Relaxation and Energy Transfer in the (Double) Quantum Sine-Gordon Model

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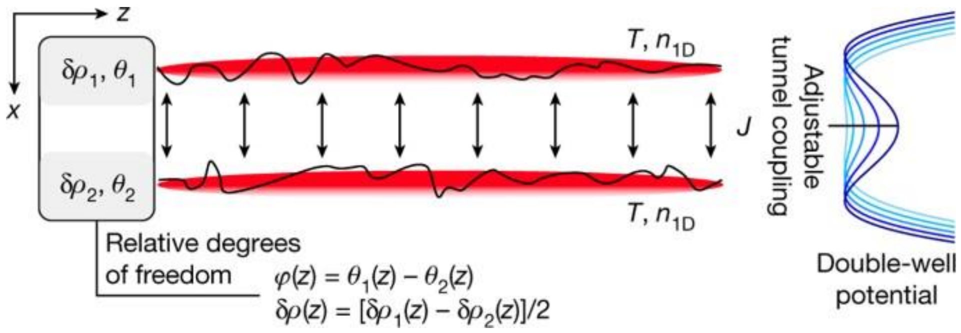
MOTIVATION

❖ EXPERIMENTAL SETUP:

two Josephson-coupled one-dimensional bosonic quasi-condensates[1]



❖ THEORY:

Bosonisation provides a **low-energy description**[2]:

$$\hat{H}_{\mathrm{sG}} = \int dx : \left(\frac{1}{2}(\partial_t \hat{\varphi})^2 + \frac{1}{2}(\partial_x \hat{\varphi})^2 - \lambda \cos \beta \hat{\varphi} \right) :$$

sine-Gordon model ≡ quantum pendulum coupled to a set of interacting, non-linear phononic modes

❖ Quantum pendulum: $\hat{H}_{\mathrm{QP}} = \frac{1}{2L}\hat{\pi}_0^2 - \lambda L \left(\frac{L}{2\pi}\right)^{2\Delta} \cos(\beta\hat{\varphi}_0)$

GOALS and QUESTIONS:

Simulating **the out-of-equilibrium dynamics** of the condensates in terms **of the sine-Gordon model** close to **the weakly interacting regime** available to the experiments

i. Range of validity of semi-classical methods

ii. Determining the relevant degrees of freedom in the experiments ⟶ importance of $k \neq 0$ modes:

- **Effects of the $k \neq 0$ modes on the** dynamics of the **quantum pendulum** (and the sine-Gordon model)
- How the **breakdown of integrability** affects the **energy transfer** between the zero-mode and non-zero modes

double sine-Gordon model: **non-integrable**

$$\hat{H}_{\mathrm{dsG}} = \int dx : \left(\frac{1}{2}(\partial_t \hat{\varphi})^2 + \frac{1}{2}(\partial_x \hat{\varphi})^2 - \lambda \cos \beta \hat{\varphi} - \alpha \cos 2\beta\hat{\varphi} \right) :$$

[1] T. Schweigler et. al., *Nature* **545** (2017) 323-326 [2] V. Gritsev et. al., *Phys. Rev. B* **75** (2007) 174511

METHODS USED

TRUNCATED WIGNER APPROACH

- SEMI-CLASSICAL METHOD
- NEEDS VALIDATION

Lattice regularisation of the sine-Gordon model:

$$\hat{H}_{\text{Lat}} = \frac{a}{2}\sum_{j=1}^{N}\left((\partial_t\hat{\varphi}_j)^2 + \frac{(\hat{\varphi}_j - \hat{\varphi}_{j-1})^2}{a^2}\right) - \frac{\lambda a}{N}\sum_{j=1}^{N}\cos\beta\hat{\varphi}_j$$

TWA: Dynamics is computed using Monte Carlo averaging of classical trajectories[3,4]

$$\underline{\varphi} = \{\varphi_j | j = 1, \ldots, N\}\,, \quad \underline{\pi} = \{\pi_j | j = 1, \ldots, N\}$$

quantum fluctuations are incorporated into the initial state described by the **Wigner quasi-probability distribution**:

$$W(\underline{\varphi}, \underline{\pi}) = \frac{1}{(2\pi)^{2N}}\int d\underline{\varphi}' \,\langle \underline{\varphi} - \underline{\varphi}'/2 | \hat{\rho} | \underline{\varphi} + \underline{\varphi}'/2\rangle \, e^{-i\underline{\varphi}'\cdot\underline{\pi}}$$

HOW GOOD IS THE TWA FOR DESCRIBING THE EXPERIMENTS?

