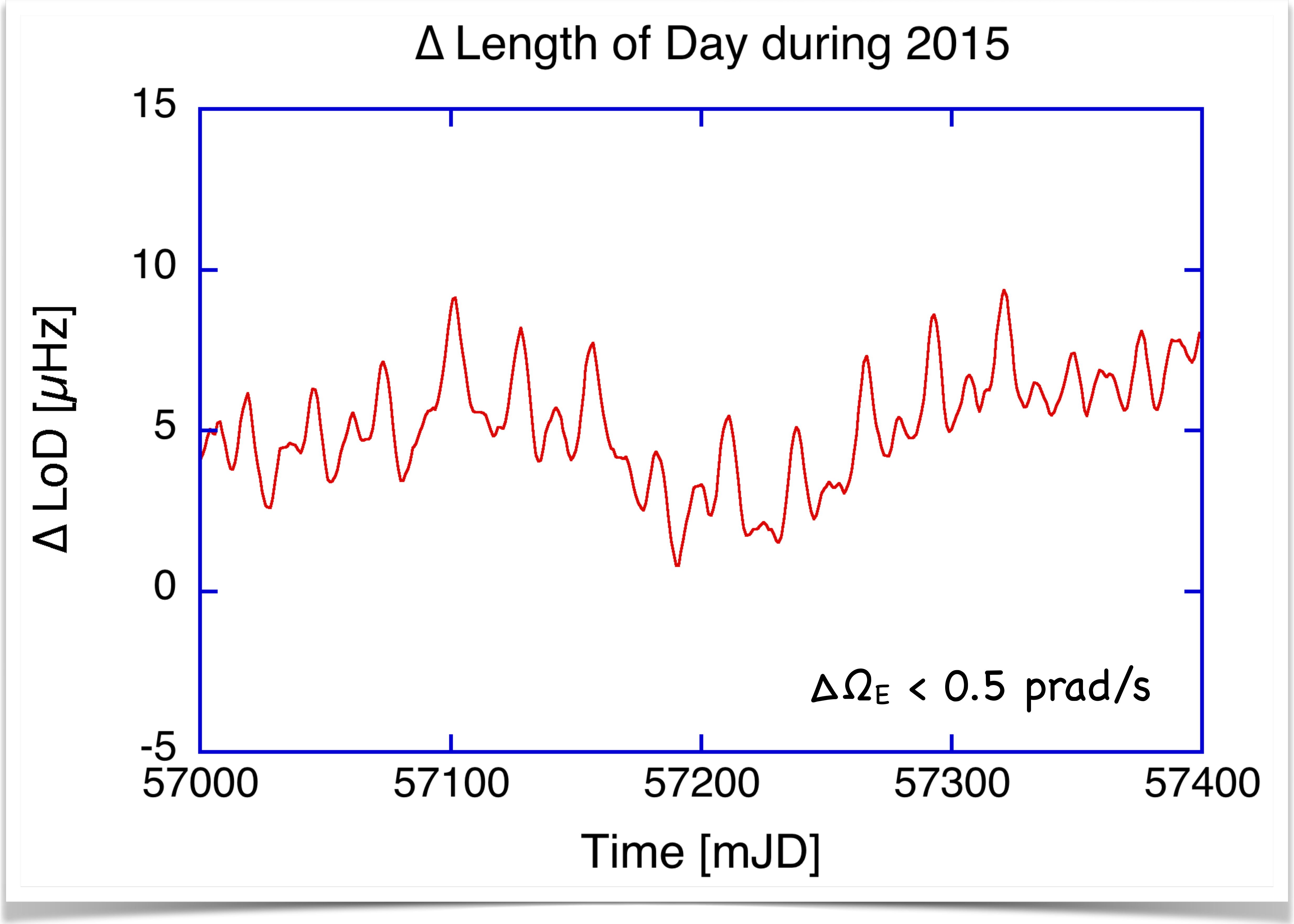


Korth et al. (2016) Classical and Quantum Gravity, 33(3)

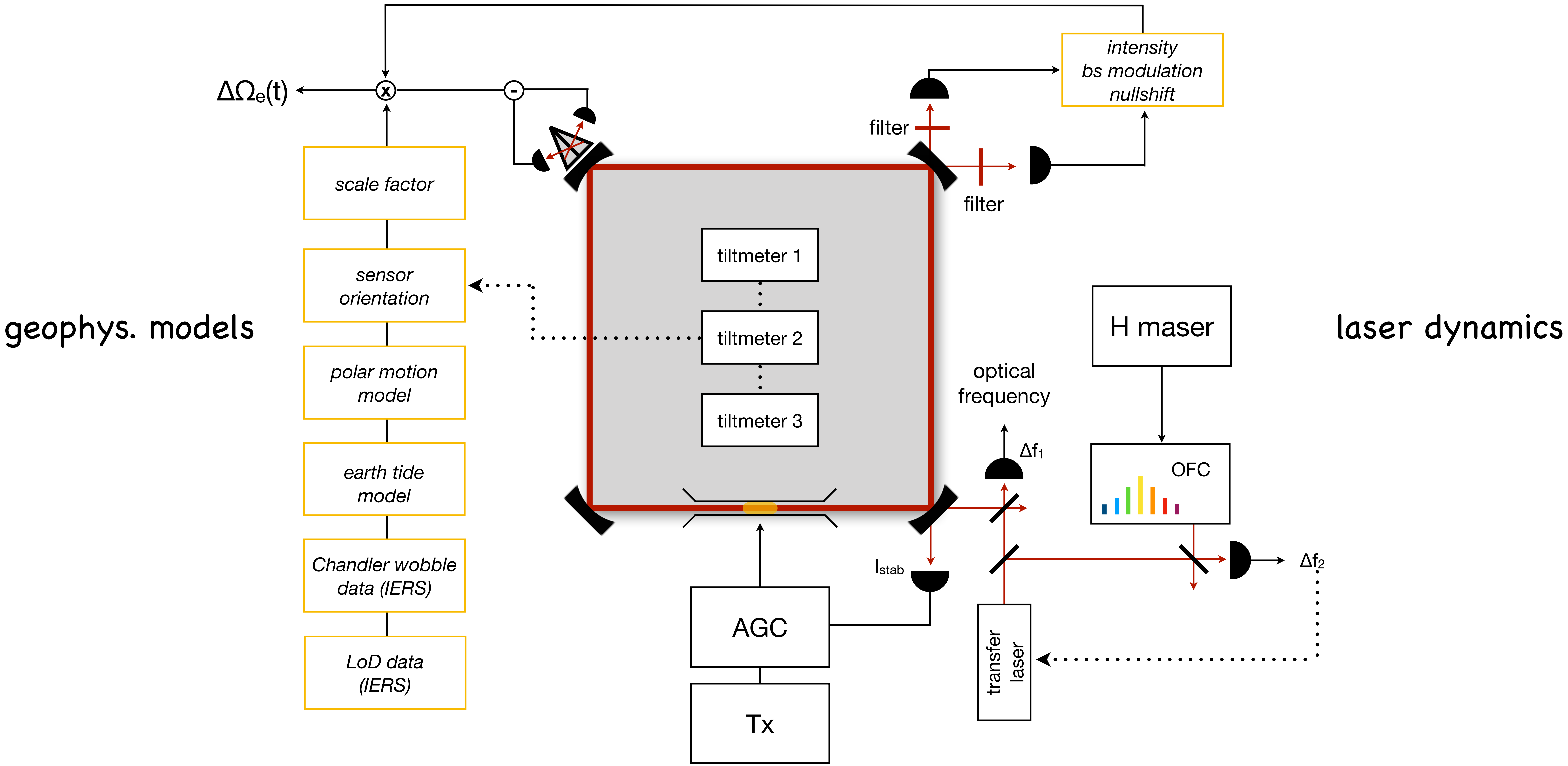
Orientation related observables for G



Ω related observable for G from mass transport phenomena



For comparison: Ω_{rel} at the level of 1% requires ≈ 35 nHz resolution and above all accuracy

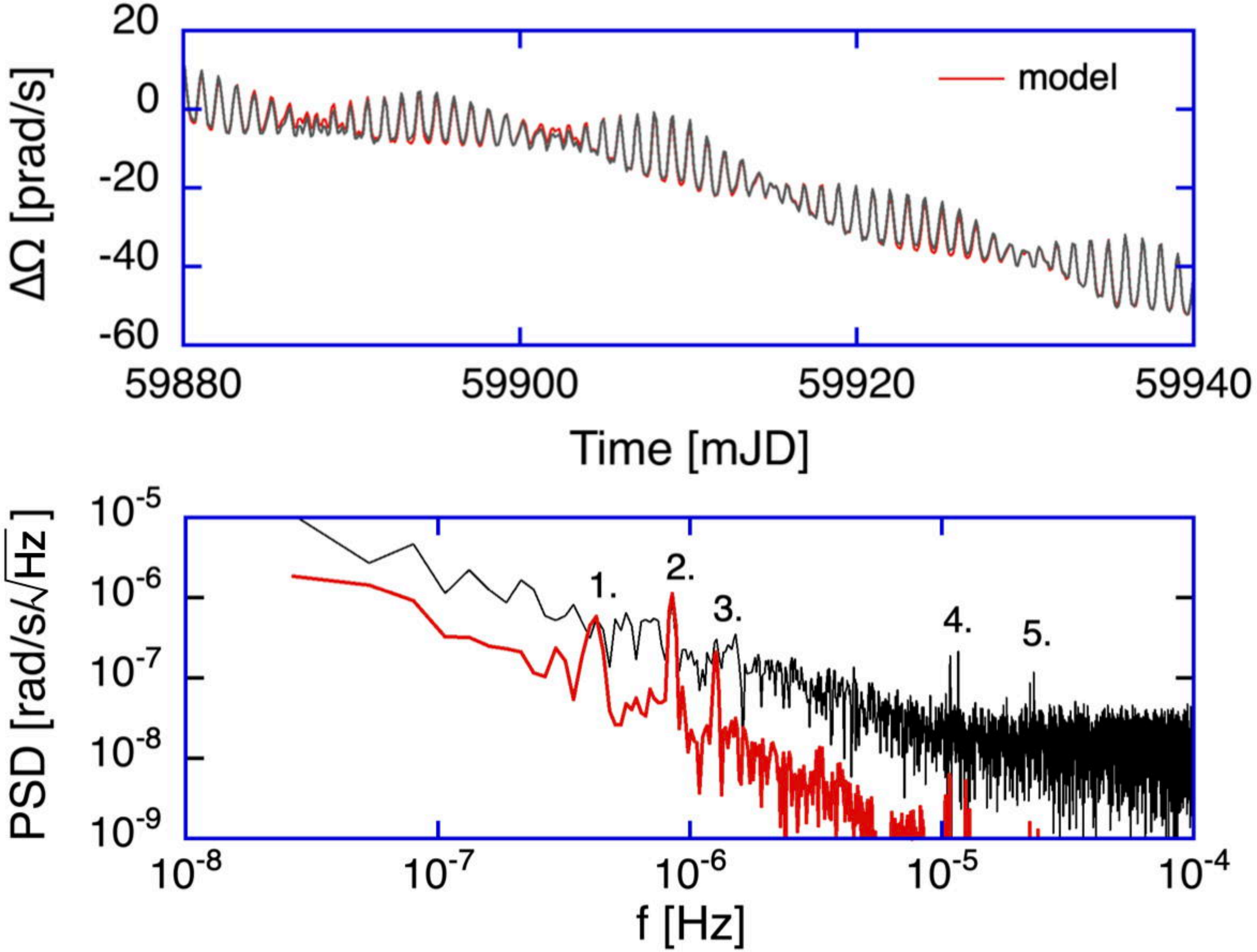


ΔΩe(t)
x
-
intensity
bs modulation
nullshift
filter
filter
scale factor
sensor
orientation
geophys. models
polar motion
model
earth tide
model
Chandler wobble
data (IERS)
LoD data
(IERS)
tiltmeter 1
tiltmeter 2
tiltmeter 3
H maser
laser dynamics
optical
frequency
Δf1
OFC
Istab
Δf2
AGC
Tx
transfer
laser

The extended ring laser equation for a single component gyro

$$\Delta\Omega_e(t) = \frac{(\Delta f(t) - \Delta f_{BS}(t) - \Delta f_{NS}(t)) - \bar{\Delta f}}{S' \sin(\theta - \Delta\theta_{tide}(t) - \Delta\theta_{pole}(t) - \Delta\theta_{IERS}(t) - \Delta\theta_{tilt}(t) + \Delta\theta_{att}(t))}$$

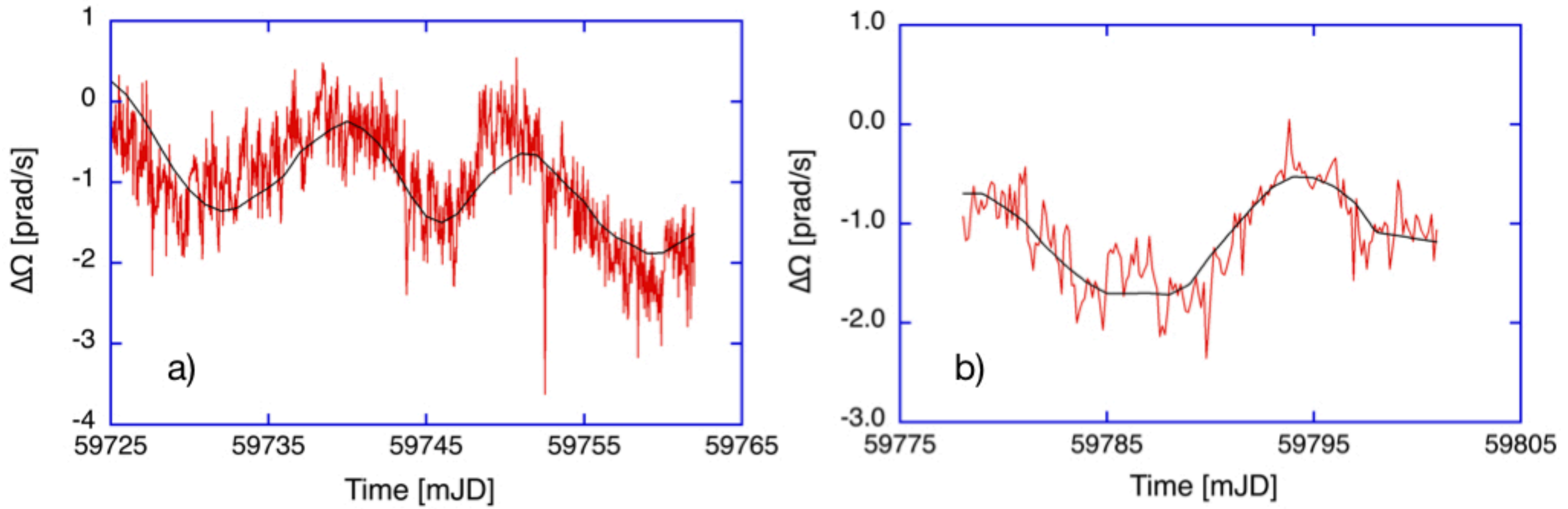
- measured beat note
- backscatter correction
- nullshift bias correction
- mean value of Sagnac frequency
- scale factor (corrected)
- latitude
- earth tides
- polar motion
- Chandler and Annual wobble
- local tilt
- mass attraction on tiltmeter



