Lax matrices for Baxter Q-operators

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Based on works with V. Bazhanov, G. Ferrando, V. Kazakov, T. Lukowski, C. Meneghelli, M. Staudacher, I. Szecsenyi and on more mathematical works with I. Karpov, V. Pestun, A. Tsymbaliuk

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Overview

Content

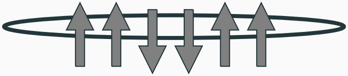
1. Introduction (Q-operators for XXX Heisenberg chain)
2. Lax matrices for A-type Q-operators
3. Extension to BCD-type and $osp(N|2m)$
4. Oscillator-type Lax matrices and the shifted Yangian
5. Outlook

Introduction

Heisenberg XXX spin chain

Prime example of a rational spin chain:

Closed XXX Heisenberg chain



$$H_{\text{closed}} = H_{\text{bulk}} + (1 - \vec{\sigma}_N \cdot \vec{\sigma}_1)$$

- Nearest neighbor bulk Hamiltonian
  $H_{\text{bulk}} = \sum_{i=1}^{N-1}(1 - \vec{\sigma}_i \cdot \vec{\sigma}_{i+1})$
- Hamiltonian is a $2^N \times 2^N$ matrix
- Solve eigenvalue problem $H_{\text{closed}} \cdot |\psi_m\rangle = E_m|\psi_m\rangle$
- Example: $m = 4$ and $|\Omega\rangle = |\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle = \binom{0}{1} \otimes \ldots \otimes \binom{0}{1}$
- Solution by Hans Bethe in 1931 (CBA)
- Here algebraic Bethe ansatz (ABA) [Faddeev et al.]

Quantum inverse scattering method

Starting point: Yang-Baxter equation

$$R_{12}(z_1 - z_2)R_{13}(z_1 - z_3)R_{23}(z_2 - z_3) = R_{23}(z_2 - z_3)R_{13}(z_1 - z_3)R_{12}(z_1 - z_2)$$

- Fundamental relation underlying integrable systems
- Each R-matrix R_{ij} acts on the tensor product of three spaces $V_1 \otimes V_2 \otimes V_3$ with

$$R_{12}(z) = R(z) \otimes I, \ldots$$

- Fundamental R-matrix for the XXX Heisenberg spin chain

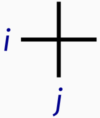
$$R(z) = z + P \quad \text{with} \quad P = \sum_{a,b=1}^{2} e_{ab} \otimes e_{ba}$$

where $(e_{ab})_{cd} = \delta_{ac}\delta_{bd}$, $z \in \mathbb{C}$ and P acts as a permutation

Graphical notation

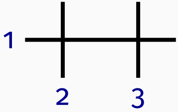
- R-matrix:

$$R_{ij}(z_i - z_j) =$$

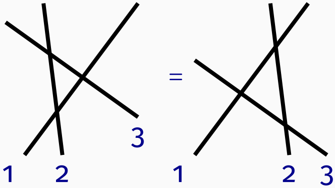


- Multiplication of R-matrices:

$$R_{12}(z_1 - z_2)R_{13}(z_1 - z_3) =$$



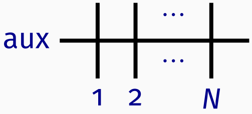
- Yang-Baxter equation:



Spin chain monodromy

Spin chain monodromy

$$\mathcal{M}(z) = \mathcal{L}_1(z)\mathcal{L}_2(z)\cdots\mathcal{L}_N(z) =$$

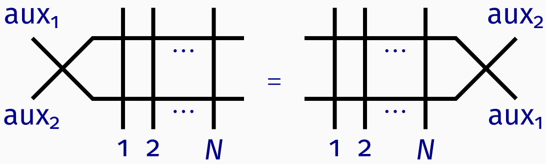


Lax operator: $\mathcal{L}_i(z) \equiv R_{\text{aux},i}(z)$

- Multiplication of 2×2 matrices in auxiliary space and tensor product in quantum space
- Satisfies RTT-relation

$$R(z_1 - z_2)(\mathcal{M}(z_1) \otimes \mathbb{I})(\mathbb{I} \otimes \mathcal{M}(z_2)) = (\mathbb{I} \otimes \mathcal{M}(z_2))(\mathcal{M}(z_1) \otimes \mathbb{I})R(z_1 - z_2)$$

