Testing Theories of Gravity by GINGER Experiment

GINGER

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General Relativity

- Describes the gravitational interaction by spacetime curvature. The foundation is the Equivalence Principle.

- Successfully passes the Solar System Tests

- In a static and spherically Symmetric background

Schwarzschild Metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2\theta d\phi^2$$

Shortcomings of GR

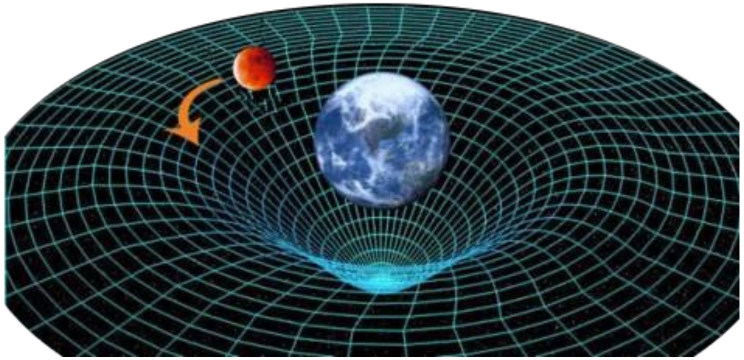
Large Scales (IR)

- Universe accelerated expansion
- Galaxy Rotation Curve
- Dark Energy
- Dark Matter
- Tensions of cosmological parameters
- H_0 tension

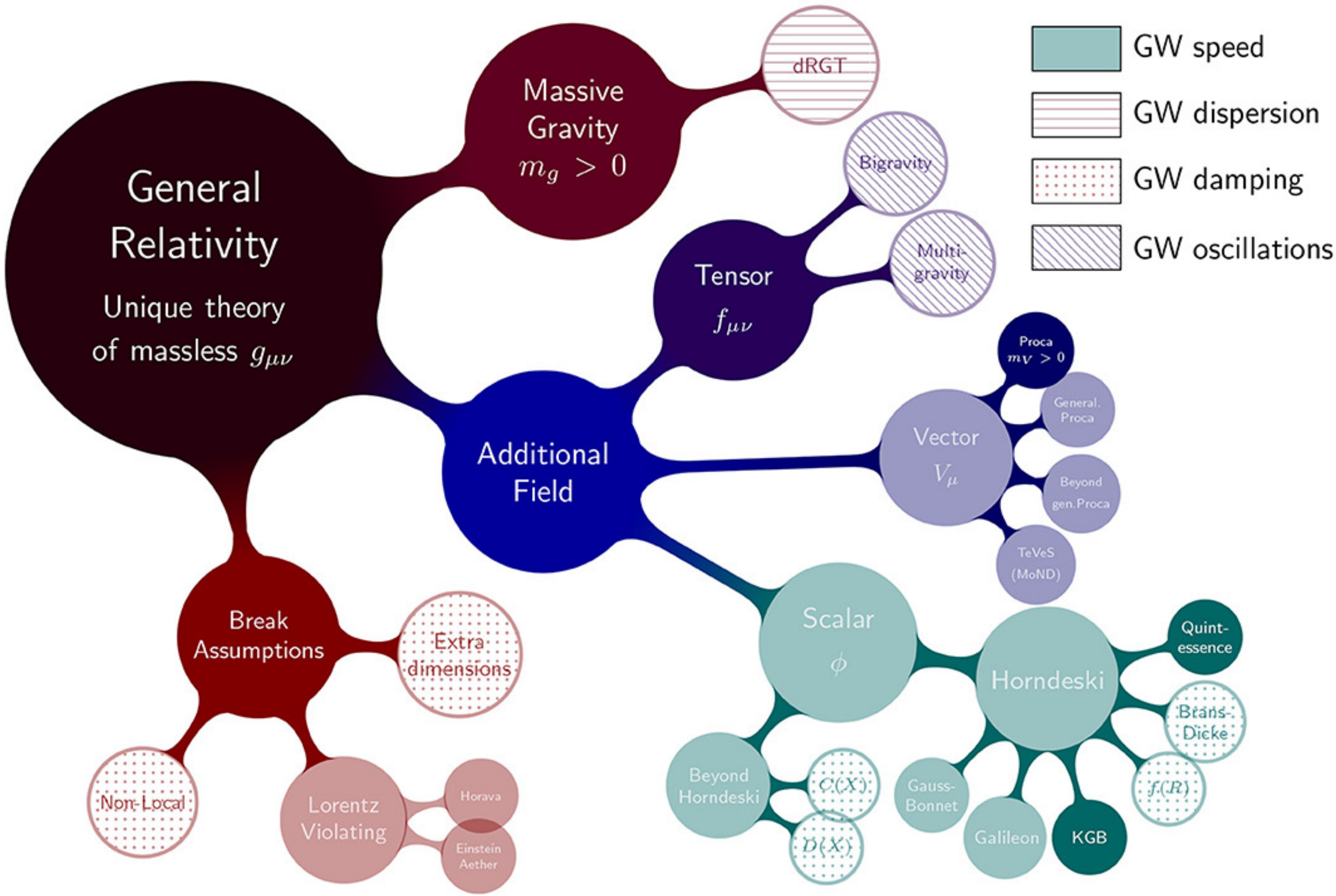
Small Scales (UV)

