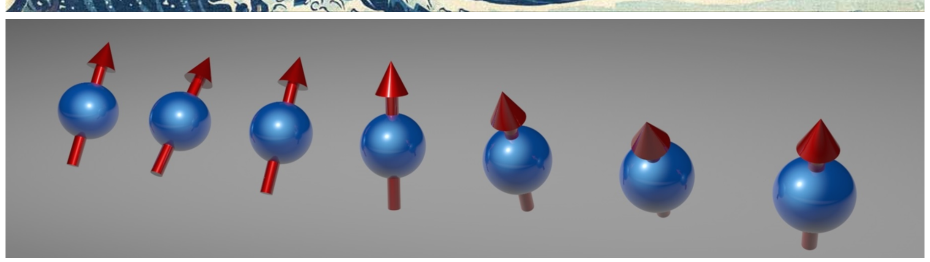
DIFFUSION, THERMALISATION AND TURBULENCE IN GENERALISED HYDRODYNAMICS



JACOPO DE NARDIS

CERGY PARIS UNIVERSITÉ

ANR AGENCE NATIONALE DE LA RECHERCHE

erc

Work done with

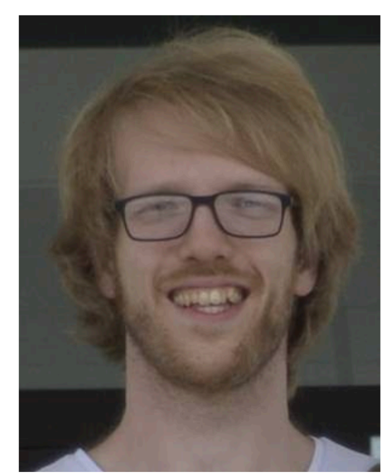



J Durnin *B Doyon* *A De Luca* *A Bastianello*

CERGY PARIS UNIVERSITÉ








Hugo Lóio

MSc Studeni

Guillaume Cecile

Anastasiia Tiutiakina

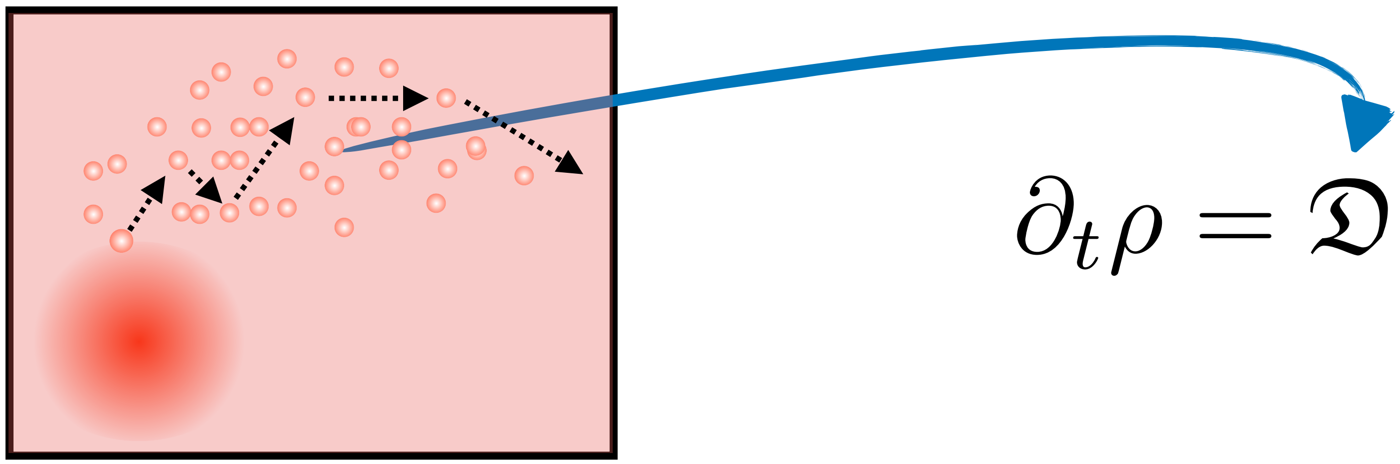
PhD Student

Leonardo Biagetti

Dr. Andrew Urilyon

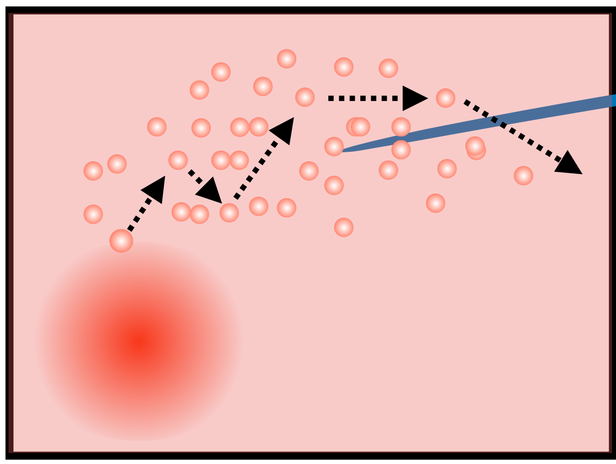
Stefano Scopa

The non-equilibrium problem

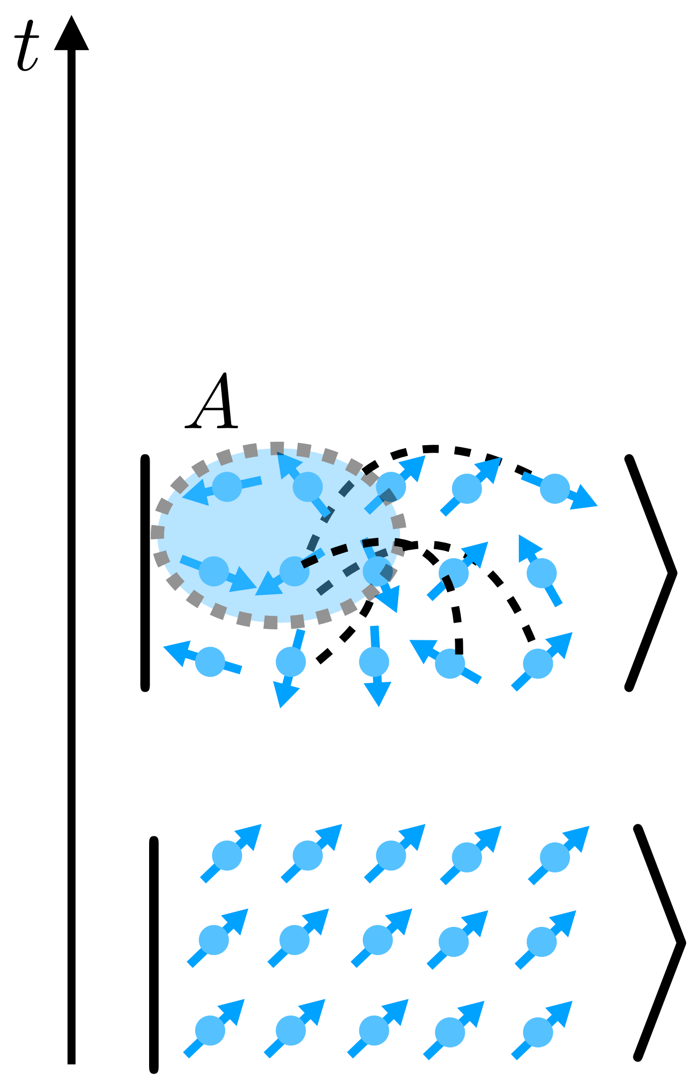


$$\partial_t \rho = \mathfrak{D} \partial_x^2 \rho$$

The non-equilibrium problem

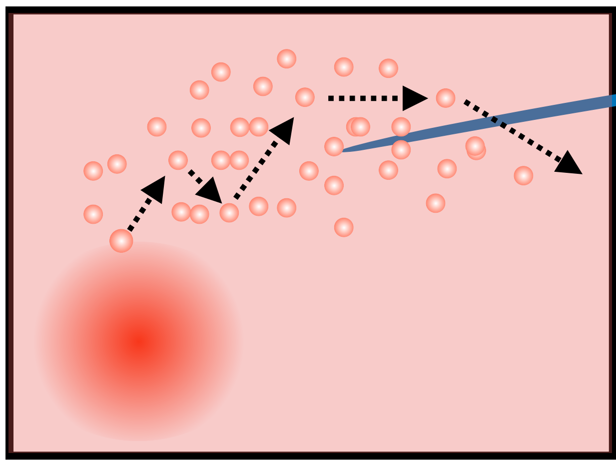


$$\partial_t \rho = \mathfrak{D} \partial_x^2 \rho$$

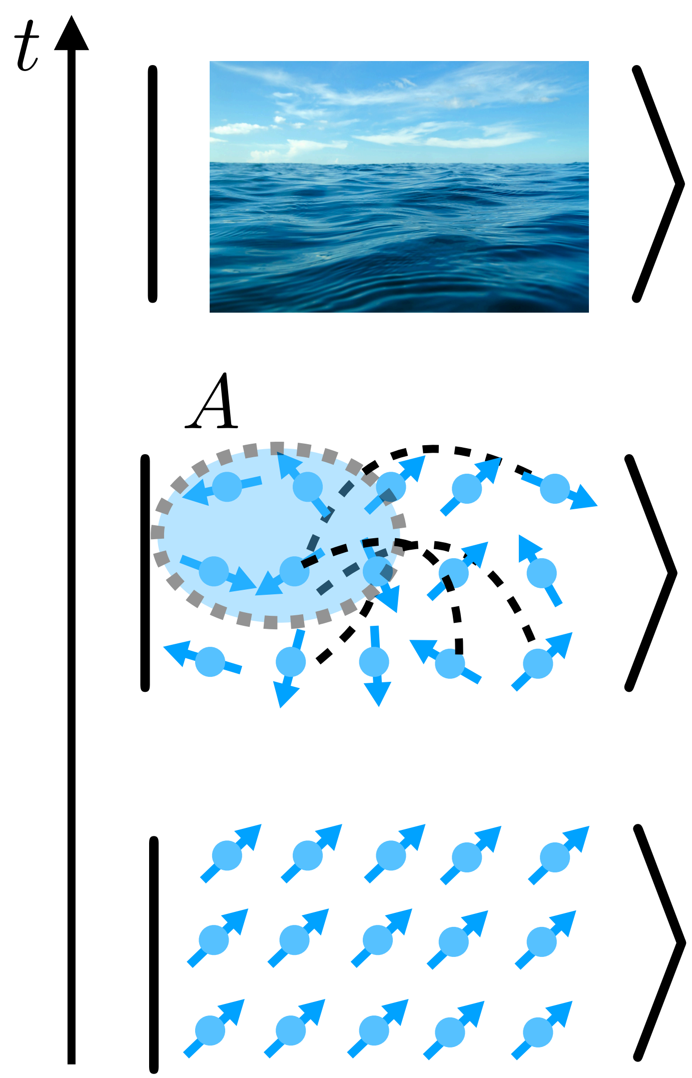


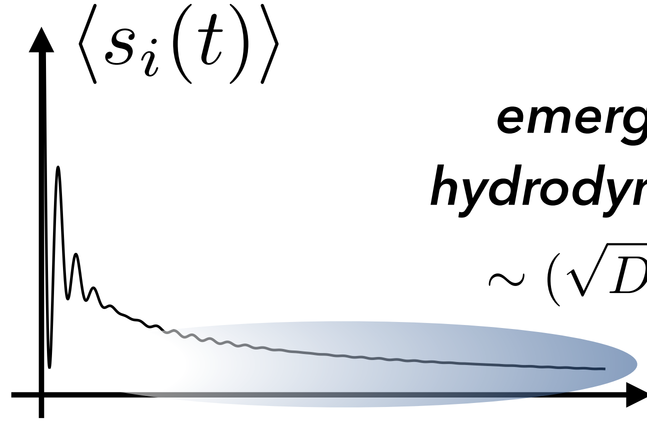
$$S_A(t) \to V_A$$

The non-equilibrium problem



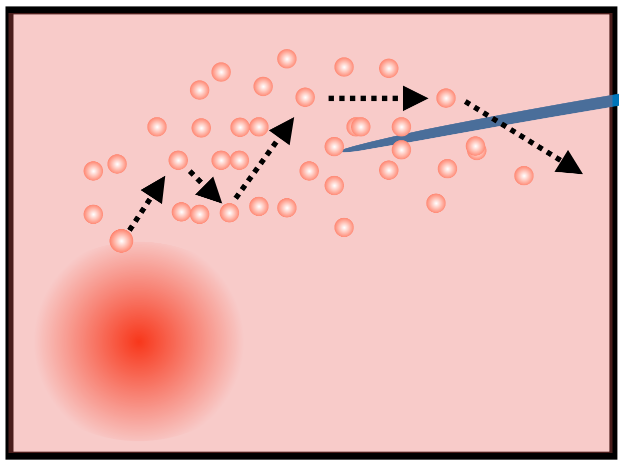
$$\partial_t \rho = \mathfrak{D} \partial_x^2 \rho$$



