Non-equilibrium Full Counting Statistics and Charged moments in Integrable models

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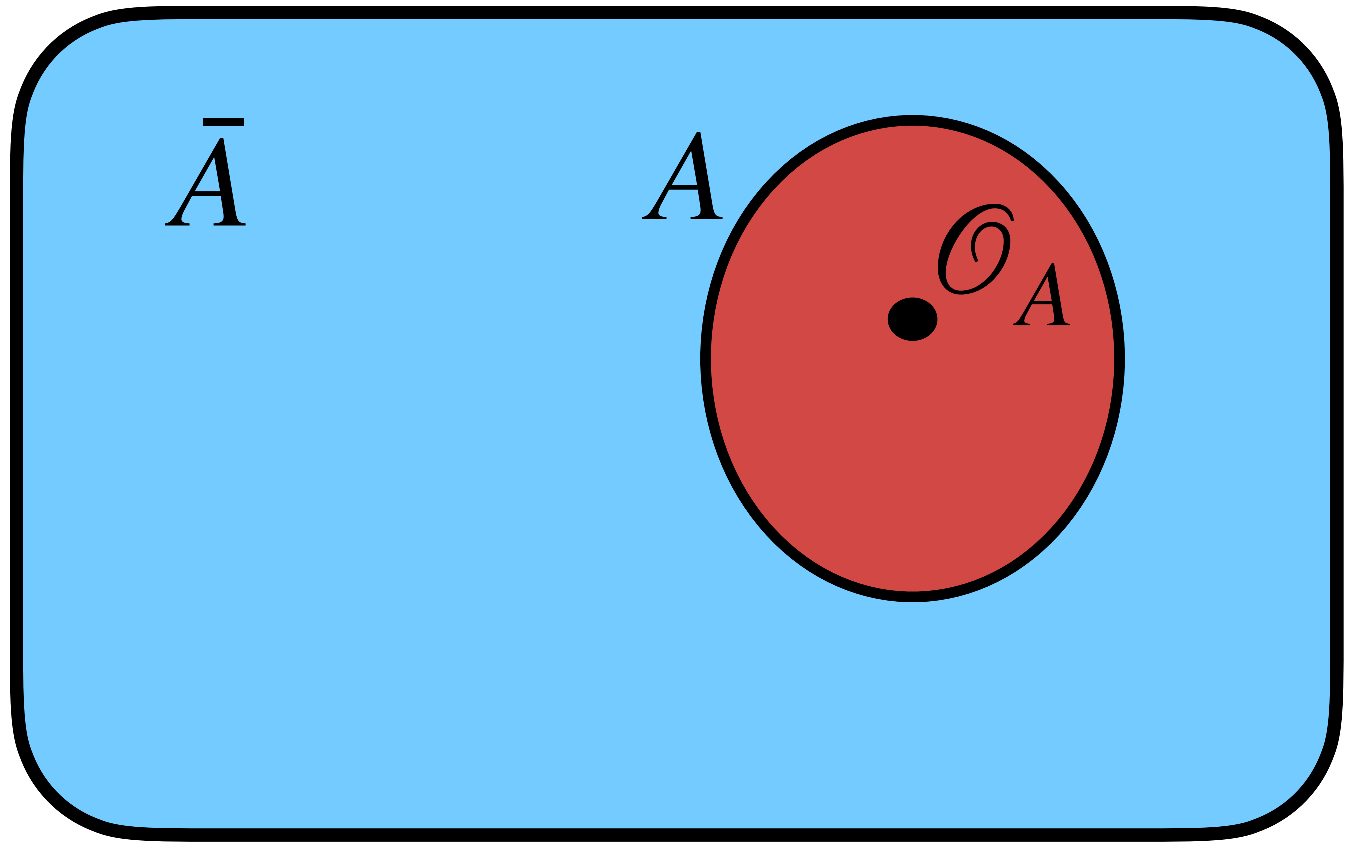
Quench Dynamics

- Universal physics in equilibrium systems is well studied. Non-equilibrium?

$$|\Psi_0(t)\rangle = e^{-iHt}|\Psi_0\rangle$$

- Complicated problem, explore full spectrum not just low energy
  - Breakdown of low energy/equilibrium methods (RG, Fermi Liquid, bosonization)
- Nevertheless, a lot known at long time $t \gg |A|$



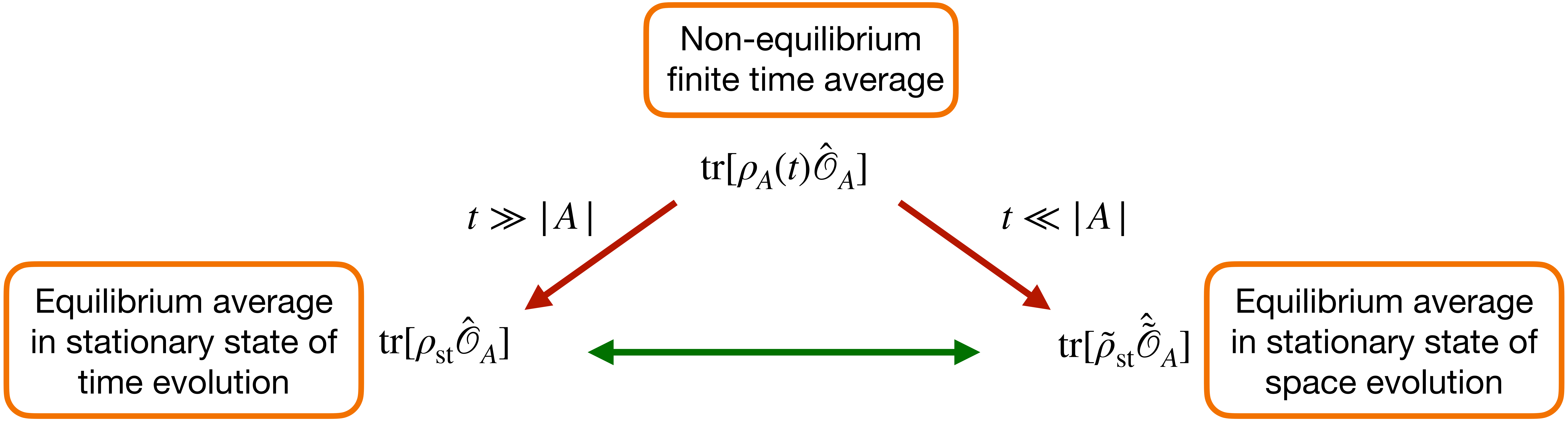
Thermalisation:

$$\lim_{|A|\to\infty} \lim_{t\to\infty} \mathrm{tr}[\rho_A(t)\hat{\mathcal{O}}_A] \to \mathrm{tr}[\rho_{\mathrm{st}}\hat{\mathcal{O}}_A]$$

ρ_{st} is stationary state e.g. GE or GGE

What about at finite time?

- At times $t \ll |A|$ can we say anything?
- Space-time duality approach to quantum dynamics can help (Bertini et. al 2019,2022)
- Maps finite time problem to an dual system which is at ***equilibrium***



- *This talk:* Calculate non equilibrium Charged Moments for integrable models

Outline

- Charged moments
- Space time duality
- Generic Results
- Explicit predictions for integrable models
- Summary/Conclusion

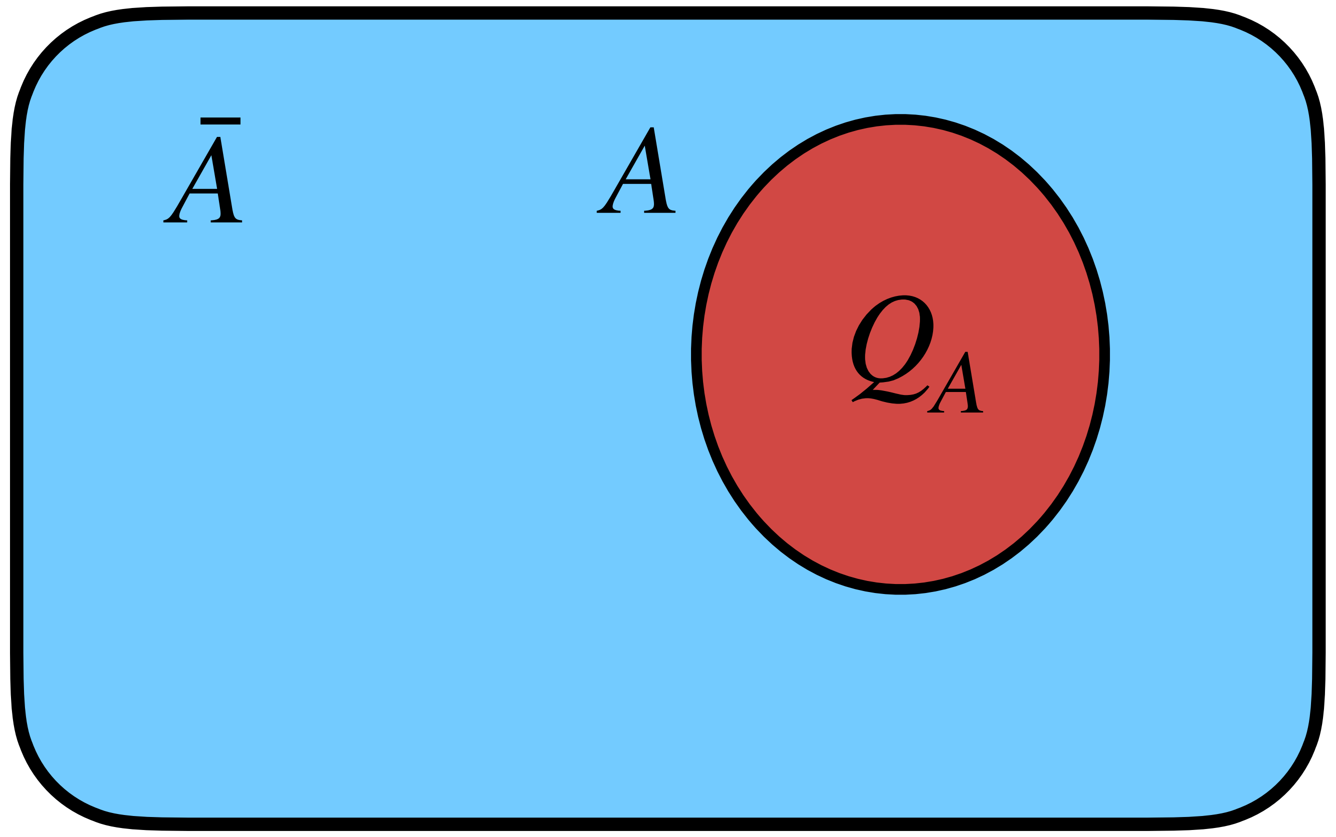
Based on:
arXiv:2212.06188 (PRL) & arXiv:2306.12404

with B. Bertini, P. Calabrese, M. Collura & K. Klobas

Charged Moments

- Consider system with $U(1)$ charge $Q = \sum_x q_x$, $[H, Q] = 0$

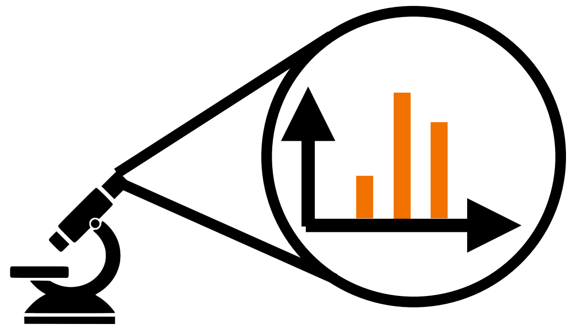
$$Z_{\beta}(A, t) = \mathrm{tr}[\prod_{j=1}^{n} \rho_A(t) e^{\beta_j Q_A}]$$



- For $\beta_j = 0,\ n > 1$: Rényi entanglement entropy

- For $n = 1$: Full counting statistics/charge probability distribution

$$P(Q_A = q, t) = \int_{-\pi}^{\pi} \frac{\mathrm{d}\beta}{2\pi} Z_{i\beta}(A, t) e^{-i\beta q}$$



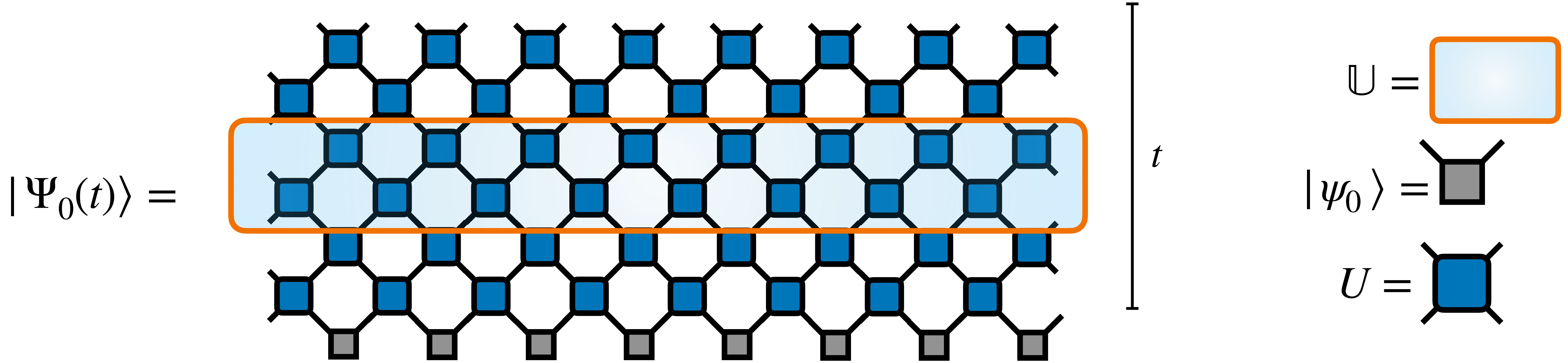
- For $\beta_j \neq 0,\ n > 1$: Symmetry resolved entanglement (Goldstein & Sela 2018) or Entanglement Asymmetry (c.f. Talks of Luca Capizzi & Michele Fossatti) (Ares, Murciano & Calabrese 2022)

Brickwork Quantum Circuits

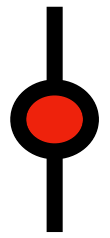
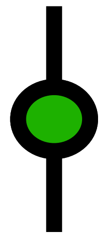
- Simplified model of dynamics, discrete space and time evolution (Nahum et. al 2017)

$$|\Psi_0(t)\rangle = \mathbb{U}^t |\Psi_0\rangle$$

- Time evolution $\mathbb{U} = U^{\otimes |A|/2}\Pi_{\text{shift}}U^{\otimes |A|/2}\Pi_{\text{shift}}$ made of local operators U
- Symmetric product initial state $|\Psi_0\rangle = |\psi_0\rangle^{\otimes |A|/2}$



Charged moments in QC

- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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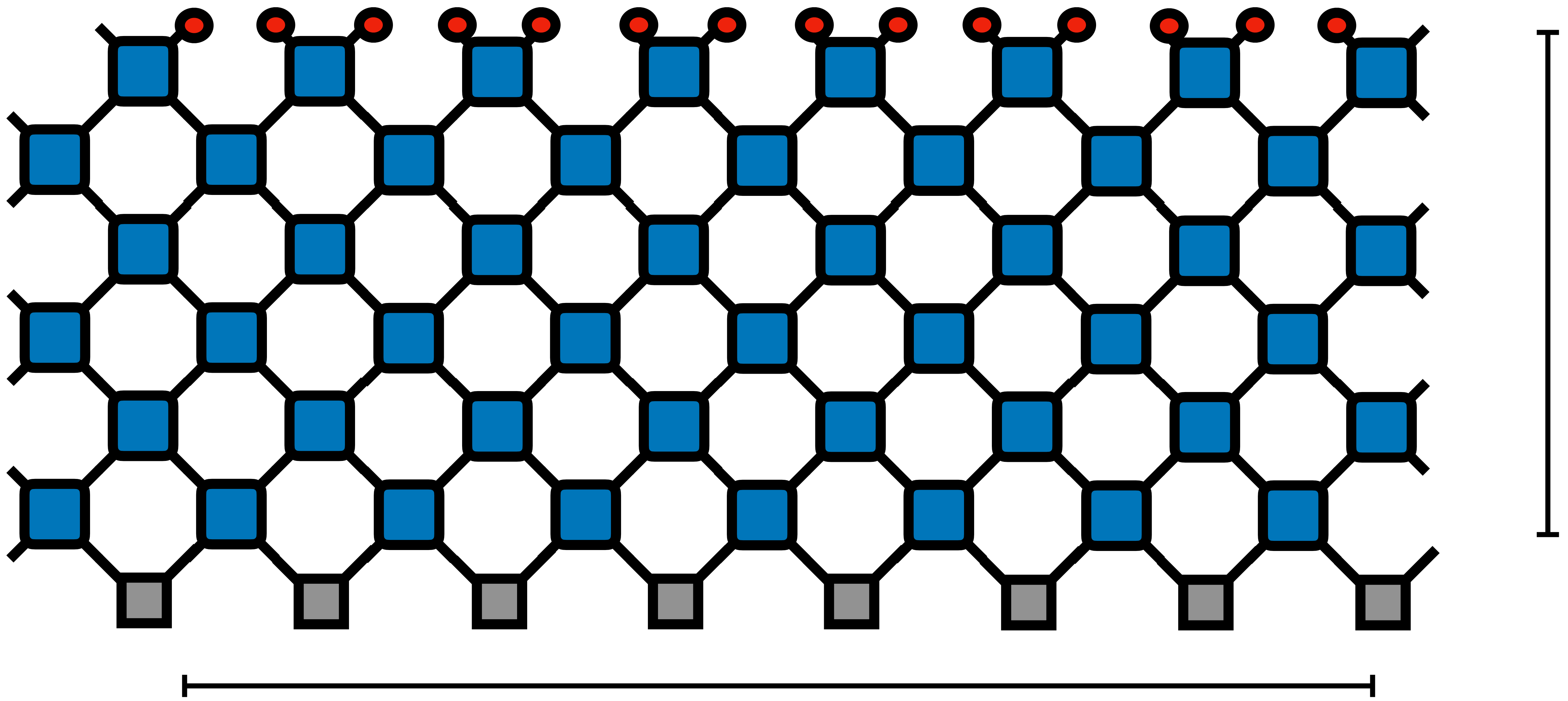

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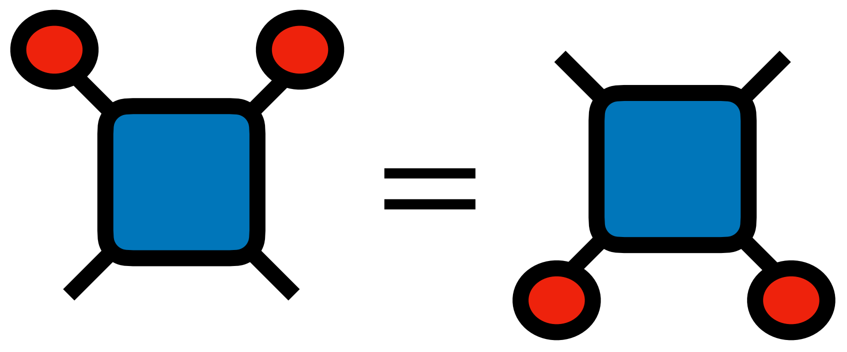

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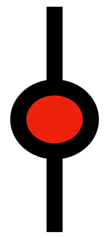
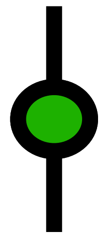
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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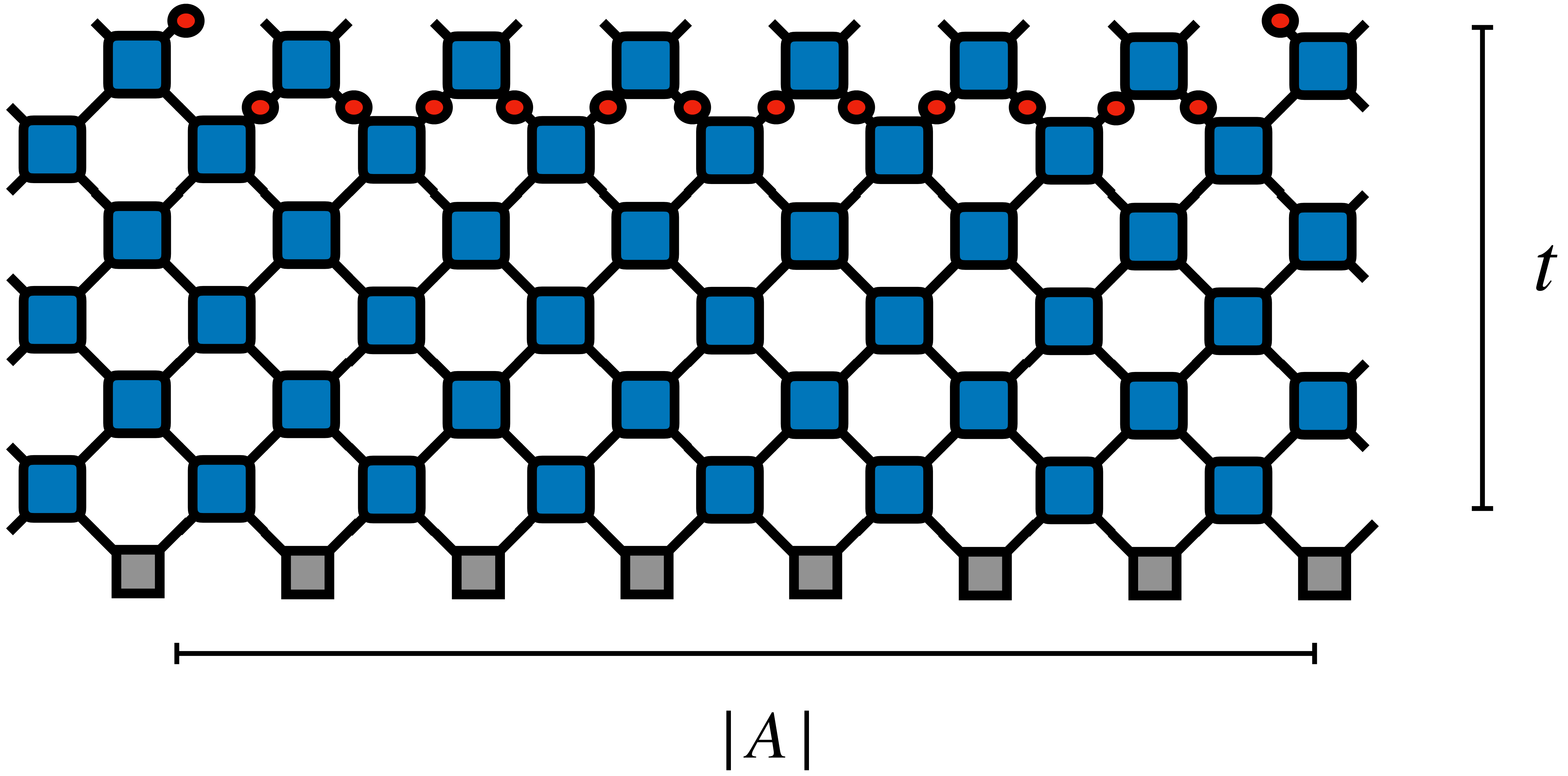

