Integrated correlators with line defects

CFT and Integrable Models
Bologna, 07/09/23

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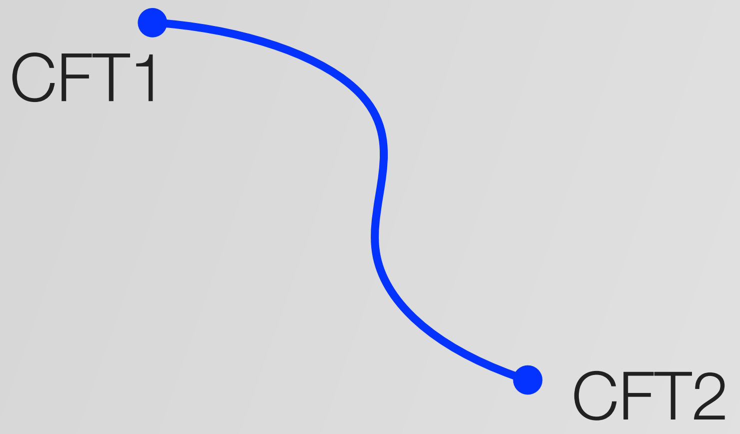
Based on:
[2301.07035], LG
JHEP 04 (2022) 171 with Penati S., Yaakov I.
work in progress with Armanini E., Griguolo L.

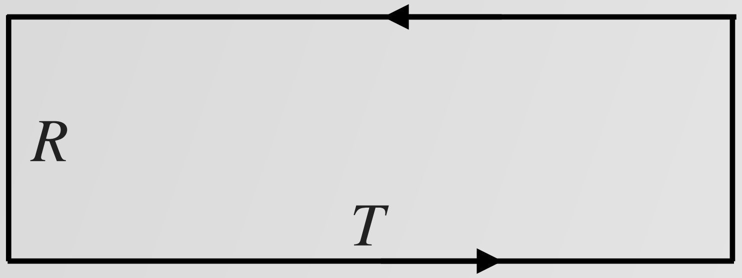
Introduction

- Goal: “solve” QFT —> “solve” CFT
  - Critical phenomena
  - End point of RG flow



- Beyond local correlation functions: defects
  - Global structure of gauge theories
  - Detect phase transition (e.g. confinement)

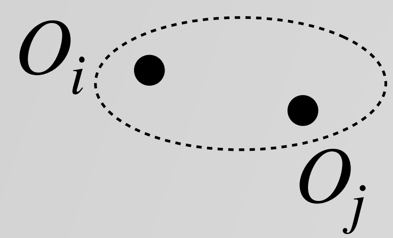
$$W = \mathrm{Tr}\left[P e^{\oint dx^\mu A_\mu}\right] \sim e^{-TV(R)}$$



- Conformal defects: magnetic impurities, particle worldlines

CFTs with line defects

- CFT data —> correlation functions $\langle O(x_1)O(x_2)\ldots O(x_n)\rangle$
  - Spectrum Δ_i and spin s
  - OPE coefficients C_{ijk}



$$\simeq \sum_k C_{ijk}\, O_k$$

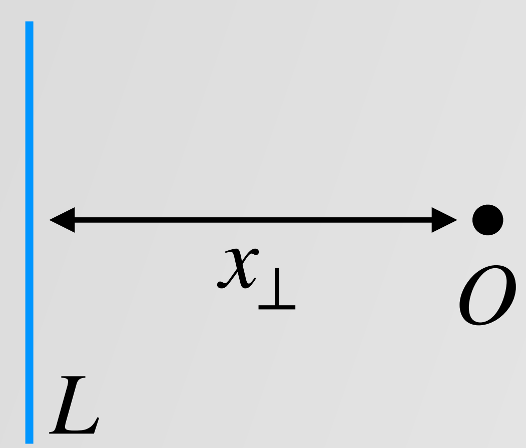
- Conformal lines preserve worldline conformal symmetry

