NEGATIVE TRIPARTITE INFORMATION AFTER QUANTUM QUENCHES IN INTEGRABLE MODELS

CFT & INTEGRABLE MODELS

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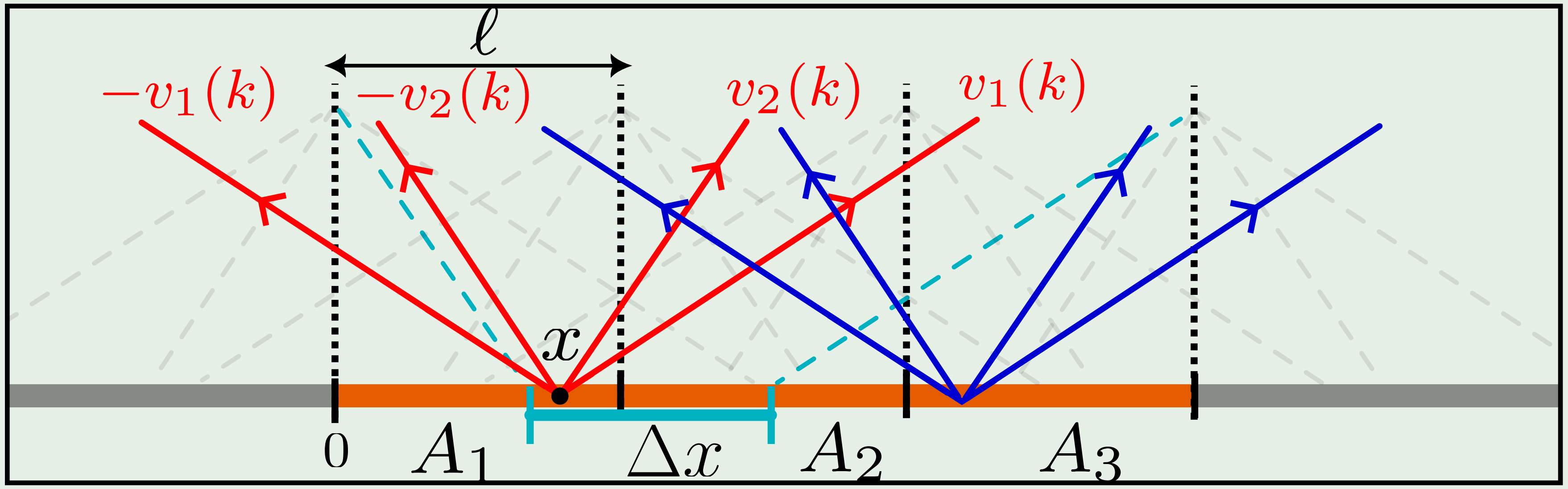


STATE VS TRAJECTORY ENTANGLEMENT

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

- Free fermions with local density monitoring: Stochastic Schroedinger Equation

$$d|\psi_t\rangle = -iHdt|\psi_t\rangle + \sum_{x=1}^{L} \left(\sqrt{\gamma}[n_x - \langle n_x\rangle_t]dW_t^{(x)} - \frac{\gamma}{2}[n_x - \langle n_x\rangle_t]^2 dt\right)|\psi_t\rangle$$

$dW_t^{(x)}$ noise

initial state $|01\rangle^{\otimes L/2}$

average density matrix $\rho_t = \overline{|\psi_t\rangle\langle\psi_t|}$

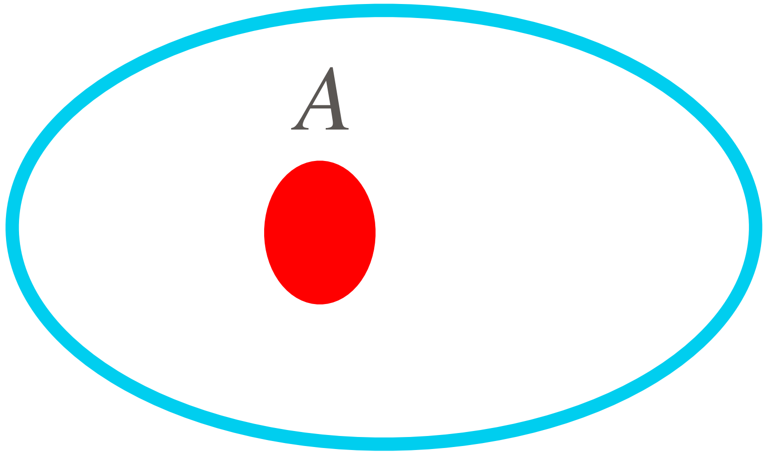
- Lindblad dynamics with dephasing

V.A. and F. Carollo, PRB 103 L020302 (2021) + …

$$d\overline{\rho_t} = -i[H, \overline{\rho_t}]dt - \gamma \sum_j [n_j, [n_j, \overline{\rho_t}]]dt$$

- Entanglement of trajectories:

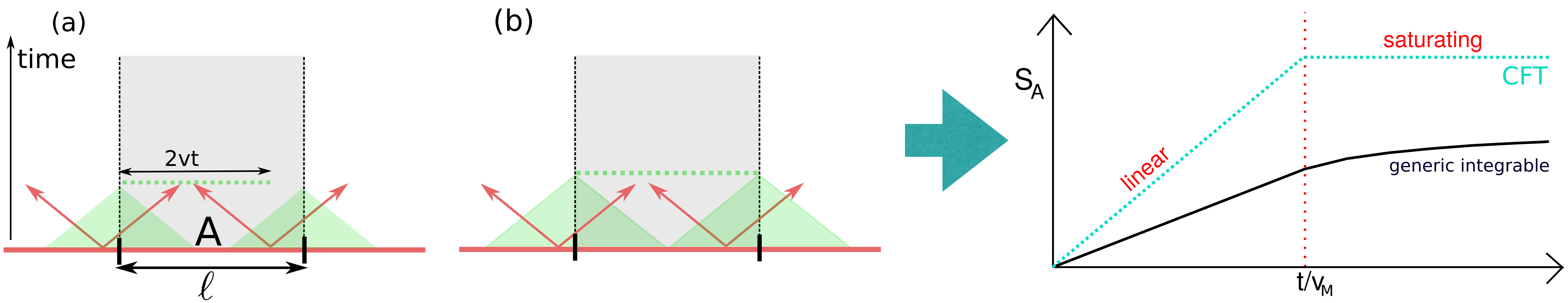
$$\overline{S_A} = \mathrm{Tr}_A \overline{\rho_{t,A} \ln(\rho_{t,A})}$$