- Pictorially



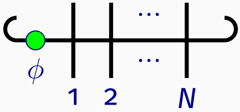
Transfer matrix

Transfer matrix with twisted boundary conditions

$$T(z) = \mathrm{tr}_{aux}\, \mathcal{D}(\phi)\mathcal{M}(z), \qquad \mathcal{D}(\phi) = \begin{pmatrix} e^{+i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$$

Breaks global $\mathfrak{gl}(2)$ symmetry → regulate Bethe roots

$T(z) =$



Commuting family of operators (common eigenstates)

$$[T(z), T(z')] = 0, \qquad [T(z), H^{\phi}_{\text{closed}}] = 0$$

- Related to untwisted Hamiltonian by $\vec{\sigma}_N \cdot \vec{\sigma}_1 \to \mathcal{D}_N \vec{\sigma}_N \mathcal{D}_N^{-1} \cdot \vec{\sigma}_1$
- Untwisted eigensystem recovered in the limit $\phi \to 0$

Algebraic Bethe ansatz for $\mathfrak{gl}(2)$

For $\mathfrak{gl}(2)$ the monodromy $\mathcal{M}(z)$ is a 2×2 matrix in the auxiliary space

$$\mathcal{M}(z) = \begin{pmatrix} A(z) & B(z) \\ C(z) & D(z) \end{pmatrix} \qquad T(z) = e^{+i\phi}A(z) + e^{-i\phi}D(z)$$

Define reference state $|\Omega\rangle$ such that

$$A(z)|\Omega\rangle = \alpha(z)|\Omega\rangle \qquad D(z)|\Omega\rangle = \delta(z)|\Omega\rangle \qquad C(z)|\Omega\rangle = 0$$

Ansatz for the wave function

- Off-shell Bethe vectors $|\psi_m\rangle = B(z_1)\cdots B(z_m)|\Omega\rangle$
- Each B-operator creates an excitation on the chain

Fundamental commutation relations

Act with transfer matrix $T(z) = e^{+i\phi}A(z) + e^{-i\phi}D(z)$ on off-shell vector $|\psi_m\rangle = B(z_1)\cdots B(z_m)|\Omega\rangle$

- Fundamental commutation relations from RTT-relation

$$[B(z_1), B(z_2)] = 0$$

$$A(x)B(y) = f(y,x)B(y)A(x) - g(y,x)B(x)A(y)$$

$$D(x)B(y) = f(x,y)B(y)D(x) - g(x,y)B(x)D(y)$$

with $f(x,y) = \frac{1+x-y}{x-y}$ and $g(x,y) = \frac{1}{x-y}$

- One finds $T(z)|\psi_m\rangle = t(z)|\psi_m\rangle$ + unwanted terms

The Baxter equation

Eigenvalues of $T(z)$ given by the Baxter equation

$$t(z) = (z+1)^N \frac{Q_\pm(z-1)}{Q_\pm(z)} + z^N \frac{Q_\pm(z+1)}{Q_\pm(z)}$$

- Baxters Q-function $Q_\pm(z) = e^{\pm iz\phi} \prod_{i=1}^{m_\pm}(z - z_i^\pm)$ ↪Q-operators
- Bethe roots $z_k^\pm$ for $k = 1, \ldots, m$ satisfy Bethe equations

$$\left(\frac{z_k^\pm + 1}{z_k^\pm}\right)^N = e^{\pm 2i\phi} \prod_{\substack{j=1 \\ j \neq k}}^{m_\pm} \frac{z_k^\pm - z_j^\pm + 1}{z_k^\pm - z_j^\pm - 1}$$

- $Q_\pm$ depending on choice of vaccuum

$$|\Omega\rangle = |\downarrow\downarrow \cdots \downarrow\rangle \text{ vs. } |\bar{\Omega}\rangle = |\uparrow\uparrow \cdots \uparrow\rangle$$