[Billò, Gonçalves, Lauria, Meineri 2016]

$$SO(d+1,1) \to SL(2,\mathbb{R}) \times SO(d-1)$$

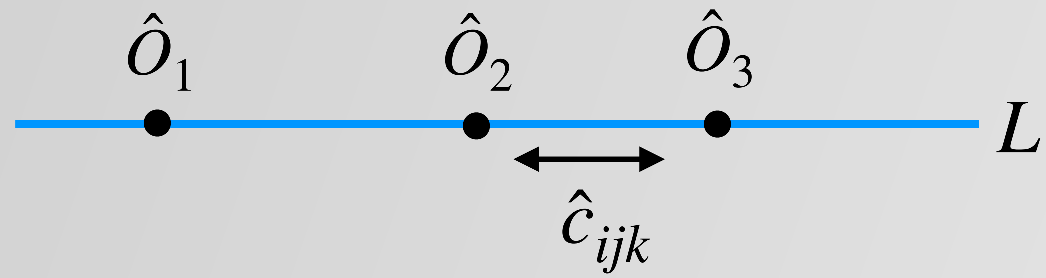
- One-point functions $\{h_O\}$

$$\langle O(x)\rangle_L \equiv \frac{\langle O(x)\, L\rangle}{\langle L\rangle} = \frac{h_O}{|x_\perp|^{\Delta_O}}$$



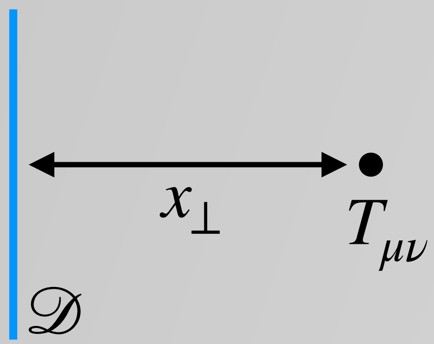
CFTs with line defects

- Defect operators —> $\{\hat{\Delta}_i\,, \hat{c}_{ijk}\}$



- (defect) CFT data & crossing symmetry

- **Stress tensor**



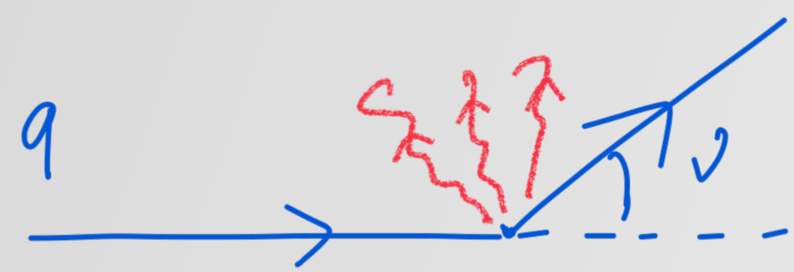
$$\langle T_{\mu\nu}\rangle_{\mathscr{D}} = \frac{h_T}{|x_\perp|^d} H_{\mu\nu}$$