QUASIPARTICLE PICTURE

- Hydrodynamic description of entanglement dynamics via emergent quasiparticles

P. Calabrese and J. Cardy, JSTAT 2005



M. Fagotti and P. Calabrese, Phys. Rev. A **78**, 010306(R) (2008)

V.A.,P. Calabrese, PNAS,114, 7947 (2017) + a lot more

Universality from thermodynamics

$$S_A = \sum_k \left[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \right]$$

scaling limit

$$t, \ell \to \infty, t/\ell \text{ fixed}$$

s_k GGE thermodynamic entropy

v_k Entangling quasiparticles are the excitations over the GGE

COLLAPSED QUASIPARTICLE ANSATZ

- Assuming pairwise correlations: Collapsed Quasiparticle Ansatz

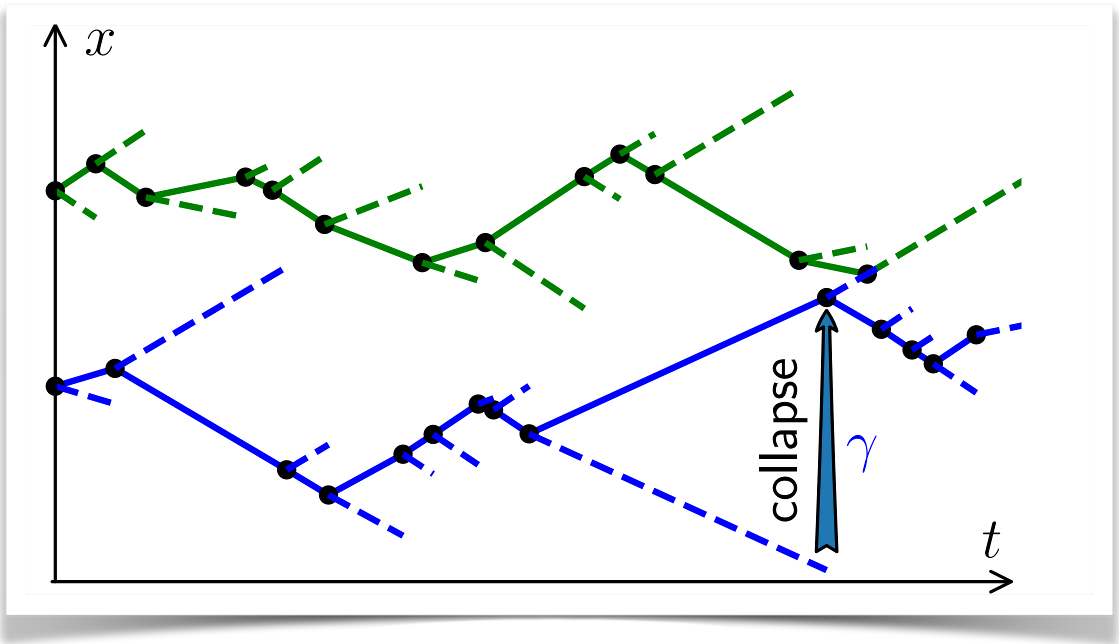
X. Cao, A. Tilloy, and A. De Luca, SciPost Phys. 7, 024 (2019)

Entanglement "decay"

Entanglement "production"

$$\overline{S}_\ell^{(\alpha)}(t) = e^{-\gamma t} S^{(\alpha),0}(t) + \gamma \int_0^t du\, e^{-\gamma u} S^{(\alpha),0}(u)$$

$$S_\ell^{(\alpha),0} = \frac{\ell}{2} \int_{-\pi}^{\pi} \frac{dq}{2\pi} s_q^{(\alpha)} \min(|v_q| t/\ell, 1)$$



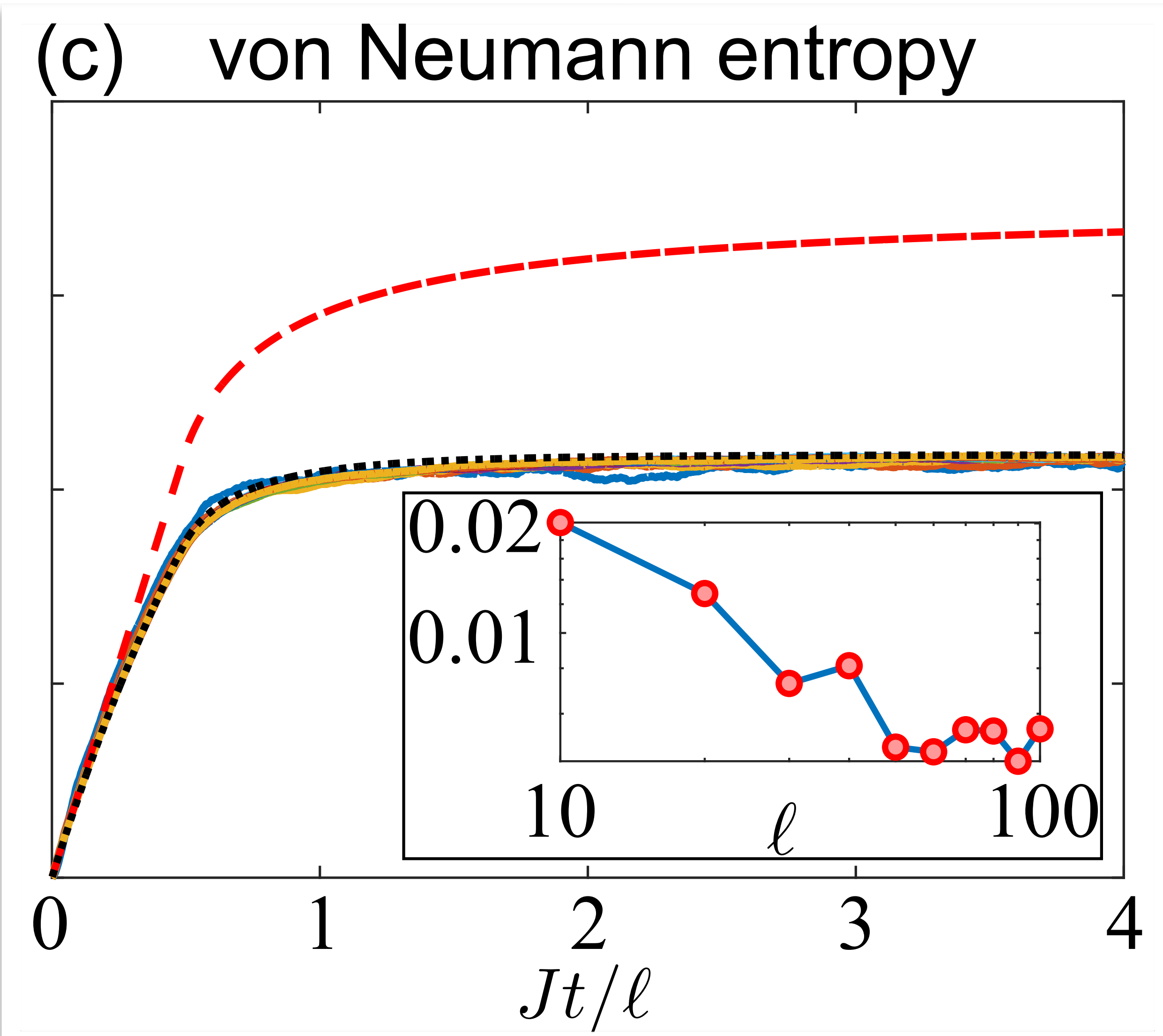
- Measurement induced entanglement production