- Renormalizability
- GR cannot be quantized
- GR cannot be treated under the same standard of other gauge theories
- Discrepancy between theoretical and experimental value of Λ
- Spacetime singularities
- No quantum description of spacetime

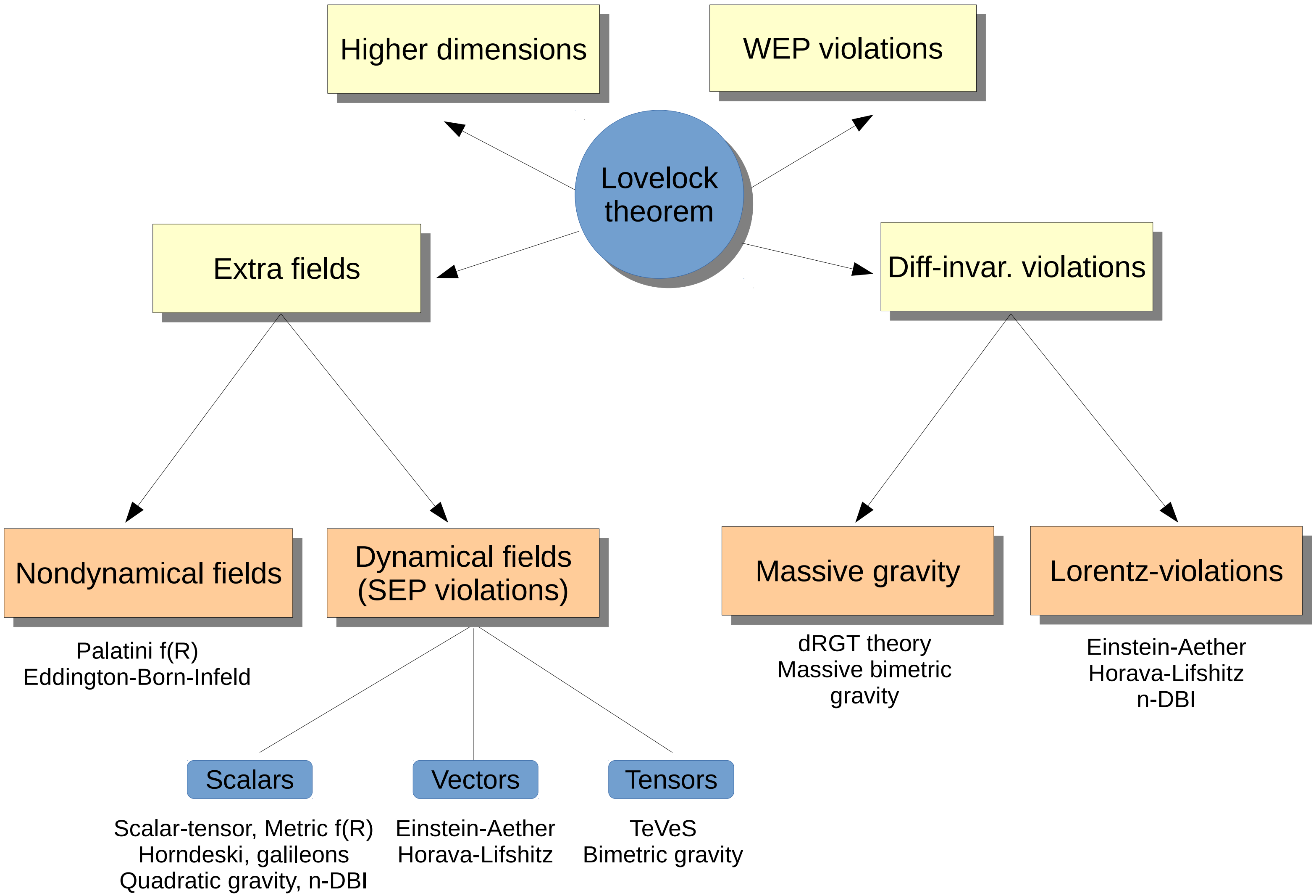
No theory is capable of solving these problems at once so far