Excellent agreement between model and measurements

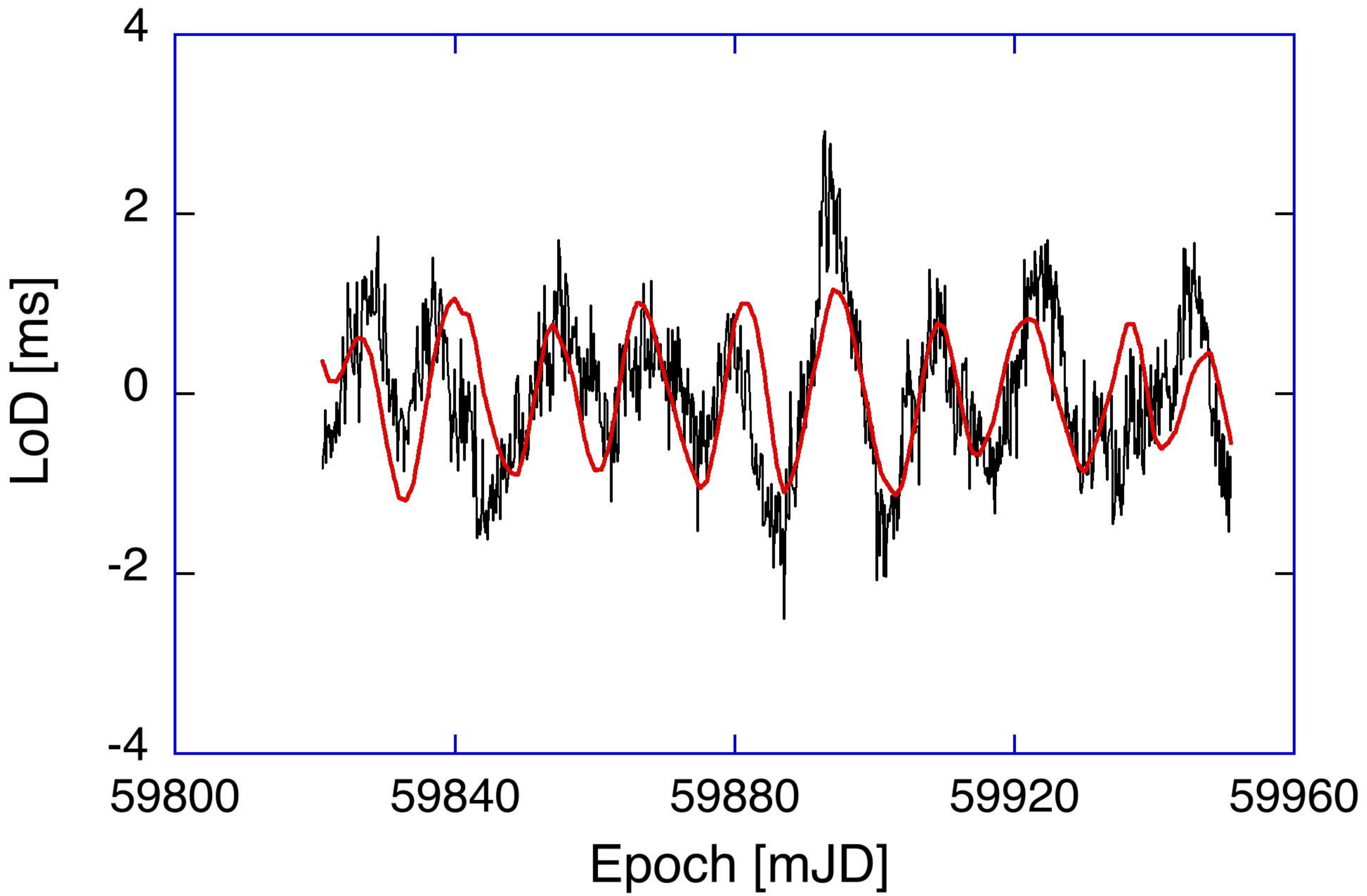
1. LoD component (28 days)
2. LoD component (14 days)
3. LoD component (9 days)
4. Ocean loading
5. Atmospheric tides

Global versus local Measurements (time domain)



- Measurements 1 data point per hour (a) and 1 data point in 3 hours (b)
- LoD signal (black) derived from IERS daily finals (one value per day)

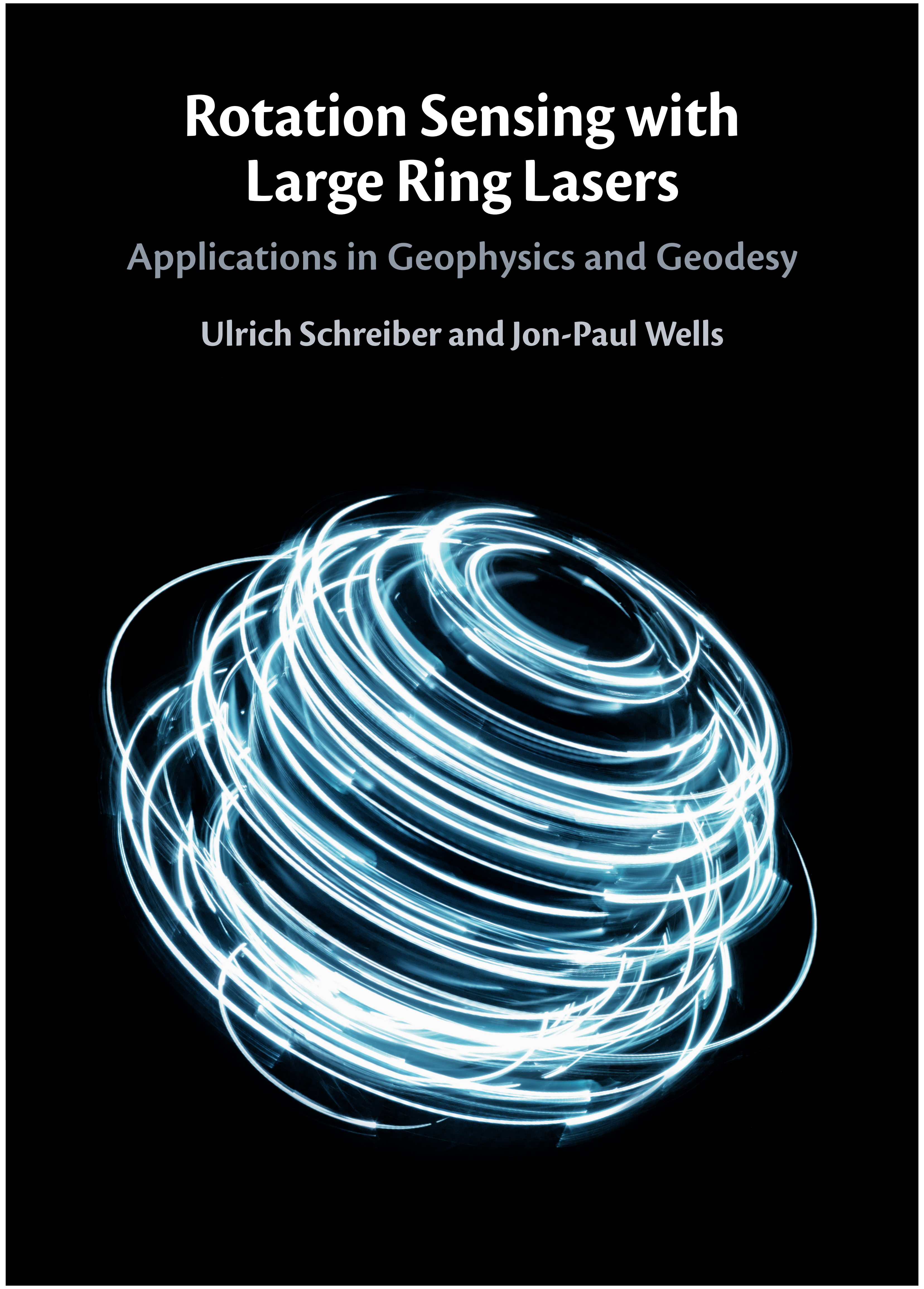
Ring Laser Observation of LoD



The black curve shows the measurements

The red curve indicates the LoD signal from the IERS website

However, we still have a mix of global and local rotation to deal with



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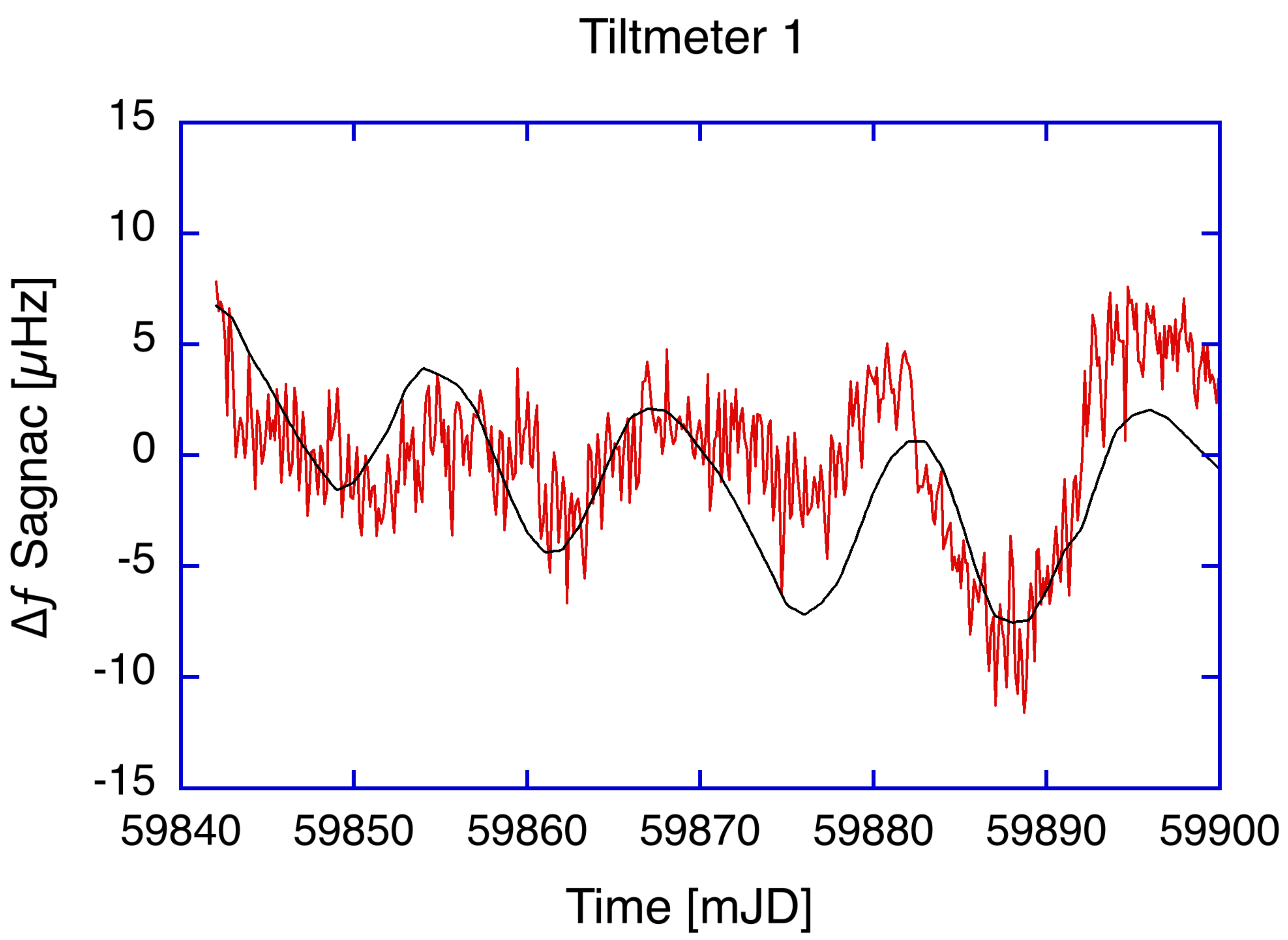
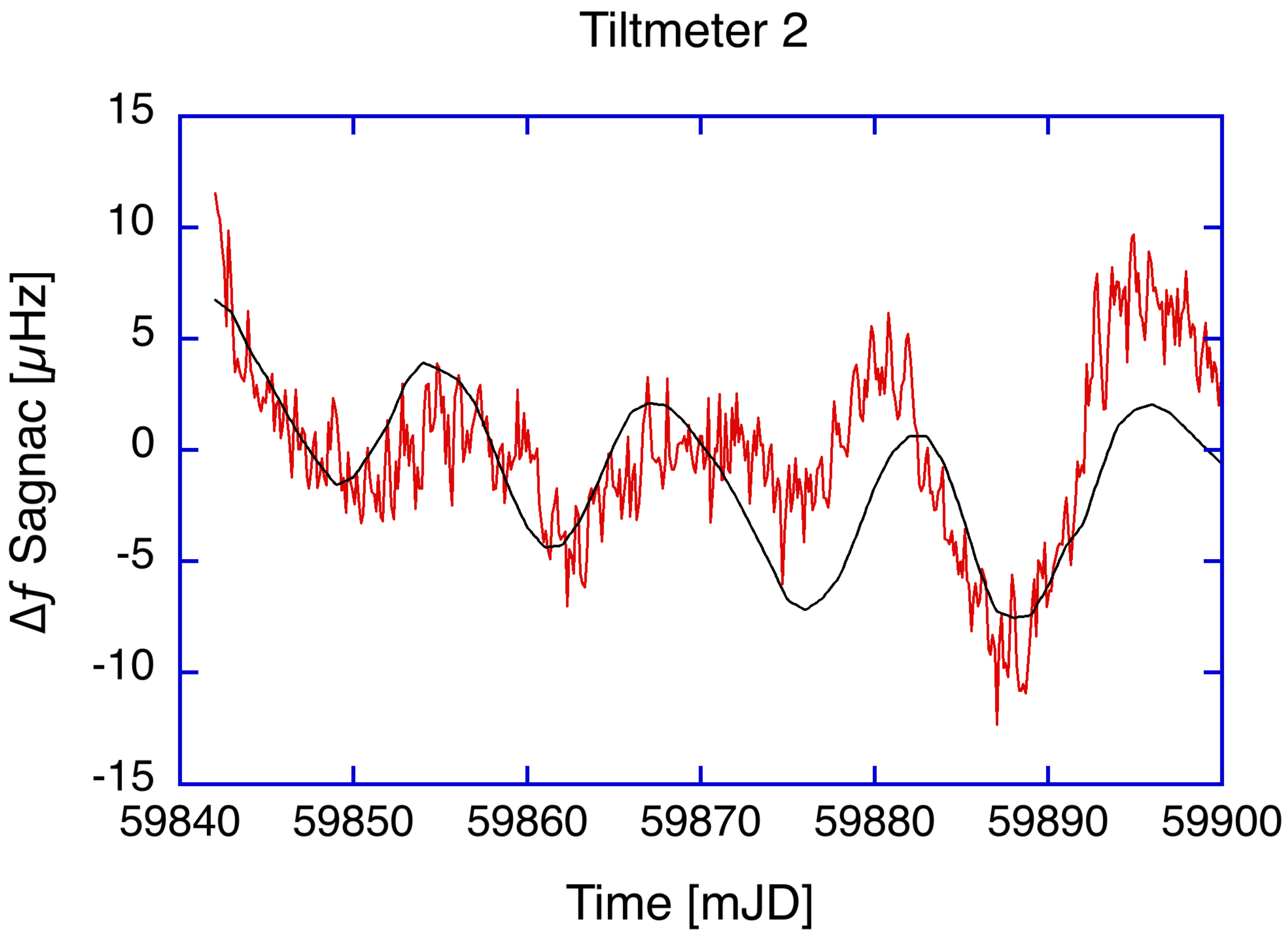
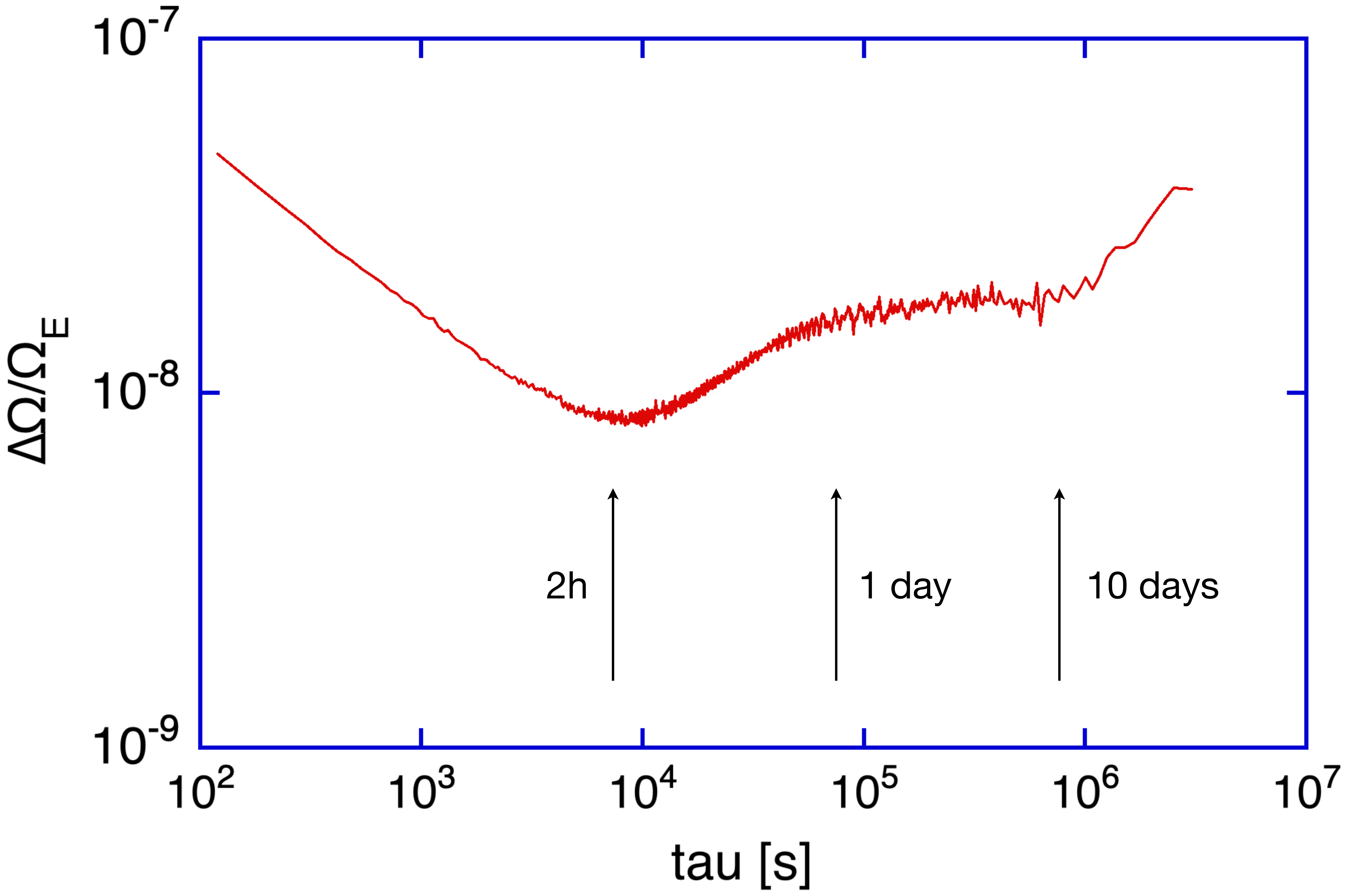
Variability introduced by the tilt measurement

