Chang Duality in Landau-Ginzburg Quantum Field Theories

10$^{\text{th}}$ Bologna Workshop on Conformal Field Theory and Integrable Models

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work in progress with Giuseppe Mussardo

"Many years later, as he faced the firing squad, Colonel Aureliano Buendía was to remember that distant afternoon when his father took him to discover ice."

Gabriel Garcia Marquez, *One Hundred Years of Solitude*

Sine-Gordon and Francesco

N·H
ELSEVIER
2 January 1997
PHYSICS LETTERS B
Physics Letters B 390 (1997) 243–251

Excited state Destri–De Vega equation for sine-Gordon and restricted sine-Gordon models

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Received 3 September 1996
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Abstract

We derive a generalization of the Destri–De Vega equation governing the scaling functions of some excited states in the sine-Gordon theory. In particular, configurations with an even number of holes and no strings are analyzed and their UV limits found to match some of the conformal dimensions of the corresponding compactified massless free boson. Quantum group reduction allows to interpret some of our results as scaling functions of excited states of Restricted sine-Gordon theory, i.e. minimal models perturbed by ϕ_{13} in their massive regime. In particular we are able to reconstruct the scaling functions of the off-critical deformations of all the scalar primary states on the diagonal of the Kac table.

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24 December 1998
PHYSICS LETTERS B
Physics Letters B 444 (1998) 442–450

Scaling functions in the odd charge sector of sine-Gordon/massive Thirring theory

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Editor: L. Alvarez-Gaumé

Abstract

A non-linear integral equation (NLIE) governing the finite size effects of excited states of even topological charge in the sine-Gordon (sG) / massive Thirring (mTh) field theory, deducible from a light-cone lattice formulation of the model, has been known for some time. In this letter we conjecture an extension of this NLIE to states with odd topological charge, thus completing the spectrum of the theory. The scaling functions obtained as solutions to our conjectured NLIE are compared successfully with Truncated Conformal Space data and the construction is shown to be compatible with all other facts known about the local Hilbert spaces of sG and mTh models. With the present results we have achieved a full control over the finite size behaviour of energy levels of sG/mTh theory. © 1998 Published by Elsevier Science B.V. All rigths reserved.

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Nuclear Physics B 540 [FS] (1999) 543–586

Non-linear integral equation and finite volume spectrum of sine-Gordon theory

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Received 21 August 1998; accepted 5 November 1998

Abstract

We examine the connection between the non-linear integral equation (NLIE) derived from light-cone lattice and sine-Gordon quantum field theory, considered as a perturbed $c = 1$ conformal field theory. After clarifying some delicate points of the NLIE deduction from the lattice, we compare both analytic and numerical predictions of the NLIE to previously known results in sine-Gordon theory. To provide the basis for the numerical comparison we use data from Truncated Conformal Space method. Together with results from analysis of infrared and ultraviolet asymptotics, we find evidence that it is necessary to change the rule of quantization proposed by Destri and de Vega to a new one which includes as a special case that of Fioravanti et al. This way we find strong evidence for the validity of the NLIE as a description of the finite size effects of sine-Gordon theory. © 1999 Elsevier Science B.V.

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NUCLEAR PHYSICS B
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Exact finite size spectrum in super sine-Gordon model

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Coleman 1975

PHYSICAL REVIEW D VOLUME 11, NUMBER 8 15 APRIL 1975

Quantum sine-Gordon equation as the massive Thirring model*

Sidney Coleman

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(Received 6 January 1975)

The sine-Gordon equation is the theory of a massless scalar field in one space and one time dimension with interaction density proportional to $\cos\beta\varphi$, where β is a real parameter. I show that if β^2 exceeds 8π, the energy density of the theory is unbounded below; if β^2 equals 4π, the theory is equivalent to the zero-charge sector of the theory of a free massive Fermi field; for other values of β, the theory is equivalent to the zero-charge sector of the massive Thirring model. The sine-Gordon soliton is identified with the fundamental fermion of the Thirring model.

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Coleman 1975

Sine-Gordon Model:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\alpha_0}{\beta^2}\cos\beta\phi + \gamma_0$$

All divergences that occur in any order of perturbation theory can be removed by normal-ordering the Hamiltonian.

