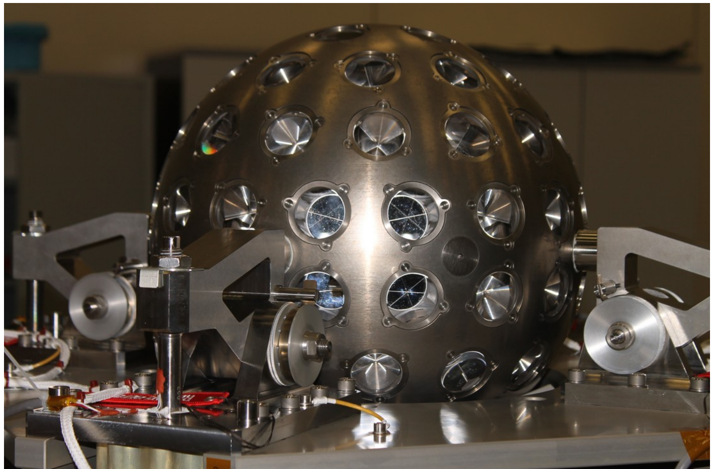
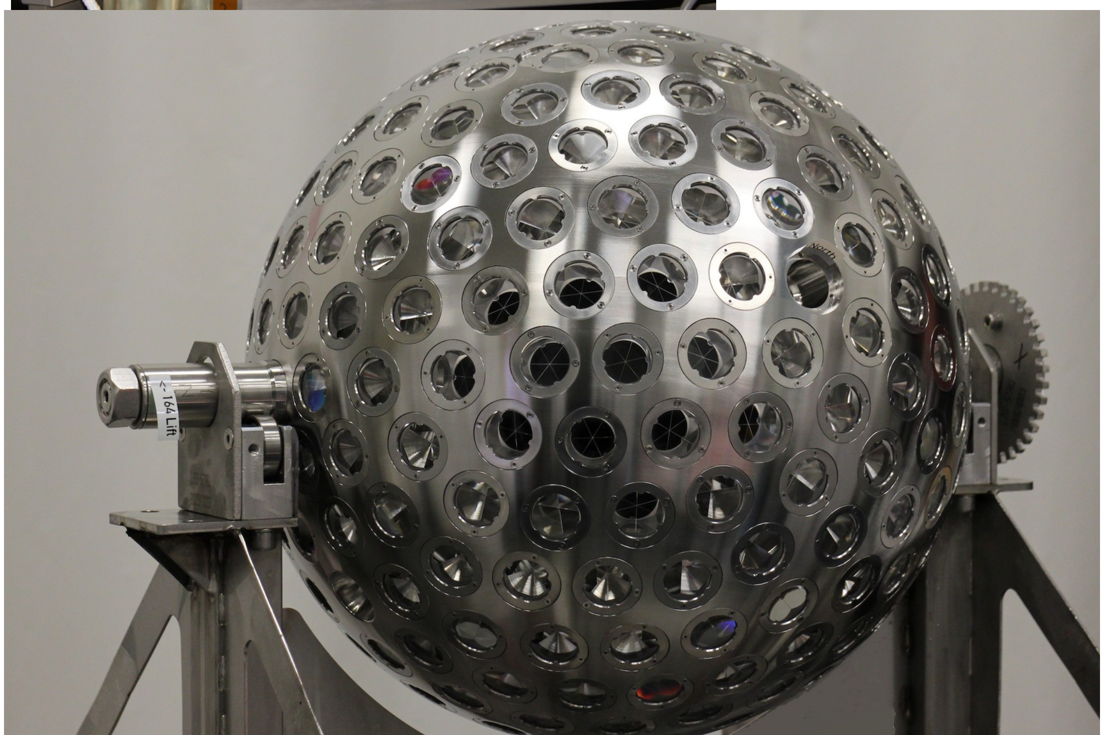
The LARES and LARES 2 satellites
for tests of gravitational physics and frame-dragging

Ignazio Ciufolini,
Antonio Paolozzi
and the LARES team

LARES

IV GRM Workshop, Pisa, 14-16 June 2023.





Ignazio Ciufolini

- *LARES (ASI, 2012)*

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- *LARES 2 (ASI, 2022)*


LARES
vega-c

OVERVIEW

- **Brief introduction on frame-dragging (dragging of inertial frames) in theoretical physics**

- **The LARES satellite (2012) and its main results:**

TEST of frame-dragging

with *about* 3% to 1% accuracy (2019/2021)

and

TEST of weak equivalence principle at a new range

and with new materials with 10^{-9} accuracy (2019).

- **The LARES 2 satellite (2022) and its objectives**

Characterization of frame-dragging by scalar curvature invariants

To distinguish between intrinsic frame-dragging phenomena generated by the angular momentum of a spinning body (Kerr metric) and effects simply due to the motion on a static background (de Sitter effect), we proposed (I.C. 1994, I.C. and Wheeler 1995) to use the Chern-Pontryagin pseudo-invariant, that, for the Kerr metric, is:

$$\frac{1}{2}\, \varepsilon_{\alpha\beta\sigma\rho}\, R^{\sigma\rho}{}_{\mu\nu}\, R^{\alpha\beta\mu\nu} = 1536\ J\ M \cos\theta\ (r^5 r^{-6} - r^3 r^{-5} + 3/16\ r\ r^{-4})$$

In weak-field and slow-motion:

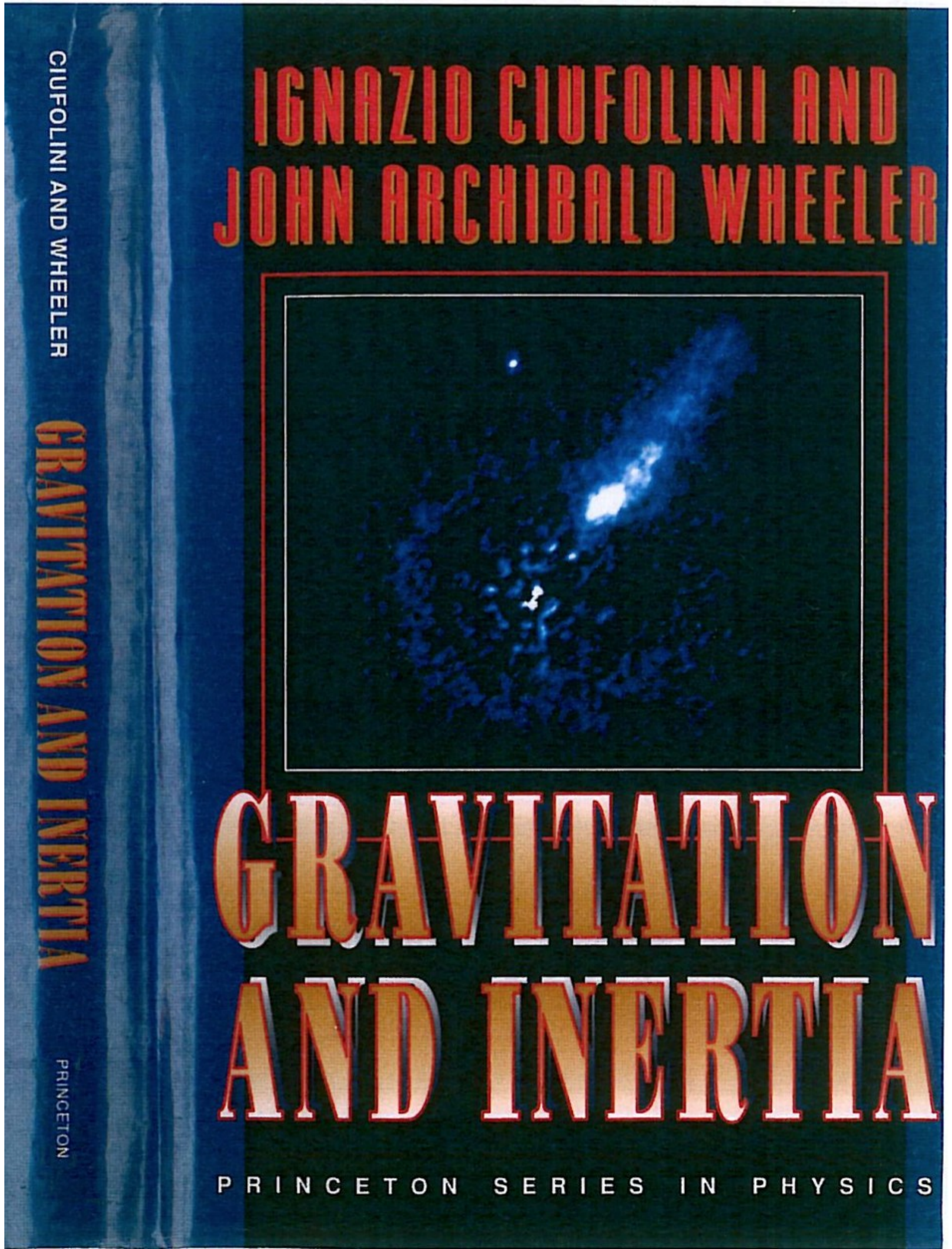
$$^{*}R \cdot R = 288\ (J\ M)/r^7 \cos\theta\ + \cdots$$

Where J = aM = angular momentum

$^{*}R \cdot R$ is also different from zero in the case of two massive bodies moving with respect to each other (calculated using the PPN metric).

See also: A. Matte, Canadian J. Math. 5, 1 (1953), K. Thorne (1986) and B. Mashhoon (2008)

For a comprehensive characterization of gravitomagnetism, using scalar curvature invariants, see: F. Costa, L. Wylleman and J. Natario (2021)


CIUFOLINI AND WHEELER
GRAVITATION AND INERTIA
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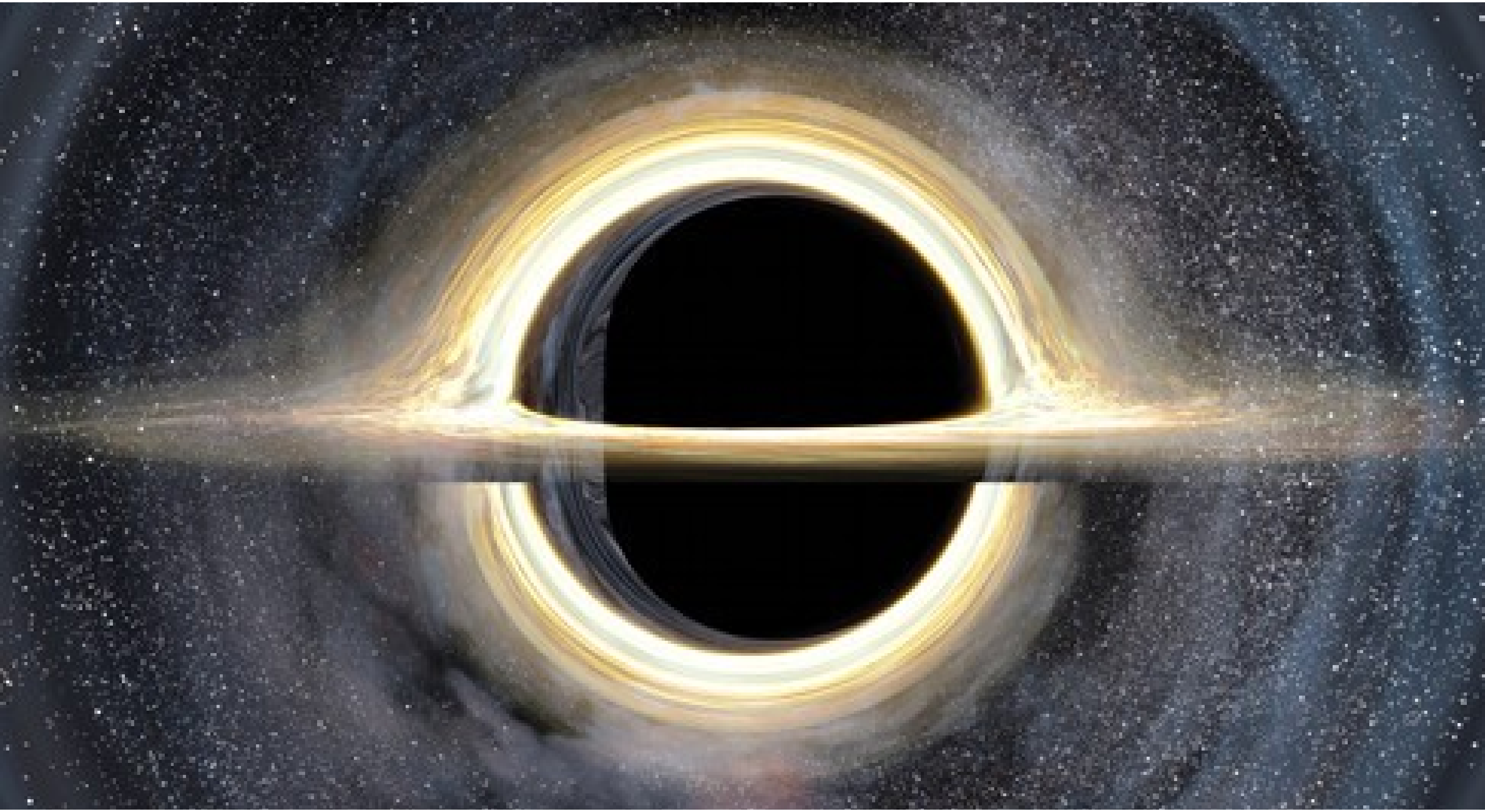
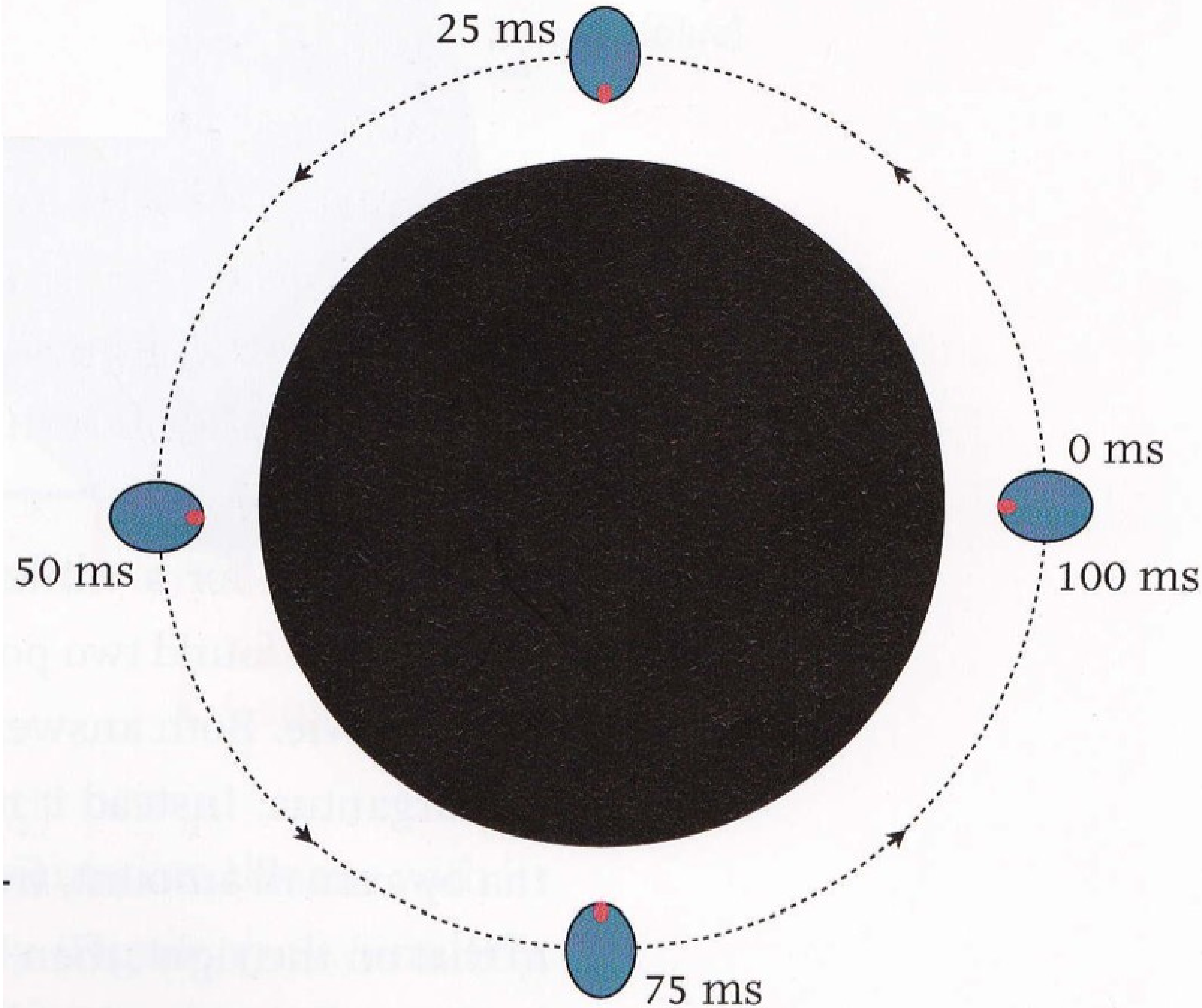
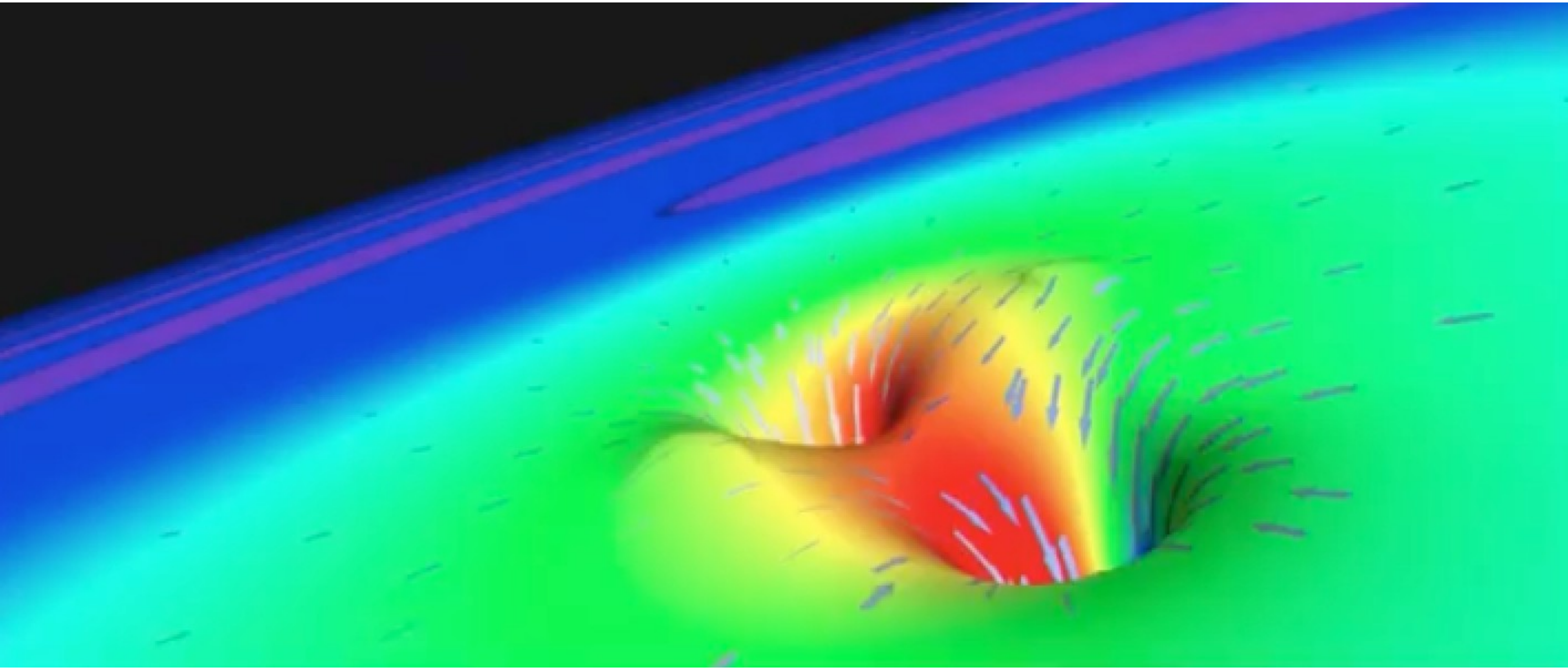
IGNAZIO CIUFOLINI AND
JOHN ARCHIBALD WHEELER
GRAVITATION
AND INERTIA
PRINCETON SERIES IN PHYSICS



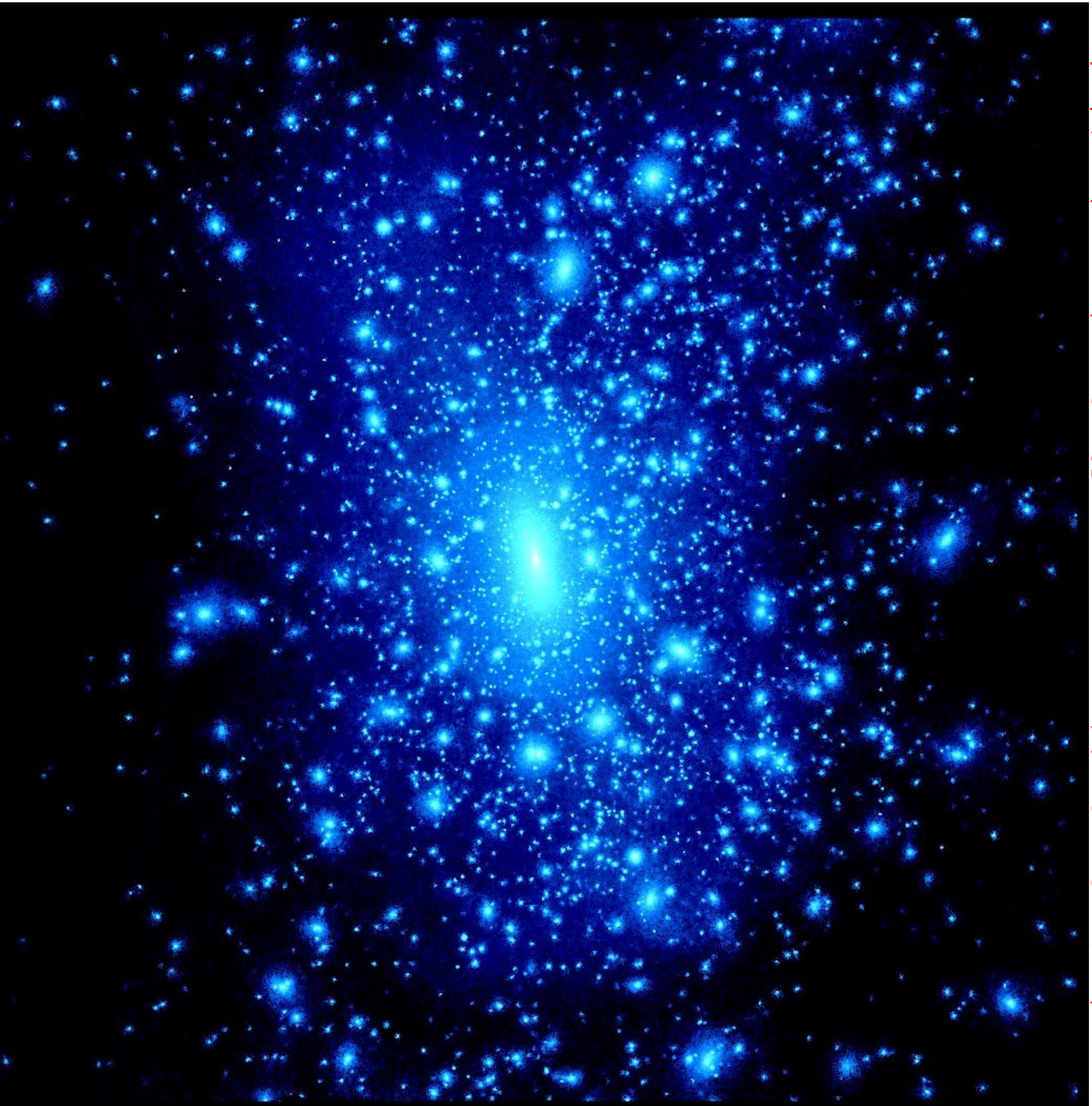
GARGANTUA: a supermassive rotating black hole: frame-dragging


25 ms
0 ms
100 ms
50 ms
75 ms

Frame-dragging and Kerr metric play a key role in the analysis of the emission of gravitational waves due to the coalescence of two spinning black holes to form a Kerr spinning black hole



From the 2017 Nobel Prize talk of Kip Thorne



A great and fascinating XX century discovery: the accelerating supernovae; dark energy or quintessence + dark matter might constitute about 95 % of the universe!

In *Chern-Simons gravity* there is a scalar field possibly related to quintessence

(Alexander and Yunes. *Phys. Rev. Lett.* 2007;
Smith, Erickcek, Caldwell and Kamionkowski. *Phys. Rev. D* 2008).

The Chern-Simons field equation is:

$$G_{\alpha\beta} - \frac{16\pi}{3} l C_{\alpha\beta} = 8\pi T_{\alpha\beta} \, ,$$

Where C_{ab} is the Cotton-York tensor:

$$C^{\alpha\beta} = \frac{1}{2}\left[(\partial_\sigma\theta)\left(\epsilon^{\sigma\alpha\mu\nu}\nabla_\mu R^\beta_\nu + \epsilon^{\sigma\beta\mu\nu}\nabla_\mu R^\alpha_\nu\right) + \nabla_\rho(\partial_\sigma\theta)\left({}^*R^{\rho\alpha\sigma\beta} + {}^*R^{\rho\beta\sigma\alpha}\right)\right]$$

Chern-Simons Gravity

The modified action of Chern–Simons theory is then:

$$S_{CS} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}R - \frac{l}{12}\theta^*\mathbf{R}\cdot\mathbf{R} - \frac{1}{2}(\partial\theta)^2 - V(\theta) + L_{mat}\right]$$

$^*\mathbf{R}\cdot\mathbf{R} = \frac{1}{2}\,\varepsilon_{\alpha\beta\sigma\rho}\,R^{\sigma\rho}{}_{\mu\nu}\,R^{\alpha\beta\mu\nu}$ is the Chern-Pontryagin pseudoscalar, θ is a scalar field, g the determinant of the metric, R the Ricci scalar, l is a new length parameter, L_{mat} the matter Lagrangian density.

In the weak-field and slow-motion approximation we then get:

$$\Delta h_{0i} + \frac{1}{m_{cs}} \Box H_i \cong 16\pi\rho v^i$$

where:

$$\mathbf{H} = \nabla \times \mathbf{h}$$

For a homogeneous sphere with mass density ρ, of radius R, rotating with angular velocity **ω**, outside the sphere we have:

$$\mathbf{H} = \mathbf{H}_{GR} + \mathbf{H}_{CS}$$

Where the General Relativity contribution is:

$$\mathbf{H_{GR}} = \frac{-16\pi G\rho R^5}{15r^3}[2\omega + 3\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \omega)]$$

And the Chern-Simons contribution is:

$$\mathbf{H_{CS}} = -16\pi G\rho R^2 \{D_1(r)\omega + D_2(r)\hat{\mathbf{r}} \times \omega + D_3(r)\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \omega)\}$$

where:

$$D_1(r) = \frac{2R}{r} j_2(m_{cs}R) y_1(m_{cs}r) \,,$$

$$D_2(r) = m_{cs}R j_2(m_{cs}R) y_1(m_{cs}r) \,,$$

$$D_3(r) = m_{cs}R j_2(m_{cs}R) y_2(m_{cs}r) \,,$$

By integrating the Lorentz force equation for a test particle:

$$m\frac{d^2\mathbf{x}}{dt^2} \cong m\left(\mathbf{G} + \frac{d\mathbf{x}}{dt} \times \mathbf{H}\right)$$

We find the ratio of the nodal drag of an orbiting test particle with semimajor axis *a*, between Chern-Simons gravity and General Relativity (for a homogeneous sphere with mass density ρ, of radius R, rotating with angular velocity **ω)**:

$$\frac{\dot{\Omega}_{\rm CS}}{\dot{\Omega}_{\rm GR}} = 15\frac{a^2}{R^2} j_2(m_{\rm CS}R) y_1(m_{\rm CS}a),$$

Where j_2 and y_1 are spherical Bessel functions and m_{cs} is the Chern-Simons mass:

$$m_{\rm cs} \equiv -3/(\ell \kappa^2 \dot{\theta}).$$

where $k^2 = 8\pi$

$\dot{\theta}$ may be related to quintessence

- **Chern-Simons gravity is equivalent to a type of String Theory (**Smith, Erickcek, Caldwell and Kamionkowski Phys. Rev. D 2008). In Smith et al. is shown that the 4-D string action for a type of string theory may reduce to the Chern-Simons gravity action. See also: Yagi K., Yunes N. and Tanaka T., Phys. Rev. D., 86 (2012) 044037 and references therein.
- **Then, on the basis of our 2004-2010 measurements of frame-dragging, using the LAGEOS satellites, in 2008, Smith, Erickcek, Caldwell and Kamionkowski (Phys. Rev. D 77, 024015, 2008) have placed limits on some possible low-energy consequences of string theory that may be related to dark energy and quintessence.**
- **There is a *new* PPN (Post-Newtonian Parametrized) parameter of Chern-Simons theory with limits from LAGEOS, LARES and GP-B.** S. **Alexander and N. Yunes. "New Post-Newtonian Parameter to Test Chern-Simons Gravity." *Physical review letters* (2007)**
- Radicella, Lambiase, Parisi, and Vilasi, Constraints on Covariant Horava-Lifshitz gravity from frame-dragging experiment, JCAP (2014)
- S. Alexander and N. Yunes “Chern-Simon Modified General Relativity”, Physics Reports, Volume 480, 2009, p. 1-55.
- T. Clifton, P.Ferreira, A. Padilla and C. Skordis, “Modified Gravity and Cosmology”.
- K. Yagi, N. Yunes and T. Tanaka, Phys. Rev. D., 86 (2012) 044037.