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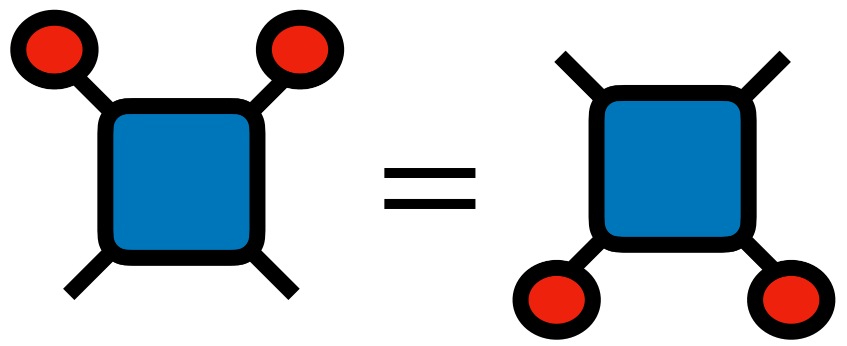

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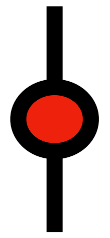
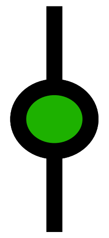
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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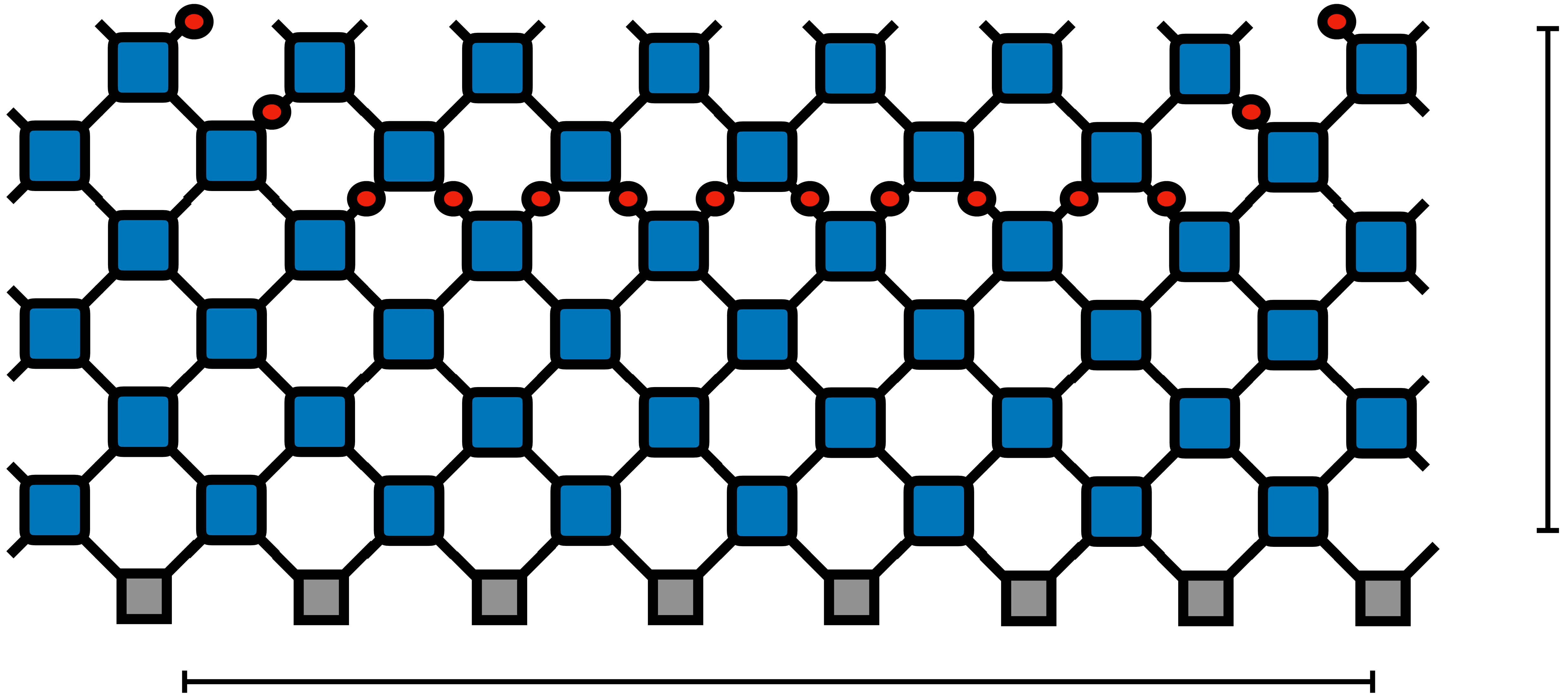

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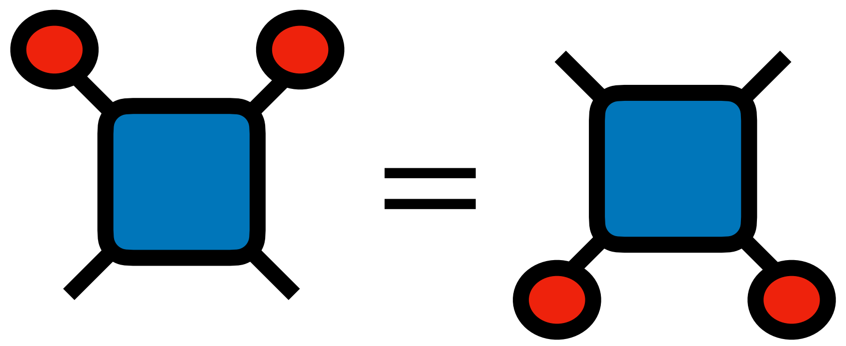

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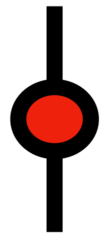
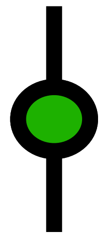
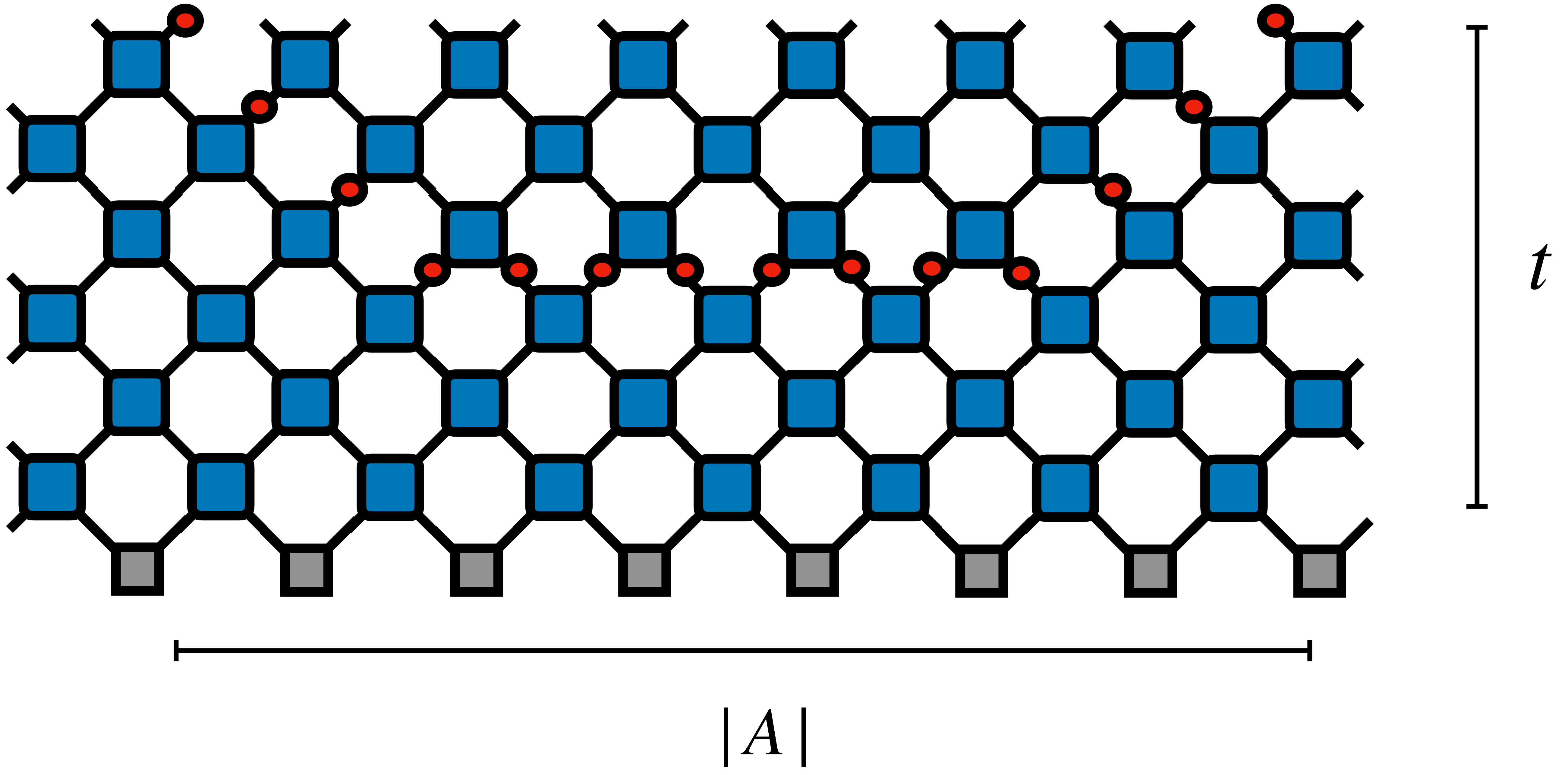
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

$e^{\beta Q_A} | \Psi_0(t) \rangle =$ 

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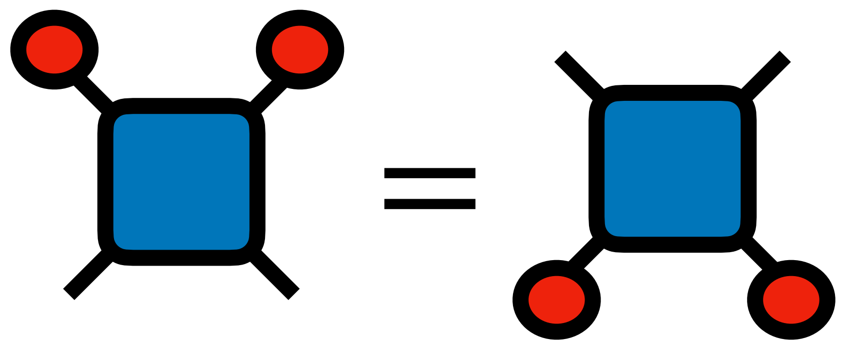

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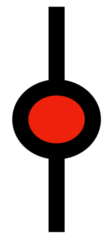
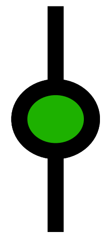
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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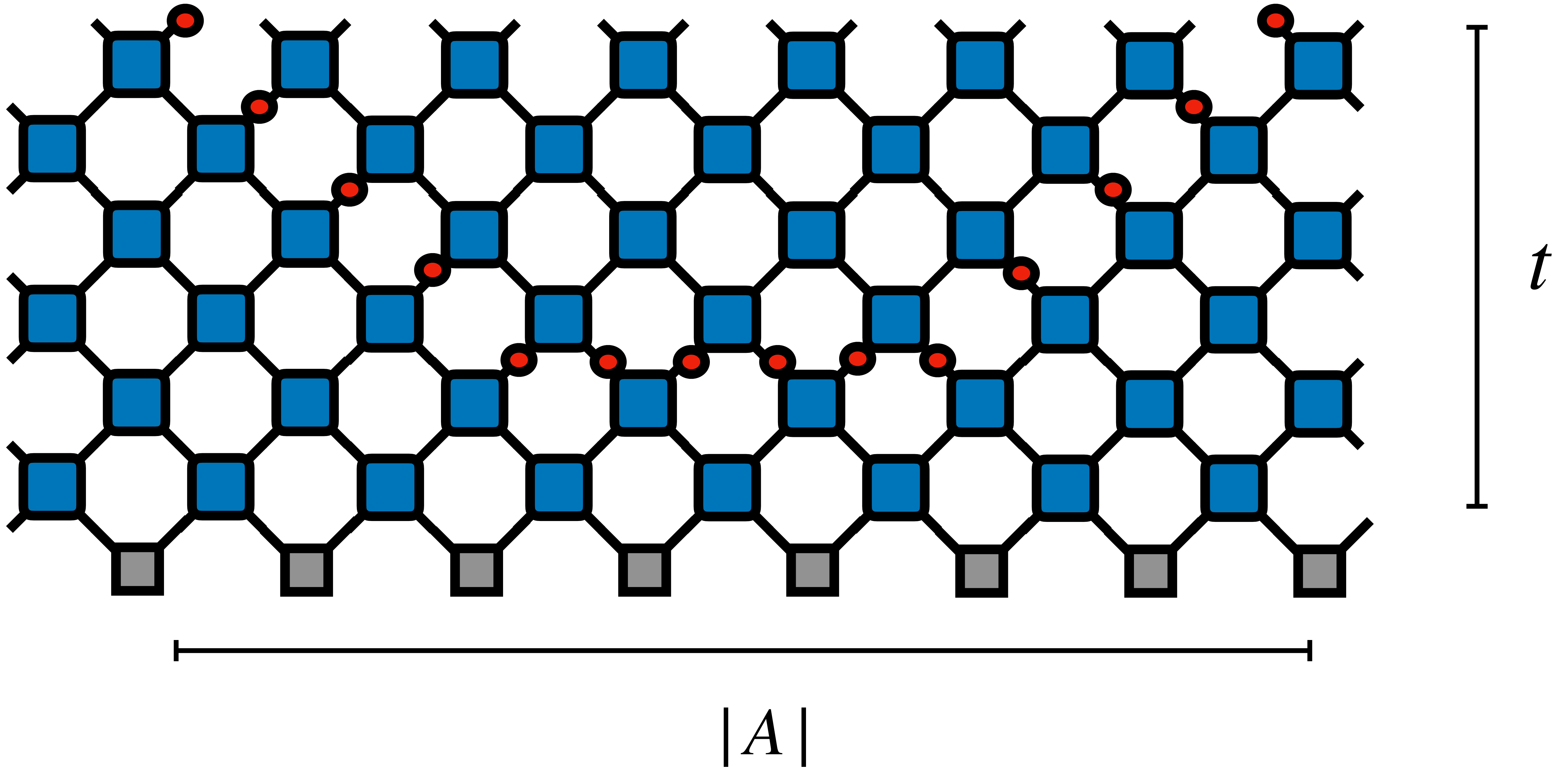

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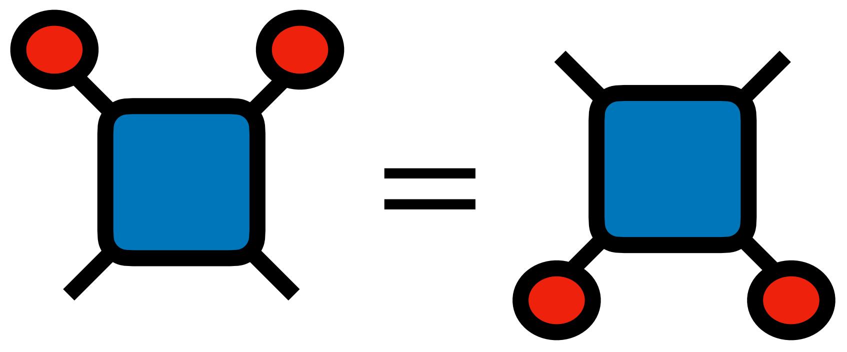

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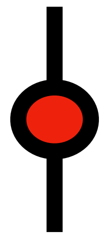
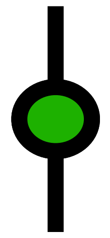
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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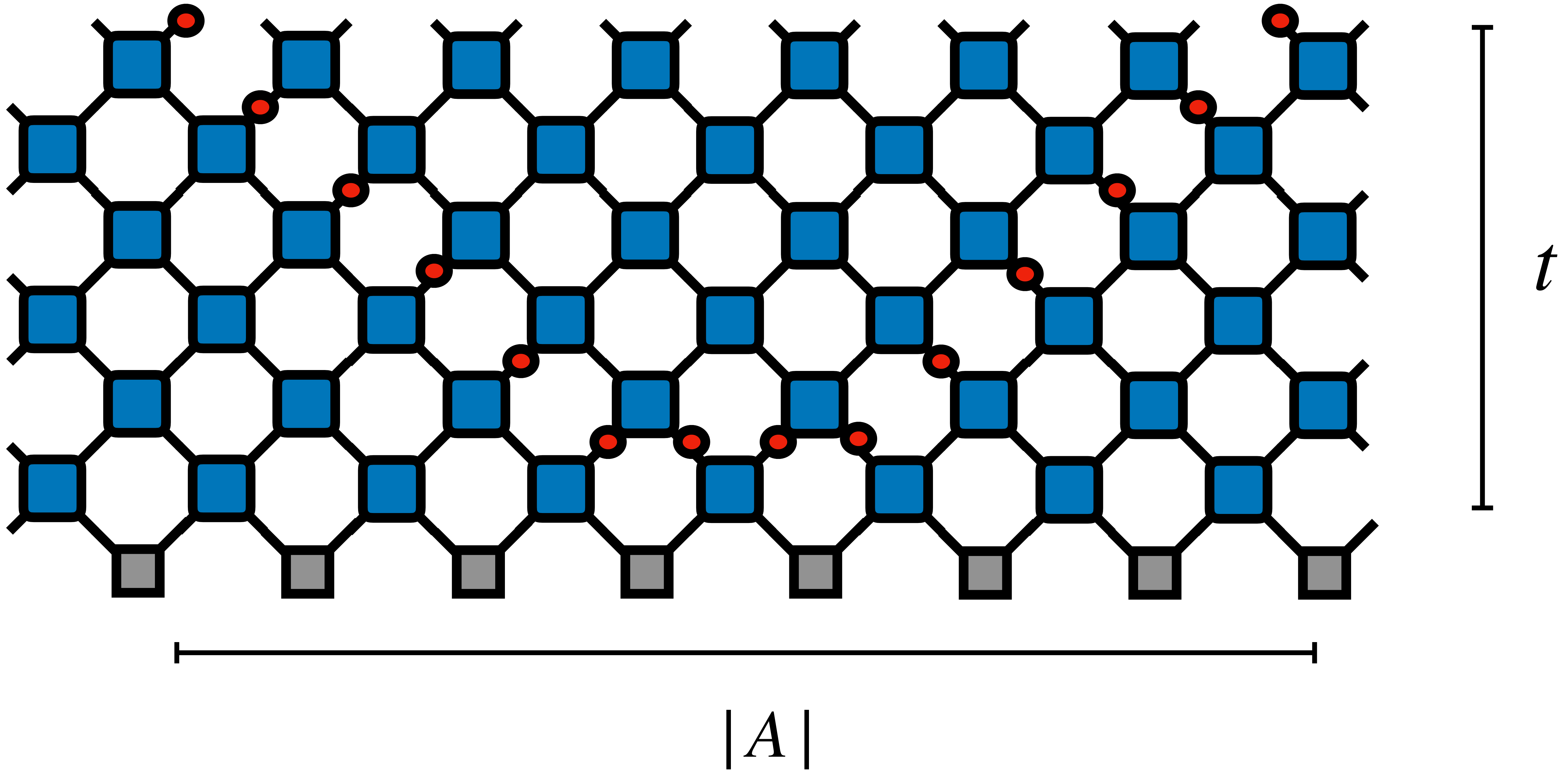