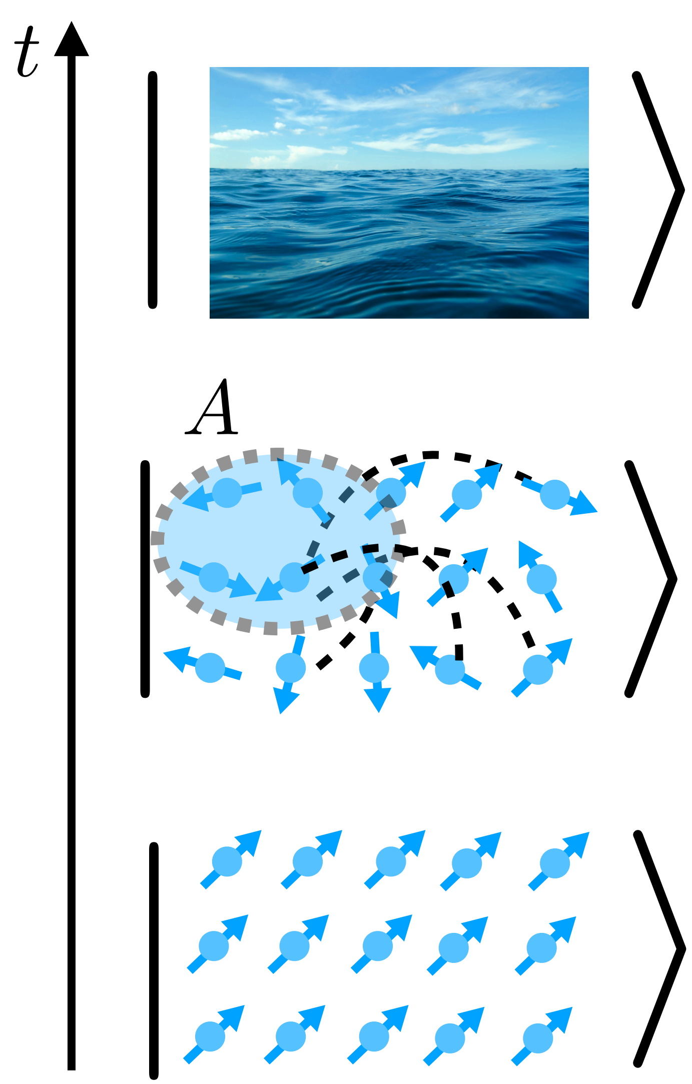


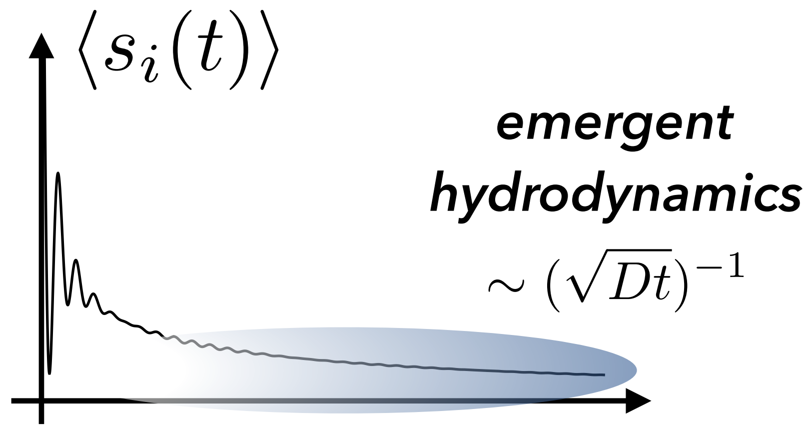
$$S_A(t) \to V_A$$

The non-equilibrium problem

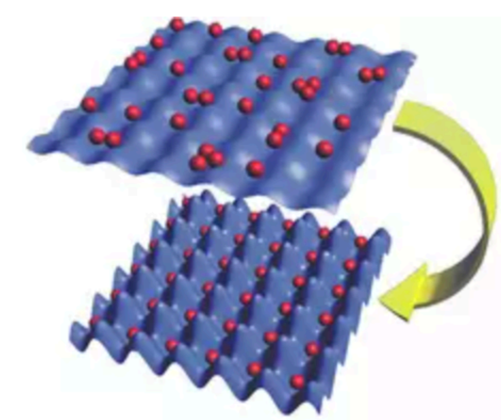


$$\partial_t \rho = \mathfrak{D} \partial_x^2 \rho$$

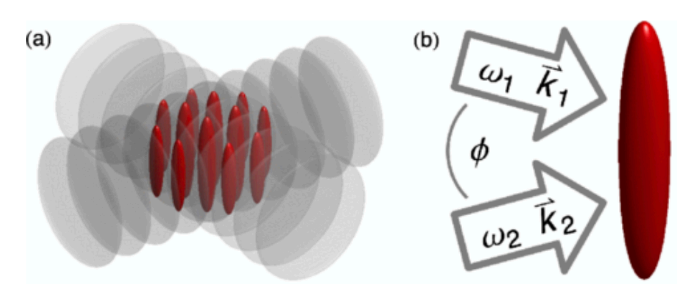


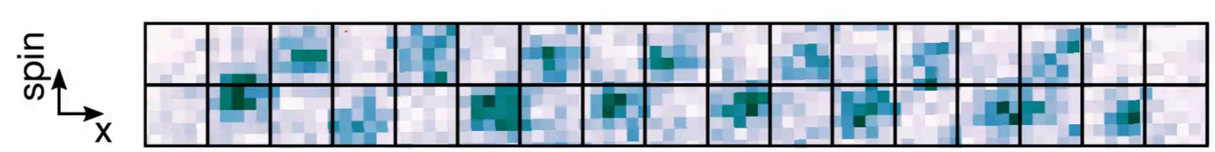


$$S_A(t) \to V_A$$



Innsbruck quantum group





TUM Munich

Hydrodynamics

A time evolved quantum state get lost in an exponentially large jungle

$$\langle \psi(t) | O | \psi(t) \rangle$$

Approximate the time-evolved state to a simpler object, where only fewer operators are needed

$$|\psi(t)\rangle\langle\psi(t)| \to \rho_{\rm hydro}(t)$$

Hydrodynamics: describe the state with the slowest operators at a certain hydrodynamic order

Most obvious choice: local densities of conserved quantities

$$[H, Q_i] = 0$$

$$Q_i = \int dx \;\, q_i(x)$$

$$\partial_t q_i(x,t) + \partial_x j_i(x,t) = 0$$

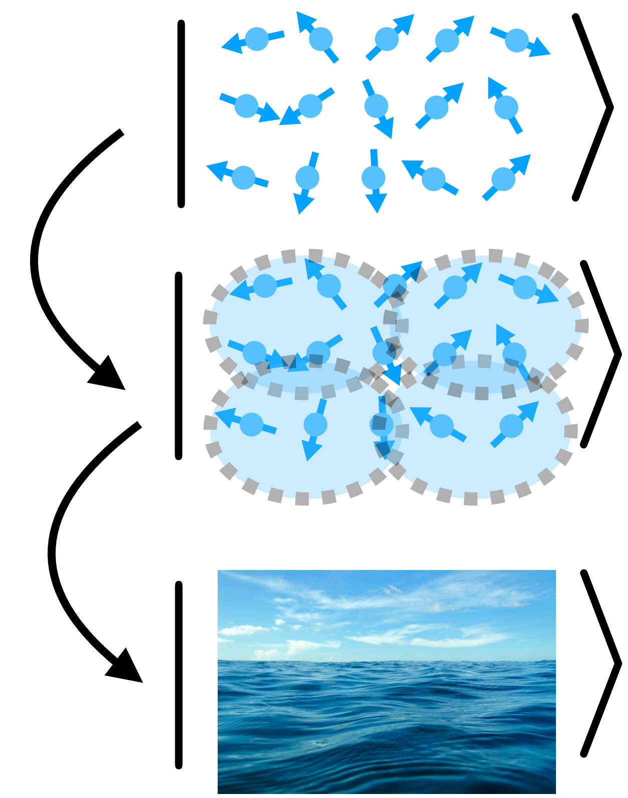
Hydrodynamics

- $$Q_i = \int dx q_i(x) \qquad \partial_t q_i(x,t) = i[H, q_i(x,t)] = -\partial_x j_i(x,t)$$ **Conserved densities**

- $$\partial_t \langle q_i(x,t) \rangle + \partial_x \langle j_i(x,t) \rangle = 0$$ **Continuity equations**

- $$\langle \cdot \rangle = \mathrm{Tr}\Big(\cdot \, e^{-\int dx \beta^i(x,t) q_i(x \quad)} \Big) / Z$$ **Fluid cells approximation**

$$\beta(x)|_{x \sim x_0} = \sum_{n \geq 0} \partial_x^n \beta|_{x=x_0} \frac{(x-x_0)^n}{n!}$$



- $$\langle j_i(x,t) \rangle = \mathcal{F}[\langle q_\cdot(x,t) \rangle, \partial_x \langle q_\cdot(x,t) \rangle, \partial_x^2 \langle q_\cdot(x,t) \rangle, \ldots]$$

Example

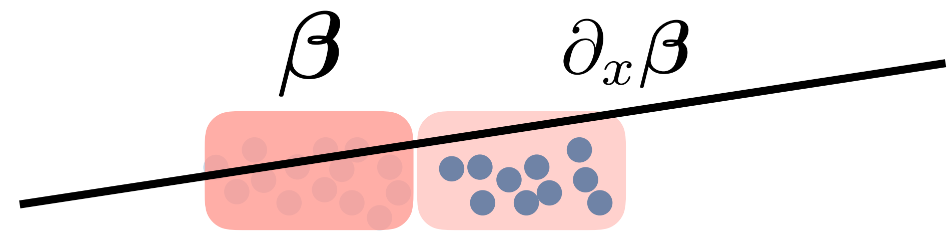
$$\langle j_i(x,t) \rangle = -\mathfrak{D} \partial_x \langle q_i(x,t) \rangle$$

How to derive it

Start from

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$

$$x_0 = 0 \quad ; \quad t_0 = 0$$



Expand expectation values of densities and currents

$$e^{A+\epsilon B} = e^A + \epsilon \int_0^1 d\tau e^{\tau A} B e^{(1-\tau)A} + O(\epsilon^2)$$

Hydrodynamic variables

$$\langle q_i(0,t)\rangle_{\tilde{\rho}} = \langle q_i\rangle - \partial_x\beta^k \int dy\, y\,(q_k(y), q_i(0,t)) \qquad = \mathtt{q}_i(0,t)$$

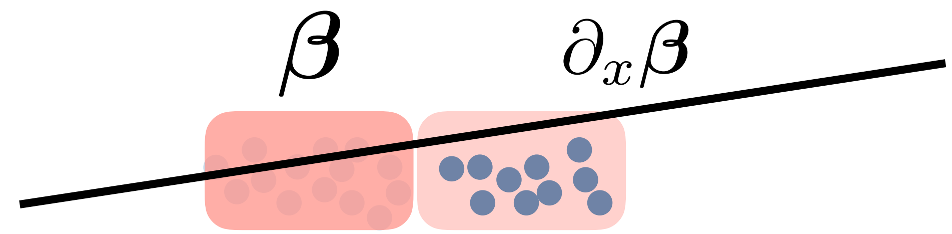
$$\langle j_i(0,t)\rangle_{\tilde{\rho}} = \langle j_i\rangle - \partial_x\beta^k \int dy\, y\,(q_k(y), j_i(0,t))$$

How to derive it

Start from

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$

$$x_0 = 0 \quad ; \quad t_0 = 0$$



Expand expectation values of densities and currents

$$e^{A+\epsilon B} = e^A + \epsilon \int_0^1 d\tau e^{\tau A} B e^{(1-\tau)A} + O(\epsilon^2)$$

Hydrodynamic variables

$$\langle q_i(0,t)\rangle_{\tilde{\rho}} = \langle q_i \rangle - \partial_x \beta^k \int dy\, y\, (q_k(y), q_i(0,t)) \qquad = \mathtt{q}_i(0,t)$$

$$\langle j_i(0,t)\rangle_{\tilde{\rho}} = \langle j_i \rangle - \partial_x \beta^k \int dy\, y\, (q_k(y), j_i(0,t))$$

Currents must be expressed as function of hydrodynamic variables

$$\langle j_i \rangle \equiv \langle j_i \rangle[\langle q \rangle] = \langle j_i \rangle[\mathtt{q}] - \partial_x \beta^k \frac{\delta \langle j_i \rangle}{\delta \langle q_l \rangle} \int dy\, y\, (q_k(y), q_l(0,t)) + \dots$$

$$\frac{\mathfrak{L}_{i,k}}{2} = \lim_{t\to\infty} \left[\int dy\, y\, (q_k(y), j_i(0,t)) - \frac{\delta \langle j_i \rangle}{\delta \langle q_l \rangle} \int dy\, y\, (q_k(y), q_i(0,t)) \right]$$

Hydrodynamics and gauge fixing

$$\langle j_i(0,t)\rangle_{\widetilde{\rho}} = \mathcal{J}_i + \frac{1}{2}\partial_x \beta^k \mathfrak{L}_{i,k}$$

Onsager Coefficients

$$\frac{\mathfrak{L}_{i,k}}{2} = \lim_{t\to\infty}\Big[\int dy\, y\,(q_k(y), j_i(0,t)) - \frac{\delta\langle j_i\rangle}{\delta\langle q_l\rangle}\int dy\, y\,(q_k(y), q_i(0,t))\Big]$$

For now it is not positive neither symmetric

Hydrodynamics and gauge fixing

$$\langle j_i(0,t)\rangle_{\widetilde{\rho}} = \mathcal{J}_i + \frac{1}{2}\partial_x\beta^k \mathfrak{L}_{i,k}$$

Onsager Coefficients

$$\frac{\mathfrak{L}_{i,k}}{2} = \lim_{t\to\infty}\left[\int dy\, y\,(q_k(y), j_i(0,t)) - \frac{\delta\langle j_i\rangle}{\delta\langle q_l\rangle}\int dy\, y\,(q_k(y), q_i(0,t))\right]$$

For now it is not positive neither symmetric

Durnin et al, 2021

De Nardis, Doyon, 2022

Must choose the right gauge! $q_i(x,t) \to q_i(x,t) + \partial_x o_i(x,t) \quad ; \quad j_i(x,t) \to j_i(x,t) + \partial_t o_i(x,t)$

This is fixed (at this order in derivative) by imposing PT invariance $\mathcal{PT}(q_i(x)) = q_i(-x)$

$$\frac{\mathfrak{L}_{i,k}}{2} = \int_0^\infty dt \int dy\,(j_k(y), j_i(0,t))^C$$

This guarantees positive entropy increase

$$\dot{S} = \int dx\ \partial\beta^i \mathfrak{L}_{i,k}\partial\beta^k$$

Densities of conserved charges are not uniquely fixed

$$Q_i = \int dx\ q_i(x)$$

Hydrodynamics and gauge fixing

De Nardis et al, 2018

Durnin et al, 2021

De Nardis, Doyon, 2022

PT symmetric densities $\mathcal{PT}(q_i(x)) = q_i(-x)$

$$\partial_t \mathtt{q}_i + \partial_x(\mathcal{J}_i + \mathfrak{L}_i^k \partial_x \beta_k) = 0$$