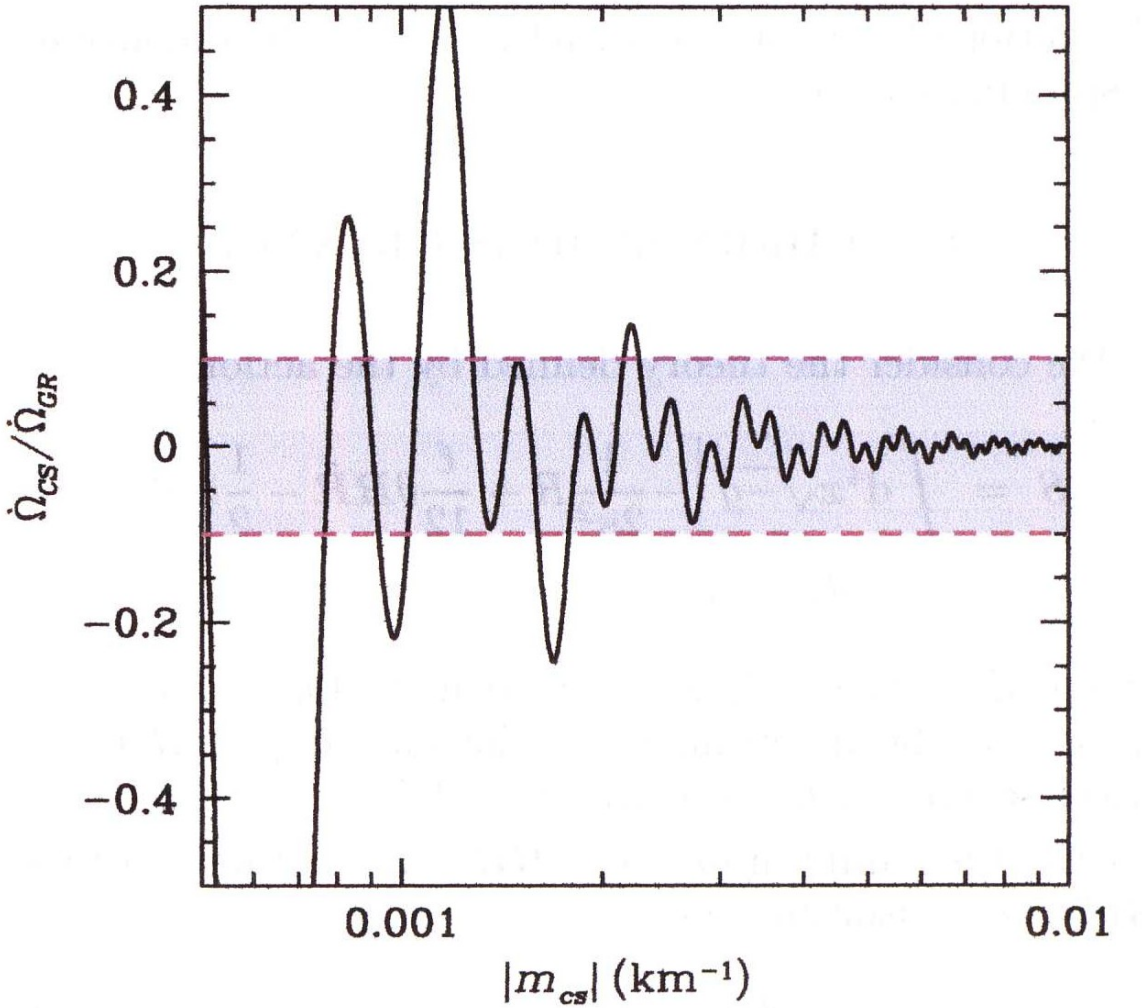


FIG. 1: The ratio $\dot{\Omega}_{\rm CS}/\dot{\Omega}_{\rm GR}$ for the LAGEOS satellites orbiting with a semimajor axis of $a \approx 12,000$ km. A 10% verification of general relativity [16] (the shaded region) leads to a lower limit on the Chern-Simons mass of $|m_{\rm cs}| \gtrsim 0.001$ km^{-1}. A 1% verification of the Lense-Thirring drag will improve this bound on $m_{\rm cs}$ by a factor of roughly five.

$m_{cs} \geq 2 \times 10^{-22}$ GeV

A brief history of the main tests of frame-dragging

GRAVITY PROBE B: since 1960 the GRAVITY PROBE B space mission was under development in USA with the goal of a 0.1% test of frame-dragging. Gravity Probe B was then launched in 2004.

LAGEOS (LAser GEOdynamics Satellite) was launched in 1976 by NASA for space geodetic measurements.

LAGEOS 3: in 1984-1989, we proposed a new laser-ranged satellite called "LAGEOS 3", identical to the LAGEOS satellite (launched in 1976 by NASA) with orbital parameters identical to those of LAGEOS but a *supplementary inclination*, that is with *inclination I = 70.16 °* and *semimajor axis = 12270 km*. A number of ASI and NASA studies confirmed its feasibility to measure frame-dragging (I.C. 1984/1986/1989, B.Tapley, I.C et al. NASA/ASI study 1989/1990, J. Ries 1989 …).

LAGEOS 2: the LAGEOS 2 satellite was launched in 1992 by ASI and NASA for space geodetic measurements.

LAGEOS and LAGEOS 2, 1997/1998: the first *rough observation* of frame-dragging was obtained using the data of LAGEOS and LAGEOS 2 (*I.C. et al. CQG 1997*, *Science 1998*).
However, the use of the perigee of LAGEOS II introduces

GRACE, 2002: the DLR (GFZ) and NASA (CSR) space mission GRACE was launched to accurately measure the Earth's gravity field.

LAGEOS and LAGEOS 2, 2004-2010: the first measurement (with accuracy of about 10%) of frame-dragging was published (*I.C. et al. Nature 2004, General Relativity and J.A Wheeler book 2010, etc.*) using GEODYN. *Independently* confirmed by the Univ. of Texas at Austin (2008/2009, with UTOPIA) and GFZ-DLR (2010, with EPOSOC).

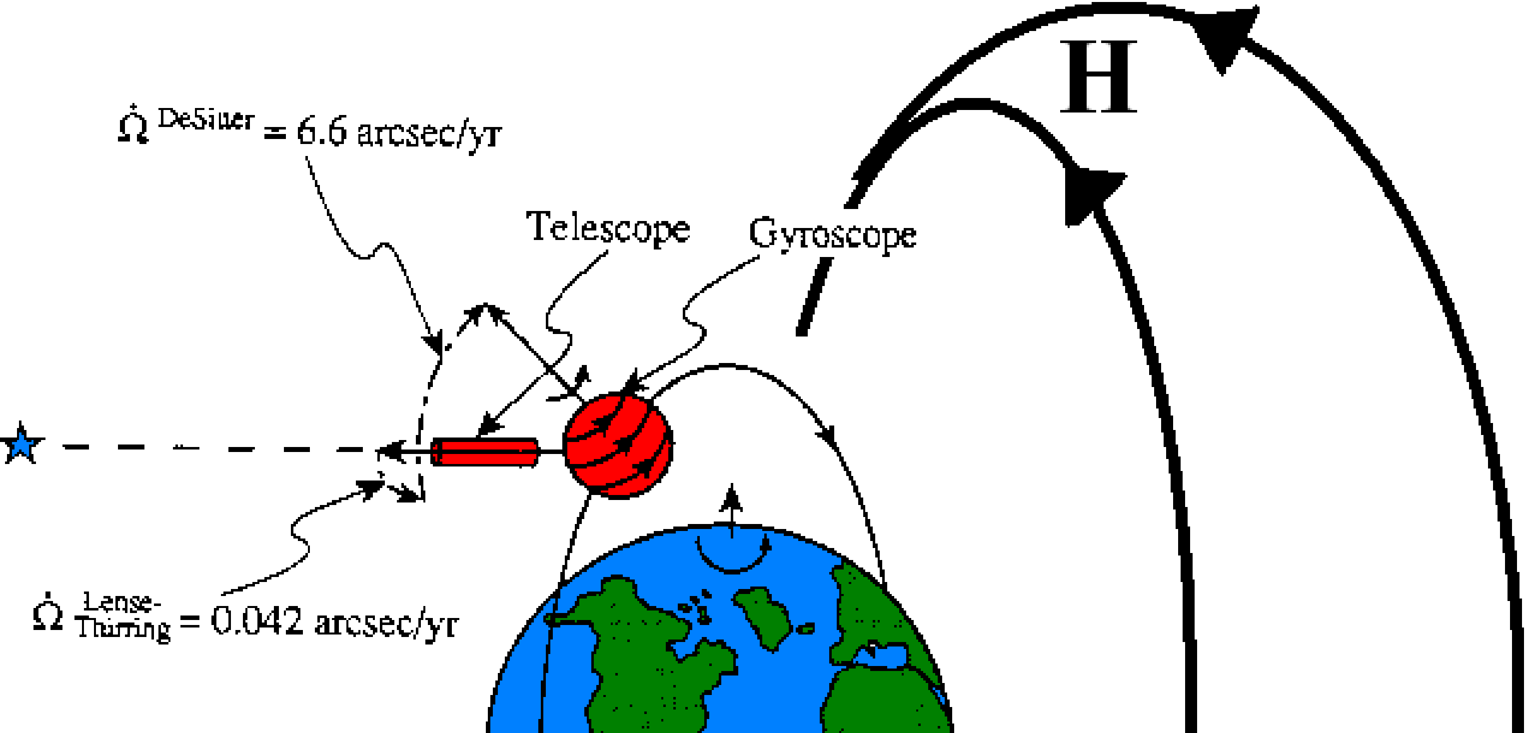
Gravity Probe result, 2011-2015: a measurement of frame-dragging was published with about 19% accuracy (*Phys. Rev. Lett. 2011 and CQG 2015*).

LARES first results, with LAGEOS and LAGEOS 2, 2016: a measurement of frame-dragging was published with approximately 5% accuracy (I.C. et al. *Eur. Phys. J. C, 2016*).

LARES 7-year results, with LAGEOS and LAGEOS 2: a measurement of frame-dragging was published with accuracy between 2% and 1% (I.C. et al. *Eur. Phys. J. C, 2019*).
Independently confirmed in 2020 by Lucchesi et. al.

LARES 10-year results, with LAGEOS and LAGEOS 2: a recent measurement of frame-dragging with accuracy between 2% and 1% (I.C. et al., 2023).

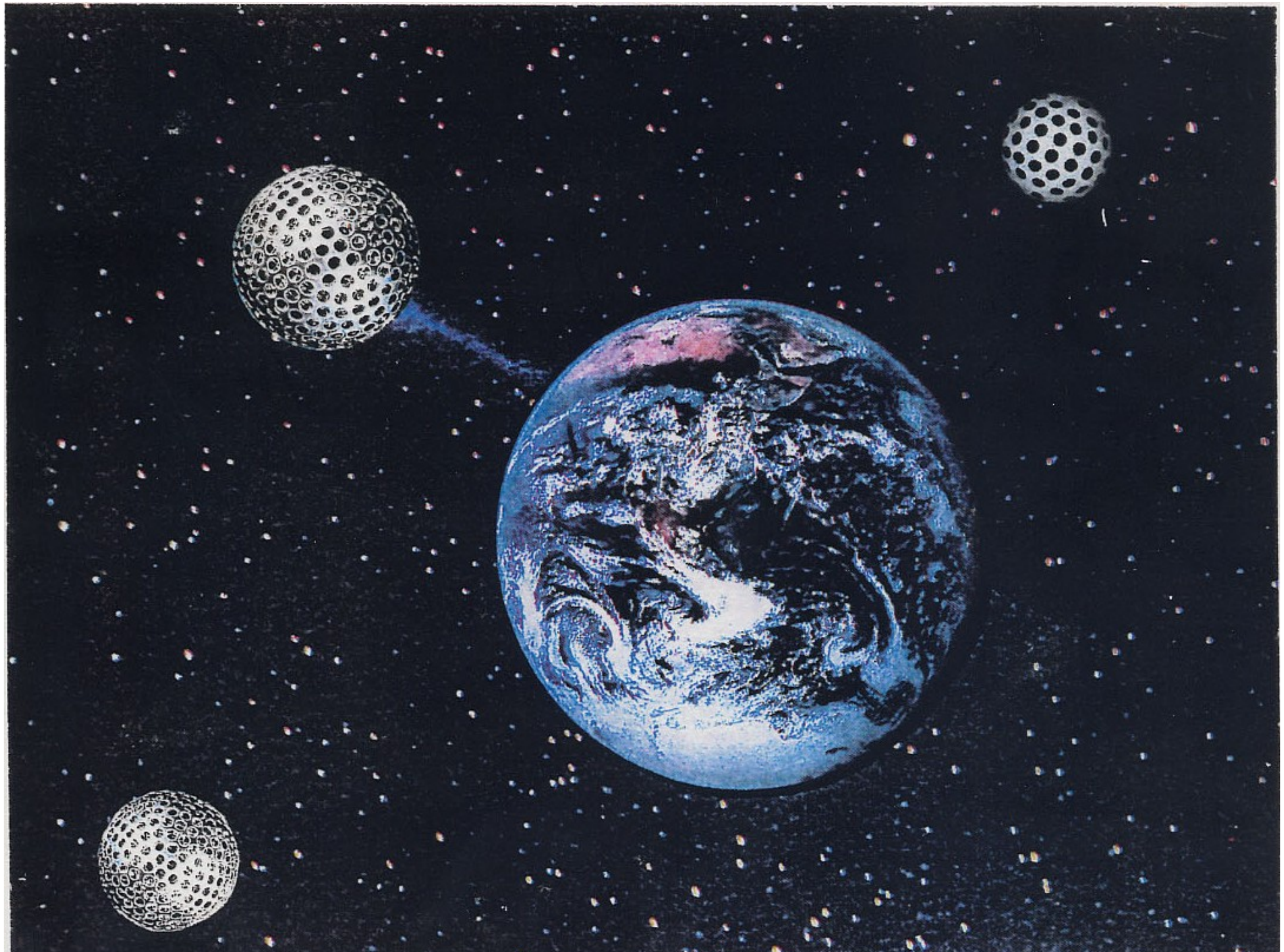
GRACE-FO, was launched on May 22, 2018



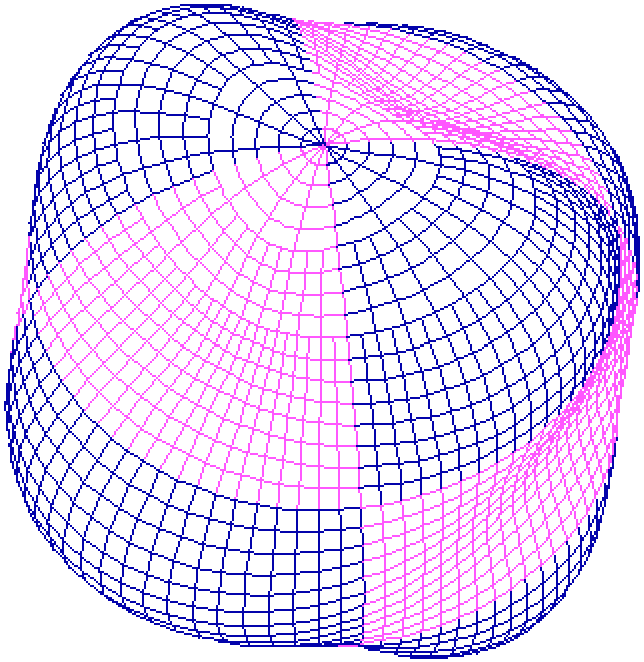
GRAVITY PROBE B, 1960-2015

GRAVITY PROBE B

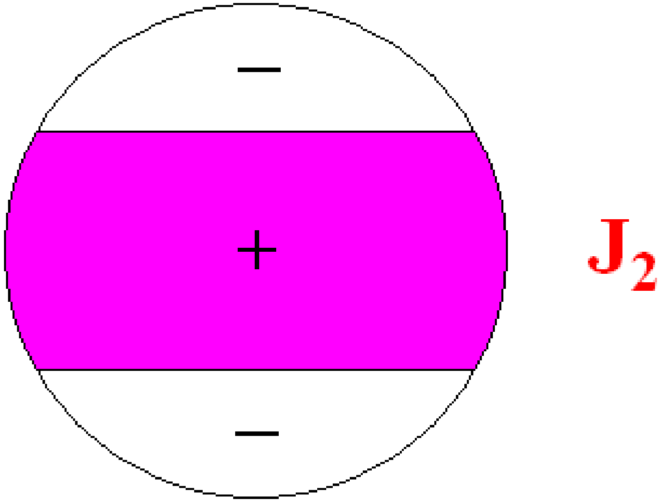
Satellite Laser Ranging
(and Lunar Laser Ranging)

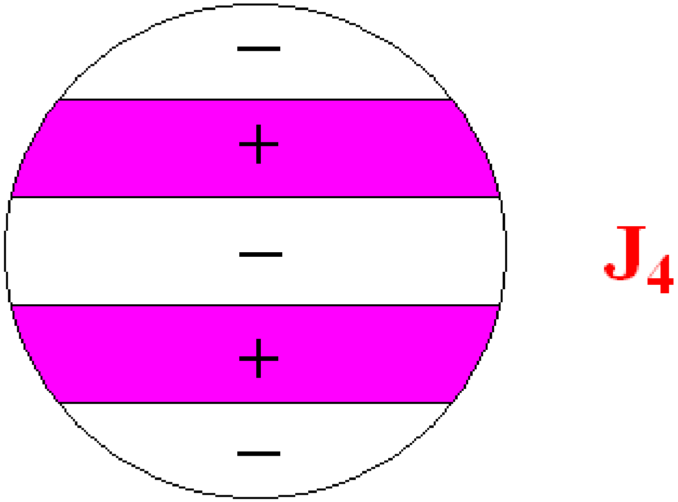


l=3, m=1



EVEN ZONAL HARMONICS





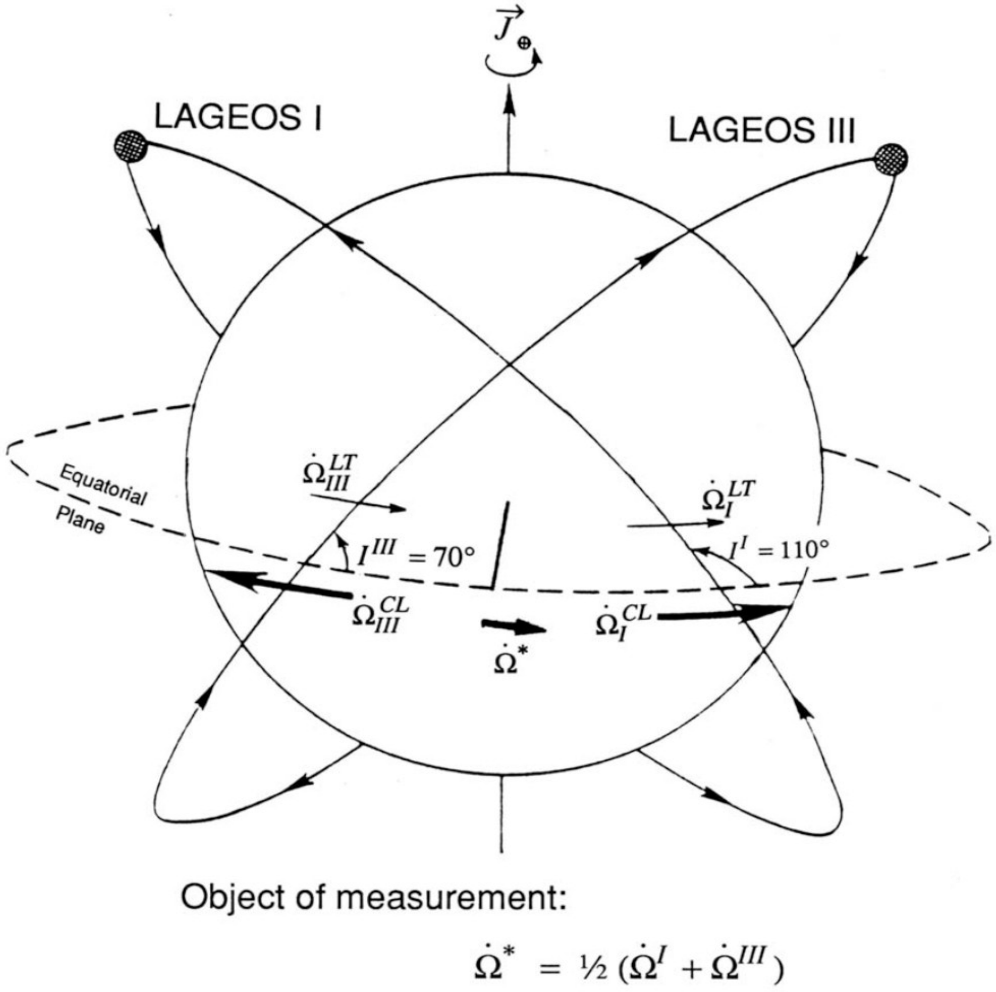
Using two satellites with supplementary inclinations, one can eliminate all the uncertainty due to all the even zonal harmonics J_{2n} (of even degree and zero order), i.e. all the axially symmetric deviations of the Earth potential from spherical symmetry also symmetric with respect to the Earth's equatorial plane.

The classical rate of change of the node of a satellite is a function of its orbital parameters, *a*, *I*, *e*, and Earth's parameters: mass, radius and even zonal harmonics J_2, J_4, …

$$\dot{\Omega}_{Class} = -\frac{3}{2}\, n\, \frac{\cos I}{(1-e^2)^2}\left\{ J_2 \left(\frac{R_\oplus}{a}\right)^2 + J_4 \left(\frac{R_\oplus}{a}\right)^4 \left[\frac{5}{8}\,(7\,\sin^2 I - 4)\,\frac{(1+\frac{3}{2}e^2)}{(1+e^2)^2}\right]\right\}$$

Whereas frame-dragging does **not** depend on the inclination *I* of a satellite

$$\dot{\Omega}^{\text{Lense–Thirring}} = \frac{2J}{a^3(1-e^2)^{3/2}}$$



The idea of the LARES 2/LAGEOS 3 experiment: **I.C. Phys. Rev. Lett. 1986,** I.C. Ph.D. dissertation 1984, I.C. IJMPA 1989, B. Tapley, I.C. et al, NASA and ASI studies 1989, J. Ries 1989).

VOLUME 56, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JANUARY 1986

Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

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(Received 16 October 1984; revised manuscript received 19 April 1985)

We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

PACS numbers: 04.80.+z

In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ precesses at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\boldsymbol{\Omega}} \times \mathbf{S}$ where

$$\dot{\boldsymbol{\Omega}} \equiv -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \tag{2}$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} \equiv d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static —$g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

All these effects are quite small for an artificial satellite orbiting the Earth.

We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8–10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field —quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2 (7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \dots\right\}, \tag{3}$$

IC, PRL 1986: Use of the nodes of two laser-ranged satellites to measure the Lense-Thirring effect

International Journal of Modern Physics A, Vol. 4, No. 13 (1989) 3083–3145

A COMPREHENSIVE INTRODUCTION TO THE LAGEOS GRAVITOMAGNETIC EXPERIMENT: FROM THE IMPORTANCE OF THE GRAVITOMAGNETIC FIELD IN PHYSICS TO PRELIMINARY ERROR ANALYSIS AND ERROR BUDGET

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Revised 7 October 1988

The existence of the gravitomagnetic field, generated by mass currents according to Einstein geometrodynamics, has never been proved. The author of this paper, after a discussion of the importance of the gravitomagnetic field in physics, describes the experiment that he proposed in 1984 to measure this field using LAGEOS (Laser geodynamics satellite) together with another non-polar, laser-ranged satellite with the same orbital parameters as LAGEOS but a supplementary inclination.

The author then studies the main perturbations and measurement uncertainties that may affect the measurement of the Lense-Thirring drag. He concludes that, over the period of the node of ~ 3 years, the maximum error, using two nonpolar laser ranged satellites with supplementary inclinations, should not be larger than ~ 10% of the gravitomagnetic effect to be measured.

Table of Contents

IC, IJMPA 1989: Analysis of the orbital perturbations affecting the nodes of LAGEOS-type satellites

(1) Use two LAGEOS satellites with supplementary inclinations to eliminate the effect of all the J_{2n}

OR:

Use n satellites of LAGEOS-type to measure the first n-1 even zonal harmonics: J_2, J_4, … and the frame-dragging effect (IC IJMPA 1989)

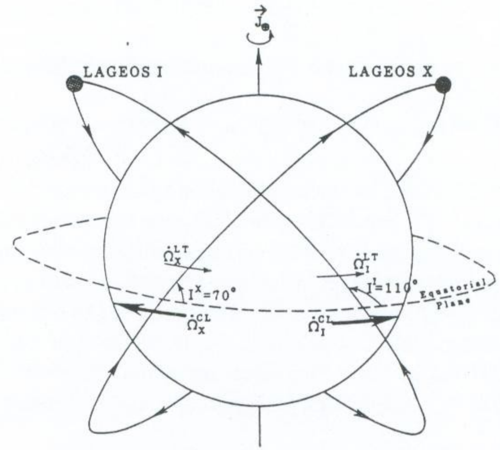

Fig. 5. The LAGEOS and LAGEOS *X* orbits and their classical and gravitomagnetic nodal precessions. A new[17] configuration to measure the Lense-Thirring effect.

For J_2, this corresponds, from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore, the uncertainty in $\dot{\Omega}^{\text{Class}}_{\text{Lageos}}$ is more than ten times larger than the Lense-Thirring precession.

A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (3.2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero. As mentioned before, Yilmaz proposed the use of polar satellites in 1959.[40,41] In 1976, Van Patten and Everitt[46,47] proposed an experiment with two drag-free, guided, counter-rotating, polar satellites to avoid inclination measurement errors.

A new solution[15,16,17,21,22,23] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS (see Fig. 5). Therefore, "LAGEOS X" should have the following orbital parameters:

$$I^X \cong \pi - I^I \cong 70°\,, \qquad a^X \cong a^I\,, \qquad e^X \cong e^I\,. \tag{3.3}$$

With this choice, since the classical precession $\dot{\Omega}^{\text{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\text{Class}}$ would be equal and opposite for the two satellites:

$$\dot{\Omega}^{\text{Class}}_X = -\dot{\Omega}^{\text{Class}}_I\,. \tag{3.4}$$

By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$ is independent of the inclination (Eq. (3.1)), $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites:

IL NUOVO CIMENTO Vol. 109 A, N. 12 Dicembre 1996

On a new method to measure the gravitomagnetic field using two orbiting satellites

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(ricevuto il 20 Settembre 1996; approvato il 15 Novembre 1996)

Summary. — We describe a new method to obtain the first direct measurement of the Lense-Thirring effect, or dragging of inertial frames, and the first direct detection of the gravitomagnetic field. This method is based on the observations of the orbits of the laser-ranged satellites LAGEOS and LAGEOS II. By this new approach one achieves a measurement of the gravitomagnetic field with accuracy of about 25%, or less, of the Lense-Thirring effect in general relativity.

PACS 11.90 – Other topics in general field and particle theory.
PACS 04.80.Cc – Experimental test of gravitational theories.

1. – The gravitomagnetic field, its invariant characterization and past attempts to measure it

Einstein's theory of general relativity [1, 2] predicts the occurrence of a «new» field generated by mass-energy currents, not present in classical Galilei-Newton mechanics. This field is called the gravitomagnetic field for its analogies with the magnetic field in electrodynamics.

In general relativity, for a stationary mass-energy current distribution $\varrho_m \boldsymbol{v}$, in the weak-field and slow-motion limit, one can write [2] the Einstein equation in the Lorentz gauge: $\Delta \boldsymbol{h} \cong 16\pi\varrho_m \boldsymbol{v}$, where $\boldsymbol{h} \equiv (h_{01}, h_{02}, h_{03})$ are the $(0i)$-components of the metric tensor; $\boldsymbol{h}$ is called the gravitomagnetic potential. For a localized, stationary mass-energy distribution, in the weak-field and slow-motion limit, we can then write: $\boldsymbol{h} \cong -2((\boldsymbol{J} \times \boldsymbol{x})/r^3)$, where $\boldsymbol{J}$ is the angular momentum of the central body. In general relativity, one can also define [2] a gravitomagnetic field $\boldsymbol{H}$ given by $\boldsymbol{H} = \nabla \times \boldsymbol{h}$.