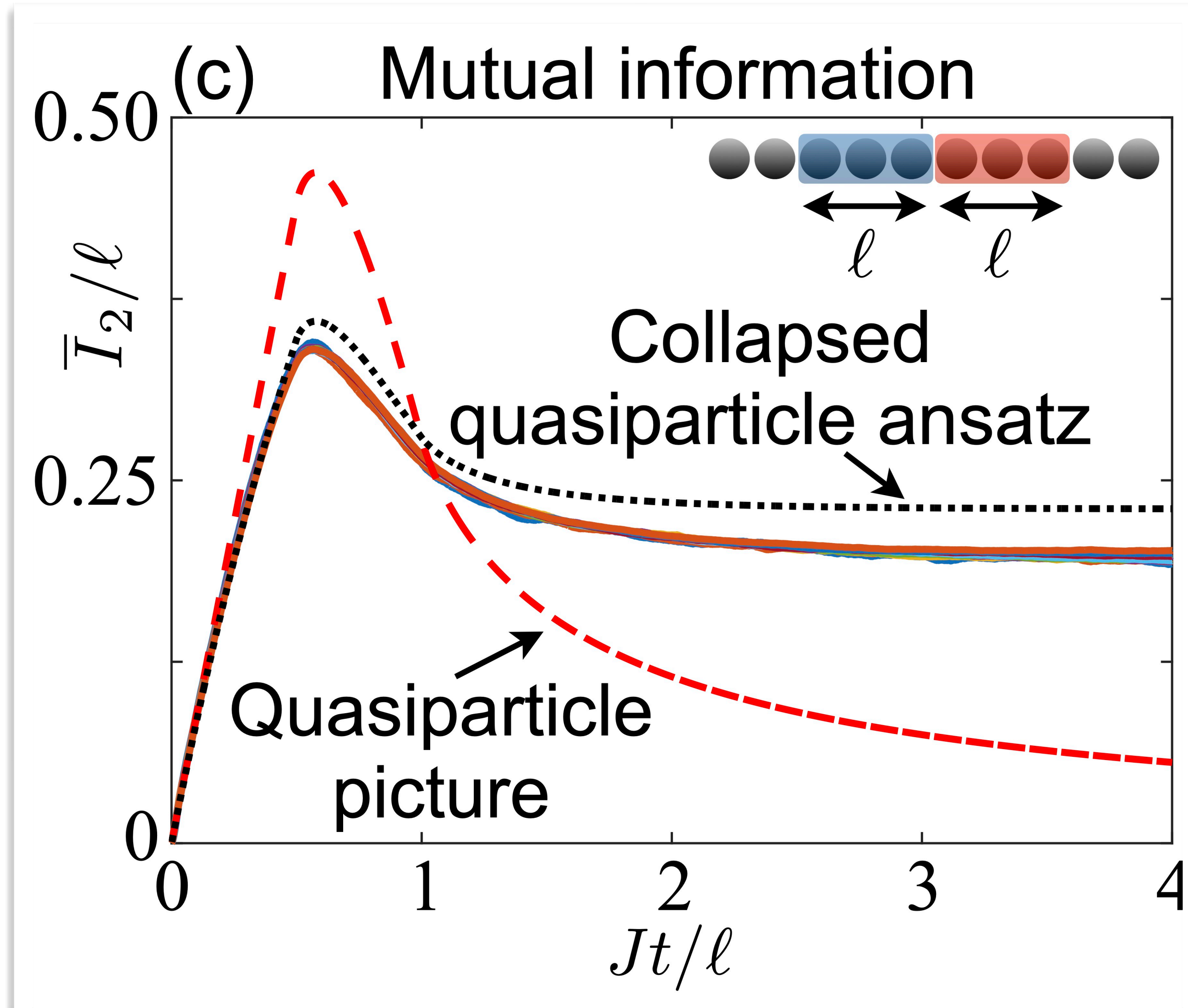
NUMERICAL CHECKS: BEYOND PAIR CORRELATIONS

$|01\rangle^{\otimes L/2}$

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

F. Carollo and V. A.,PRB 106 L220304 (2022)

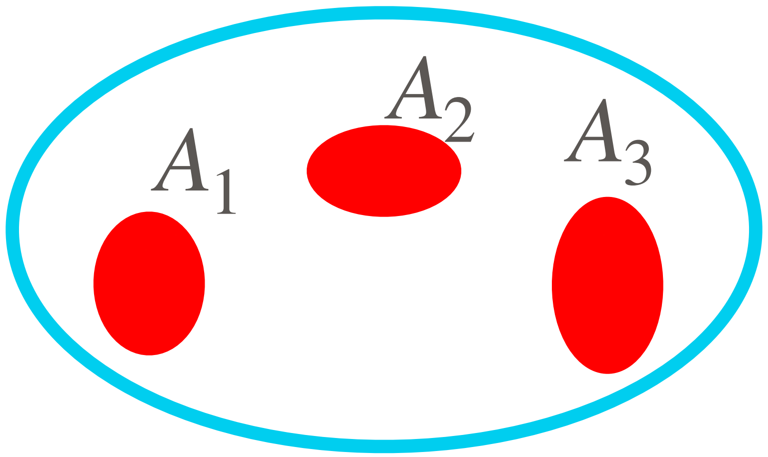




- Incomplete description of entanglement dynamics via entangled pairs

TRIPARTITE MUTUAL INFORMATION

- Quantum correlations between three subsystems



$$S_A = -\,\mathrm{Tr}\rho_A \ln(\rho_A)$$ Von Neumann entropy

$$I_2(A,B) = S(A) + S(B) - S(A \cup B)$$

Kitaev and Preskill, PRL 96 110404 (2006), Levin and Wen, PRL 96 110405 (2006)

