Theory and Phenomenology of Fundamental Interactions
UNIVERSITY AND INFN · BOLOGNA

Istituto Nazionale di Fisica Nucleare
Sezione di Bologna

Entanglement and Quantum Information Classifiers for Quantum Phases of Matter

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10th Bologna Workshop

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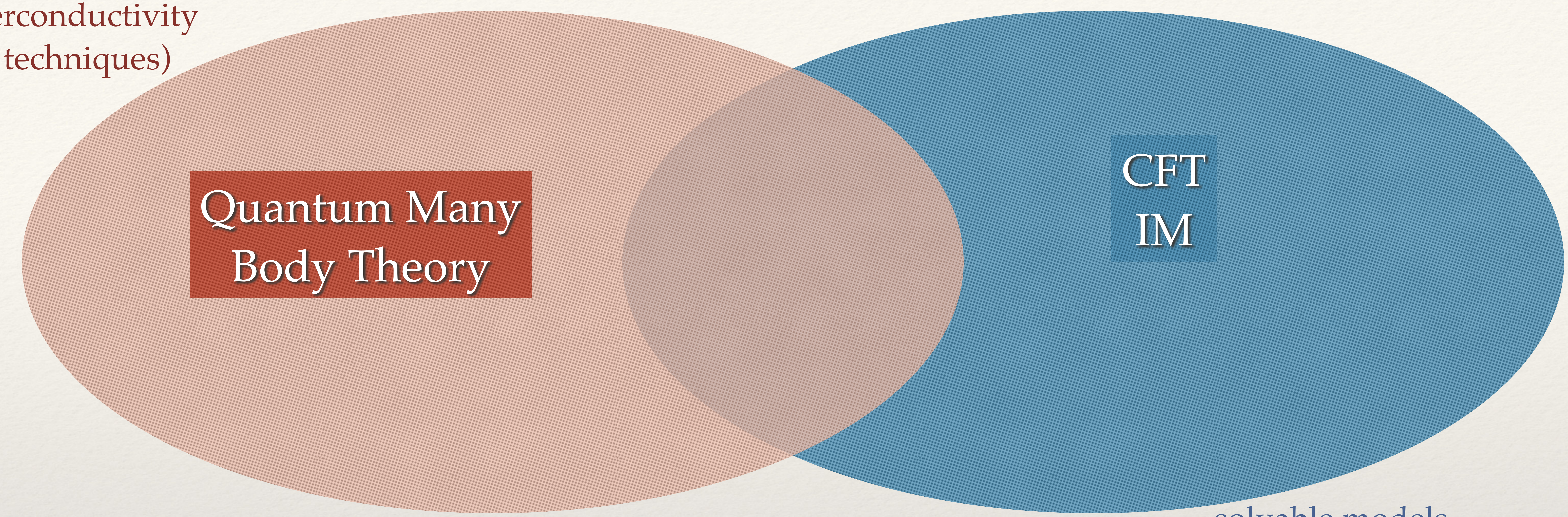
paradigm: superconductivity
(perturbative techniques)

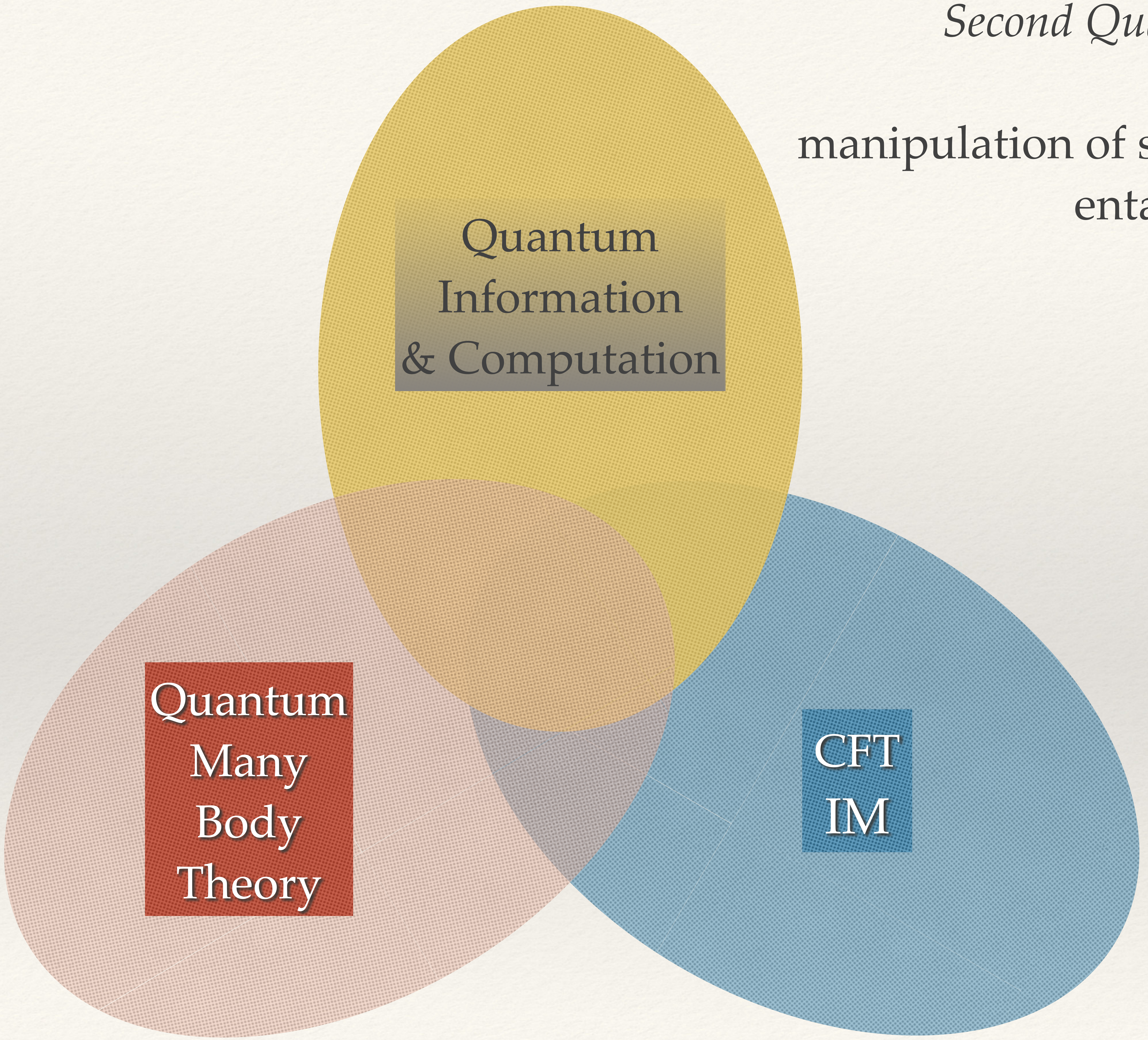


solvable models
(effective) field theories
universality and classification of QPT