- Multiplicative renormalization of α_0

$$\alpha = \alpha_0 \left(\frac{m^2}{\Lambda^2}\right)^{\beta^2/8\pi}$$

- Additive renormalization of γ_0

$$\gamma = \gamma_0 + \int \frac{dk}{8\pi}\frac{2k^2+m^2}{\sqrt{k^2+m^2}}$$

- β does not renormalize

S. Coleman - Phys. Rev. D 11, 2088 (1975)

Coleman 1975

$$H_0 = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2$$

$$N_\mu\left[H_0\right] = N_m\left[H_0\right] + \frac{1}{8\pi}(m^2 - \mu^2)$$

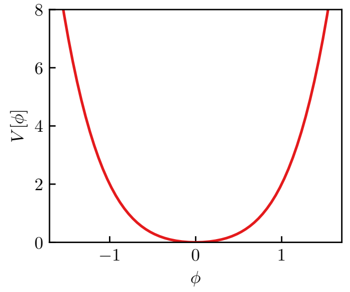
$$N_\mu\left[e^{i\beta\phi}\right] = \left(\frac{m^2}{\mu^2}\right)^{\beta^2/8\pi} N_m\left[e^{i\beta\phi}\right]$$

$$H = N_m\left[H_0 - \frac{\alpha}{\beta^2}\cos\beta\phi - \gamma\right]$$

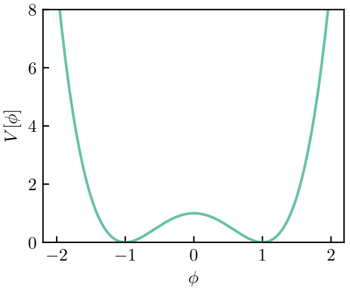
S. Coleman - Phys. Rev. D 11, 2088 (1975)

ϕ^4 QFT in $(1+1)$ dimensions

$$H = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{2}m^2\phi^2 + g\phi^4$$



$$H' = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + g(\phi^2 - c^2)^2$$



Regularization

$$H = N_m \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{2} m^2 \phi^2 + g\phi^4 \right]$$

$$\begin{aligned} H' &= N_\mu \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + g(\phi^2 - c^2)^2 \right] \\ &= N_\mu \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 - \frac{1}{4} \mu^2 \phi^2 + g\phi^4 \right] \end{aligned}$$

$$\mu^2 = \left. \frac{\delta^2 V}{\delta \phi^2} \right|_{\phi = \pm c}$$

Change of ordering

$$H = N_m\left[H_0 + \frac{1}{2}m^2\phi^2 + g\phi^4\right]$$

Change of ordering

$$H = N_\mu \left[H_0 + \frac{1}{2} m^2 \phi^2 + g\phi^4 \right] + \,?$$

Change of ordering: Coleman's prescription

$$H = N_\mu \left[H_0 + \frac{1}{2} m^2 \phi^2 + g\phi^4 \right] + \,?$$

$$N_\mu [H_0] = N_m [H_0] + \frac{1}{8\pi}(m^2 - \mu^2)$$

$$N_\mu [e^{i\beta\phi}] = \left(\frac{m^2}{\mu^2} \right)^{\beta^2/8\pi} N_m [e^{i\beta\phi}]$$

Change of ordering: Coleman's prescription

$$H = N_\mu\left[H_0 + \frac{1}{2}m^2\phi^2 + g\phi^4\right] + \,?$$

$$N_\mu\left[H_0\right] = N_m\left[H_0\right] + \frac{1}{8\pi}(m^2 - \mu^2)$$

$$\boxed{N_\mu\left[e^{i\beta\phi}\right] = \left(\frac{m^2}{\mu^2}\right)^{\beta^2/8\pi} N_m\left[e^{i\beta\phi}\right]}$$

Change of ordering: Coleman's prescription

$$\boxed{N_\mu\left[e^{i\beta\phi}\right] = \left(\frac{m^2}{\mu^2}\right)^{\beta^2/8\pi} N_m\left[e^{i\beta\phi}\right]}$$

$$N_\mu\left[\phi^2\right] = N_m\left[\phi^2\right] + \frac{1}{4\pi}\ln\frac{\mu^2}{m^2}$$

$$N_\mu\left[\phi^4\right] = N_m\left[\phi^4\right] + 6\left(\frac{1}{4\pi}\ln\frac{\mu^2}{m^2}\right)N_m\left[\phi^2\right] + 3\left(\frac{1}{4\pi}\ln\frac{\mu^2}{m^2}\right)^2$$