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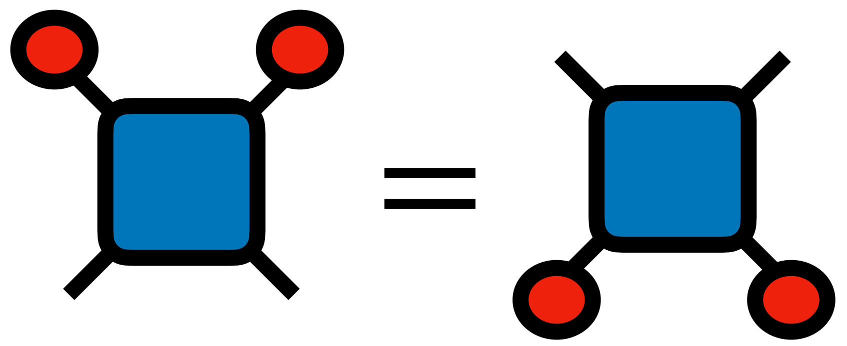

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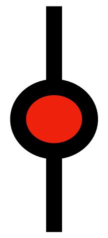
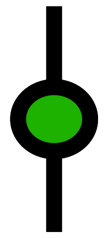
- Charge is sum of local operators $Q_A = \sum q,$ s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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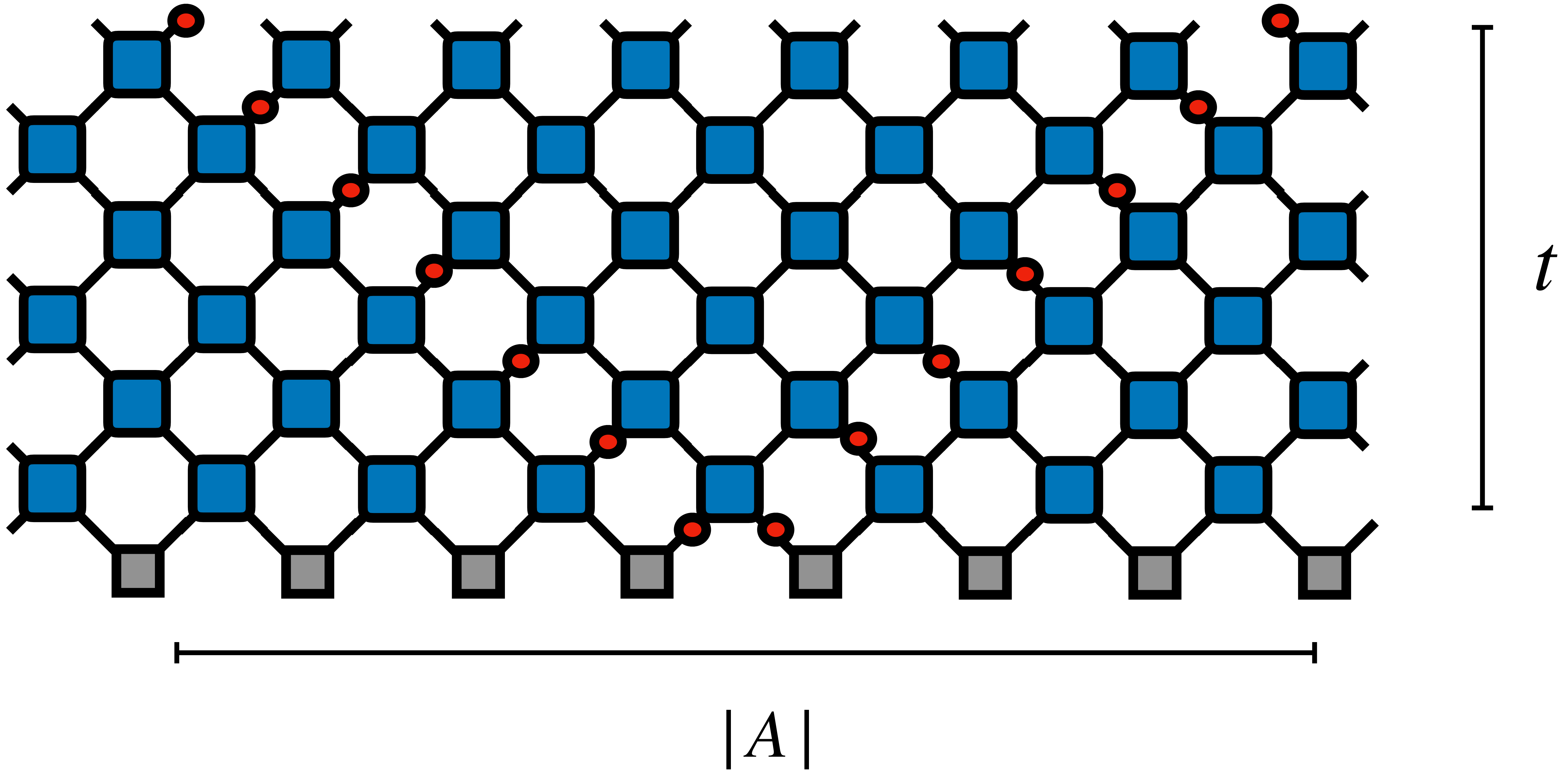

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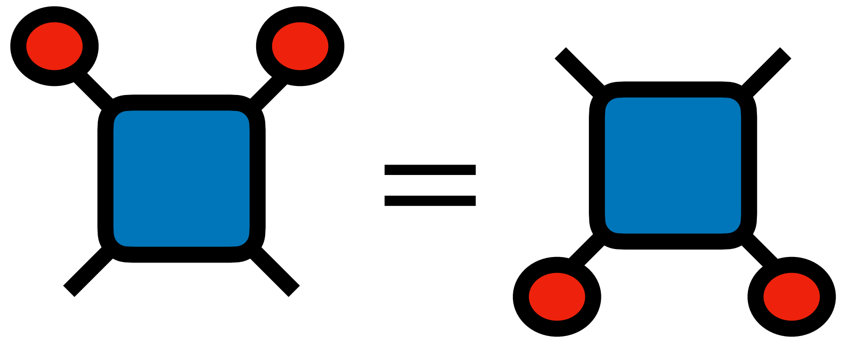

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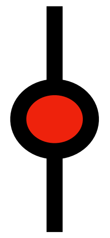
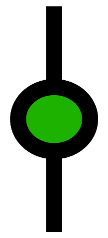
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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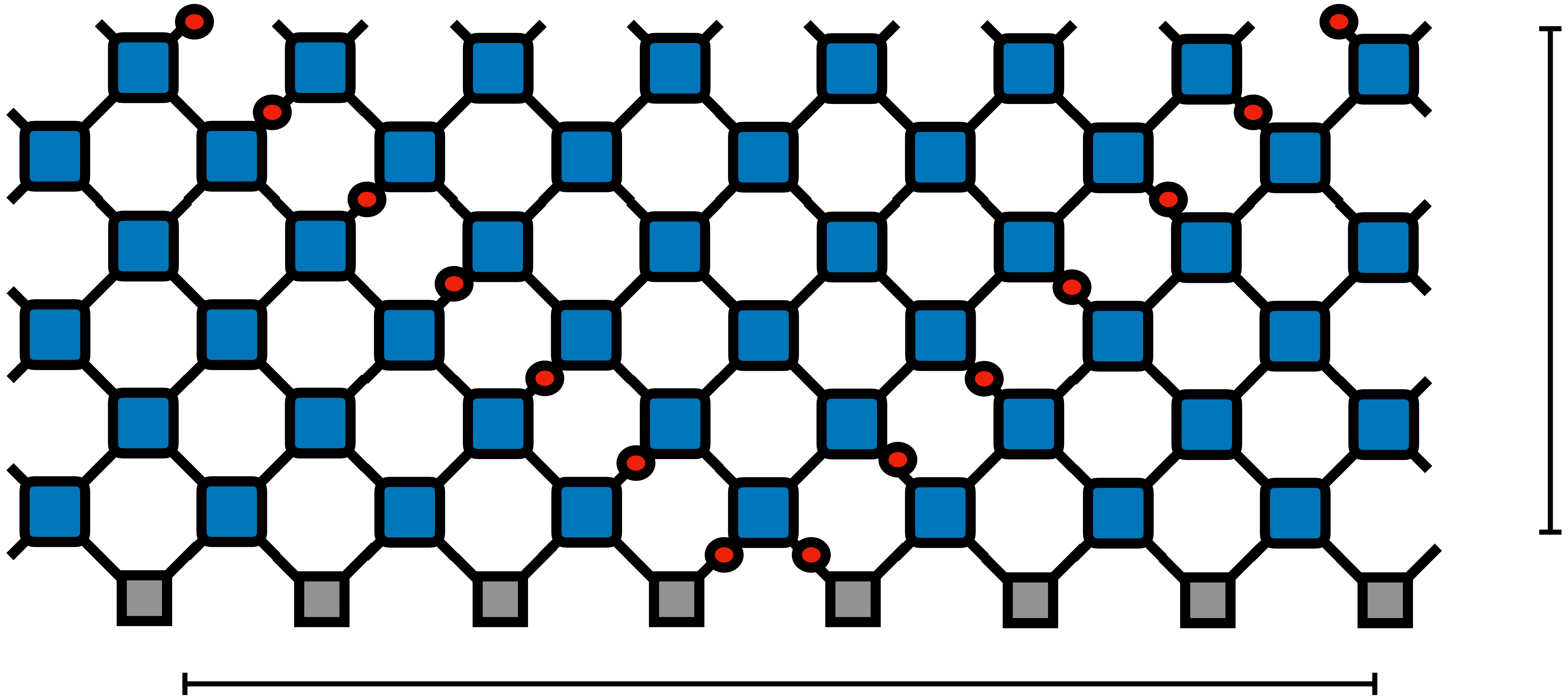

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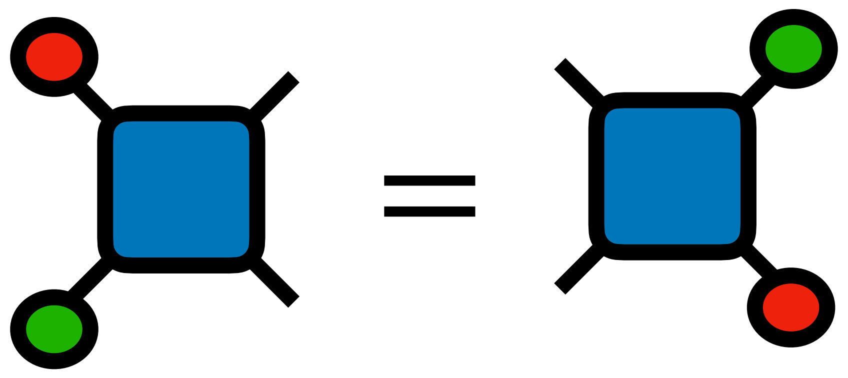

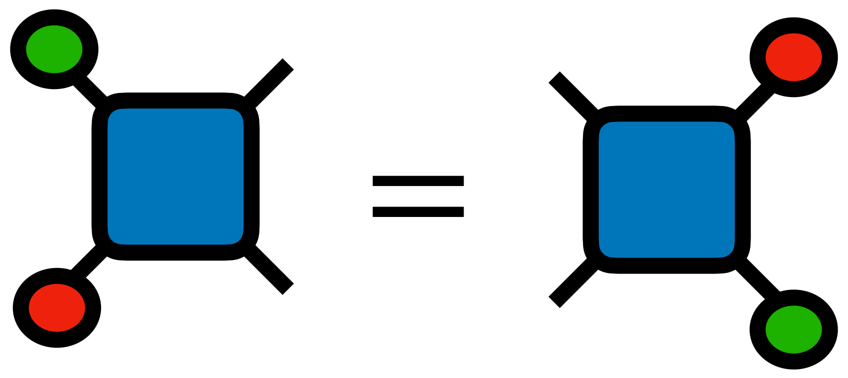

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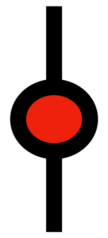
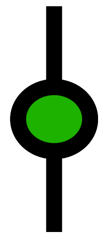
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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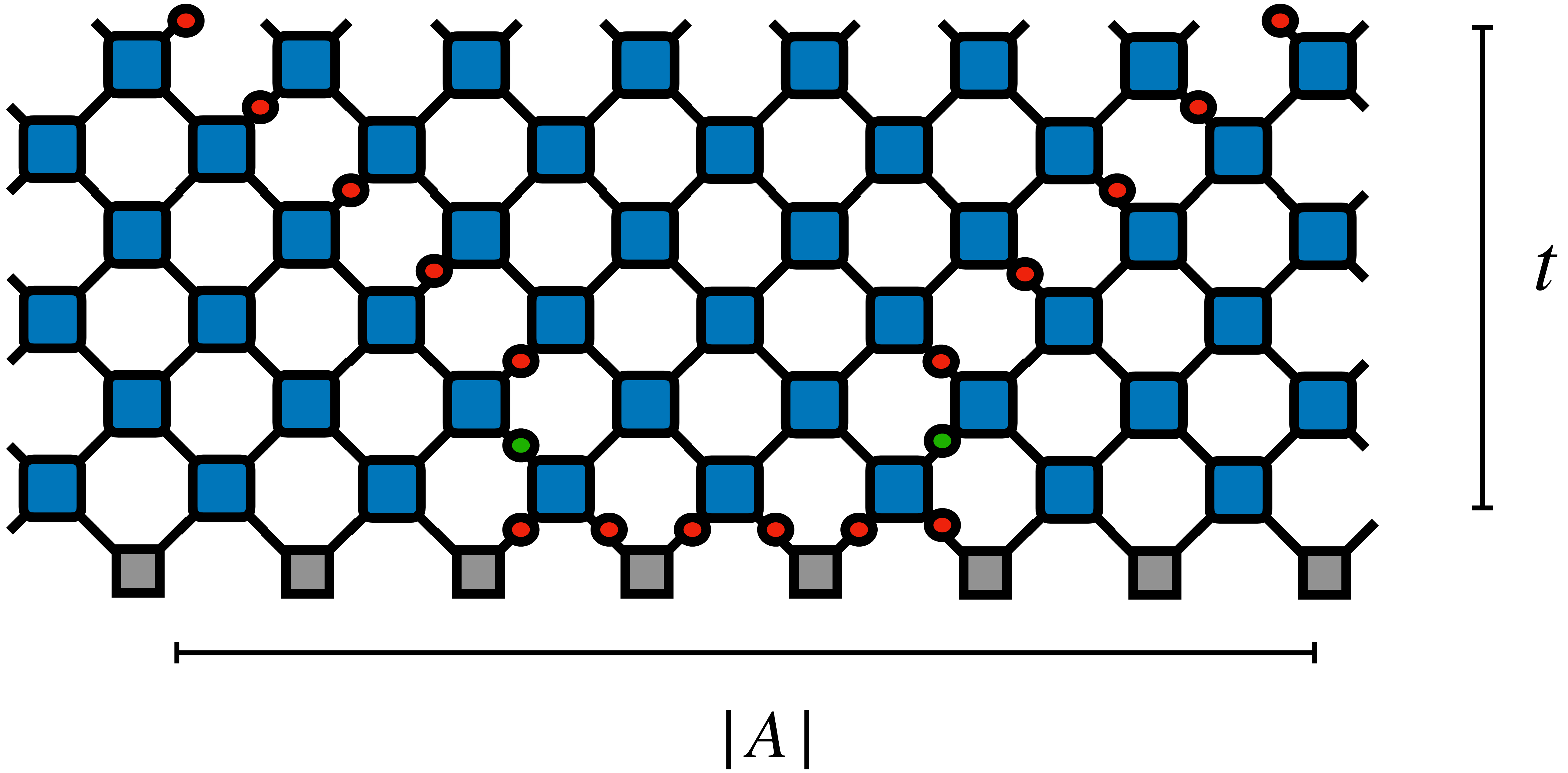

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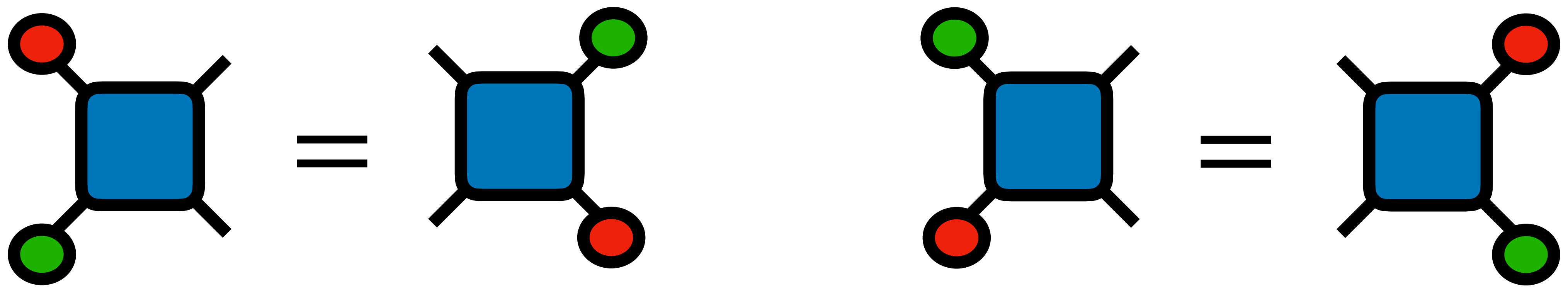

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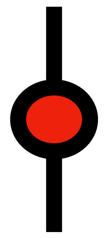
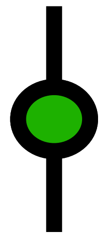
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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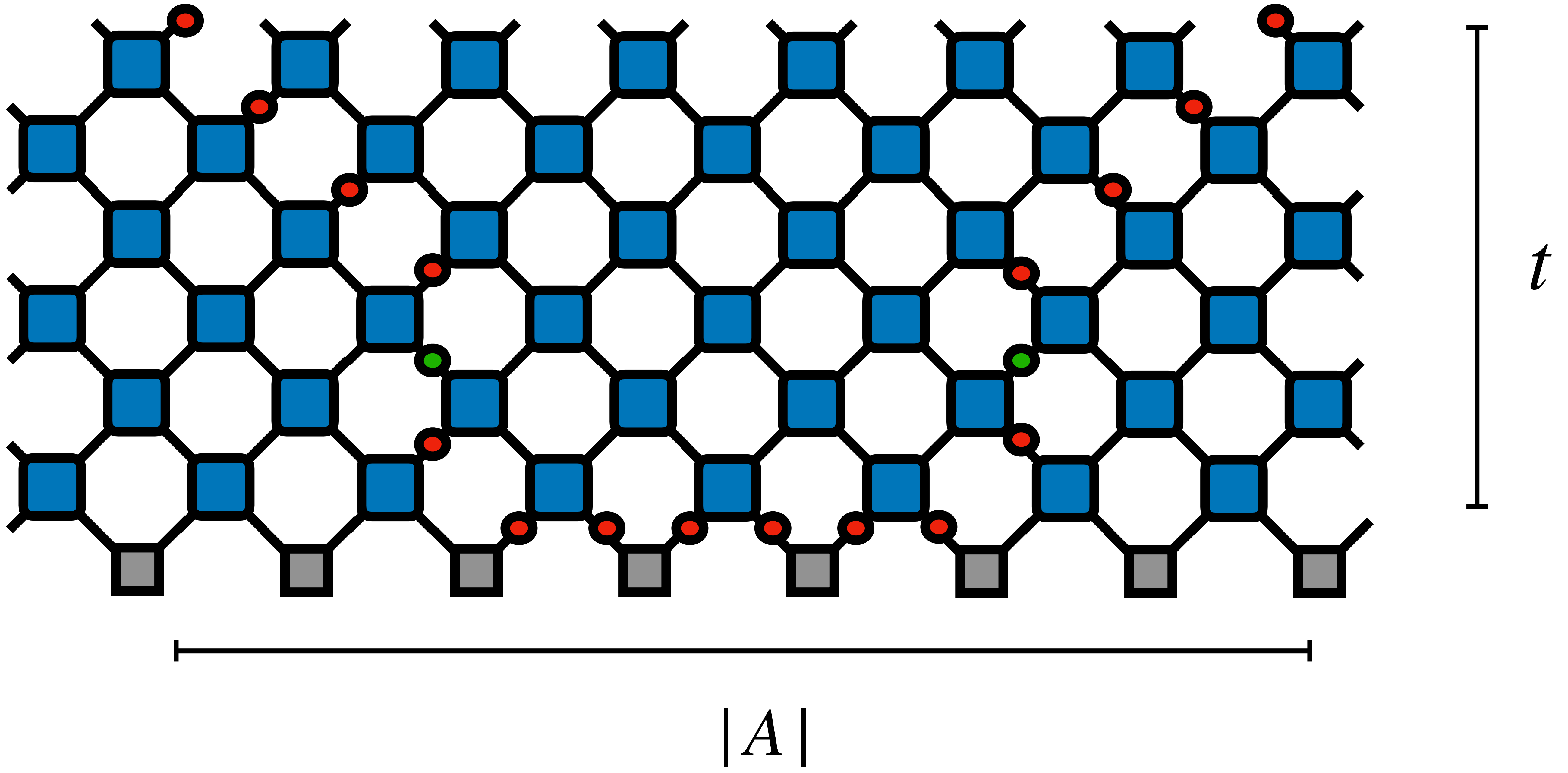

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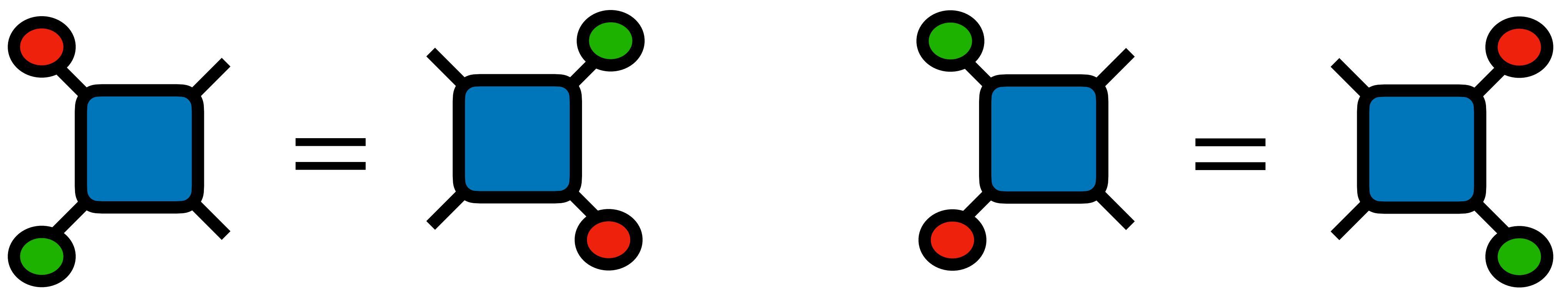

