Can cuspy dark matter halos hold cored stellar mass distributions?

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"Una manera de hacer Europa"

(Fornax Dwarf - ESO)

Outline

1.- Introduction

- My pathway to SIDM
- Motivation of the work

2.- Eddington inversion method comes to help

3.- Application to real galaxies (work in progress)

4.- Future pathways

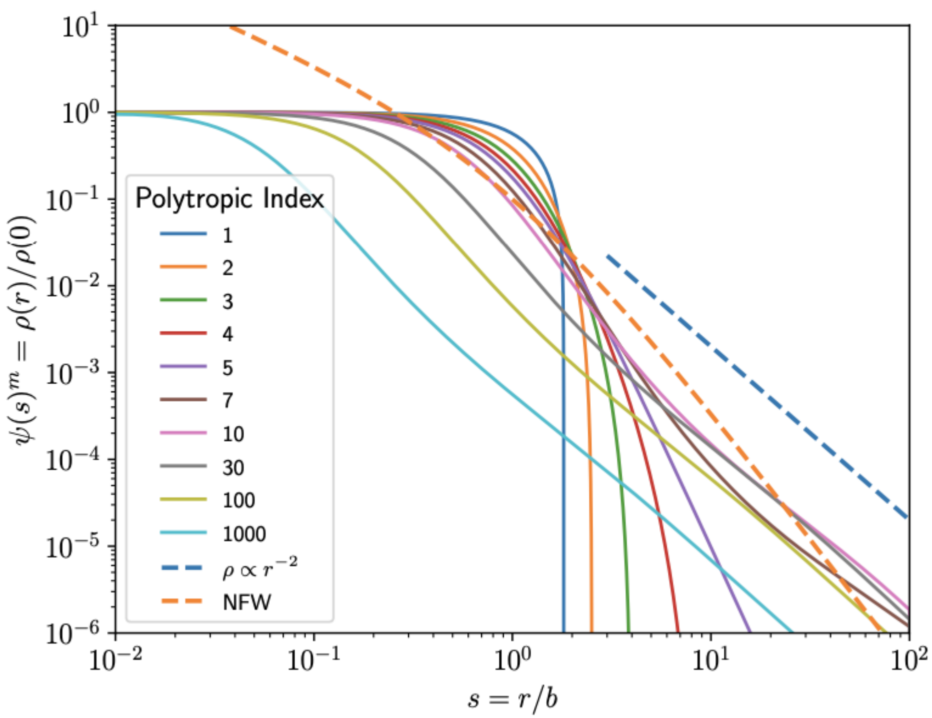
5.- Conclusions

Introduction: my pathway to SIDM

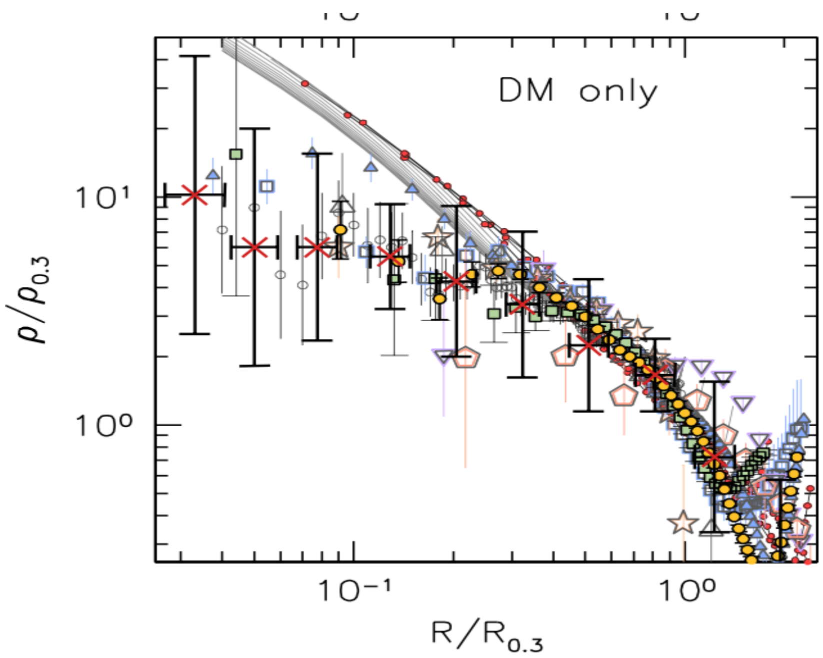
Plastino & Plastino (1993; Physics Letters A 1993, 174) proved that a self-gravitation N-body system in thermodynamic equilibrium, as defined by the Tsallis entropy, has a density profile given by a **polytrope.**

- Tsallis entropy describes systems with long range forces (e.g., self gravity).