low-dimensional + strongly interacting systems
quantum nonperturbative and geometrical effects
quantum phases of matter

numerics - DMRG

e.g.
XXZ spin ½ chain
fermionic models
sine-gordon



Second Quantum Revolution

manipulation of single quantum systems
entanglement

Quantum Simulations

new analytical tools for theoretical investigation
new experimental platforms

⇩

new insight for the study of quantum phase transitions and phases of matter

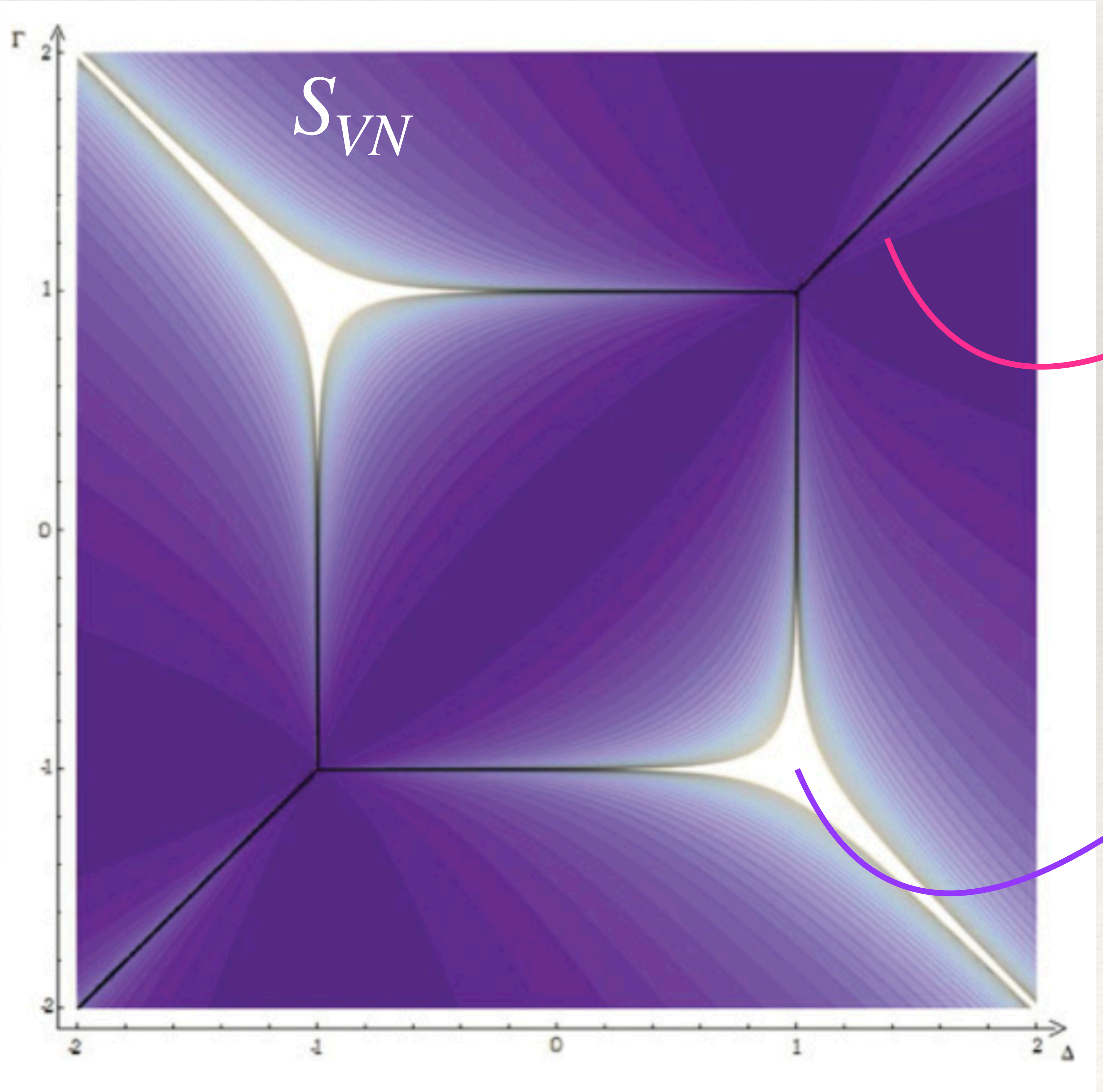
- Bipartite entanglement via Renyi (von Neumann) entropy

$$\mathcal{S} = A \cup B \qquad \rho_A = Tr_B[\rho_T]$$

$$S_\alpha = \frac{1}{1-\alpha} Tr[\rho_A^\alpha]$$

E.E., Stefano Evangelisti, Fabio Franchini, Francesco Ravanini
PLA274 (2010) 2101, PRB 83 (2011) 012402, PRB 85 (2012) 115428 , PRB 88 (2013) 104418