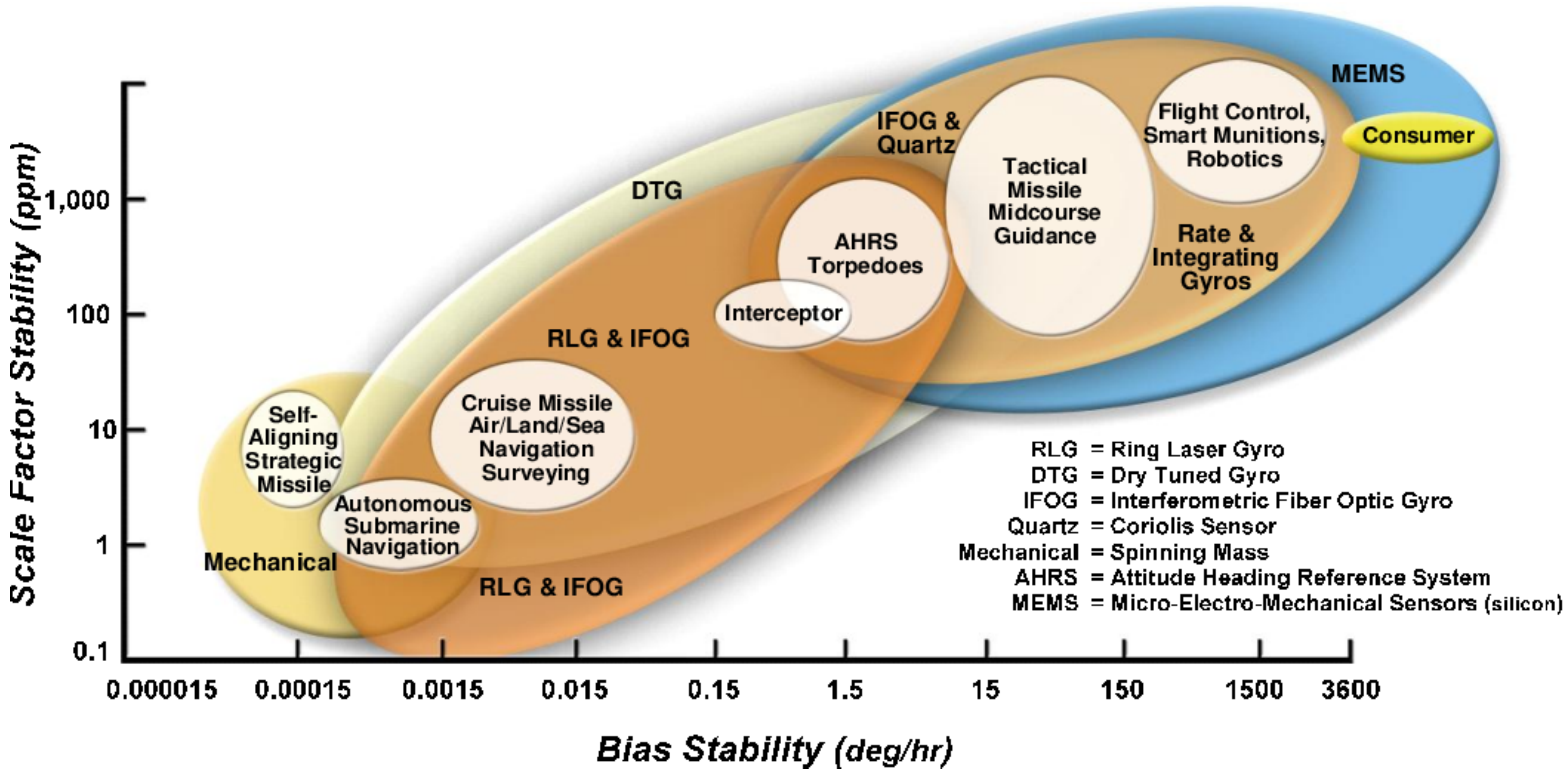




This all said: This is the first ever measurement of the variation in the rotation rate of the Earth with a local inertial sensor at much higher rate and without smoothing. Compared to the time series obtained from GNSS and VLBI one has to expect differences.



10^{-7}
10^{-8}
10^{-9}
$\Delta\Omega/\Omega_E$
2h
1 day
10 days
10^2
10^3
10^4
10^5
10^6
10^7
tau [s]

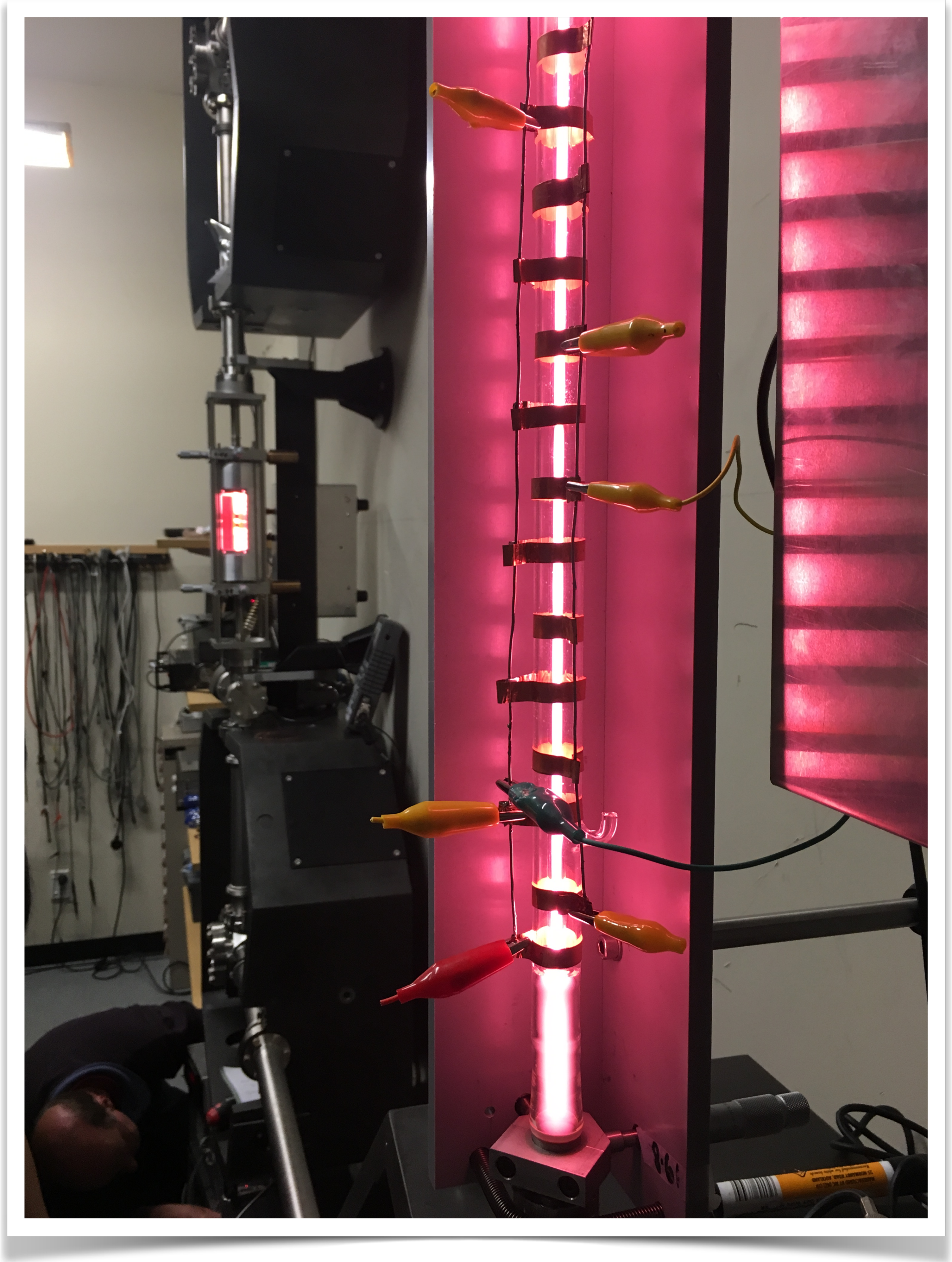

MEMS
IFOG & Quartz
DTG
Flight Control, Smart Munitions, Robotics
Consumer
Tactical Missile Midcourse Guidance
AHRS Torpedoes
Rate & Integrating Gyros
Interceptor
RLG & IFOG
Self-Aligning Strategic Missile
Cruise Missile Air/Land/Sea Navigation Surveying
Autonomous Submarine Navigation
Mechanical
RLG & IFOG
RLG = Ring Laser Gyro
DTG = Dry Tuned Gyro
IFOG = Interferometric Fiber Optic Gyro
Quartz = Coriolis Sensor
Mechanical = Spinning Mass
AHRS = Attitude Heading Reference System
MEMS = Micro-Electro-Mechanical Sensors (silicon)
Scale Factor Stability (ppm)
1,000
100
10
1
0.1
0.000015
0.00015
0.0015
0.015
0.15
1.5
15
150
1500
3600
Bias Stability (deg/hr)

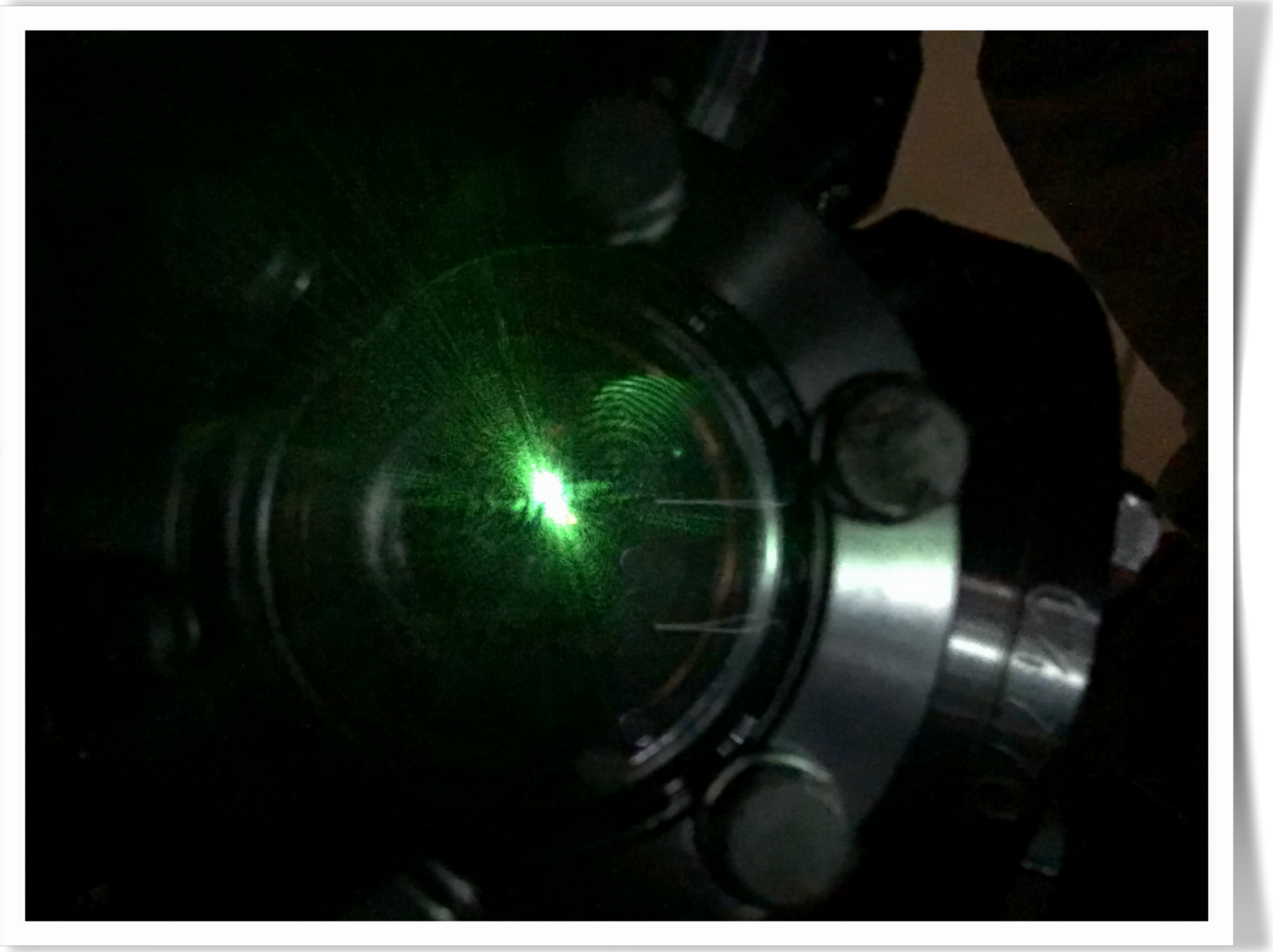
o.oo5/4e-10

G

Significantly reduced gain, but scale factor up to 20% larger and backscatter much reduced, due to smaller spot size.