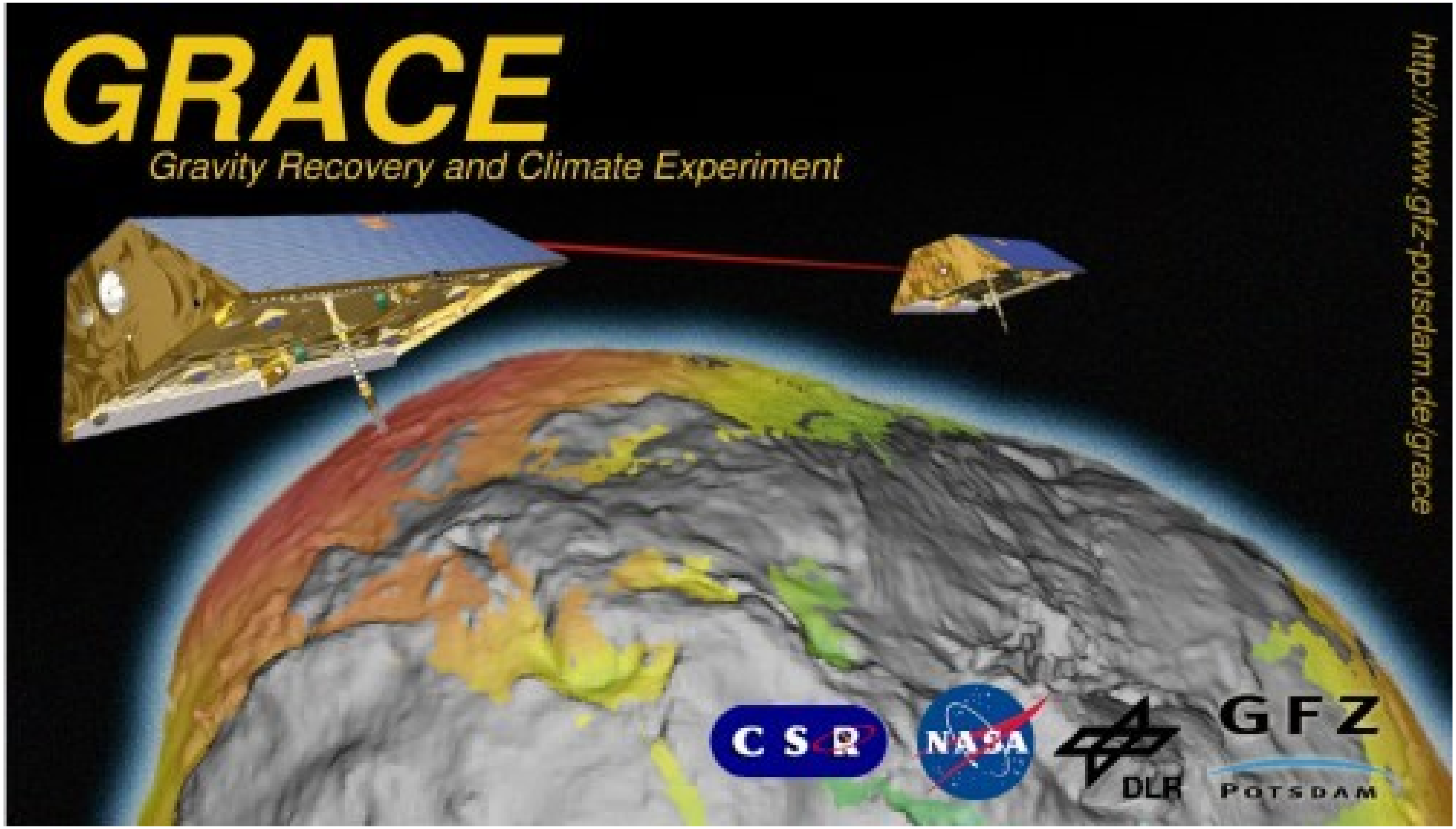
The Lense-Thirring effect is a consequence of the gravitomagnetic field and consists of a tiny perturbation of the orbital elements of a test particle due to the angular momentum of the central body. To characterize the gravitomagnetic field generated by the angular momentum of a body, and the Lense-Thirring effect, and distinguish it from other relativistic phenomena, such as the de Sitter effect, due to the

1709

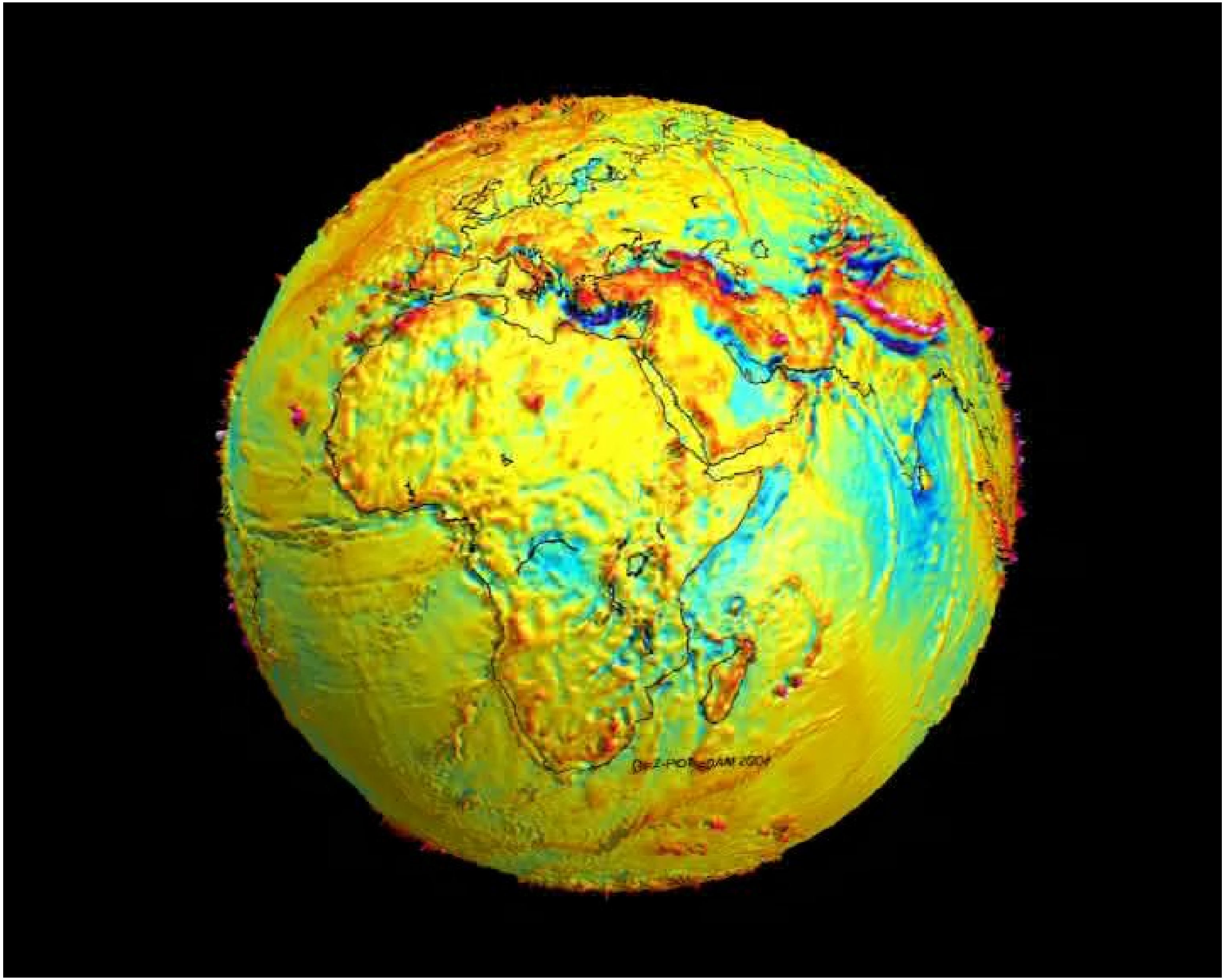
IC NCA 1996:
use the node of LAGEOS and the node of LAGEOS II to measure the Lense-Thirring effect

However, in 1996 the two nodes were not enough to measure the Lense-Thirring effect

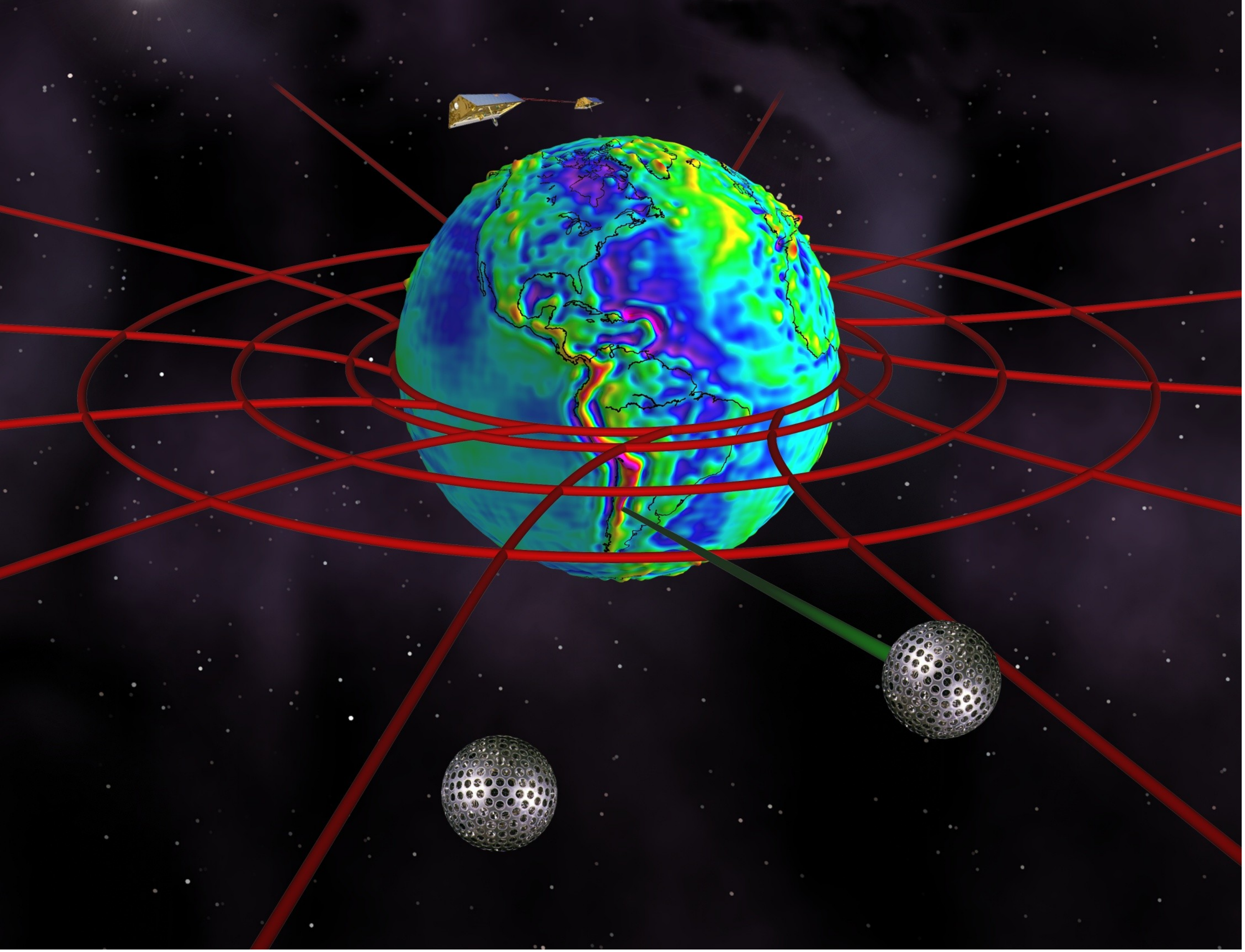
The LAGEOS 3 satellite was never funded but in 2002 the GRACE space mission was launched.



Use of GRACE to test Lense-Thirring at a few percent level:
J. Ries, et al. 2003 (1999), E. Pavlis 2002 (2000)

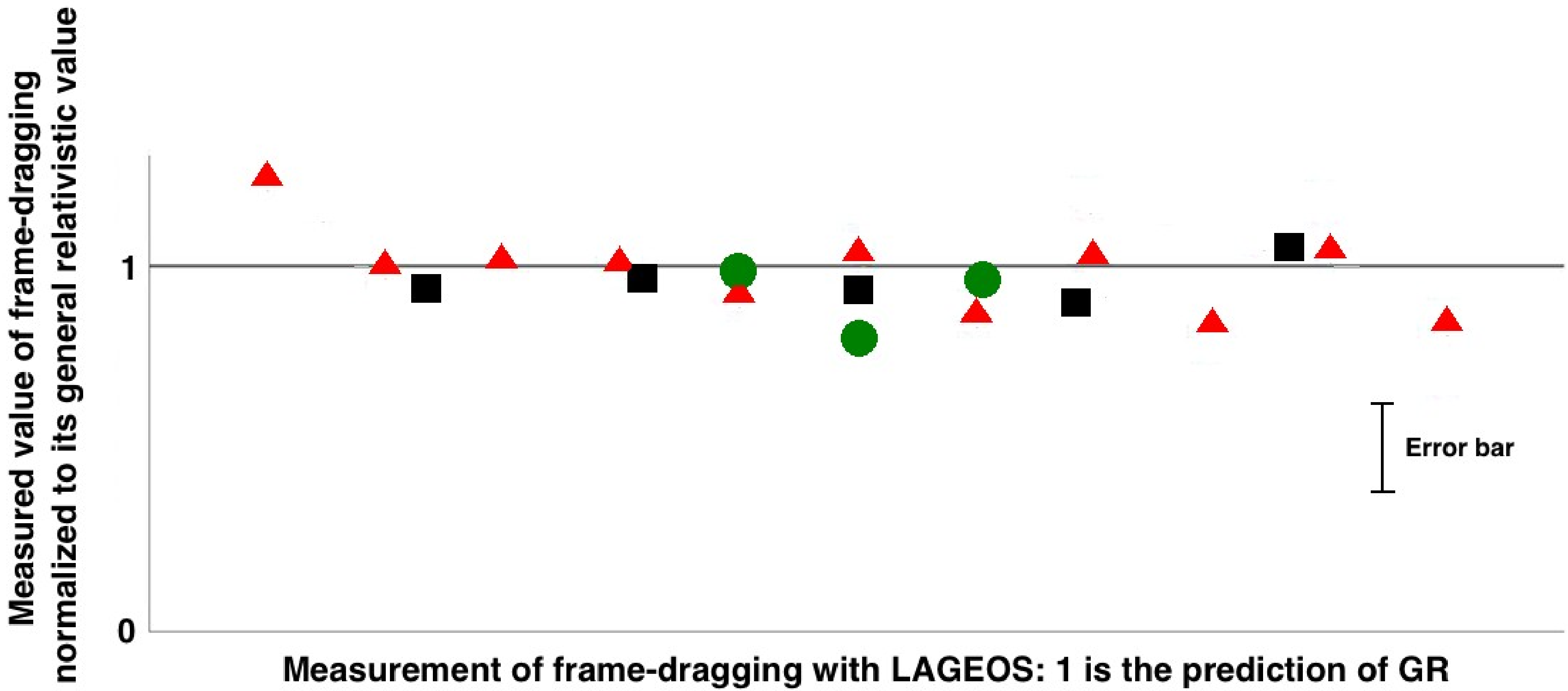


Earth gravity field anomalies by GRACE



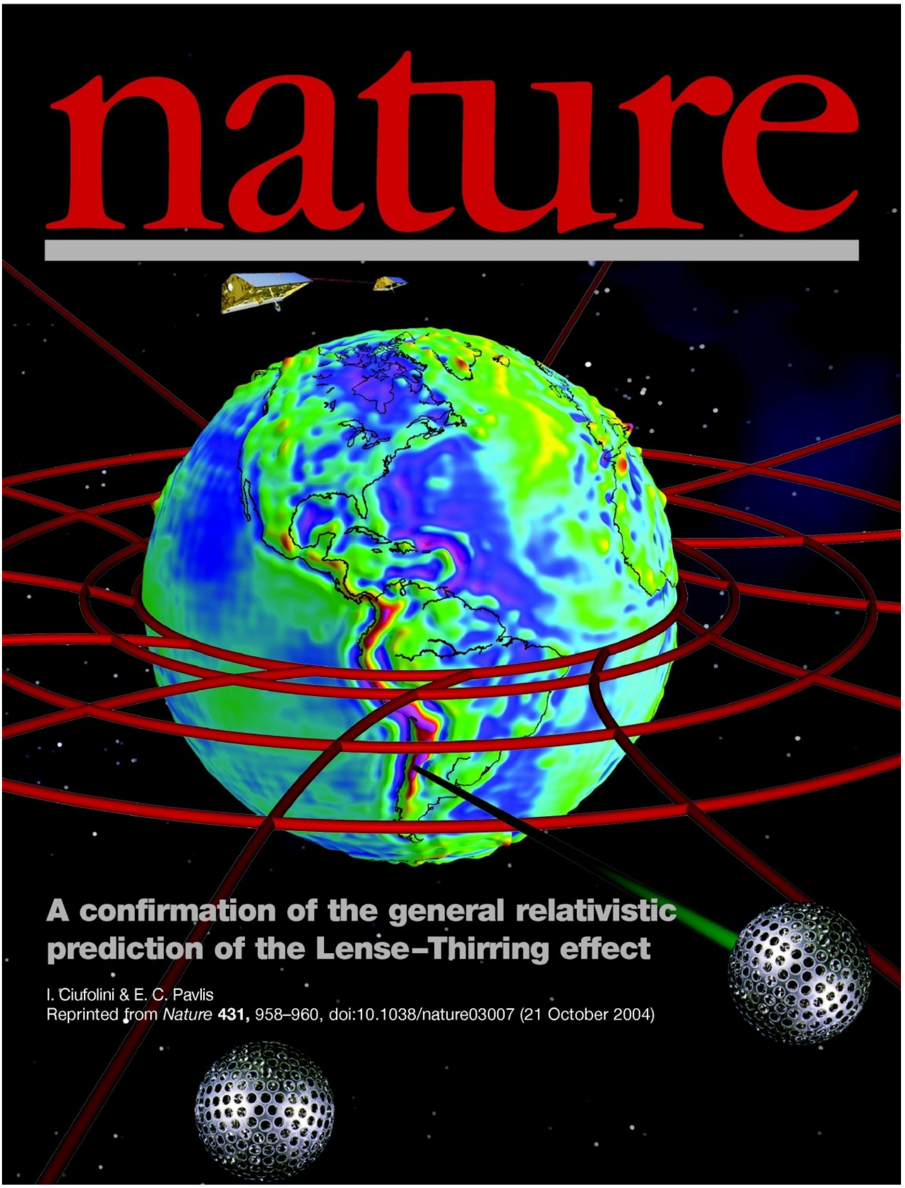
From 2004 to 2010, using LAGEOS + LAGEOS 2 and the GRACE determinations of the Earth gravitational field we were able to measure the frame-dragging effect and eliminate the uncertainties in J_2

A number of independent measurements were obtained and published from 2004 to 2010



Triangles: **CSR UT-Austin**: with orbital estimator **UTOPIA**
Squares: **GFZ-Helmholtz Inst.**-Germany: with orbital estimator **EPOSOC**
Circles: **Univ. Salento-Rome-Maryland (NASA Goddard)**: with orbital estimator **GEODYN**

The first results with GRACE and LAGEOS were published in Nature Letters in 2004 and 2007



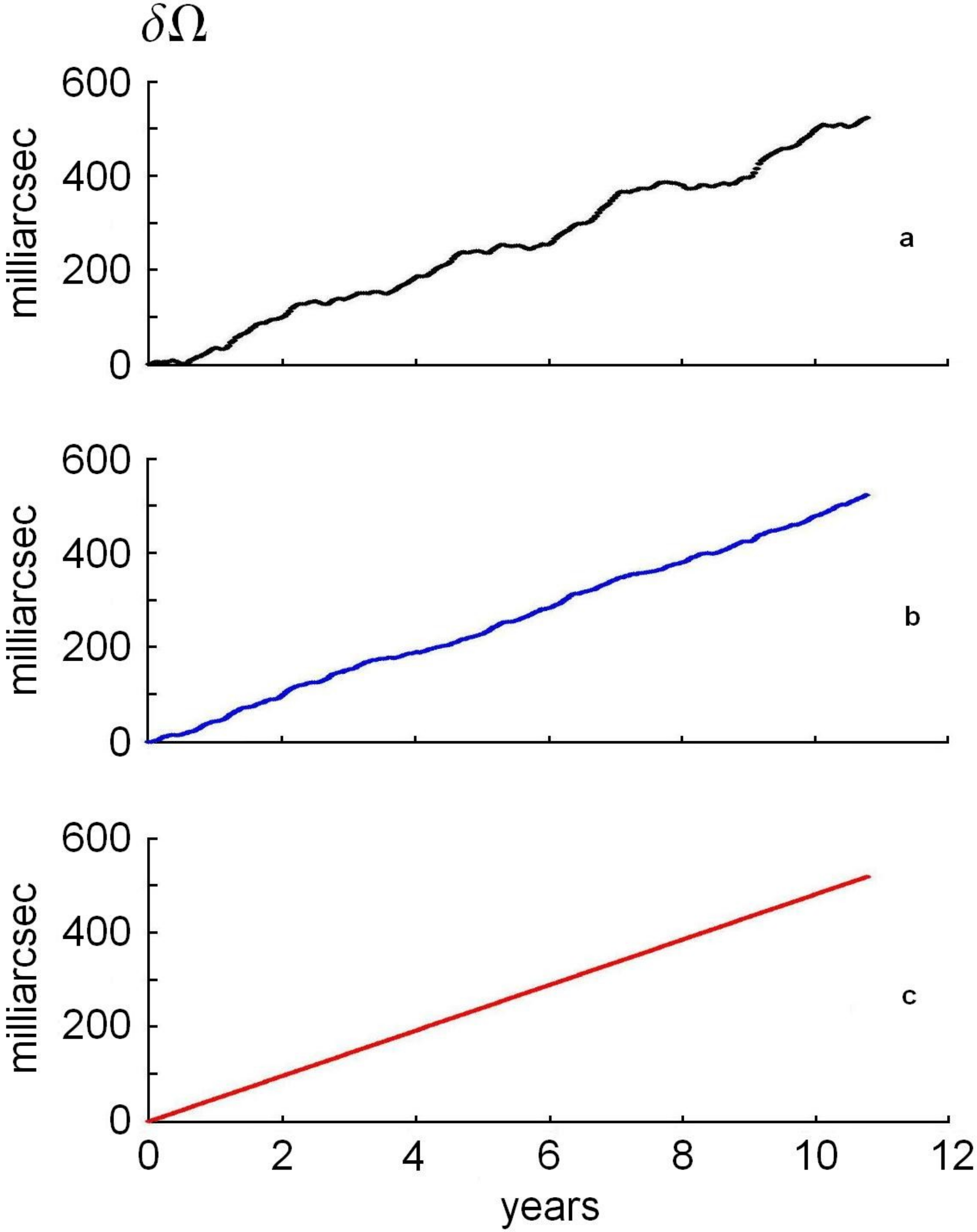


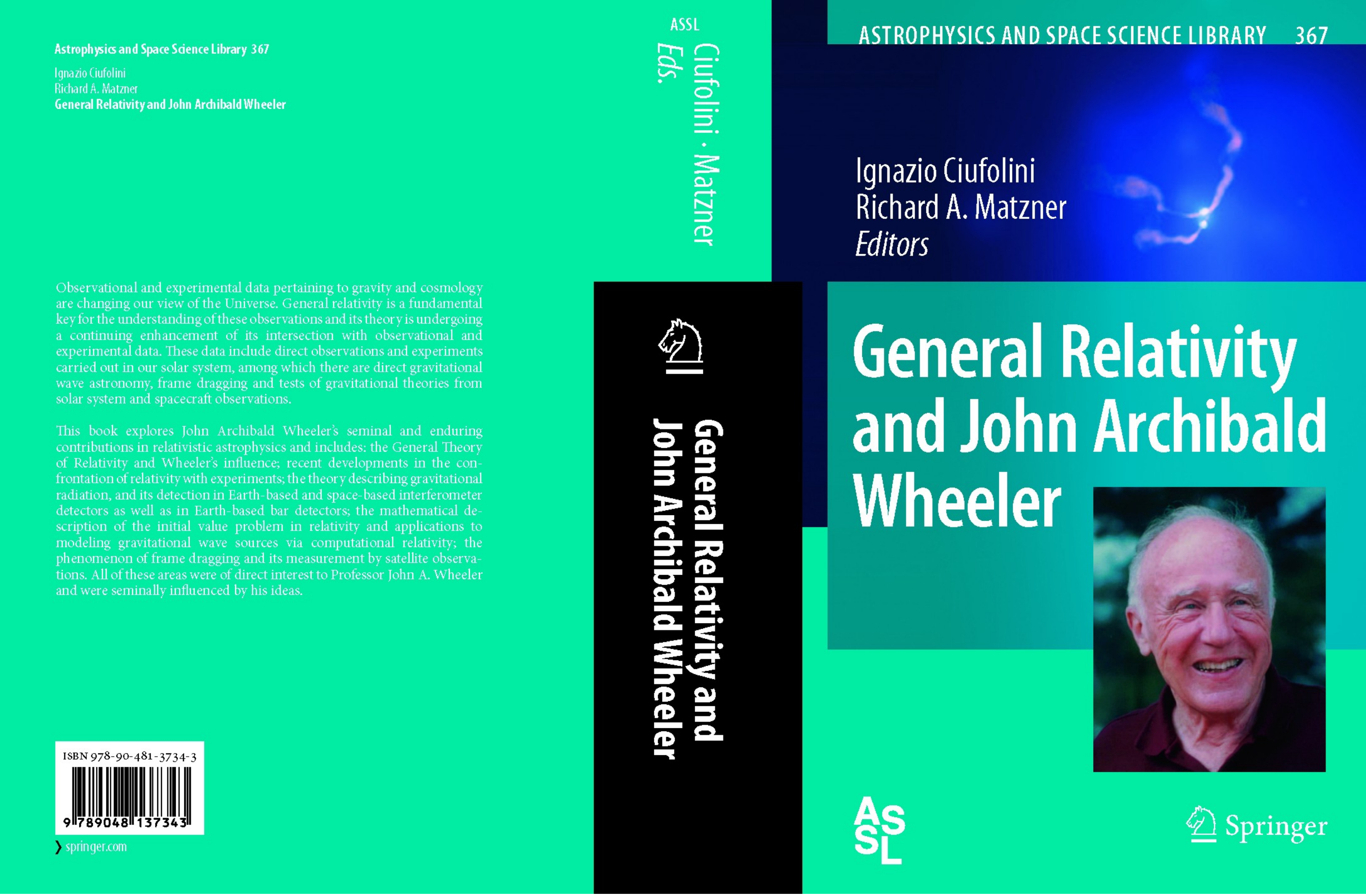
Figure 2

Observed value of Lense-Thirring effect using The combination of the LAGEOS nodes and GRACE

Observed value of Lense-Thirring effect = 99% of the general relativistic prediction. Fit of linear trend plus 6 known frequencies

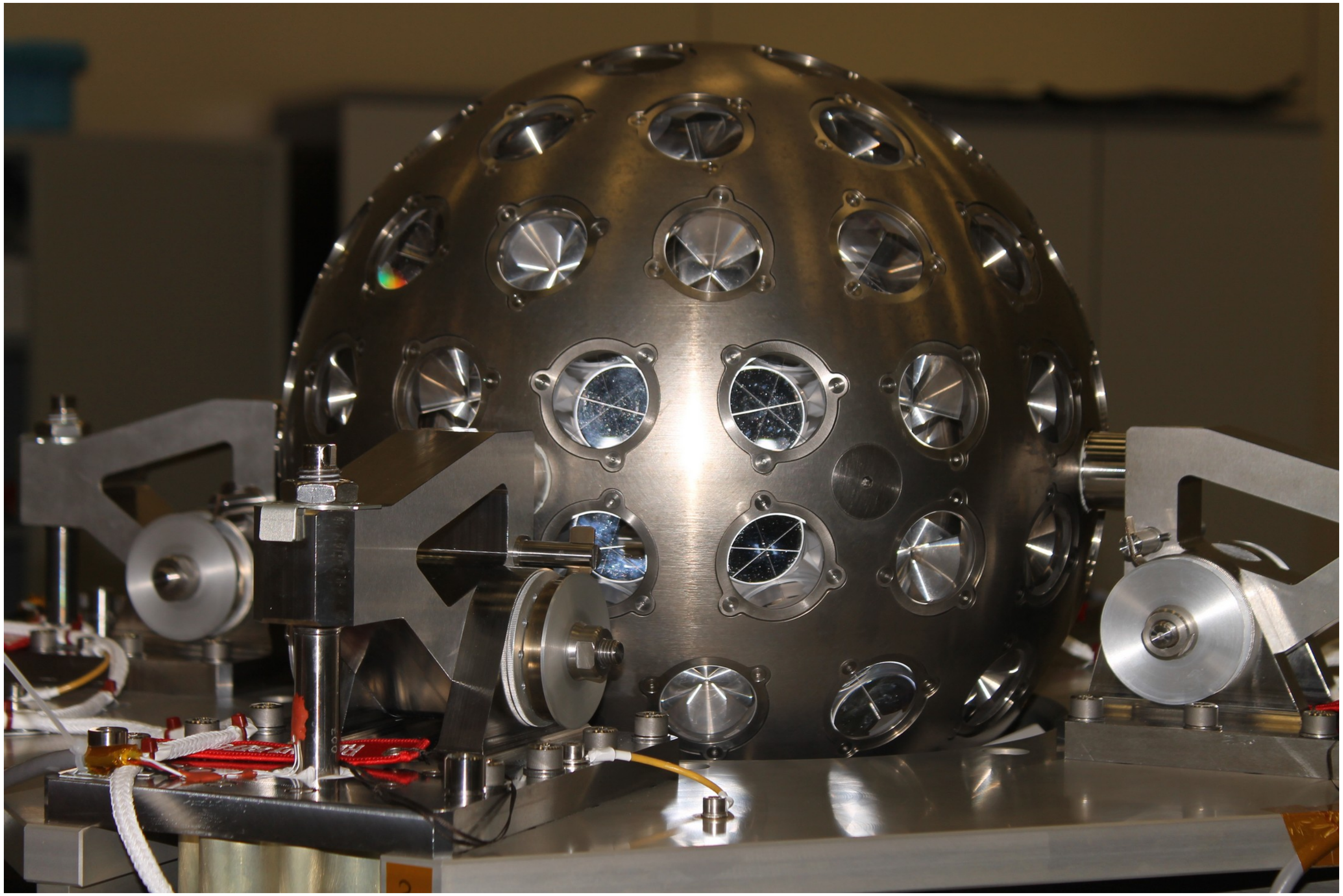
General relativistic Prediction = 48.2 mas/yr

I.C. & E.Pavlis, Letters to NATURE, 431, 958, 2004.