H and H' equivalence

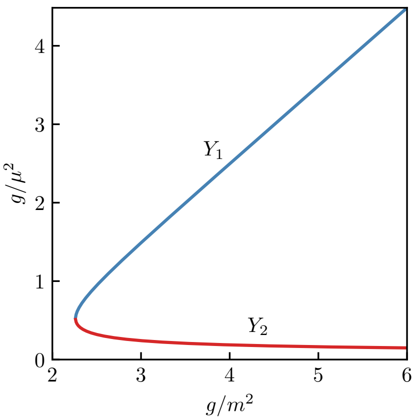
$$H = N_\mu \left[H_0 + g\left(\phi^2 + \frac{3}{4\pi}\ln\frac{m^2}{\mu^2} + \frac{m^2}{4g}\right)^2 \right] + \frac{1}{8\pi}(\mu^2 - m^2)$$

$$H' = N_\mu \left[H_0 + g\left(\phi^2 - c^2\right)^2 \right]$$

$$\boxed{\frac{m^2}{g} + \frac{3}{\pi}\ln\frac{m^2}{g} = \frac{3}{\pi}\ln\frac{\mu^2}{g} - \frac{\mu^2}{2g}}$$

H and H' equivalence

$$\boxed{\frac{m^2}{g} + \frac{3}{\pi}\ln\frac{m^2}{g} = \frac{3}{\pi}\ln\frac{\mu^2}{g} - \frac{\mu^2}{2g}}$$



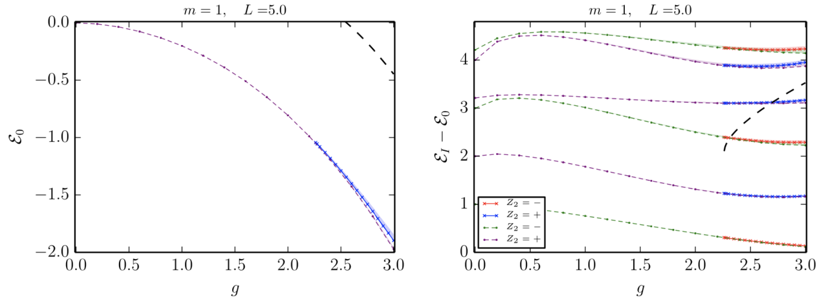
S-J. Chang - Phys. Rev. D 13, 2778 (1976)

$$Y_2: \quad \frac{g}{\mu^2} \xrightarrow[g/m^2\to\infty]{} 0$$

A strong-coupling theory in terms of H is identical to a weak-coupling theory in terms of H'.

$$\langle\phi\rangle = \left(\frac{\mu^2}{2g}\right)^{1/2} \simeq \left(\frac{3}{\pi}\ln\frac{g}{m^2}\right)^{1/2} \neq 0$$

Test of Chang Duality



S. Rychkov, L. G. Vitale - Phys. Rev. D 93, 065014 (2016)

Extension to Ginzburg-Landau Theories

Back to Coleman

$$\boxed{e^{i\beta\phi_{\mu}} = \left(\frac{m^2}{\mu^2}\right)^{\beta^2/8\pi} e^{i\beta\phi_m}}$$

$$\phi_m^l = \sum_{k=0}^{[l/2]} \frac{l!}{2^k k!(l-2k)!}\alpha^k(m^2/\mu^2)\phi_\mu^{l-2k}, \qquad \alpha(m^2/\mu^2) = \frac{1}{4\pi}\ln\frac{m^2}{\mu^2}$$

Given $\mathbb{Z}_2$ symmetric Ginzburg-Landau Theory ϕ^{2n}:

$$H = N_m\left[H_0\right] + \sum_{k=1}^{n} g_{2k}\phi_m^{2k}$$

$$H' = N_\mu\left[H_0\right] + \sum_{k=1}^{n} h_{2k}\phi_\mu^{2k},$$

what is the analogue of Chang's relation in order to have the equivalence of H and H'?