$$H_{ij} = \frac{x_i x_j}{|x_\perp|^2} - \frac{2}{d}\delta_{ij}$$

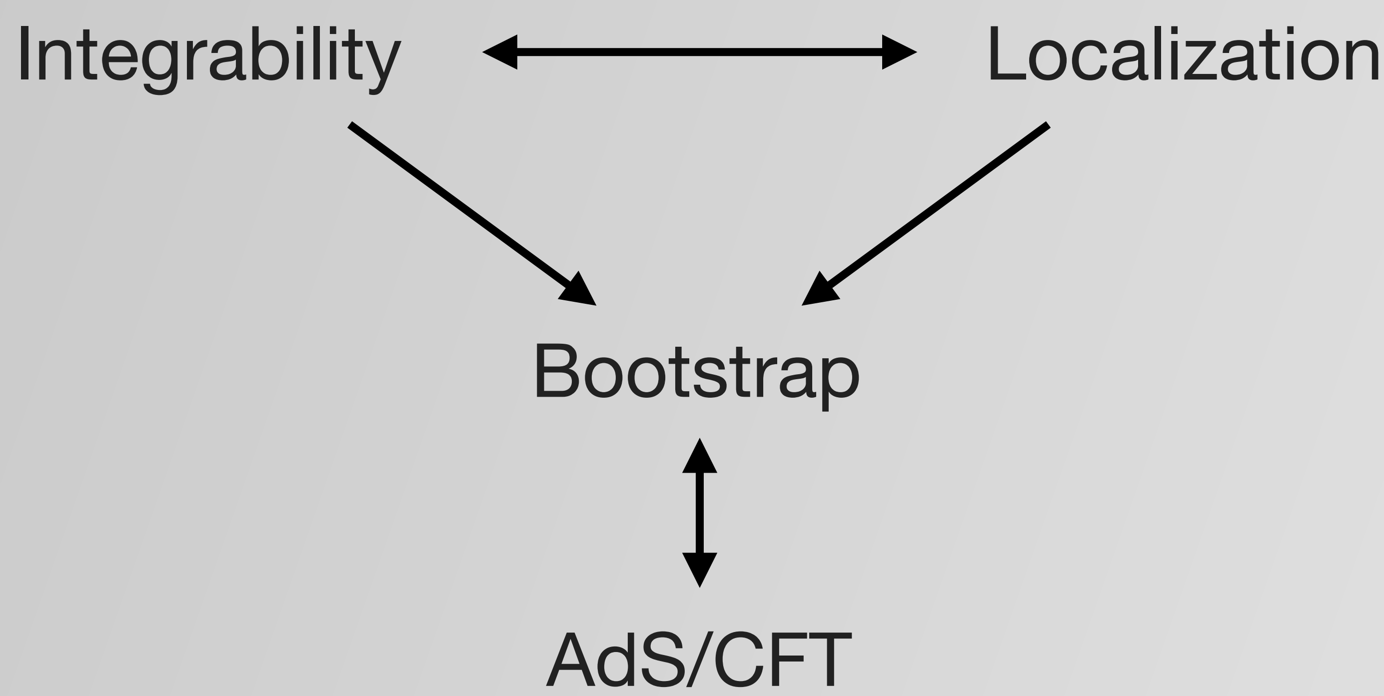
$$H_{\|} = \frac{d-2}{d}$$

- **Displacement operator** $\partial_\mu T^{\mu i} = \delta(\mathscr{D})\hat{D}^i$

$$\langle \hat{D}^i(x_{\|})\hat{D}^j(0)\rangle_{\mathscr{D}} = \frac{12\,B}{|x_{\|}|^4}\delta^{ij}$$



Why SUSY?



- Deform CFT by a relevant deformation $S_{CFT} \to S_{CFT} + m \int d^d x \; O(x)$

- Generating functional for **integrated correlators**

$$\langle \int d^d x_1 \, O(x_1) \ldots \int d^d x_n \, O(x_n) \rangle = \frac{1}{Z} \frac{\partial^n Z[m]}{\partial^n m} \Bigg|_{m=0}$$

ABJM

- $\mathcal{N} = 6$ Chern-Simons matter theory with gauge group $U(N_1)_k \times U(N_2)_{-k}$
  - Gauge fields: $A_\mu \in Adj(U(N_1))$ $\hat{A}_\mu \in Adj(U(N_2))$
  - Matter fields: $C_I, \bar{\psi}^I \in (N_1, \bar{N}_2)$ $I = 1,\ldots,4 \in SU(4)_R$
- Dual to M-theory on $\mathrm{AdS}_4 \times S^7/Z_k$ *[Aharony, Bergman, Maldacena, Jafferis 08]*
- Real mass deformations

$$S_{\text{mass}} \propto m_1 \int d^3x \, \mathrm{Tr}\left[i\left(\bar{C}^1 C_1 - \bar{C}^3 C_3\right) + \bar{\psi}^1 \psi_1 - \bar{\psi}^3 \psi_3)\right]$$
$$+ m_2 \int d^3x \, \mathrm{Tr}\left[i\left(\bar{C}^2 C_2 - \bar{C}^4 C_4\right) + \bar{\psi}^2 \psi_2 - \bar{\psi}^4 \psi_4)\right]$$

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Localization for mABJM

[Kapustin, Willett, Yaakov 2009]

[Jafferis 2010]

[Hama, Hosomichi, Lee 2010]