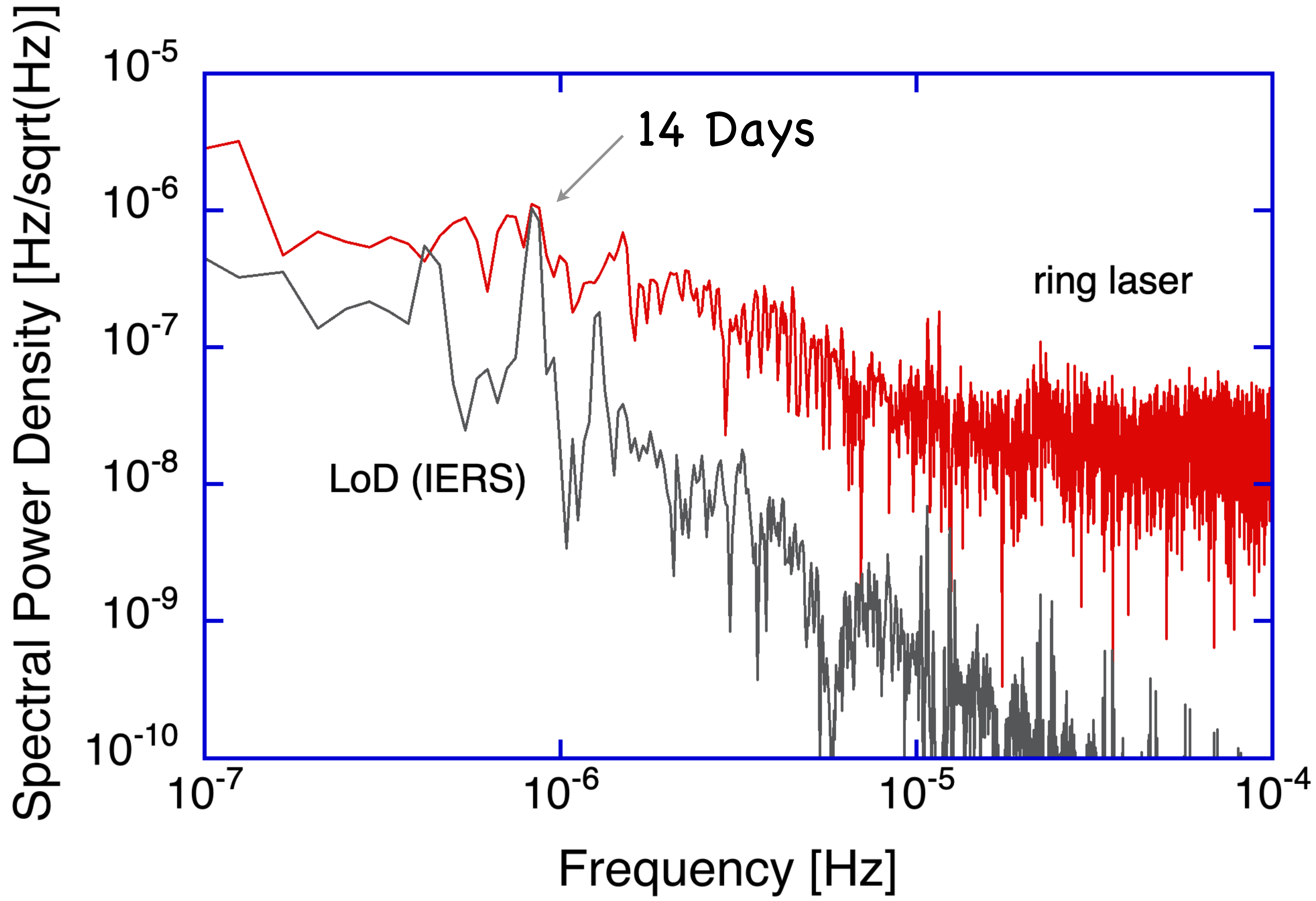
612 nm
604 nm
594 nm





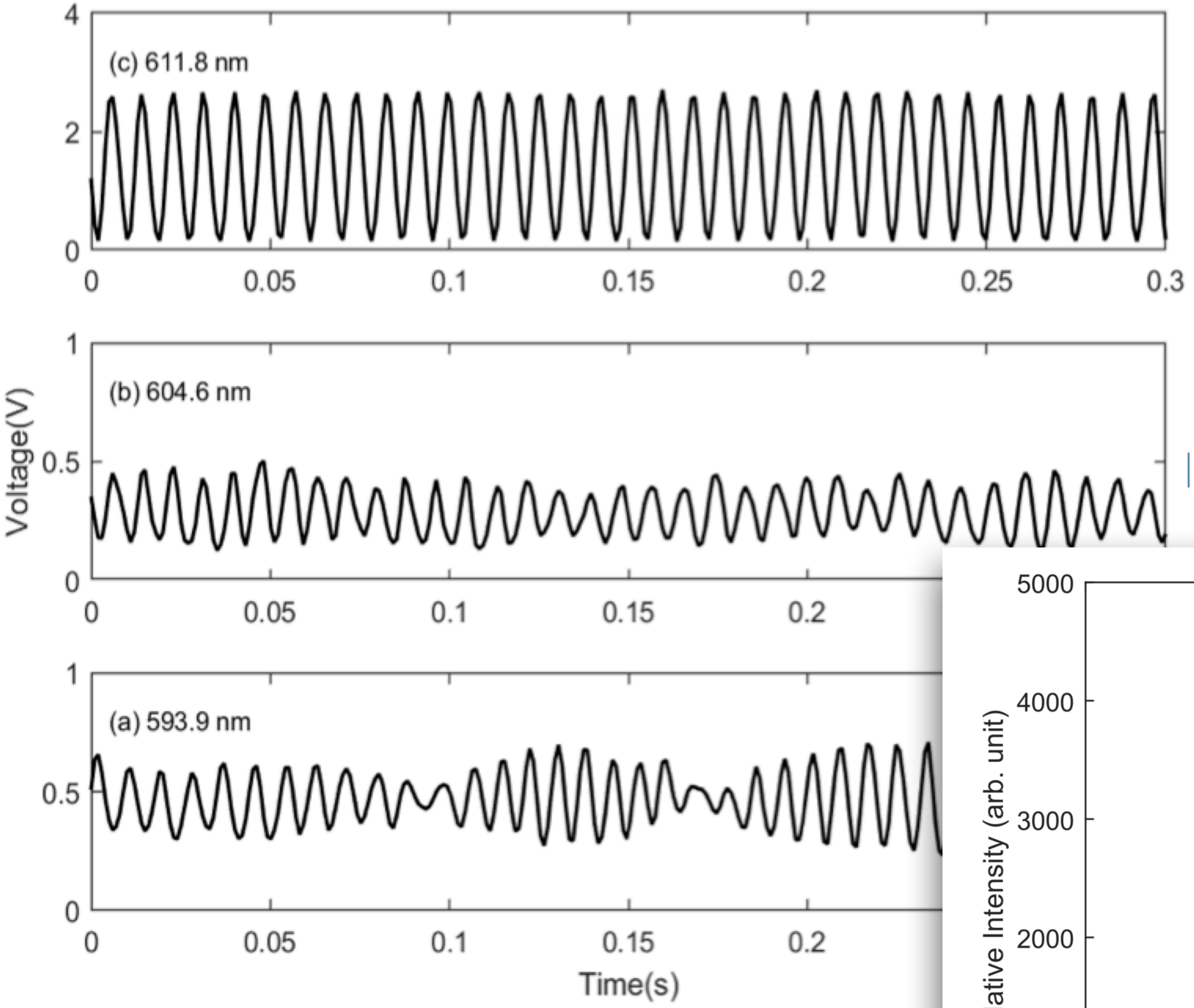
543 nm

The highest frequency component in “LoD” is at 1 μHz (fortnightly period)



- A joint plot of IERS data and ring laser observations clearly shows that G detects the contribution of LoD in its measurements
- The SNR is not yet overwhelming, nevertheless it is the first ever measurement of LoD from an inertial sensor
- The seasonal part of LoD is larger but about 1 decade lower in frequency