XYZ MODEL



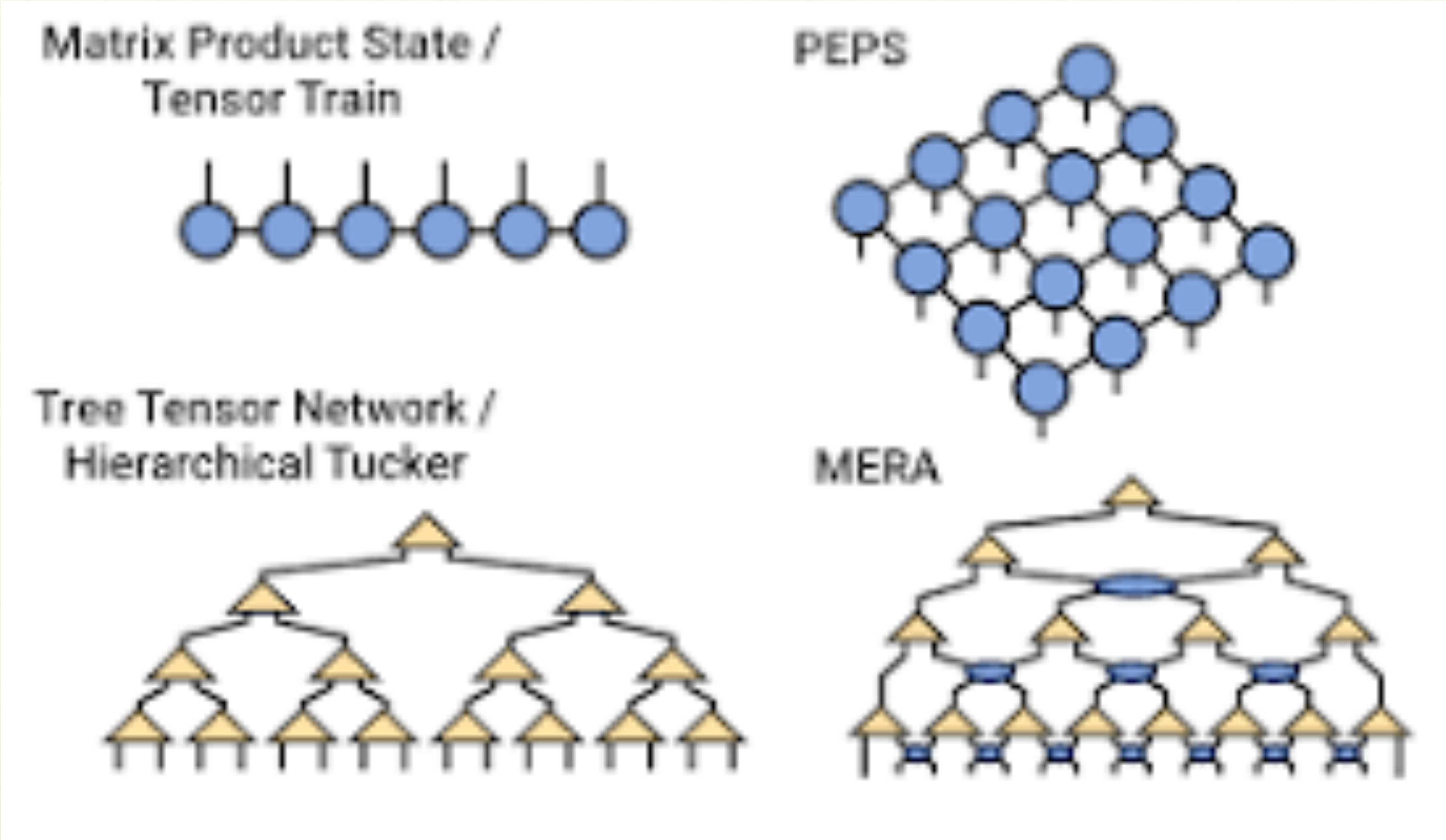
QPT divergence

essential singularity

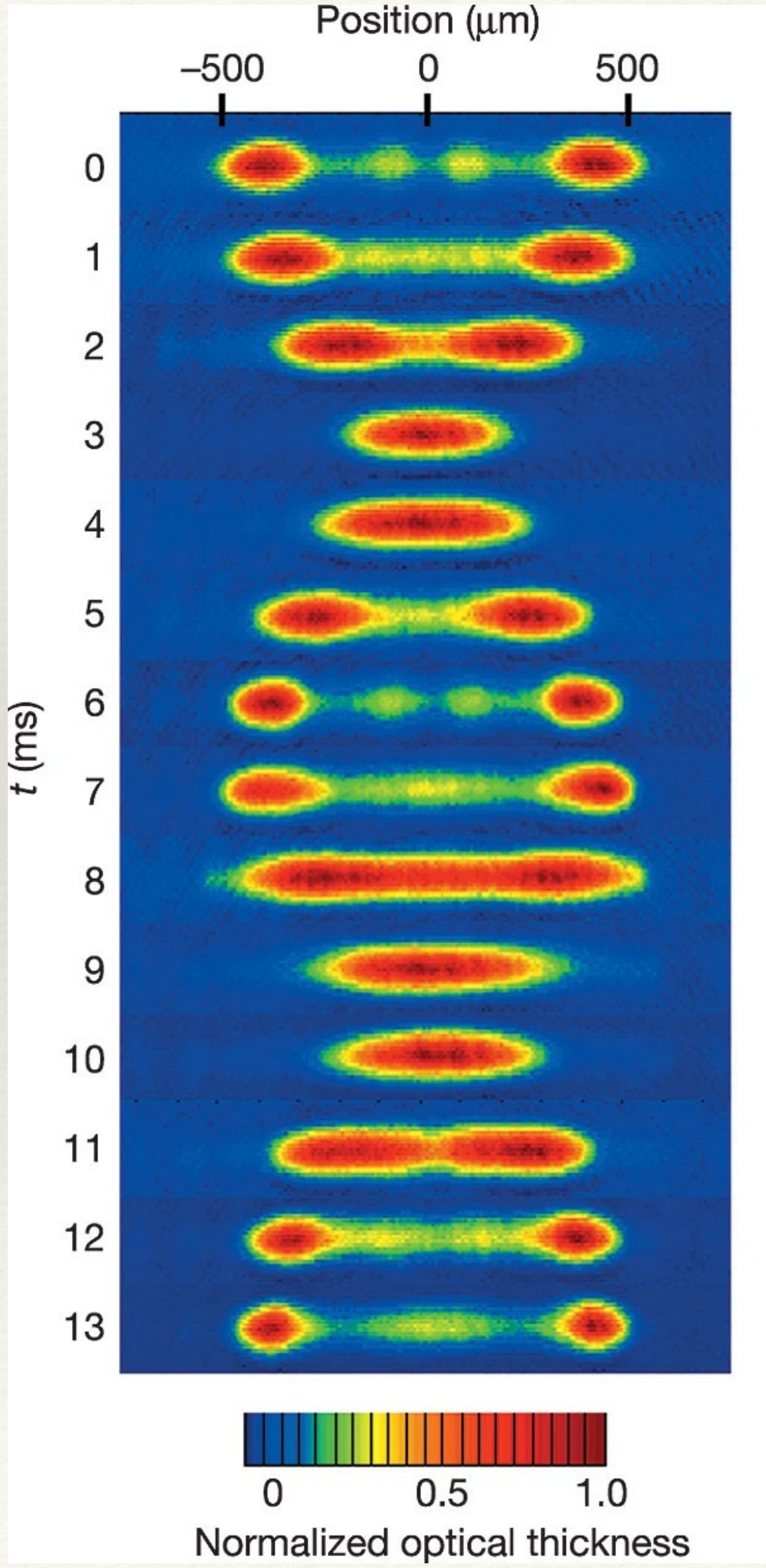
Continuum Limit to Sine-Gordon model

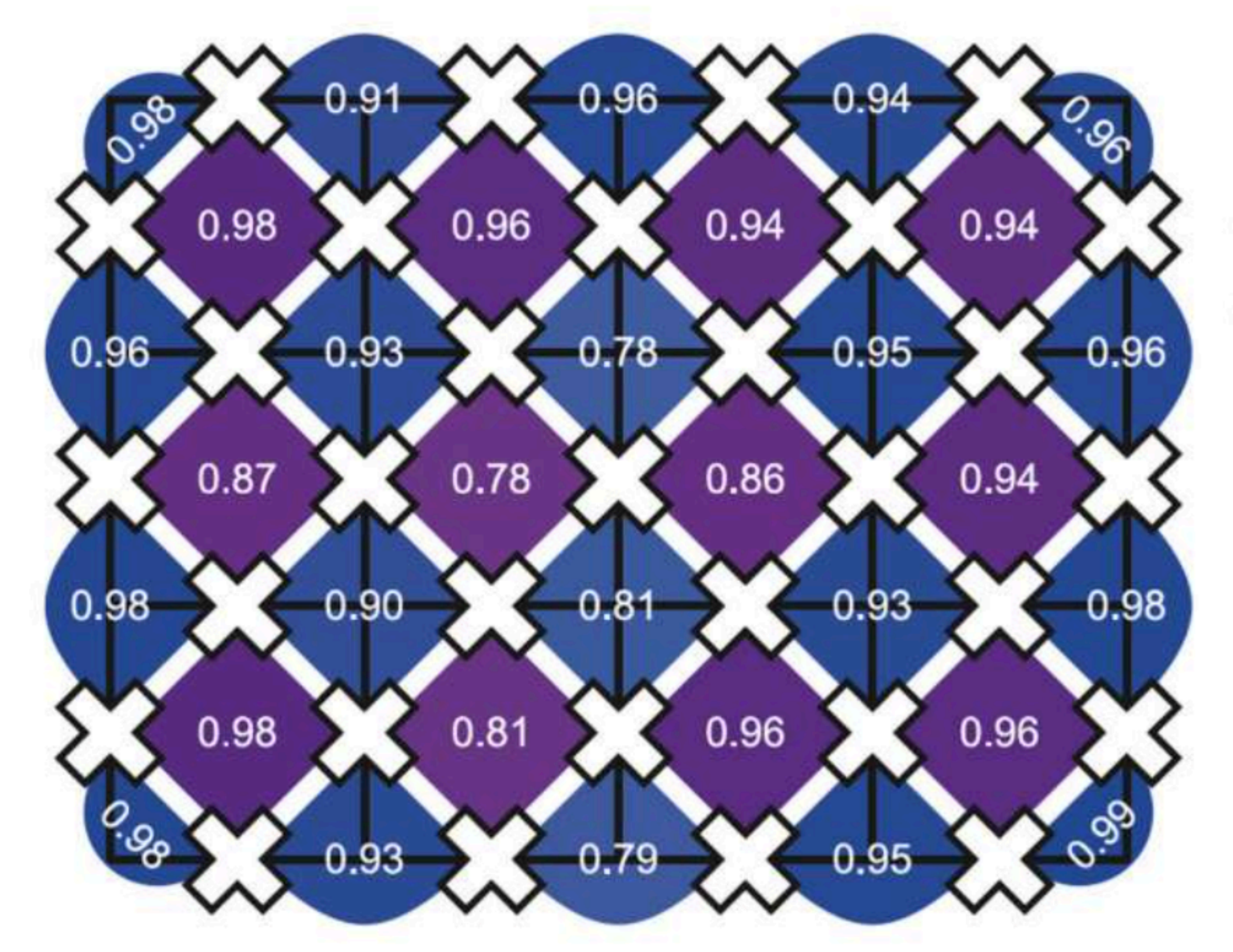
$$S_{sG} = \frac{1}{6}\ln\left(\frac{1}{Ma}\right) + \frac{1}{6}\ln\left(\frac{\sin[\pi(1-\frac{\beta^2}{8\pi})]}{(1-\frac{\beta^2}{8\pi})}\right) + O(1/\ln(a))$$

quantum-inspired numerics
DMRG
MPS
Tensor Networks



out-of equilibrium phenomena
quench protocols & Kibble-Zurek
thermalisation
real-time dynamics





topological
phases and transitions
classification
edge states/anyons

LONG RANGE KITAEV CHAIN

$$H_L = -t\sum_{j=1}^{L}(a_j^\dagger a_{j+1} + \text{H.c.}) - \mu\sum_{j=1}^{L}\left(n_j - \frac{1}{2}\right) + \frac{\Delta}{2}\sum_{j=1}^{L}\sum_{\ell=1}^{L-1} d_\ell^{-\alpha}(a_j a_{j+\ell} + a_{j+\ell}^\dagger a_j^\dagger).$$