aka *topological entropy*

$$I_3(A_1, A_2, A_3) = I_2(A_2, A_1) + I_2(A_2, A_3) - I_2(A_2, A_1 \cup A_3)$$

- Holographic theories → $I_3 < 0$

P. Hayden, M. Headrick, and A. Maloney, PRD 87, 046003 (2013)

V. Balasubramanian, A. Bernamonti, N. Copland, B. Craps, F. Galli, PRD 84 105017 (2011)

- Quantum information scrambling

Random circuits $I_3 < 0$

B. Bertini and L. Piroli, PRB 102 064305 (2020)

- TMI & chaotic dynamics

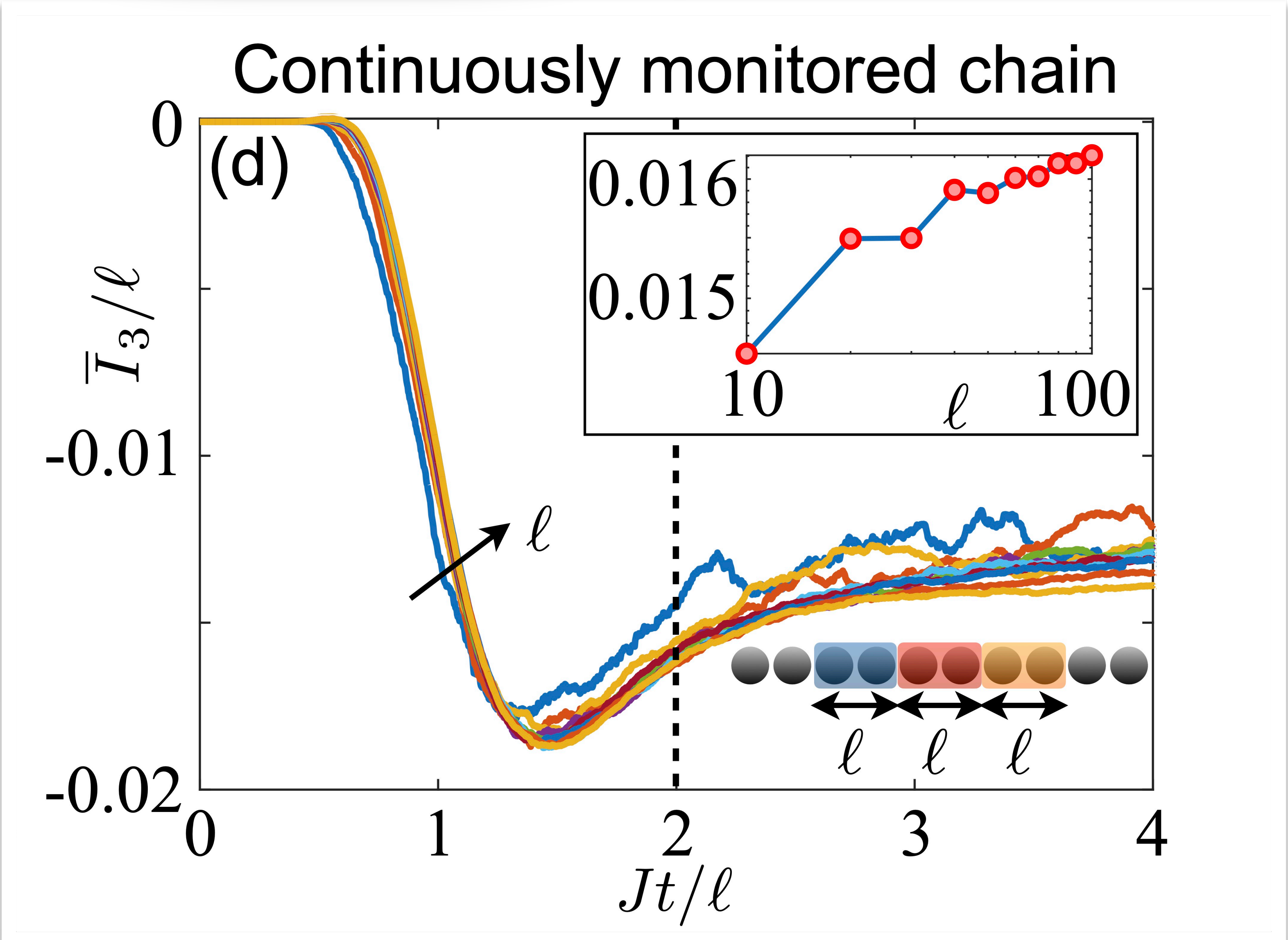
A. Zabalo, M. Gullans, J. Wilson, S. Gopalakrishnan, D. Huse, J. Pixley, PRB 101 060301 (2020)

NUMERICAL CHECKS: BEYOND PAIR CORRELATIONS

F. Carollo and V. A.,PRB 106 L220304 (2022)

$$I_3 < 0$$

Tripartite information



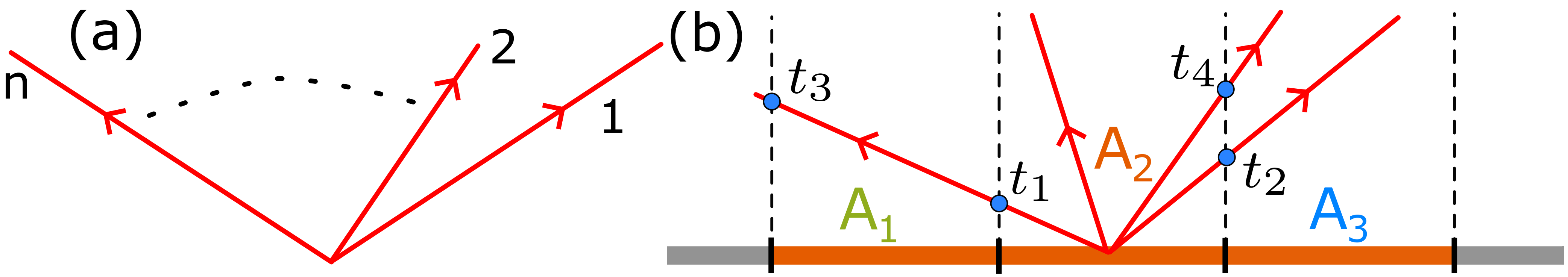
What is going on?

How do we get nonzero TMI?

Can we get negative TMI in integrable systems?