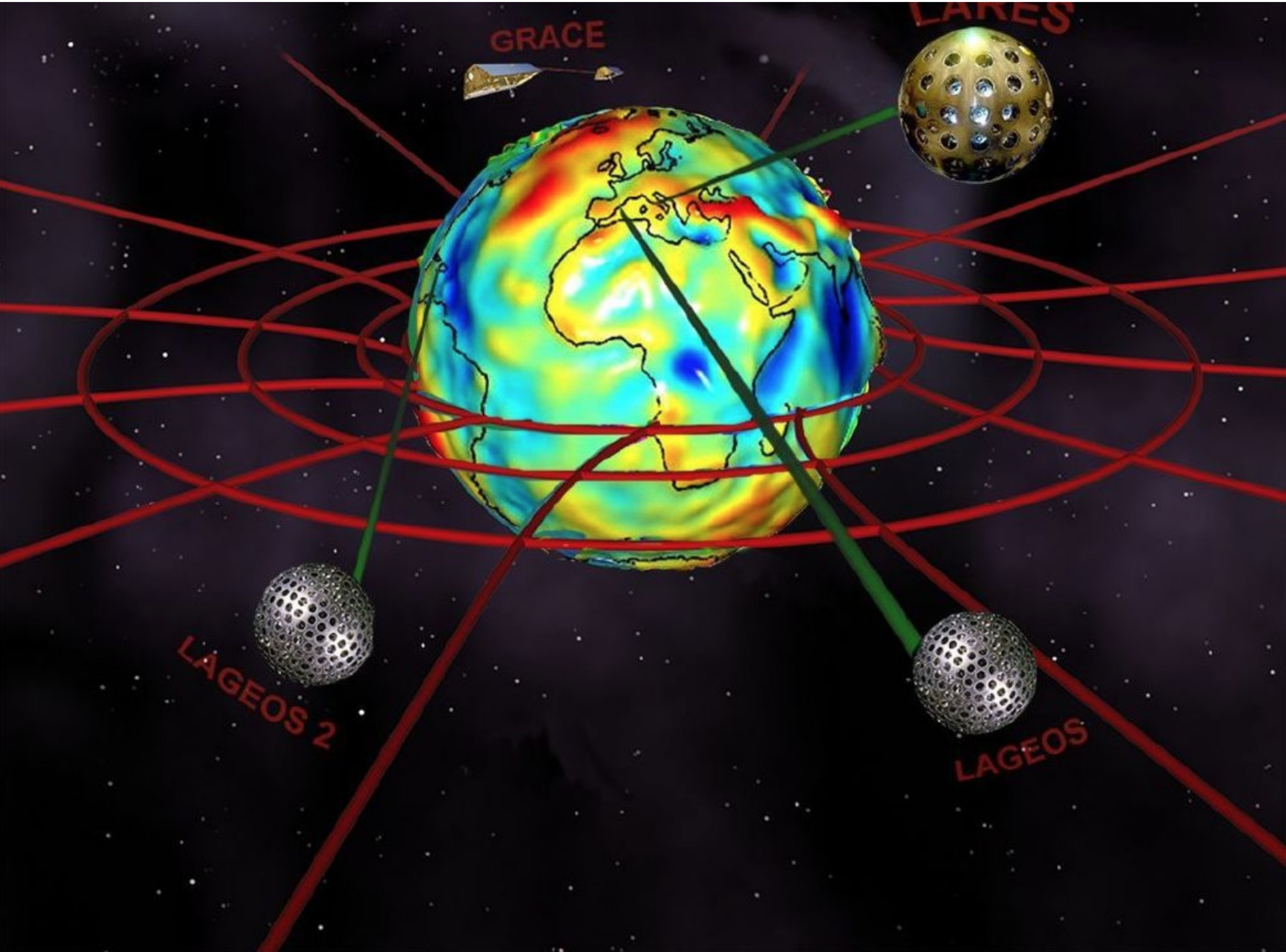


Detailed results with GRACE and LAGEOS were published in J.A Wheeler Book (I.C. et al., 2010)

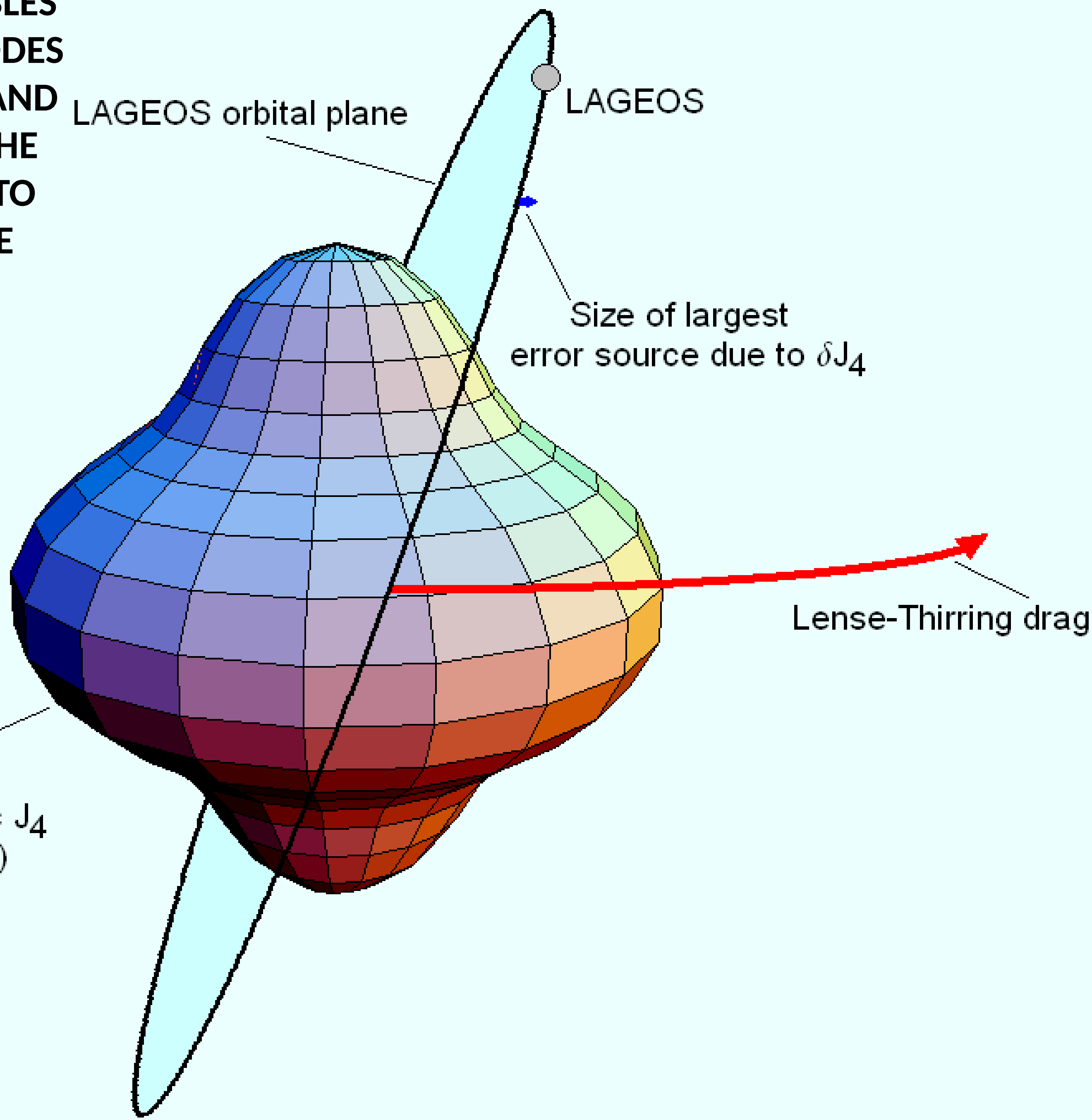
LARES (LAser RElativity Satellite)



LARES was successfully launched and very accurately injected in the nominal orbit on the 13th of February 2012 with the VEGA launching vehicle.


GRACE
LARES
LAGEOS 2
LAGEOS

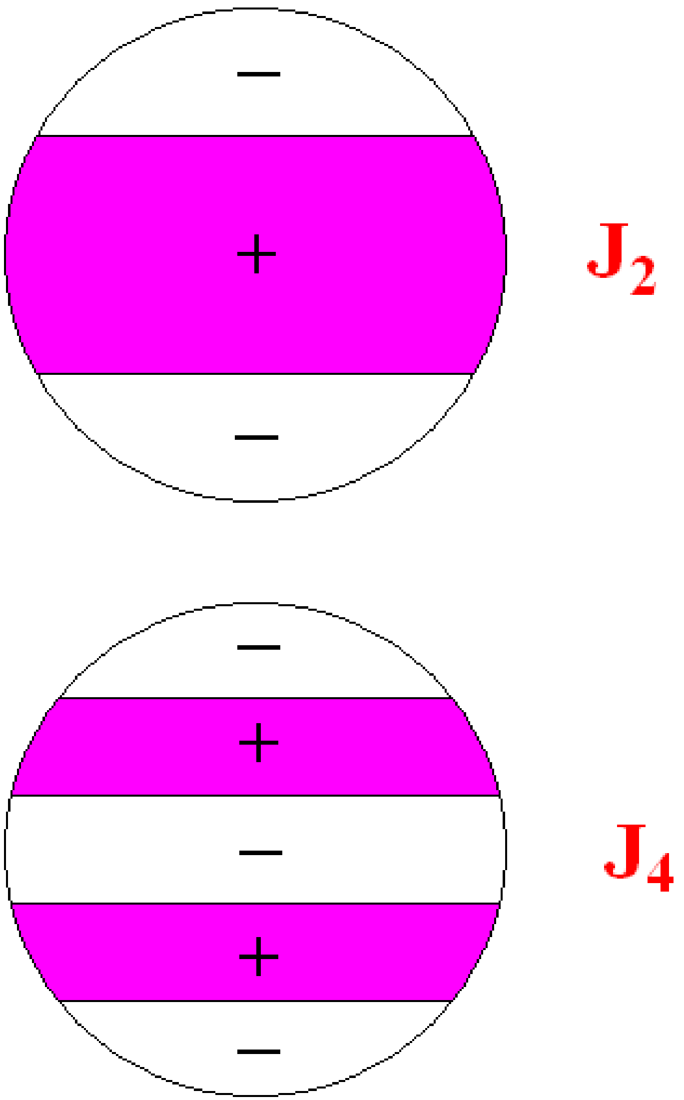
USING THE 3 OBSERVABLES PROVIDED BY THE 3 NODES OF *LAGEOS*, *LAGEOS 2* AND *LARES* WE ELIMINATE THE 2 UNCERTAINTIES DUE TO J_2 AND J_4 AND MEASURE FRAME-DRAGGING



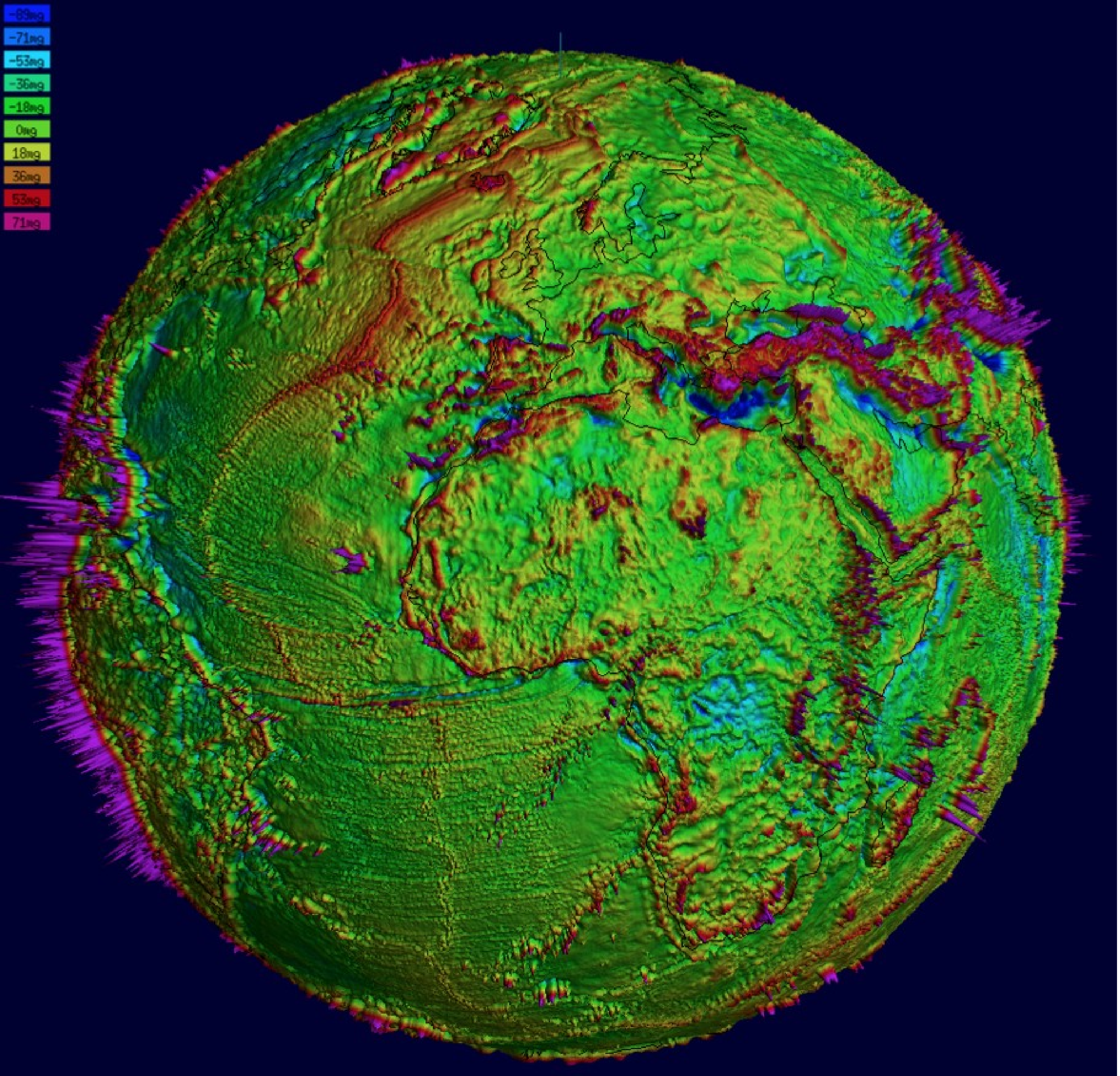
Using LARES + LAGEOS +LAGEOS 2 and the GRACE determinations of the Earth gravitational field we were able to measure the frame-dragging effect and eliminate the uncertainties in J_2 and J_4.

LARES, combined with the LAGEOS and LAGEOS 2 orbital data and using the GRACE Earth gravity field determinations, provided measurements of frame-dragging, with accuracy between 1% and 3% (depending on the estimate of the systematic errors) and a test of the weak equivalence principle with accuracy of about 10^{-9}.

EVEN ZONAL HARMONICS

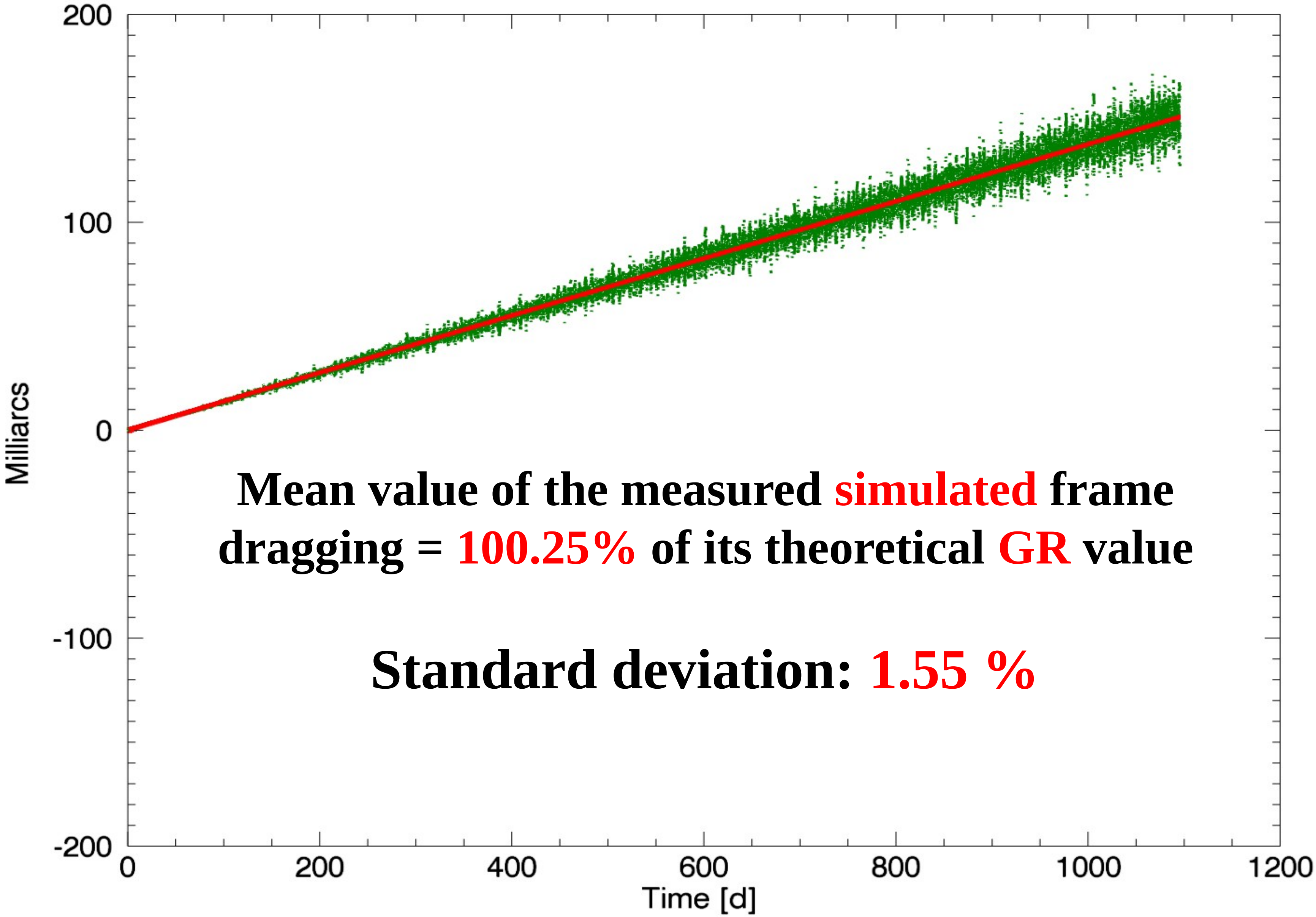


The GRACE gravity field model GGM05S



Parameter	Nominal value	1-Sigma
GM	0.3986004415E+15	8E+05
C20	-.484165112E-03	2.5E-10
C40	0.539968941E-06	0.12280000E-11
C60	-.149966457E-06	0.73030000E-12
C80	0.494741644E-07	0.53590000E-12
C10 0	0.533339873E-07	0.43780000E-12
C20-dot	0.116275500E-10	0.01790000E-11
C40-dot	0.470000000E-11	0.33000000E-11
Cr LAGEOS 1	1.13	0.00565
Cr LAGEOS 2	1.12	0.0056
Cr LARES	Cr_L	0.0054

Main parameters of the Monte Carlo simulation (100 simulations with GFZ)
I.C. et al., Class. and Quantum Grav., 2013



Residuals (in green) of the 100 Monte Carlo simulation of the LARES experiment. In red is the theoretical prediction of GR

Eur. Phys. J. C (2019) 79:872
https://doi.org/10.1140/epjc/s10052-019-7386-z

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

An improved test of the general relativistic effect of frame-dragging using the LARES and LAGEOS satellites

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Received: 27 September 2019 / Accepted: 8 October 2019

Abstract We report the improved test of frame-dragging, an intriguing phenomenon predicted by Einstein's General Relativity, obtained using 7 years of Satellite Laser Ranging (SLR) data of the satellite LARES (ASI, 2012) and 26 years of SLR data of LAGEOS (NASA, 1976) and LAGEOS 2 (ASI and NASA, 1992). We used the static part and temporal variations of the Earth gravity field obtained by the space geodesy mission GRACE (NASA and DLR) and in particular the static Earth's gravity field model GGM05S augmented by a model for the 7-day temporal variations of the lowest degree Earth spherical harmonics. We used the orbital estimator GEODYN (NASA). We measured frame-dragging to be equal to 0.9910 ± 0.02, where 1 is the theoretical prediction of General Relativity normalized to its frame-dragging value and ±0.02 is the estimated systematic error due to modelling errors in the orbital perturbations, mainly due to the errors in the Earth's gravity field determination. Therefore, our measurement confirms the prediction of General Relativity for frame-dragging with a few percent uncertainty.

1 General relativity, dragging of inertial frames and the objectives of the LARES space mission

Einstein's gravitational theory of General Relativity is fundamental to understand our universe [1–4]. It has a number of outstanding experimental verifications [4–6], among which are the recent impressive LIGO laser interferometers direct detections of gravitational waves and observation of black holes, and of their collision, through the emission of gravitational waves [7,8].

LARES (LAser RElativity Satellite) [9] is a laser-ranged satellite of ASI, the Italian Space Agency, dedicated to test General Relativity and fundamental physics, and to measurements of space geodesy and geodynamics. Among the tests of General Relativity, the main objective of LARES is a measurement of dragging of inertial frames, or frame-dragging, with an accuracy of a few percent. In addition to the test of frame-dragging, LARES, together with the LAGEOS (LAser GEOdynamcs Satellite of NASA) [11] and LAGEOS 2 (of ASI and NASA), has recently provided a test of the weak equivalence principle [10], at the foundations of General Relativity and other viable gravitational theories, with an accuracy of about 10^{-9}, at a previously untested range between about 7820 and 12270 km, and using previously untested materials of a tungsten alloy (the material of LARES) and aluminum–brass (the material of LAGEOS and LAGEOS 2). The orbital parameters and characteristics of the LARES and LAGEOS satellites are provided in the next section.

Frame-dragging [12] is an intriguing phenomenon of General Relativity: in Einstein's gravitational theory the inertial frames, which can only be defined locally (according to the equivalence principle [1,2,4]), have no fixed direction with respect to the distant stars but are instead dragged by the currents of mass-energy such as the rotation of a body, e.g., the rotation of the Earth (the axes of the local inertial frames are determined in General Relativity by local test-gyroscopes.) For a detailed description of such intriguing phenomenon and

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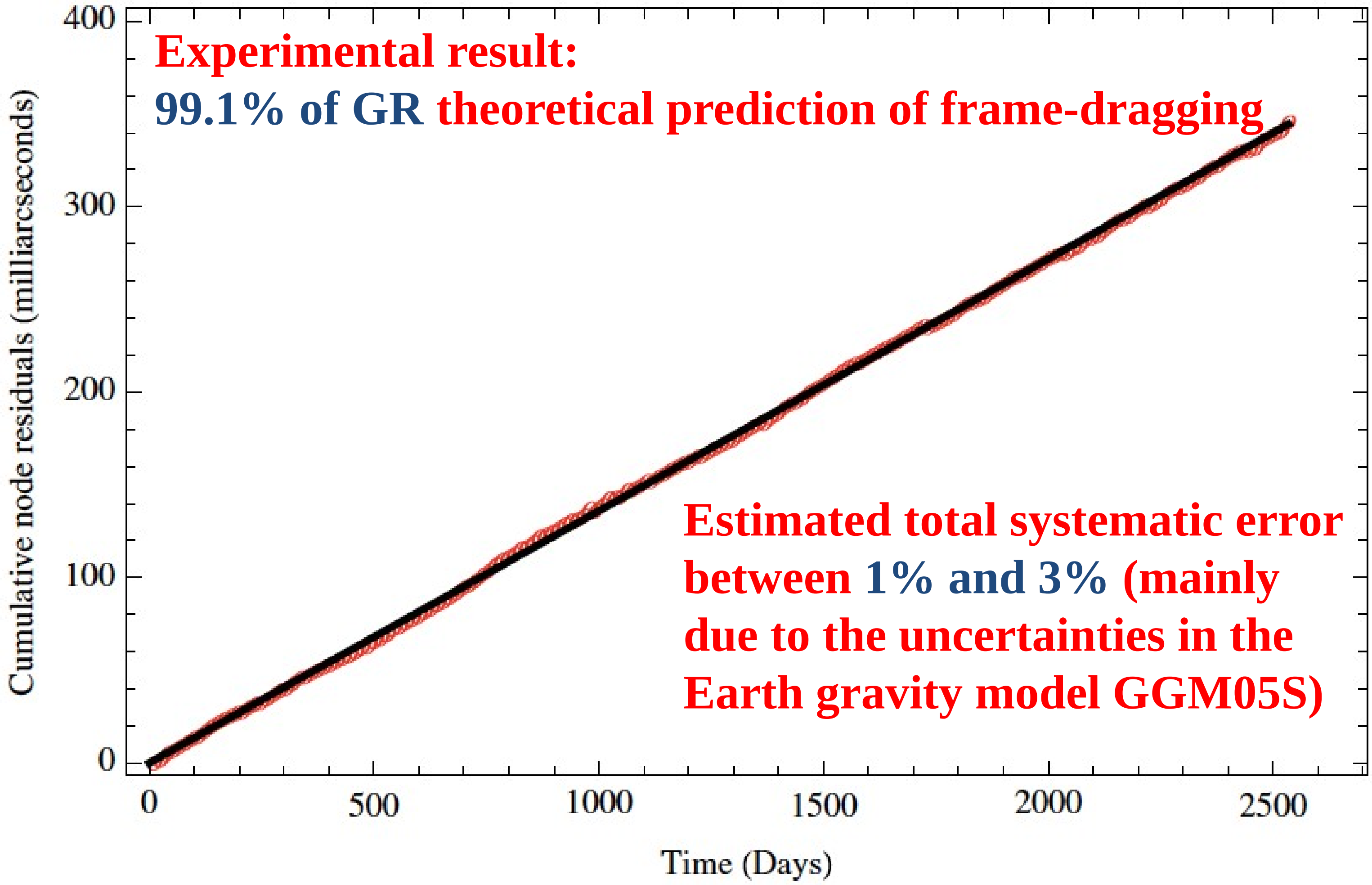
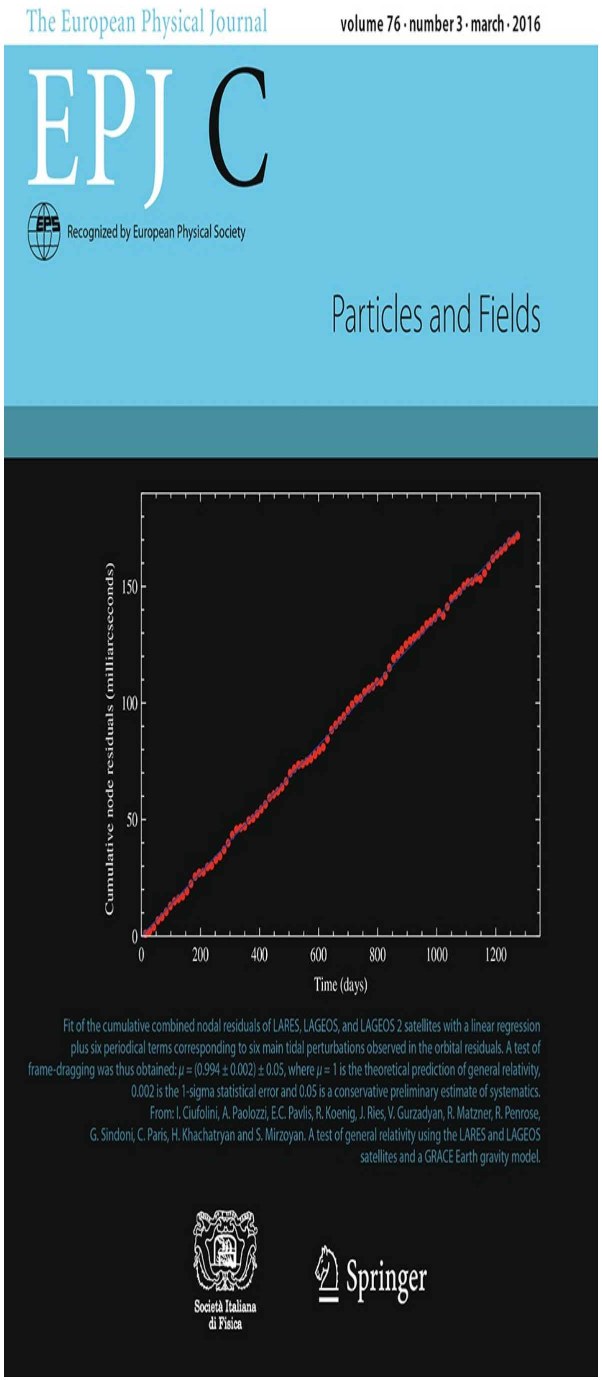
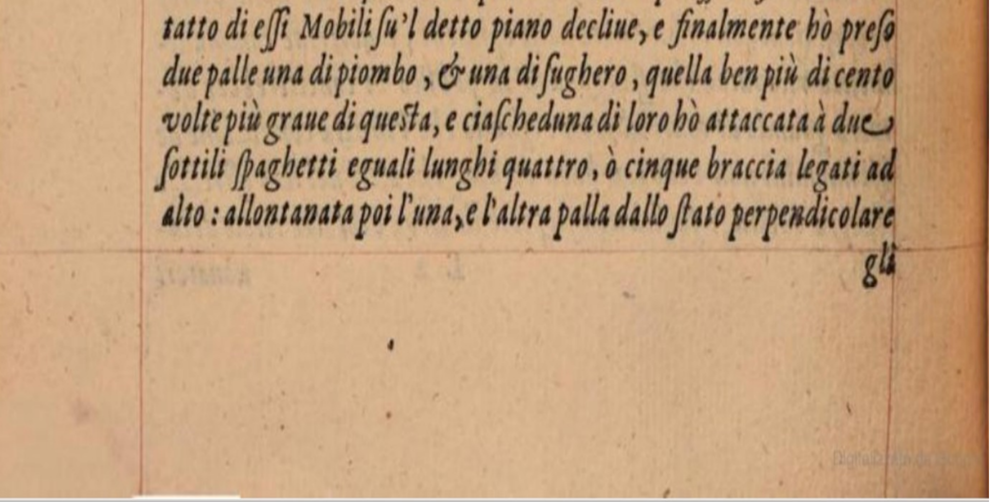