short range $\alpha \to \infty$
long range $\alpha < 1$

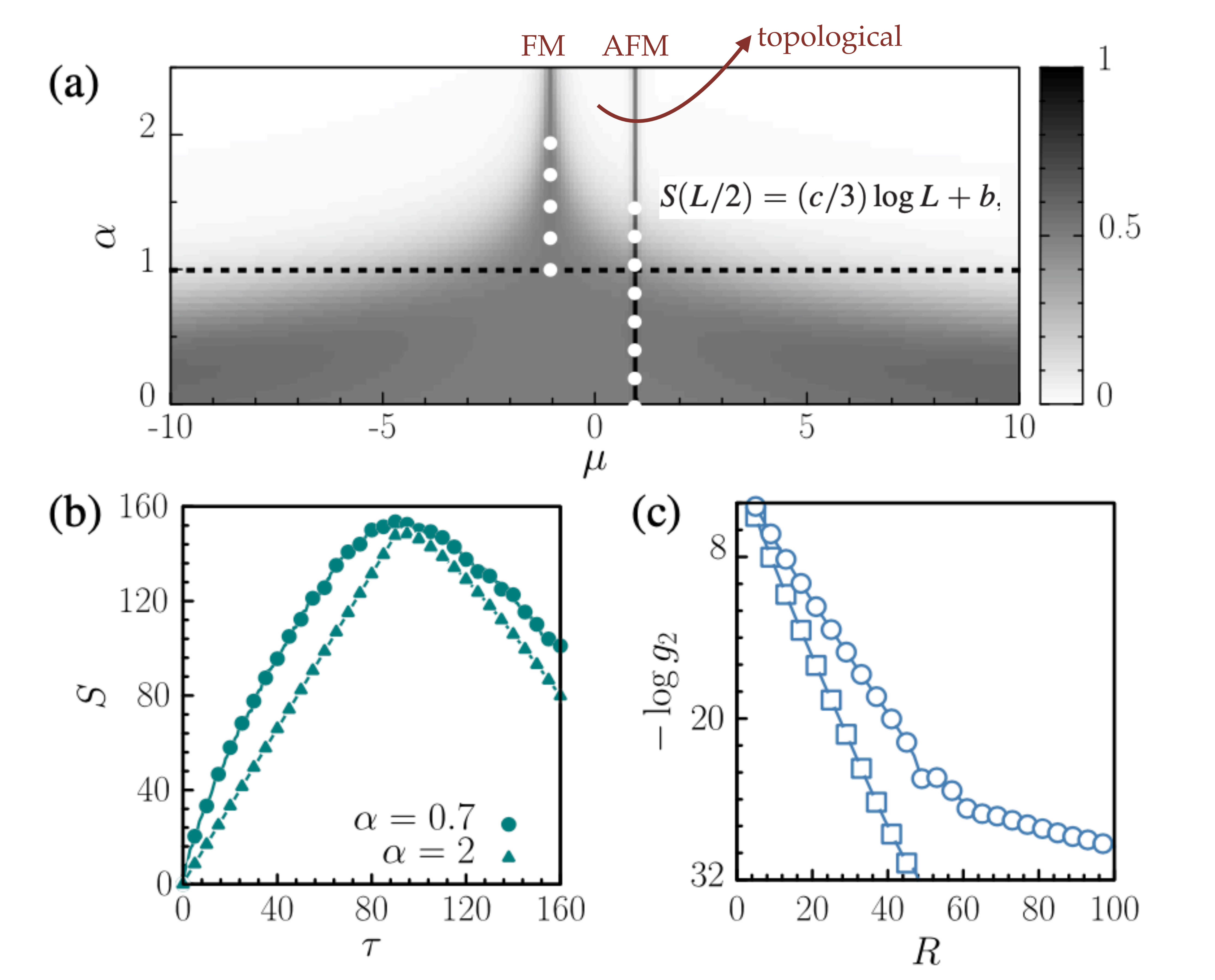


FIG. 1 (color online). (a) Effective central charge c_{eff} obtained by fitting $S(L/2)$. Two gapless conformal field theories with $c = 1/2$ are visible for $\mu = 1$ ($\alpha > 3/2$) and $\mu = -1$ ($\alpha > 2$). White vertical dotted lines: gapless lines with broken conformal symmetry. Horizontal dashed line separates two regions: correlation functions display a hybrid exponential-algebraic ($\alpha > 1$) and purely algebraic decay ($\alpha < 1$). (b) Time evolution of $S(L/2)$ after a quench from a product state with $\mu \gg 1$ to $\mu = 1$: $\alpha > 1$, $S(L/2)$ grows linearly, $\alpha < 1$, $S(L/2)$ grows logarithmically. (c) $g_2(R)$ correlation function for $\mu = 2$ and $\alpha = 10$ (squares), showing exponential behavior and $\alpha = 7$ (circles), showing an exponential with an algebraic tail even in the gapped region.

D. Vodola et al., PRL 113 (2014) 156402New J. Phys. 18 (2015) 015001

MULTIPARTITE ENTANGLEMENT & FISHER INFORMATION

k-producible state

$$|\psi_{k-\mathrm{prod}}\rangle = \bigotimes_{l=1}^{M} |\psi_l\rangle$$

with $|\psi_l\rangle$ with $N_l \leq k$ particles, entangled

k-entangled state = k-producible but not (k-1)-producible

Quantum Fisher Information

$$F_{\mathrm{Q}}[\rho(\theta)] \equiv \max_{\hat{E}_\mu} F[\rho(\theta), \hat{E}_\mu]$$

with: $\rho(\theta) = e^{-i\theta\hat{O}}\rho e^{i\theta\hat{O}}$; $\{\hat{E}_\mu\}_\mu$ is a POVM and $F[\rho(\theta, \hat{E}_\mu] = \sum_\mu \frac{(\partial_\theta Tr[\rho(\theta)\hat{E}_\mu])^2}{Tr[\rho(\theta)\hat{E}_\mu]}$

for pure states

$$F_{\mathrm{Q}}[|\psi\rangle, \hat{O}] = 4(\Delta\hat{O})^2$$

for k-producible states

$$F_{\mathrm{Q}}[|\psi\rangle_{k-\mathrm{prod}}, \hat{O}] \leq sk^2 + r^2$$

$(N = sk + r)$

P. Hyllus et al. PRA 85 (2012) 022321

F. Dell'Anna et al., arXiv:2307.02407

what is QFI saying about quantum passes of matter and transitions?