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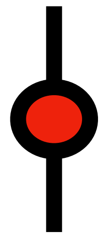
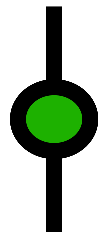
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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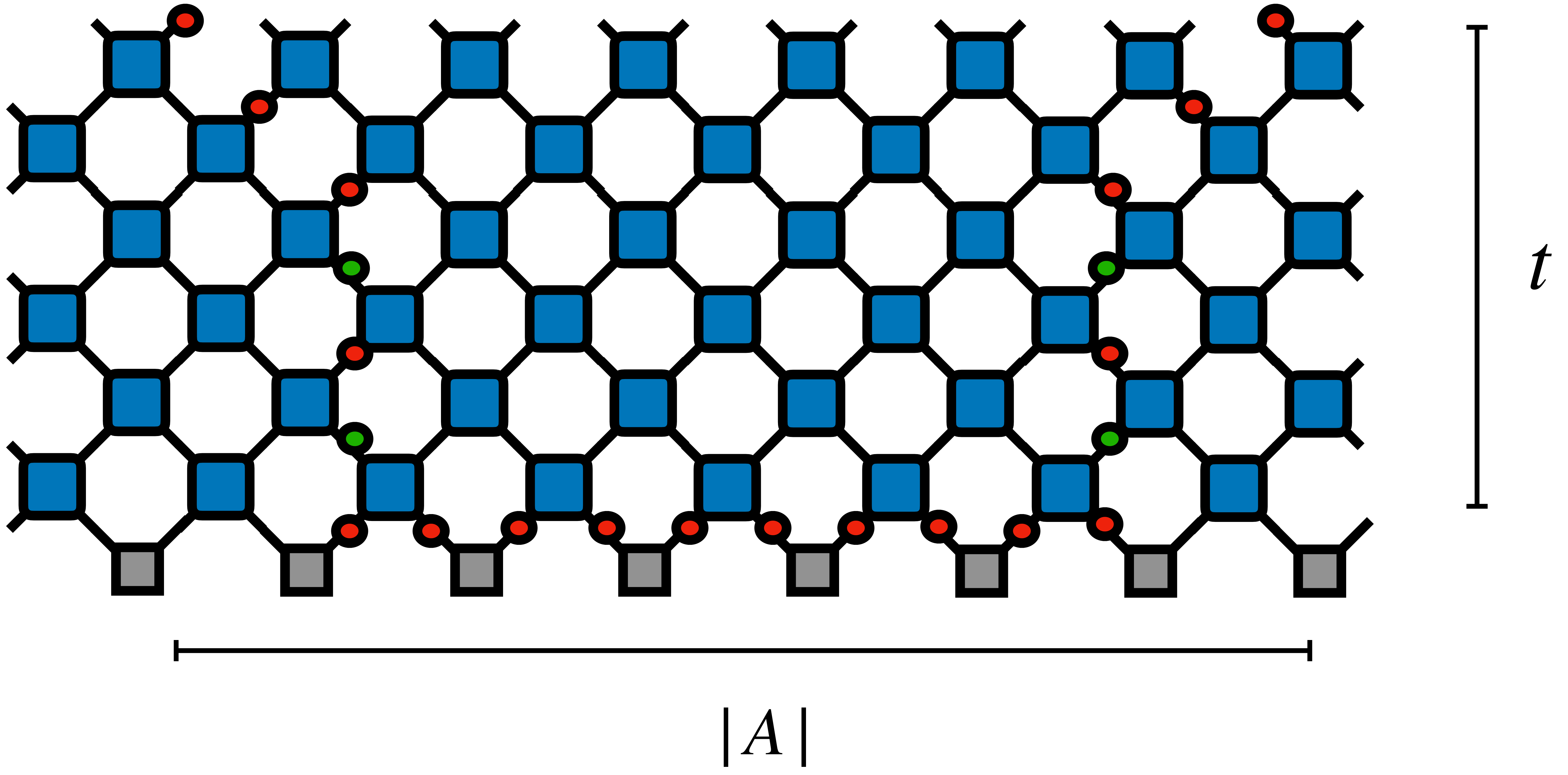

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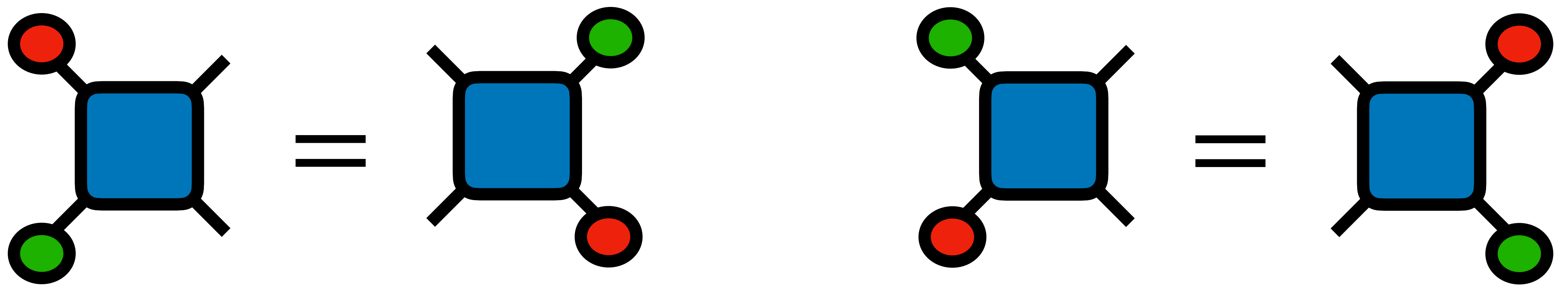

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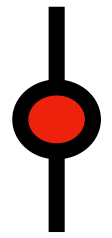
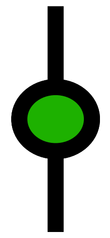
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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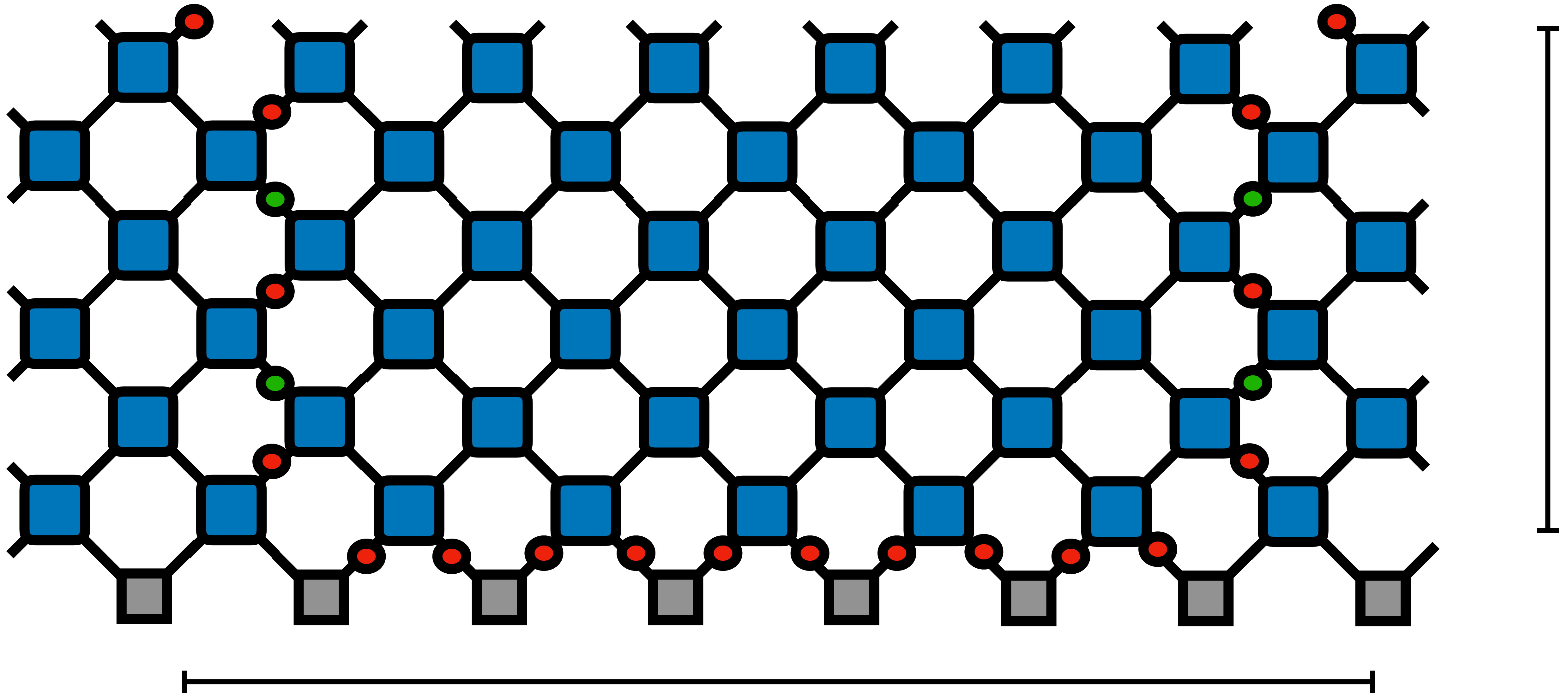

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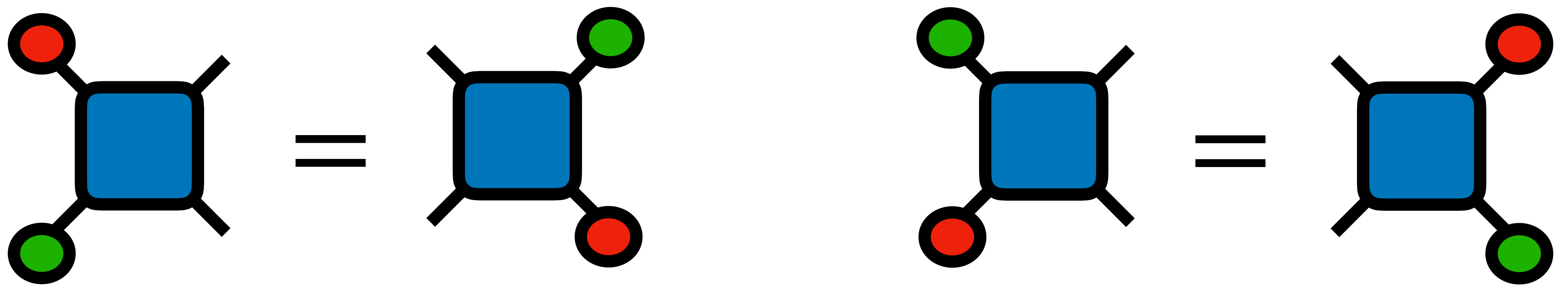

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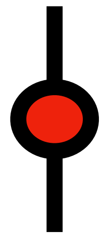
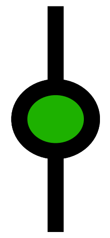
- Charge is sum of local operators $Q_A = \sum q,$ s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

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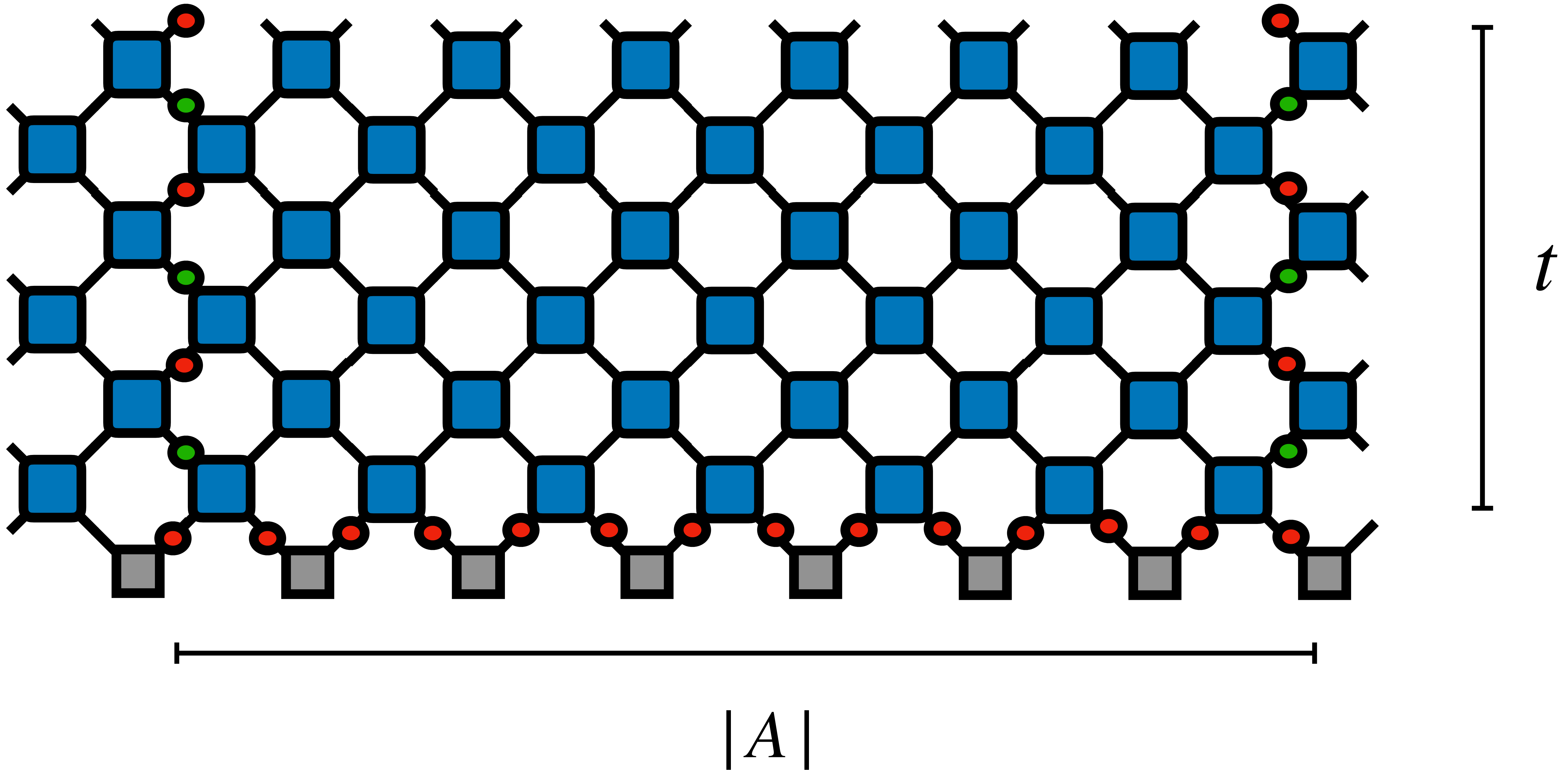

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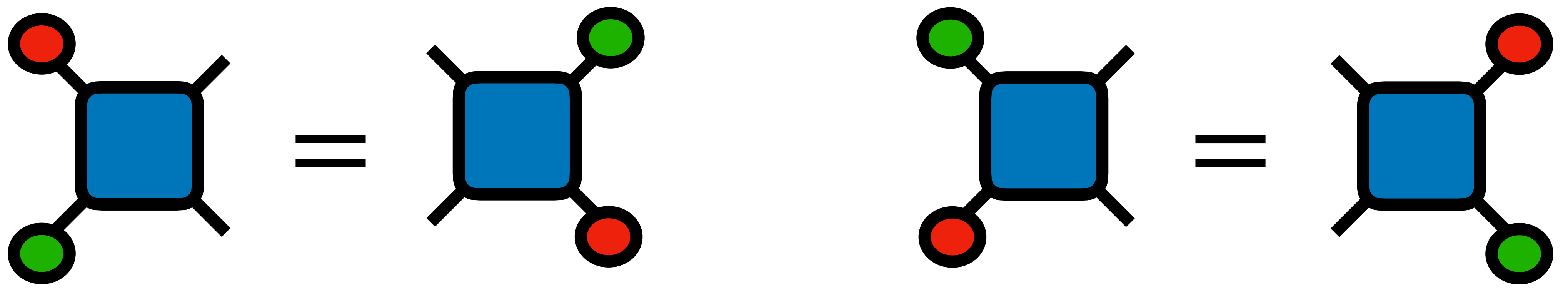

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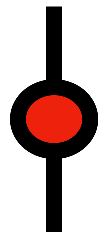
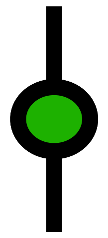
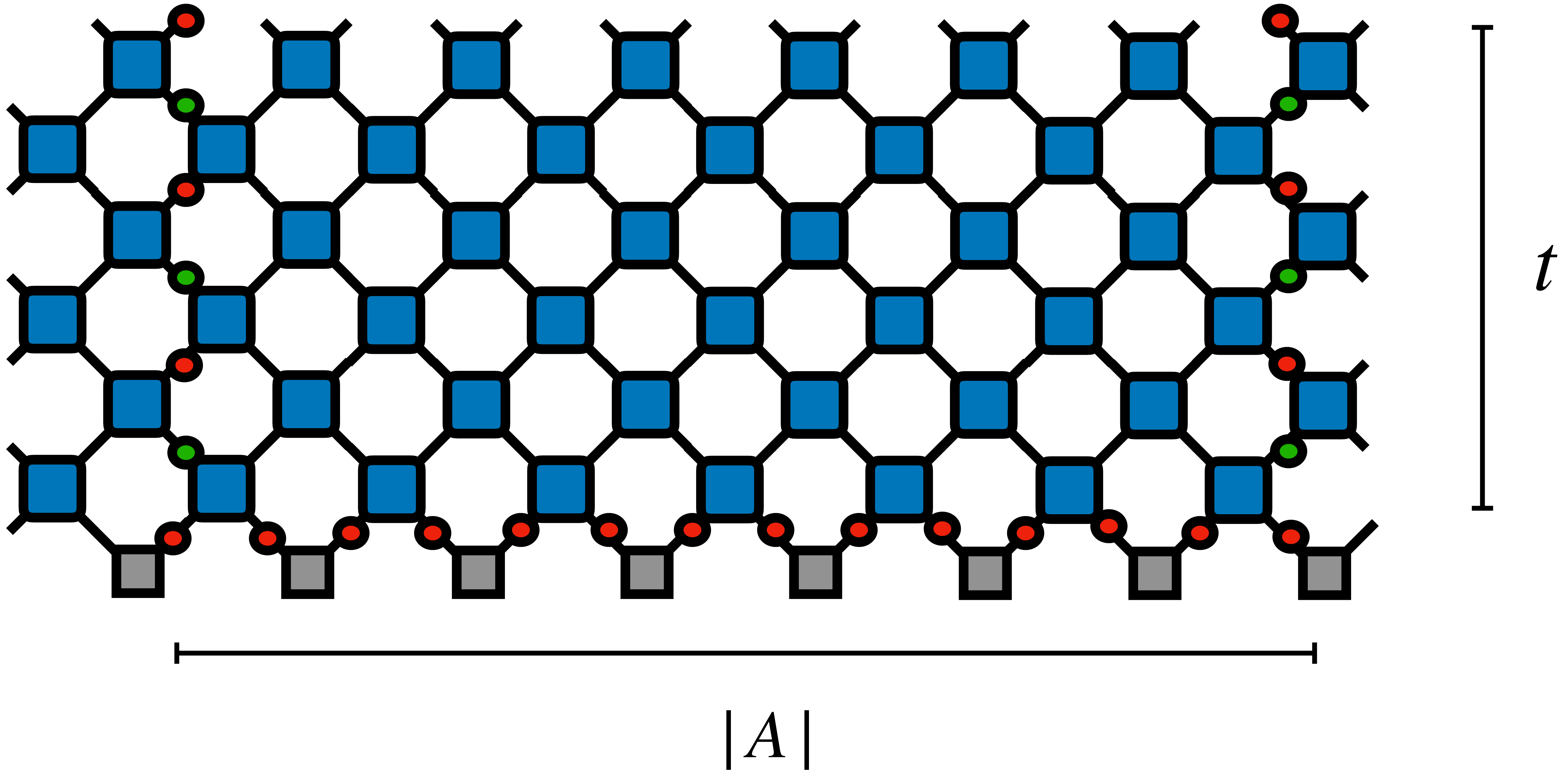
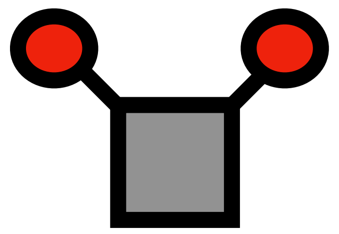
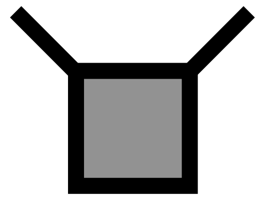
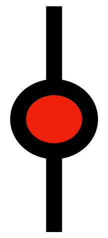
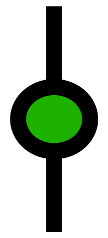
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