- QQ-relations

$$Q_+(z+\frac{1}{2})Q_-(z-\frac{1}{2}) - Q_+(z-\frac{1}{2})Q_-(z+\frac{1}{2}) = \left(z+\frac{1}{2}\right)^N$$

Alternative to Bethe equations [Pronko,Stroganov]

Q-operator construction for XXX

Oscillator type solutions

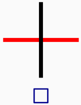
Employ degenerate solutions of the Yang-Baxter equation

$$R(x-y)\,(L_\pm(x)\otimes\mathbb{I})\,(\mathbb{I}\otimes L_\pm(y)) = (\mathbb{I}\otimes L_\pm(y))\,(L_\pm(x)\otimes\mathbb{I})\,R(x-y)$$

given by [Izergin,Korepin ’84; BLZ ’96; Antonov,Feigin ’96; Rossi, Weston ’02; Korff ’04; Bazhanov et al ’10]

$$L_+(z) = \begin{pmatrix} z-\bar{\mathbf{a}}\mathbf{a} & \bar{\mathbf{a}} \\ -\mathbf{a} & 1 \end{pmatrix} \qquad L_-(z) = \begin{pmatrix} 1 & \bar{\mathbf{a}} \\ \mathbf{a} & z+\mathbf{a}\bar{\mathbf{a}} \end{pmatrix},$$

- $V_1 = V_2 = \mathbb{C}^2$ and $V_3 = \textit{oscillator}$
- $L_\pm$ is a 2×2-matrix with operatorial entries $[\mathbf{a},\bar{\mathbf{a}}]=1$
- Diagrammatic expression

$L_\pm(z)$ = osc 

Red: oscillator space (auxiliary space)
Black: $\mathfrak{gl}(2)$ space (quantum space)

Q-operator construction for XXX spin chain

Define Q-operators as [Bazhanov, Łukowski, Meneghelli, Staudacher '10]

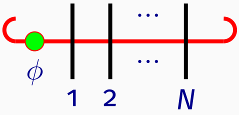
$$Q_{\pm}(z) = e^{\pm iz\phi}\, Z_{\pm}^{-1}\, \mathrm{tr}_{osc}\, \mathcal{D}_{\pm}\, M_{\pm}(z)$$

with the monodromy

$$M_{\pm}(z) = L_{\pm}(z) \otimes \ldots \otimes L_{\pm}(z)$$

- $\mathcal{D}_{\pm} = e^{\mp 2i\phi \bar{\mathbf{a}}\mathbf{a}}$ depends on twist field ϕ and regulates the trace over the infinite-dimensional oscillator space
- Normalization $Z_{\pm} = \mathrm{tr}_{osc}\, \mathcal{D}_{\pm}$

Diagrammatic form of the Q-operators

$Q_{\pm}(z) =$ 

Weyl permutations

$Q_\pm$ can be generated from $Q_\mp$ using Weyl permutations

Define

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

such that

$$L_+(z) = \sigma_x L_-(z)\sigma_x^{-1}|_{ph}$$

with the particle hole transformation $\mathbf{a} \to \bar{\mathbf{a}}$ and $\bar{\mathbf{a}} \to -\mathbf{a}$.

The Q-operators are related via

$$Q_+(z) = (\sigma_x \otimes \ldots \otimes \sigma_x)Q_-(z)(\sigma_x^{-1} \otimes \ldots \otimes \sigma_x^{-1})|_{\phi\to-\phi}$$

→ Distinguished Lax/Q-operators (L_+,Q_+)

Factorisation and prove of QQ-relations

Lax matrices satisfy remarkable factorisation formula

$$L_+^{[1]}(x+\lambda_1)L_-^{[2]}(x+\lambda_2-1) = S\mathfrak{L}_+^{[1]}(x)B^{[2]}S^{-1}$$

for two sets of oscillators $[1]$ and $[2]$. Here

$$\mathfrak{L}_+^{[1]}(x) = \begin{pmatrix} x+\lambda_1-\bar{\mathbf{a}}^{[1]}\mathbf{a}^{[1]} & -\bar{\mathbf{a}}^{[1]}(\lambda_1-\lambda_2-\bar{\mathbf{a}}^{[1]}\mathbf{a}^{[1]}) \\ -\mathbf{a}^{[1]} & x+\lambda_2+\bar{\mathbf{a}}^{[1]}\mathbf{a}^{[1]} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & \bar{\mathbf{a}}^{[2]} \\ 0 & 1 \end{pmatrix}$$

with $S = \exp[\bar{\mathbf{a}}^{[1]}\mathbf{a}^{[2]}]$.