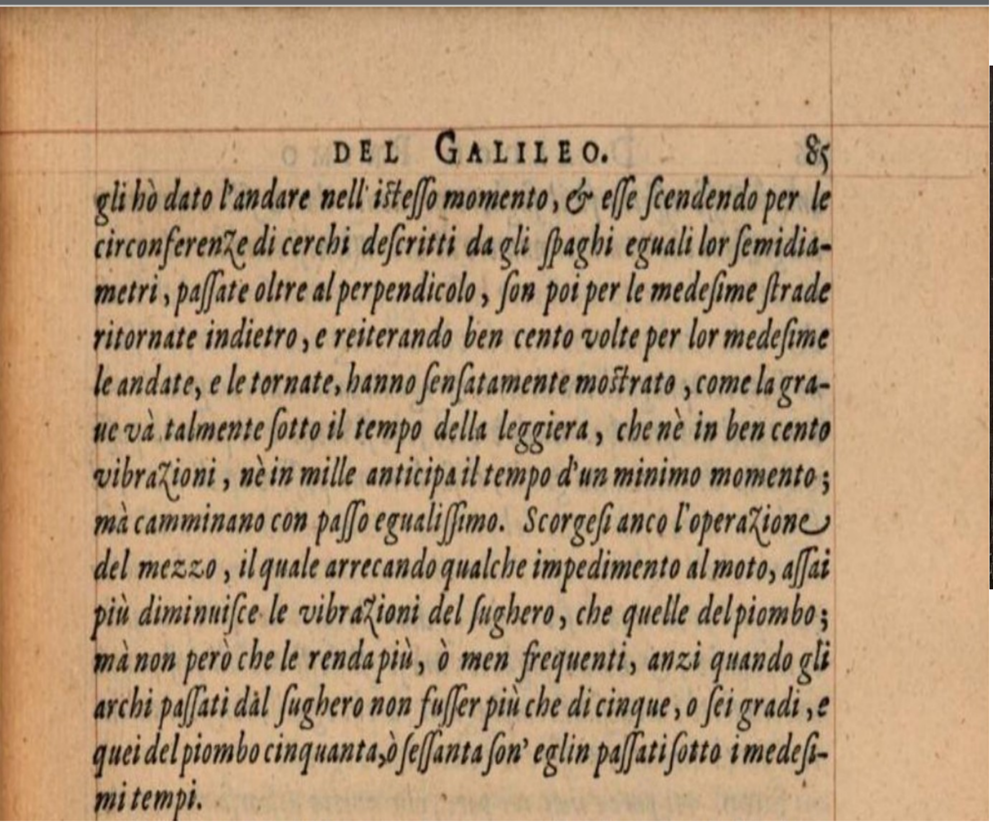
Fit of the cumulative combined observed nodal residuals of LARES, LAGEOS and LAGEOS 2 with a linear regression plus the five main periodical terms corresponding to the five main tidal perturbations (I.C., et al., EPJC, 2019)



Cover of EPJC (03.2016) dedicated to tests of frame-dragging with LARES

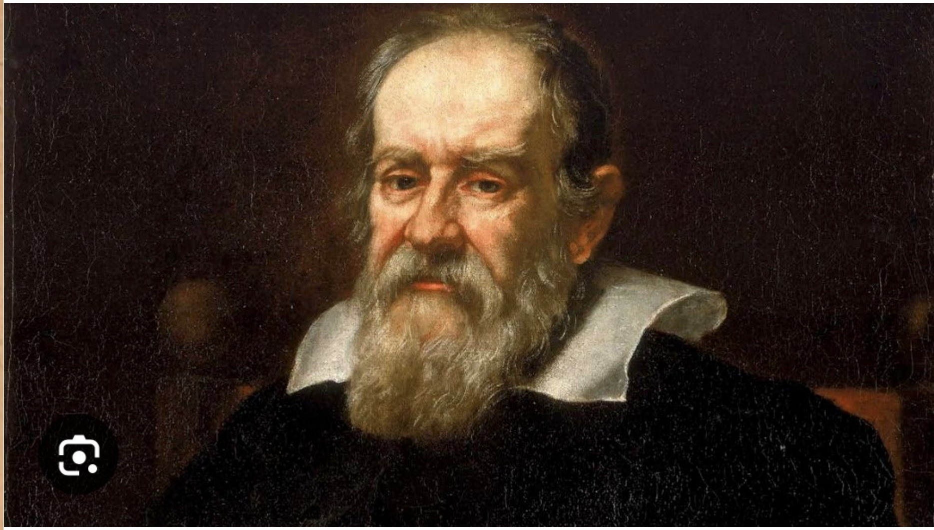


tatto di essi Mobili su'l detto piano decliue, e finalmente hò preso due palle una di piombo, & una di sughero, quella ben più di cento volte più graue di questa, e ciascheduna di loro hò attaccata à due sottili spaghetti eguali lunghi quattro, ò cinque braccia legati ad alto: allontanata poi l'una, e l'altra palla dallo stato perpendicolare gli



gli hò dato l'andare nell'istesso momento, & esse scendendo per le circonferenze di cerchi descritti da gli spaghi eguali lor semidiametri, passate oltre al perpendicolo, son poi per le medesime strade ritornate indietro, e reiterando ben cento volte per lor medesime le andate, e le tornate, hanno sensatamente mostrato, come la graue và talmente sotto il tempo della leggiera, che nè in ben cento vibrazioni, nè in mille anticipa il tempo d'un minimo momento; mà camminano con passo egualissimo. Scorgesi anco l'operazione del mezzo, il quale arrecando qualche impedimento al moto, assai più diminuisce le vibrazioni del sughero, che quelle del piombo; mà non però che le renda più, ò men frequenti, anzi quando gli archi passati dal sughero non fusser più che di cinque, o sei gradi, e quei del piombo cinquanta, ò sessanta son' eglin passati sotto i medesimi tempi.

Section of "*Giornata Prima*" (First Day) of Galileo Galilei, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attenenti alla meccanica e i movimenti locali* (1638, Leiden, Netherlands)



«Galilei: The father of modern physics and modern science» (Albert Einstein, Stephen Hawking et al.)

A test of the *Weak Equivalence Principle* with LAGEOS, LAGEOS 2 and LARES

Using the three laser-ranged satellites LAGEOS, LAGEOS 2 and LARES, we obtained a new confirmation to approximately one part in a billion of the weak equivalence principle ("uniqueness of free fall") in the Earth's gravitational field, at previously untested range and with previously untested materials:

$$\delta(m_g/m_i) = 2.0 \times 10^{-10} \pm 1.1 \times 10^{-9}$$

Range: from about 7820 km to 12220 km

Materials: Aluminum and brass (LAGEOS and LAGEOS 2) versus sintered tungsten (LARES)

$$\delta\left(\frac{m_g}{m_i}\right) = \left.\frac{m_g}{m_i}\right|_{tungsten} - \left.\frac{m_g}{m_i}\right|_{aluminum/brass}.$$

I.C. et al. Scientific Reports-Nature, 9, 1-10 (2019).

www.nature.com/scientificreports

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Nature Scientific Reports 2019

OPEN

Satellite Laser-Ranging as a Probe of Fundamental Physics

Ignazio Ciufolini[1], Richard Matzner[2*], Antonio Paolozzi[3], Erricos C. Pavlis[4], Giampiero Sindoni[3], John Ries[5], Vahe Gurzadyan[6] & Rolf Koenig[7]

Satellite laser-ranging is successfully used in space geodesy, geodynamics and Earth sciences; and to test fundamental physics and specific features of General Relativity. We present a confirmation to approximately one part in a billion of the fundamental weak equivalence principle ("uniqueness of free fall") in the Earth's gravitational field, obtained with three laser-ranged satellites, at previously untested range and with previously untested materials. The weak equivalence principle is at the foundation of General Relativity and of most gravitational theories.

General Relativity (GR) describes gravitational interaction via the geometry of spacetime whose dynamical curvature is determined by the distribution and motion of mass-energy; concurrently the motion of mass-energy is determined by the spacetime geometry. "Mass tells spacetime how to curve and spacetime tells mass how to move" (Wheeler[1]). However, for such a geometrical picture to work, any two particles, independently of their mass, composition and structure, must follow the same geometrical path of spacetime[2–4]. The weak equivalence principle states that the motion of any test particle due to the gravitational interaction with other bodies is independent of the mass, composition and structure of the particle. [A test particle is an electrically neutral particle, with negligible gravitational binding energy, negligible angular momentum and small enough that the inhomogeneities of the gravitational field within its volume have negligible effect on its motion.] Thus, the motion of planets, stars, and galaxies in the universe is simply dictated by the geometry of spacetime: they all follow purely geometrical curves of the spacetime called geodesic[1,2,5,6]. A geodesic is the generalization to a curved spacetime of a straight line of the flat Euclidean geometry. [The surface of a sphere is an example of a non-Euclidean geometry with positive curvature.] For example the motion of an artificial satellite around the Earth is not determined by the gravitational force that the Earth's mass exerts on the satellite as in Newtonian theory. Rather the satellite is simply following a geometrical curve in spacetime, a geodesic, independent of its properties such as mass, composition, and structure, depending only on its initial conditions of position and velocity[5]. Then, for example, the observed (approximately) elliptical orbit of a satellite around the Earth is just the projection to our three dimensional space of the geodesic followed by the satellite in the four-dimensional curved spacetime geometry generated by the Earth's mass (see Fig. 1a).

There are a number of different formulations of the equivalence principle. The weak equivalence principle, also known as the Galilei equivalence principle, is based on the principle that the ratio of the inertial mass to the passive gravitational mass is the same for all bodies. This last formulation is also known as the Newton equivalence principle. The weak form is at the basis of most known viable theories of gravity. The medium form states that locally, in freely falling frames, all the non-gravitational laws of physics are the laws of special relativity[6]; the strong form includes gravitation itself in the local laws of physics, meaning that an external gravitational field cannot be detected in a freely falling frame by its influence on local gravitational phenomena. The medium form is at the basis of any gravitational theory based on a spacetime geometry described by a symmetric metric tensor, the so-called metric theories of gravitation, and the strong form is a cornerstone of GR. Since the weak equivalence principle underlies the geometrical structure of GR as well as our understanding of the dynamics of the universe and of astrophysical bodies, it has been tested in very accurate experiments[2–4]. Its tests go from the pendulum experiments (and inclined tables) of Galileo Galilei (about 1610), Christian Huygens (1673), Isaac Newton (1687) and Bessel (1832), to the classic torsion balance experiments of Eötvos[7] (1889 and 1922) in the gravitational field of Earth (at a range from the center of ~6370 km). Roll, Krotkov and Dicke[8] (1964) used

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LARES 2 (LAGEOS 3)

The LARES 2 (LAGEOS 3) satellite was launched in July 2022 for tests of frame-dragging at a level of accuracy of about 0.1% accuracy and for other tests of General Relativity and Fundamental Physics (and measurements in space geodesy and geodynamics.

MAIN PAPERS ON LARES 2

- **I.C., A. Paolozzi, E. C.Pavlis, G. Sindoni, R. Koenig, J. C. Ries, R. Matzner, V. Gurzadyan, R. Penrose, D. Rubincam and C. Paris, I. An introduction to the LARES 2 space experiment, EPJ P 132: 336 (2017).**
- **I.C, et al., II. Monte Carlo simulations and covariance analyses of the LARES 2 experiment, EPJ P 132: 337 (2017).**
- **I.C., Richard Matzner, Vahe Gurzadyan and Roger Penrose, III. de Sitter effect and the LARES 2 space experiment, EPJ C 77:819 (2017).**
- **I.C, et al., IV. Thermal drag and the LARES 2 space experiment EPJ P, 133, 2018.**
- **A. Paolozzi, et al., A. JOURNAL OF GEODESY, 1-10 (2019).**
- **M. Pearlman, et al., JOURNAL OF GEODESY, 1-14 (2019).**
- **A. Paolozzi, et al., AER. MISS. SPAZIO, 97, 135-144 (2018),**
- **F. Felli, et al., PROCEDIA STRUCTURAL INTEGRITY, 9, 295-302 (2018).**
- **D. Pilone, et al., FRATT. INTEGR. STRUTT., 56, 56-64 (2021)**
- **I.C. and C. Paris, THE EUROPEAN PHYSICAL JOURNAL PLUS, 136(10), 1030 (2021).**
- **I.C., A. Paolozzi, E.C. Pavlis, J.C. Ries, R. Matzner, C. Paris, E. Ortore, V. Gurzadyan, and R. Penrose, The LARES 2 satellite, general relativity and fundamental physics. THE EUROPEAN PHYSICAL JOURNAL C, *83*:87 (2023).**
- **Based on earlier proposal: I.C., PHYS. REV. LETT. (1986); I.C., Ph.D. dissertation (1984); I.C., IJMP A (1989); B. Tapley, I.C. et al, NASA and ASI studies (1989), J. Ries Ph.D. (1989).**

Eur. Phys. J. C (2023) 83:87
https://doi.org/10.1140/epjc/s10052-023-11230-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

The LARES 2 satellite, general relativity and fundamental physics

Ignazio Ciufolini[2,1,a], **Antonio Paolozzi**[2], **Erricos C. Pavlis**[3], **John C. Ries**[4], **Richard Matzner**[5], **Claudio Paris**[2], **Emiliano Ortore**[2], **Vahe Gurzadyan**[6], **Roger Penrose**[7]

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[5] Center for Gravitational Physics, Weinberg Theory Center, University of Texas at Austin, Austin, USA
[6] Center for Cosmology and Astrophysics, Alikhanian National Laboratory, Yerevan State University, Yerevan, Armenia
[7] Mathematical Institute, University of Oxford, Oxford, UK

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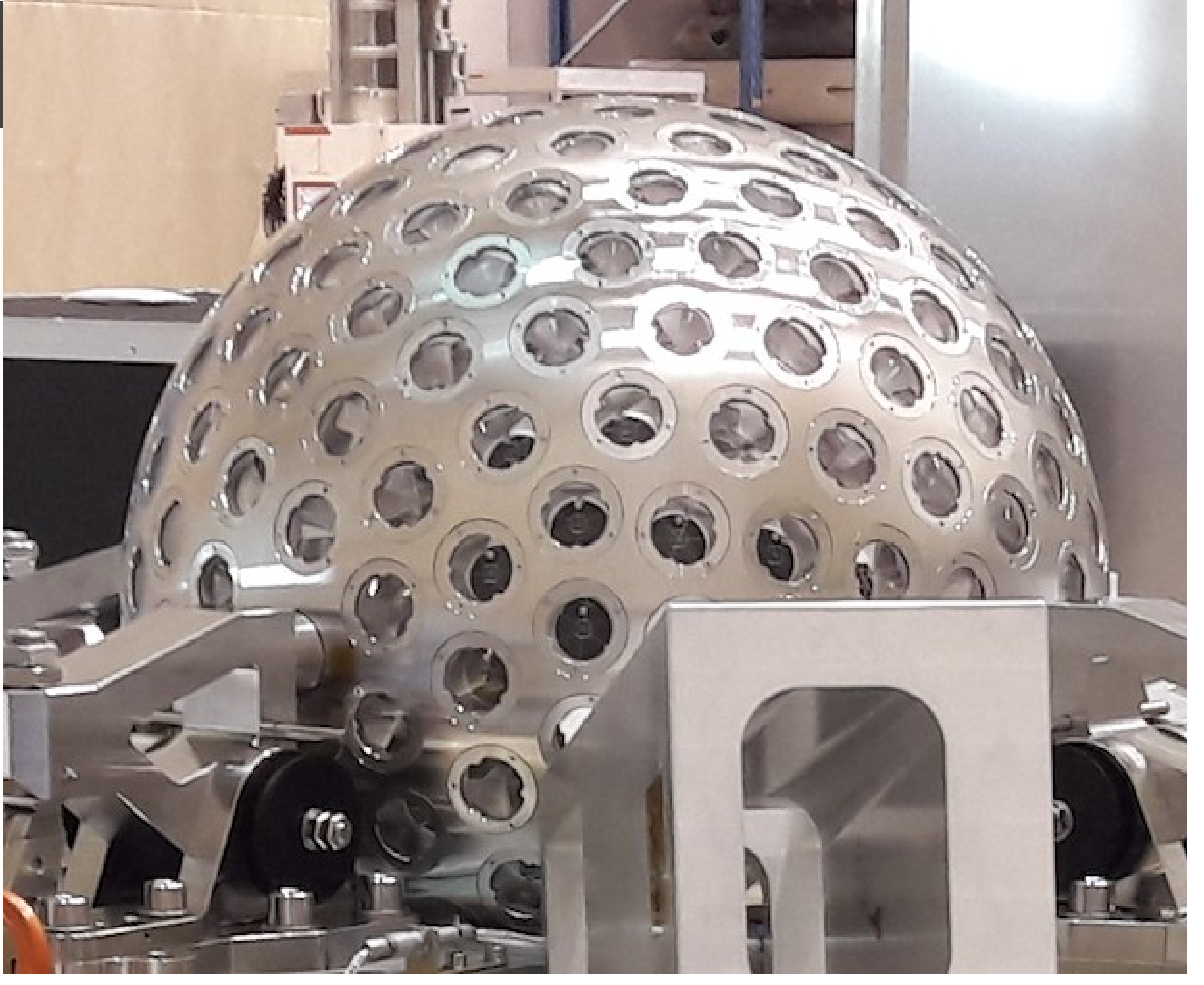
Abstract LARES 2, successfully launched on July 13, 2022, is a new generation laser-ranged satellite. LARES is an acronym for LAser RElativity Satellite. The first LARES satellite was successfully launched on February 13, 2012 with the ESA-ASI-AVIO launch vehicle VEGA. LARES 2 was injected with extremely high precision onto a high-

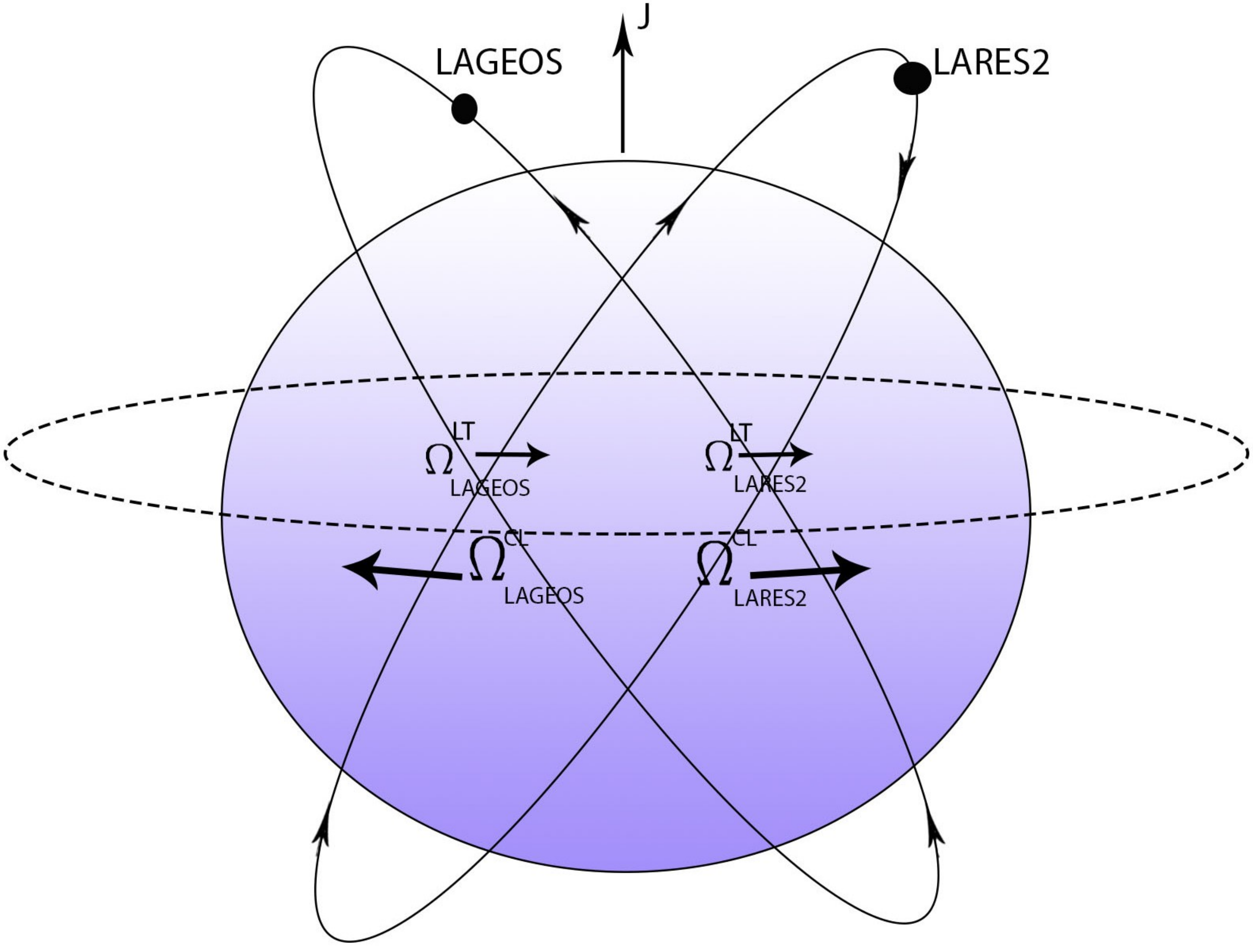
Frame (ITRF) by improving the determination of the Earth center of mass and by contributing to a better determination of its rotation axis.

1 The LARES 2 launch and its orbit



LARES 2 of the Italian Space Agency ready for launch with VEGA C

LARES 2/LAGEOS 3



About 35 years ago in the office of John Archibald Wheeler

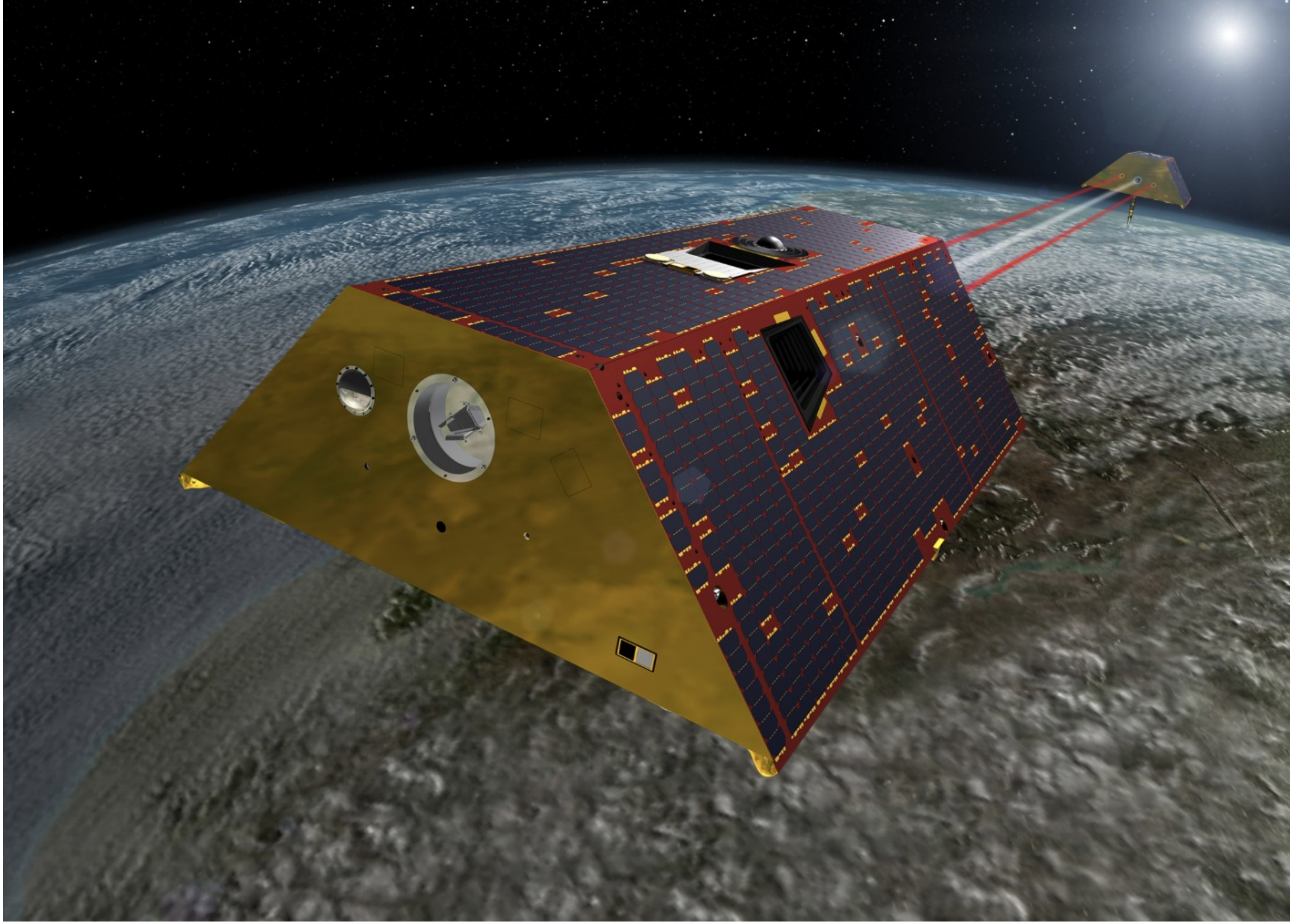
LARES 2: what is new with respect to LAGEOS 3?

1) The Earth gravity field knowledge and the even zonal harmonics determinations are today extremely improved thanks to the GRACE and to the GRACE Follow On (May 22, 2018) space missions.
The Earth quadrupole moment, J_2, is improved by a factor of more than 100 with respect to the Earth gravity field determinations in 1984!

2) The knowledge of other orbital perturbations, such as the Earth's tidal perturbations, is greatly improved with respect to 1984.

3) The satellite structure is quite improved with respect to all the other laser-ranged satellites: using the new 1 inch retroreflectors we can today reach less than 1 mm precision in ranging.

4) The satellite LARES 2 has been injected by VEGA C into the special orbit supplementary to LAGEOS with better accuracy than in 1984.