Modified gravity roadmap



Most theories can be reduced to GR +scalar fields by the Lovelock Theorem



E. Berti et al. CQG 32 (2015) 243001

Most theories can be reduced to the following classes

Horava-Lifshits Gravity

$$S = \int d^3x\, dt \sqrt{-g} \left\{ \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^4} \left(\nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk} \nabla^j R^{ik} - \frac{1}{8} \nabla_i R \nabla^i R \right) \right\}$$

$$ds^2 = N^2 dt^2 - g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \qquad K^2 = g_{ij} K^{ij}$$

Scalar-Tensor Gravity

$$S = \int \sqrt{-g} \left[f\left(R, R_{\alpha\beta} R^{\alpha\beta}, \phi \right) + \omega(\phi) \nabla_\alpha \phi \nabla^\alpha \phi \right] d^4x,$$

$$Y \equiv R^{\mu\nu} R_{\mu\nu}$$

$\omega(\phi)$ → Kinetic term: general function of ϕ

Both provide the Schwarzschild solution as a particular limit

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2 \theta d\phi^2$$

Scalar-Tensor Gravity

$$S=\int\sqrt{-g}\left[f\left(R,R_{\alpha\beta}R^{\alpha\beta},\phi\right)+\omega(\phi)\nabla_\alpha\phi\nabla^\alpha\phi\right]d^4x$$

Field equations

$$f_R R_{\mu\nu}-\frac{f+\omega(\phi)\nabla^\alpha\phi\nabla_\alpha\phi}{2}g_{\mu\nu}-\nabla_\mu\nabla_\nu f_R+g_{\mu\nu}\Box f_R+2f_Y R^\alpha_\mu R_{\alpha\nu}$$
$$-2f_Y\left(\nabla_\alpha\nabla_\nu R^\alpha_\mu+\nabla_\alpha\nabla_\mu R^\alpha_\nu\right)+\Box\left(f_Y R_{\mu\nu}\right)+g_{\mu\nu}\nabla_\beta\nabla_\alpha\left(f_Y R^{\alpha\beta}\right)+\omega(\phi)\nabla_\mu\phi\nabla_\nu\phi=0$$

Klein-Gordon equation

$$2\omega(\phi)\Box\phi+\omega_\phi(\phi)\nabla_\alpha\phi\nabla^\alpha\phi-f_\phi=0$$

Properties:

- **Explain late and early time evolution without DM and DE**
- **Fit the experimental observations at the astrophysical level**

Horava-Lifshitz Theory

Properties:

- Effective quantum gravity capable of solving the small-scale shortcomings of GR
- Lorentz-Invariance emerges at large distances
- Successfully passes the Solar System Tests