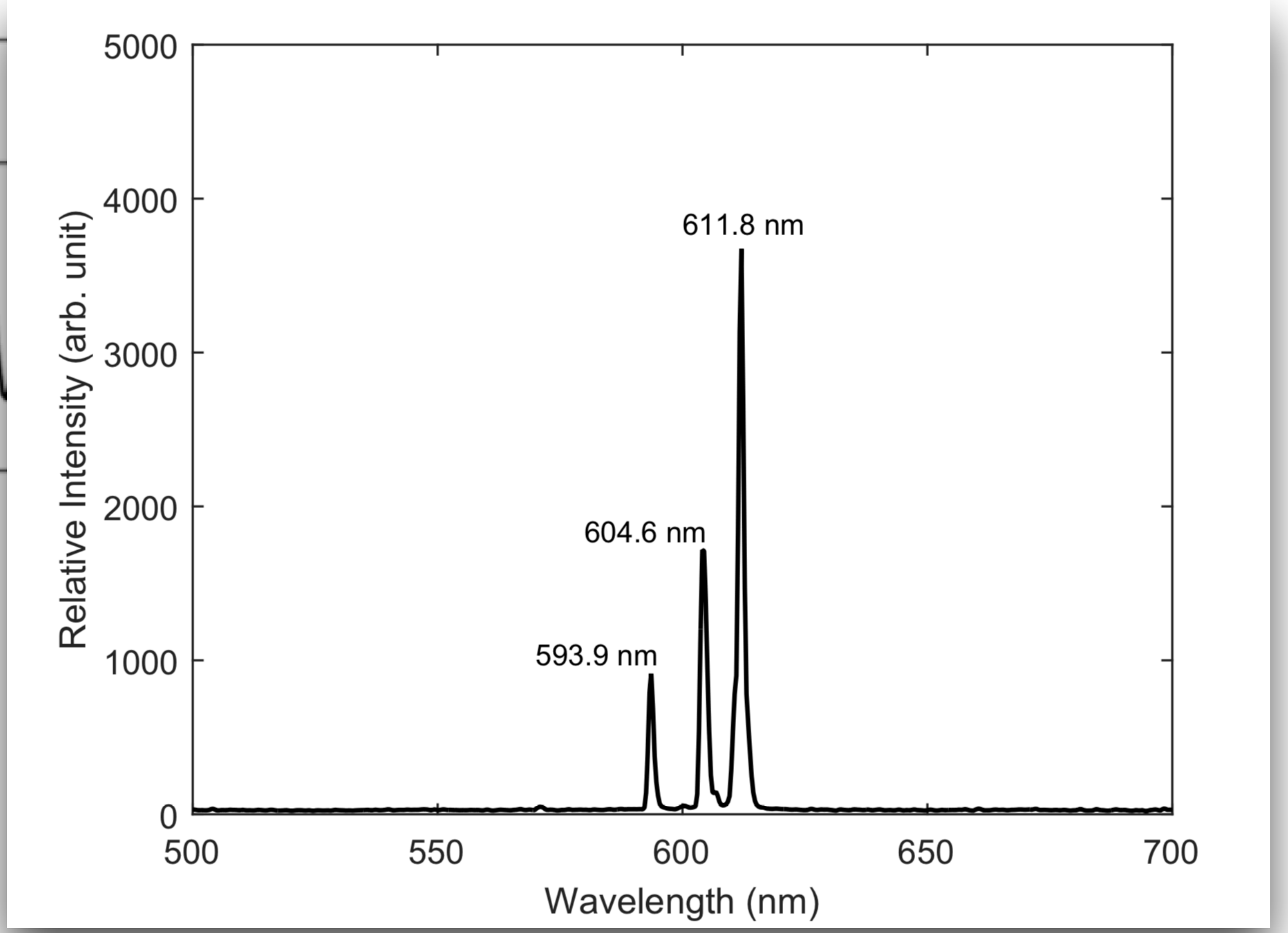
Upscaling by Wavelength…



$\Delta f \approx 117.4$ Hz

$\Delta f \approx 118.9$ Hz

$\Delta f \approx 121$ Hz

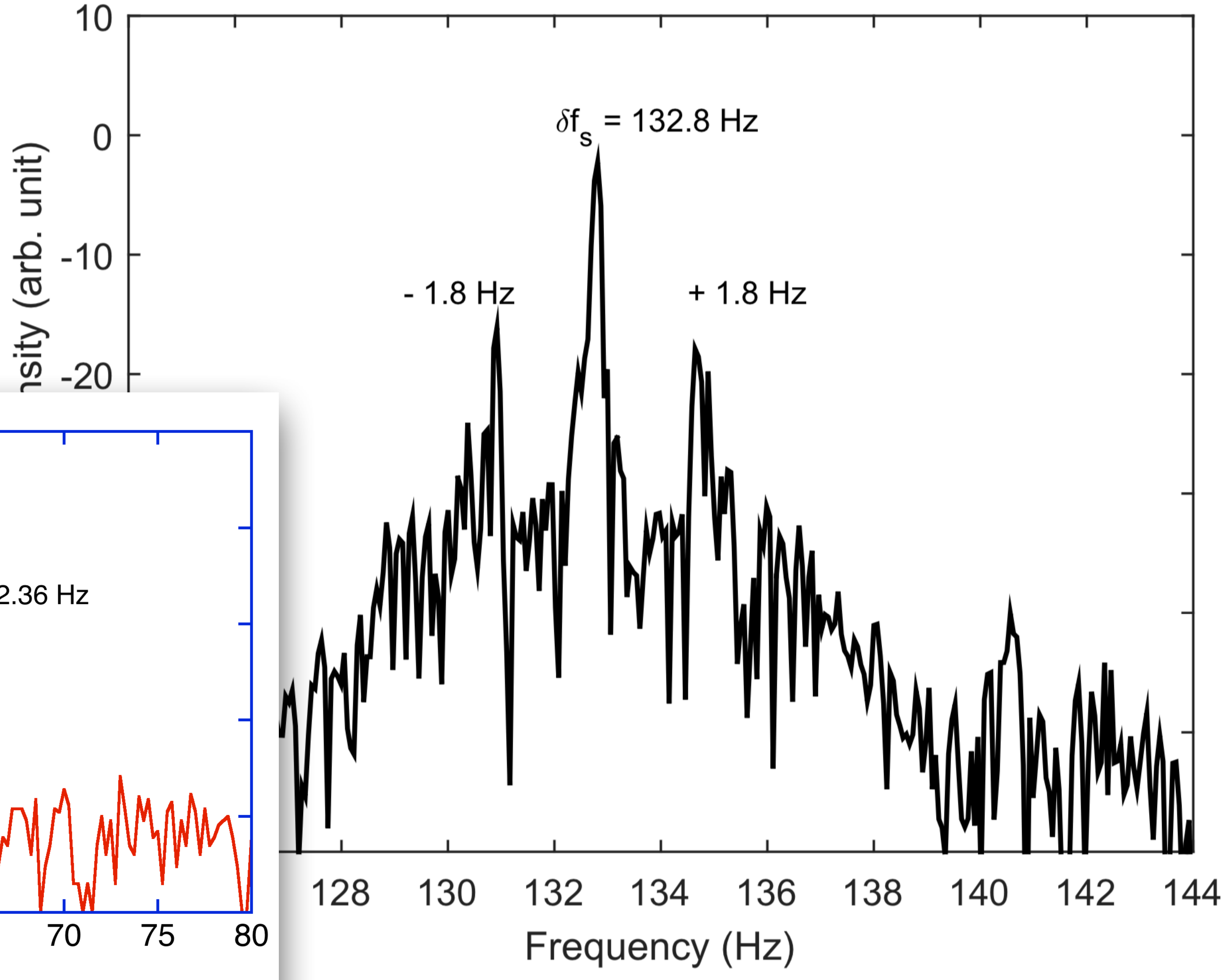


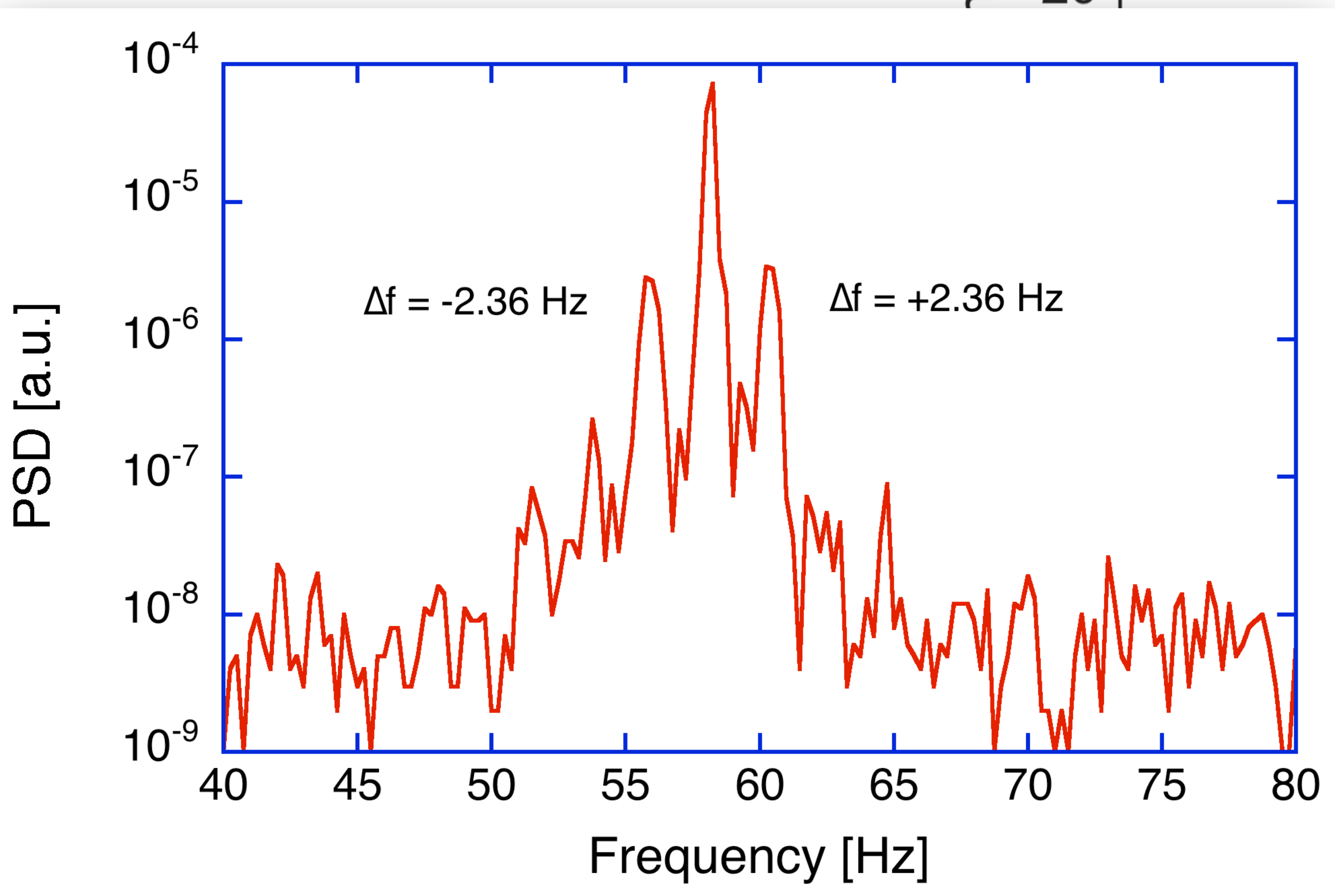
…but signal SNR depends on the available gain

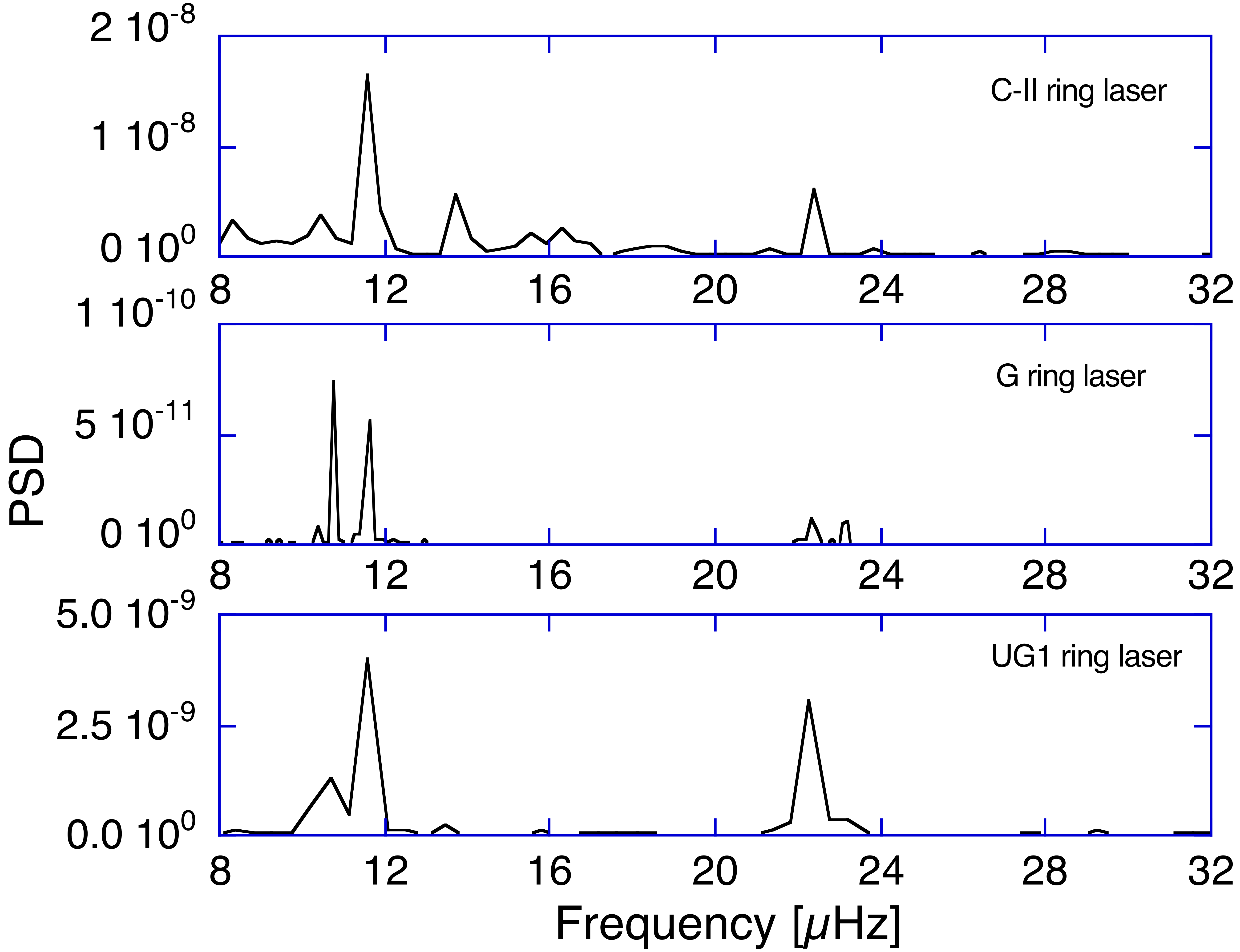
Going to 543 nm is promising, but comes at a price. The gain is 1/17 that of 632.8 nm.
It does not only require a long gain tube, it also causes fancy gas dynamics in the form acoustic resonances in t
electro magnetic field of the laser excitation.