Leads infinite-dimensional transfer matrix

$$\Delta T_\lambda^+(x) = Q_+(x+\lambda_1)Q_-(x+\lambda_2-1)$$

Reducible for $\lambda_1-\lambda_2\in\mathbb{N}\rightarrow$ BGG-resolution

$$\Delta T_\lambda(x) = Q_+(x+\lambda_1)Q_-(x+\lambda_2-1) - Q_+(x+\lambda_2-1)Q_-(x+\lambda_1)$$

Baxter Q-operators for open chain

Baxter Q-operators for open chain

Baxter Q-operators [Szecsenyi, RF '15]

$$Q_\pm(z) = tr\, K_\pm(z) U_\pm(z)$$

Double-row monodromy

$$U_\pm(z) = \underbrace{L_\pm^{[1]}(z) L_\pm^{[2]}(z) \cdots L_\pm^{[N]}(z)}_{M_\pm(z)} \hat{K}_\pm(z) \underbrace{L_\pm^{[N]}(z) \cdots L_\pm^{[2]}(z) L_\pm^{[1]}(z)}_{\hat{M}_\pm(z)}$$

Lax operators $L_\pm^{[i]}(z)$ at site i.

K-matrices for diagonal boundaries

$$\hat{K}_\pm(z) = \frac{\Gamma(\pm q - z)}{\Gamma(\pm q - z + 1 + \bar{\mathbf{a}}\mathbf{a})} \qquad K_\pm(z) = \frac{\Gamma(\mp p - z + \bar{\mathbf{a}}\mathbf{a})}{\Gamma(\mp p - z)}$$

Trigonometric generalisations [Tsuboi,Baseilhac '17] [Tsuboi '17 &'19] [Vlaar,Weston '20]

Boundary factorisation [Cooper,Vlaar,Weston '23]

Where are we?

Generalisations

Generalisations for the closed chain

- Higher rank A_r [Bazhanov,RF,Lukowski,Meneghelli,Staudacher]
- Other simple Lie algebras: $BCDE$-type,...

  [RF],[RF,Tsymbaliuk],[RF,Karpov,Tsymbaliuk],[Costello,Gaiotto,Yagi],[Boujakhrout,Saidi]
- Susy: $su(n|m)$, $osp(N|2m)$,... [RF,Lukowski,Meneghelli,Staudacher],[RF,Tsymbaliuk]
- Non-compact representations [Derkachov et al]
- Trigonometric-deformation [Boos,Klümper,Göhmann,Nirov,Razumov],

  [Bazhanov,Tsuboi], [Tsuboi]
- Roots of unity [Miao,Lamers,Pasquier]
- Elliptic-deformation [Felder,Zhang]

Generalisations for the open chain

Same story but extra complications because of the boundaries

$\rightarrow$ Robert's talk

More comments

Relation to shifted Yangian of Braverman, Finkelberg and Nakajima [RF,Pestun],[RF,Pestun,Tsymbaliuk],[RF,Tsymbaliuk]

Other ways to the QQ-system:

- q-character approach [Hernandez,Jimbo], [Frenkel,Hernandez],[Hernandez]
- ODE/IM [Dorey,Tateo], [Masoero,Raimondo,Valeri],[Ekhammar,Shu,Volin],[Fioravanti,Rossi]

Use of QQ-systems:

- Solve Bethe equations [Razumov,Stroganov],[Marboe,Volin]
  $su(n) \subset su(n|m)$, (same for $so(N), sp(2m) \subset osp(N|2m)$?)
- QSC of AdS/CFT correspondence [....]
  $su(4|4), osp(4|6), d(2,1;\alpha)$

sl(n) spin chains

Rational sl(n) spin chains

Start again from famous solution to YBE

$$R(x) = x + P \qquad \text{with} \qquad P = \sum_{i,j=1}^{n} e_{ij} \otimes e_{ji}$$