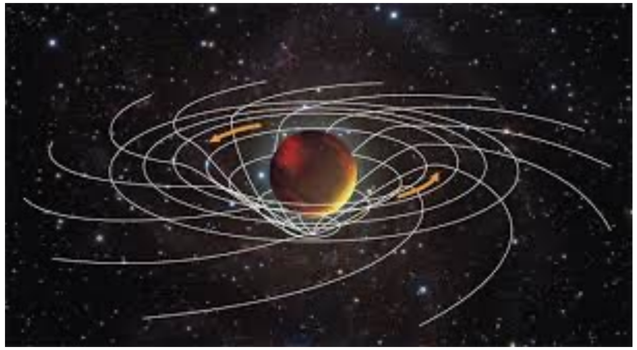
spherically symmetric solution:

$$g_{00} = (g_{11})^{-1} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}$$

$\omega \longrightarrow$ Constant

Schwarzschild solution: $4M/\omega r^3 \ll 1$

Is it possible to find out probes and test-beds for Theories of Gravity?



- *Geodesic motions around compact objects (e.g- SgrA*)*
- **Lense-Thirring effect**
- *Torsion-balance experiments*
- *Microgravity experiments*
- *Free-fall in atomic physics*
- *Violation of Equivalence Principle*
- *effective masses related to further gravitational degrees of freedom*

***G. Tino, L. Cacciapuoti, S. Capozziello et al. Prog.Part.Nucl.Phys.* 112 (2020) 103772**

Lense Thirring Effect

This effect can be obtained starting from a Kerr-like metric

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$ds^2 = \left(1 - \frac{r_S}{r}\right)dt^2 - \frac{1}{1 - \frac{r_S}{r}\frac{J^2}{M^2 r^2}}dr^2 - r^2 d\theta^2 - \left(r^2 + \frac{J^2}{M^2} + \frac{r_S J^2}{M^2 r}\right)d\phi^2 - \frac{2 r_S J}{M r}dt\,d\phi$$

Angular Momentum

Correction to the precession of a gyroscope near a large rotating mass, due to the dragging of the spacetime

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4Mr^3}J$$

Weak-Field limit

Motivations:

Often exact solutions in TGs cannot be found analytically

The typical values of the Newtonian gravitational potential Φ are larger than 10^{-5} in the Solar System (in geometrized units, Φ is dimensionless).

Scheme:

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{2}(\partial_\sigma \partial_\mu h^\sigma_\nu + \partial_\sigma \partial_\nu h^\sigma_\mu - \partial_\mu \partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu} \Box h)$$

Weak field in Scalar-Tensor Gravity

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix} = \begin{pmatrix} 1 + 2\phi + 2\Xi & 2A_i \\ 2A_i & -\delta_{ij} + 2\Psi\delta_{ij} \end{pmatrix}$$

- *Three potentials arise: two scalar potentials and one vector potential*
- *Φ, Ψ are proportional to the power c^{-2} (Newtonian limit) while A_i is proportional to c^{-3} and Ξ to c^{-4} (post-Newtonian limit)*

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

Kerr spacetime

$$g_{00} \equiv \mathcal{A}(t,r,\theta)$$
$$g_{0i} = \mathcal{E}(t,r,\theta)$$
$$g_{ij}\delta^{ij} = \mathcal{B}(t,r,\theta) + \mathcal{C}(t,r,\theta) + \mathcal{D}(t,r,\theta)$$

Weak field in Scalar-Tensor Gravity

By means of the decomposition of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$\begin{array}{ccc} h_{00} & \sim & \mathcal{O}(2) \\ h_{0i} & \sim & \mathcal{O}(3) \\ h_{ij} & \sim & \mathcal{O}(2), \end{array}$$

The function f, up to the c^{-4} order, can be developed as:

$$f(R, R_{\alpha\beta}R^{\alpha\beta}, \phi) = f_R(0,0,\phi^{(0)})R + \frac{f_{RR}(0,0,\phi^{(0)})}{2}R^2 + \frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2}(\phi - \phi^{(0)})^2 + f_{R\phi}(0,0,\phi^{(0)})R\phi + f_Y(0,0,\phi^{(0)})R_{\alpha\beta}R^{\alpha\beta},$$

Weak field in Scalar-Tensor Gravity

Vector potential

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1-(1+m_Y|\mathbf{x}|)\,e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}}\times\mathbf{J}$$

