Charge Imbalance resolved Rényi negativity of free compact boson

10th Bologna Workshop on CFT and Integrable Models
Dept. of Physics and Astronomy, University of Bologna

Himanshu Gaur
collaborator Urjit A. Yajnik

Department of Physics, Indian Institute of Technology Bombay, Mumbai, India

September 4, 2023

Symmetry Resolved Entanglement

- For mixed states the Negativity measures are used to study entanglement

$$\mathcal{E} = \lim_{n_e \to 1} \ln R_{n_e}, \qquad \mathcal{N} = \lim_{n_e \to 1} \frac{1}{2}(R_{n_e} - 1),$$

where $R_n \equiv \mathrm{Tr}\left(\rho_A^{T_2}\right)^n$ is the Rényi negativity.

- When a system possesses a global internal symmetry, entanglement measures maybe resolved into local charge sectors. In case of negativity measures we have

$$[\rho^{T_2}, \hat{\mathcal{Q}}] = 0, \qquad \text{where} \;\; \hat{\mathcal{Q}} = \hat{Q}_{A_1} - \hat{Q}_{A_2}^{T}.$$

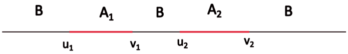

Figure: Disjoint intervals.

Free Compact Boson

- We may define the multi-charged moments and Symmetry resolved Rényi negativity [E. Cornfeld...(2018)]

$$R_n(\alpha, \beta) = \mathrm{Tr}\left(\rho_A e^{i\alpha \hat{Q}_{A_1} - i\beta \hat{Q}_{A_2}^T}\right),$$
$$\mathcal{R}_n(q) = \int_{-\pi}^{\pi} \mathrm{d}\alpha \, e^{-i\left(q - \langle \hat{\mathcal{Q}} \rangle\right)\alpha} R_n(\alpha).$$

- The action of $2d$ free boson is given by

$$\mathcal{S} = \frac{1}{8\pi} \int \mathrm{d}^2 x \, \partial_\mu \varphi \partial^\mu \varphi,$$
$$\varphi \sim \varphi + 2\pi k R \qquad \text{where } k \in \mathbb{Z}.$$

- Compact boson is the CFT of Luttinger liquid. Compactification radius R is related to the Luttinger parameter K via $R = \sqrt{\frac{1}{2K}}$.
- Free compact boson has a global $U(1)$ symmetry due to the invariance under $\varphi \to \varphi + a$.

Results

- The charge imbalance resolved Rényi negativity for disjoint interval is

$$\mathcal{R}_n(q) = \frac{\sqrt{\pi n} R_n}{\sqrt{2K \ln\left(\tilde{\ell}_1 \tilde{\ell}_1 |1-x|\right)}} \exp\left\{ -\frac{\pi^2 n \left(q - \langle \hat{\mathcal{Q}} \rangle\right)^2}{2K ln\left(\tilde{\ell}_1 \tilde{\ell}_1 |1-x|\right)} \right\}.$$

- We see that the charge imbalance resolved Rényi negativity has Gaussian distribution in charge imbalance sectors.
- We also note that the Luttinger parameter K appears as an overall constant in the exponent.
- In the case of adjacent interval it is given by

$$\mathcal{R}_n(q) = \frac{\sqrt{\pi n} R_n}{\sqrt{2K \ln\left(\tilde{\ell}_1^{\,2} \tilde{\ell}_2^{\,2} / (\tilde{\ell}_1 + \tilde{\ell}_2)\right)}} \exp\left\{ -\frac{\pi^2 n \left(q - \langle \hat{\mathcal{Q}} \rangle\right)^2}{2K ln\left(\tilde{\ell}_1^{\,2} \tilde{\ell}_2^{\,2} / (\tilde{\ell}_1 + \tilde{\ell}_2)\right)} \right\}$$

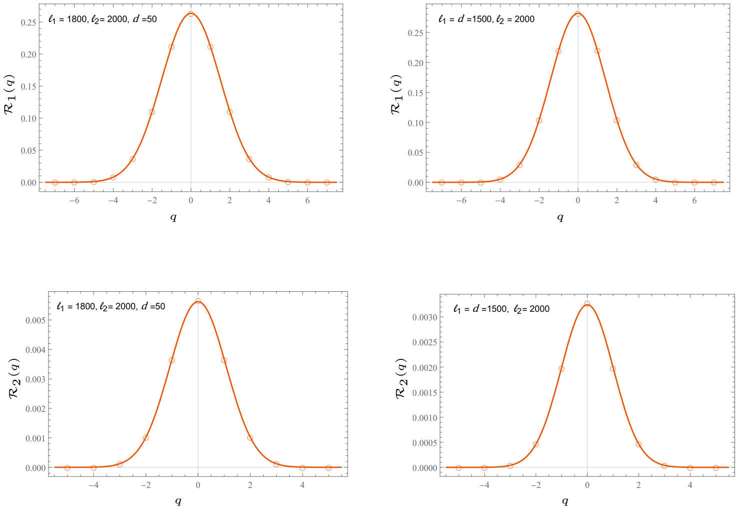

Figure: Plots for charged imbalance resolved Rényi negativity (top is for $n = 1$ and bottom is for $n = 2$). Solid lines are analytical results for compact free boson. Discrete plot is the numerical result for tight binding model.