ENTANGLED MULTIPLETS

- TMI is zero if entangled pairs are present



- Generic quench → much more subtle entanglement pattern

$I_3 < 0$ → information is highly delocalised

- Shared multiplets contribute to the TMI

CLASSICALLY ENTANGLED MULTIPLETS

- Different classes of entangled multiplets

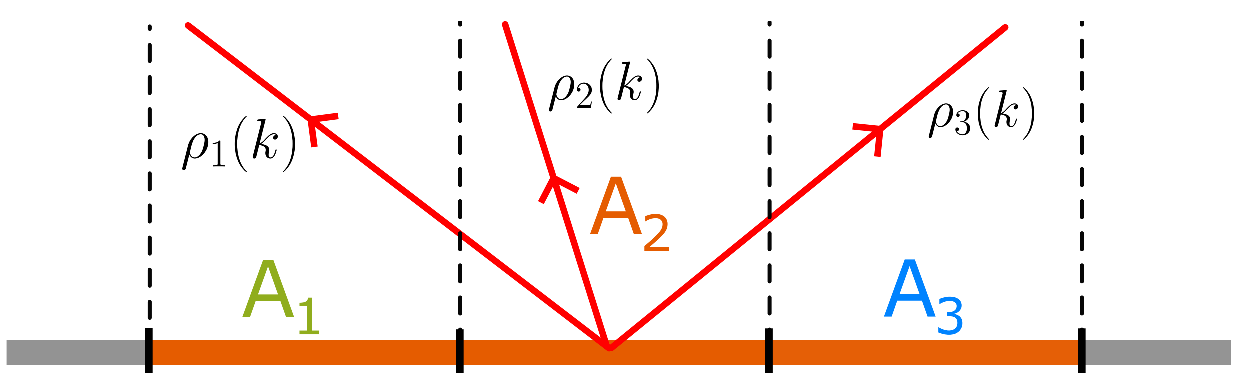
$$H = \sum_{j=1}^{L} \frac{1}{2}(S_j^+ S_{j+1}^- + h.c.)$$

- Classically entangled multiplets

B. Bertini et al., J. Stat. Mech. (2018) 063104

$|\uparrow\downarrow\downarrow\rangle^{\otimes L/3}$ $\quad$ $|\uparrow\downarrow\downarrow\downarrow\rangle^{\otimes L/4}$ $\quad$ ● ● ●

$n = 3$ $\quad$ $n = 4$



$$\sum_j \rho_j = 1 \qquad \rho_{in}(k) = \sum_{j\in A} \rho_j(k)$$

$$s(k) = s_{YY}(\rho_{in})$$

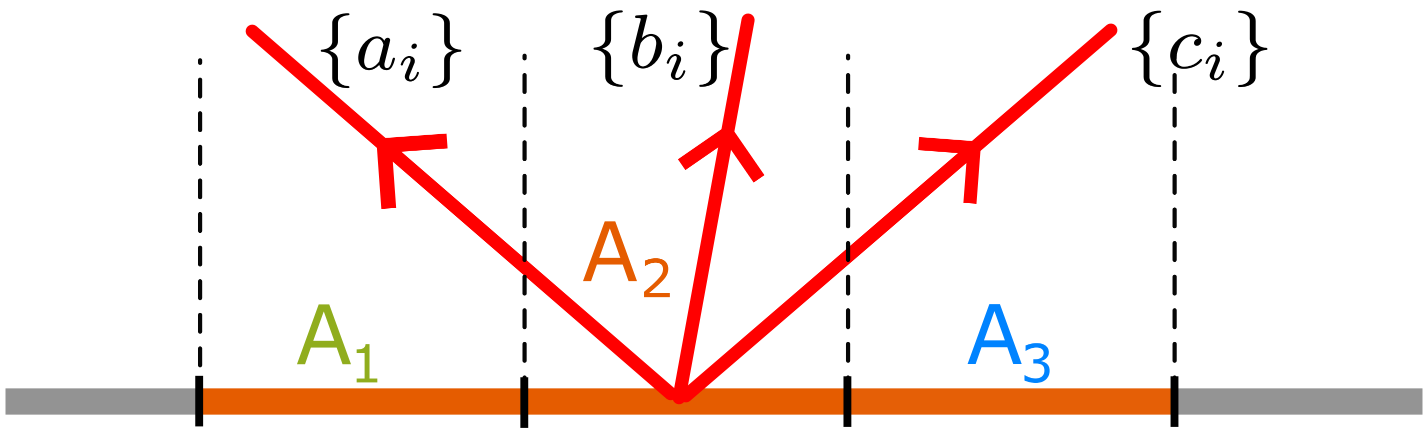
$$s_{YY}(x) = -x\ln(x) - (1-x)\ln(1-x)$$

F. Caceffo and V.A., arXiv:2305.10245

- Entanglement dynamics determined by the GGE

$$I_3 > 0$$

POSITIVE TMI FOR CLASSICALLY ENTANGLED MULTIPLETS



$$I_3(t) = \int \frac{dk}{2\pi} \tau_3(k) \mathscr{D}(k, \ell, t))$$

- For n particles multiplets:

$$\tau_3(k) = s_{\{a_i\}\cup\{b_i\}\cup\{c_i\}}(k) - s_{\{a_i\}\cup\{b_i\}}(k) - s_{\{a_i\}\cup\{c_i\}}(k) - s_{\{b_i\}\cup\{c_i\}}(k) + s_{\{a_i\}}(k) + s_{\{b_i\}}(k) + s_{\{c_i\}}(k)$$

$$a = \sum_{j=1}^{p} \rho_{a_j}(k) \qquad b = \sum_{j=1}^{q} \rho_{b_j}(k) \qquad c = \sum_{j=1}^{r} \rho_{c_j}(k)$$

$$p + q + r \leq n$$

$$f(x) := -x\ln(x) - (1-x)\ln(1-x)$$

$$\tau_3(k) = f(a+b+c) - f(a+b) - f(a+c) - f(b+c) + f(a) + f(b) + f(c)$$

- $\tau_3(k)$ vanishes at boundaries and is positive at the unique stationary point.