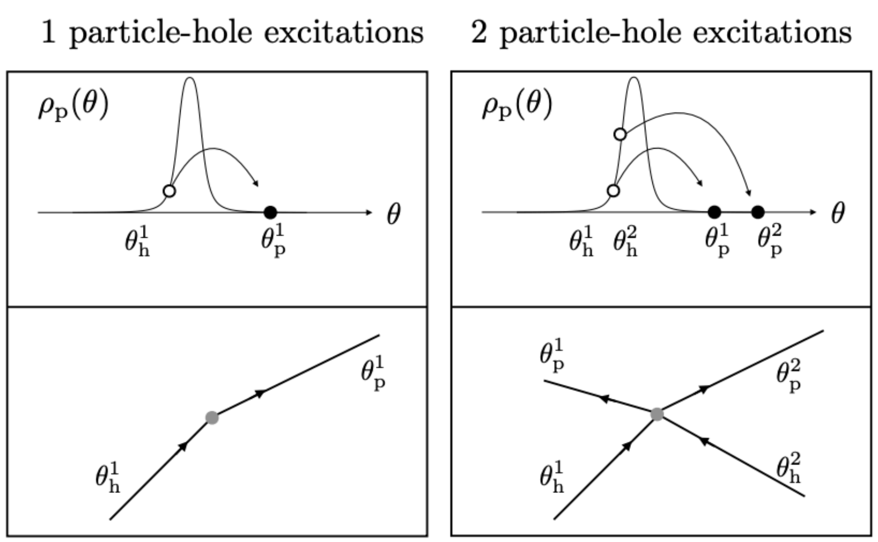
$$\mathfrak{L}_{i,k} = \frac{1}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{+\infty} dx (j_i(x,t), j_k(0,0))^C$$

Form factor expansion

$$\mathfrak{L}_{i,k} = \frac{1}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{+\infty} dx (j_i(x,t), j_k(0,0))^C$$

$$\langle \rho | q_k | \rho \rangle = \int d\theta \rho(\theta) h_k(\theta) \qquad \langle \rho | j_k | \rho \rangle = \int d\theta \rho(\theta) v^{\mathrm{eff}}_{[\rho]}(\theta) h_k(\theta)$$

$$\mathbf{1} = \sum_{\mu} |\mu\rangle\langle\mu| \simeq \sum_{n>1} \frac{1}{(n!)^2} \int d\boldsymbol{\theta}_p \int d\boldsymbol{\theta}_h \mathcal{F}[\boldsymbol{\theta}_p, \boldsymbol{\theta}_h] |\rho; \boldsymbol{\theta}_p, \boldsymbol{\theta}_h\rangle\langle\rho; \boldsymbol{\theta}_p, \boldsymbol{\theta}_h|$$



Form factor expansion

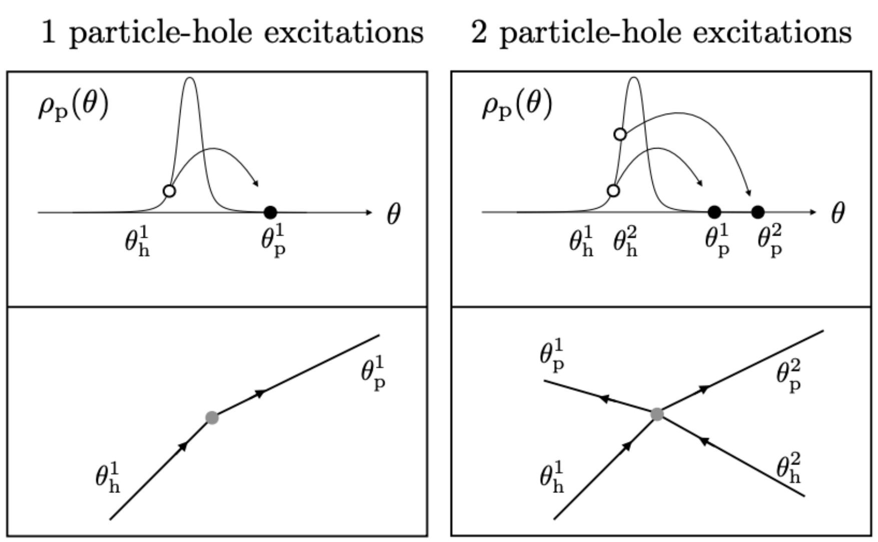
$$\mathfrak{L}_{i,k} = \frac{1}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{+\infty} dx (j_i(x,t), j_k(0,0))^C$$

$$\langle\rho|q_k|\rho\rangle = \int d\theta\rho(\theta)h_k(\theta) \qquad \langle\rho|j_k|\rho\rangle = \int d\theta\rho(\theta)v^{\text{eff}}_{[\rho]}(\theta)h_k(\theta)$$

$$\mathbf{1} = \sum_\mu |\mu\rangle\langle\mu| \simeq \sum_{n>1}\frac{1}{(n!)^2}\int d\boldsymbol{\theta}_p \int d\boldsymbol{\theta}_h \mathcal{F}[\boldsymbol{\theta}_p,\boldsymbol{\theta}_h]|\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h\rangle\langle\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h|$$

thermodynamic form factors

De Nardis et al 2018



$$\langle\rho|q_k|\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h\rangle \sim k_{[\rho]}[\boldsymbol{\theta}_p,\boldsymbol{\theta}_h]\ f_k[\boldsymbol{\theta}_p,\boldsymbol{\theta}_h]$$

$$\langle\rho|j_k|\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h\rangle \sim \varepsilon_{[\rho]}[\boldsymbol{\theta}_p,\boldsymbol{\theta}_h]\ f_k[\boldsymbol{\theta}_p,\boldsymbol{\theta}_h]$$

$$\langle\rho|o|\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h\rangle_n \sim \frac{1-e^{2\pi i\sum_j S^{\text{dr}}_{[\rho]}(\theta_h^1,\theta^j)}}{\theta_p^1-\theta_h^1}\langle\rho|o|\rho;\boldsymbol{\theta}_p,\boldsymbol{\theta}_h\rangle_{n-1}$$

thermodynamic form factors bootstrap!

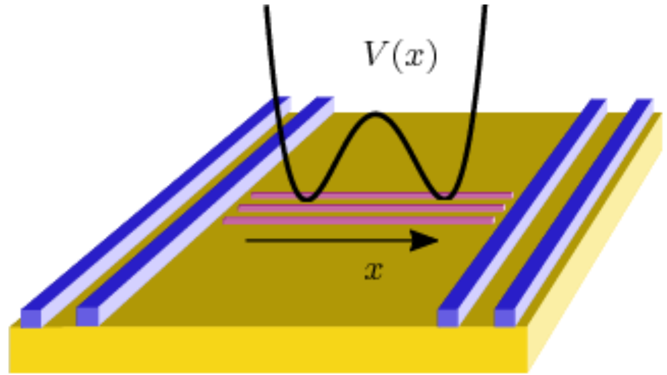
Panfil Cubero 2020

Panfil Cubero 2021

Panfil Konik 2023

Inclusion of external forces

$$H = H_0 + \int V^k(x) q_k(x)$$



$$H_{x_0} = H_0 + V^k(x_0) Q_k + \partial_x V^k|_{x=x_0} \int dx (x - x_0) q_k(x) + \dots$$

$$\partial_t q_i(x_0) = i[H_{x_0}, q_i] = -\partial_x j_{H_0,i} + \mathfrak{f}^k j_{k,i}$$

$$\mathfrak{L}_{i,k} = \frac{1}{2}\int_{-\infty}^{\infty} dt \int_{-\infty}^{+\infty} dx (j_i(x, t$$

generalised currents

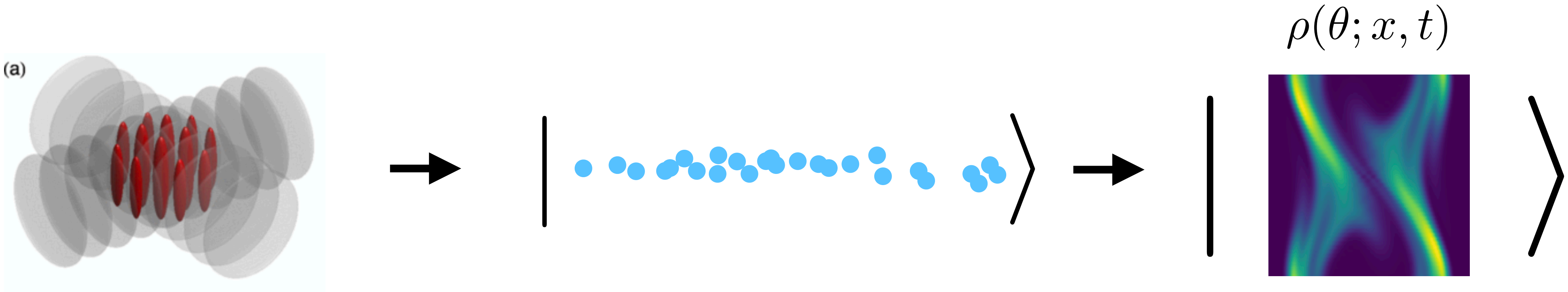
$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

Durning et al 2021

$$\mathfrak{L}[i, k; j, l] = \frac{1}{2}\int_{-\infty}^{+\infty} dt \int dx \, (j_{i,k}(x,t), j_{j,l})$$

generalised Onsager coefficients

Generalised hydrodynamics



Bertini et al, 2016

Castro-Alvaredo et al, 2016

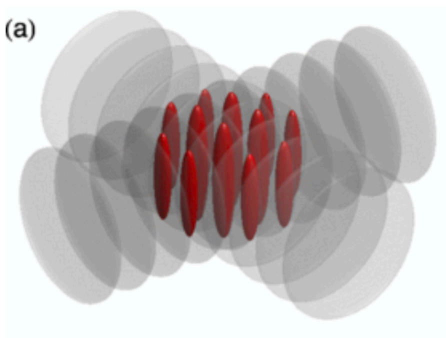
Durning et al 2021

Biagetti et al 2023

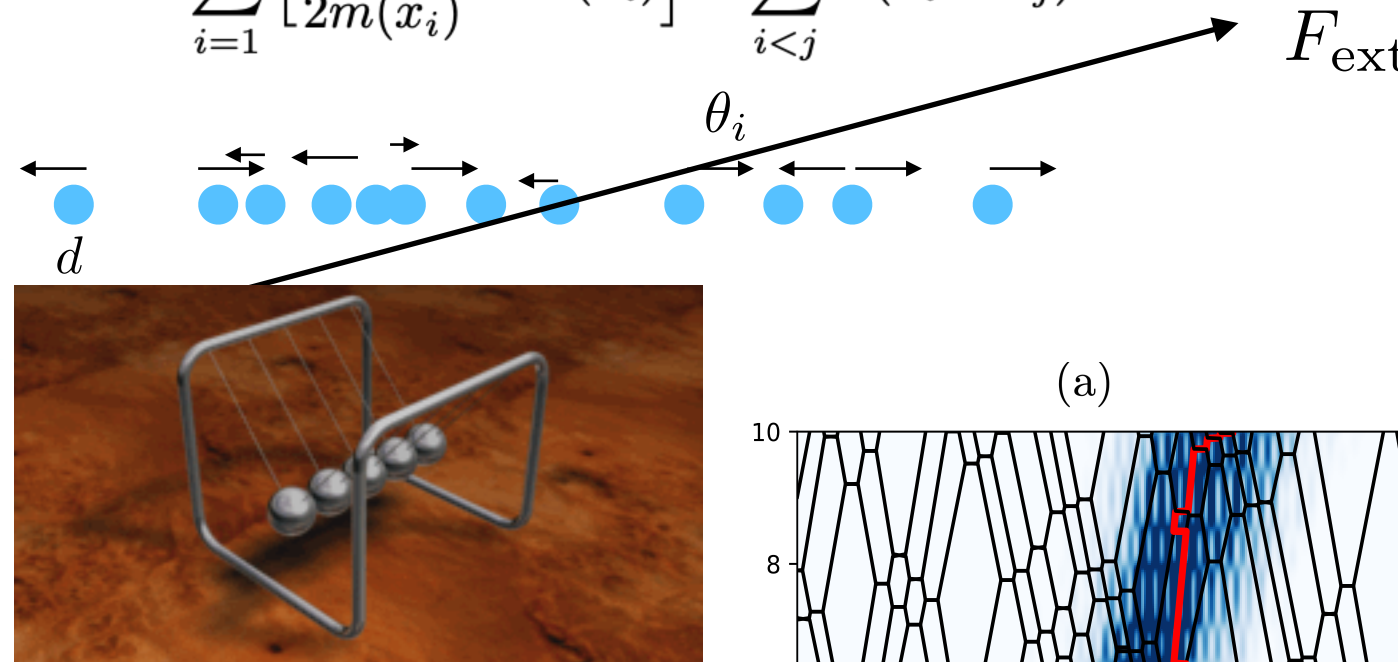
$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\mathrm{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\boldsymbol{\mathfrak{D}}_{[\rho]} \nabla \rho)$$

$$\boldsymbol{J}^{\mathrm{eff}}_{[\rho]} = \begin{pmatrix} v^{\mathrm{eff}}_{[\rho]} \\ a^{\mathrm{eff}}_{[\rho]} \end{pmatrix} \qquad \boldsymbol{\mathfrak{D}}_{[\rho]} = \begin{pmatrix} \mathfrak{D}^{(1,1)}_{[\rho]} & \mathfrak{D}^{(1,2)}_{[\rho]} \\ \mathfrak{D}^{(1,2)}_{[\rho]} & \mathfrak{D}^{(2,2)}_{[\rho]} \end{pmatrix} \qquad \nabla = \begin{pmatrix} \partial_x \\ \partial_\theta \end{pmatrix}$$

Everything is hard rods (billiard balls)

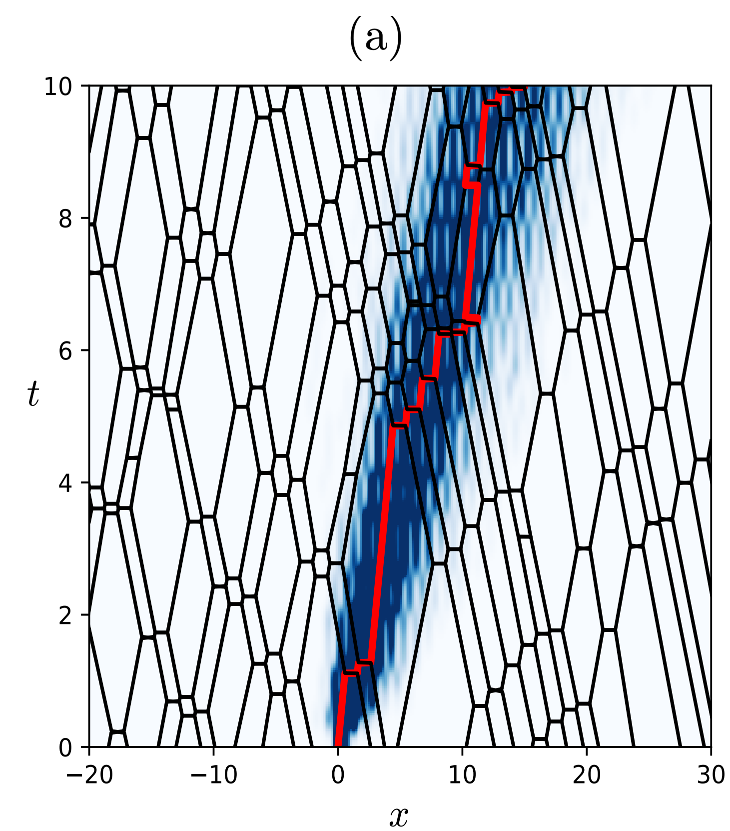


$$H = \sum_{i=1}^{N} \left[\frac{\theta_i^2}{2m(x_i)} + V(x_i) \right] + \sum_{i<j} U(x_i - x_j)$$