$n^2 \times n^2$ matrix, x spectral parameter and $(e_{ij})_{kl} = \delta_{ik}\delta_{jl}$

- Degenerate solutions to RLL

$$R(x-y)(L_I(x) \otimes \mathbb{I})(\mathbb{I} \otimes L_I(y)) = (\mathbb{I} \otimes L_I(y))(L_I(x) \otimes \mathbb{I})R(x-y)$$

In total there are 2^n degenerate solutions that construct Q-operators

Weyl permutations $\rightarrow$ $n+1$ distiguished ones

Lax matrix for Q-operators

For the distinguished Lax matrices we get

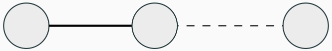
[Bazhanov,RF,Lukowski,Meneghelli,Staudacher]

$$L_{(a)}(x) = \left(\begin{array}{c|c} xI_a - \bar{\mathbf{A}}\mathbf{A} & \bar{\mathbf{A}} \\ \hline -\mathbf{A} & I_{n-a} \end{array}\right), \qquad a = 0, \ldots, n$$

with $\mathbf{A}_{\dot{\alpha}\alpha} = \mathbf{a}_{\dot{\alpha}\alpha}$ and $\bar{\mathbf{A}}_{\alpha\dot{\alpha}} = \bar{\mathbf{a}}_{\alpha\dot{\alpha}}$ with $[\mathbf{a}_{\dot{\alpha}\alpha}, \bar{\mathbf{a}}_{\beta\dot{\beta}}] = \delta_{\alpha\beta}\delta_{\dot{\alpha}\dot{\beta}}$

Indices: $\alpha, \beta = 1, \ldots, a$ and $\dot{\alpha}, \dot{\beta} = a+1, \ldots, n$

$A_{n-1}:$



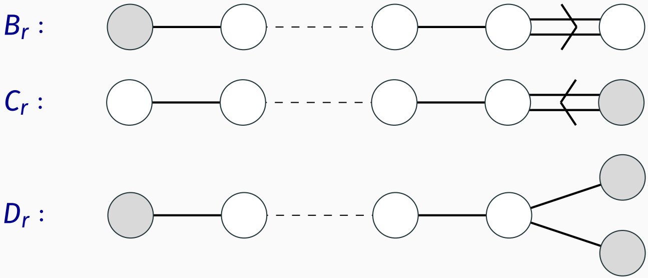
$L_{(1)}$ $L_{(2)}$ $L_{(n-1)}$

$L_{(a)}$ arises as degenerations of Lax matrices $\mathcal{L}_{a,s}$ for $s \to \infty$

BCD-type and orthosymplectic case

BCD-type

Lax matrices for gray nodes [RF,Karpov, Tsymbaliuk]



Unified within

$$osp(N|2m) = \begin{cases} B_n & \text{if } m = 0 \quad \& \quad N = 2n+1 \\ D_n & \text{if } m = 0 \quad \& \quad N = 2n \\ C_m & \text{if } N = 0 \end{cases}$$

Q-operators from oscillator realisation

Lax matrices for first node

$$L_{(1)}(z) = \left(\begin{array}{c|c|c} z^2 + z(2 - r - \bar{w}w) + \frac{1}{4}\bar{w}J\bar{w}^t w^t J w & z\bar{w} - \frac{1}{2}\bar{w}J\bar{w}^t w^t J & -\frac{1}{2}\bar{w}J\bar{w}^t \\ \hline -zw + \frac{1}{2}J\bar{w}^t w^t J w & zI - J\bar{w}^t w^t J & -J\bar{w}^t \\ \hline -\frac{1}{2}w^t J w & w^t J & 1 \end{array}\right).$$