$$\tau_3(k) \geq 0$$

NEGATIVE TMI: QUASIPARTICLE PICTURE

- A more complex entanglement pattern

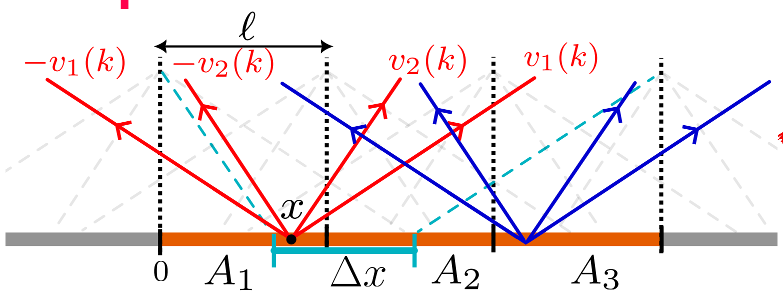
F. Caceffo and V.A., arXiv:2305.10245

$$H = \sum_{j=1}^{L} \frac{1}{2}(S_j^+ S_{j+1}^- + h.c.)$$

$$I_3(A_1, A_2, A_3) = I_2(A_2, A_1) + I_2(A_2, A_3) - I_2(A_2, A_1 \cup A_3)$$

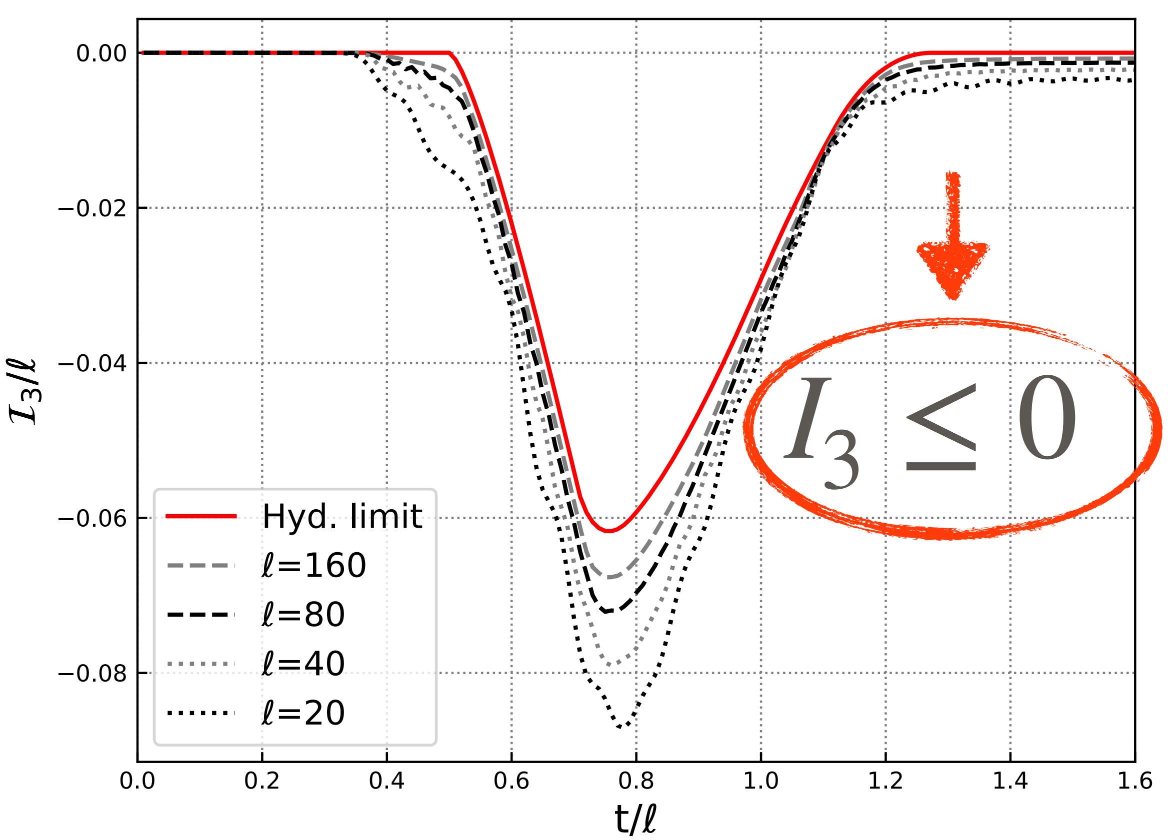
Entangled quadruplets

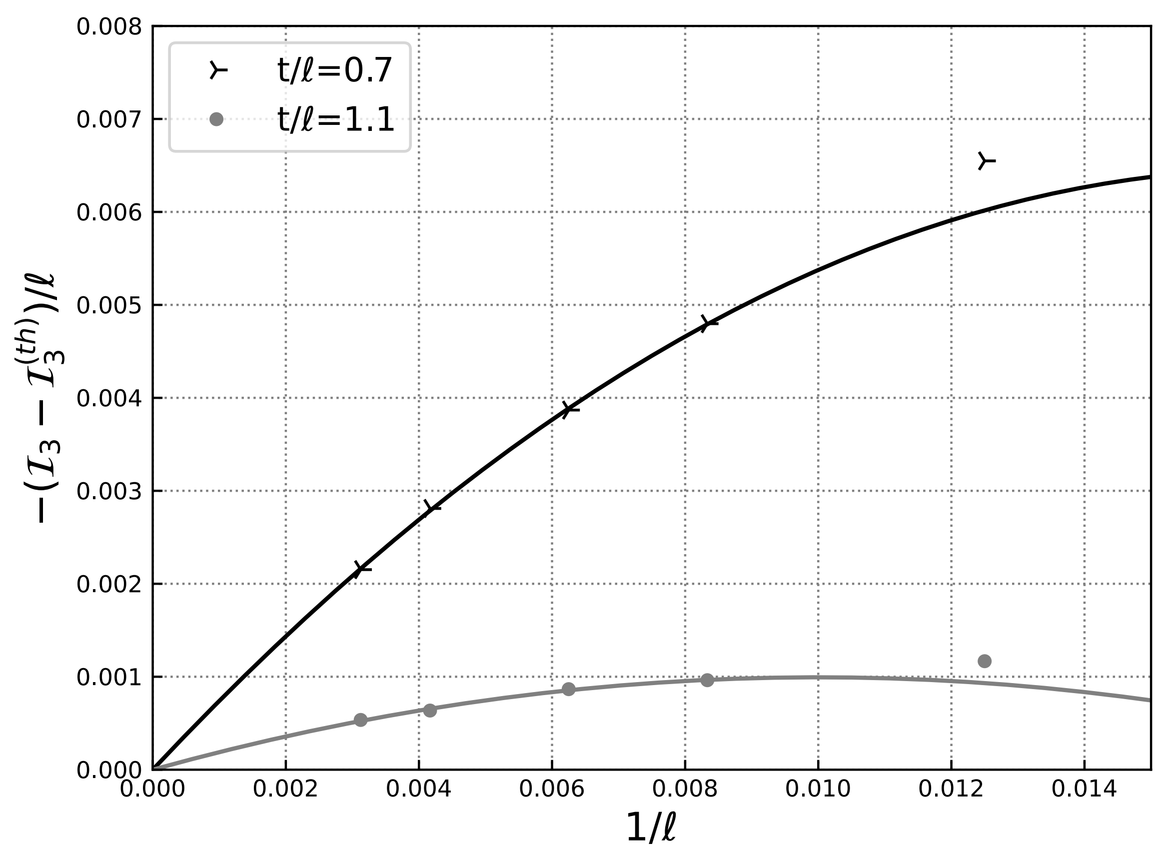
$$| \uparrow \uparrow \downarrow \downarrow \rangle^{\otimes L/4}$$



$$I_3(t) = \int_{\pi/2}^{3\pi/4} \frac{dk}{2\pi} \tau_3(k)(\mathcal{D}_1(k, \ell, t) + \mathcal{D}_2(k, \ell, t))$$