The sidebands are caused
by building oscillations

$\Delta f \approx 59$ Hz @ 1.1523 μm

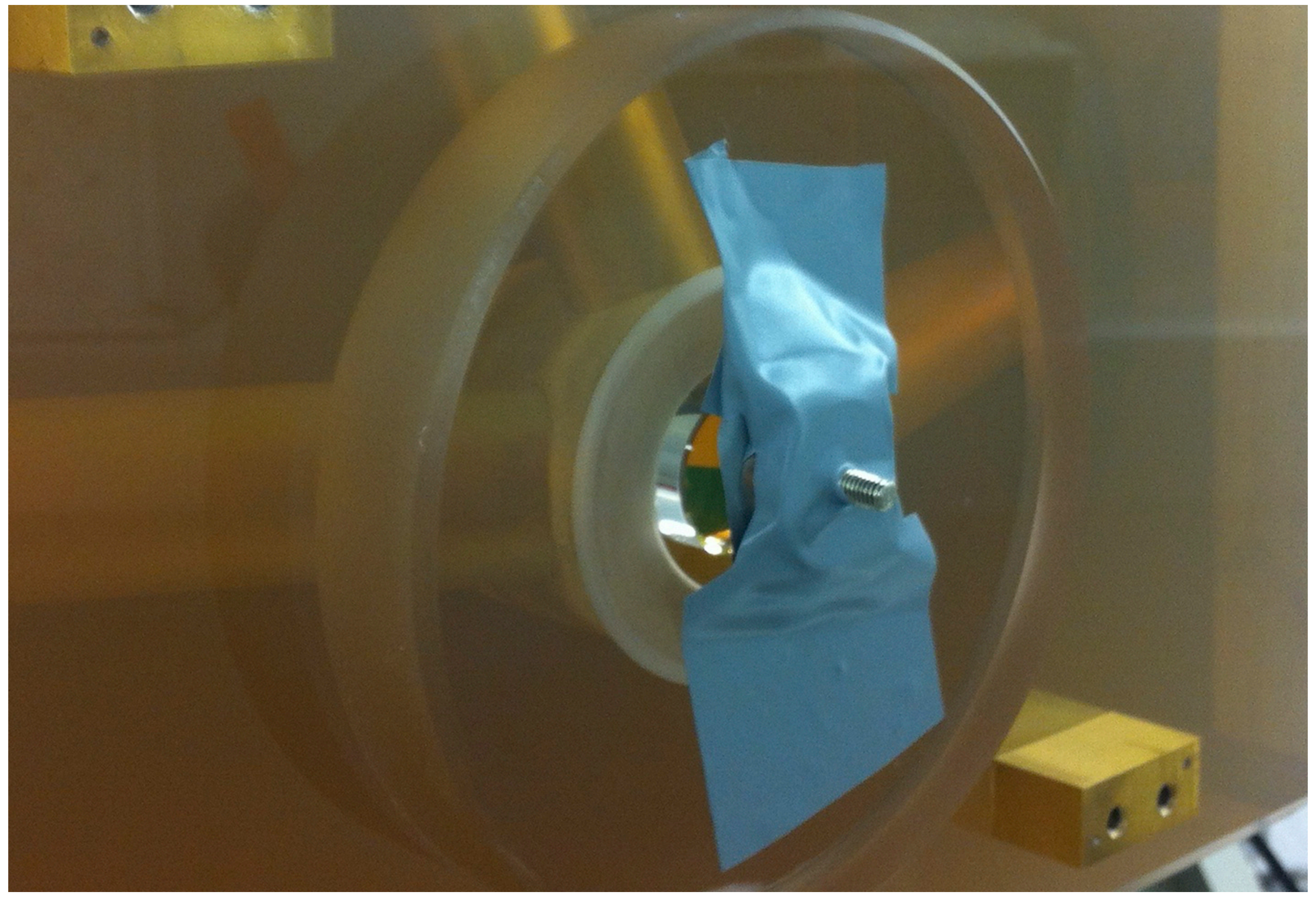


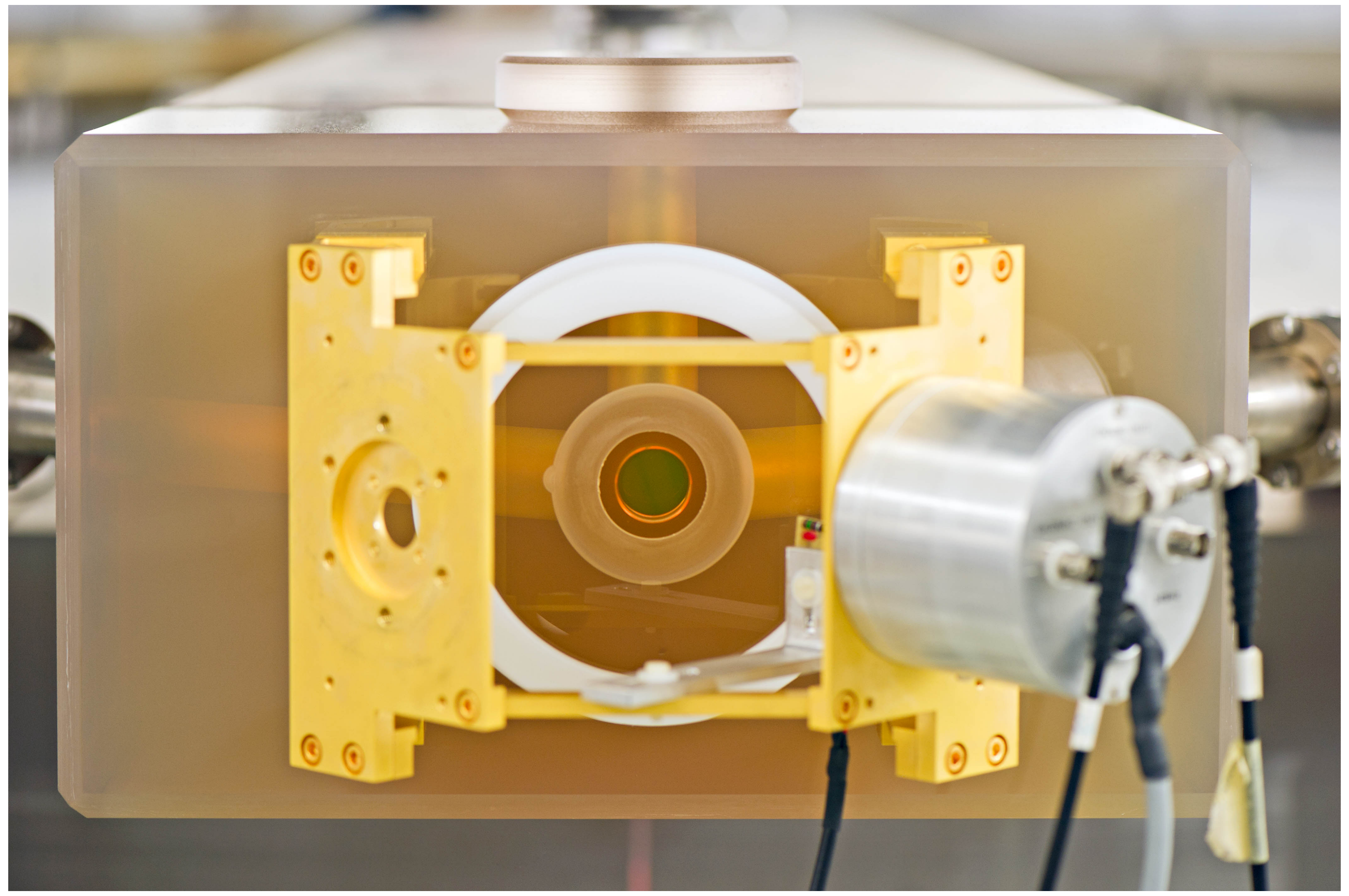


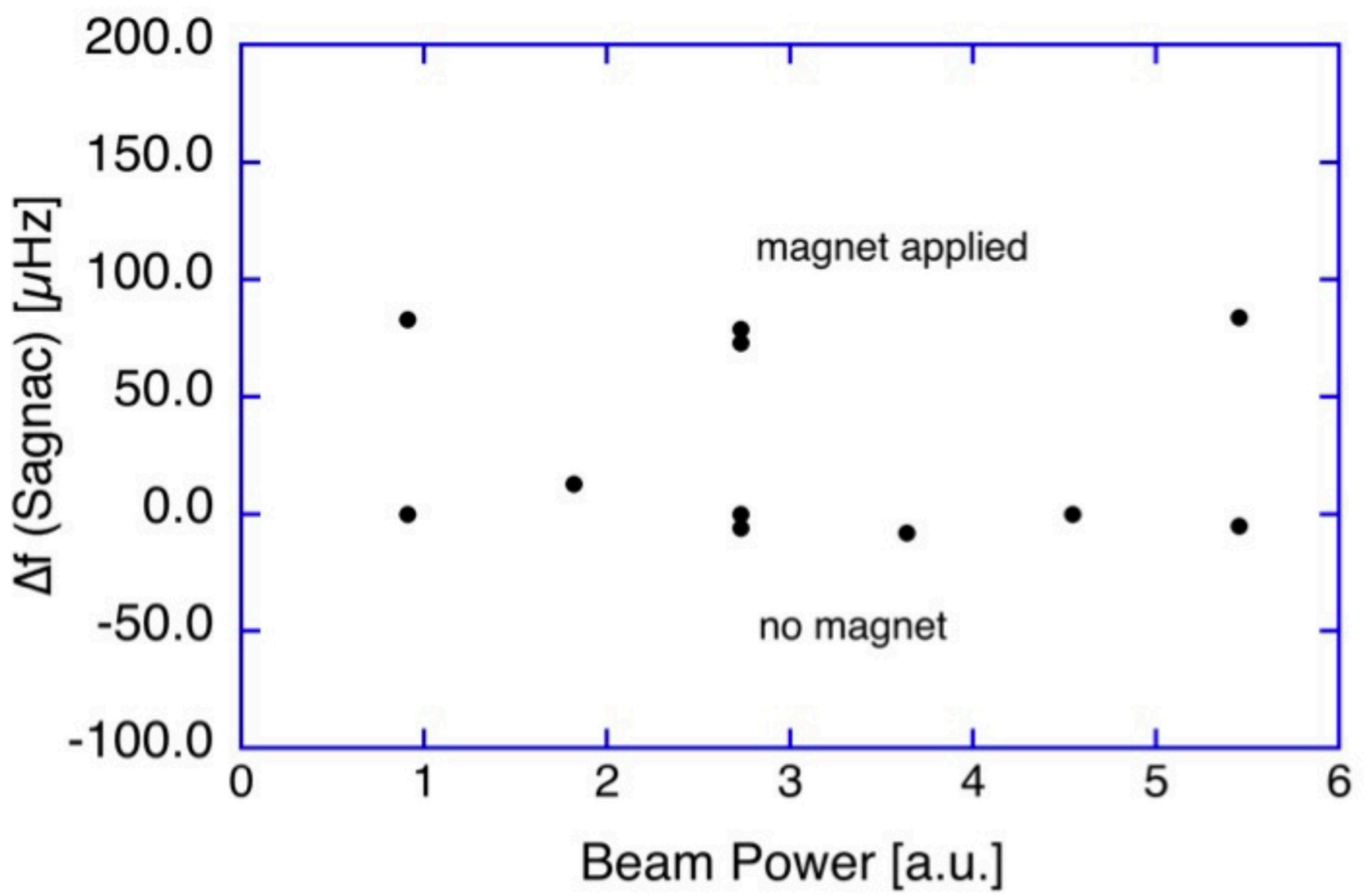


Polar Motion and Solid Earth Tides show up in all Gyros from 1 - 400 m² with progressive difficulties from lacking stability above 16 m²

Ring lasers and mirrors







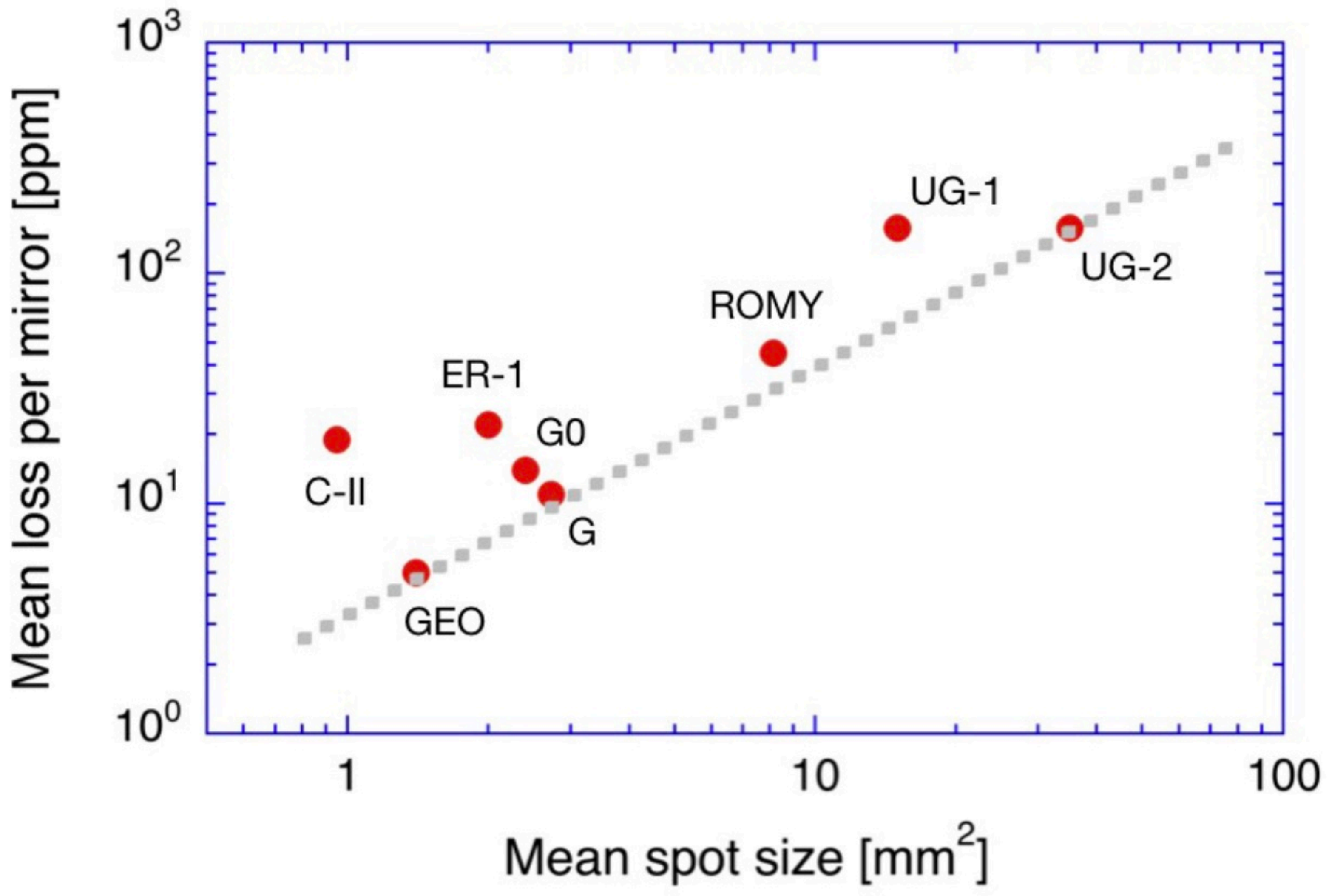


Figure 4.39 The placement of a strong rare earth magnet behind the mirror holder (see fig. 4.38) caused an offset of about 80 μ, which did not depend on the actual laser beam power.

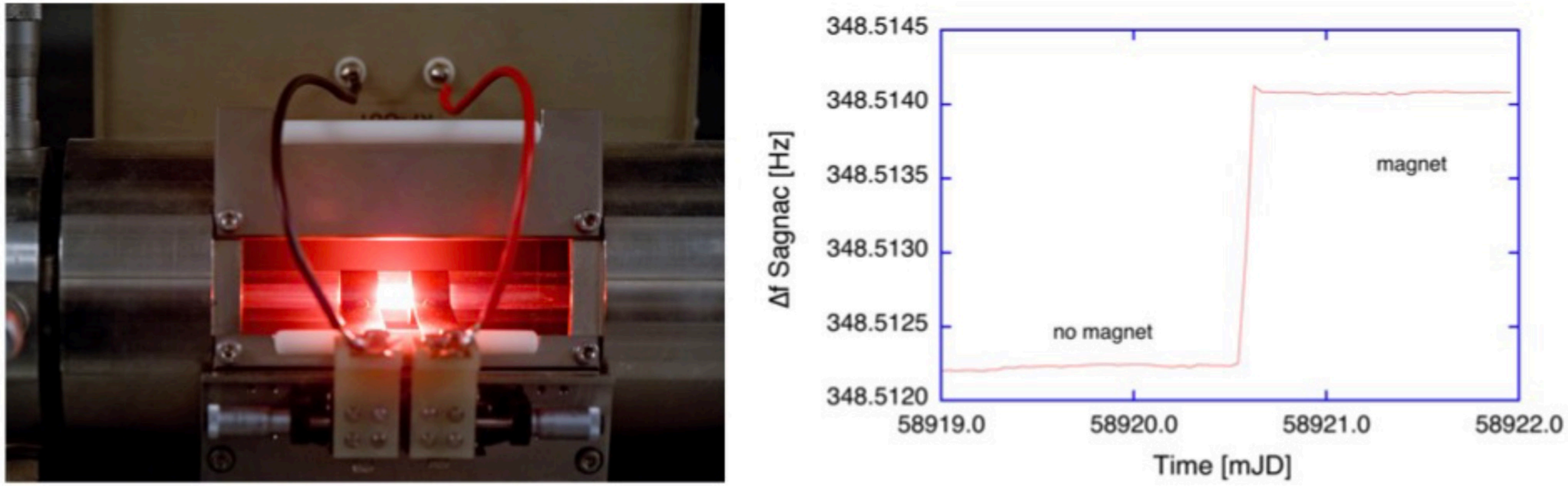
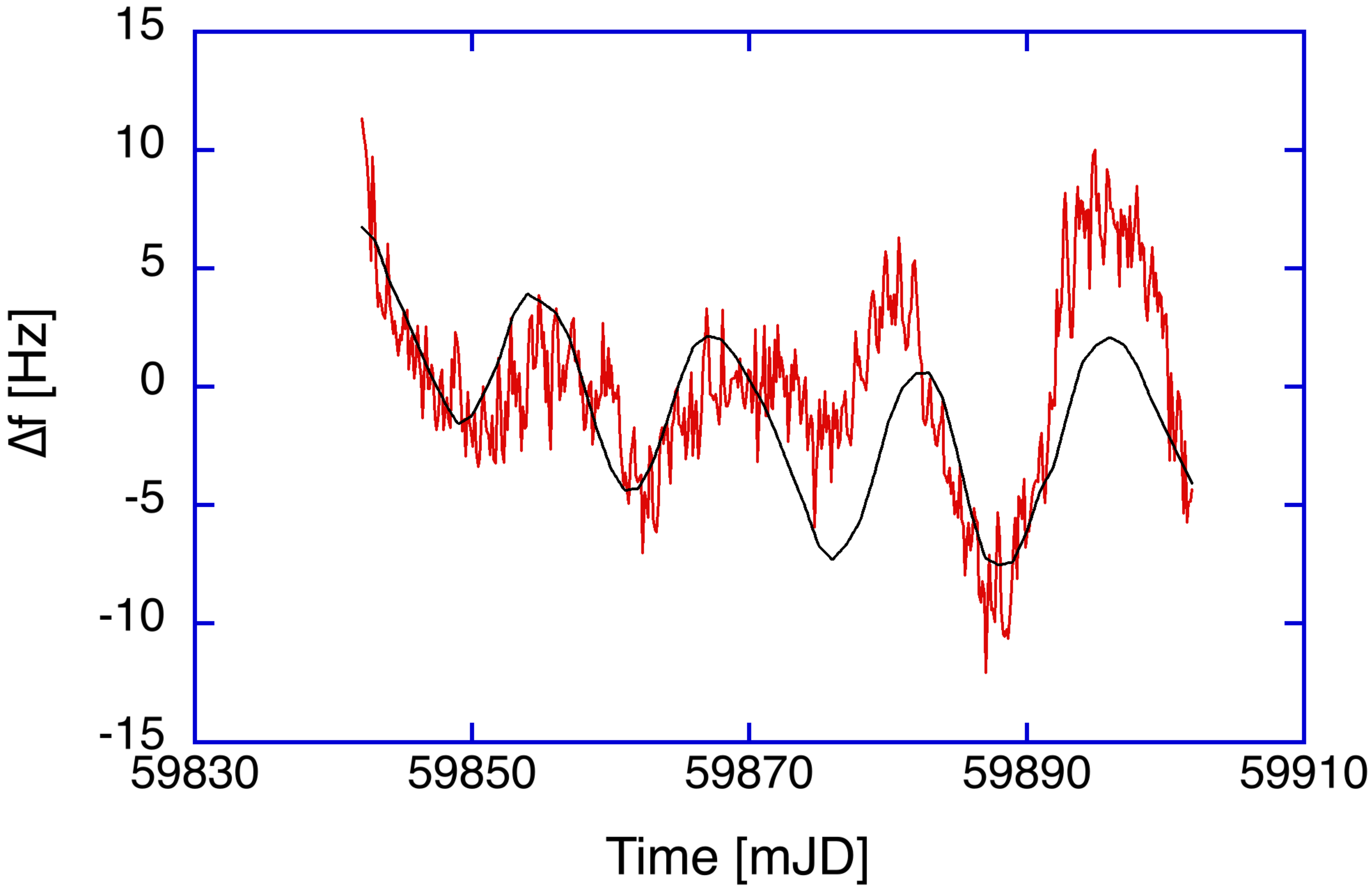
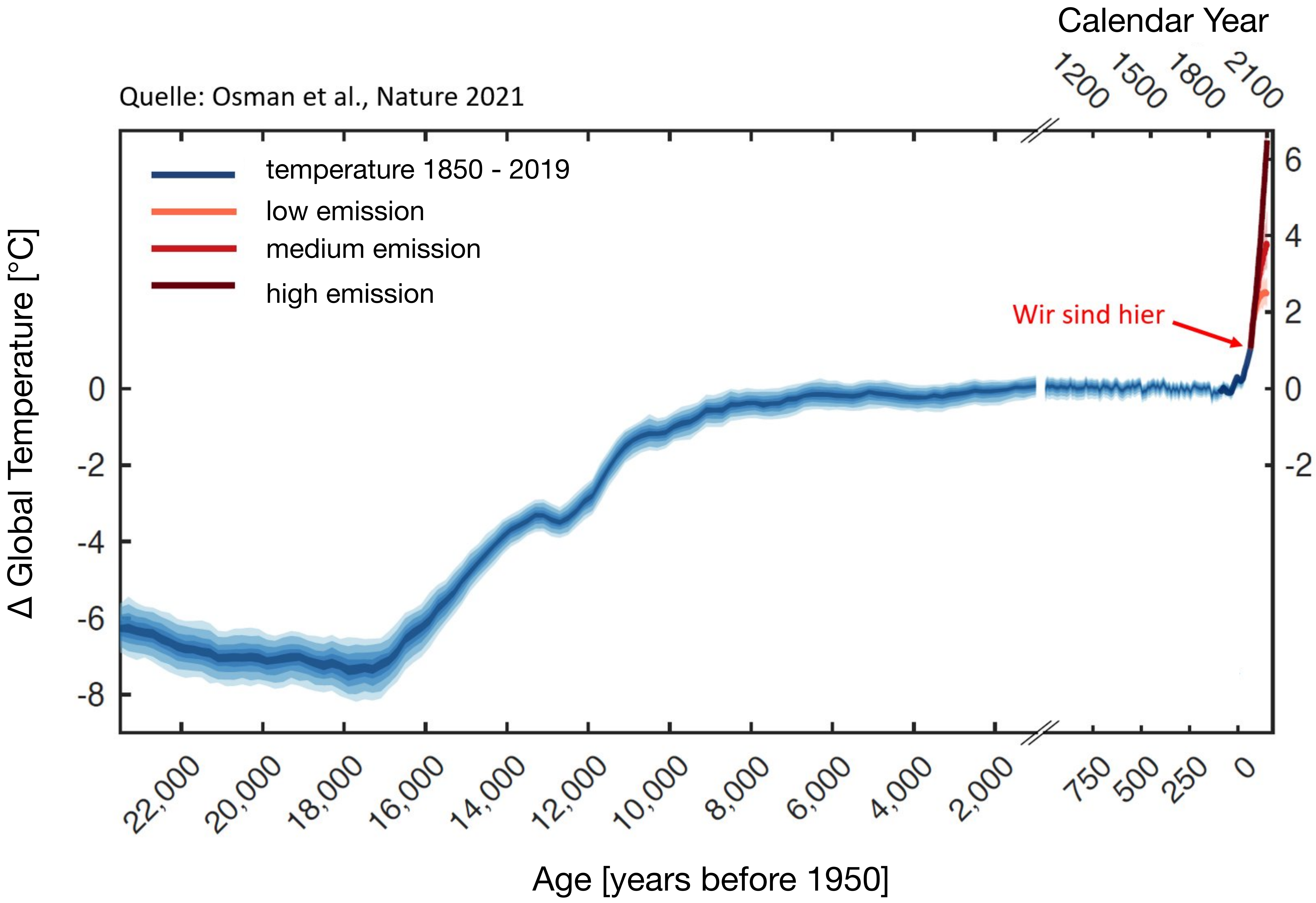