with $N + 2m - 2$ pairs of oscillators

$$\bar{w} = (\bar{\mathbf{a}}_2, \ldots, \bar{\mathbf{a}}_n, \bar{\mathbf{c}}_{n+1}, \cdots, \bar{\mathbf{c}}_{n+m}, \bar{\mathbf{a}}_{n+m+1}, \bar{\mathbf{c}}_{(n+m)'}, \cdots, \bar{\mathbf{c}}_{(n+1)'}, \bar{\mathbf{a}}_{n'}, \ldots, \bar{\mathbf{a}}_{2'}) .$$

and

$$w = \bar{w}^\dagger , \qquad J = \sum_i e_{ii'}$$

Commutation relations

$$[\mathbf{a}, \bar{\mathbf{a}}] = 1 , \qquad \{\mathbf{c}, \bar{\mathbf{c}}\} = 1$$

Q-operators from oscillator realisation

Lax matrices at spinorial nodes of $osp(2n|2m)$

$$L(z) = \left(\begin{array}{c|c} z\,I - \bar{\mathbf{A}}\mathbf{A} & \bar{\mathbf{A}} \\ \hline -\mathbf{A} & I \end{array}\right),$$

with

$$\bar{\mathbf{A}} = \left(\begin{array}{c|c} \bar{\mathbf{C}} & \bar{\mathbf{A}}_{D_n} \\ \hline \bar{\mathbf{A}}_{C_m} & -J\bar{\mathbf{C}}^t J \end{array}\right), \qquad \mathbf{A}_{ij} = \bar{\mathbf{A}}_{ji}^\dagger \qquad \bar{\mathbf{C}}_{ij} = \bar{\mathbf{c}}_{ij}$$

and

$$\bar{\mathbf{A}}_{D_n} = \begin{pmatrix} \bar{\mathbf{a}}_{1,1} & \cdots & \bar{\mathbf{a}}_{1,n-1} & 0 \\ \vdots & \iddots & 0 & -\bar{\mathbf{a}}_{1,n-1} \\ \bar{\mathbf{a}}_{n-1,1} & 0 & \iddots & \vdots \\ 0 & -\bar{\mathbf{a}}_{n-1,1} & \cdots & -\bar{\mathbf{a}}_{1,1} \end{pmatrix}, \quad \bar{\mathbf{A}}_{C_m} = \begin{pmatrix} \bar{\mathbf{b}}_{1,1} & \cdots & \bar{\mathbf{b}}_{1,m-1} & \sqrt{2}\,\bar{\mathbf{b}}_{1,m} \\ \vdots & \iddots & \sqrt{2}\,\bar{\mathbf{b}}_{2,m-1} & \bar{\mathbf{b}}_{1,m-1} \\ \bar{\mathbf{b}}_{m-1,1} & \sqrt{2}\,\bar{\mathbf{b}}_{m-1,2} & \iddots & \vdots \\ \sqrt{2}\,\bar{\mathbf{b}}_{m,1} & \bar{\mathbf{b}}_{m-1,1} & \cdots & \bar{\mathbf{b}}_{1,1} \end{pmatrix},$$

Commutation relations

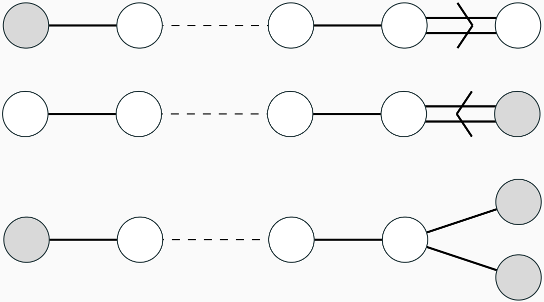
$$[\mathbf{a}, \bar{\mathbf{a}}] = 1, \qquad [\mathbf{b}, \bar{\mathbf{b}}] = 1, \qquad \{\mathbf{c}, \bar{\mathbf{c}}\} = 1$$

BCD-type

B_r :

C_r :

D_r :



- Factorisation and functional relations [RF,Karpov,Tsymbaliuk]
- What is going on for the white nodes?
  Reps of the Lie algebra do not lift to reps of the Yangian!