- Partition function as a matrix model

$$\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_{N_1}) \,, \quad M = \mathrm{diag}(\mu_1, \ldots, \mu_{N_2})$$

- ABJM matrix model:

$$Z(m) = \frac{1}{N_1!N_2!} \int d\lambda_i \, d\mu_j \; e^{i\pi k(\lambda_i^2 - \mu_i^2)} \, Z_{vec}(\lambda_i) \, Z_{vec}(\mu_j) \; Z_{mat}(\lambda_i, \mu_j)$$

- Vector determinants:

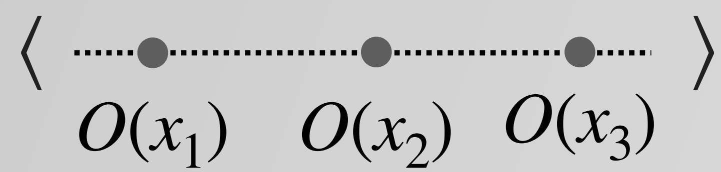
$$Z_{vec}^{ABJM}(\lambda_i) = \prod_{i<j} 2 \sinh^2 \pi(\lambda_i - \lambda_j)$$

- Matter sector:

$$Z_{mat}^{ABJM}(\lambda_i, \mu_j) = \frac{1}{\prod_{i,j} 4 \cosh \pi(\lambda_i - \mu_j) \cosh \pi(\lambda_i - \mu_j + m)}$$

Topological sector in 3d

- $\mathcal{N} = 4$ SCFTs in 3d have a topological sector



- Preserves a supercharge $\mathcal{Q} = Q + S$ such that $\hat{P} = P + R_+ = \{\mathcal{Q}, X\}$

- Example: ABJM

$$\mathcal{C}(x) = C_1(x) + xC_3(x) \qquad\qquad \bar{\mathcal{C}}(x) = x\bar{C}_1(x) - \bar{C}^3(x)$$

$$S = \mathrm{Tr}\left[\bar{\mathcal{C}}(x)\mathcal{C}(x)\right]$$

- Superconformal primary of the stress tensor multiplet

$$S_I^{\ J} \xrightarrow{Q} \chi \xrightarrow{Q} P \xrightarrow{Q} \psi \xrightarrow{Q} T_{\mu\nu}$$

Generating functionals

- The following cohomological equivalence holds

[LG, Penati, Yaakov 2021]
[Bomans, Pufu 2021]

$$\frac{\partial}{\partial m} S_{mass}[m] = 4\pi r^2 \oint_{S^1_\tau} \mathrm{S}(\tau) + \{\mathcal{Q}, \ldots\}$$

- $\mathrm{S}(\tau)$ integrated correlators from mass derivatives

[Agmon, Chester, Pufu 20]

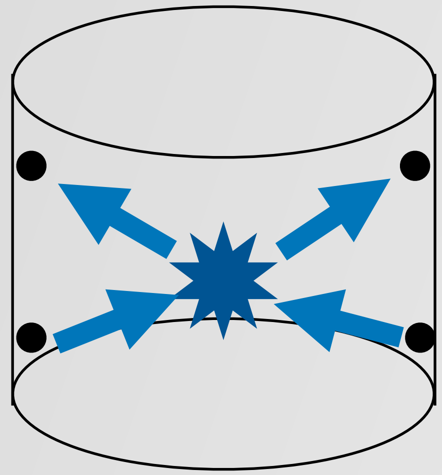
$$\frac{1}{(4\pi r)^n} \frac{1}{Z} \frac{d^n Z(m)}{d^n m}\bigg|_{m=0} = \langle \int d\tau_1\, \mathrm{S}(\tau_1) \ldots \int d\tau_n\, \mathrm{S}(\tau_n) \rangle$$

- Application: central charge

[Gorini, Griguolo, LG, Penati, Seminara Soresina 20]
[Binder, Chester, Pufu 20]

$$c_T = -\frac{64}{\pi^2} \frac{\partial^2}{\partial m^2} \log Z[m]\bigg|_{m=0}$$