GL: H and H' equivalence

	$g_2\phi_m^2$	$g_4\phi_m^4$	$g_6\phi_m^6$	$g_8\phi_m^8$	
1	$C_0^2\alpha$	$C_0^4\alpha^2$	$C_0^6\alpha^3$	$C_0^8\alpha^4$	.
ϕ_μ^2	1	$C_2^4\alpha$	$C_2^6\alpha^2$	$C_2^8\alpha^3$	h_2
ϕ_μ^4		1	$C_4^6\alpha$	$C_4^8\alpha^2$	h_4
ϕ_μ^6			1	$C_6^8\alpha$	h_6
ϕ_μ^8				1	h_8

$$\begin{aligned} g_2 + C_2^4\alpha\ g_4 + C_2^6\alpha^2 g_6 + C_2^8\alpha^3 g_8 &= h_2 \\ g_4 + C_4^6\alpha\ g_6 + C_4^8\alpha^2 g_8 &= h_4 \\ g_6 + C_6^8\alpha\ g_8 &= h_6 \\ g_8 &= h_8 \end{aligned}$$

$$C_j^i = \frac{i!}{2^{\frac{i-j}{2}}(\frac{i-j}{2})!j!}.$$

GL: H and H' equivalence

For a ϕ^{2n} theory

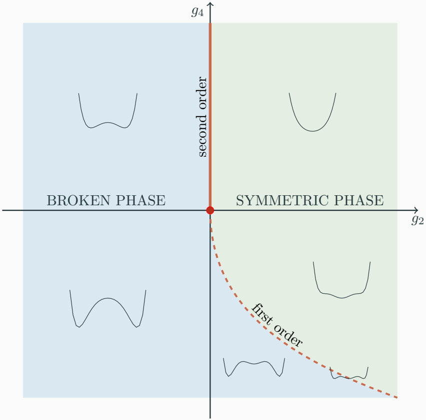
$$\boxed{\frac{m^2}{g} + 2\sum_{j=1}^{n}(-1)^j K_{2j}\alpha^{j-1}(m^2/\mu^2)\tilde{h}_{2j} = 0}$$

$$\begin{aligned}
g_4 &= h_4 - L_6\alpha h_6 + L_8\alpha^2 h_8 - \dots \\
g_6 &= h_6 - M_8\alpha h_8 + \dots \\
g_8 &= h_8 - \dots \\
&\vdots
\end{aligned}$$

Chang Duality in GL ϕ^6

Duality in GL ϕ^6

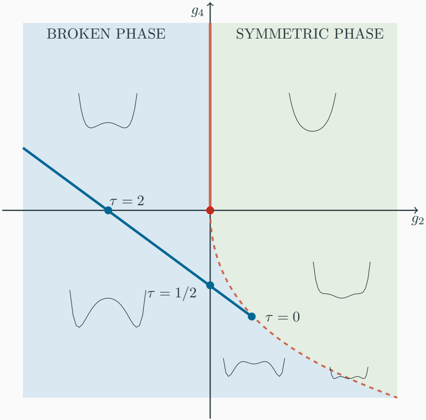
Class of universality of the Tricritical Ising Model ($\mathcal{M}_4$, $c = \frac{7}{10}$)



$$H = N_m[H_0] + \frac{m^2}{2}\phi_m^2 + g_4\phi_m^4 + g\phi_m^6$$

$$H' = N_\mu[H_0] + g(\phi_\mu^2 + a^2)(\phi_\mu^2 - c^2)^2$$

Duality in GL ϕ^6



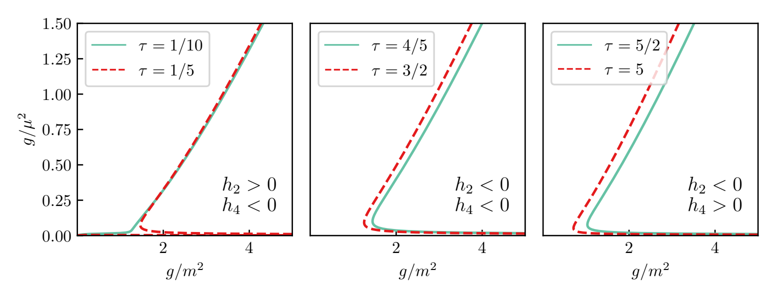
	$\frac{m^2}{2}\phi_m^2$	$g_4\phi_m^4$	$g\phi_m^6$	
1	α	$3\alpha^2$	$15\alpha^3$	.
ϕ_μ^2	1	6α	$45\alpha^2$	$gc^2(c^2 - 2a^2)$
ϕ_μ^4		1	15α	$g(a^2 - 2c^2)$
ϕ_μ^6			1	g