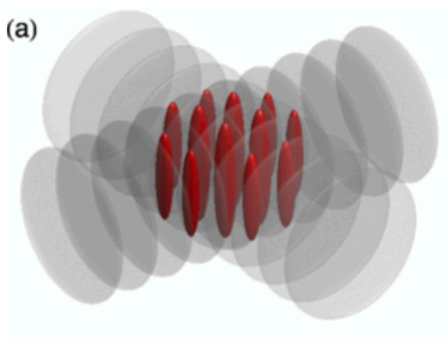
$$\dot{x}_i = \theta_i m(x)$$

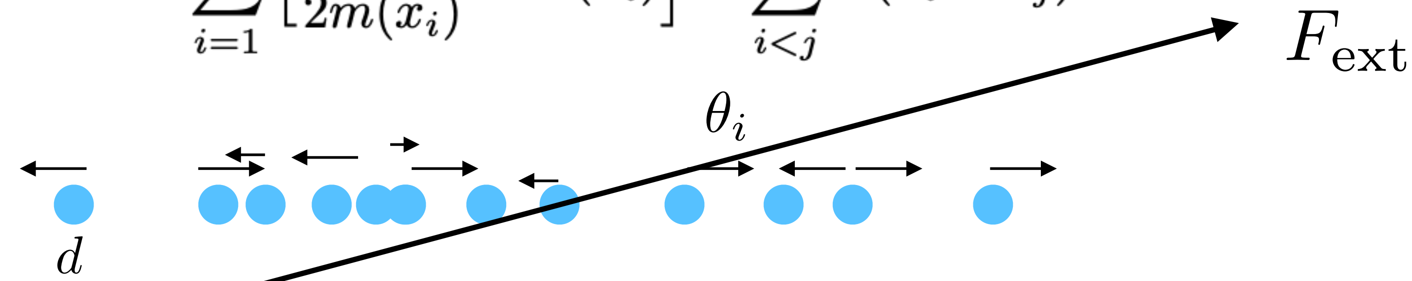
$$\dot{\theta}_i = -\partial_x V(x) - \frac{\theta^2}{2} \partial_x m$$

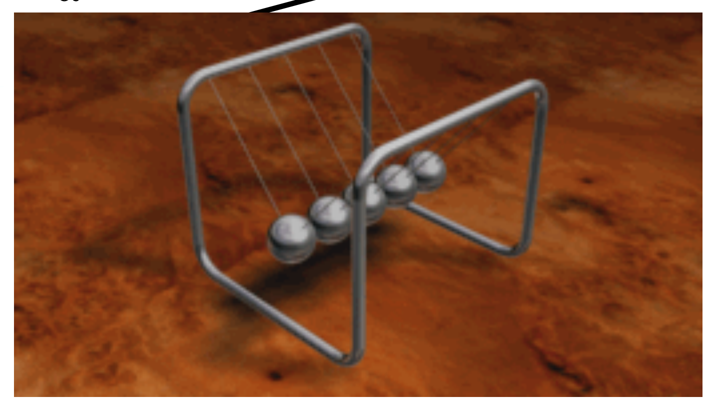


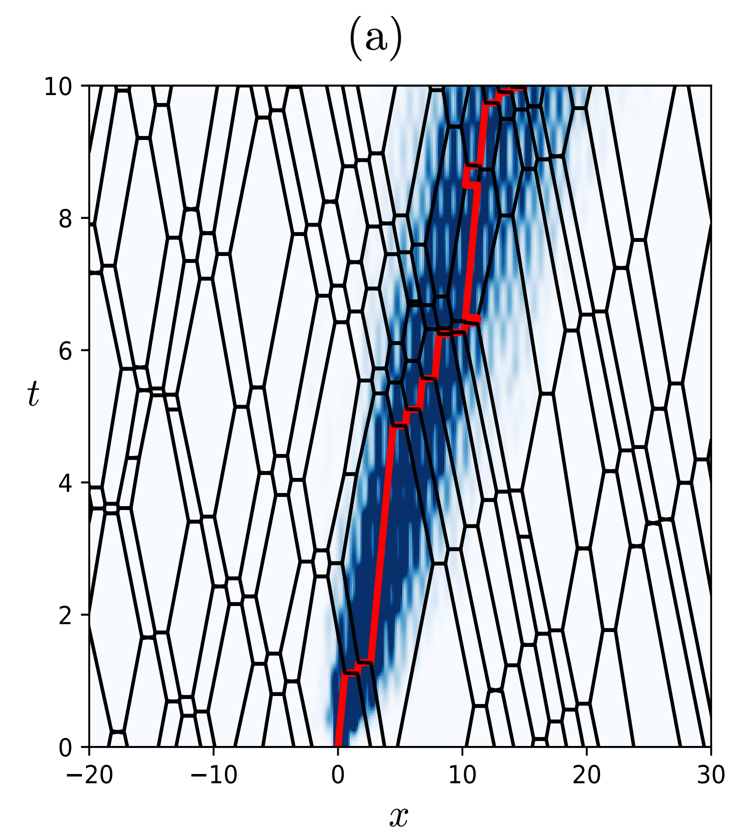
Everything is hard rods (billiard balls)

$$H = \sum_{i=1}^{N} \left[\frac{\theta_i^2}{2m(x_i)} + V(x_i) \right] + \sum_{i<j} U(x_i - x_j)$$





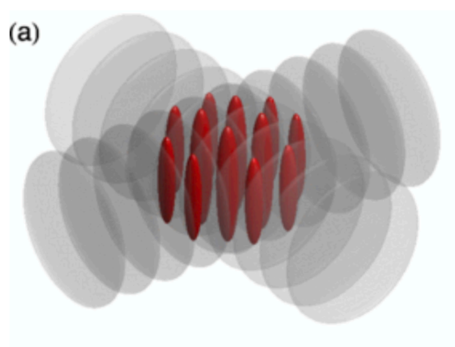




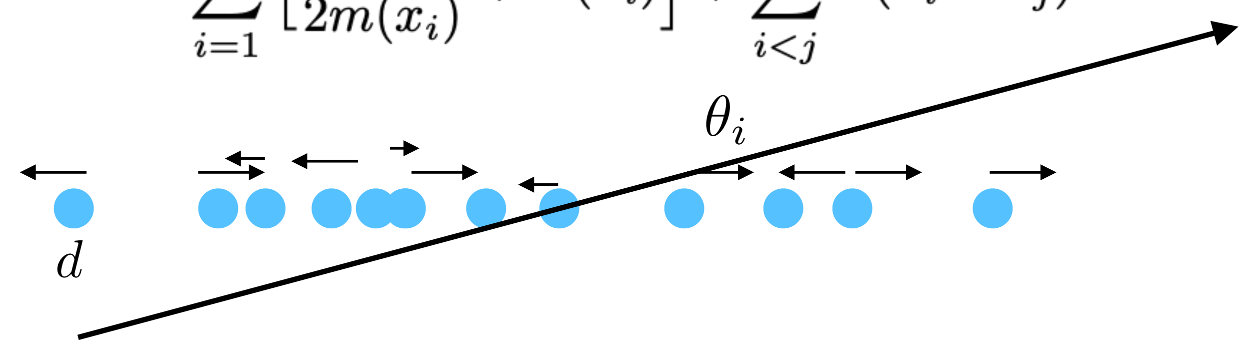
$$\dot{x}_i = \theta_i m(x)$$

$$\dot{\theta}_i = -\partial_x V(x) - \frac{\theta^2}{2} \partial_x m$$

Everything is hard rods (billiard balls)

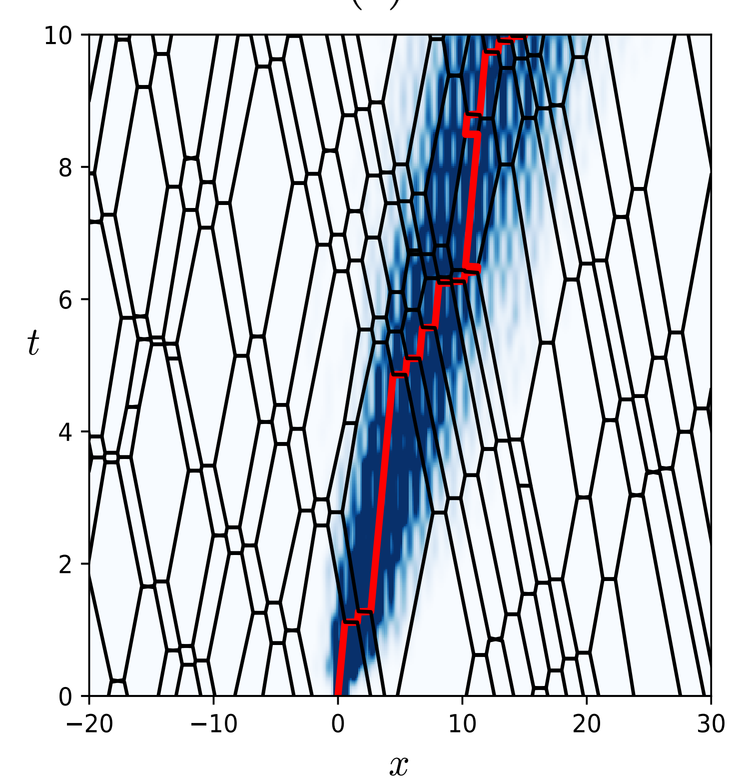


$$H = \sum_{i=1}^{N} \left[\frac{\theta_i^2}{2m(x_i)} + V(x_i) \right] + \sum_{i<j} U(x_i - x_j)$$



$$\dot{x}_i = \theta_i m(x)$$

$$\dot{\theta}_i = -\partial_x V(x) - \frac{\theta^2}{2} \partial_x m$$



$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$

$$\dot{x} = v^{\text{eff}}_{[\rho]}(\theta)$$

$$\dot{\theta} = a^{\text{eff}}_{[\rho]}(\theta)$$

$$\mathfrak{D}_{x,x} \sim \langle (\delta x)^2 \rangle$$

$$\mathfrak{D}_{x,\theta} \sim \langle \delta x \delta \theta \rangle$$

$$\mathfrak{D}_{\theta,\theta} \sim \langle (\delta \theta)^2 \rangle$$

Everything is hard rods (billiard balls)

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\boldsymbol{\mathfrak{D}}_{[\rho]} \nabla \rho)$$

$$\langle \delta x^2(\theta, x, t) \rangle^c|_{\theta'} = t^2 \langle (\delta v^{\text{eff}})^2 \rangle^c|_{\theta'} = t^2 \left(\frac{\delta v^{\text{eff}}(\theta)}{\delta n(\theta')} \right)^2 \langle \delta n(\theta')^2 \rangle^c$$

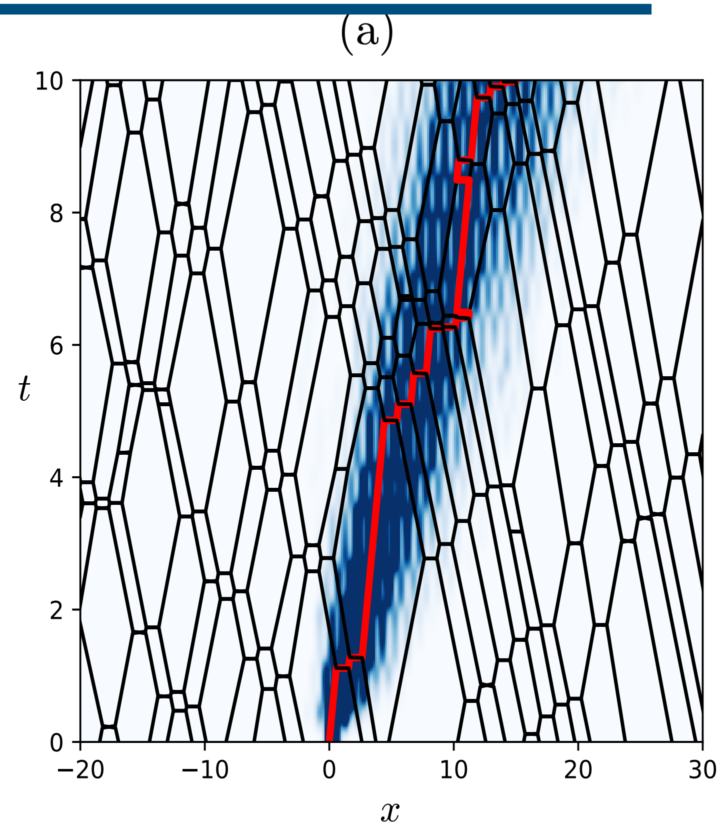
$$\langle \delta n(\theta)^2 \rangle^c = \frac{n(\theta)}{\Lambda} \qquad \Lambda = t|v^{\text{eff}}(\theta) - v^{\text{eff}}(\theta')|$$

$$\langle \delta x^2(\theta, x, t) \rangle^c = t \int_{\mathbb{R}} \mathrm{d}\theta' \rho(\theta', x, t) \boldsymbol{g}_{1,1}(\theta, \theta') = t \mathrm{diag}(\boldsymbol{\mathfrak{D}}^{(1,1)}_{[\rho]})$$

$$\mathfrak{D}^{(1,1)} = \delta_{\theta,\theta'} \left[\int d\kappa \rho(\kappa) \boldsymbol{g}_{1,1}(\theta, \kappa) \right] - \rho(\theta') \boldsymbol{g}_{1,1}(\theta, \theta')$$

$$\langle \delta \theta^2(\theta, x, t) \rangle^c|_{\theta'} = t^2 \langle (\delta a^{\text{eff}})^2 \rangle^c|_{\theta'} = t^2 \left(\frac{\delta a^{\text{eff}}(\theta)}{\delta n(\theta')} \right)^2 \langle \delta n(\theta')^2 \rangle$$

$$\langle \delta \theta^2(\theta, x, t) \rangle^c = t \int_{\mathbb{R}} \mathrm{d}\theta' \rho(\theta', x, t) \boldsymbol{g}_{2,2}(\theta, \theta') = t \mathrm{diag}(\boldsymbol{\mathfrak{D}}^{(2,2)}_{[\rho]}).$$