Oscillator construction and shifted Yangian

Famous oscillator-type solutions for $gl(2)$

Introduced solutions are polynomials in oscillators:

$$\mathcal{L}(x) = \begin{pmatrix} x + x_1 - \bar{\mathbf{a}}\mathbf{a} & -\bar{\mathbf{a}}(x_1 - x_2 - \bar{\mathbf{a}}\mathbf{a}) \\ -\mathbf{a} & x + x_2 + \bar{\mathbf{a}}\mathbf{a} \end{pmatrix}, \quad L(x) = \begin{pmatrix} 1 & -\mathbf{a} \\ \bar{\mathbf{a}} & x + x_2 - \bar{\mathbf{a}}\mathbf{a} \end{pmatrix}$$

Another well known solution is the Toda Lax

$$L_{Toda}(x) = \begin{pmatrix} 0 & -e^{-q} \\ e^{q} & x - p \end{pmatrix}$$

with $[p, e^{\pm q}] = \pm e^{\pm q}$ and $[p, q] = 1$.

Toda Lax is not polynomial in oscillators .

Q-operators from BFN

All Lax matrices for Q-operators (and many more) can be obtained from Drinfeld's current realisation (A-type)

[Bravermann,Finkelberg,Nakajima] [Kamnitzer,Webster,Weekes,Yacobi][Gerasimov,Kharchev,Lebedev,Oblezin]

$$E_i(x) = -\sum_{r=1}^{a_i} \frac{\prod_{s=1}^{a_{i-1}}(p_{i,r} - p_{i-1,s} - 1)}{(x - p_{i,r})\prod_{s\neq r}(p_{i,r} - p_{i,s})} \mathcal{Z}_i(p_{i,r}) e^{q_{i,r}}$$

$$F_i(x) = \sum_{r=1}^{a_i} \frac{\prod_{s=1}^{a_{i+1}}(p_{i,r} - p_{i-1,s} + 1)}{(x - p_{i,r} - 1)\prod_{s\neq r}(p_{i,r} - p_{i,s})} e^{-q_{i,r}}$$

$$G_i(x) = \frac{\prod_{s=1}^{a_i}(x - p_{i,s})}{\prod_{s=1}^{a_{i-1}}(x - p_{i,s} - 1)} \mathcal{Z}_1(x)\cdots\mathcal{Z}_{i-1}(x)$$

with $[p, e^{\pm q}] = \pm e^{\pm q}$ and $\mathcal{Z}_i(x) = \prod_k (x - x_{i,k})$

- a_i dictates order of spectral parameters
- Yields Lax matrices at any order of spectral parameter

  [RF,Pestun,Tsymbaliuk '20]

Q-operators from BFN

Degenerate examples for $gl(2)$

$$L(x) = \begin{pmatrix} 1 & -e^{-q} \\ (p - x_2)e^q & x - p \end{pmatrix}$$

Relation between oscillators

$$\bar{\mathbf{a}} = (p + x_2)e^q \qquad \mathbf{a} = e^{-q}$$

But: Mapping to polynomial oscillators $(\mathbf{a}, \bar{\mathbf{a}})$ not possible for all Lax matrices, e.g.

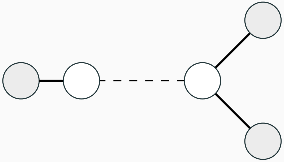
$$L_{Toda}(x) = \begin{pmatrix} 0 & -e^{-q} \\ e^q & x - p \end{pmatrix}$$

Q-operators from BFN

Lax matrices can be evaluated for BCD-type

[Nakajima,Weekes],[RF,Tsymbaliuk]

D_r :



Lax operators with correct asymptotics for Q-operators exist at white nodes but trace prescription still under construction!

Outlook

Outlook

QQ-system

- Construction of Q-operators from BFN
- Open spin chains (shifted twisted Yangian)

Generalisations

- Different representations in matrix space
- Generalisation to EFG-type
- Supersymmetric $osp(N|2m)$ and $D(2,1;\alpha)$
- Relation to QSC for AdS_4/CFT_3 and AdS_3/CFT_2

  [Bombardelli,Cavaglià,Conti,Fioravanti,Gromov,Tateo], [Cavaglià,Gromov,Stefański jr.,Torrielli]

Thank you for your attention!