- Higher derivatives terms in SUGRA action



Fermionic Wilson line

- Fermionic Wilson loops as a superholonomy of $U(N_1\,|\,N_2)$ *[Drukker, Trancanelli 2009]*

$$W[\gamma] = Tr_{\mathscr{R}}\, P\exp\left(-i\int_\gamma \mathscr{L}\right) \qquad \mathscr{L} = \begin{pmatrix} A_3 - \frac{2\pi i}{k} M_J{}^I C_I \bar{C}^J & i\sqrt{\frac{2\pi}{k}}\eta_I^\alpha \bar{\psi}_\alpha^I \\ -i\sqrt{\frac{2\pi}{k}}\psi_I^\alpha \bar{\eta}_\alpha^I & \hat{A}_3 - \frac{2\pi i}{k}\hat{M}_J{}^I \bar{C}^J C_I \end{pmatrix}$$

- Maximally supersymmetric solution on the line

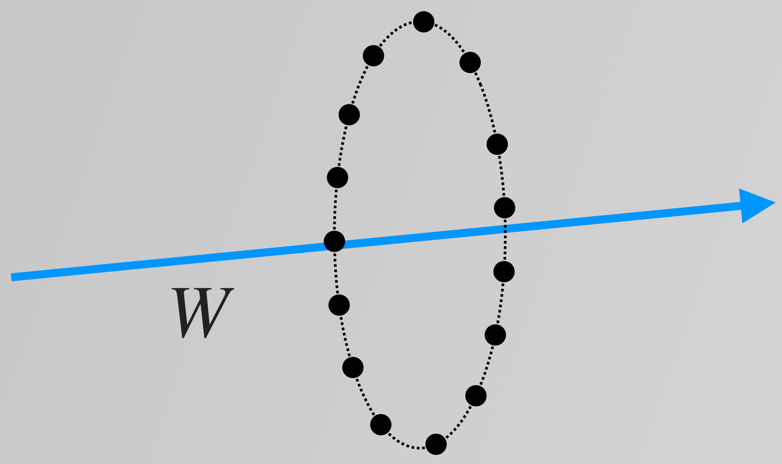
$$M = \hat{M} = \mathrm{diag}(-1,1,1,1) \qquad \eta_I^\alpha = \sqrt{2}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}_I (1,0)^\alpha, \qquad \bar{\eta}_\alpha^I = i\sqrt{2}\ (1,0,0,0)^I \begin{pmatrix}1\\0\end{pmatrix}_\alpha$$

- Dual to M2-brane

[Roadmap on Wilson loops in 3d Chern–Simons-matter theories, 2020]

Adding line defects

- Add a line defect preserving supersymmetry [LG 23]



$$\mathscr{C}(\tau) = e^{-\frac{i}{2}\tau} C_1 + e^{\frac{i}{2}\tau} C_3\,,$$

$$\bar{\mathscr{C}}(\tau) = e^{\frac{i}{2}\tau} \bar{C}^1 - e^{-\frac{i}{2}\tau} \bar{C}^3$$

- Simplest example $\mathrm{S}(\tau) = \mathrm{Tr}\left(\bar{\mathscr{C}}(\tau)\mathscr{C}(\tau)\right)$

- Exact formula for defect correlation functions

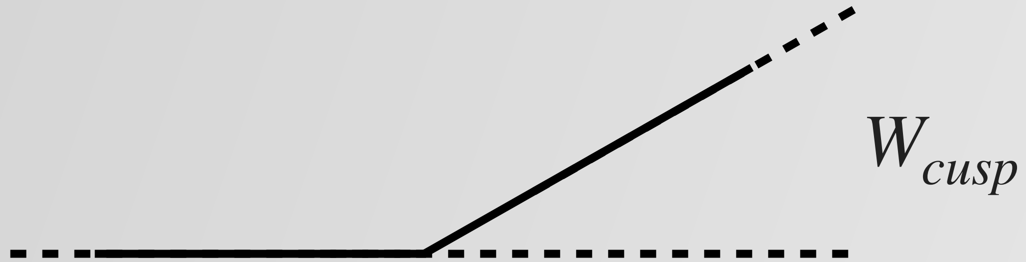
$$\frac{1}{(4\pi r)^n}\frac{1}{\langle W\rangle}\frac{d^n\langle W\rangle(m)}{d^n m}\bigg|_{m=0} = \langle \int d\tau_1 \mathrm{S}(\tau_1)\dots\int d\tau_n \mathrm{S}(\tau_n)\,\rangle_W$$