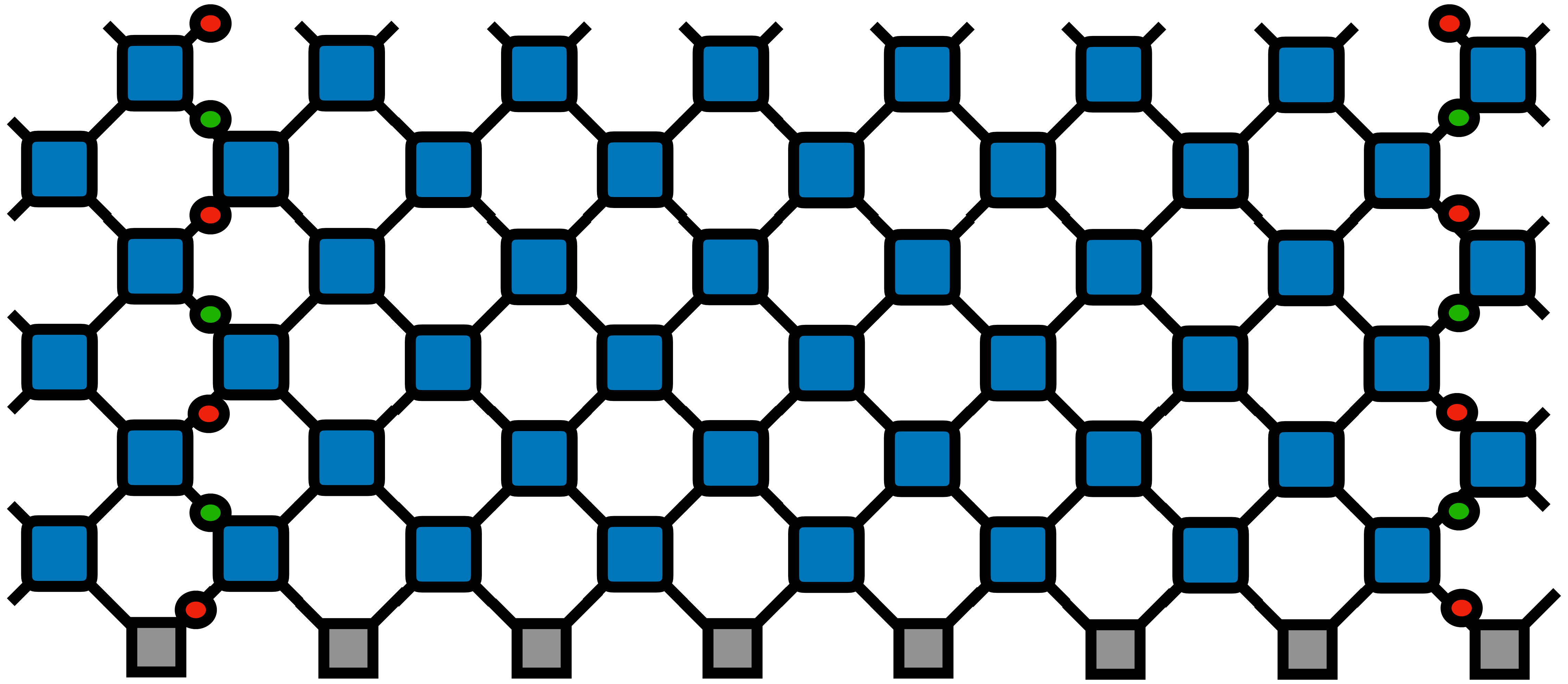
$e^{\beta Q_A} | \Psi_0(t) \rangle =$ 

- For a symmetric initial state

 $=$  $\times\, e^{2\beta q_0}$

Charged moments in QC

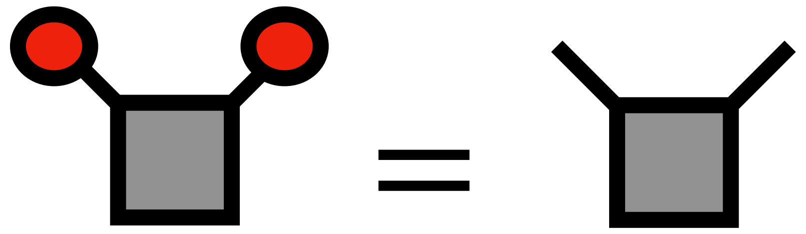
- Charge is sum of local operators $Q_A = \sum q$, s.t $e^{\beta q} =$  $e^{-\beta q} =$ 

$e^{\beta Q_A} | \Psi_0(t) \rangle =$  $\times e^{|A| q_0}$

t

$|A|$

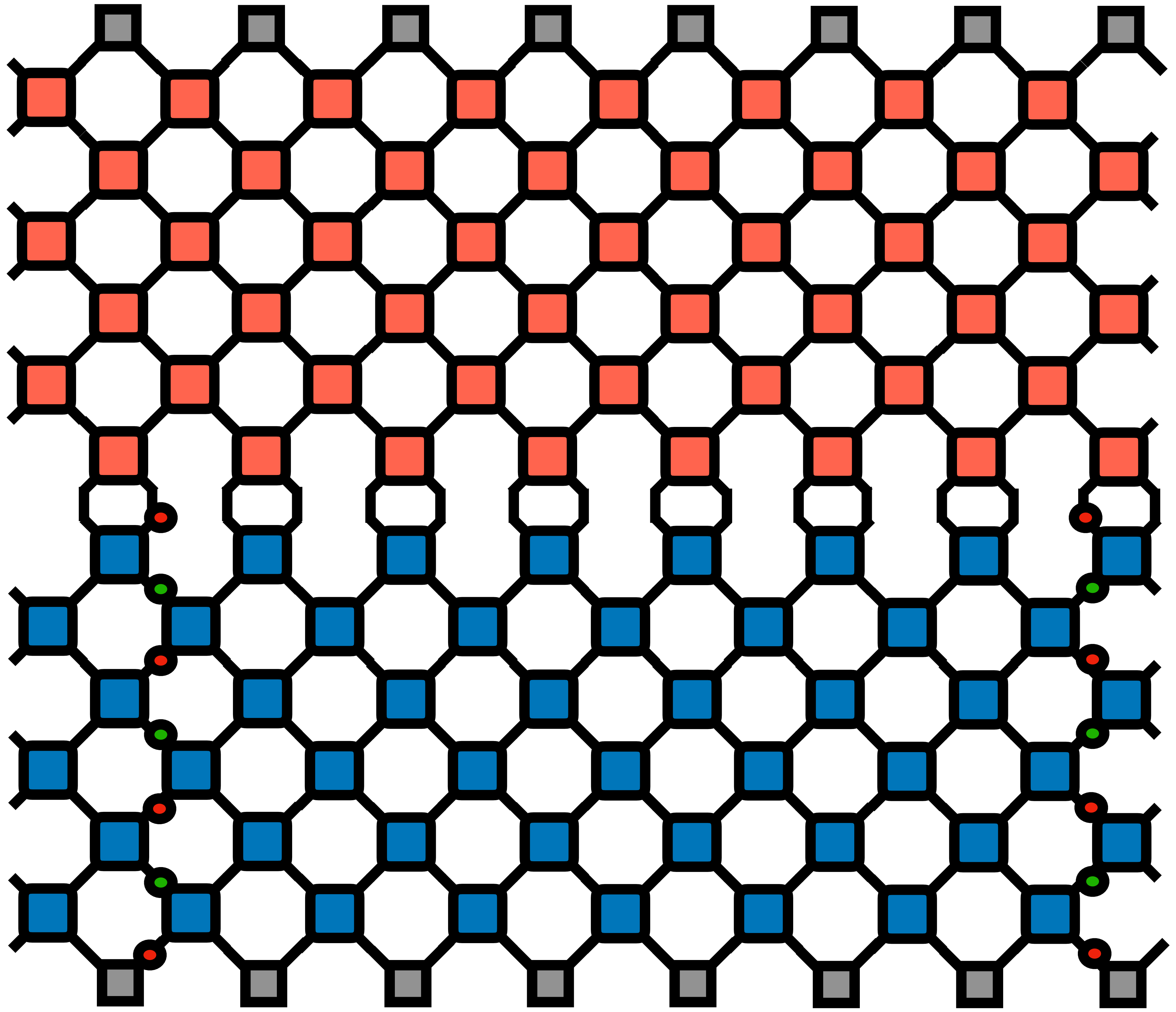
- For a symmetric initial state

 $\times e^{2\beta q_0}$

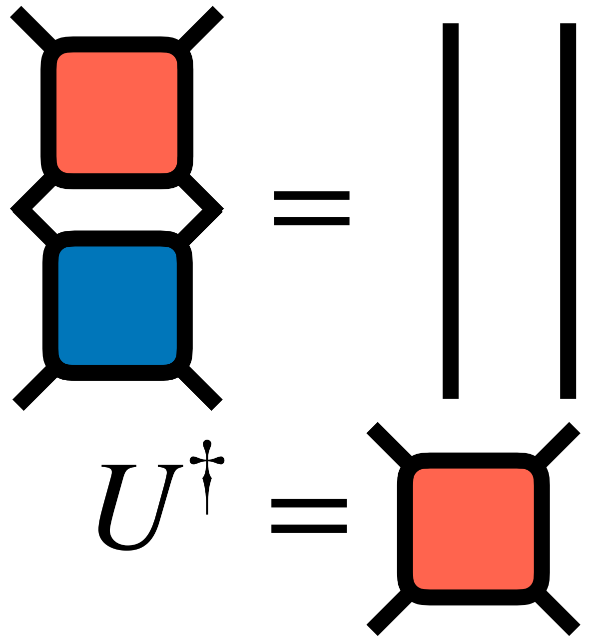
Charged moments in QC

- The FCS for $2t < |A|, |\bar{A}|$ are then

$Z_\beta(A, t) =$



$\times \operatorname{tr}[\rho_A(0) e^{\beta Q_A}]$



- Interpret this as evolution in space like direction

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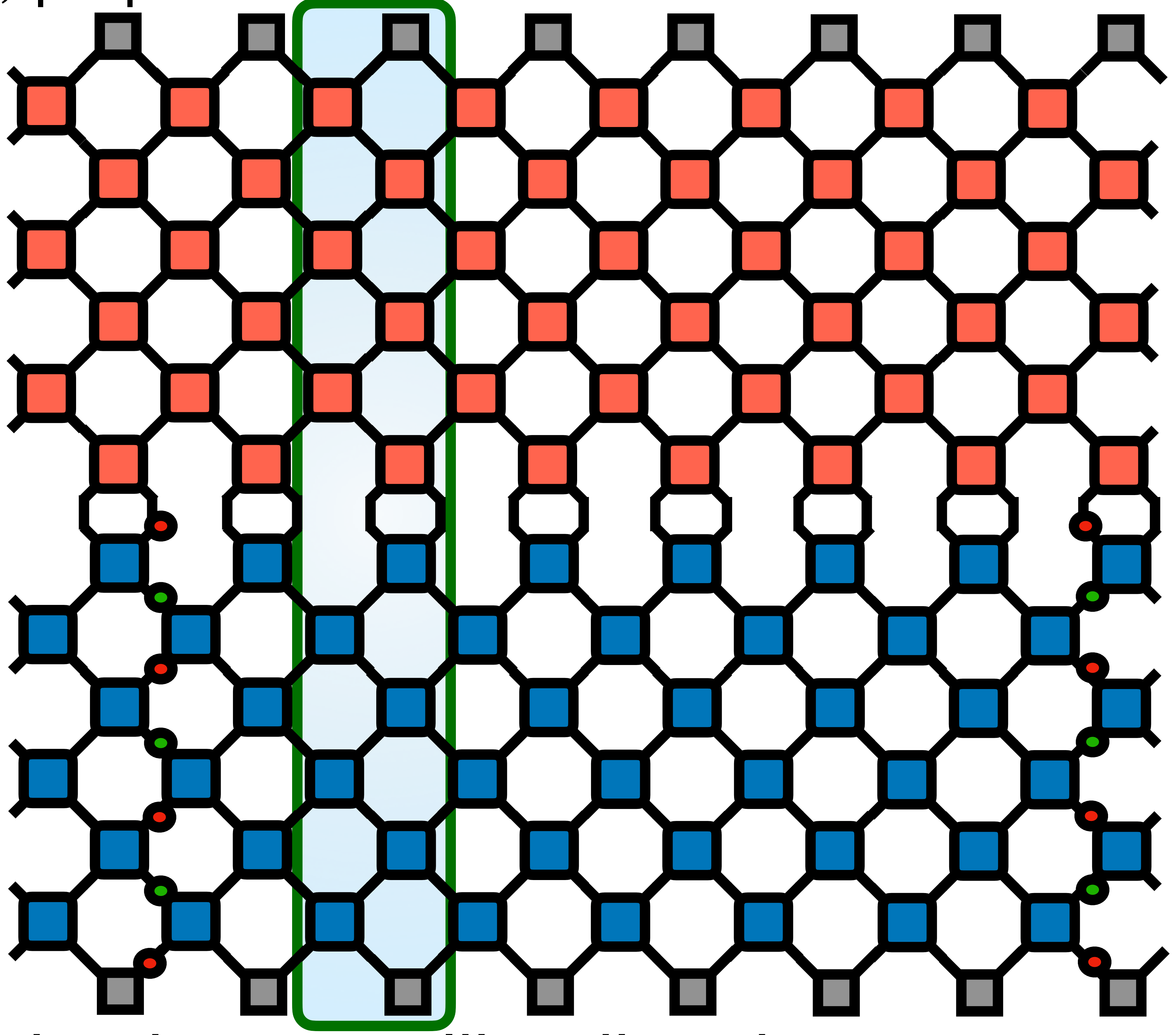

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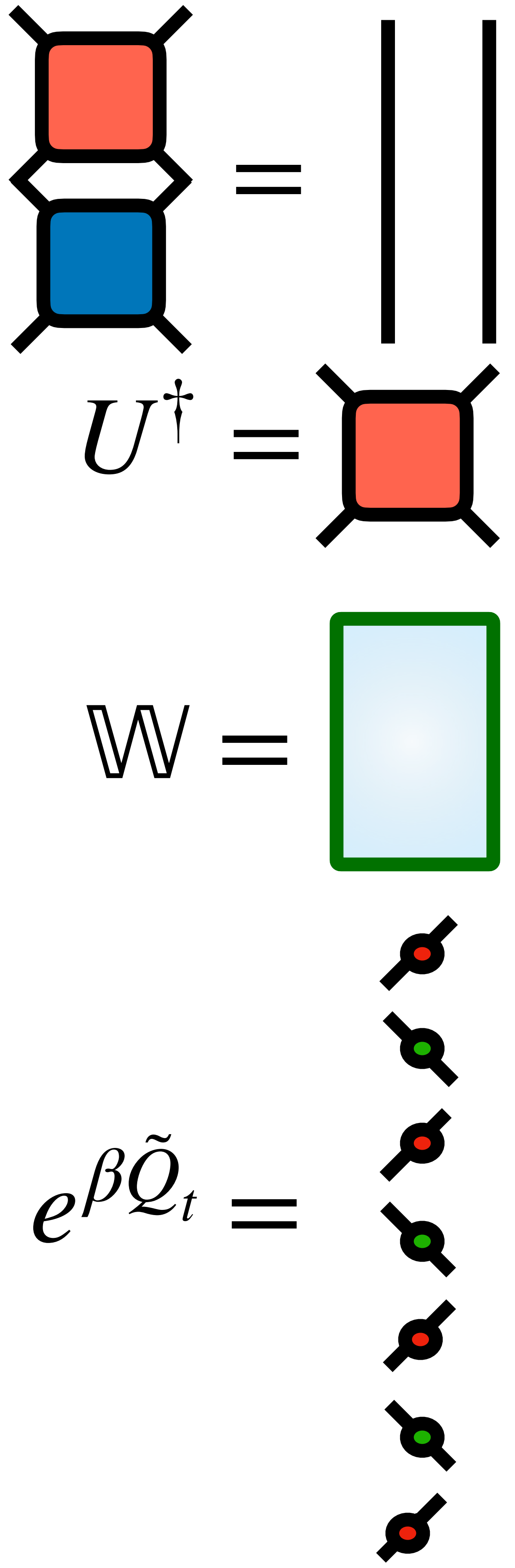


$U^\dagger =$, $\mathbb{W} =$, $e^{\beta \tilde{Q}_t} =$

- Interpret this as evolution in space like direction

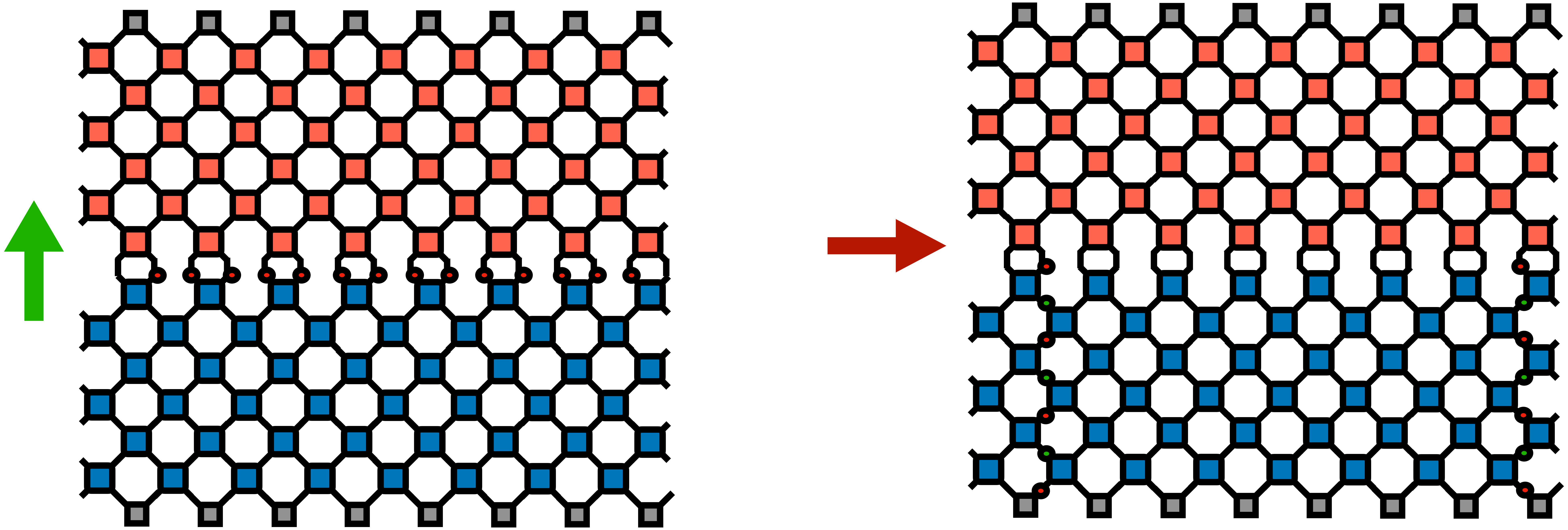
$$Z_\beta(A,t) = Z_\beta(A,0)\operatorname{tr}[\mathbb{W}^{|\bar{A}|}e^{\beta\tilde{Q}_t}\mathbb{W}^{|A|}e^{-\beta\tilde{Q}_t}]$$

Spacetime swap

- Two viewpoints for $Z_\beta(A, t)$

$$\mathrm{tr}[\mathbb{U}^t \rho_A(0) \mathbb{U}^{\dagger t} e^{\beta Q_A}]$$

$$\mathrm{tr}[\mathbb{W}^{|\bar{A}|} e^{\beta \tilde{Q}_t} \mathbb{W}^{|A|} e^{-\beta \tilde{Q}_t}]$$



- The dual perspective offers simplifications for $2t < |A|, |\bar{A}|$

Generic properties

- The space transfer matrix has unique largest eigenvector $\mathbb{W}^{|A|} \to \tilde{\rho}_{\text{st}}$

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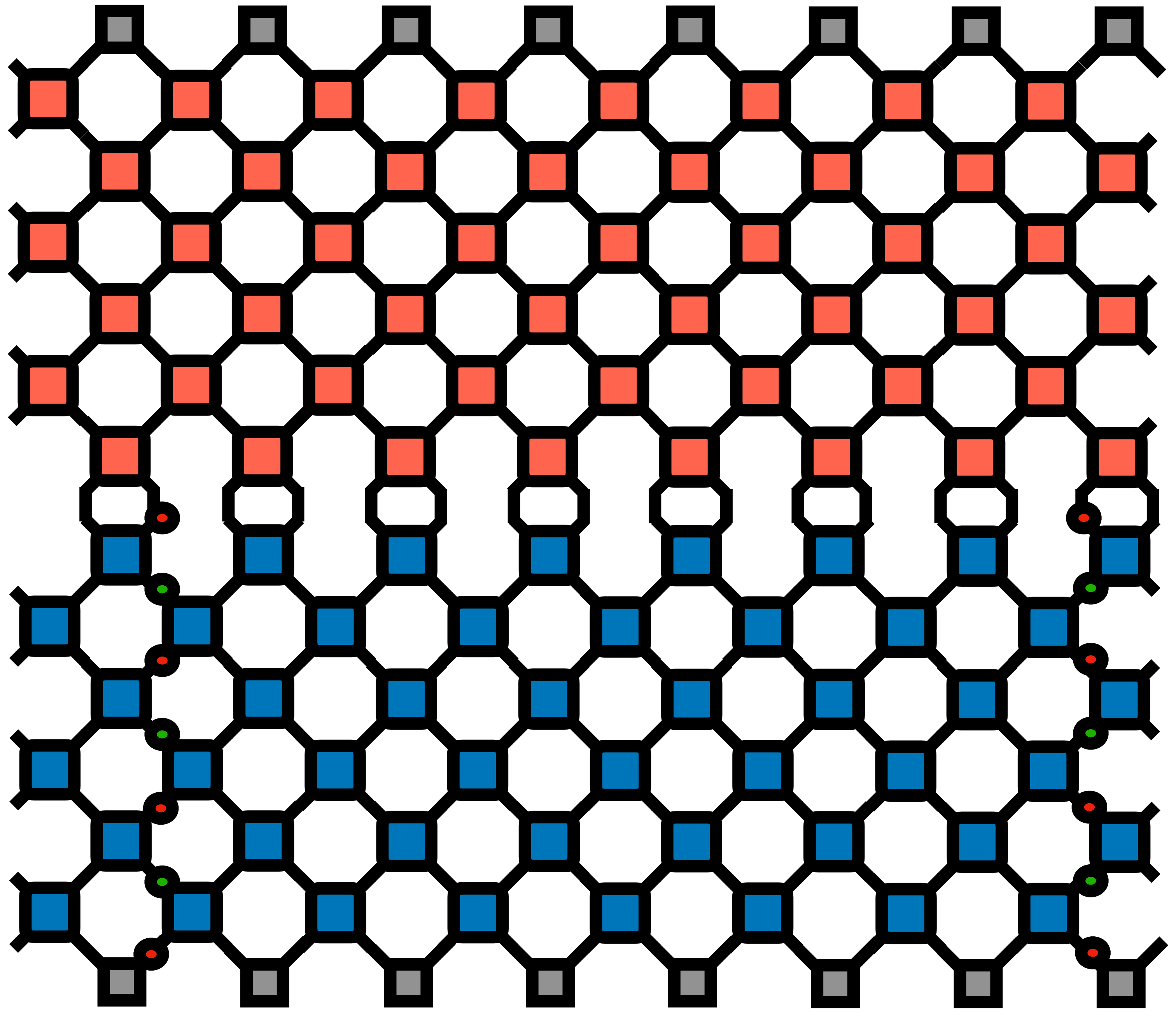