Scalar potential

$$\phi(r) = -\frac{GM}{r}\Big[1+g(\xi,\eta)e^{-m_R\tilde{k}_R r} + [1/3-g(\xi,\eta)]e^{-m_R\tilde{k}_\phi r} - \frac{4}{3}e^{-m_Y r}\Big]$$

with the definitions:

$$m_R^2 = -\frac{1}{3f_{RR}\left(0,0,\phi^{(0)}\right)+2f_Y\left(0,0,\phi^{(0)}\right)}$$

$$m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)} \qquad \eta = \frac{m_\phi}{m_R}$$

$$m_\phi^2 = -\frac{f_{\phi\phi}\left(0,0,\phi^{(0)}\right)}{2\omega\left(\phi^{(0)}\right)} \qquad g(\xi,\eta) = \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$$

$$\xi = \frac{3f_{R\phi}\left(0,0,\phi^{(0)}\right)^2}{2\omega\left(\phi^{(0)}\right)} \qquad \tilde{k}^2_{R,\phi} = \frac{1-\xi+\eta^2\pm\sqrt{\left(1-\xi+\eta^2\right)^2-4\eta^2}}{2}$$

Lense-Thirring precession in Scalar-Tensor Gravity

$$\boldsymbol{\Omega}_{\rm LT}^{\rm (EG)} = \frac{1}{2}(\epsilon^{ijk}\partial_i A_k)(\epsilon_{\ell nk}\partial^\ell A^k) = \frac{G}{r^3}\sqrt{\left(\epsilon_{\ell km}\partial^m \epsilon^{ijk} J_i x_j\right)^2} = -e^{-m_Y r}\left(1 + m_Y r + m_Y^2 r^2\right)\boldsymbol{\Omega}_{\rm LT}^{\rm (GR)}$$

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1 - (1 + m_Y|\mathbf{x}|)\, e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}} \times \mathbf{J} \qquad Y \equiv R^{\mu\nu}R_{\mu\nu}$$

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4Mr^3} J \qquad m_Y^2 = \frac{1}{f_Y\left(0, 0, \phi^{(0)}\right)}$$

For $f_Y \to 0$ i.e. $m_Y \to \infty$, we obtain the same outcome for the gravitational potential of $f(R, \phi)$-theory

Experimental constraints

Experimental constrains by GP-B

$$\Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + {m_Y}^2 r^2)\Omega_{\rm LT}^{\rm (GR)} \qquad \textbf{\textit{and}} \qquad \Omega_{\rm LT}^{\rm (GR)} = \frac{G}{2r^3}\mathbf{J}$$

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)}$$

GR=general relativity
EG= extended gravity

The Gravity Probe B (GP-B) four gyroscopes aboard an Earth-orbiting satellite allowed to measure the frame-dragging effect with an error of about 19%

Effect	Measured (mas/y)	Predicted (mas/y)
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

$$\left|\frac{\Omega^{LT}_{obs} - \Omega^{LT}_{GR}}{\Omega^{LT}_{GR}}\right| = 0.05$$

The changes in the direction of spin gyroscopes, contained in the satellite orbiting at h = 650 km of altitude and crossing directly over the poles, have been measured with extreme precision

Experimental constrains: GP-B

Results

1) $(1 + m_Y r^* + {m_Y}^2 r^{*2})\, e^{-m_Y r^*} \lesssim \frac{\delta|\mathbf{\Omega}_{\rm LT}|}{|\mathbf{\Omega}_{\rm LT}^{\rm (GR)}|} \simeq 0.19$

2) $m_Y \geq 7.3 \times 10^{-7} m^{-1}$

$$m_Y^2 = \frac{1}{f_Y\left(0, 0, \phi^{(0)}\right)}$$

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)}$$

$$\Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + {m_Y}^2 r^2)\Omega_{\rm LT}^{\rm (GR)}$$

Experimental constrains by LARES

The Laser Relativity Satellite (LARES) mission of the Italian Space Agency is designed to test the frame dragging and the Lense-Thirring effect, to within 1% of the value predicted in the framework of GR