Gopalakrishnan et al 2018

Biagetti et al 2023

From GHD to CHD (Bose gas)

$$\partial_t \rho + \partial_x (v^{\text{eff}} \rho) = \frac{1}{2} \partial_x (\mathfrak{D} \partial_x \rho)$$

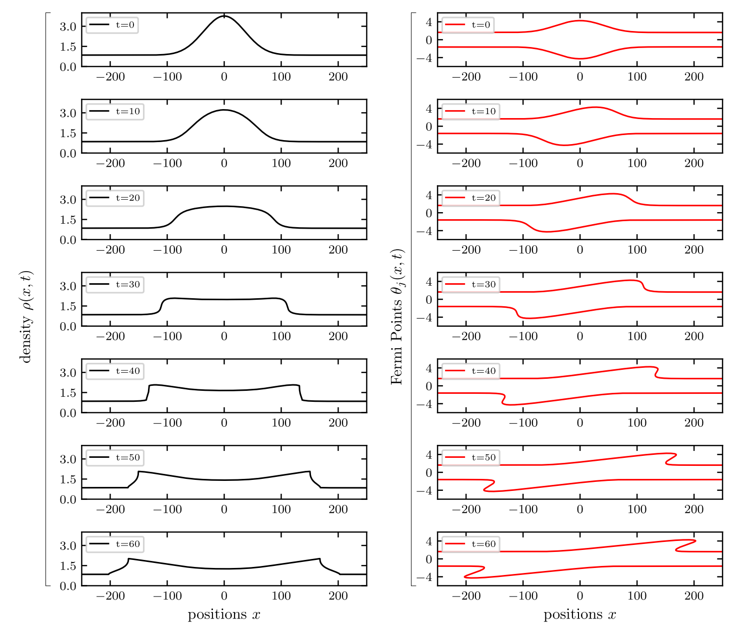
$$\delta n \to \sum_\sigma \sigma \delta(\theta - \theta^\sigma(x,t)) \delta\theta^\sigma(x,t)$$

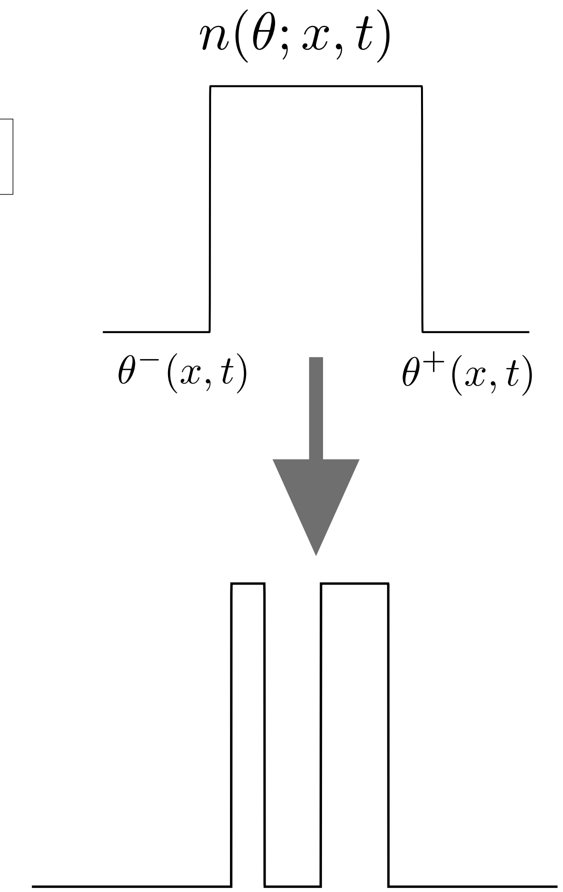
$$\partial_t \theta^\sigma + v^{\text{eff}}(\theta^\sigma) \partial_x \theta^\sigma = 0$$

$$\partial_t \rho \; + \; \partial_x(\eta\rho) = 0$$

$$\partial_t \eta \; + \; \eta \partial_x \eta + \rho^{-1} \partial_x \mathcal{P}_s^{(0)}(\rho) = 0$$

Doyon et al 2017





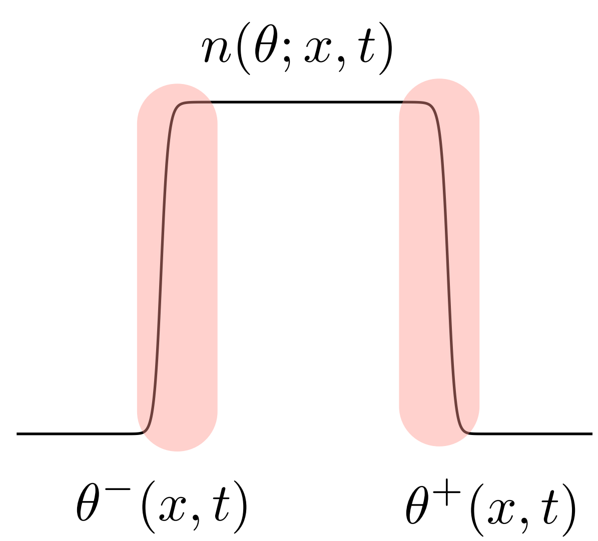
Urichuk et al 2023

$$\partial_t \rho \; + \; \partial_x(\eta\rho) = 0,$$

$$\partial_t \eta \; + \; \eta \partial_x \eta + \frac{\partial_x \mathcal{P}_s(\rho, T)}{\rho} = \frac{\partial_x(\mu(\rho,T) \partial_x \eta)}{\rho}$$

$$\mu(\rho, T) = \rho \frac{T}{4\pi} K (\partial_\rho \log K)^2 + O(T^3)$$

$$\partial_t T \; + \; \eta \partial_x T = -\mathcal{P}_s^T T \partial_x \eta + \frac{\mu(\rho, T)}{T \rho \tilde{\chi}_e} (\partial_x \eta)^2$$



From GHD to CHD (Bose gas)

$$\partial_t \rho + \partial_x (v^{\text{eff}} \rho) = \frac{1}{2} \partial_x (\mathfrak{D} \partial_x \rho)$$

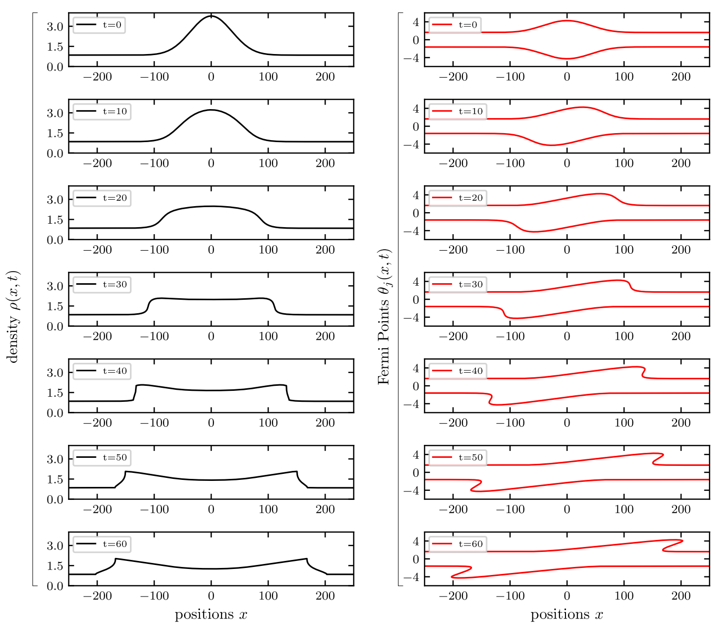
$$\delta n \to \sum_\sigma \sigma \delta(\theta - \theta^\sigma(x,t)) \delta\theta^\sigma(x,t)$$

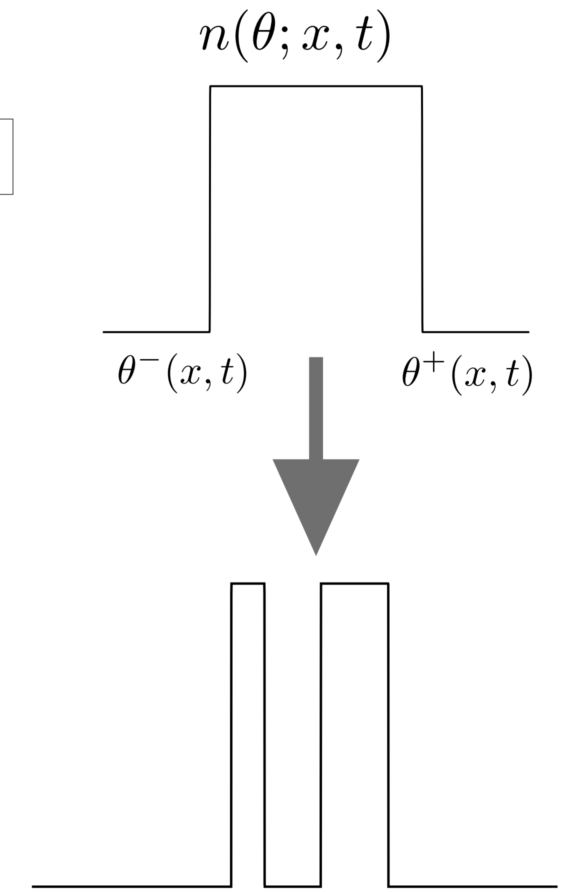
$$\partial_t \theta^\sigma + v^{\text{eff}}(\theta^\sigma) \partial_x \theta^\sigma = 0$$

$$\partial_t \rho \; + \; \partial_x(\eta\rho) = 0$$

$$\partial_t \eta \; + \; \eta \partial_x \eta + \rho^{-1} \partial_x \mathcal{P}_s^{(0)}(\rho) = 0$$

Doyon et al 2017





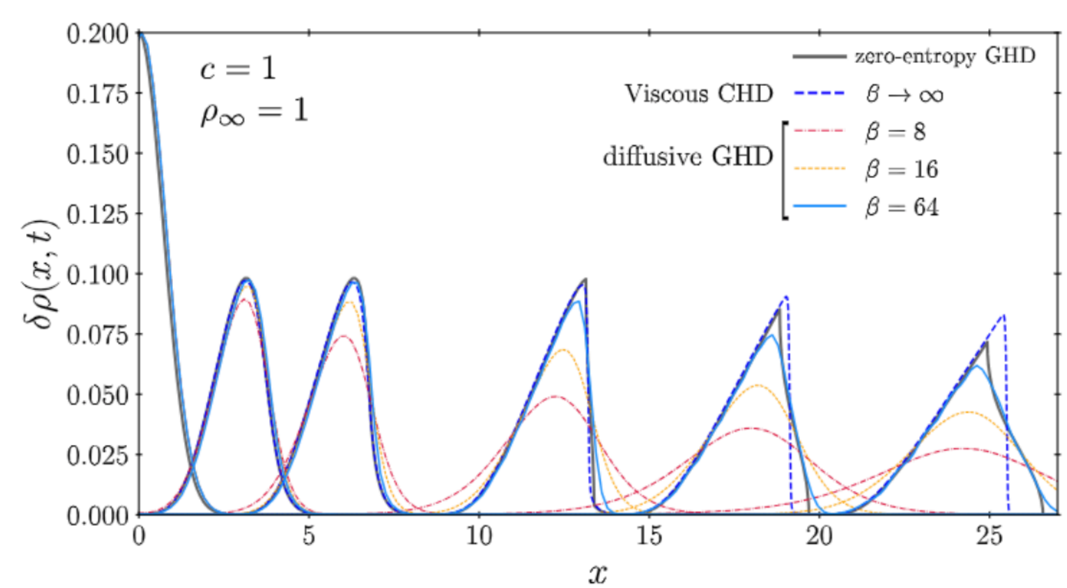
Urichuk et al 2023

$$\partial_t \rho \; + \; \partial_x(\eta\rho) = 0,$$

$$\partial_t \eta \; + \; \eta \partial_x \eta + \frac{\partial_x \mathcal{P}_s(\rho, T)}{\rho} = \frac{\partial_x(\mu(\rho,T) \partial_x \eta)}{\rho}$$

$$\mu(\rho, T) = \rho \frac{T}{4\pi} K (\partial_\rho \log K)^2 + O(T^3)$$

$$\partial_t T \; + \; \eta \partial_x T = -\mathcal{P}_s^T T \partial_x \eta + \frac{\mu(\rho,T)}{T \rho \tilde{\chi}_e} (\partial_x \eta)^2$$



From GHD to CHD (Bose gas)

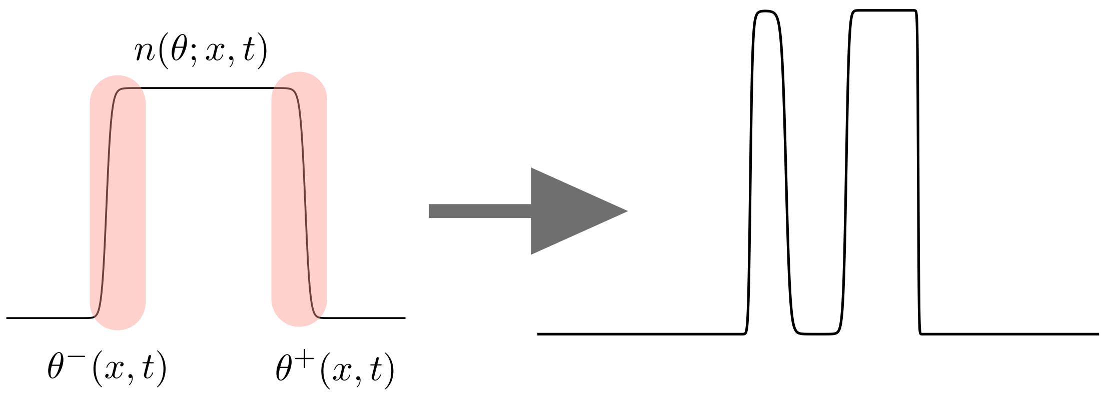
Urichuk et al 2023

$$\partial_t \rho \; + \; \partial_x(\eta\rho) = 0,$$

$$\partial_t \eta \; + \; \eta\partial_x\eta + \frac{\partial_x \mathcal{P}_s(\rho,T)}{\rho} = \frac{\partial_x(\mu(\rho,T)\partial_x\eta)}{\rho}$$

$$\mu(\rho,T) = \rho\frac{T}{4\pi}K(\partial_\rho \log K)^2 + O(T^3)$$

$$\partial_t T \; + \; \eta\partial_x T = -\mathcal{P}_s^T T\partial_x\eta + \frac{\mu(\rho,T)}{T\rho\tilde{\chi}_e}(\partial_x\eta)^2$$