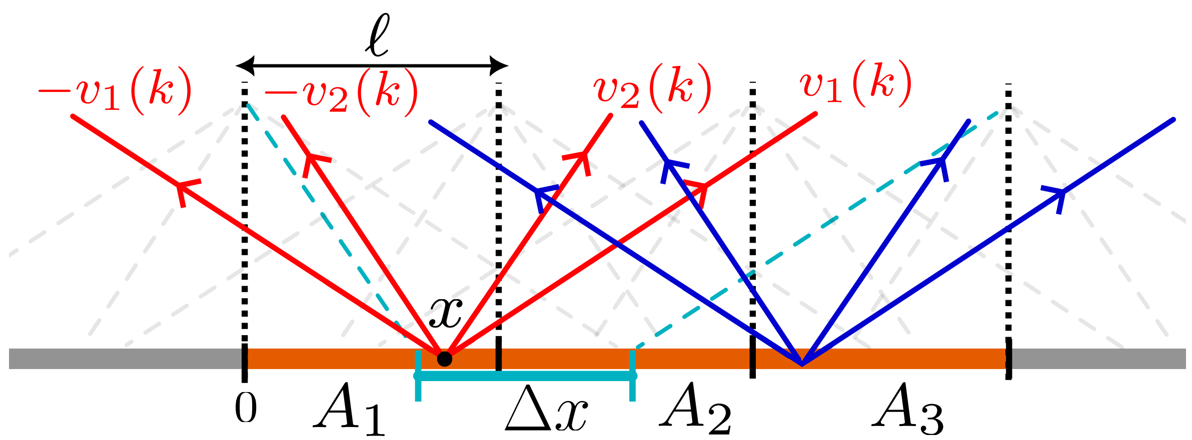
not easily related to GGE





TMI: QUASIPARTICLE PICTURE

F. Caceffo and V.A., arXiv:2305.10245



$$I_3(t) = \int \frac{dk}{2\pi} \tau_3(k) \mathscr{D}(k, \ell, t))$$

$$\mathscr{D}_1(k, \ell, t) = \max\{\min\{v_1 t, \ell + v_2 t, 2\ell - v_2 t, 3\ell - v_1 t\} - \max\{v_2 t, \ell - v_2 t, 2\ell - v_1 t\}, 0\}$$

$$\mathscr{D}_2(k, \ell, t) = \max\{\min\{\ell + v_1 t, 2\ell + v_2 t, 3\ell - v_2 t\} - \max\{v_1 t, \ell + v_2 t, 2\ell - v_2 t, 3\ell - v_1 t\}, 0\}$$

$$v_j(k) = 2\sin(k - (j-1)\pi/2)$$

- Fermionic correlation matrix in multiplet space

$$G(k) = \frac{1}{4}\begin{pmatrix} 2 & -1-i & 0 & -1+i \\ -1+i & 2 & -1-i & 0 \\ 0 & -1+i & 2 & -1-i \\ -1-i & 0 & -1+i & 2 \end{pmatrix}$$

A FORMULA TO REMEMBER

F. Caceffo and V.A., arXiv:2305.10245

$$S_A(t) = \int_{\pi/2}^{3/4\pi} \frac{dk}{2\pi} \Big\{ (s_{\{1\}} + s_{\{3\}}) \Big[(v_1 - v_2)t \; \Theta(\ell - (v_1 - v_2)t) +$$

$$\ell \Theta((v_1 - v_2)t - \ell) + (v_4 - v_3)t \; \Theta(\ell - (v_1 - v_3)t) + (\ell - (v_1 - v_4)t)\chi\big(\ell/(v_1 t - v_3 t), \ell/(v_1 t - v_4 t)\big) \Big] +$$

$$(s_{\{2\}} + s_{\{4\}}) \Big[((v_1 - v_4)t - \ell)\chi\big(\ell/(v_1 t - v_4 t), \min\{\ell/(v_1 t - v_2 t), \ell/(v_2 t - v_4 t)\}\big) +$$

$$(v_1 - v_2)t\chi\big(\ell/(v_2 t - v_4 t), \ell/(v_1 t - v_2 t)\big) + (v_2 - v_4)t\chi\big(\ell/(v_1 t - v_2 t), \ell/(v_2 t - v_4 t)\big) +$$

$$\ell \Theta\Big(t - \max\big\{\ell/(v_1 t - v_2 t), \ell/(v_2 t - v_4 t)\big\}\Big) \Big] + (s_{\{1,2\}} + s_{\{3,4\}}) \Big[(v_2 - v_4)t\Theta(\ell - (v_1 - v_4)t) + (\ell - (v_1 - v_2)t) +$$

$$\chi\big(\ell/(v_1 t - v_4 t), \ell/(v_1 t - v_2 t)\big) \Big] + s_{\{1,3\}} \Big[((v_1 - v_3)t - \ell)\chi\big(\ell/(v_1 t - v_3 t), \ell/(v_1 t - v_3 t)\big) +$$

$$(\ell - (v_2 - v_4)t)\chi\big(\ell/(v_1 t - v_4 t), \ell/(v_2 t - v_4 t)\big) \Big] + \int_{3/4\pi}^{\pi} \frac{dk}{2\pi} \{1 \leftrightarrow 2,3 \leftrightarrow 4\}$$