Like GRACE, the twin GRACE-FO satellites will follow each other in orbit around the Earth, separated by about 137 miles (220 km). Seen in an artist's rendering. Credit: NASA

Table 1 Mean of the orbital elements of LARES 2 and LAGEOS over 127 days and the estimated corresponding error in measuring frame-dragging due to the Earth even zonal harmonics

	Mean orbital inclination	Mean semimajor axis	Mean eccentricity
LARES 2	70.1615°	12266.1359395 km	0.00027
LAGEOS	109.8469°	12270.020705 km	0.00403
Deviation of LARES 2 from the optimal orbit	Sum of the two satellites' inclinations – 180° ≅ 0.0084°	Difference of the two satellites' semimajor axes ≅ 3.88477 km	
Error in the test of frame-dragging due to deviations from the optimal orbit	Less than 0.006%	Less than 0.02%	

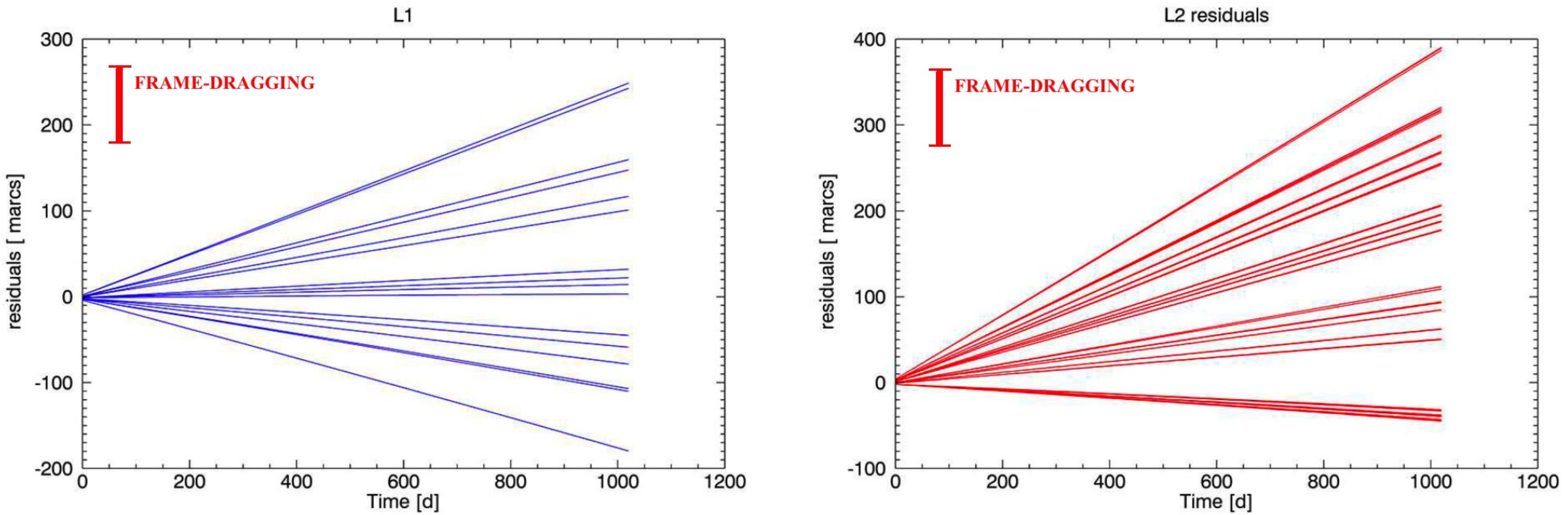
Error Budget of test of frame-dragging with LARES 2

Source of Error	Estimated error
Injection Error and Even Zonal Harmonics	≅ 0.1\% of frame-dragging
Non-zonal harmonics and tides	≅ 0.1\% of frame-dragging
Albedo	≅ 0.1\% of frame-dragging
Thermal Drag and Satellites Eclipses	≅ 0.1\% of frame-dragging
Measurement Error of the LAGEOS and LARES 2 Orbital Parameters	≅ 0.1\% of frame-dragging
Total RSS Error	≅ 0.2\% of frame-dragging

Monte Carlo simulation
main parameters with their sigmas (gravity field GOCO05S: 2015)

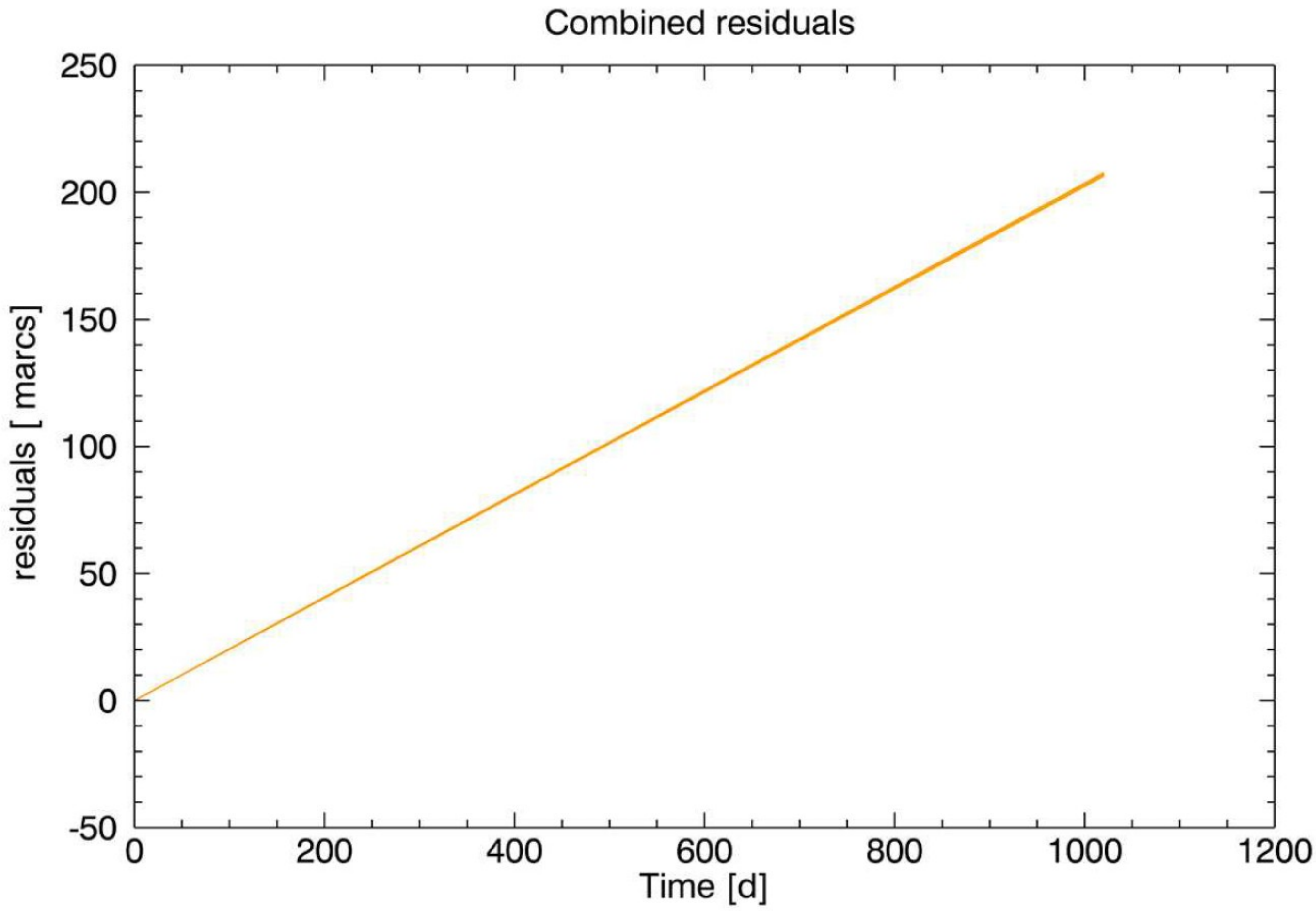
Parameter	Nominal Value	1-Sigma
GM	$0.3986004415 \cdot 10^{15}$	$8 \cdot 10^5\ m^3/s^2$
$C_{2,0}$	$-0.484165217062 \cdot 10^{-3}$	$0.5 \cdot 10^{-11}$
$C_{4,0}$	$0.539998760761 \cdot 10^{-6}$	$0.0614 \cdot 10^{-11}$
$C_{6,0}$	$-0.149975578413 \cdot 10^{-6}$	$0.36515 \cdot 10^{-12}$
$C_{8,0}$	$0.494771161193 \cdot 10^{-7}$	$0.26795 \cdot 10^{-12}$
$C_{10,0}$	$0.533423124477 \cdot 10^{-7}$	$0.2189 \cdot 10^{-12}$
$\dot{C}_{2,0}$	$1.207500000000 \cdot 10^{-11}$	$0.00895 \cdot 10^{-11}$
$\dot{C}_{4,0}$	$0.470000000000 \cdot 10^{-11}$	$0.165 \cdot 10^{-12}$
$C_{30,0}$	$9.571735690410 \cdot 10^{-7}$	$0.6531 \cdot 10^{-11}$
$C_{50,0}$	$6.864653382320 \cdot 10^{-8}$	$1.61115 \cdot 10^{-12}$
C_r LAGEOS	1.13	$0.3 \cdot 10^{-2}$
C_r LARES 2	1.10	$0.3 \cdot 10^{-2}$

I.C., et al., II. Monte Carlo simulations and covariance analyses of the LARES 2 experiment, EPJ P 132: 337 (2017).



Monte Carlo simulation of the LARES 2 experiment: size of the node residuals of LARES 2 and LAGEOS after about 1000 days with respect to frame-dragging (about 90 milliarcsec/1000-day)

I.C., et al., II. Monte Carlo simulations and covariance analyses of the LARES 2 experiment, EPJ P 132: 337 (2017).

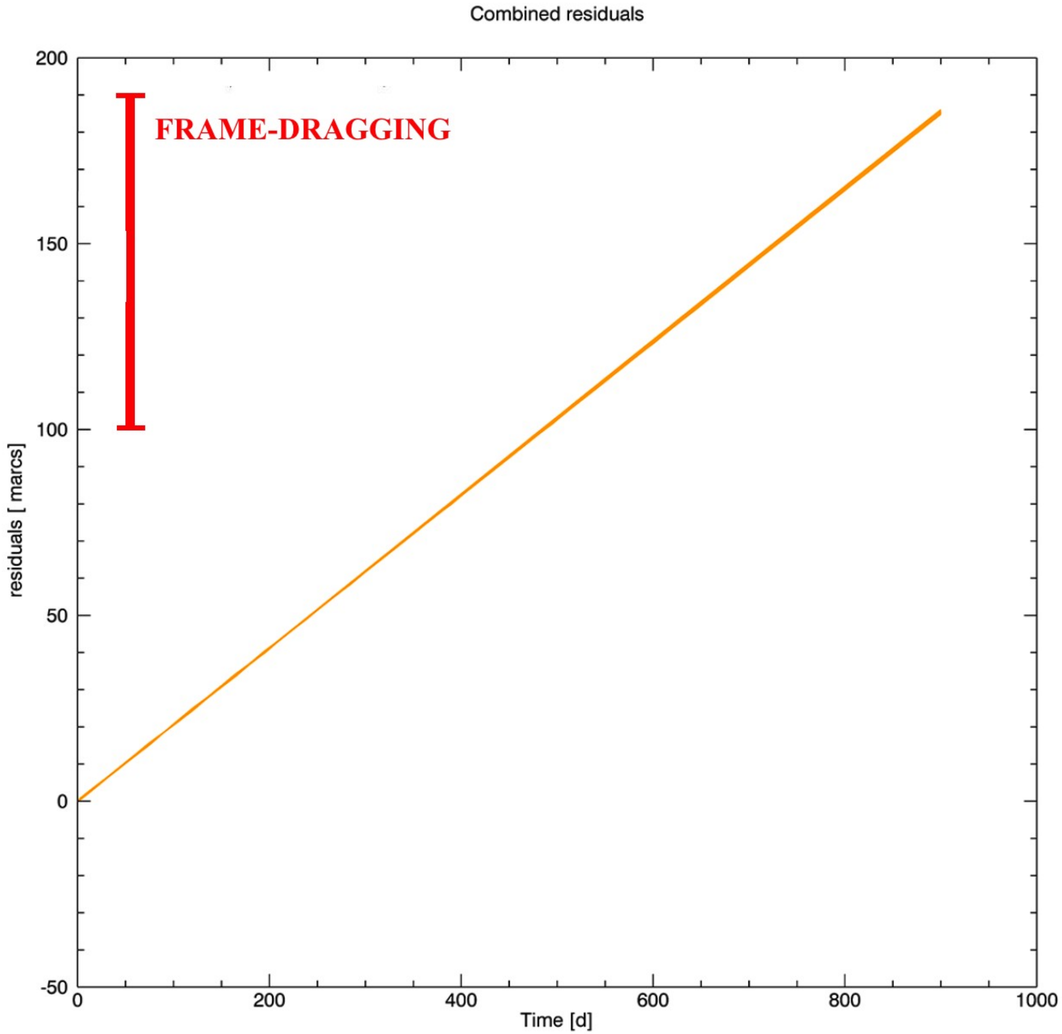


Result of the Monte Carlo simulation: when the LAGEOS and LARES 2 orbits are combined the spread is at the level of about 0.15% of frame-dragging!

I.C., et al., II. Monte Carlo simulations and covariance analyses of the LARES 2 experiment, EPJ P 132: 337 (2017).

Similarly, with a covariance analysis of the LARES 2-LAGEOS experiment, we found:

frame-dragging = 1.0007±

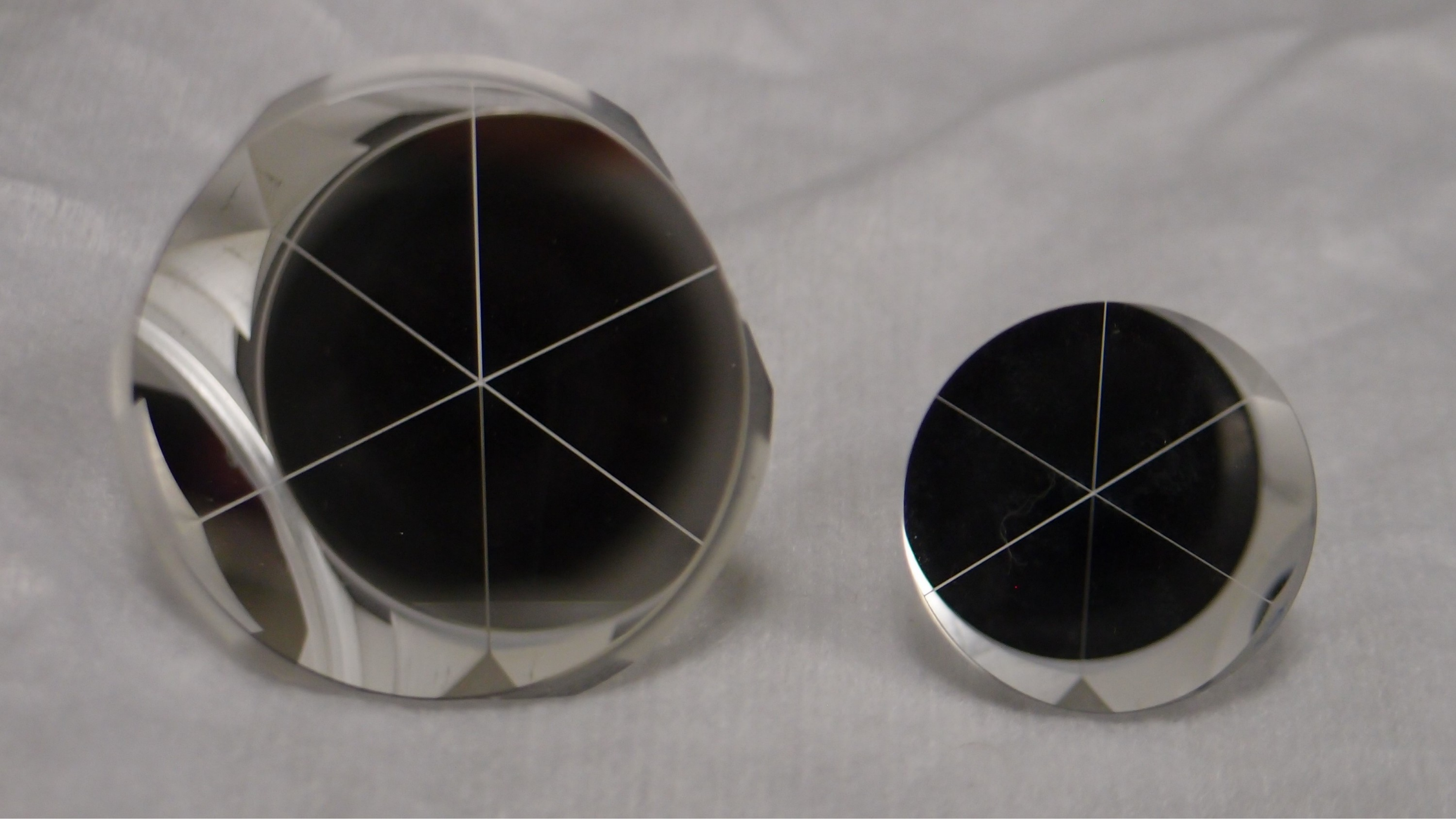


I.C, et al., II. Monte Carlo simulations and covariance analyses of the LARES 2 experiment, EPJ P 132: 337 (2017).

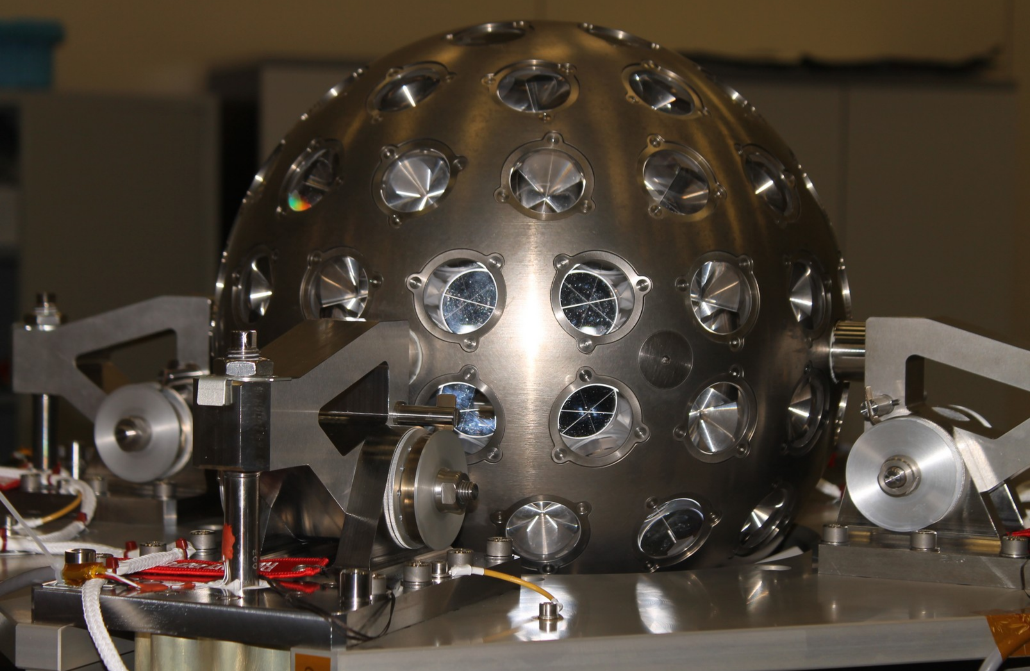
*** Frame-dragging was measured in 2019/2021 with accuracy (*systematic errors*) of *about* 2%-1% using almost 10 years of LARES + LAGEOS + LAGEOS 2 observations**

*** LARES 2 was launched in 2022 and after a number of years of data we may reach an accuracy of *about* 0.2%-0.1% in testing frame-dragging, plus other tests of fundamental physics**

Using LAGEOS, LAGEOS 2 and LARES we obtained a test of the equivalence principle at a new range and with new materials with an uncertainty (systematic errors) of about 10^{-9}

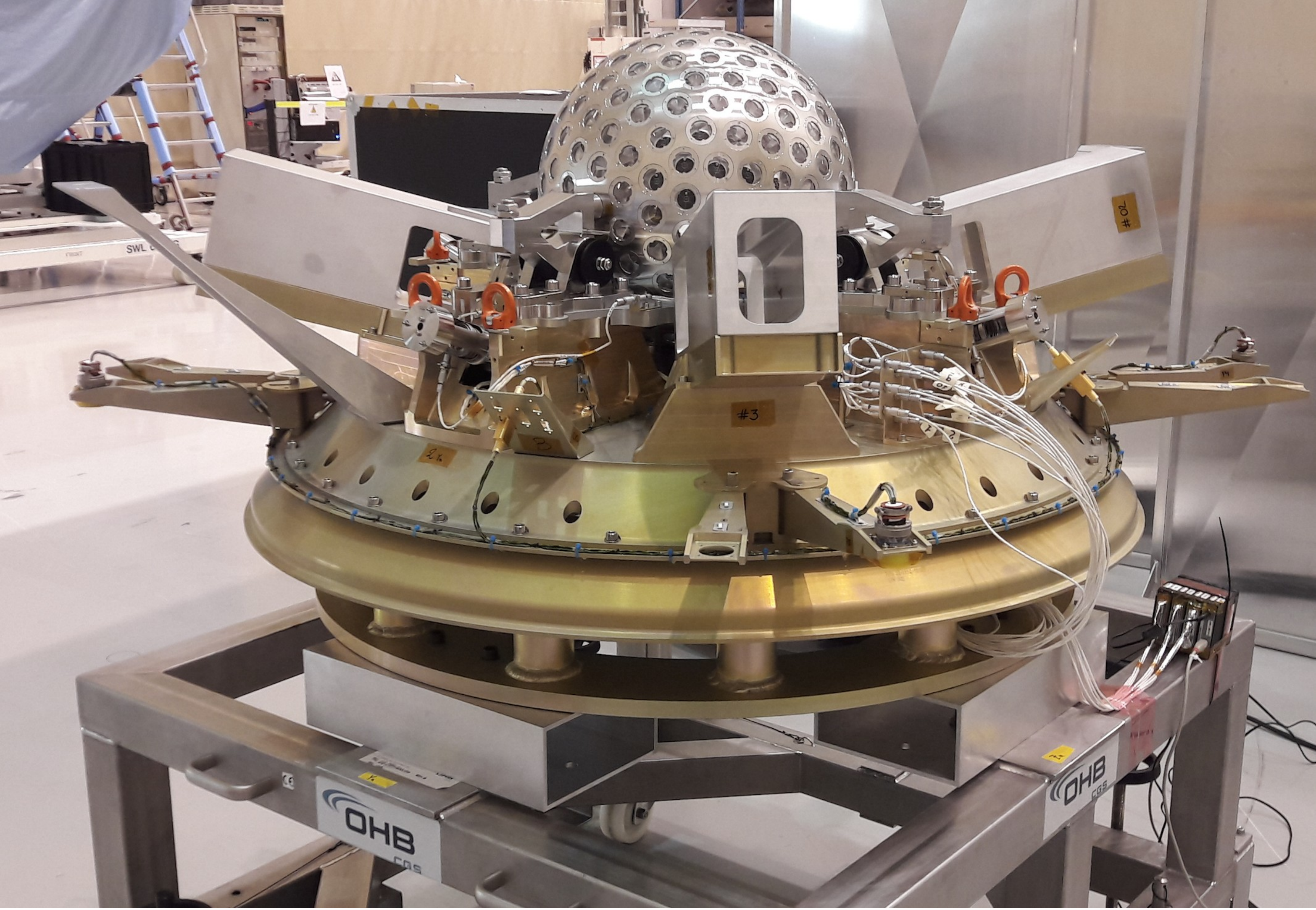


The 1.5 inches retro-reflector on LAGEOS, LAGEOS 2, LARES and the 1 inch retro-reflector on LARES 2

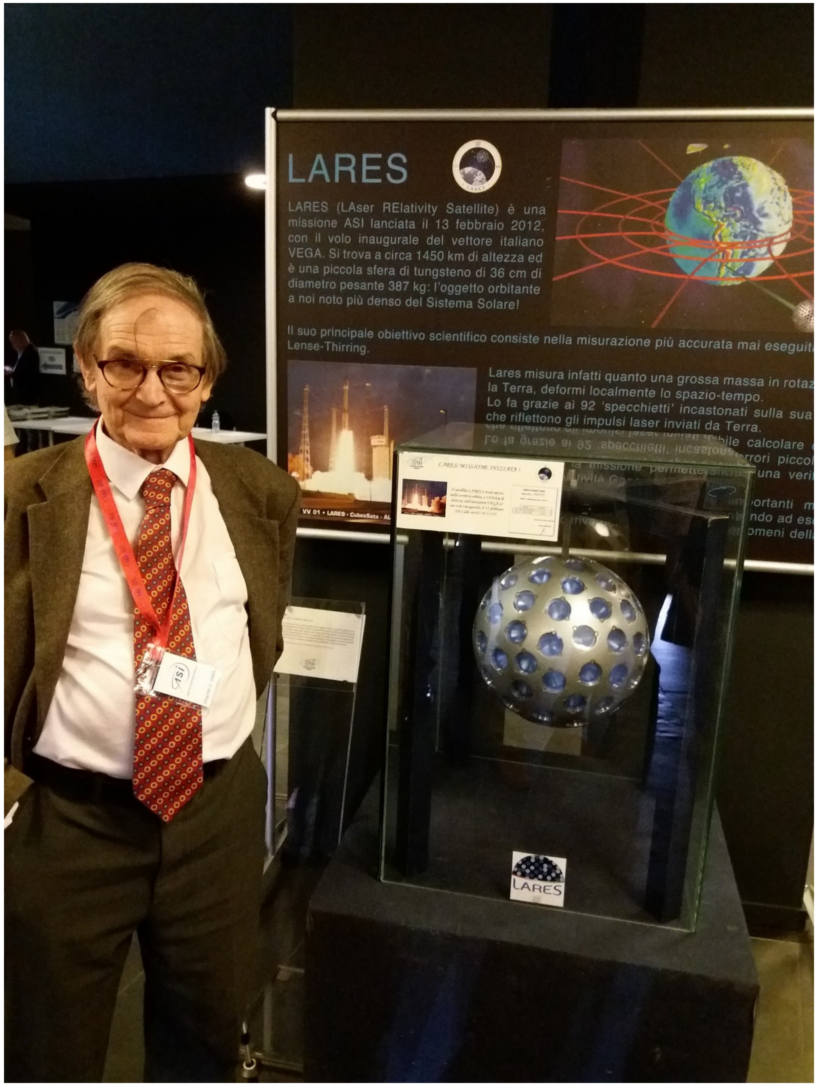


 LARES 2012

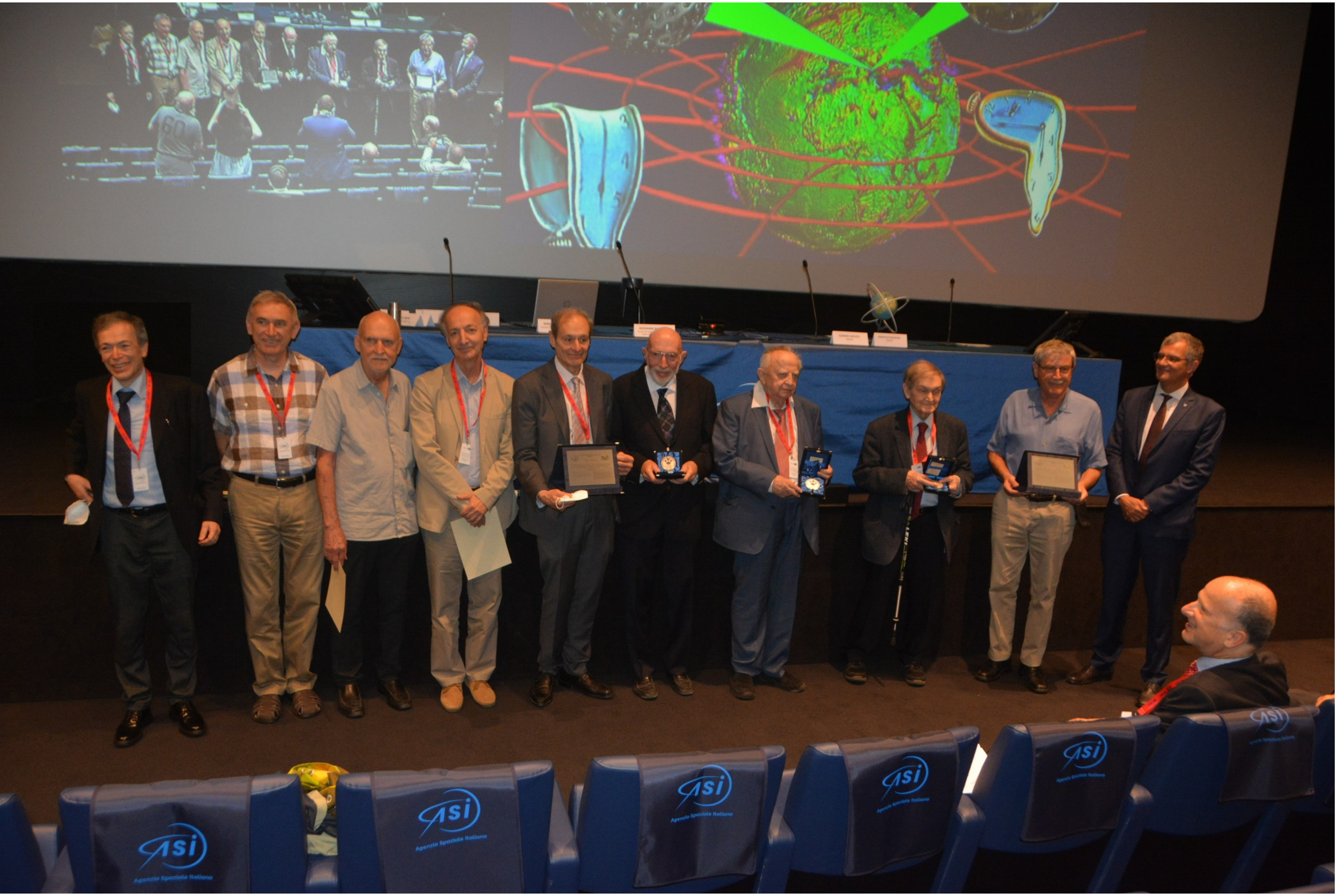
 LARES 2, 2020



LARES 2: ready for launch in 2022



Roger Penrose at ASI International LARES meeting 2019



ASI
2022

with:

Giorgio Saccoccia, Mario Cosmo, Kip Thorne, Roger Penrose, Igor Novikov, Paul Davis, Richard Matzner, Vahe Gurzadyan, Sergei Kopeikin

THANK YOU!