$$a^2 = \tau c^2$$

$$\mu^2 = 8gc^4(\tau + 1).$$

Duality in GL ϕ^6: masses equation

$$\frac{m^2}{g} - \frac{1-2\tau}{4(\tau+1)}\frac{\mu^2}{g} + \frac{3(\tau-2)}{2\pi\sqrt{2(\tau+1)}}\sqrt{\frac{\mu^2}{g}}\ln\frac{m^2}{\mu^2} - \frac{45}{8\pi^2}\ln^2\frac{m^2}{\mu^2} = 0$$



G. Mussardo, OP - In preparation

Duality in GL ϕ^6: change of variables

After setting $x = g/m^2$ and $y = g/\mu^2$:

$$u = \frac{y}{x}$$
$$v = \sqrt{y}$$

Polynomial equation in v:

$$\boxed{v^2 + \frac{k_1}{k_2 \ln u} v + \frac{u + k_0}{k_2 \ln^2 u} = 0}$$

with

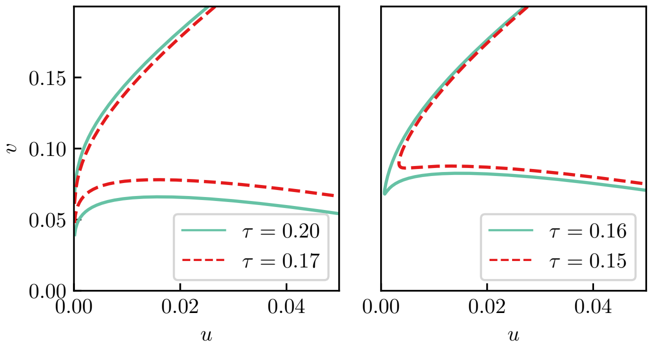
$$k_0 = \frac{2\tau - 1}{4(\tau + 1)}$$
$$k_1 = \frac{3(\tau - 2)}{2\pi\sqrt{2(\tau + 1)}}$$
$$k_2 = -\frac{45}{8\pi^2}$$

Duality in GL ϕ^6: change of variables

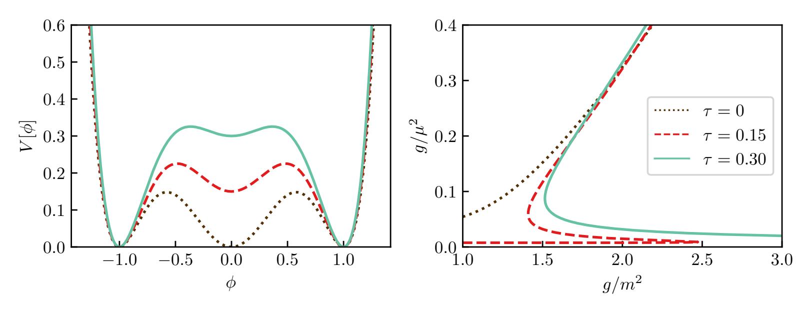
$$v_\pm = -\frac{k_1}{2k_2 \ln u}\left(1 \mp \sqrt{1 - \frac{4k_2}{k_1^2}(k_0 + u)}\right),$$

which approaches zero when $u \to 0$, provided that

$$4k_2 k_0 < k_1^2 \qquad \Longrightarrow \qquad \tau > \sqrt{10} - 3 \equiv \tau^* \sim 0.1623$$