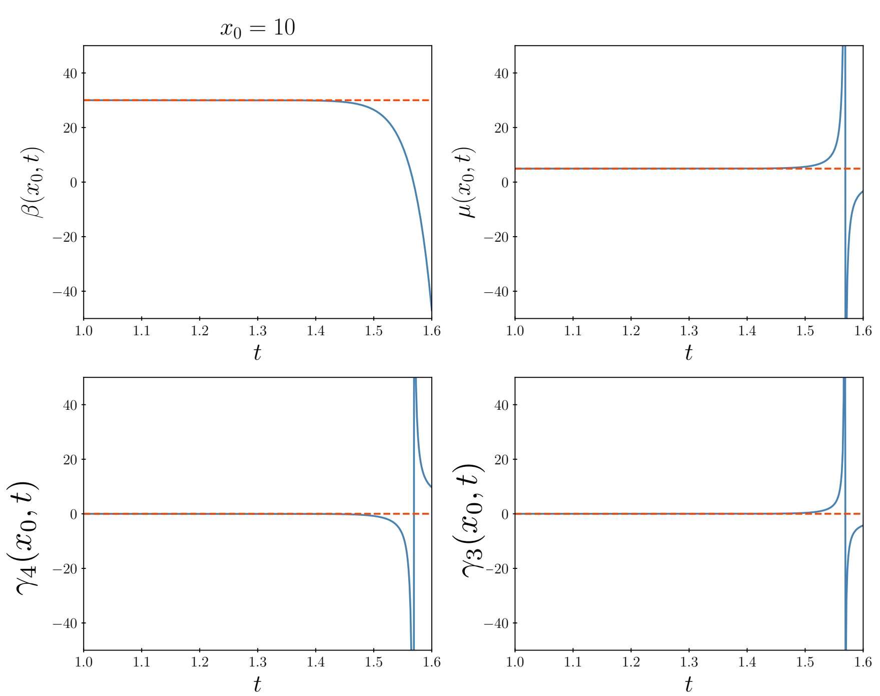


$$e^{-\beta(H-\mu N)} \to e^{-\beta(H+\gamma_3 Q_3+\gamma_4 Q_4-\mu N)}$$

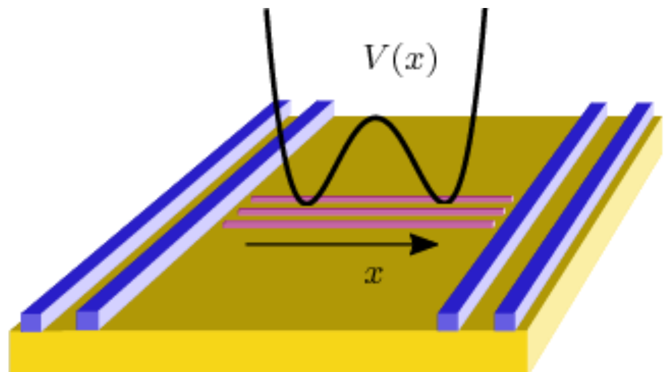
$$\widehat{\mathcal{D}}T \;=\; \frac{1}{T}\frac{G_2\chi_T^{(3)} - G_3\chi_T^{(2)}}{\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)}} + \left(\frac{\chi_{\gamma_4}^{(3)}\chi_{\gamma_3}^{(2)} - \chi_{\gamma_4}^{(2)}\chi_{\gamma_3}^{(3)}}{\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)}}\right)T\widehat{\mathcal{D}}\gamma_4,$$

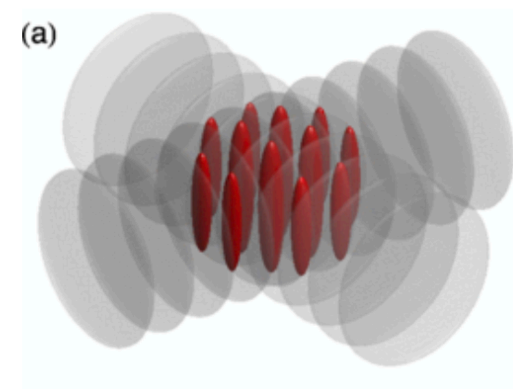
$$\widehat{\mathcal{D}}\gamma_3 \;=\; \frac{1}{T^2}\frac{G_3\chi_T^{(2)} - G_2\chi_T^{(3)}}{\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)}} - \frac{\chi_{\gamma_4}^{(3)}\chi_T^{(2)} - \chi_{\gamma_4}^{(2)}\chi_T^{(3)}}{\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)}}\widehat{\mathcal{D}}\gamma_4,$$

$$\widehat{\mathcal{D}}\gamma_4 \;=\; \frac{G_4(\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)}) - (\chi_T^{(4)} - \chi_{\gamma_3}^{(4)})\chi_T^{(3)}G_2 + (\chi_T^{(4)} - \chi_{\gamma_3}^{(4)})\chi_T^{(2)}G_3}{T^2\chi_{\gamma_4}^{(4)}(\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)})} + \frac{\chi_T^{(4)}(\chi_{\gamma_4}^{(3)}\chi_{\gamma_3}^{(2)} - \chi_{\gamma_4}^{(2)}\chi_{\gamma_4}^{(3)}) - \chi_{\gamma_3}^{(4)}(\chi_{\gamma_4}^{(3)}\chi_T^{(2)} - \chi_{\gamma_4}^{(2)}\chi_T^{(3)})}{\chi_{\gamma_4}^{(4)}(\chi_{\gamma_3}^{(3)}\chi_T^{(2)} - \chi_{\gamma_3}^{(2)}\chi_T^{(3)})}.$$



Thermalisation induced by external forces





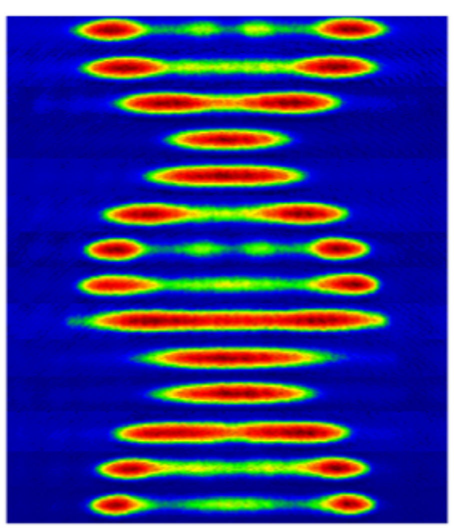
MENU nature

Letter | Published: 13 April 2006

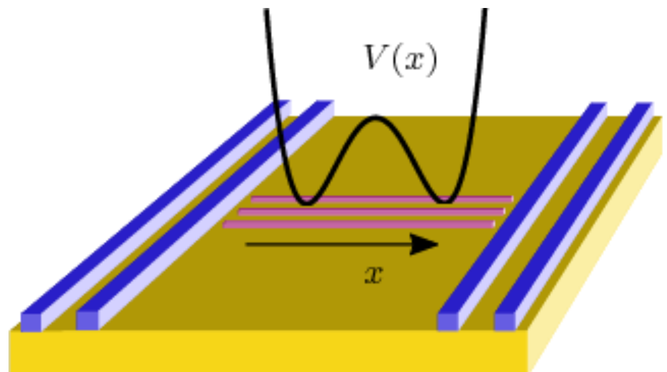
A quantum Newton's cradle

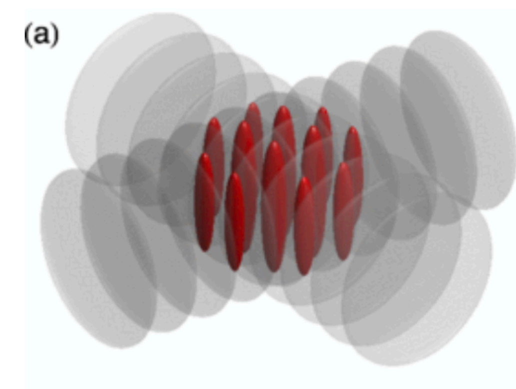
Toshiya Kinoshita, Trevor Wenger & David S. Weiss

Nature **440**, 900–903(2006) | Cite this article



Thermalisation induced by external forces





MENU
nature

Letter | Published: 13 April 2006

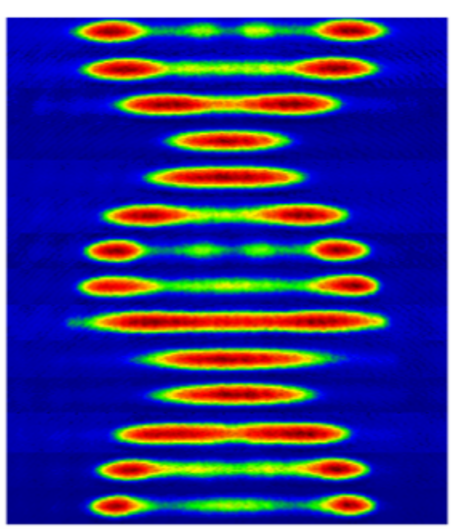
A quantum Newton's cradle

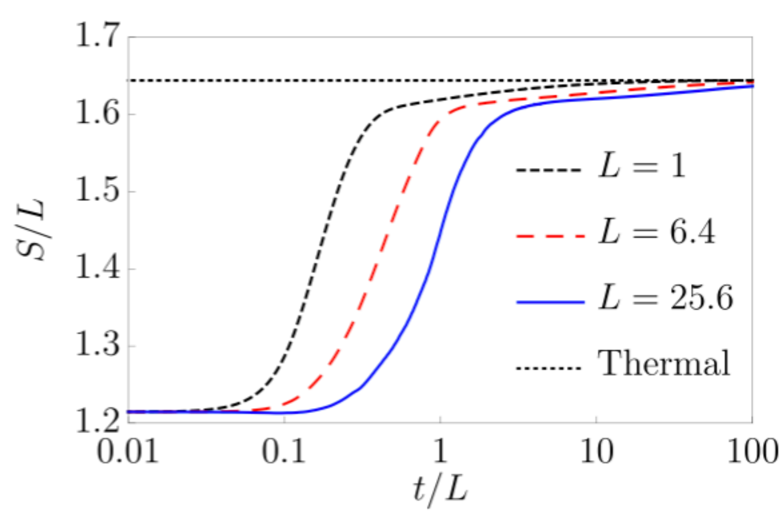
Toshiya Kinoshita, Trevor Wenger & David S. Weiss

Nature **440**, 900–903(2006) | Cite this article

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$

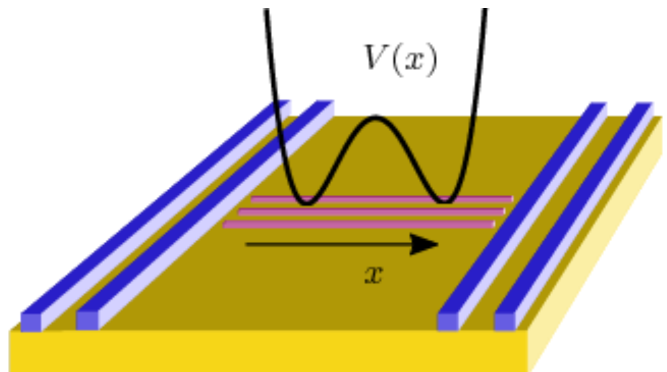
$$\partial_t S \geq 0 \qquad \rho \to \rho_{\text{thermal}}$$