Figure 4.40 The magnet was placed with about 20 mm clearance over the plasma in the gaintube section. This caused an offset in the in the observed beatnote of more than 1.845 mHz.

In the end we have to find out what is the true "Length of Day" variation

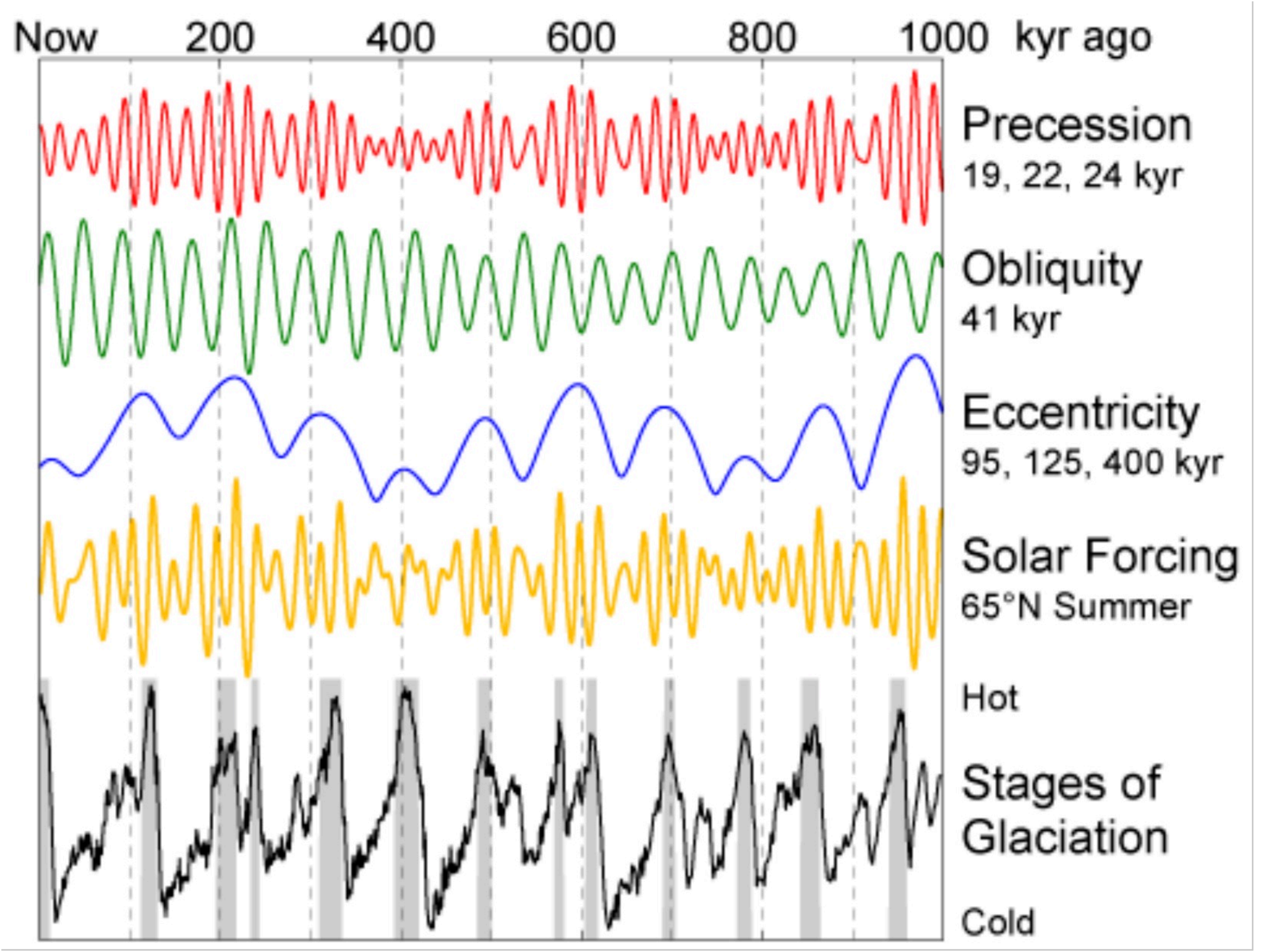


Global mean surface temperature over the last 24000 years



Driver

- Solar radiation and Earth albedo and greenhouse gases
- Ocean circulation
- Variation in Earth orbit and solar radiation (Milancovic-cycles)



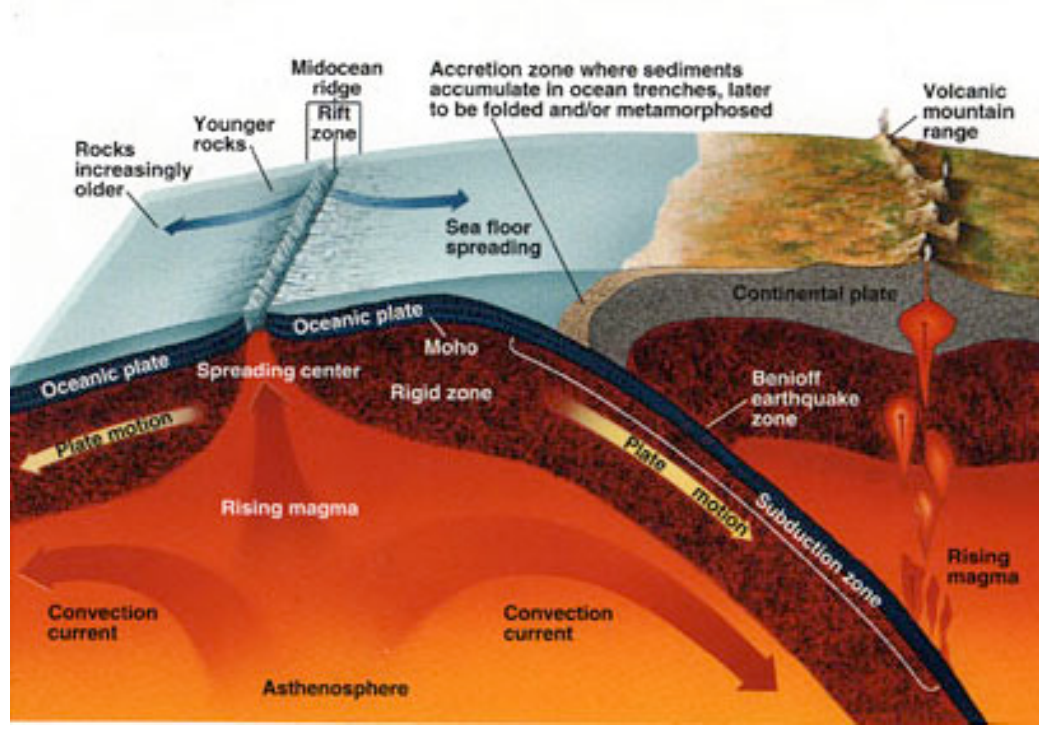
System Earth - Relevant Timescales

Lithosphere:

Plate Tectonics ↔ Earthquakes

Millions of Years ↔ several Seconds

cm/year ↔ km/s



Hydrosphere:

Sea Level Rise ↔ Tsunami

3 mm/year ↔ 300 m/s

Atmosphere:

Climate ↔ Weather

years - decades ↔ hours - days

→ Requirements

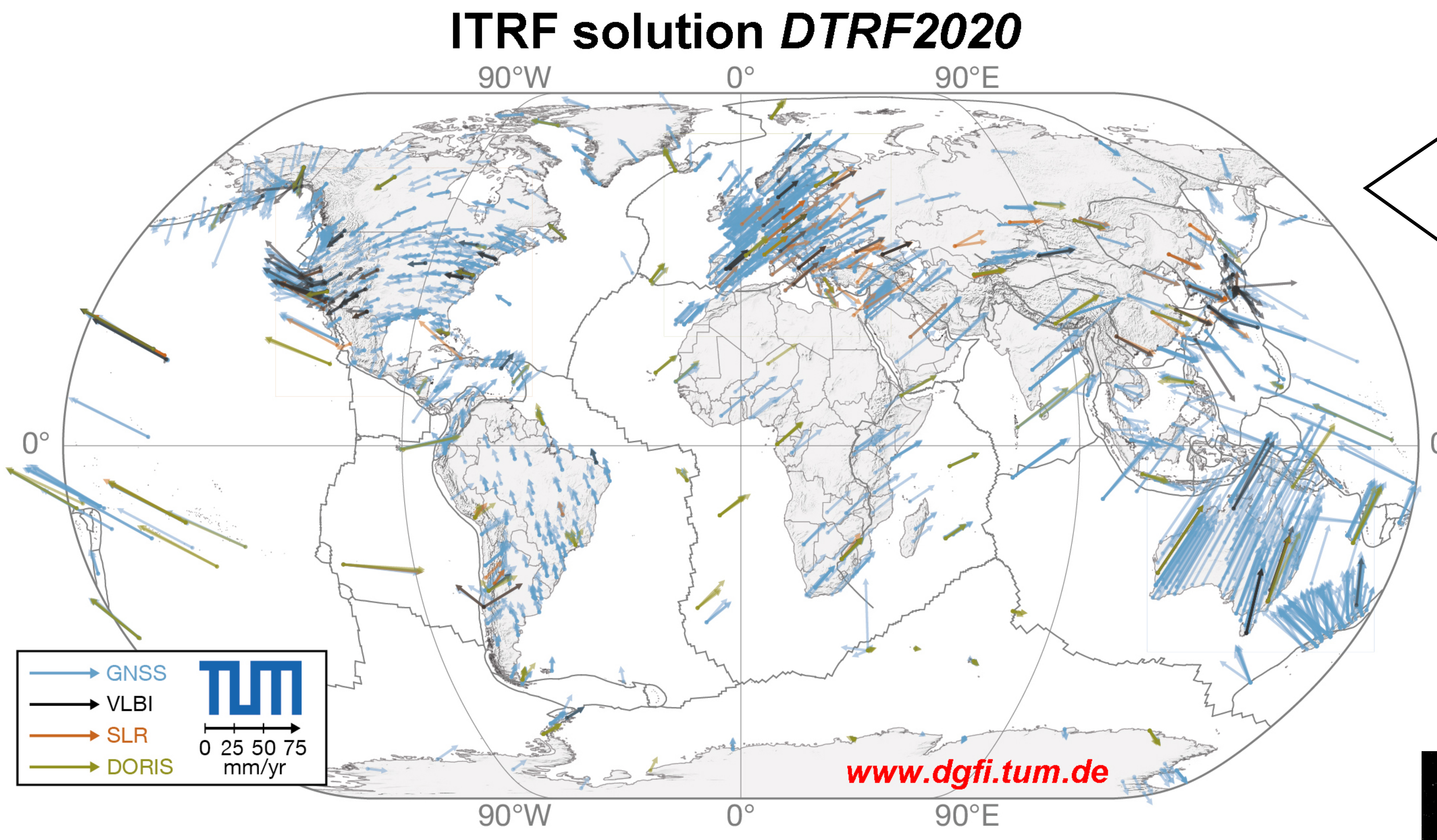
Measurement techniques of extremely high resolution and stability

Quantification of very small and slow processes vs. highly dynamic realtime

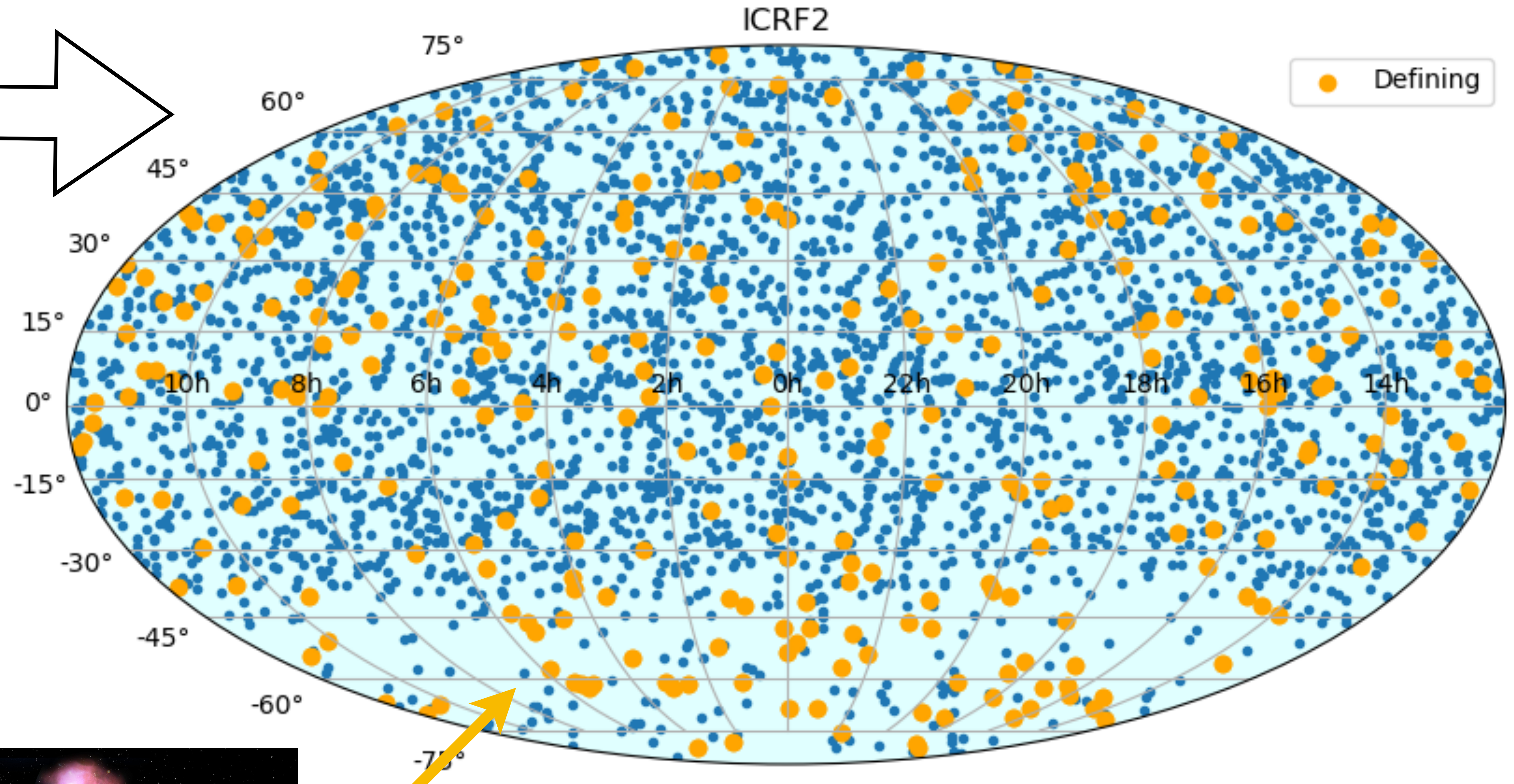
Friedrich Robert Helmert:

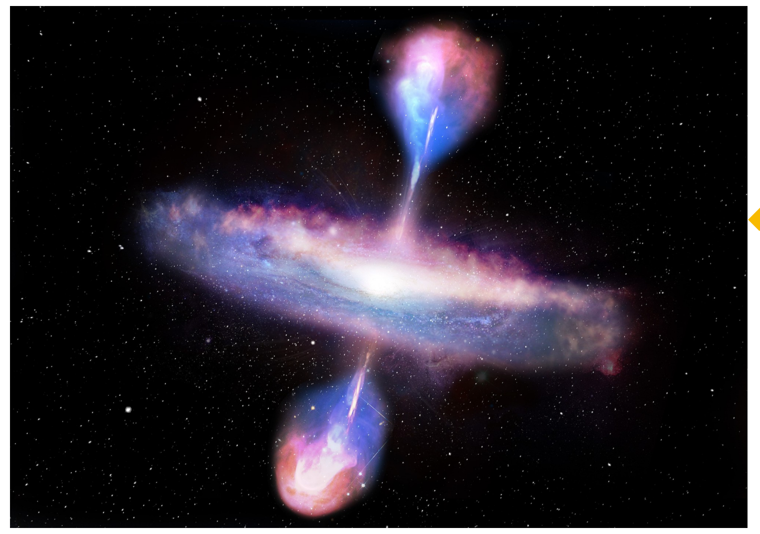
“Geodesy is the science of the measurement and mapping of the earth surface”

We want positioning (navigation) on earth, but our observed targets (satellites, quasars and stars) are represented in space



ICRF: Space fixed Reference Frame





ITRF: Earth fixed Reference Frame

- „Internal" Goal

Evolution of GGOS and the geodetic observation technologies to establish an Earth fixed reference frame with a relative accuracy of at least

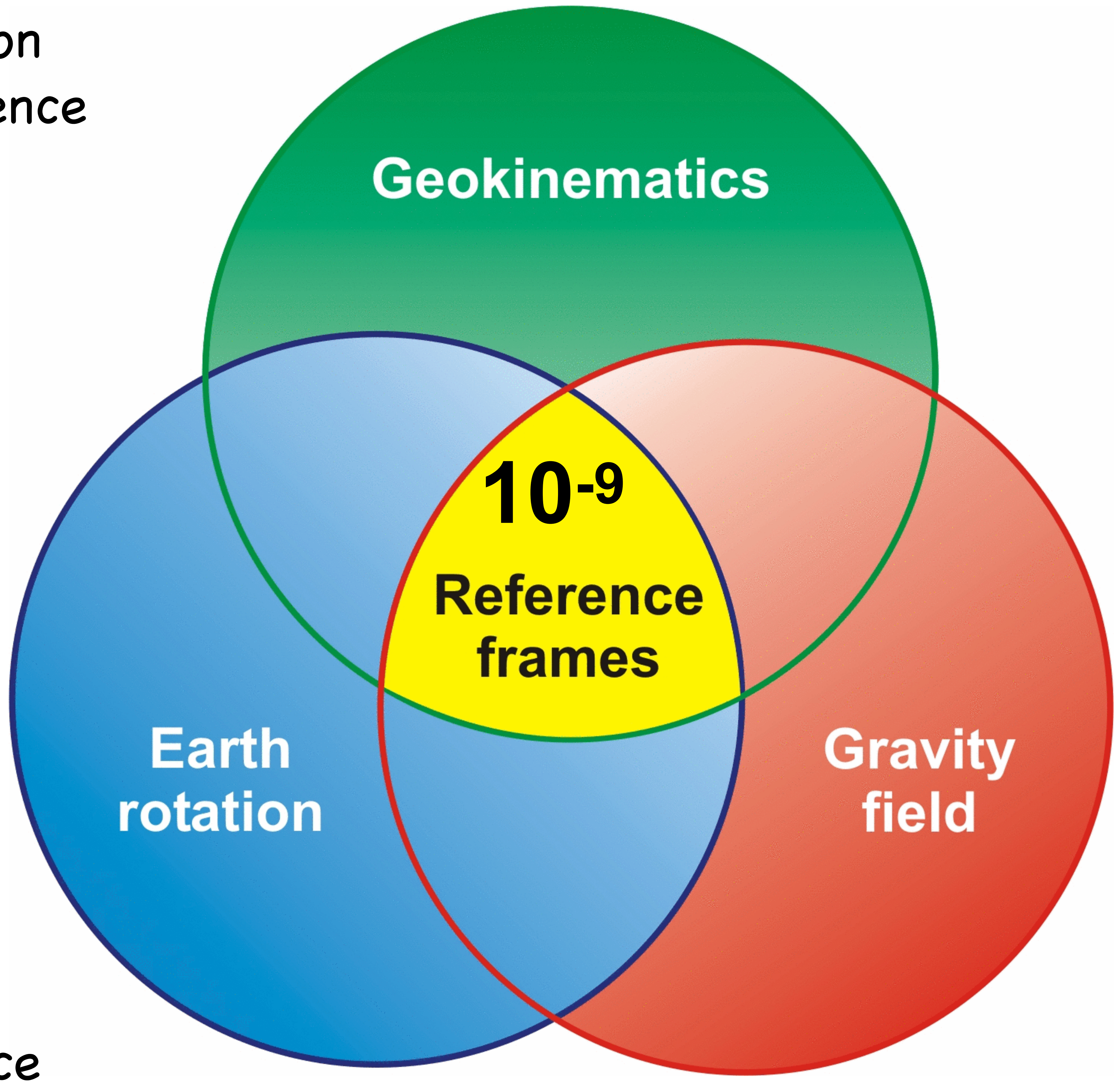
10^{-9} = 1 ppb

with high spatial and temporal resolution.

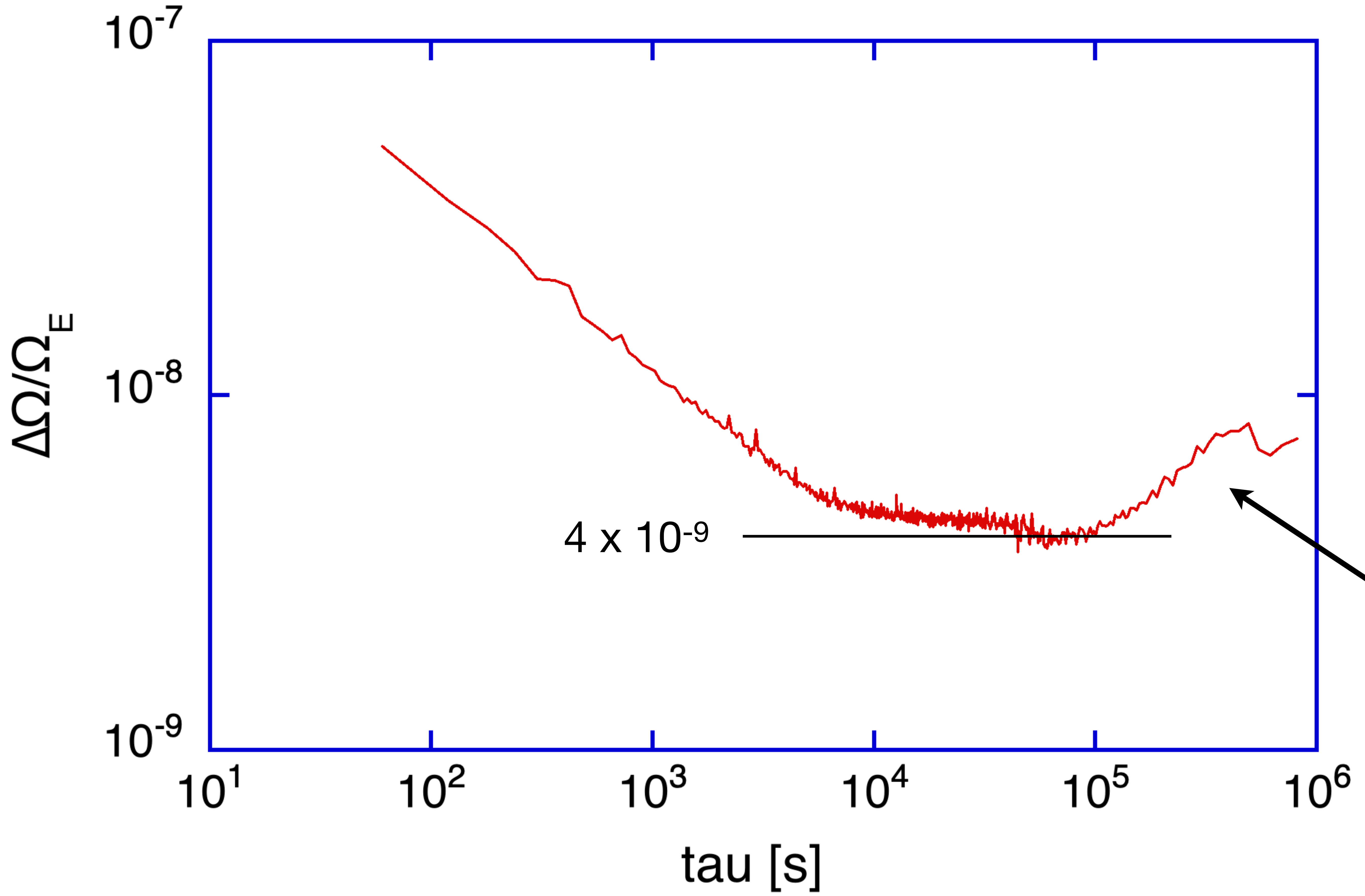
- „External" Goal

Integration of GGOS as an important contributor into **Earth System Research** (Modeling of physical, chemical and biological processes).

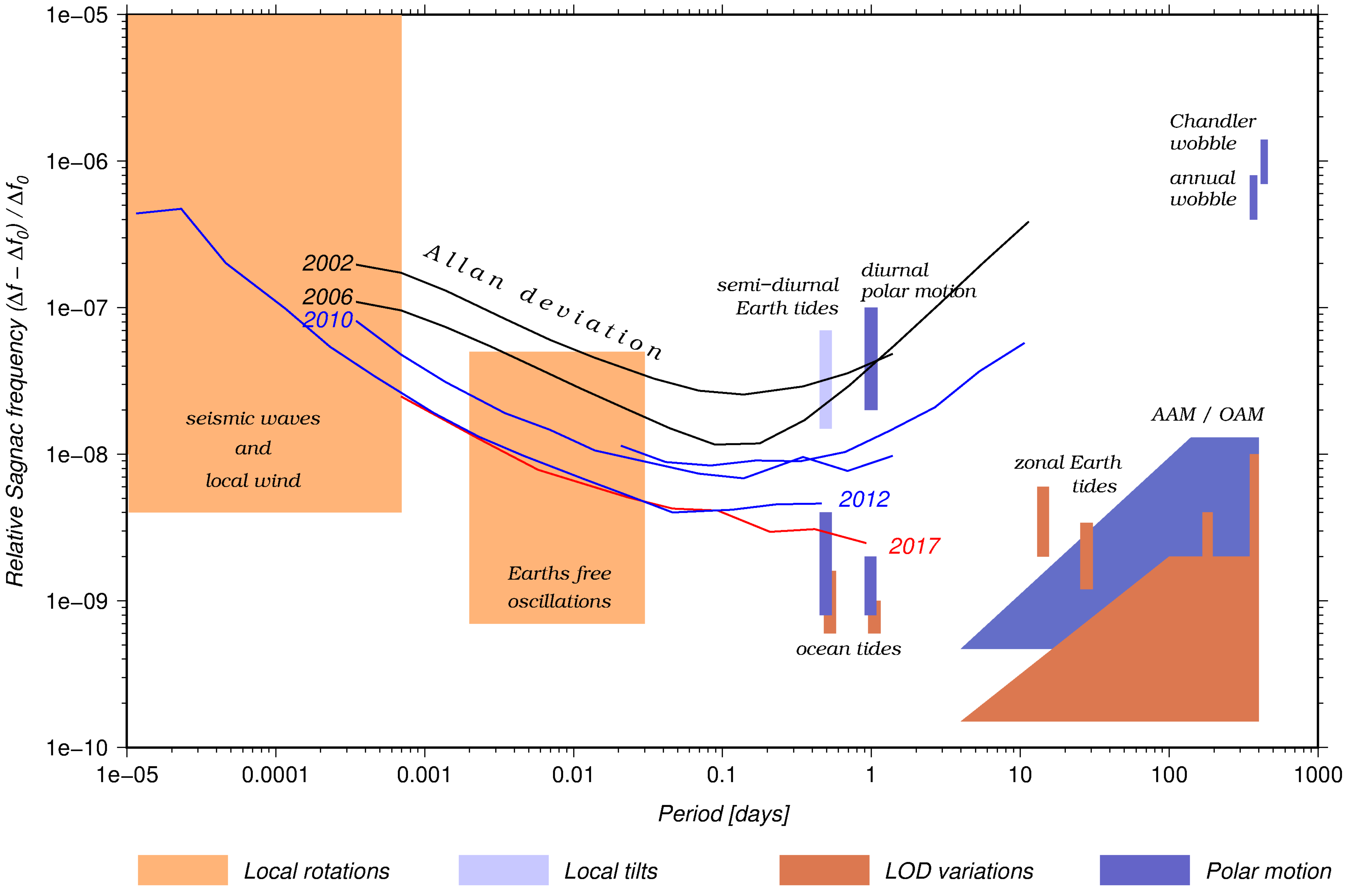
Contributions: Mass transport, dynamics, surface deformations.



G performance in 2022 (post error correction)



Model correction deficiencies
(tilt, laser rate equation)


1e−05
1e−06
1e−07
1e−08
1e−09
1e−10
Relative Sagnac frequency (Δf − Δf0) / Δf0
1e−05
0.0001
0.001
0.01
0.1
1
10
100
1000
Period [days]
seismic waves
and
local wind
2002
2006
2010
2012
2017
Allan deviation
Earths free
oscillations
semi−diurnal
Earth tides
diurnal
polar motion
ocean tides
Chandler
wobble
annual
wobble
AAM / OAM
zonal Earth
tides
Local rotations
Local tilts
LOD variations
Polar motion