The body of this satellite has a diameter of about 36.4 cm and weights about 400 kg

It was inserted in an orbit with 1450 km of perigee, an inclination of 69.5 ± 1 degrees and eccentricity 9.54×10^{-4}

It allows to obtain a **stronger constraint** for m_Y:

$$(1 + m_Y r^* + m_Y{}^2 r^{*2}) e^{-m_Y r^*} \lesssim \frac{\delta|\Omega_{\rm LT}|}{|\Omega_{\rm LT}^{\rm (GR)}|} \simeq 0.01$$

From which we obtain $m_Y \geq 1.2 \times 10^{-6} \mathrm{m}^{-1}$

S. Capozziello, G. Lambiase et al.
Phys.Rev.D 91 (2015) 4, 044012

LARES and GP-B

Summing up, using data from the Gravity Probe B and LARES missions, we obtain constraints on m_Y.

$$\mathbf{\Omega}_{\rm LT} = \mathbf{\Omega}_{\rm LT}^{\rm (GR)} + \mathbf{\Omega}_{\rm LT}^{\rm (EG)} \qquad \Omega_{\rm LT}^{\rm (EG)} = -e^{-m_Y r}(1 + m_Y r + {m_Y}^2 r^2)\Omega_{\rm LT}^{\rm (GR)} \qquad m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)}$$

GP-B

$$m_Y \geq 7.3 \times 10^{-7} m^{-1}$$

LARES

$$m_Y > 1.2 \times 10^{-6} \mathrm{m}^{-1}$$

These results show that space-based experiments can be used to test extensively parameters of fundamental theories

Further limits by GINGER

GINGER: the case of Horava-Lifshitz Gravity

GINGER

Weak field limit in Horava-Lifshitz Gravity

$$S=\int d^3x\,dt\,\sqrt{-g}\left\{\frac{2}{\kappa^2}\left(K_{ij}K^{ij}-\lambda K^2\right)-\frac{\kappa^2}{2w^4}\left(\nabla_i R_{jk}\nabla^i R^{jk}-\nabla_i R_{jk}\nabla^j R^{ik}-\frac{1}{8}\nabla_i R\nabla^i R\right)\right\}$$

$$K_{ij}=\frac{1}{2N}\left(\dot g_{ij}-\nabla_i N_j-\nabla_j N_i\right)\qquad K^2=g_{ij}K^{ij}$$

Linearization of the metric tensor

$$g_{\mu\nu}\sim\begin{pmatrix} 1+g_{00}^{(2)}+g_{00}^{(4)}+\ldots & g_{0i}^{(3)}+\ldots \\ g_{0i}^{(3)}+\ldots & -\delta_{ij}+g_{ij}^{(2)}+\ldots \end{pmatrix}$$

With similar computations as in the previous case, the ratio between the Horava-Lifshitz and General Relativity Gyroscopic precession is

$$\frac{\Omega^G_{HL}}{\Omega^G_{GR}}=\frac{1}{3}\left(1+2\frac{G}{G_N}a_1-2\frac{a_2}{a_1}\right)$$

a_1, a_2 constants *to be constrained*

Ω^G_{HL} → *Gyroscopic precession*

G is the effective gravitational constant

Constraining a_1, a_2

Motivations: It has been shown that, in order for the matter coupling to be consistent with solar system tests, the gauge field and the Newtonian potential must be coupled to matter in a specific way, but there are no indication on how to obtain the precise prescription from the action principle. Recently such a prescription has been generalised and a scalar-tensor extension of the theory has been developed to allow the needed coupling to emerge in the IR regime without spoiling the power-counting renormalizability of the theory.