bilinear-biquadratic model S=1

$$H = J' \sum_{i=1}^{N} \left[\cos(\theta)\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} - \sin(\theta)(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})^2\right]$$

$\beta = \tan\theta$

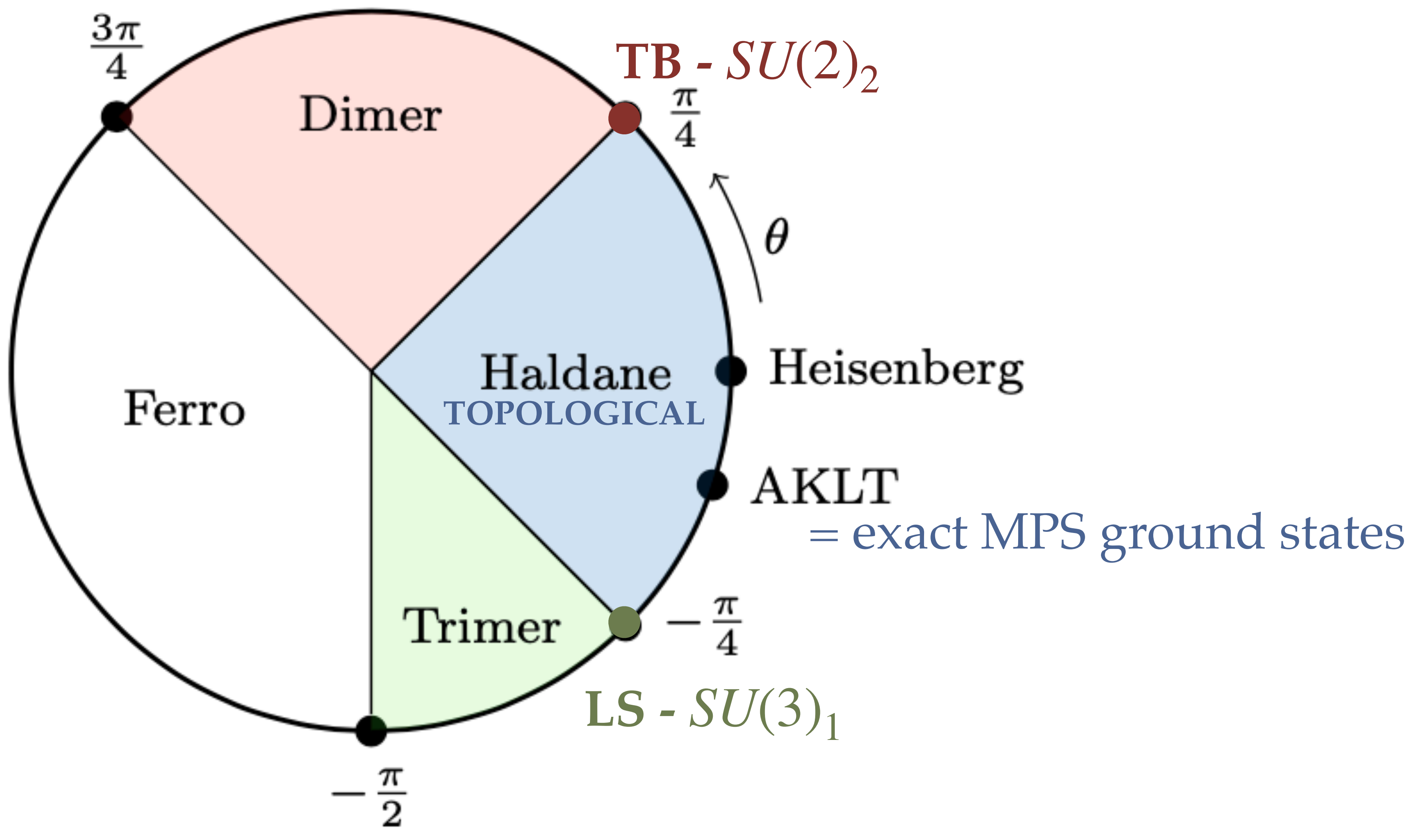
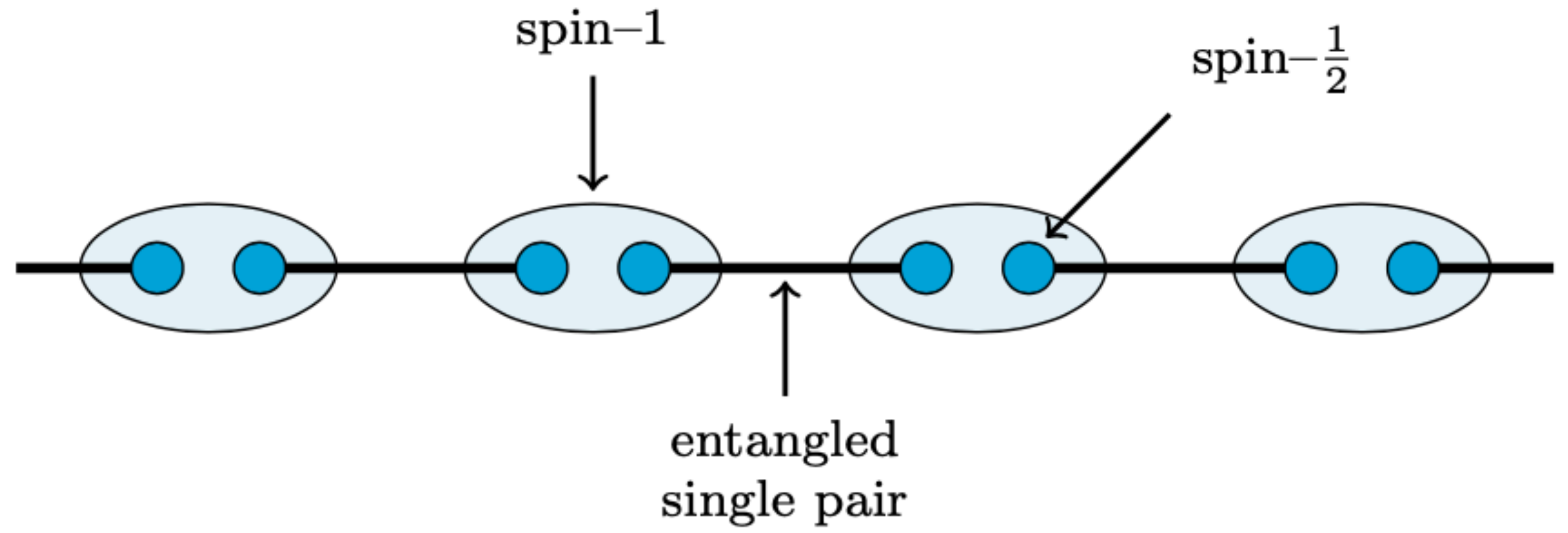
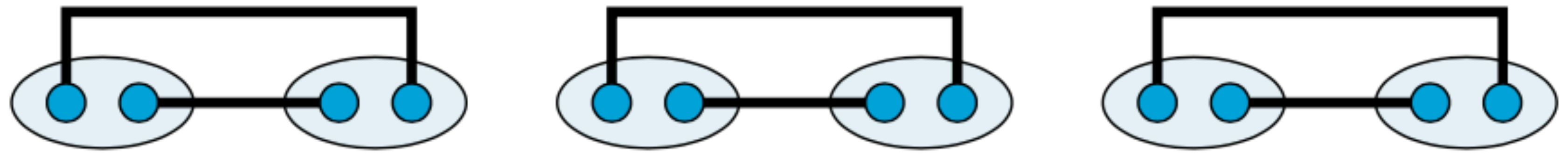