Duality in GL ϕ^6

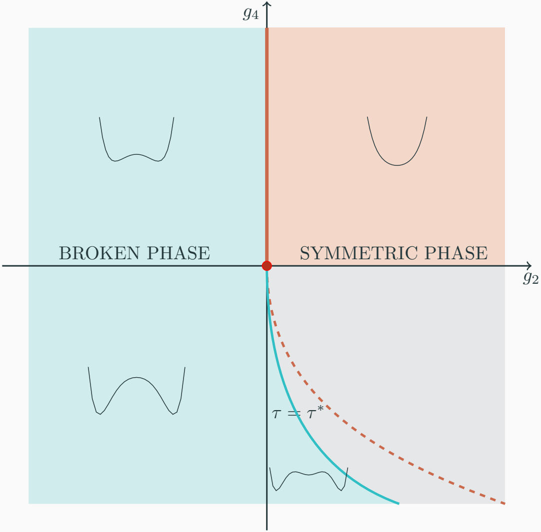


The g_4 equation:

$$\frac{g_4}{g} = \frac{\tau - 2}{2\sqrt{2(\tau+1)}} \frac{1}{v(u)} - \frac{15}{4\pi} \ln u$$

$$\frac{g_4}{g} \overset{u \to 0}{\sim} f(\tau) \ln u \qquad f(\tau) < 0 \text{ for } \tau > \tau*$$

Duality in GL ϕ^6



Thanks for the attention!

Chang Duality: general approach

Duality in GL ϕ^8 and General approach

$$H' = N_\mu[H_0] + N_\mu[V] \equiv N_\mu[H_0] + g(\phi_\mu^2 + a^2)(\phi_\mu^2 + b^2)(\phi_\mu^2 - c^2)^2$$

$$\frac{\mu^2}{g} = 8c^6(\tau_1\tau_2 + \tau_1 + \tau_2 + 1)$$

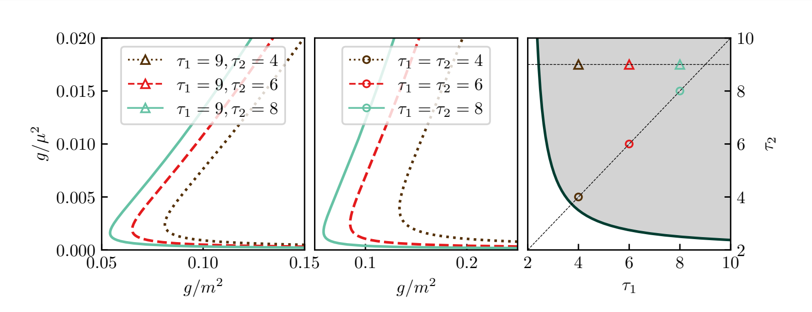
$$\boxed{\frac{m^2}{g} + k_0\frac{\mu^2}{g} + k_1\left(\frac{\mu^2}{g}\right)^{2/3}\ln\frac{m^2}{\mu^2} + k_2\left(\frac{\mu^2}{g}\right)^{1/3}\ln^2\frac{m^2}{\mu^2} + k_3\ln^3\frac{m^2}{\mu^2} = 0}$$

with

$$k_0 = \frac{2\tau_1\tau_2 - \tau_1 - \tau_2}{4(\tau_1\tau_2 + \tau_1 + \tau_2 + 1)} \qquad k_1 = \frac{3(\tau_1\tau_2 - 2\tau_1 - 2\tau_2 + 1)}{4\pi(\tau_1\tau_2 + \tau_1 + \tau_2 + 1)^{2/3}}$$

$$k_2 = -\frac{45}{16\pi^2}\frac{\tau_1 + \tau_2 - 2}{(\tau_1\tau_2 + \tau_1 + \tau_2 + 1)^{1/3}} \qquad k_3 = \frac{105}{8\pi^3}$$

Duality in GL ϕ^8 and General approach



$$\boxed{\frac{m^2}{g} + \sum_{j=0}^{n-1} k_j(\{\tau_i\}) \left(\frac{\mu^2}{g}\right)^{1-\frac{j}{n-1}} \left(\ln \frac{m^2}{\mu^2}\right)^j = 0}$$

For Ginzburg-Landau ϕ^{2n} theory:

$$\sum_{j=0}^{n-1} k_j(\{\tau_i\})\xi^j = 0, \qquad \xi = v \ln u, \qquad u = y/x \qquad v = y^{1/(n-1)}$$