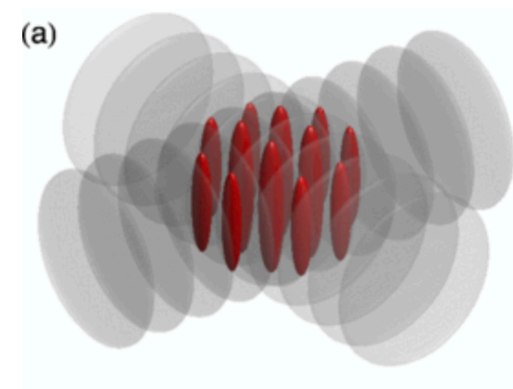




Bastianello et al, 2020

Thermalisation induced by external forces





MENU

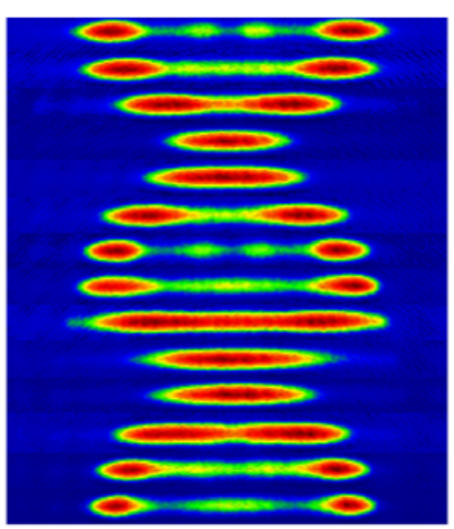
nature

Letter | Published: 13 April 2006

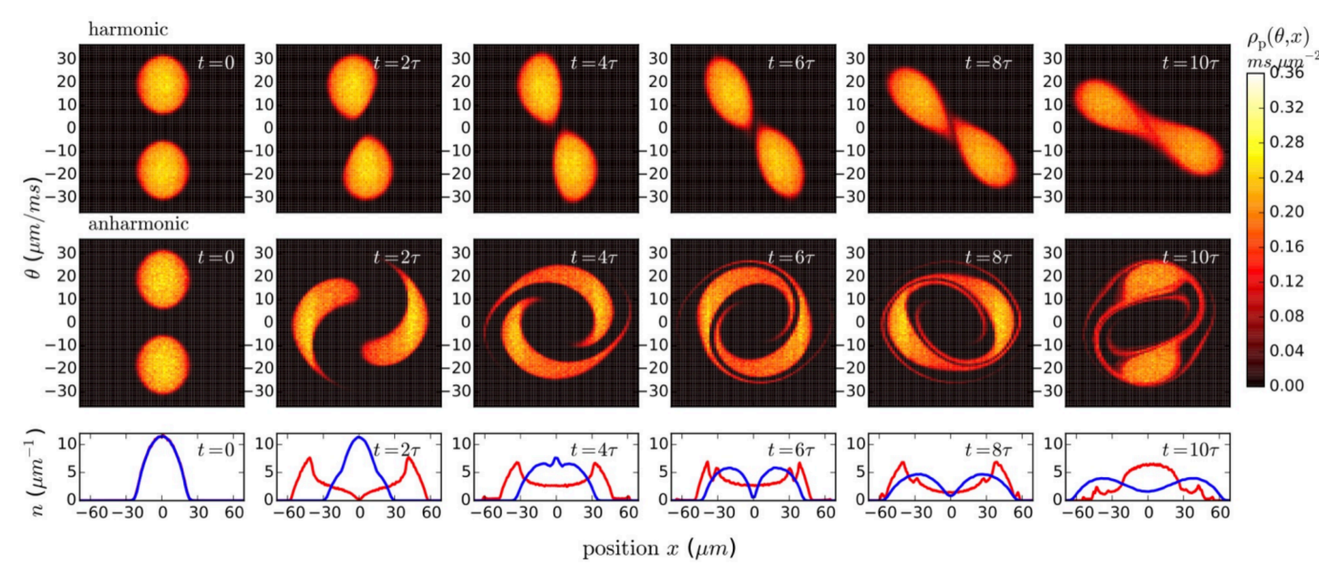
A quantum Newton's cradle

Toshiya Kinoshita, Trevor Wenger & David S. Weiss

Nature **440**, 900–903(2006) | Cite this article



$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$



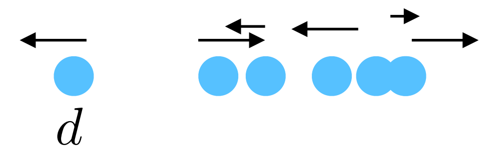
Caux et al, 2018

3-stage thermalisation in Hard rods

Biagetti et al, 2023

$$m(x) = (1 + m_0 \cos(2\pi x/\ell))^{-1}; \quad V(x) = V_0 \cos(2\pi x/\ell)$$

$$H = \sum_{i=1}^{N} \left[\frac{\theta_i^2}{2m(x_i)} + V(x_i) \right] + \sum_{i<j} U(x_i - x_j)$$

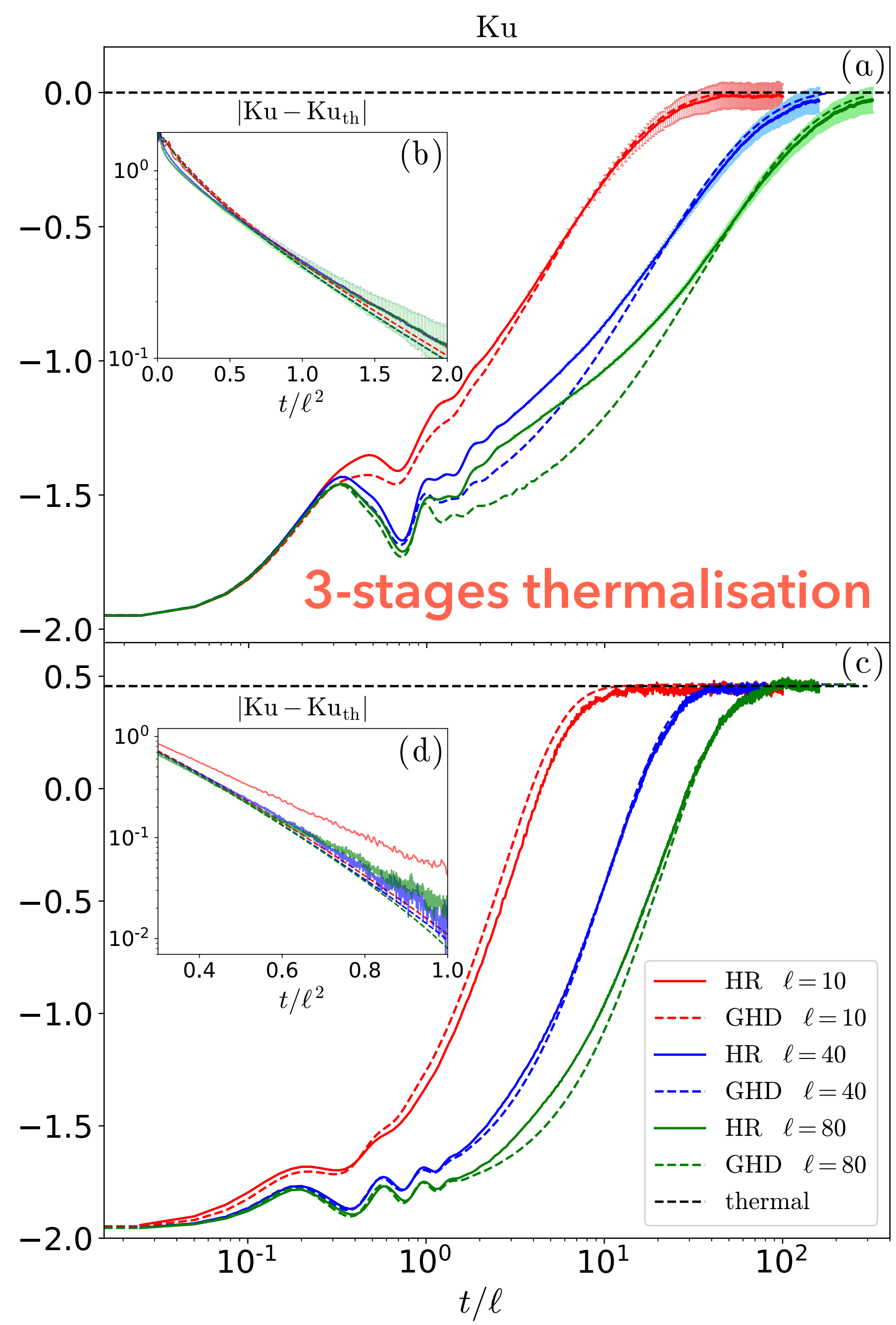


$$\rho(\theta, x, t=0) \sim e^{-\frac{1}{2}\left(\frac{\theta - \theta_{\mathrm{Bragg}}}{\sigma_{\mathrm{Bragg}}}\right)^2} + e^{-\frac{1}{2}\left(\frac{\theta + \theta_{\mathrm{Bragg}}}{\sigma_{\mathrm{Bragg}}}\right)^2},$$

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\mathrm{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$

$$\rho_{\mathrm{th}}(\theta; x) \sim e^{-\beta(m(x)\theta^2/2 + V(x) - \mu)}$$

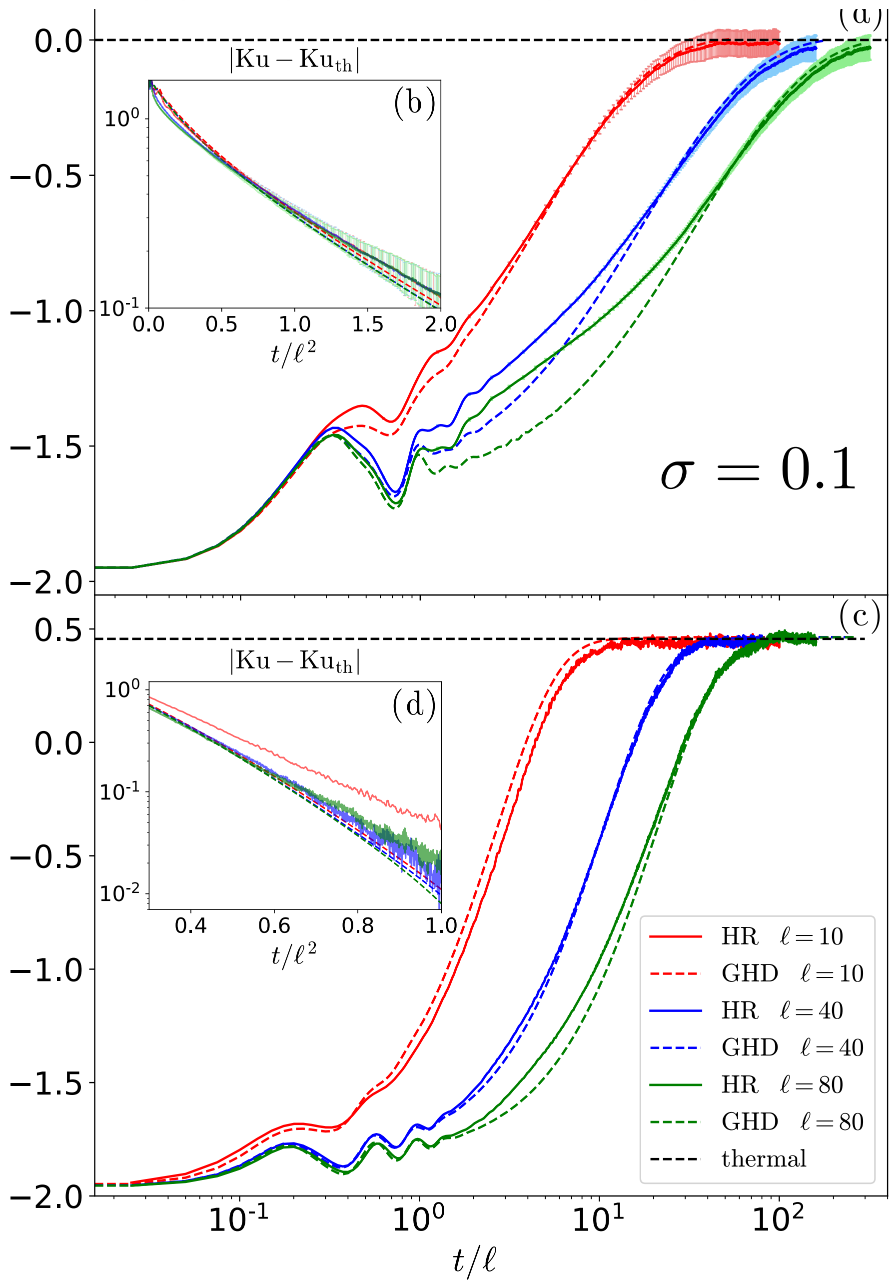
$$\hat{\rho}(\theta; t) = \int dx \rho(\theta; x, t)$$



3-stage thermalisation in Hard rods

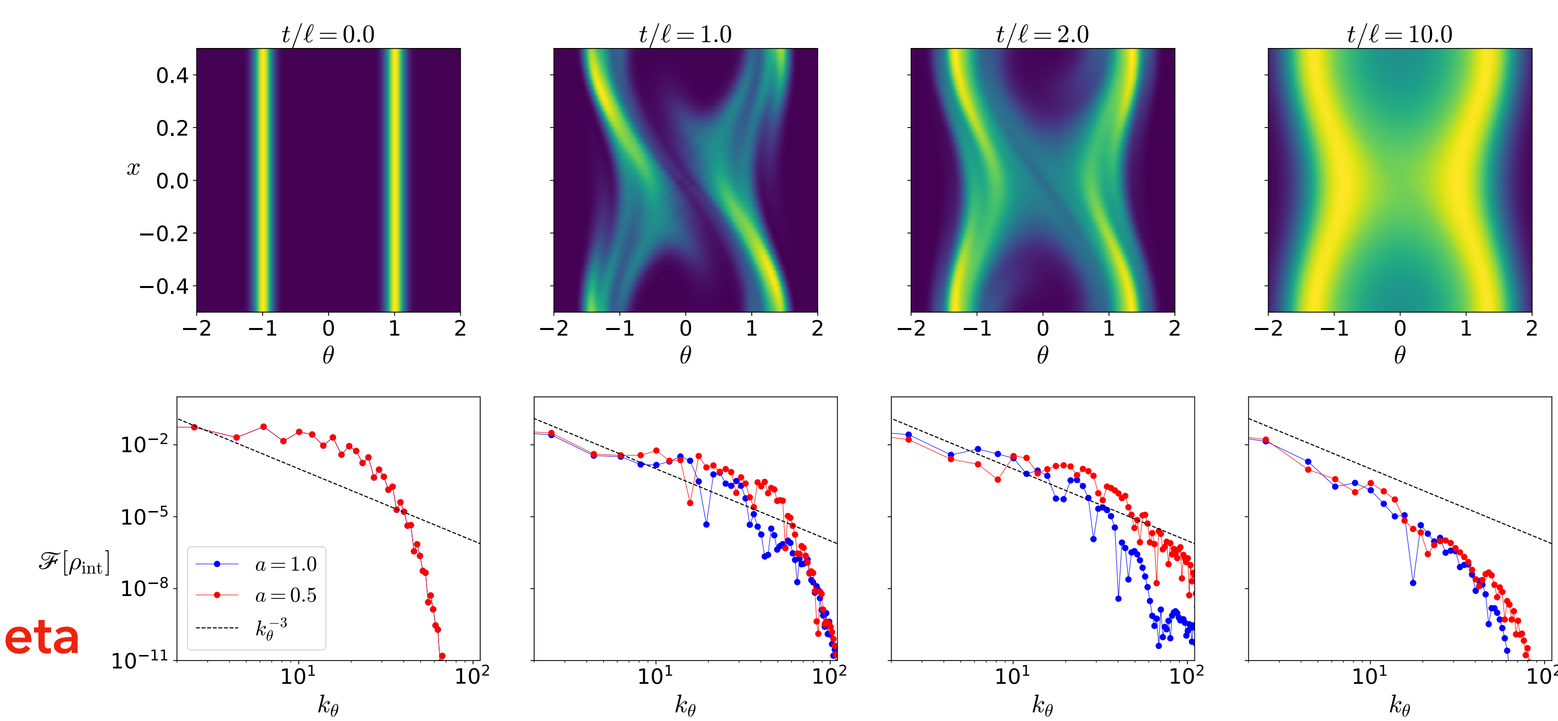
Biagetti et al, 2023

$$m(x) = (1 + m_0 \cos(2\pi x/\ell))^{-1}; \quad V(x) = V_0 \cos(2\pi x/\ell)$$



$$\rho(\theta, x, t=0) \sim e^{-\frac{1}{2}\left(\frac{\theta-\theta_{\text{Bragg}}}{\sigma_{\text{Bragg}}}\right)^2} + e^{-\frac{1}{2}\left(\frac{\theta+\theta_{\text{Bragg}}}{\sigma_{\text{Bragg}}}\right)^2},$$