ISING QUENCH

- A more complex entanglement pattern

F. Caceffo and V.A., arXiv:2305.10245

$$H = J\sum_{j=1}^{L}\sigma_j^x\sigma_{j+1}^x + \sum_{j=1}^{L} h_j\sigma_j^z$$

$$I_3(A_1,A_2,A_3) = I_2(A_2,A_1) + I_2(A_2,A_3) - I_2(A_2,A_1\cup A_3)$$

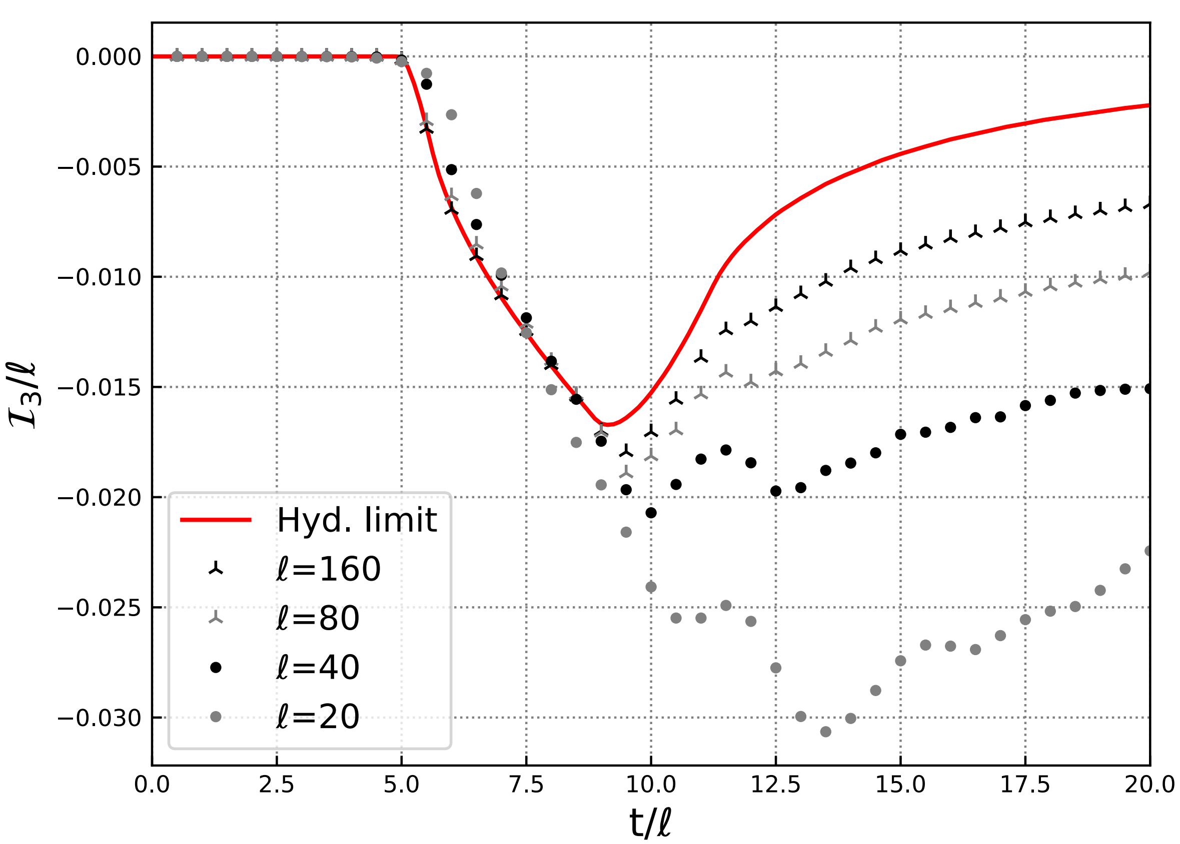
Entangled quadruplets

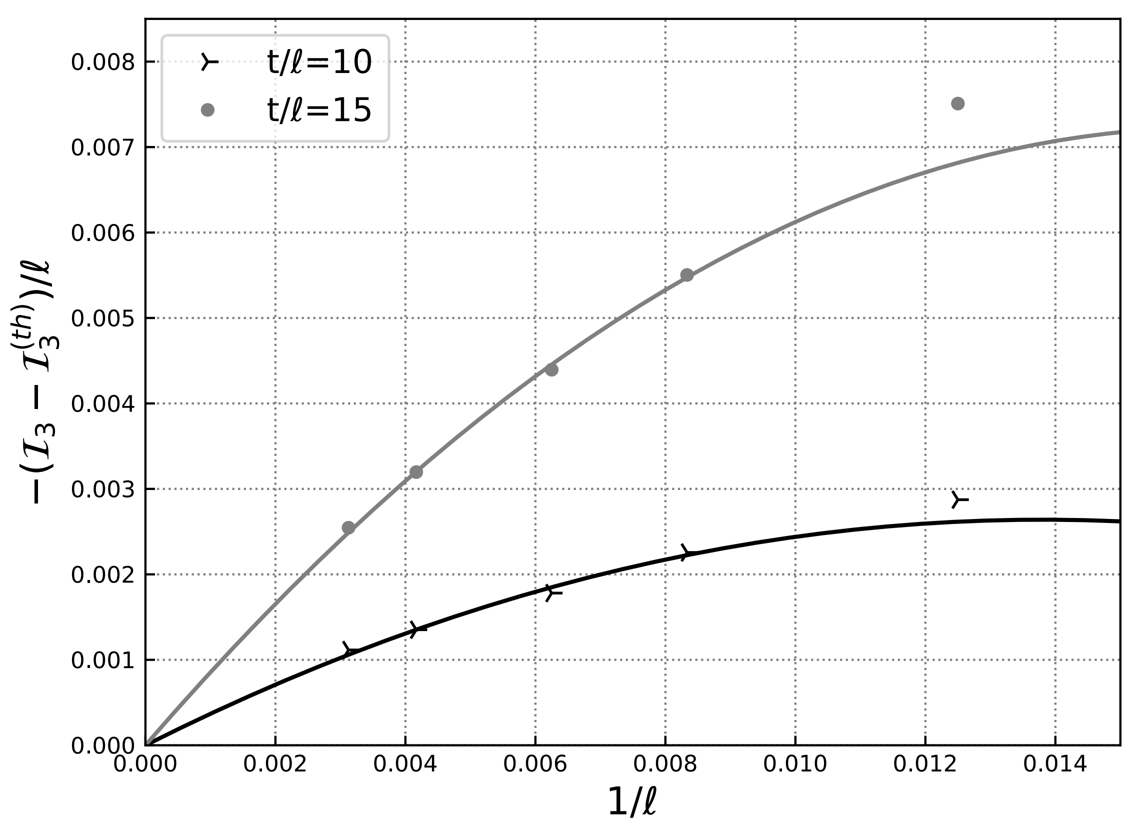
A. Bastianello and P. Calabrese, Scipost Phys. 5 033 (2018)

$$I_3(t) = \int_{\pi/2}^{3\pi/4}\frac{dk}{2\pi}\tau_3(k)(\mathcal{D}_1(k,\ell,t)+\mathcal{D}_2(k,\ell,t))$$

$$|GS(h_{even},h_{odd})\rangle$$

not easily related to GGE





CONCLUSIONS

- TMI ideal to identify the presence of entangled multiplets
- Quasiparticle picture still possible for free systems
- Link between entanglement and thermodynamics weakened
- Multiplet scrambling
- Most likely QP for monitored fermions hard to obtain

Postdoc positions available at University of Pisa!

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THANK YOU