BASIC IDEA AND METHOD

- **The potential energy of body 1 in the field of body 2 could in principle be of the type:**

$$U(r) = -\frac{GM_1M_2}{r}\left(1 + \frac{b_1b_2}{GM_1M_2}e^{-\frac{r}{\lambda}}\right)$$

Where - GM_1M_2/r is the standard Newtonian potential energy (representing the Newtonian gravitational theory as the lowest order approximation of GR), G is the gravitational constant, M_1 and M_2 are the masses of the two bodies, b_1 and b_2 are some composition dependent propertiesof bodies 1 and 2, defining the additional interaction, r is the distance between the two bodies and Lamda the Yukawa range of the additional interaction.

I.C. et al. Scientific Reports-Nature, 9, 1-10 (2019).

Mario Cosmo and Roger Penrose at ASI 2022

If there is a breakdown of the equivalence principle, the main leading terms producing a residual (unmodelled or non-modelled) additional radial acceleration of each of the three satellites LAGEOS; LAGEOS 2 and LARES, are:

$$\delta a_r \cong -\frac{GM_\oplus}{r^2}\,\delta\left(\frac{m_g}{m_i}\right) - \frac{\delta(GM_\oplus)}{r^2} + 3\frac{GM_\oplus}{r^2}\left(\frac{R_\oplus}{r}\right)^2 P_{20}\,\delta J_2 + 2\frac{GM_\oplus}{r^3}\delta r$$

So we have three unknowns:

$$\delta\left(\frac{m_g}{m_i}\right) \qquad \delta J_2 \qquad \delta(GM_\oplus)$$

I.C. et al. Scientific Reports-Nature, 9, 1-10 (2019).

For the three satellites LAGEOS, LAGEOS 2 and LARES, the residual radial accelerations are experimentally determined by the Satellite Laser Ranging observations and our orbital estimators GEODYN (EPOS-OC and UTOPIA)

Therefore, we we have three unknowns for the three observables (the three residual radial accelerations).

The error in the determination of the radial distance of each satellite is the main bias in the measurement of the three unknowns. It is at the level of a few millimeters for the three satellites.

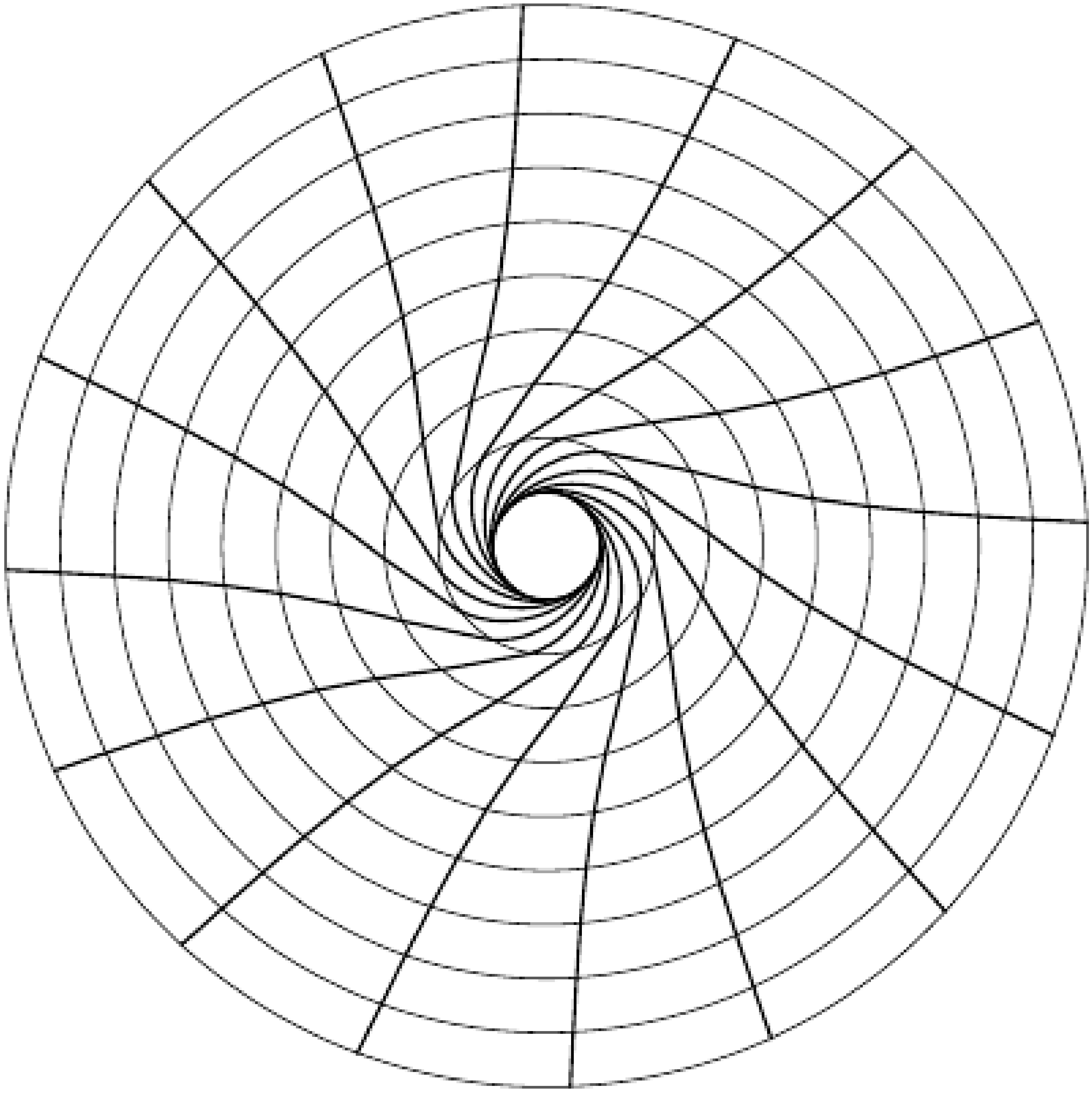
So in conclusion we found:

$$\delta(m_g/m_i) = 2.0 \times 10^{-10} \pm 1.1 \times 10^{-9}$$

where:

$$\delta\left(\frac{m_g}{m_i}\right) = \frac{m_g}{m_i}\bigg|_{tungsten} - \frac{m_g}{m_i}\bigg|_{aluminum/brass}.$$

I.C. et al. Scientific Reports-Nature, 9, 1-10 (2019).

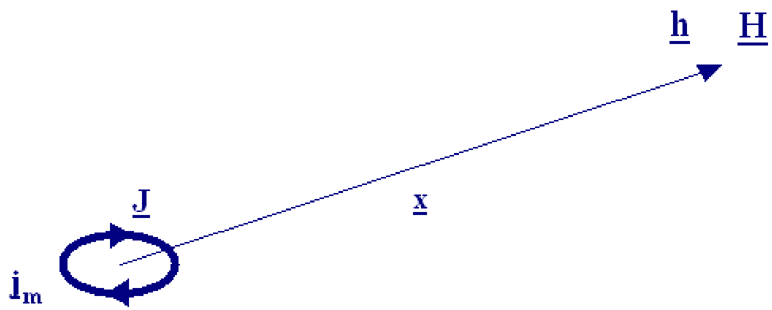


After about 7 years of laser-ranging data of the LARES satellite, together with LAGEOS and LAGEOS 2 and with the improved Earth's gravity models, we measured (2019) the frame-dragging effect with accuracy of about 2%, with other implicational for fundamental physics such as improving the limits on Chern-Simon mass and placing further limits on String Theories equivalent to Chern-Simon gravity. Furthermore, LARES provided a test of the weak equivalence principle at a new range and with new materials with about 10^{-9} accuracy.

Thus, LARES-type satellites could test other fundamental physics effects and much improve the existing limits on frame-dragging and Chern-Simon mass.

THE WEAK-FIELD AND SLOW MOTION ANALOGY WITH ELECTRODYNAMICS

Gravitomagnetic Field in General Relativity



From weak field and slow motion limit of G= χ T:

$$\Delta\, h_{0i} \cong 16\,\pi\,\rho\, v^i$$

Lorentz gauge

Electromagnetism

$$\Delta\, \mathbf{A} = -\frac{4\pi}{c}\,\mathbf{j}$$

where $\mathbf{h} \equiv (h_{01},\ h_{02}\ , h_{03})$ is the gravitomagnetic potential

$$h_{0i}\,(\mathbf{x}) \cong -\,4\int \frac{\rho(\mathbf{x}')v^i(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,d^3x'$$

$$\mathbf{A}\,(\mathbf{x}) = \frac{1}{c}\int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\,d^3x'$$

$$\mathbf{h}\,(\mathbf{x}) \cong -\,2\,\frac{\mathbf{J}\times\mathbf{x}}{|\mathbf{x}|^3}$$

$$\mathbf{A}\,(\mathbf{x}) \cong \frac{\mathbf{m}\times\mathbf{x}}{|\mathbf{x}|^3}$$

The gravitomagnetic field is:

$$\mathbf{H} = \nabla\times\mathbf{h} \cong 2\left[\frac{\mathbf{J}-3(\mathbf{J}\cdot\hat{\mathbf{x}})\,\hat{\mathbf{x}}}{|\mathbf{x}|^3}\right]$$

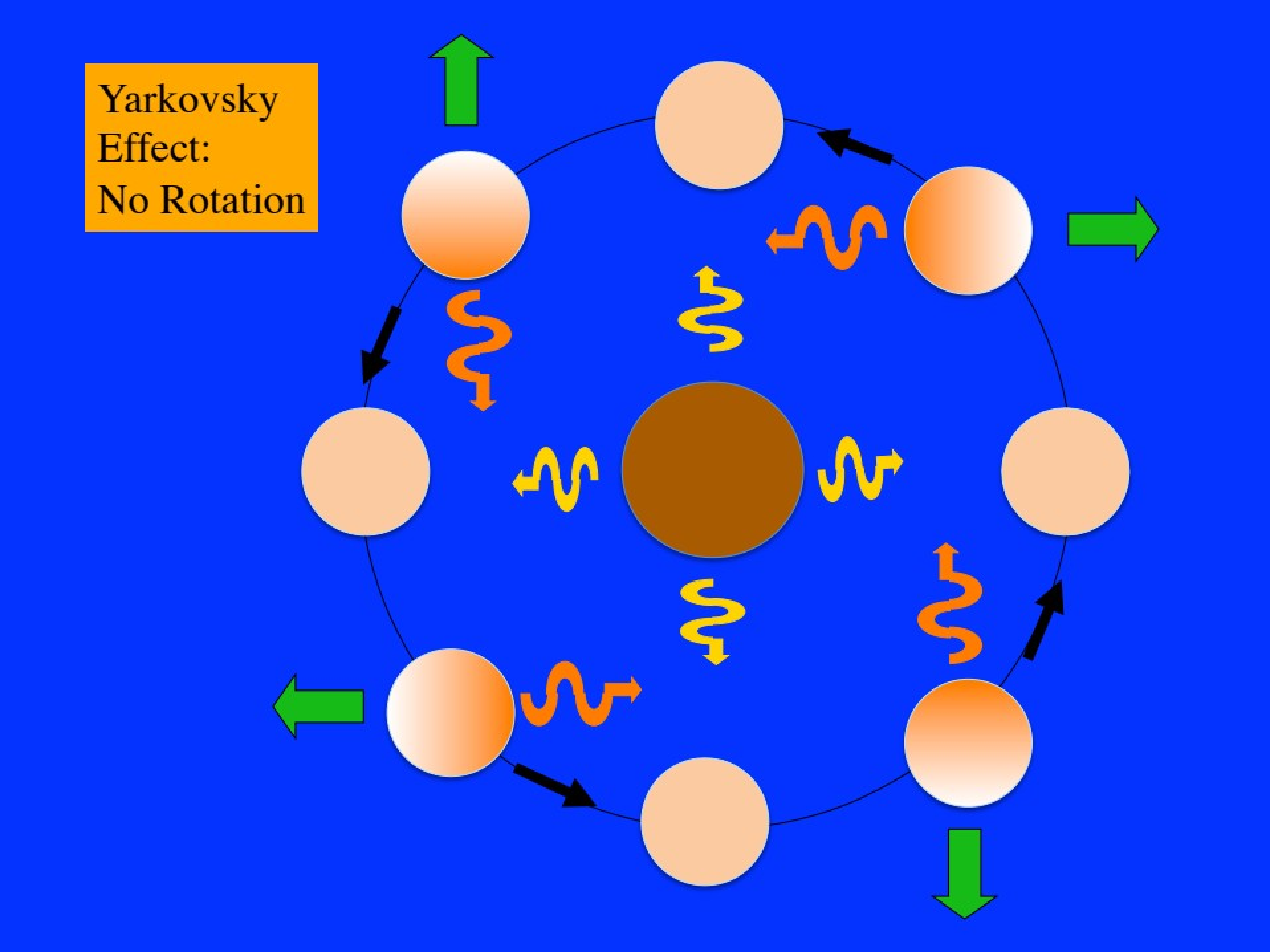
$$\mathbf{B} = \nabla\times\mathbf{A} \cong$$

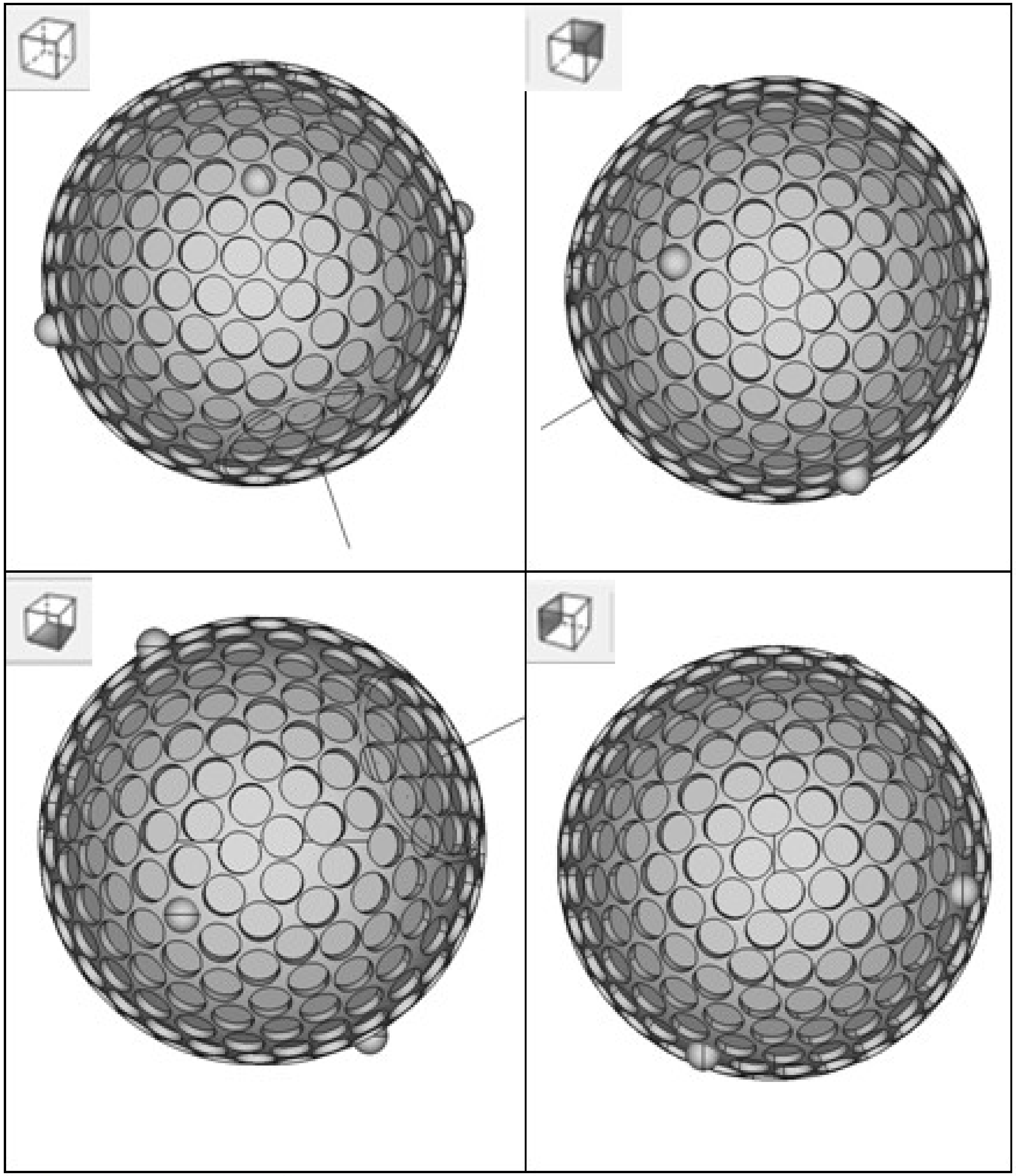
$$\cong \frac{3\,\hat{\mathbf{x}}\,(\hat{\mathbf{x}}\cdot\mathbf{m}) - \mathbf{m}}{|\mathbf{x}|^3}$$

From weak field and slow motion limit of D u=0:

$$m\,\frac{d^2\mathbf{x}}{dt^2} \cong m\,(\mathbf{G} + \frac{d\mathbf{x}}{dt}\times\mathbf{H})$$

$$m\,\frac{d^2\mathbf{x}}{dt^2} = q\,(\mathbf{E} + \frac{d\mathbf{x}}{dt}\times\mathbf{B})$$


Yarkovsky
Effect:
No Rotation

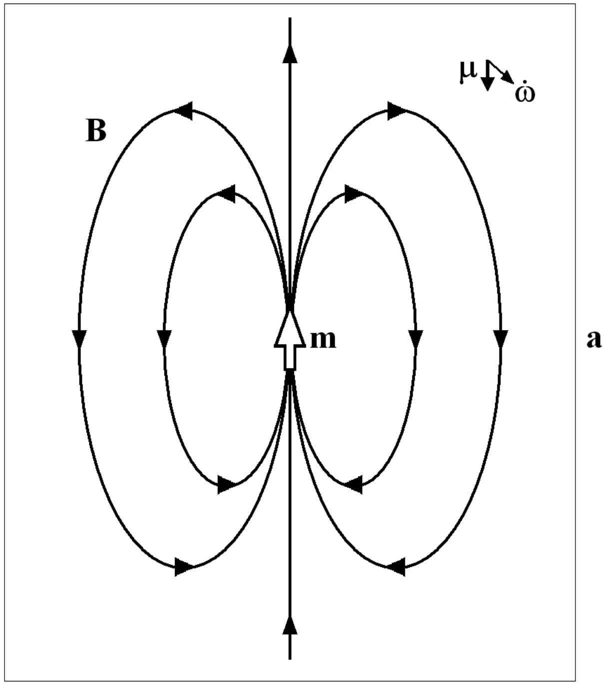


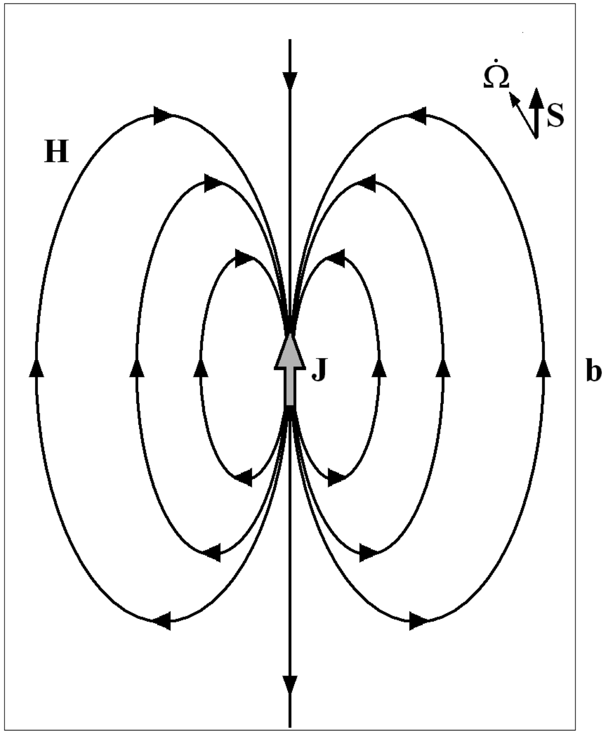
Design of LARES 2 by Antonio Paolozzi and the LARES team of Sapienza and Salento Universities

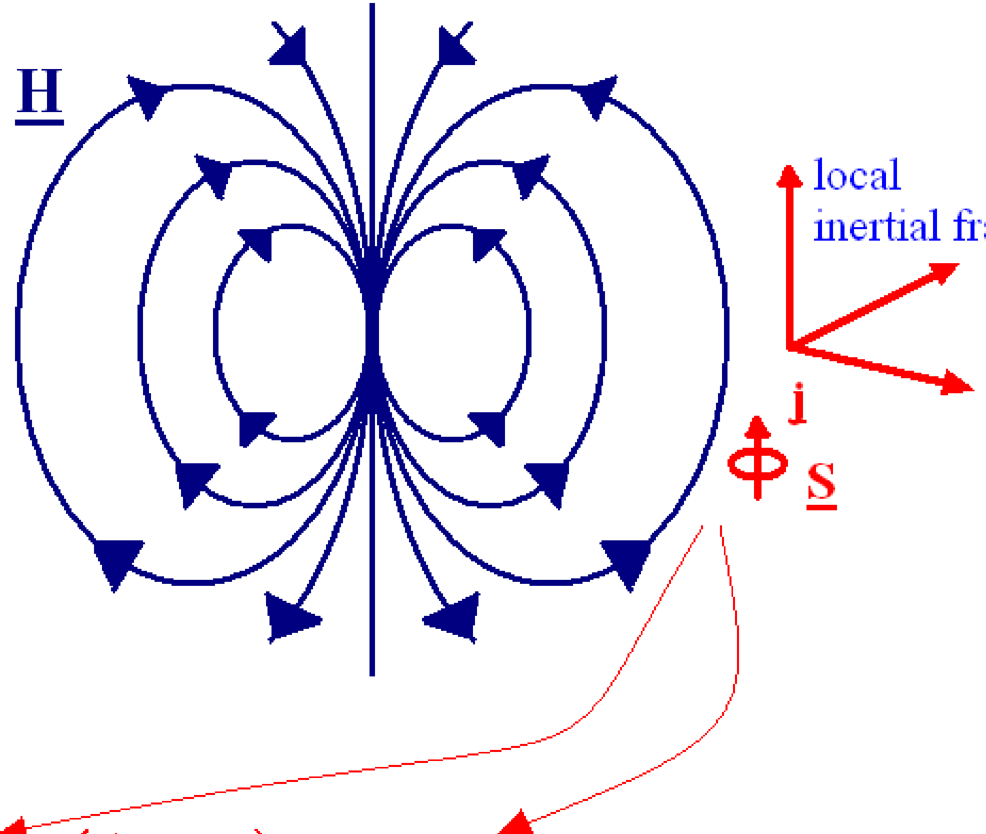
GRAVITOMAGNETISM

There is a interesting analogy of weak-field and slow-motion General Relativity with electromagnetism

Magnetic field **B**, gravitomagnetic field **H** and the precession of a magnetic dipole **μ** and of a gyroscope **S**







$$\mathbf{F} = \left(\frac{1}{2}\mathbf{S}\cdot\nabla\right)\mathbf{H} \qquad \tau \cong \tfrac{1}{2}\mathbf{S}\times\mathbf{H} = \frac{d\mathbf{S}}{dt} \equiv \dot{\mathbf{\Omega}}\times\mathbf{S}$$

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2}\mathbf{H} = \frac{-\mathbf{J} + 3(\mathbf{J}\cdot\hat{\mathbf{x}})\hat{\mathbf{x}}}{|\mathbf{x}|^3}$$

Dragging of inertial frames:

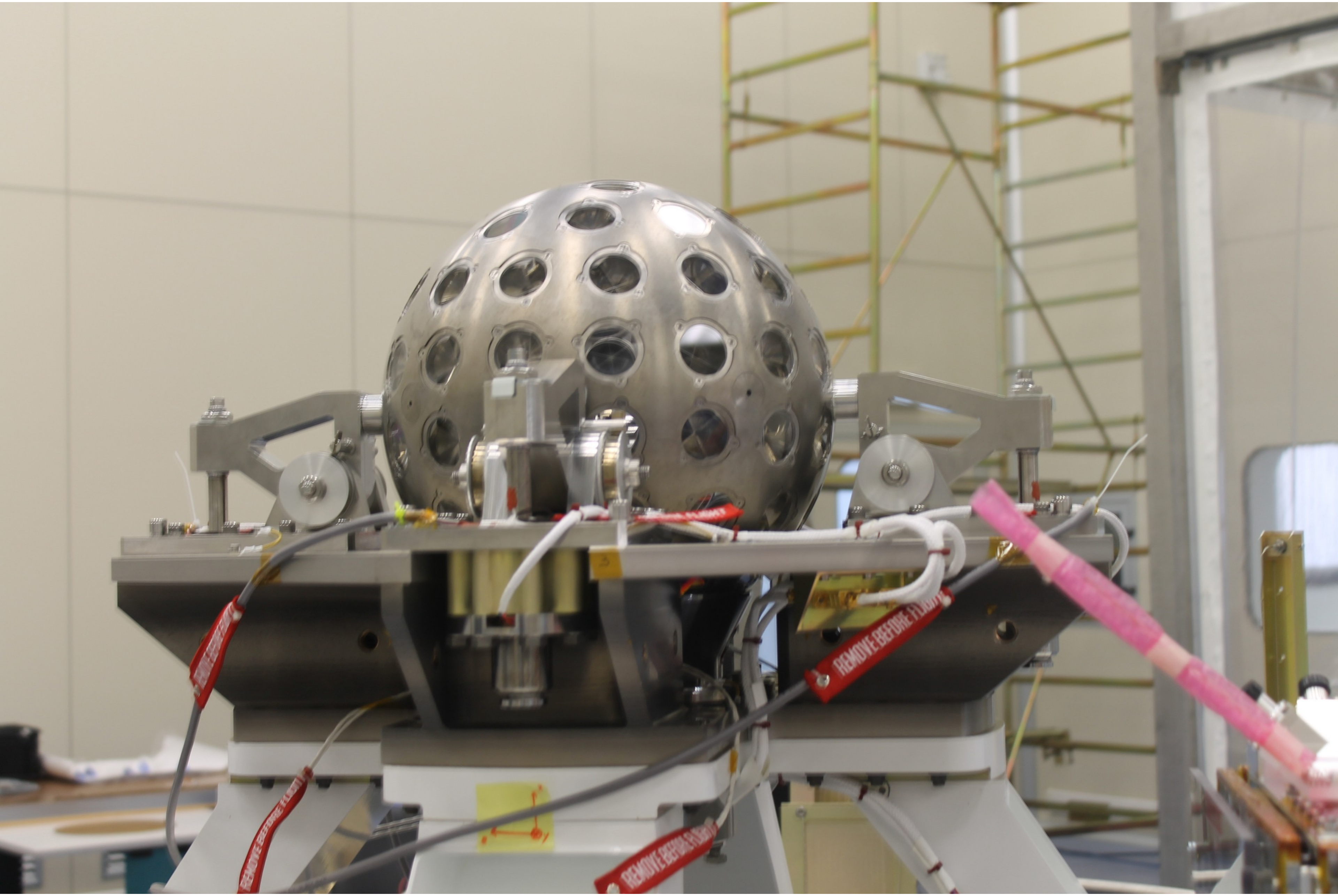
Mach principle in general relativity

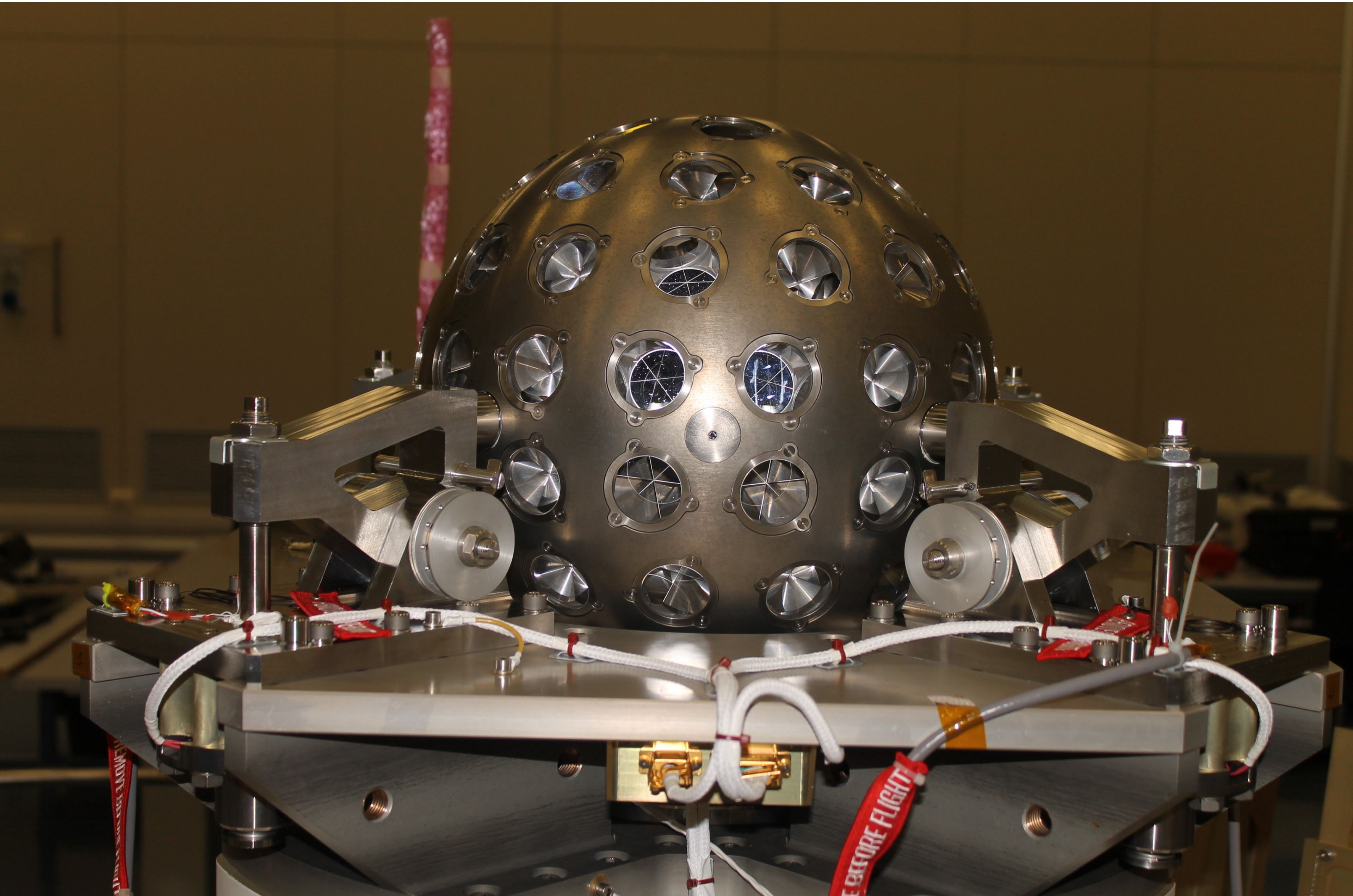
GRAVITATION AND INERTIA

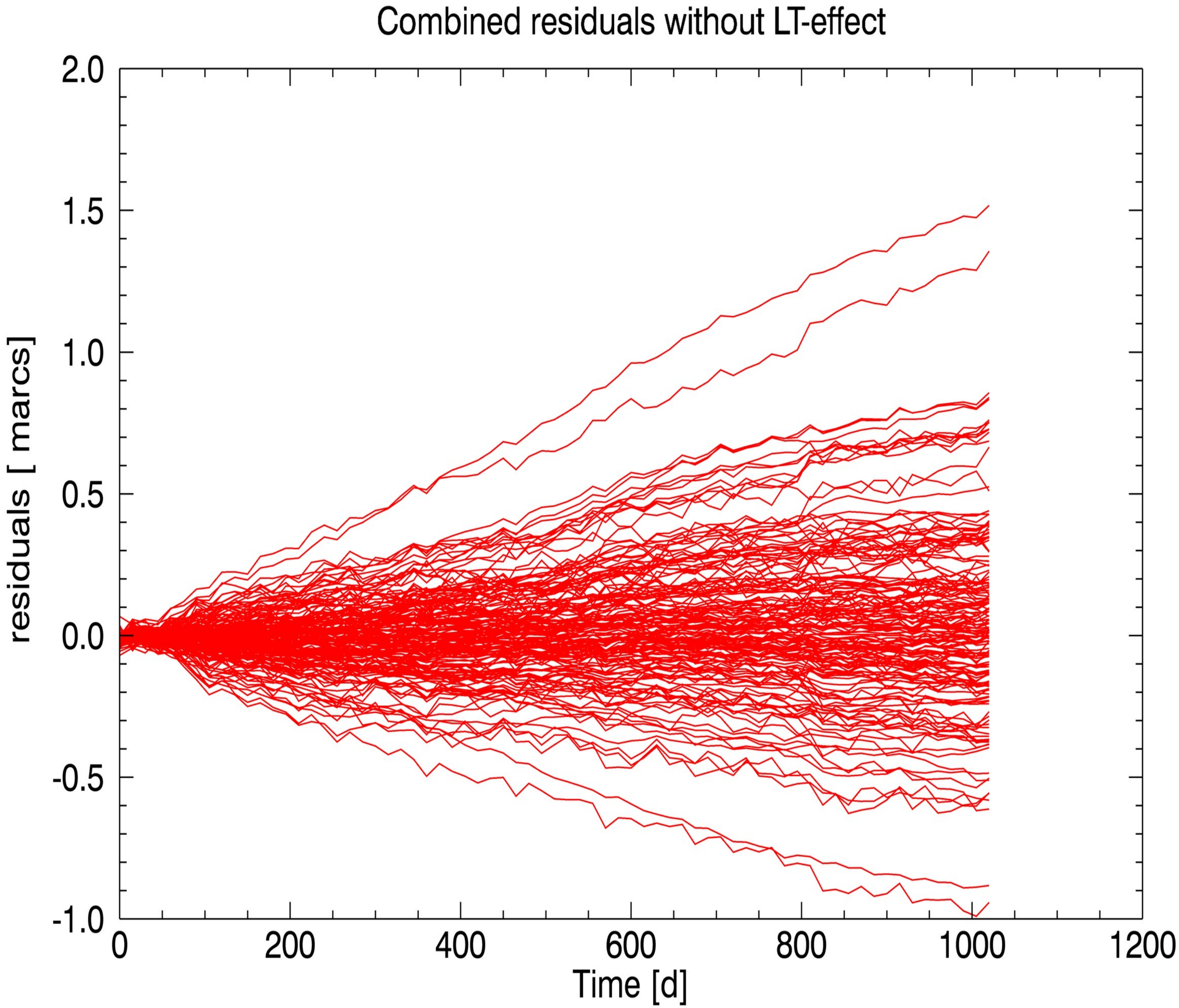
I.C. and J.A. Wheleer -1995

EIGEN-GRACE02S Model and Uncertainties

Even zonals lm	Value $\cdot 10^{-6}$	Uncertainty	Uncertainty on node I	Uncertainty on node II	Uncertainty on perigee II
20	–484.16519788	$0.53 \bullet 10^{-10}$	$1.59\ \Omega_{LT}$	$2.86\ \Omega_{LT}$	$1.17\ \omega_{LT}$
40	0.53999294	$0.39 \bullet 10^{-11}$	$0.058\ \Omega_{LT}$	$0.02\ \Omega_{LT}$	$0.082\ \omega_{LT}$
60	–.14993038	$0.20 \bullet 10^{-11}$	$0.0076\ \Omega_{LT}$	$0.012\ \Omega_{LT}$	$0.0041\ \omega_{LT}$
80	0.04948789	$0.15 \bullet 10^{-11}$	$0.00045\ \Omega_{LT}$	$0.0021\ \Omega_{LT}$	$0.0051\ \omega_{LT}$
10,0	0.05332122	$0.21 \bullet 10^{-11}$	$0.00042\ \Omega_{LT}$	$0.00074\ \Omega_{LT}$	$0.0023\ \omega_{LT}$

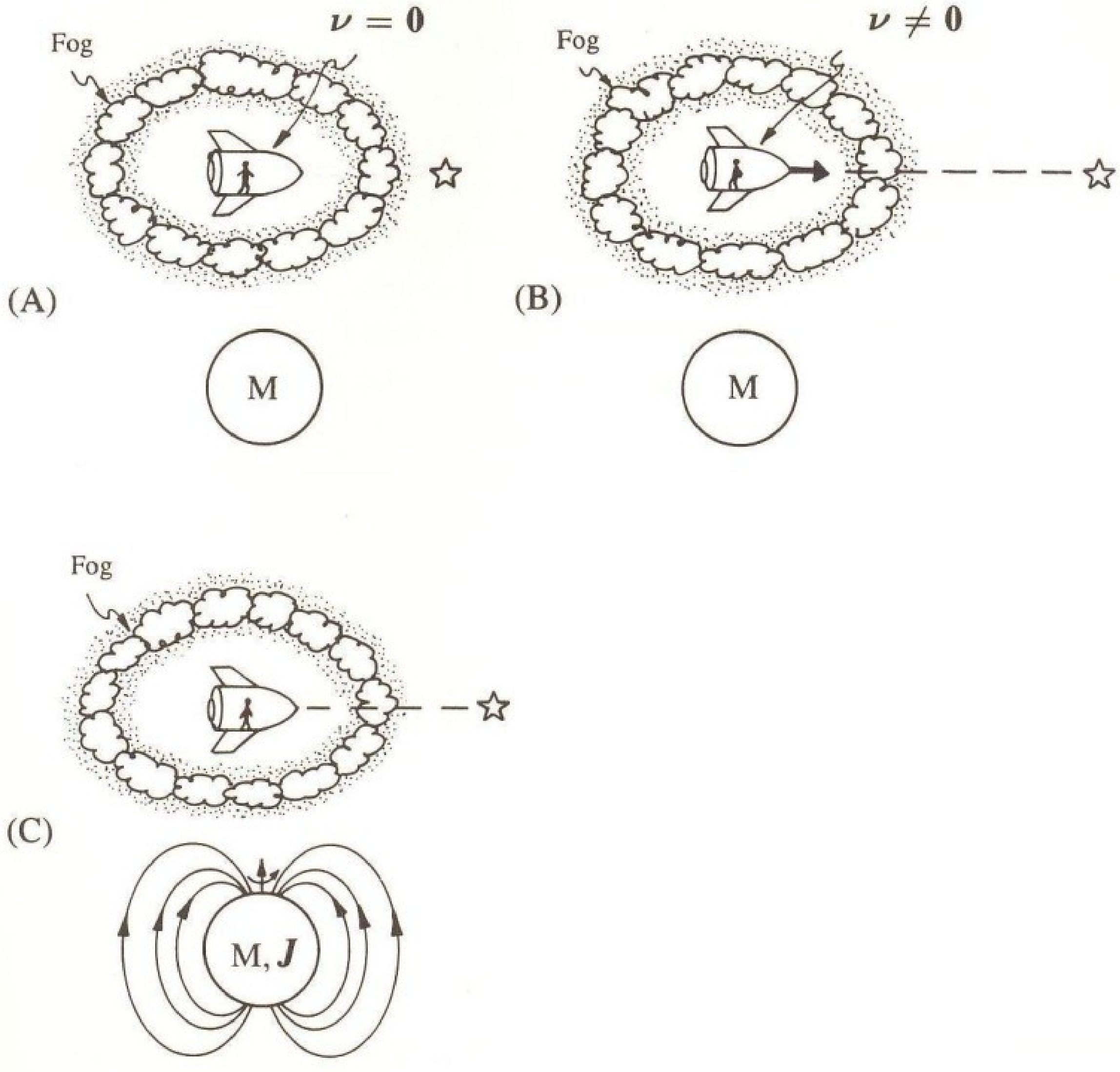

REMOVE BEFORE FLIGHT

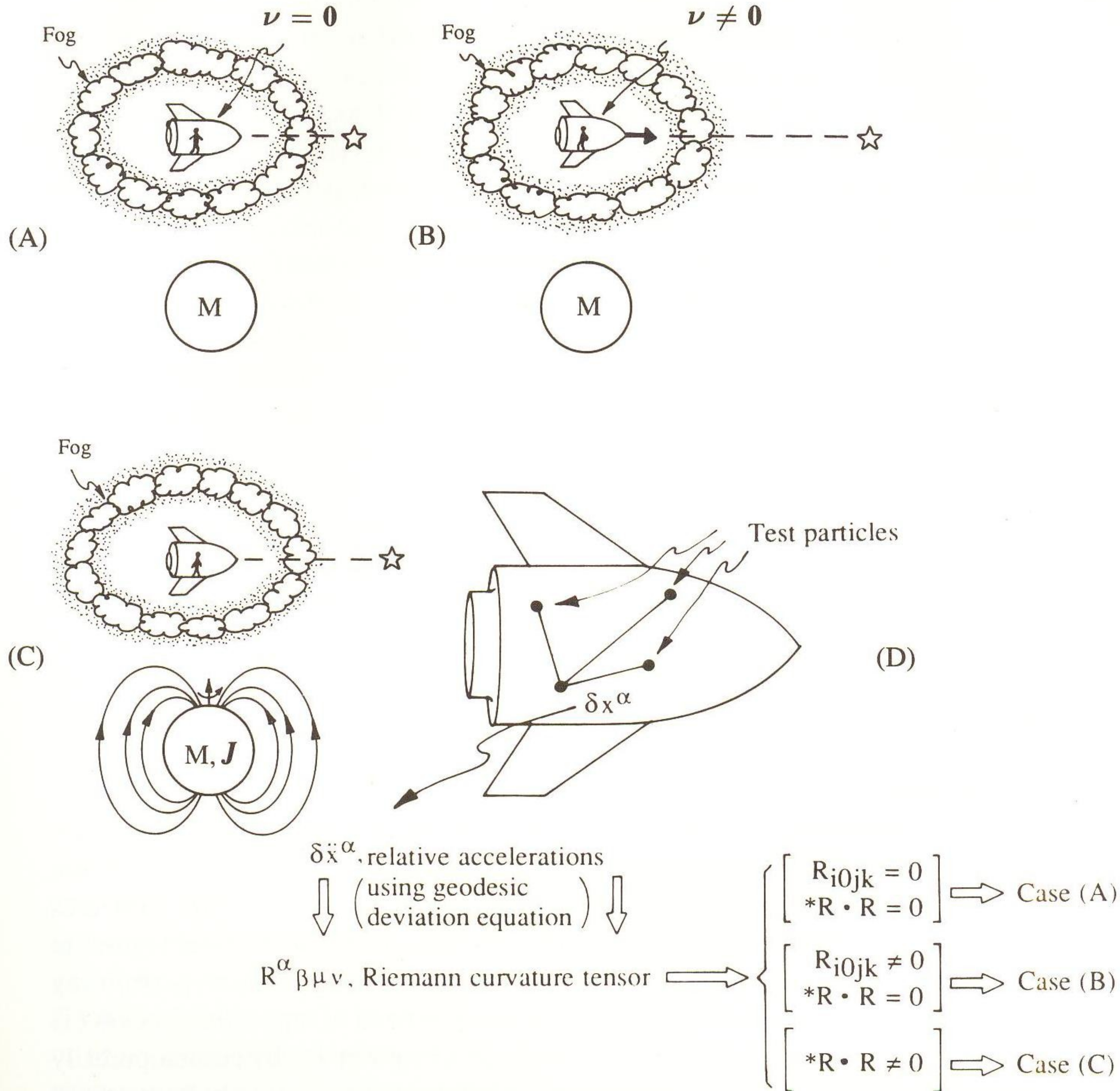

BEFORE FLIGHT



Combined residuals without LT-effect
residuals [marcs]
Time [d]
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0
200
400
600
800
1000
1200

THE GRAVITOMAGNETIC FIELD




Fog
$\nu = 0$
(A)
M
Fog
$\nu \neq 0$
(B)
M
Fog
(C)
M, J
Test particles
δx^α
(D)
$\delta \ddot{x}^\alpha$, relative accelerations
(using geodesic deviation equation)
$R^\alpha{}_{\beta\mu\nu}$, Riemann curvature tensor
$R_{i0jk} = 0$
$^*R \cdot R = 0$
Case (A)
$R_{i0jk} \neq 0$
$^*R \cdot R = 0$
Case (B)
$^*R \cdot R \neq 0$
Case (C)

Chern-Simons Gravity

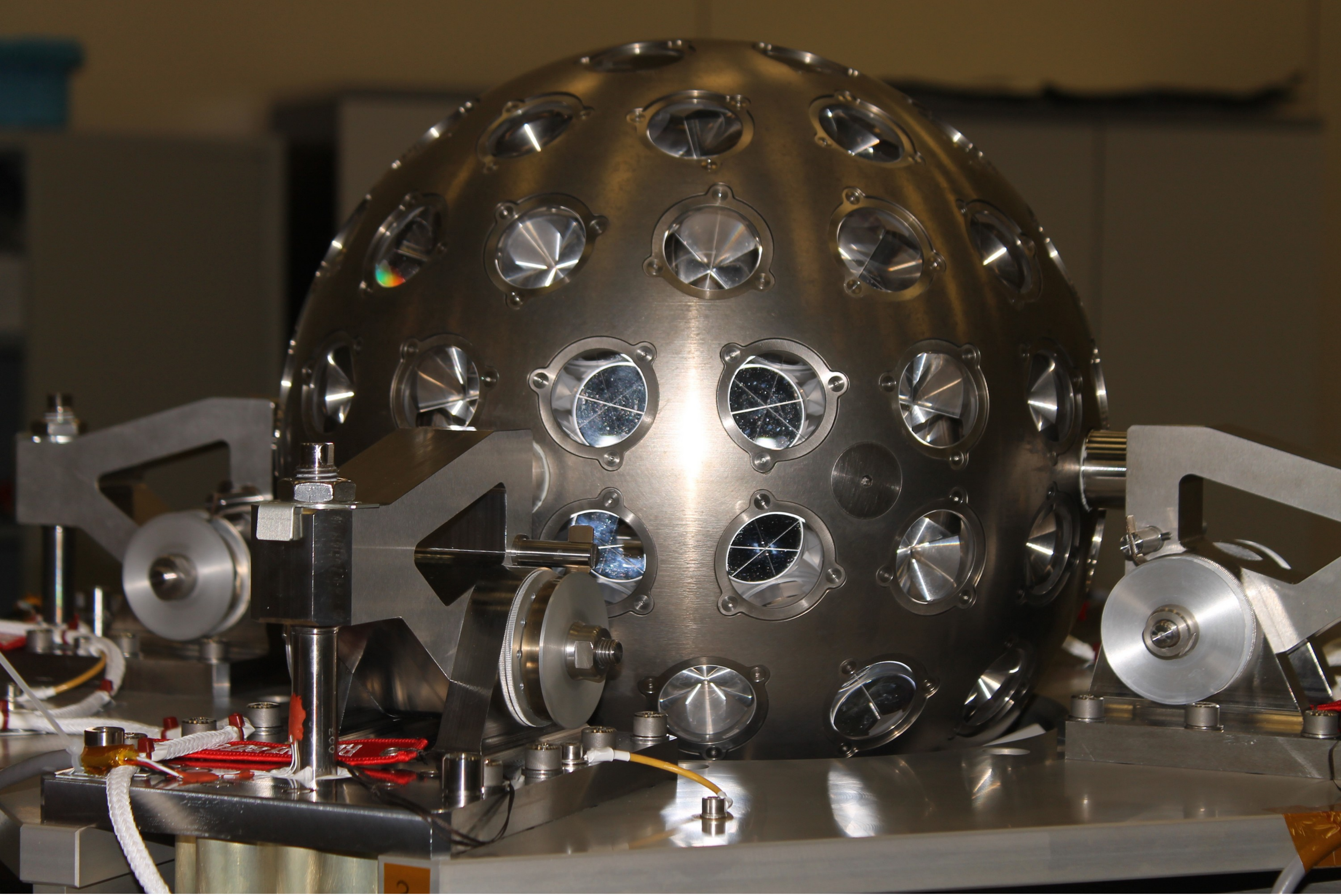
The modified action of Chern–Simons theory is then:

$$S_{CS} = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi}R - \frac{l}{12}\theta^*\mathbf{R}\cdot\mathbf{R} - \frac{1}{2}(\partial\theta)^2 - V(\theta) + L_{mat}\right]$$

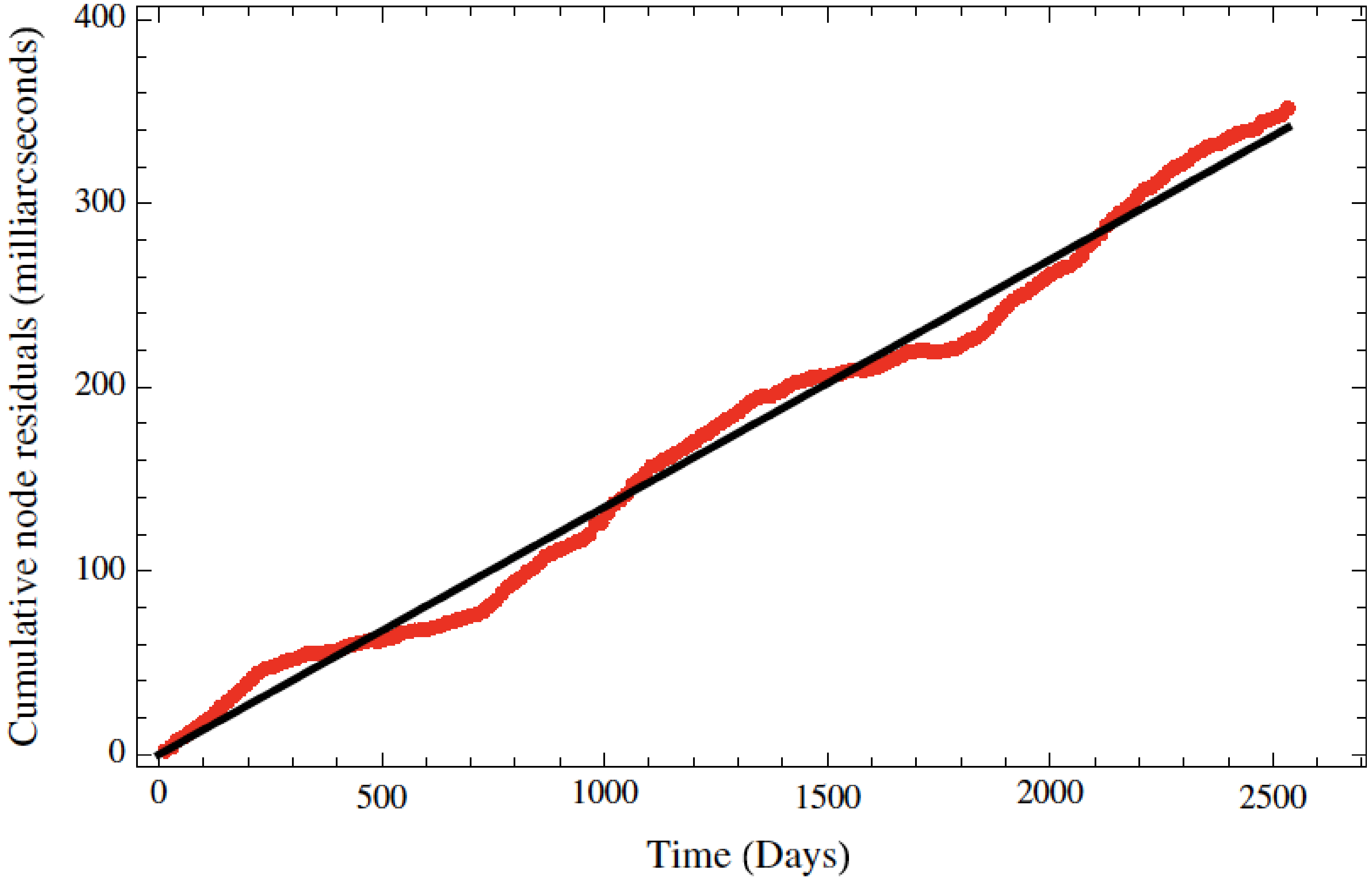
$^*R \cdot R = ½\, \varepsilon_{\alpha\beta\sigma\rho}\, R^{\sigma\rho}{}_{\mu\nu}\, R^{\alpha\beta\mu\nu}$ is the Pontryagin pseudoscalar, θ is a Scalar field, g the determinant of the metric, R the Ricci scalar, l is a new length parameter, L_{mat} the matter Lagrangian density.

The dynamical equation for the scalar field θ is:

$$\Box\theta = \frac{dV}{d\theta} + \frac{1}{12}l^*\mathbf{R}\cdot\mathbf{R}\,.$$



LARES of the Italian Space Agency, launched in 2012



ys

Fit of the cumulative combined observed nodal residuals of LARES, LAGEOS and LAGEOS 2 over about 7 years with a linear regression: published in EPJC 2019 (Ciufolini et al.)

Intuitive characterization of DRAGGING OF INERTIAL FRAMES (*FRAME-DRAGGING* as Einstein named it in 1913)

- In electromagnetism, by the Maxwell-Ampère equation, a magnetic field is generated by electric currents:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \partial_t \mathbf{E}$$

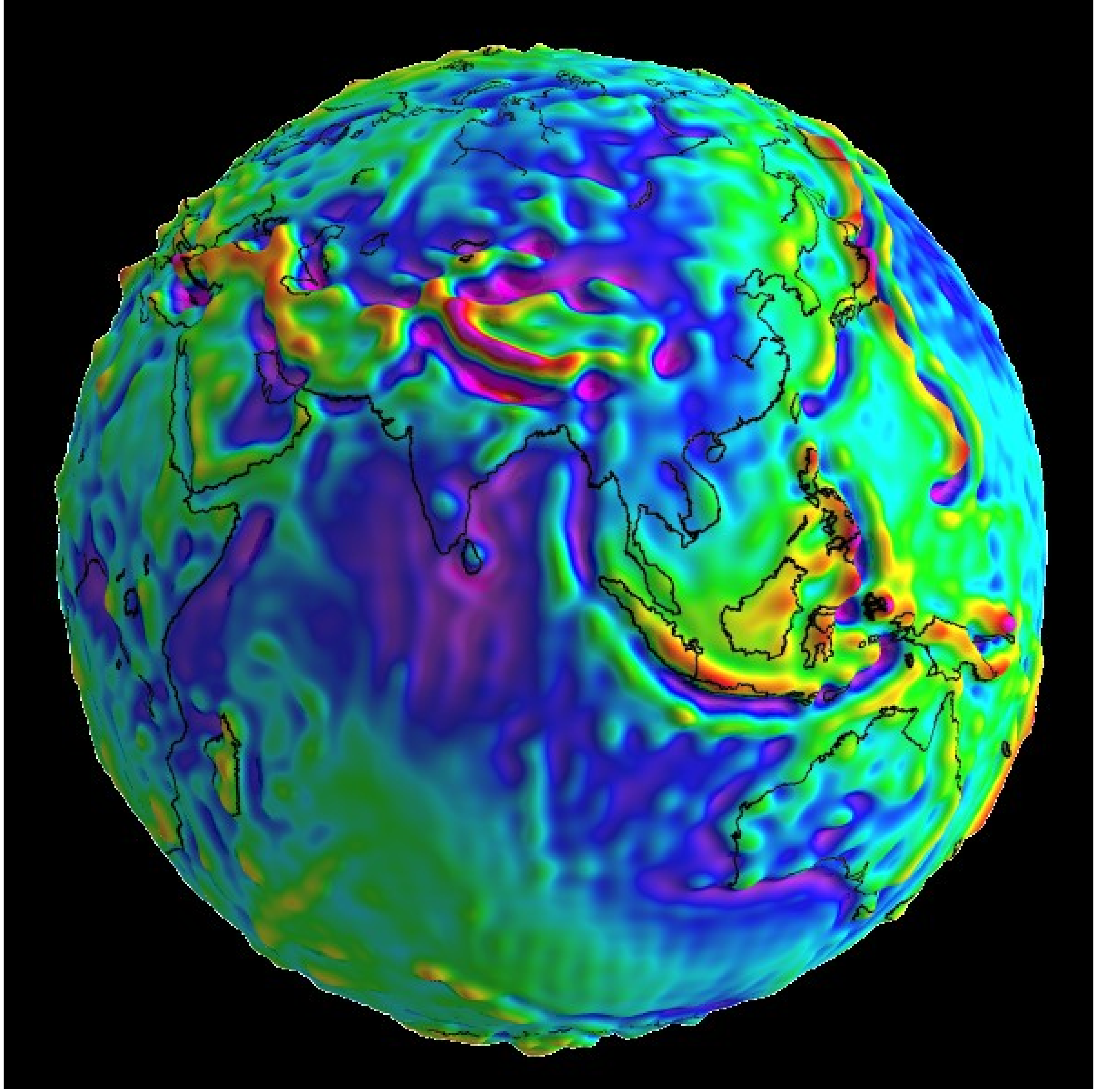
However, in Newtonion gravitation, by the Poisson equation, only mass generates a gravitational field:

$$\triangle U = 4\pi G \rho$$

- **In GR spacetime curvature is generated by mass-energy currents: εu^α**

$$G^{\alpha\beta} = \chi T^{\alpha\beta} = \chi [(\varepsilon + p) u^\alpha u^\beta + p g^{\alpha\beta}$$

It plays a key role in high energy astrophysics (Kerr metric)



EIGEN-GRACE-S (GFZ 2004)