Applications

- Supersymmetry relates h_T and the bremsstrahlung *[Lewkowycz, Maldacena 2014]*

$$B = \frac{i}{8\pi^2 r\langle W\rangle}\frac{\partial}{\partial m}\langle W\rangle(m)\Big|_{m=0}$$

- Bremsstrahlung and integrability *[Correa, Giraldo-Rivera, Lagares 23]*
    - Cusp anomalous dimension
    - Derivation of $h(\lambda)$



$$\log\langle W_{cusp}\rangle \sim B$$

- Single trace operators + fermionic WL are integrable *[Jiang, Wu, Yang 23]*

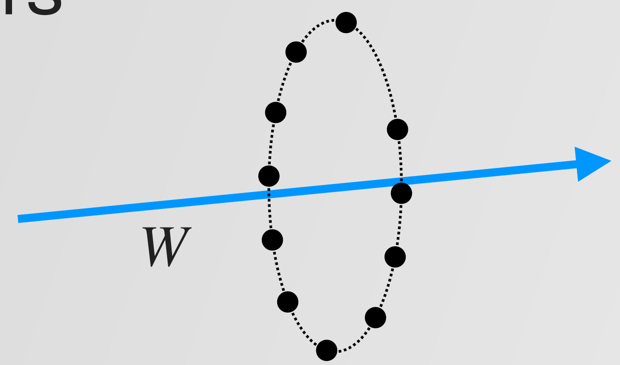
Exact defect correlators

- Vev of SUSY operators as MM average *[Drukker, Trancanelli 2009]*

$$\langle W \rangle = \langle \sum_i e^{2\pi\lambda_i} + \sum_j e^{2\pi\mu_j} \rangle_{MM}$$

- Mass derivatives give the insertions for defect correlators

$$\langle \mathrm{S} \rangle_W = -\pi r \langle \sum_{i,j} \tanh \pi(\lambda_i - \mu_j) \rangle_W$$



$$\langle \mathrm{S}(\tau_1)\mathrm{S}(\tau_2) \rangle_W = (\pi r)^2 \langle \left(\sum_{i,j} \tanh \pi(\lambda_i - \mu_j) \right)^2 - \sum_{i,j} \frac{1}{\cosh^2 \pi(\lambda_i - \mu_j)} \rangle_W$$

- Exact insertions in the matrix model for stress tensor correlators!

[Jiang, Wu, Yang 23]

Result & checks

- One-point function

$$B = \frac{N_1 N_2}{4k(N_1 + N_2)} - \frac{\left(\pi^2 N_1 N_2 \left(N_1 N_2 - 3\right)\right)}{24k^3 \left(N_1 + N_2\right)} + O(k^{-4})$$

- Agreement with previous proposal for B

[Lewkowycz, Maldacena 2014]
[Bianchi, Griguolo, Mauri, Penati, Preti, Seminara 2017]
[Bianchi, Griguolo, Mauri, Penati, Seminara 2018]
[Bianchi, Preti, Vescovi 2018]

- Two-points function

$$\langle \mathrm{S}(\tau_1)\mathrm{S}(\tau_2)\rangle_W = \frac{N_1 N_2}{64\pi r^2} - \frac{\left(N_1 N_2 \left(N_1^2 + N_2^2 - 14\right)\right)}{384 k^2 r^2} + O(k^{-4})$$

- Check with Feynman diagrams at order $1/k$

Strong coupling

- Partition function as a gas of N non-interactaging 1d fermions

[Marino, Putrov 11]

[Nosaka 15]

$$Z(m) = \frac{1}{N!} \sum_{\sigma \in S_N} (-1)^{\epsilon(\sigma)} \int d^N x \prod_i \rho(x_i, x_{\sigma(i)})$$