Matter action

$$S_M = \int dt d^3x \tilde{N} \sqrt{\tilde{g}}\, \mathcal{L}_M\, (\tilde{N}, \tilde{N}_i, \tilde{g}_{ij}; \psi_n)$$

Lapse function

Scalar Potential

$$\begin{aligned} \tilde{N} &= (1 - a_1\sigma)N, \\ \tilde{N}^i &= N^i + N g^{ij}\nabla_j\phi, \\ \tilde{g}_{ij} &= (1 - a_2\sigma)^2 g_{ij}, \end{aligned}$$

Vector

$$\sigma = \frac{A - \mathcal{A}}{N}, \quad \text{with} \quad \mathcal{A} = -\dot{\phi} + N^i\nabla_i\phi + \frac{1}{2}N\nabla^i\phi\nabla_i\phi.$$

a_1, a_2 are then related to the potentials and can be constrained by GINGER

GINGER measures the difference in frequency of light of two beams circulating in a laser cavity in opposite directions. This translates into a time difference between the right-handed beam propagation time and the left-handed one

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}}\, ds^i$$

The difference in time can be linked to the Sagnac frequence $\mathbf{\Omega}_S$, measured by GINGER

Perimeter

Laser wavelength

$$c\delta\tau = N(\lambda_+ - \lambda_-) = Nc\left(\frac{f_- - f_+}{f^2}\right) = \frac{P\lambda}{c}\,\delta f \equiv \frac{P\lambda}{c}\,\Omega_S$$

Wavelength difference

Splitting in terms of frequency between the two beams

P= 20-24 m, wave length = 632 nm

GINGER in Horava-Lifshitz Gravity

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}}\, ds^i \qquad \Omega_S = -\frac{2c^2\sqrt{g_{00}}}{P\lambda} \oint \frac{g_{0i}}{g_{00}}\, ds^i$$

In Horava-Lifshitz, it is

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Sagnac term

Lense-Thirring term

- A → Area encircled by the light beams
- α → Angle between the local radial direction and the normal to the plane of the array-laser ring
- θ → Colatitude of the laboratory
- Ω_E → Rotation rate of the Earth as measured in the local reference frame
- I_E → Momentum of Inertia
- P → Perimeter
- **λ** → Laser wavelength

Horava-Lifshitz vs General Relativity

General Relativity

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E \quad \left[\cos(\theta+\alpha) - \frac{G_N M}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

$G = G_N$

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Horava-Lifshitz Gravity

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E \quad \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}\,a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Advantages of using GINGER

- The precision of GINGERINO is 1/1000 in the geodesic term, 1/100 in the LT term
- GINGER experiment should overcome such uncertainty providing a precision of 1/1000 in the LT term
- The presence of two rings yields a dynamical measurement of the angle **α**

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E\left[\cos(\theta+\alpha) - \underbrace{\left(1+\frac{G}{G_N}\,a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta}_{\text{Geodesic Term}} - \underbrace{\frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha+\sin\theta\sin\alpha\right)}_{\text{LT Term}}\right]$$

- *Measurement of LT term constrains the value of G, measurement of geodesic term constrains a_1 and a_2*
- *The precision of GINGERINO close to 10^{-15} rad/s corresponds to a precision of 1.4 $\cdot\, 10^{-9}$ with respect to the dominant term.*

Perspectives

In $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity, GP-B and LARES satellites provide

$$\dot{m}_Y \geq 7.3 \times 10^{-7} m^{-1} \qquad\qquad m_Y > 1.2 \times 10^{-6}\mathrm{m}^{-1}$$

constraint on m_y by GINGER

constraints on a_1, a_2 by GINGER

In Horava-Lifshitz gravity, the weak-field limit provide

$$\begin{aligned} c\,\delta\tau &= \frac{4A\Omega_E}{c}\left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\theta\sin\alpha\right. \\ &- \left.\frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right] \end{aligned}$$

Perspectives

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E\left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}\,a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

- Fixing a_1 and a_2 by GINGER allows to retain or reject viable theories
- GINGER could select effective models for Quantum Gravity in the weak field limit
- With respect to satellite experiments, results can be tuned and reproduced.

S. Capozziello, C. Altucci, F. Bajardi, A.Di Virgilio et al... Euro. Phys. J. Plus 136 (2021) 5

A.Di Virgilio, U. Giacomelli, A. Simonelli et al... Euro. Phys. J. C 81 (2021) 457

A. Porzio, C. Altucci, S. Capozziello, R. Velotta, et al... PoS Corfù 2017 (2018) 181