MINI-SUPERSPACE BASED TRUNCATED HAMILTONIAN APPROACH

- CONTAINS ALL QUANTUM CORRELATIONS
- ACCURACY IS CONTROLLED

MSTHA: **Finite matrix representation of finite volume Hamiltonian** and observables on the **truncated** massless boson basis built upon the **quantum pendulum eigenstates**:

$$\mathcal{H}_{\text{FB}}^{\text{trun.}} = \text{span}\left\{ \prod_{k>0} a_{-k}^{r_k}\bar{a}_{-k}^{\bar{r}_k}\,|n\rangle \,\middle|\, n \leq n_{\max} \text{ and } \sum_{k>0} k(r_k + \bar{r}_k) \leq \ell_{\text{cut}} \right\}.$$

where $\hat{H}_{\text{QP}}\,|n\rangle = \varepsilon_n\,|n\rangle \quad n \in \mathbb{N}$

sine-Gordon Hamiltonian is a sum of products of **zero and non-zero mode matrices**:

$$\hat{H}_{\text{sG}} = \frac{1}{2}\int_0^L : \left[(\partial_t\hat{\varphi}_0)^2 + (\partial_t\hat{\tilde{\varphi}})^2 + (\partial_x\hat{\tilde{\varphi}})^2\right] : \; - \frac{\lambda}{2}\int_0^L dx : \left[e^{i\beta\hat{\varphi}_0}e^{i\beta\tilde{\varphi}} + e^{-i\beta\hat{\varphi}_0}e^{-i\beta\tilde{\varphi}}\right] :$$

[3] A. Polkovnikov, *Phys. Rev. A* **68** (2003) 053604
[4] A. Polkovnikov, *Ann. Phys.* **325** (2010) 1790-1852

ANSWERS

Strength of inter-mode **interactions** is the Luttinger parameter: $K = \frac{\pi}{\beta^2}$

Deep quantum $K \approx 1$.

K small: strong interactions

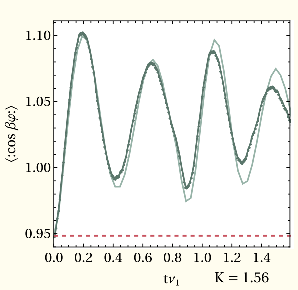
K large: weak interactions

Experiments: $K \approx 27$

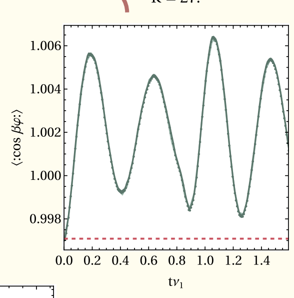
K

Weak quenches:

K small:
- ✅ **MSTHA numerically exact**
- ❌ **TWA breaks down**
- **Rapid energy transfer** for non-integrable case

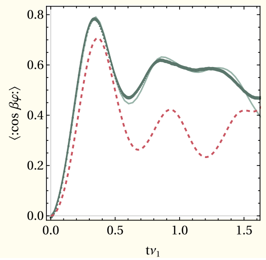


K large:
- ✅ **MSTHA numerically exact** even for $K = 27$
- ✅ **TWA follows MSTHA**
- **Slow energy transfer** is virtually unaffected by integrability breaking

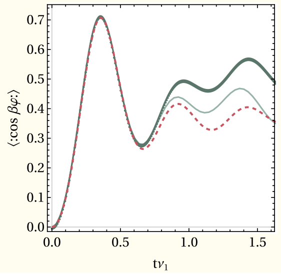


Strong quenches:

K small:
- ✅ **MSTHA numerically exact**
- ☑ **TWA improves** to a point



K large:
- ☑ **MSTHA improves,** but **fails to converge for large** K
- ✅ **TWA predicts the dynamics**



- **Mini-superspace treatment extends the validity of THA** closer to the weakly interacting regime available to the experiments[5].
- **TWA is shown to describe the dynamics in the experimental regime**.
- **TWA and THA: powerful complementary approaches**.
- large K: The $k \neq 0$ modes only affect the dynamics to a limited amount, (depending on the initial energy density)
- **Slow energy transfer indicates extremely weak coupling** of the zero and non-zero modes

[5] D. X. Horváth et. al., *Phys. Rev. A* **100** (2019) 013613