- From WKB with $\hbar = 2\pi k$ —> resum perturbative series in $1/N$
- Vev of Wilson loop from thermal average [Klemm, Marino, Schiereck, Soroush 12]
- Massive background [Armanini, Griguolo, Guerrini in progress]

$$\langle W_{1/2} \rangle = B_1(k, m_1, m_2) \frac{\mathrm{Ai}\left(C(m_1, m_2)\, N + f(m_1, m_2) - \frac{2}{k(1 - i m_2)} \right)}{\mathrm{Ai}\left(C(m_1, m_2)\, N + f(m_1, m_2) \right)} +$$

$$+ B_2(k, m_1, m_2) \frac{\mathrm{Ai}\left(C(m_1, m_2)\, N + f(m_1, m_2) - \frac{2}{k(1 + i m_1)} \right)}{\mathrm{Ai}\left(C(m_1, m_2)\, N + f(m_1, m_2) \right)}$$

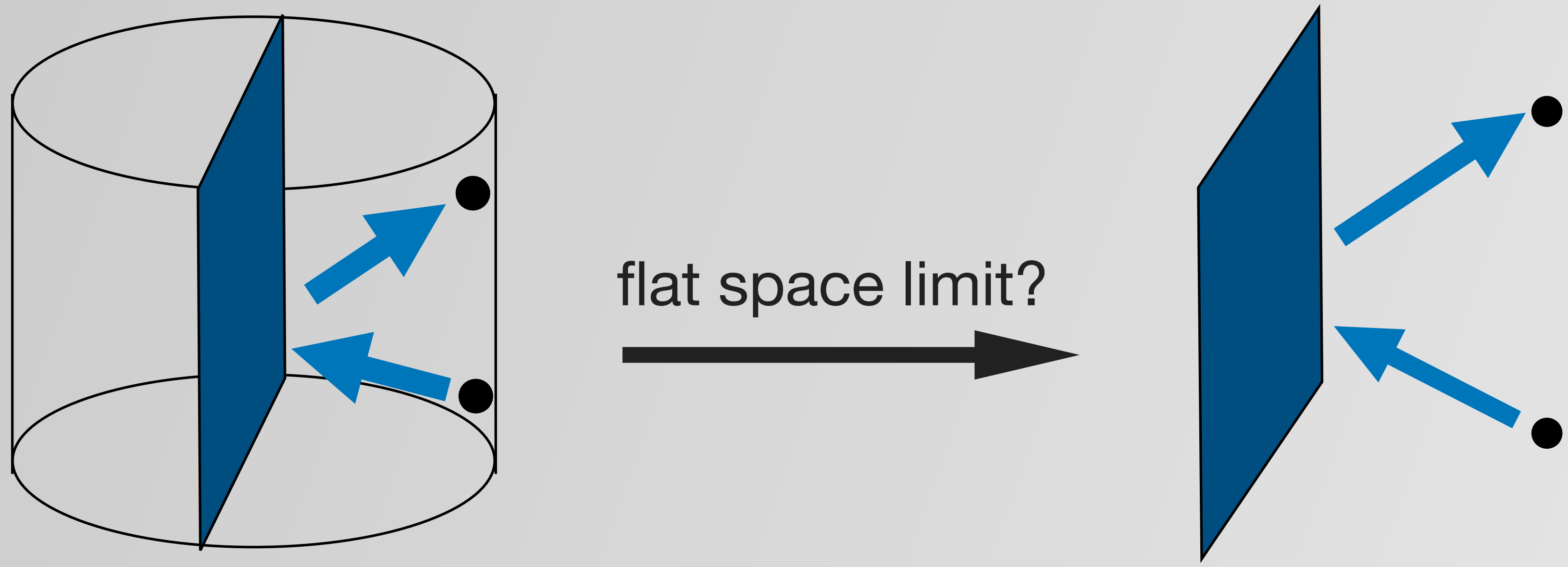
- Agreement with B at strong coupling

Conclusions

- New SUSY configuration with local operators and line defects
- Defect correlation functions from supersymmetric deformation
- Application: stress tensor defect correlators in ABJM

Future developments

- Defect bootstrap for 3d $\mathcal{N} \geq 4$ theories
- Graviton-M2 brane scattering
- Holography for the massive WL



Thank you!