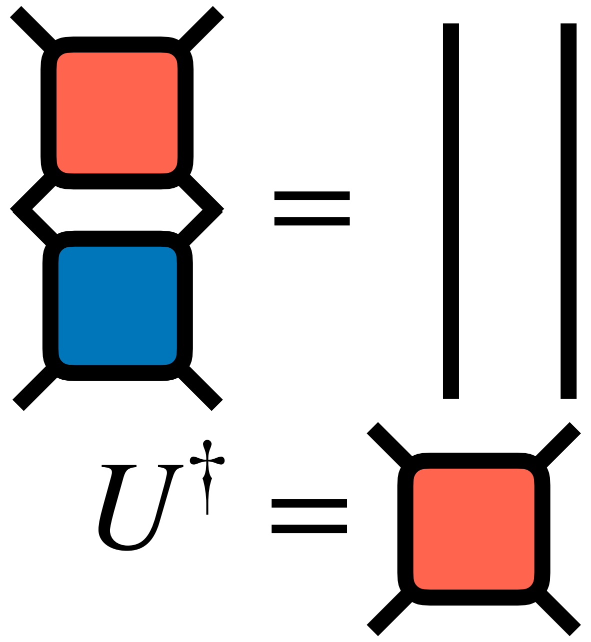

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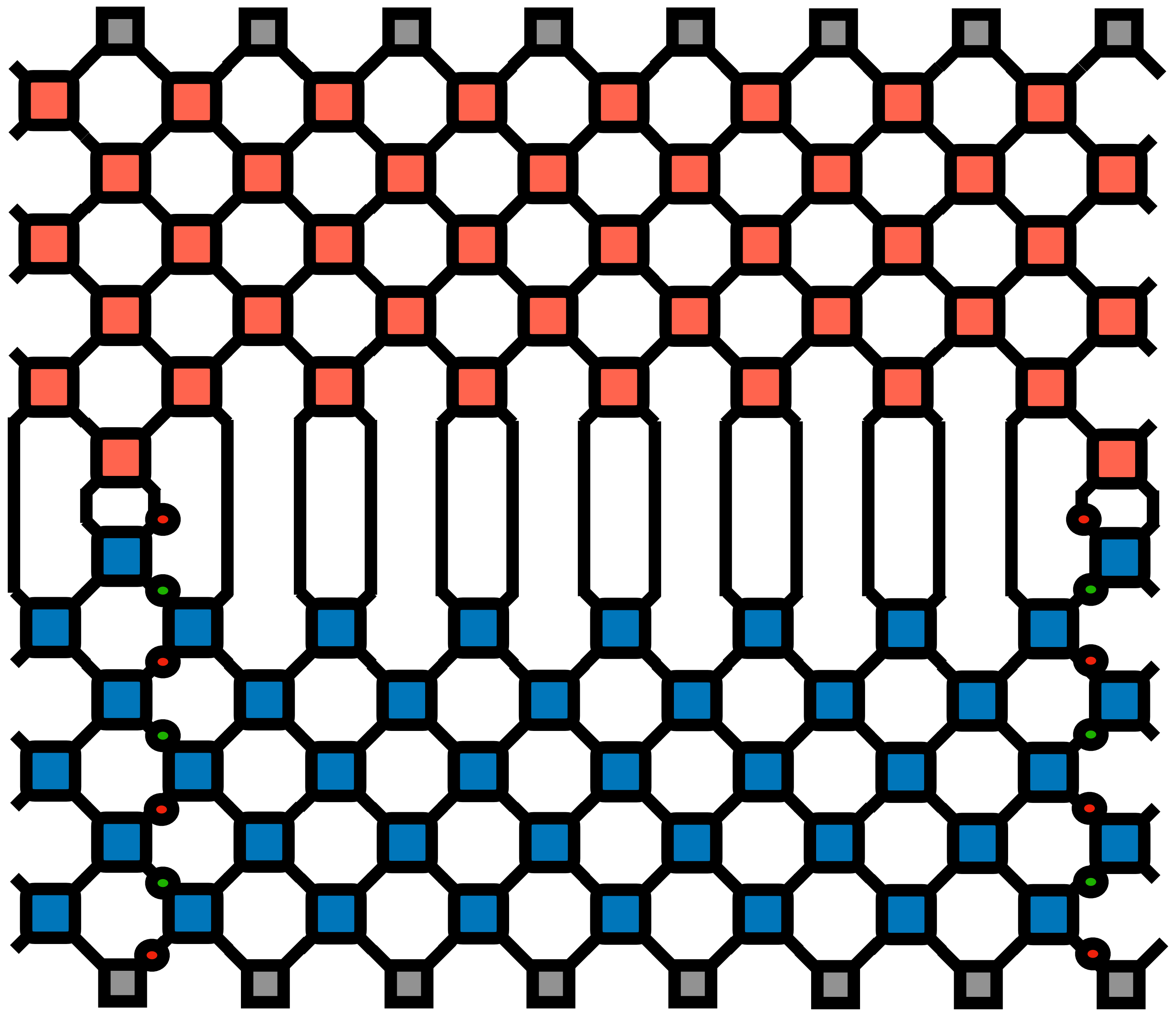

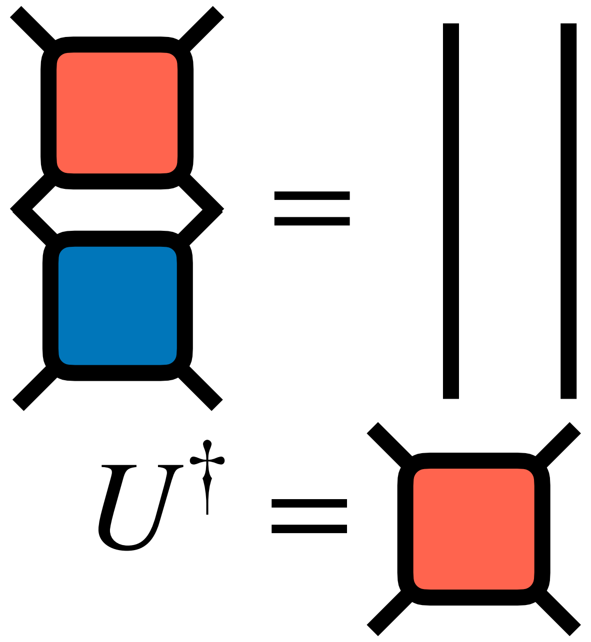

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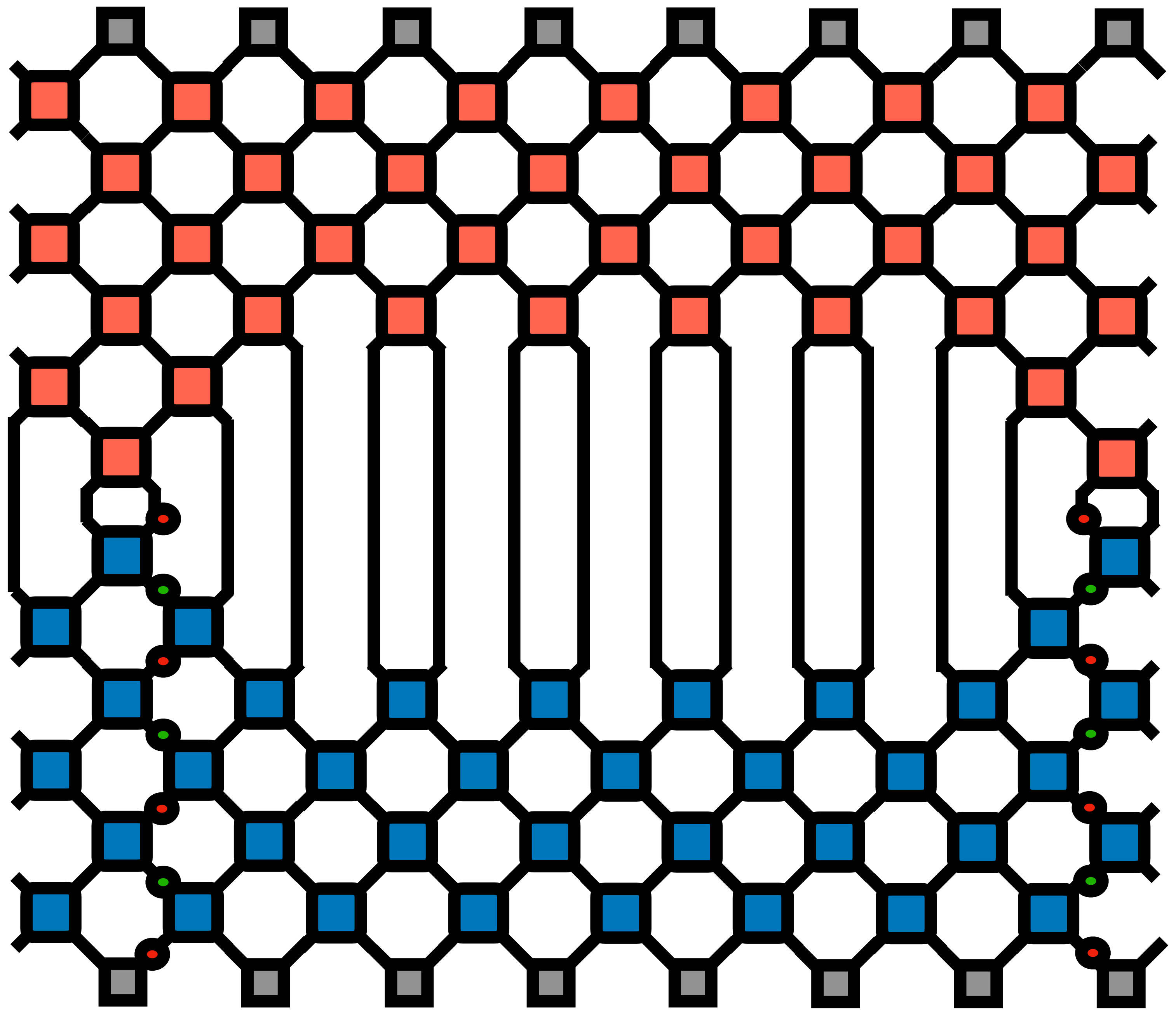

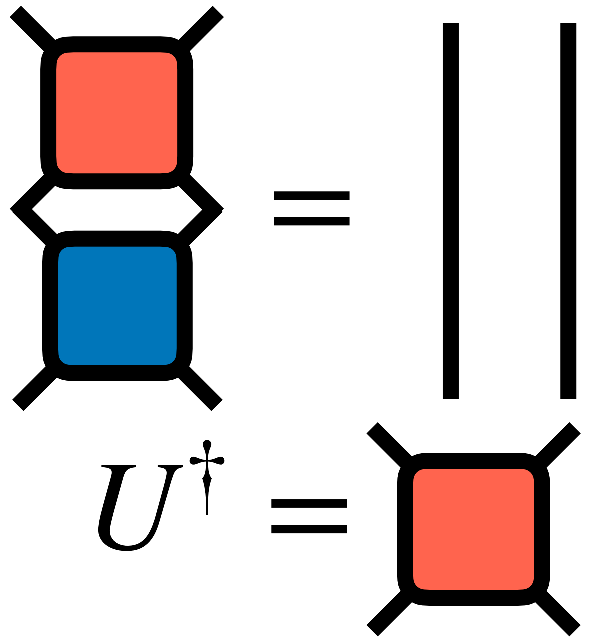

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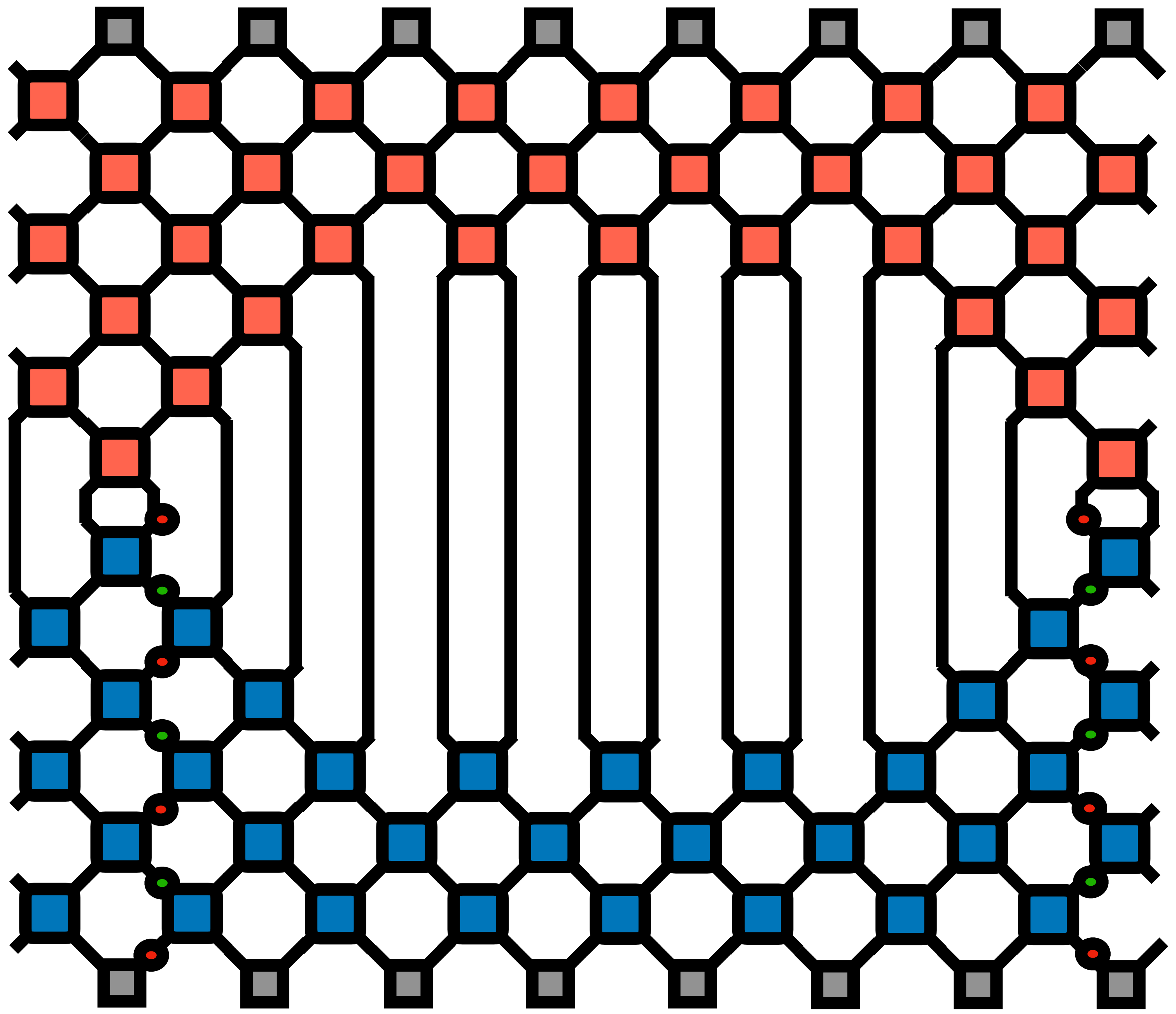

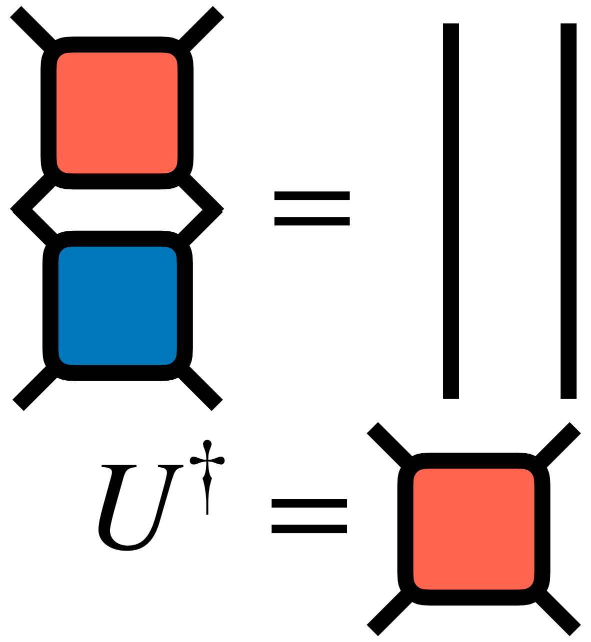

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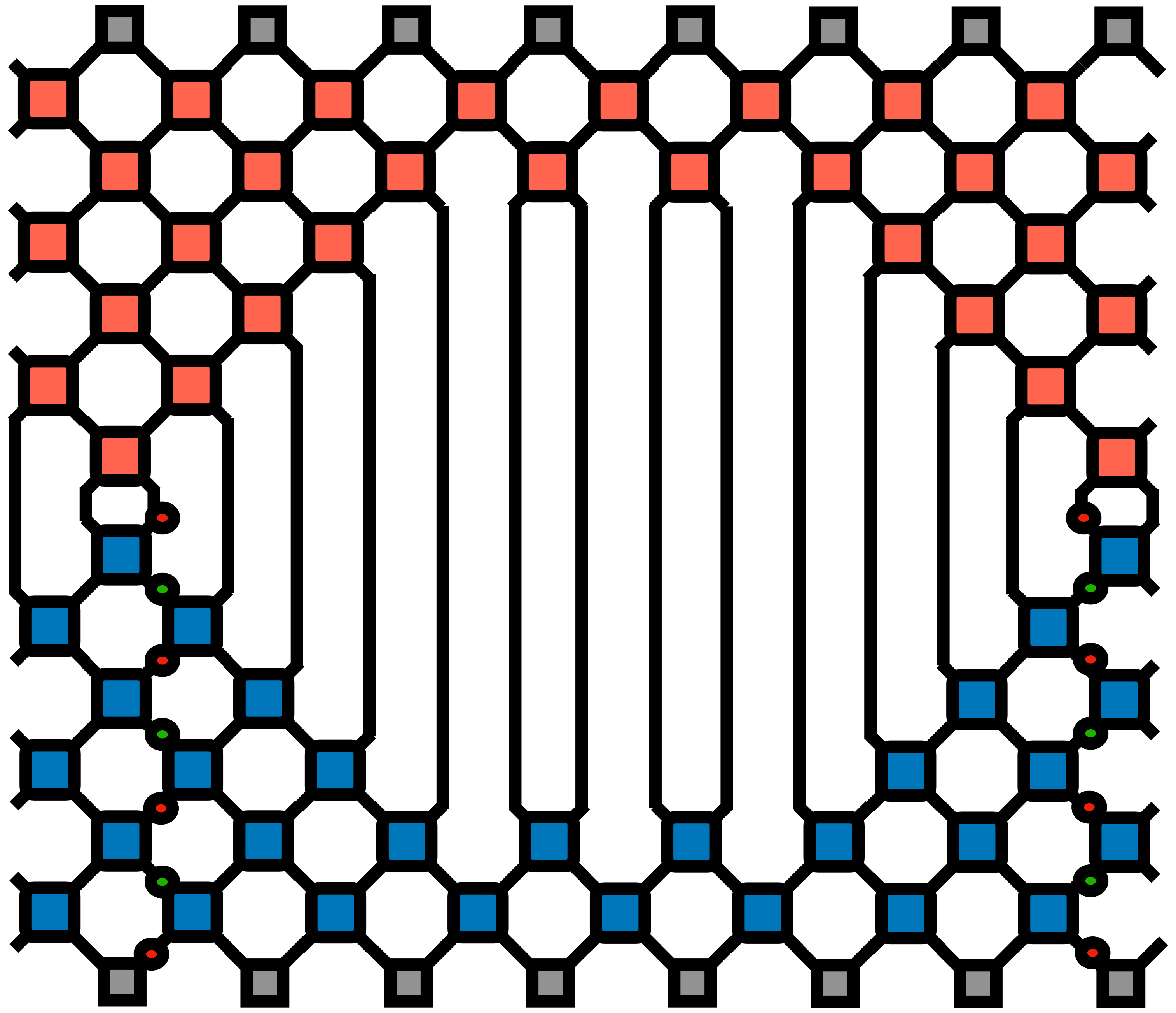