Figure 1. Phase diagram of BLBQ model with some remarkable points: $\theta = 0$ the AF Heisenberg model, $\theta = \arctan(-1/3)$ the AKLT point and, $\theta = \pm\pi/4$ the Takhtajan-Babujian models and Lai-Sutherland respectively.



all entangled

Figure 2. Entangled pair structure of the AKLT's ground state in the VBS representation: each valence bond represents a singlet state (thick black); every site, represented by an oval, contains two spin-1/2 particles (blue dots).



(k=2)-producible

Figure 3. An example of spin-1 dimer state in the VBS representation with six sites. The thick lines join the entangled spin-1/2 degrees of freedom.

$$F_Q[\,|\psi\rangle\,, O] = \left[\langle\psi| O^2 |\psi\rangle - \langle\psi| O |\psi\rangle^2\right]$$

$$O^z = \sum_{j=1}^{N} S^z \qquad \tilde{O}^z = \sum_{j=1}^{N} \tilde{S}^z \qquad \tilde{S}^z = e^{i\pi \sum_{\ell<j} S^z_\ell} S^z_j$$

$$F_Q = \sum_{i=1}^{N} M_{ii} - 2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} M_{ij} - \left(\sum_{i=1}^{N} V_i\right)^2$$

$$f_Q(O^\alpha) = F_Q/N$$

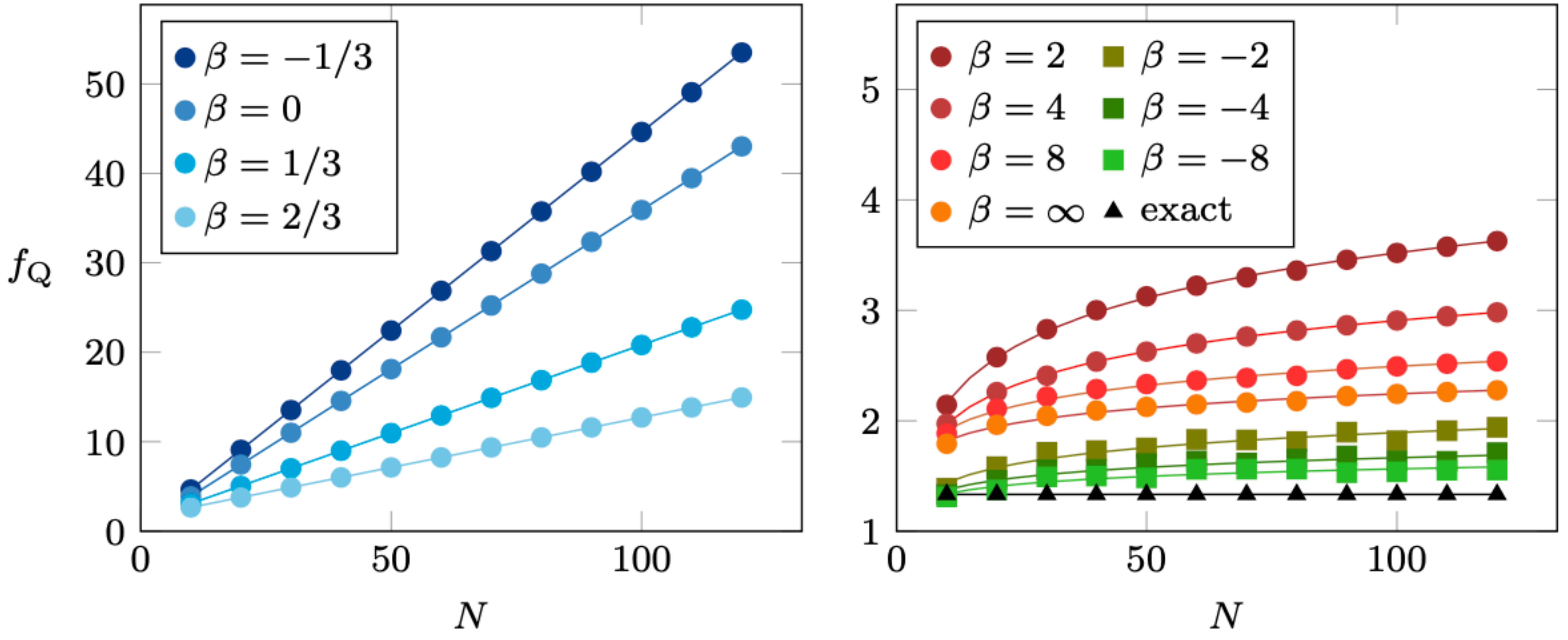
$$M = \begin{pmatrix} \langle (S_1^z)^2 \rangle & \langle S_1^z S_2^z \rangle & \langle S_1^z \Omega(2) S_3^z \rangle & \cdots & \langle S_1^z \Omega(2) \cdots \Omega(N-1) S_N^z \rangle \\ 0 & \langle (S_2^z)^2 \rangle & \langle S_2^z S_3^z \rangle & \cdots & \langle S_2^z \Omega(3) \cdots \Omega(N-1) S_N^z \rangle \\ 0 & 0 & \langle (S_3^z)^2 \rangle & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \langle S_{N-1}^z S_N^z \rangle \\ 0 & 0 & 0 & \cdots & \langle (S_N^z)^2 \rangle \end{pmatrix}$$

$$V = \left(\langle S_1^z \rangle , \langle \Omega(1) S_2^z \rangle , \ldots , \langle \Omega(1) \cdots \Omega(N-1)) S_N^z \rangle \right)$$

$$\Omega(l) = \mathbb{I} \,, e^{i\pi S_l^z}$$

Advantage is two-fold:

- sensible to multipartite entanglement
- at criticality $f_Q(O^\alpha) \sim N^{1-2\Delta_\alpha}$



(a) Haldane phase. (b) Dimer and trimer phase.

BLBQ model, Haldane phase

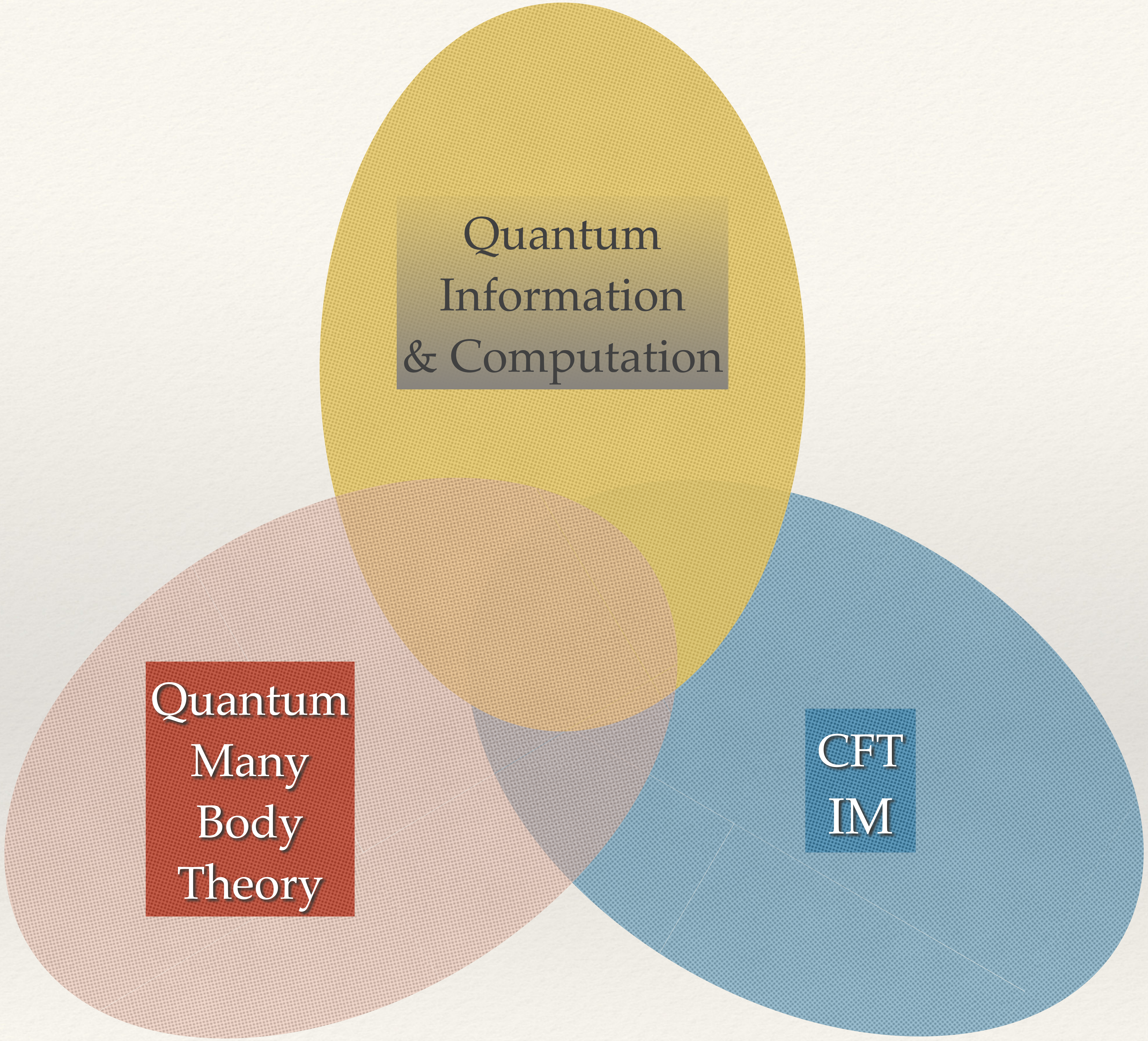
$f_Q(|\psi_\beta\rangle, \tilde{O}^z) = q + bN^\delta$

β	q	b	δ
$-1/3$	0.225 ± 0.003	0.4441 ± 0.0001	1.0002 ± 0.0001
0	0.35 ± 0.05	0.355 ± 0.002	1.002 ± 0.003
$1/3$	1.122 ± 0.009	0.197 ± 0.004	0.9999 ± 0.0005
$2/3$	1.55 ± 0.04	0.111 ± 0.020	0.997 ± 0.001

BLBQ model

$f(|\psi_\beta\rangle, \tilde{O}^z) = q + b\ln N$

Dimer phase			Trimer phase		
β	q	b	β	q	b
2	0.81 ± 0.04	0.58 ± 0.01	-2	0.98 ± 0.06	0.19 ± 0.01
4	1.03 ± 0.01	0.405 ± 0.003	-4	1.08 ± 0.06	0.12 ± 0.01
8	1.34 ± 0.05	0.24 ± 0.01	-8	1.11 ± 0.05	0.09 ± 0.01
∞	1.39 ± 0.04	0.18 ± 0.01			



CROSS-FERTILIZATION
among the different fields

EXPERIMENTAL IMPLEMENTATION
(analogue or digital computation)

THANK YOU

for your attention

and

to Francesco for the organization