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$



Fourier transform in theta

$$\mathfrak{F}[\hat{\rho}](k_\theta) = \int dk_\theta e^{ik_\theta \theta} \left[\int dx \rho(\theta; x, t) \right]$$

"turbulent" phase

3-stage thermalisation in Hard rods

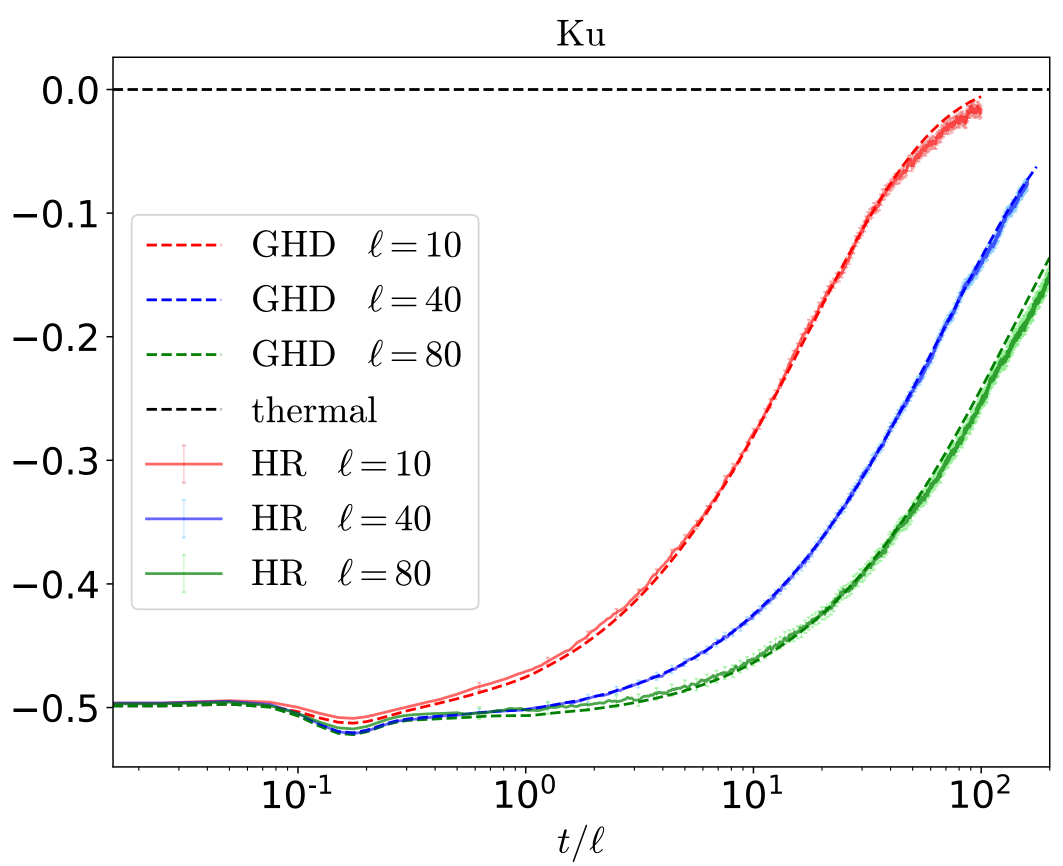
Biagetti et al, 2023

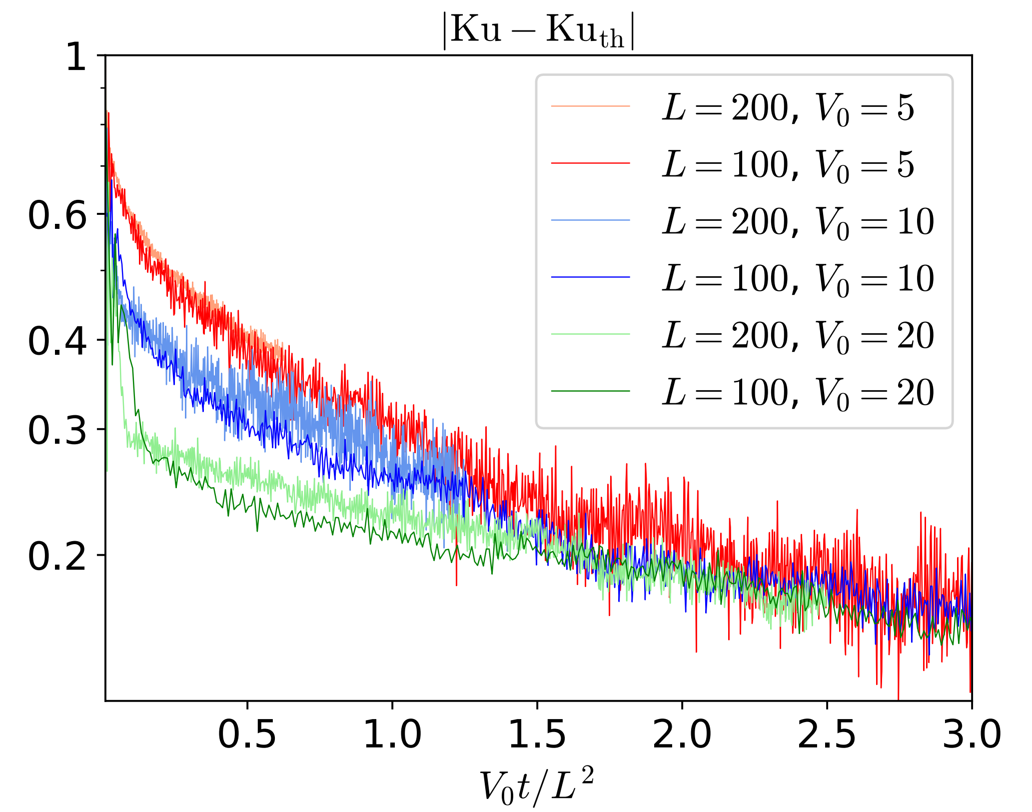
$$m(x) = (1 + m_0 \cos(2\pi x/\ell))^{-1}; \quad V(x) = V_0 \cos(2\pi x/\ell)$$

$$\rho(\theta, x, t=0) \sim e^{-\frac{1}{2}\left(\frac{\theta-\theta_{\rm Bragg}}{\sigma_{\rm Bragg}}\right)^2} + e^{-\frac{1}{2}\left(\frac{\theta+\theta_{\rm Bragg}}{\sigma_{\rm Bragg}}\right)^2},$$

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\rm eff}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$

$\sigma = 1$



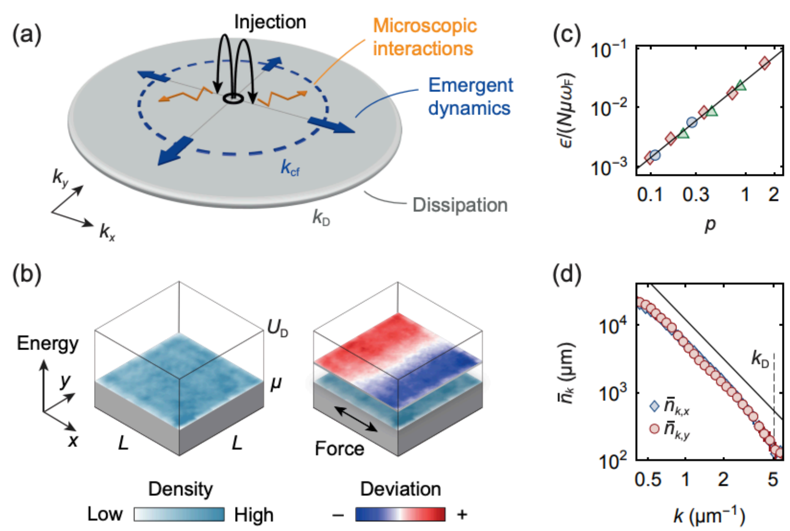


Wave turbulence?

Emergence of isotropy and dynamic scaling in 2D wave turbulence in a homogeneous Bose gas

Maciej Gałka,[1,*] Panagiotis Christodoulou,[1] Martin Gazo,[1] Andrey Karailiev,[1]
Nishant Dogra,[1] Julian Schmitt,[1,2] and Zoran Hadzibabic[1]

[1]*Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
[2]*Institut für Angewandte Physik, Universität Bonn, Wegelerstraße 8, 53115 Bonn, Germany*
(Dated: November 4, 2022)



$$i\partial_t \psi = -\frac{\nabla^2 \psi}{2m} + g|\psi|^2\psi$$

$$\psi \sim a_k e^{ikx - i\omega_k t}$$

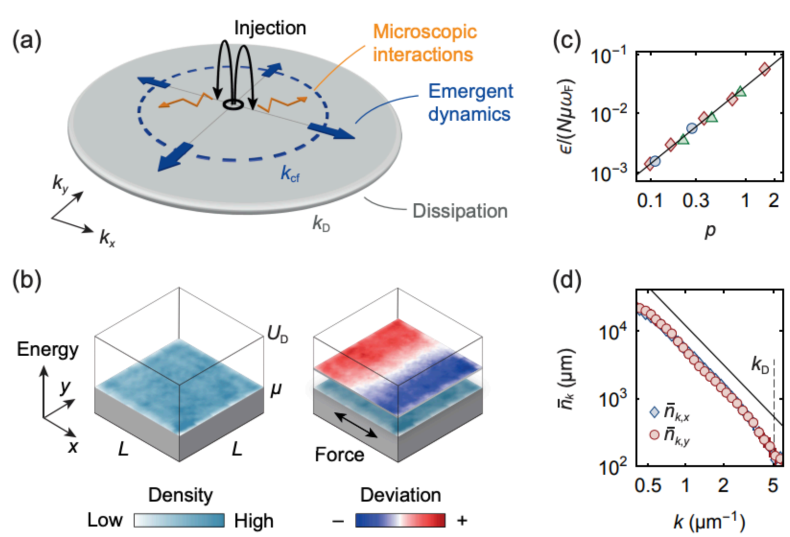
$$\partial_t \langle n_k \rangle = \mathcal{G}[g, n_k]$$

Wave turbulence also in GHD?

Emergence of isotropy and dynamic scaling in 2D wave turbulence in a homogeneous Bose gas

Maciej Gałka,[1,*] Panagiotis Christodoulou,[1] Martin Gazo,[1] Andrey Karailiev,[1]
Nishant Dogra,[1] Julian Schmitt,[1,2] and Zoran Hadzibabic[1]

[1] *Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
[2] *Institut für Angewandte Physik, Universität Bonn, Wegelerstraße 8, 53115 Bonn, Germany*
(Dated: November 4, 2022)



$$i\partial_t \psi = -\frac{\nabla^2 \psi}{2m} + g|\psi|^2\psi$$

$$\psi \sim a_k e^{ikx - i\omega_k t}$$

$$\partial_t \langle n_k \rangle = \mathcal{G}[g, n_k]$$

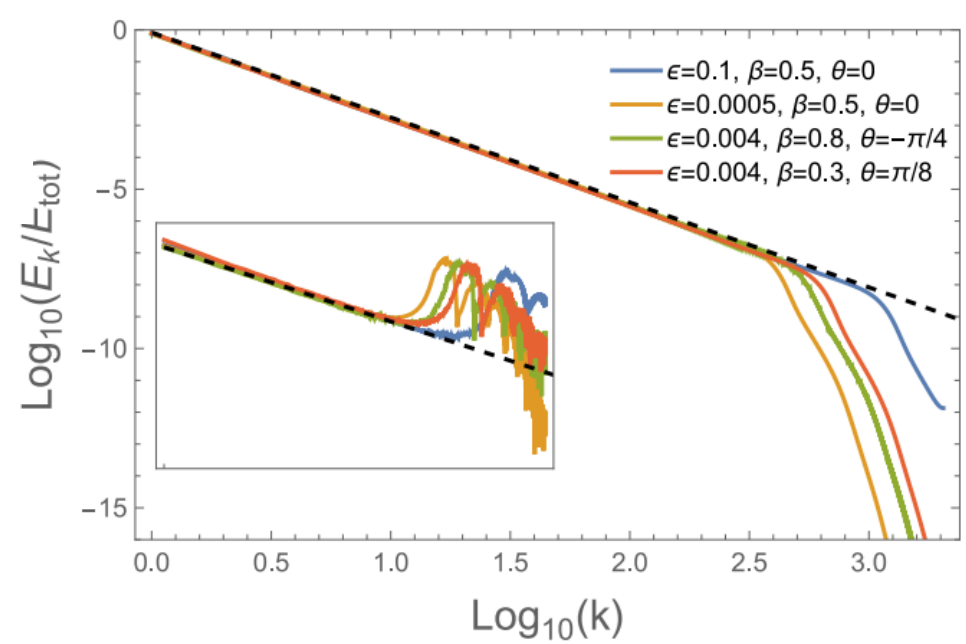
Burgers turbulence in the Fermi-Pasta-Ulam-Tsingou chain

Matteo Gallone,[1,*] Matteo Marian,[2,†] Antonio Ponno,[3,‡] and Stefano Ruffo[1,4,5,§]

[1] *SISSA, Via Bonomea 265, 34136 Trieste, Italy*
[2] *Department of Physics, University of Trieste, Via A. Valerio 2, 34127 Trieste, Italy*
[3] *Department of Mathematics "T. Levi-Civita", University of Padova, Via Trieste 63, 35121 Padova, Italy*
[4] *INFN Sezione di Trieste*
[5] *ISC-CNR, via Madonna del Piano 10, 50019 Sesto Fiorentino (Firenze)*
(Dated: August 19, 2022)

$$H = \sum_{j=1}^{N} \left[\frac{p_j^2}{2} + V(q_{j+1} - q_j) \right]$$

$$V(y) = \frac{y^2}{2} + \frac{\alpha y^3}{3} + \frac{\beta y^4}{4}$$



Conclusions

$$\partial_t \rho + \nabla \cdot (\boldsymbol{J}^{\text{eff}}_{[\rho]} \rho) = \frac{1}{2} \nabla \cdot (\mathfrak{D}_{[\rho]} \nabla \rho)$$

Diffusion or viscosities as a universal mechanism to understand integrability breaking thermalisation and heating

relations to non-linear Luttinger liquids?

More complete characterisation of integrability breaking and its relaxation dynamics

thermodynamic form factors

universal characterisation of integrability breaking terms?

A lot to explore in the full "2D" GHD equation

Is turbulence a universal phenomena?

Low-temperature viscosity and dispersive terms

relation to dispersive shock waves?