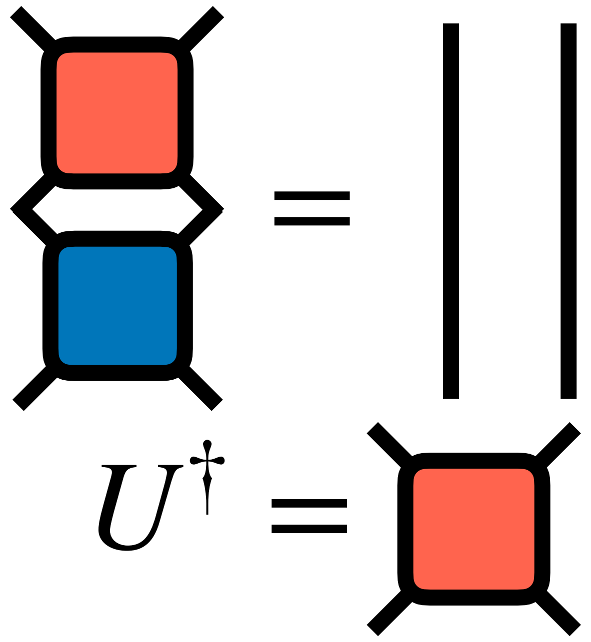

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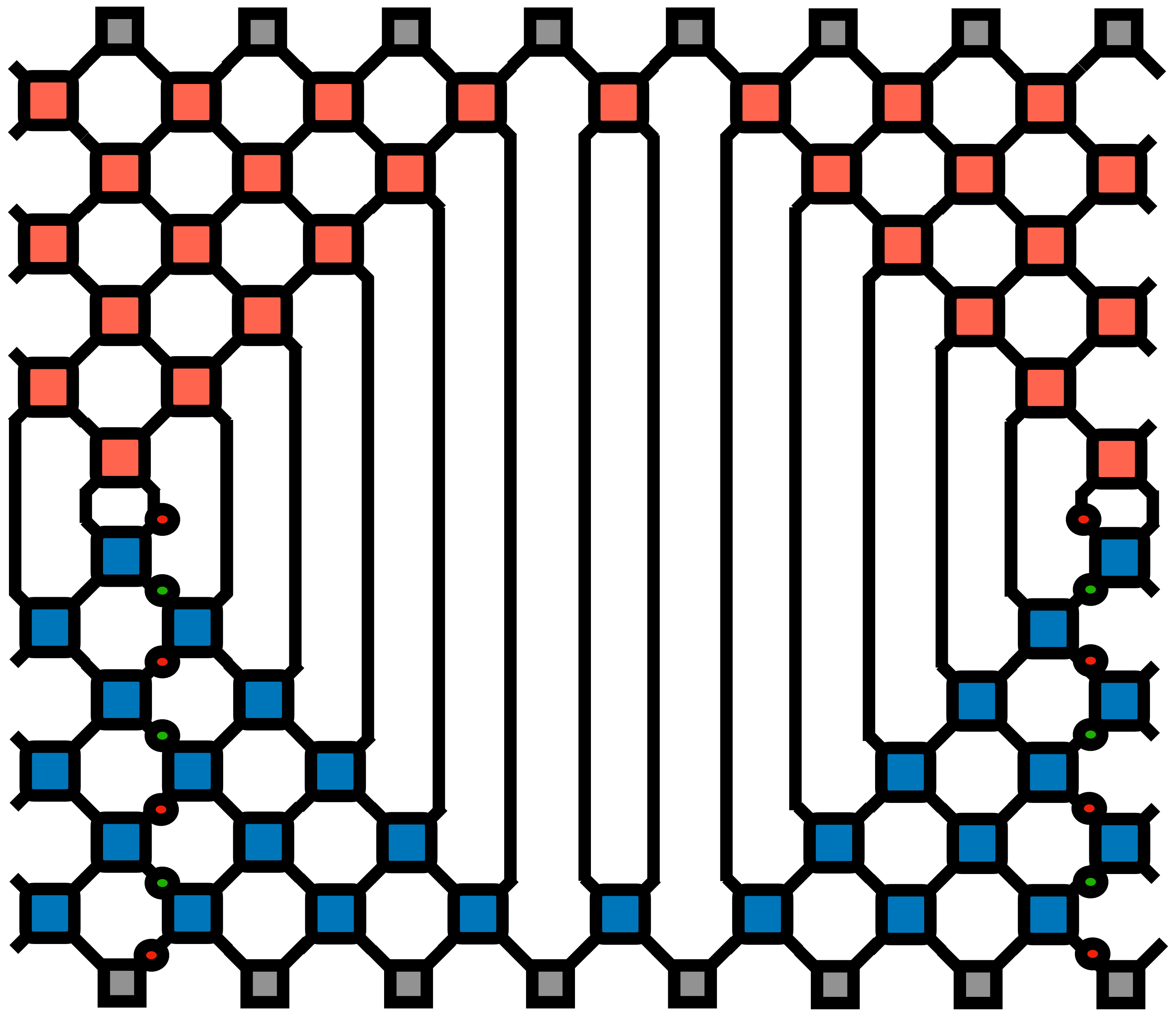

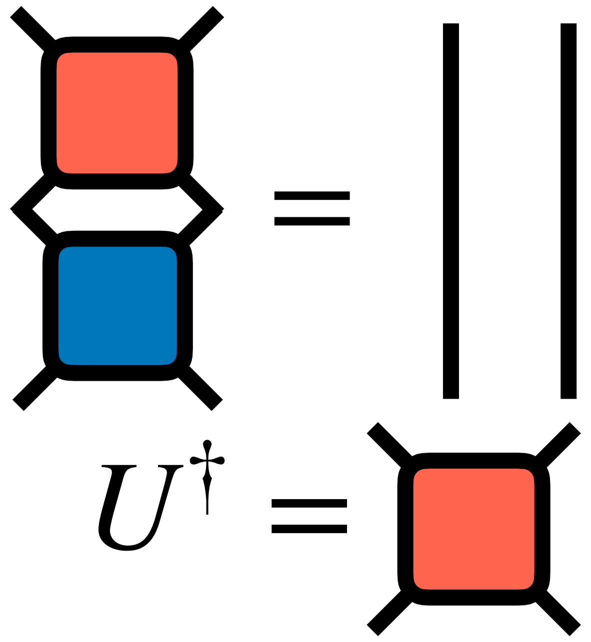

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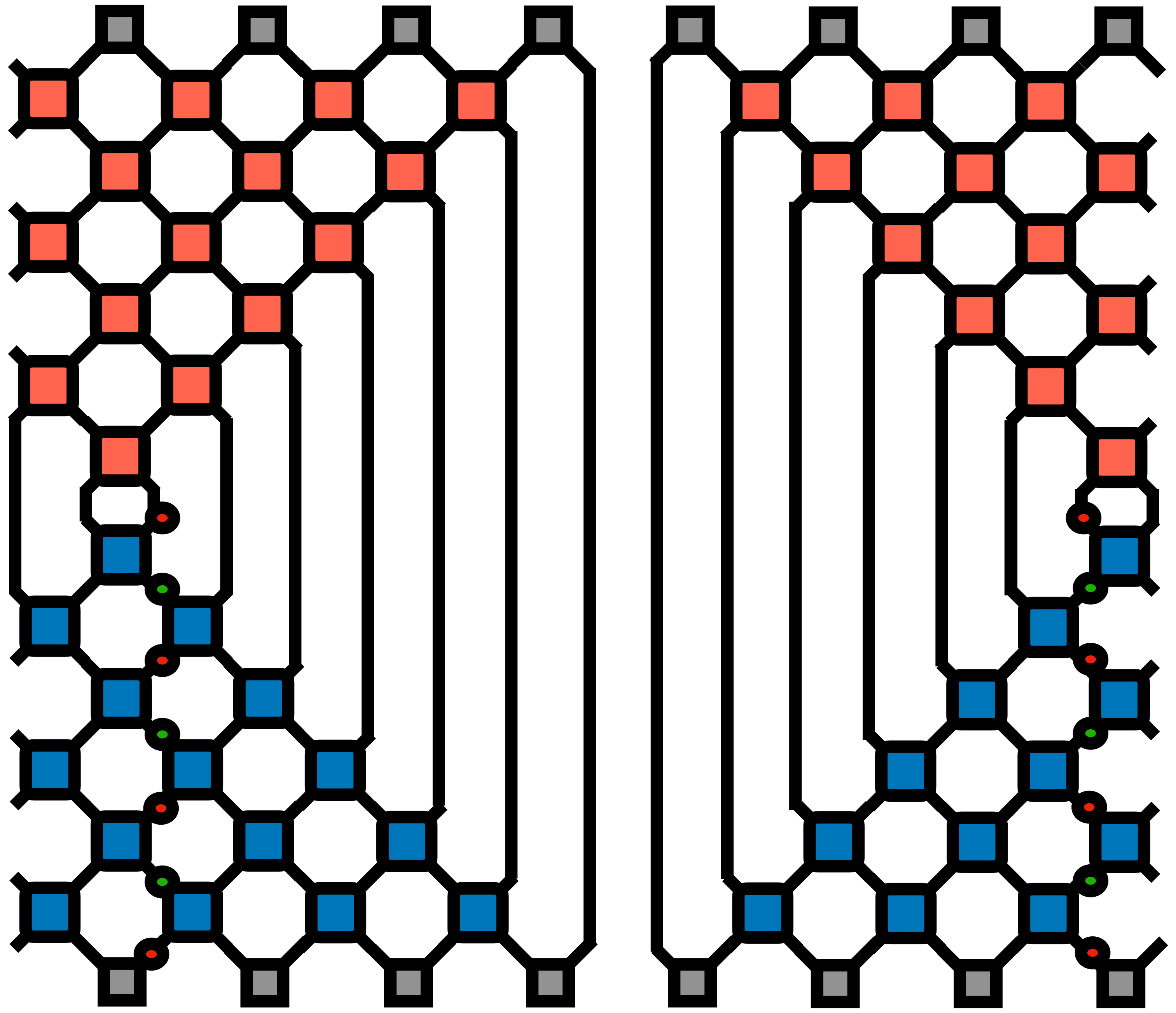

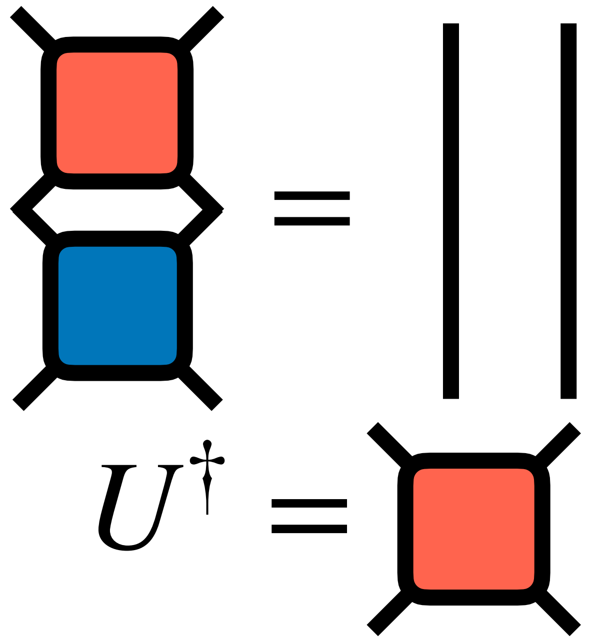

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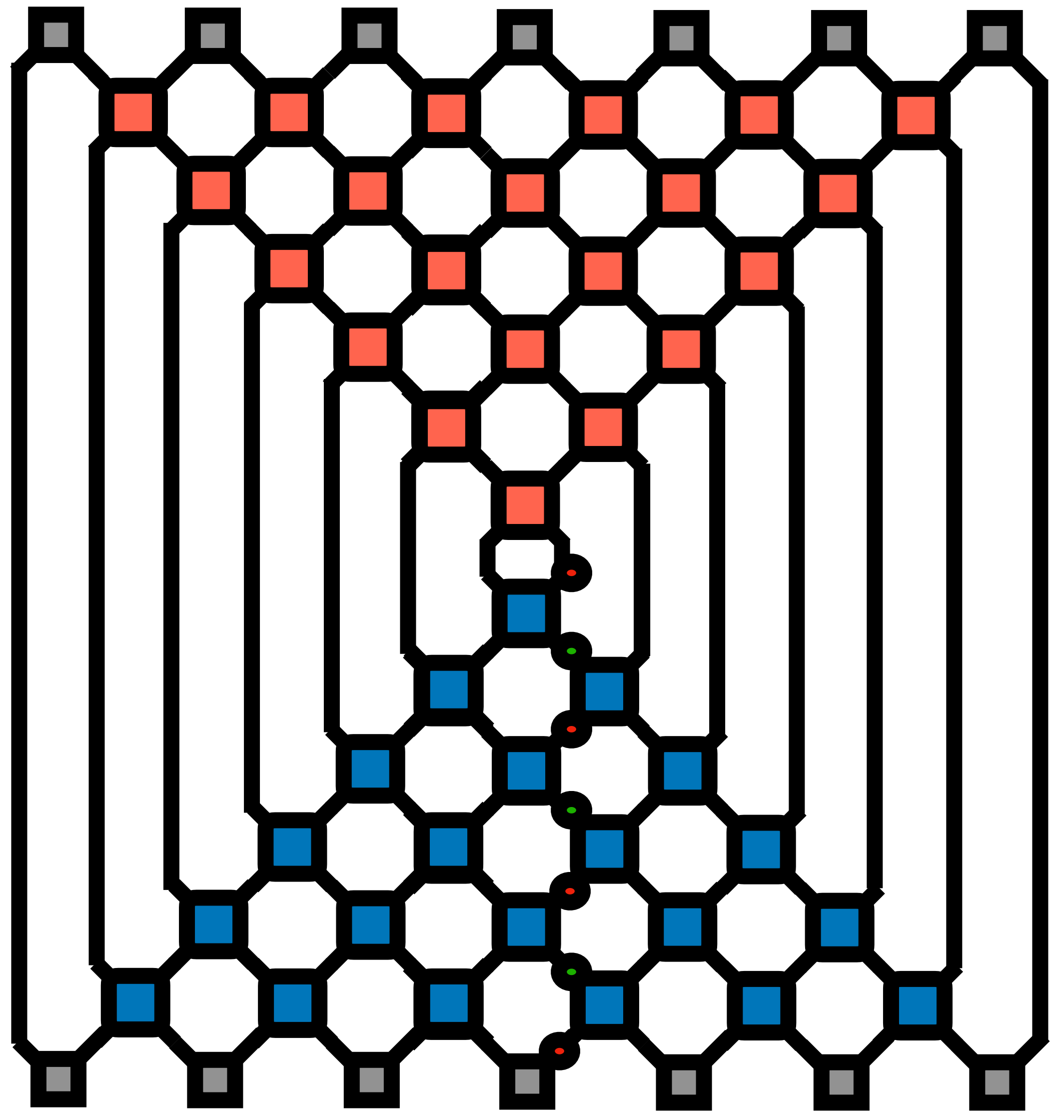
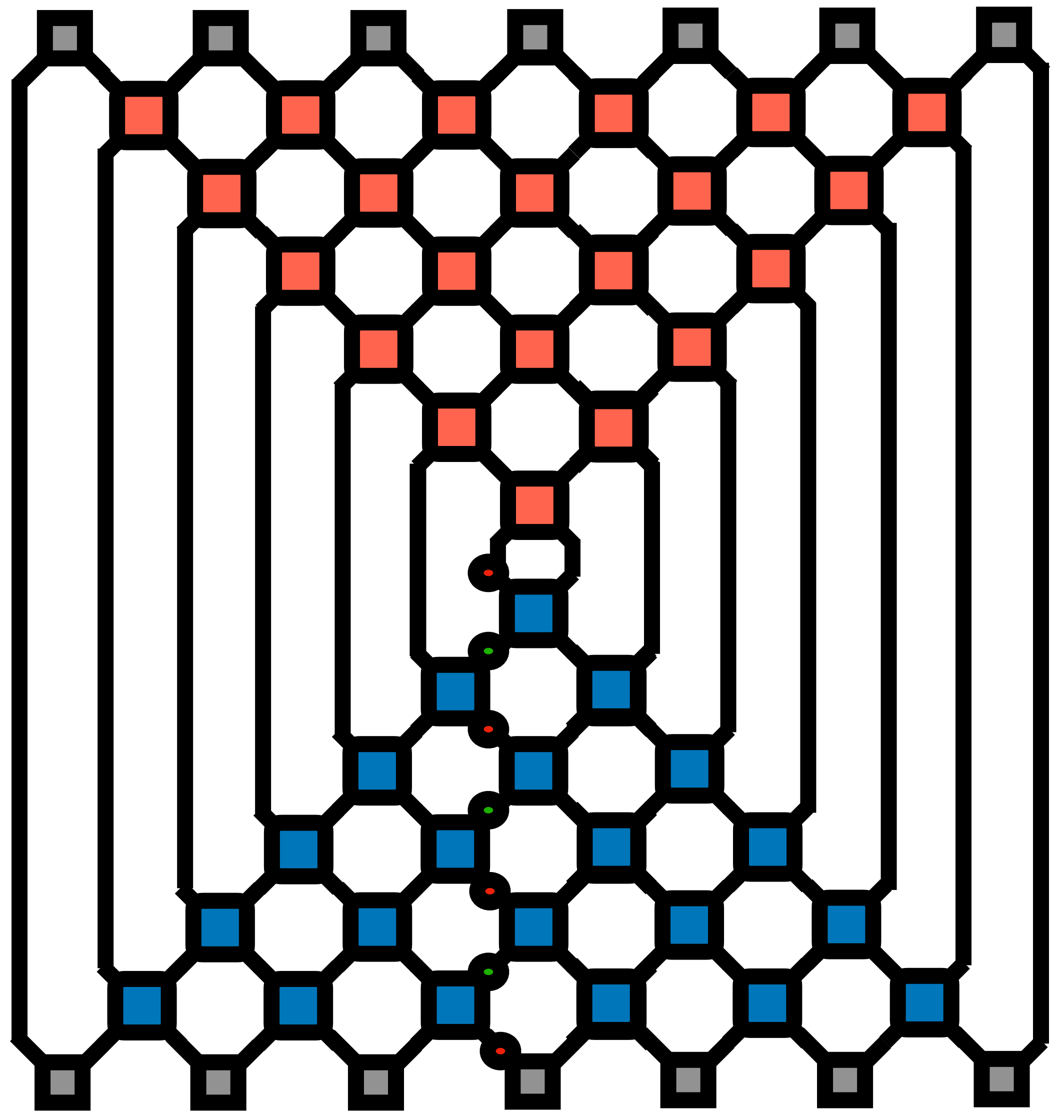


$= \mathrm{tr}[\tilde{\rho}_{\mathrm{st}}e^{\beta \tilde{Q}_t}] = \langle\, e^{\beta J_L(t)} \,\rangle$



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$$\to Z_\beta(A,0)\mathrm{tr}[\tilde{\rho}_{\mathrm{st}}e^{\beta \tilde{Q}_t}]\mathrm{tr}[\tilde{\rho}_{\mathrm{st}}e^{-\beta \tilde{Q}_t}]$$

  - Time integrated current at causally disconnected boundaries

- Extensive charge in dual system

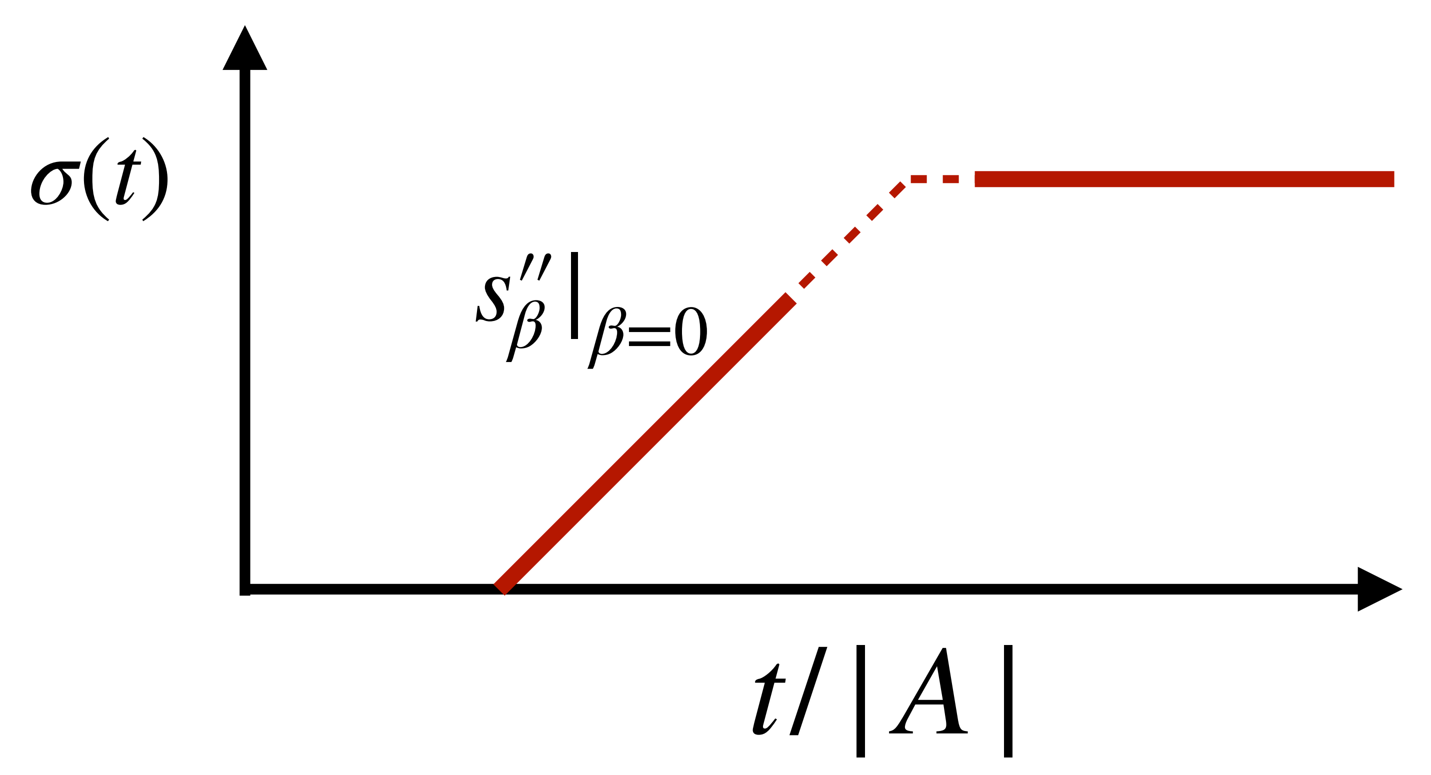
$$\lim_{t\to\infty}\lim_{|A|\to\infty}\frac{1}{t}\log \mathrm{tr}[\tilde{\rho}_{st}e^{\beta \tilde{Q}_t}] \to s_\beta$$

- Time delay for symmetric states (Parez et. al 2021)

$$P(Q_A = |A|q, t) \simeq \Theta(t - t_D)\frac{e^{-\frac{(q-q_0)^2}{2\sigma(t)}}}{\sqrt{2\pi\sigma(t)}}$$

  - If current is bounded need time to transport charge

$$t_D \sim |A||q - q_0|/\mathrm{tr}[\tilde{Q}_t]$$



TBA integrable models

- For integrable models can calculate long time limit using TBA

$$\frac{1}{|A|}\log \mathrm{tr}[\rho_{\mathrm{st}} e^{\beta Q_A}] = \sum_{m=1}^{M} \int \mathrm{d}\lambda \frac{|p_m'(\lambda)|}{2\pi} \log[1-\theta_m(\lambda)+\theta_m(\lambda)e^{-\mathrm{sgn}[p_m']\log x_m(\lambda)}]$$

  - M = # quasiparticle species, momenta $p_m(\lambda)$, energy $\epsilon_m(\lambda)$, scattering kernel $T_{mn}(\lambda-\mu)$
  - Occupation function $\theta_m(\lambda)$ from quench action (Caux Essler '13) or string-charge duality (Illievski et al. '15)

- $\log x_m(\lambda)$ related to effective quasiparticle charge $\partial_\beta \log x_m(\lambda)|_{\beta=0} = q_{m,eff}$

$$\log x_m(\lambda) = -\beta q_m + \sum_{n=1}^{M} \int T_{mn}(\lambda-\mu)\log[1-\theta_n(\mu)+\theta_n(\lambda)e^{-\mathrm{sgn}[p_n']\log x_n}]$$

- Use these to get explicit expression for dual state FCS: $\mathrm{tr}[\tilde{\rho}_{\mathrm{st}} e^{\beta \tilde{Q}_t}]$

TBA Spacetime Swap

- Space time swap $p_m(\lambda) \leftrightarrow \epsilon_m(\lambda)$ to get dual stationary state (Bertini et. al 2022)

$$\frac{1}{t}\log \mathrm{tr}[\tilde{\rho}_{\mathrm{st}} e^{\beta \tilde{Q}_t}] = \sum_{m=1}^{M} \int \mathrm{d}\lambda \frac{|\epsilon_m'(\lambda)|}{2\pi} \log[1-\theta_m(\lambda)+\theta_m(\lambda)e^{-\mathrm{sgn}[\epsilon_m']\log y_m(\lambda)}]$$

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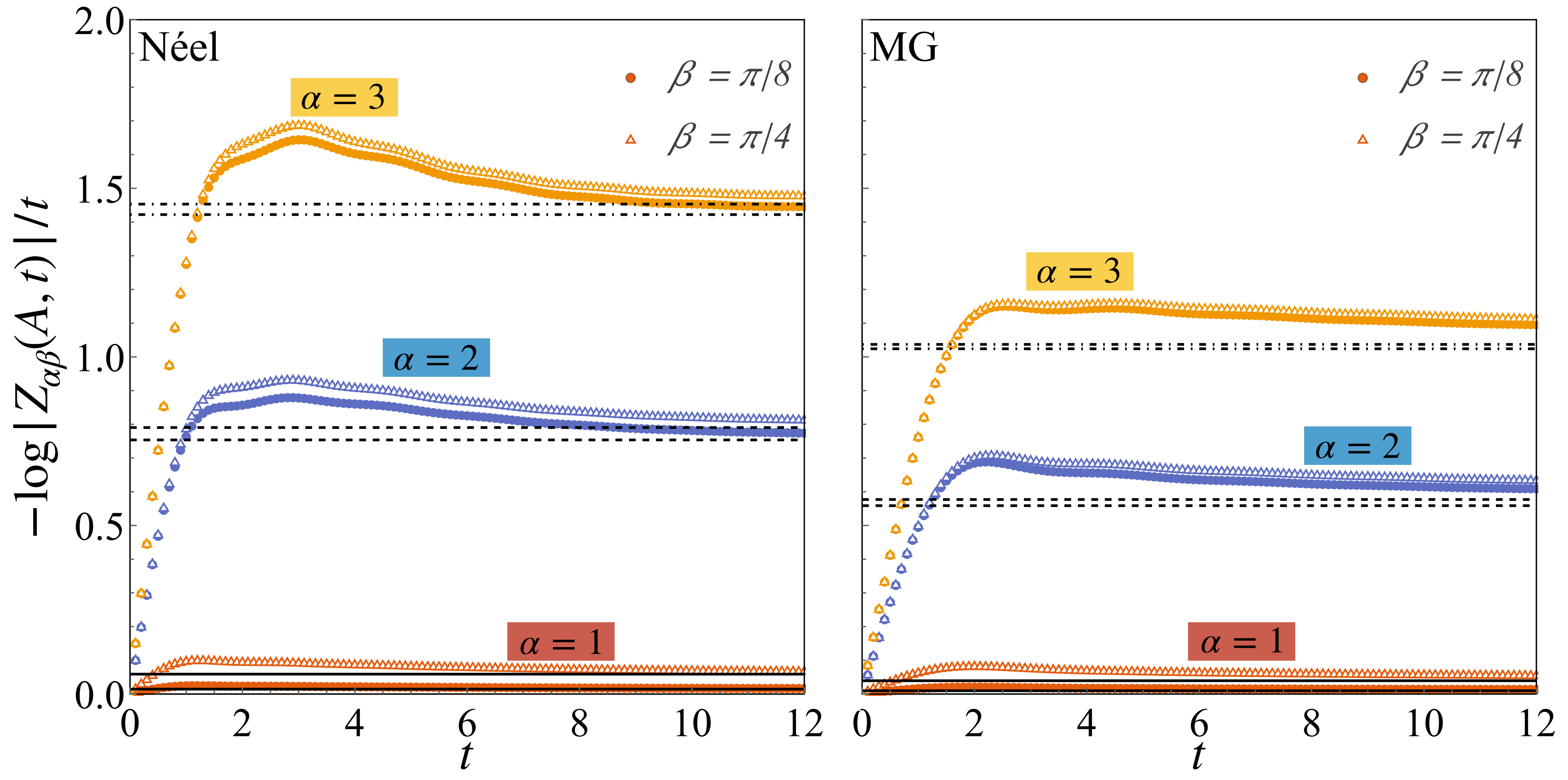
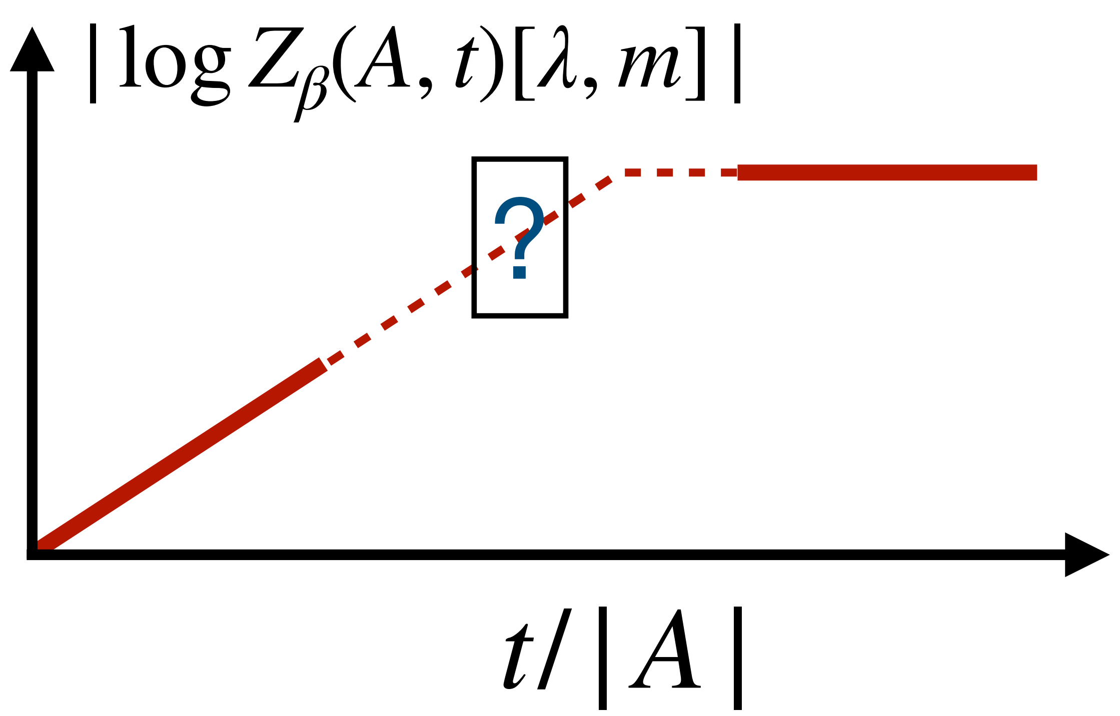
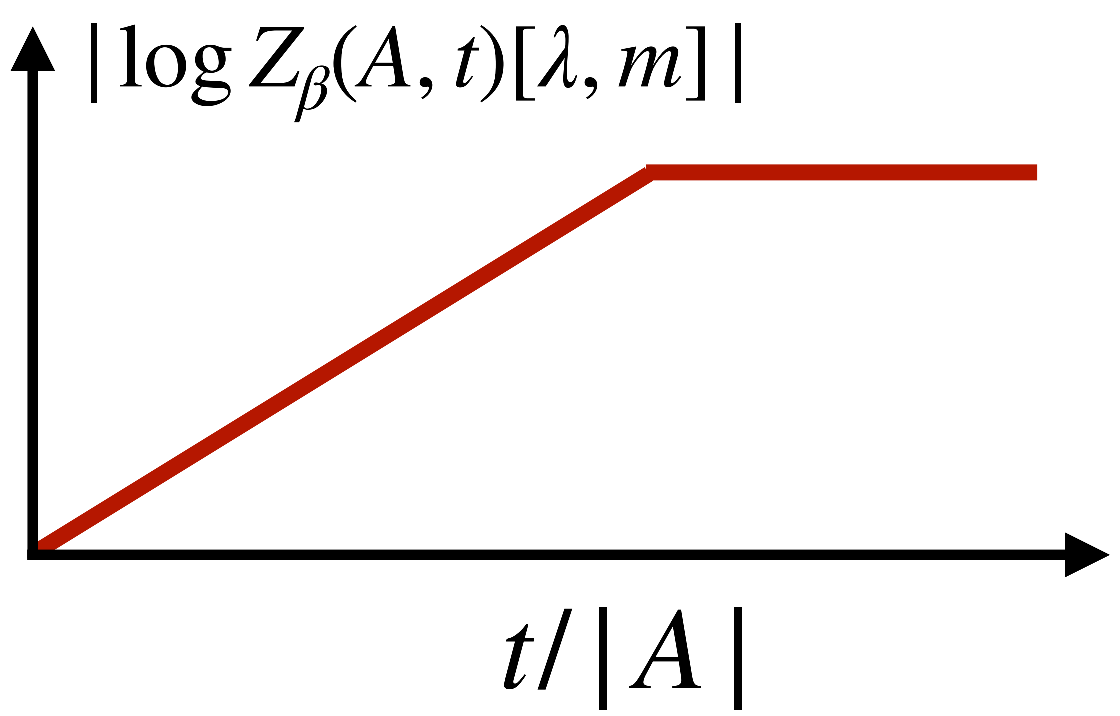
- Checked
  - Analytically in free models, Rule 54
  - Numerically in XXZ
  - Agrees with BMFT (Myers et. al 2020)



Finite time dynamics

- Use a “quasiparticle” picture to connect the two regimes (Calabrese & Cardy 2005, Alba & Calabrese 2017)
  - Each mode contributes t linear growth and then saturates





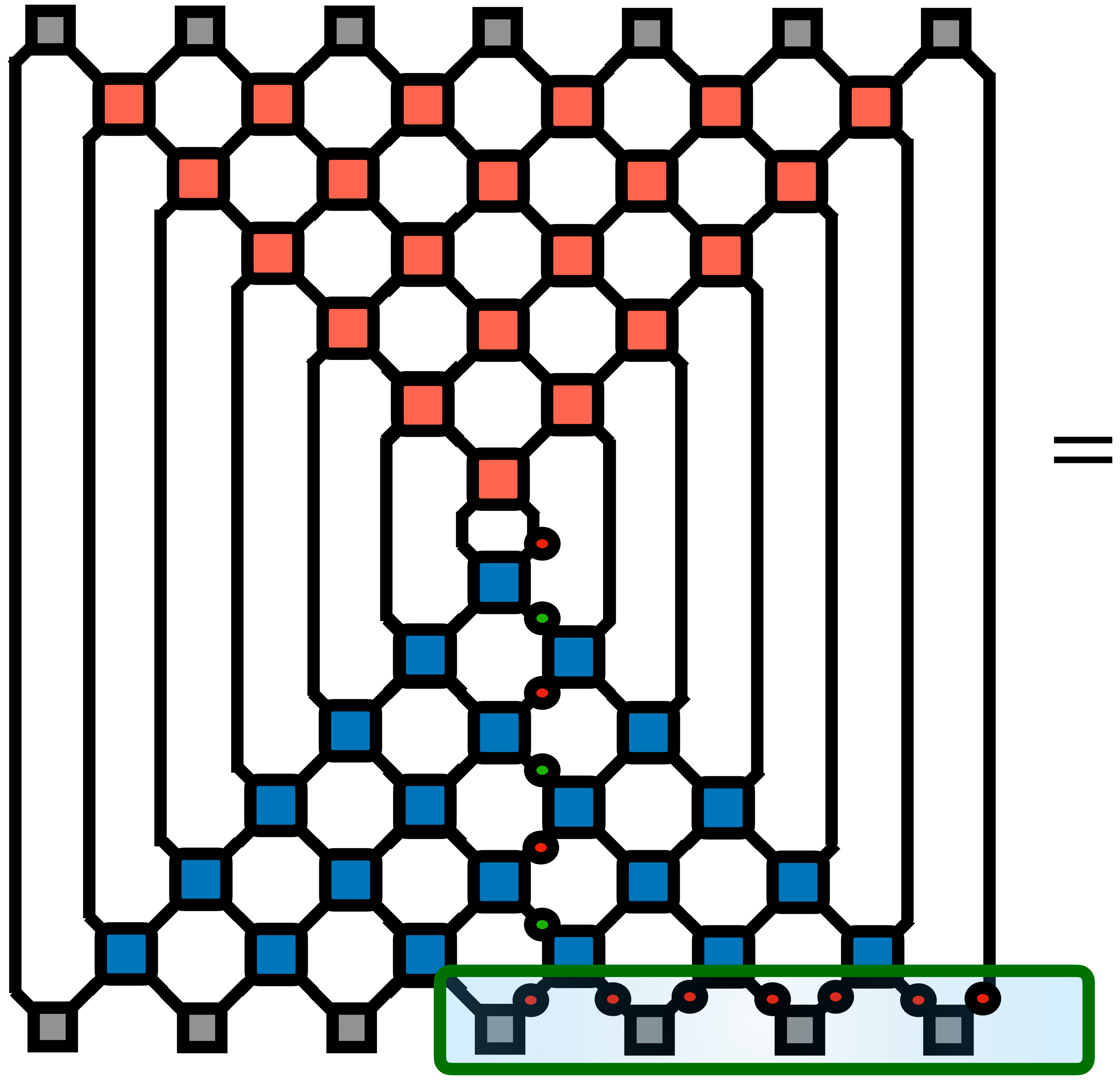
- Charge probability distribution for $t \gg t_D$, $q \approx q_0$, FT via saddle point

$$P(Q_A = |A|q, t) \simeq \frac{e^{-\frac{(q-q_0)^2}{2\sigma(t)}}}{\sqrt{2\pi\sigma(t)}} \qquad \sigma(t) = \sum_{m=1}^{M} \int \mathrm{d}\lambda \min[|A|, 2t|v_m(\lambda)]q_{m,\mathrm{eff}}^2 \rho_m[1-\theta_m]$$

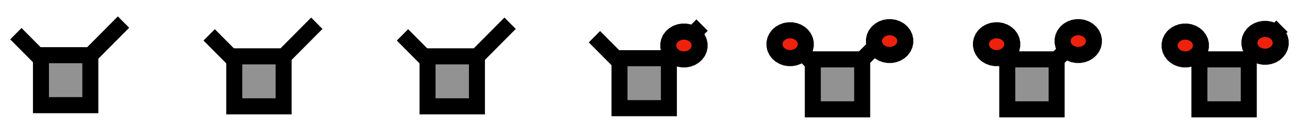
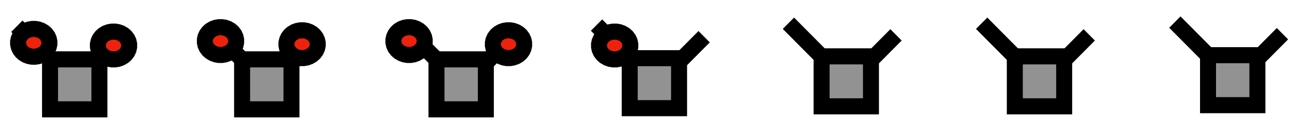
  - Quasiparticle velocity $v_m(\lambda)$, quasiparticle charge $q_{m,\mathrm{eff}}(\lambda)$

Non symmetric initial states

- Can also treat charge moments for non symmetric initial states



$$= \mathrm{tr}[\tilde{\rho}_{\mathrm{st}}(0,\beta) e^{\beta \tilde{Q}_t}] = \langle\, e^{\beta J_L(t)} \,\rangle_{\mathrm{NESS}}$$

- NESS of bipartite quench from  or 

- Use GHD solution for NESS instead of $\theta_m(\lambda)$ (Bertini et. al 2016, Castro-Alvaredo et. al. 2016)

Conclusions & Outlook

- Space-time duality maps non-equilibrium systems to equilibrium ones
  - Generic properties for charged moments based on equilibrium methods
  - Specific predictions for integrable models
  - Also works for higher Rènyi index

$$Z_\beta(A,t) \simeq Z_\beta(A,0)\mathrm{tr}[\tilde{\rho}_{\mathrm{st}}^n e^{\beta \tilde{Q}_A}]\mathrm{tr}[\tilde{\rho}_{\mathrm{st}}^n e^{-\beta \tilde{Q}_A}]$$

  - Entanglement Asymmetry
  - Quantum Mpemba Effect

Based on:
arXiv:2212.06188 (PRL) & arXiv:2306.12404

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Thank you

Entanglement asymmetry

- Can calculate entanglement asymmetry, $[\rho_A(0), Q] \neq 0$ (Ares et. al. 2022)

$$\Delta S[\rho_A(t)] = \mathrm{tr}[\rho_A(t)\{\log \rho_A(t) - \log \rho_{A,Q}(t)\}]$$

$$\rho_{A,Q}(t) = \sum_q \Pi_q \rho_A(t) \Pi_q$$

  - $\rho_{A,Q}(t)$: RDM projected onto charge symmetric space, Π_q- projector onto $Q_A = q$
  - Measures the restoration of symmetry after quench from a symmetry broken state
- Calculate with charged moments via replica trick and spacetime swap

$$\Delta S[\rho_A(t)] = \frac{1}{2} + \frac{1}{2} \log \pi |A| b(t)$$

$$b(t) = \sum_{j=1}^{M} \int \mathrm{d}\lambda \, [1 - \min[1,2|v_m|t/|A|] q_{m,\mathrm{eff}}^2 \, \rho_m(\lambda)[1 - \theta_m(\lambda)]$$

  - Use to study relaxation dynamics/symmetry restoration: *Quantum Mpemba Effect*

Quantum Mpemba effect

- Consider two different broken symmetry states, ρ_1, ρ_2 with

$$\Delta S[\rho_{A,1}(0)] > \Delta S[\rho_{A,2}(0)]$$

- Quench to an integrable Hamiltonian, generically $\lim_{t\to 0} \Delta S_A(\rho, t) \to 0$

QME if: $\Delta S_A[\rho_1(t)] < \Delta S_A[\rho_2(t)], \quad \forall t > t_{\mathrm{M}}$

- *Example*: Quench of XXZ from tilted Ferro

$$|\Psi_0\rangle = e^{i\frac{\theta}{2}S^y}|\Uparrow\rangle$$

- Faster symmetry restoration for larger tilt
    - Can be explained through transport properties