- Polytropes are solutions of the Lane-Emden equation

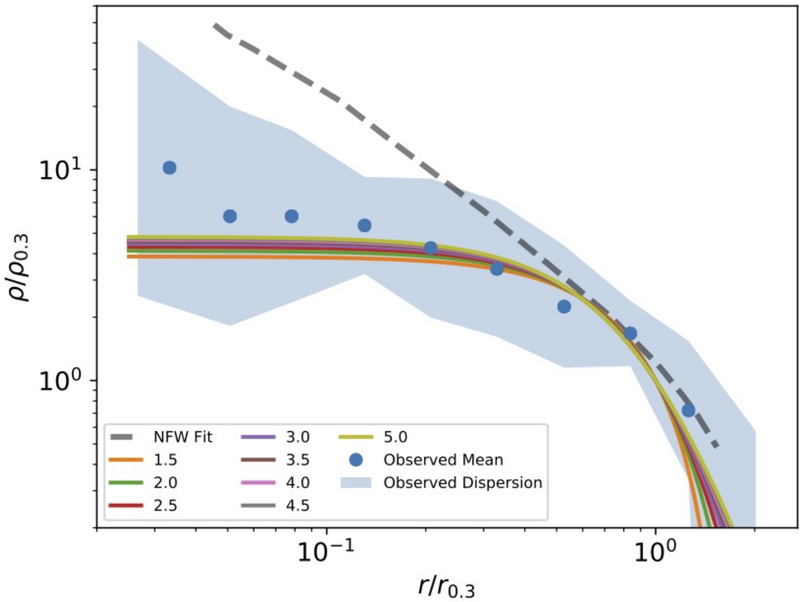


- The same exercise with the Boltzmann-Gibbs entropy yields the isothermal sphere, which is unphysical (infinite mass & energy).
- Polytropes have finite mass (m <= 5)
- Polytropes have all the same core.
- m=5 is the Plummer profile whereas m = ∞ is the isothermal sphere.



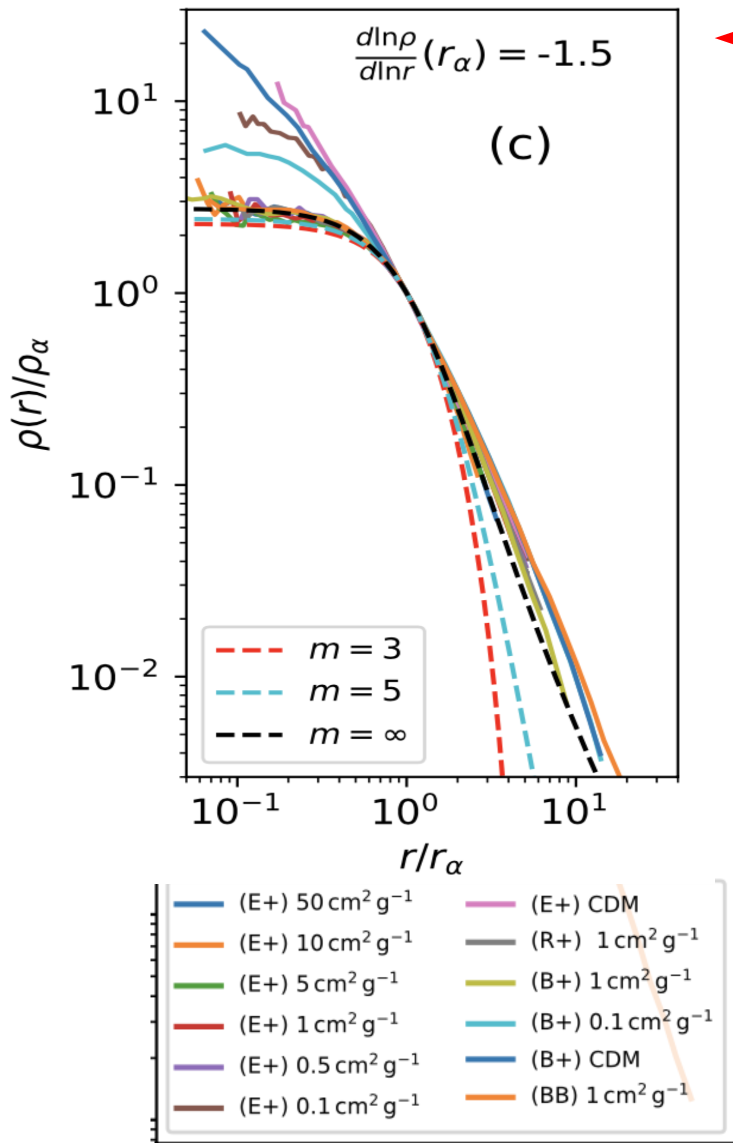
Total density profiles from HI rotation curves of dwarfs (LITTLE THINGS, Oh+15, AJ, 149, 180).

Polytropes happen to be on top of the observed density without any degree of freedom (SA+ 2020, A&A, 642, L14)



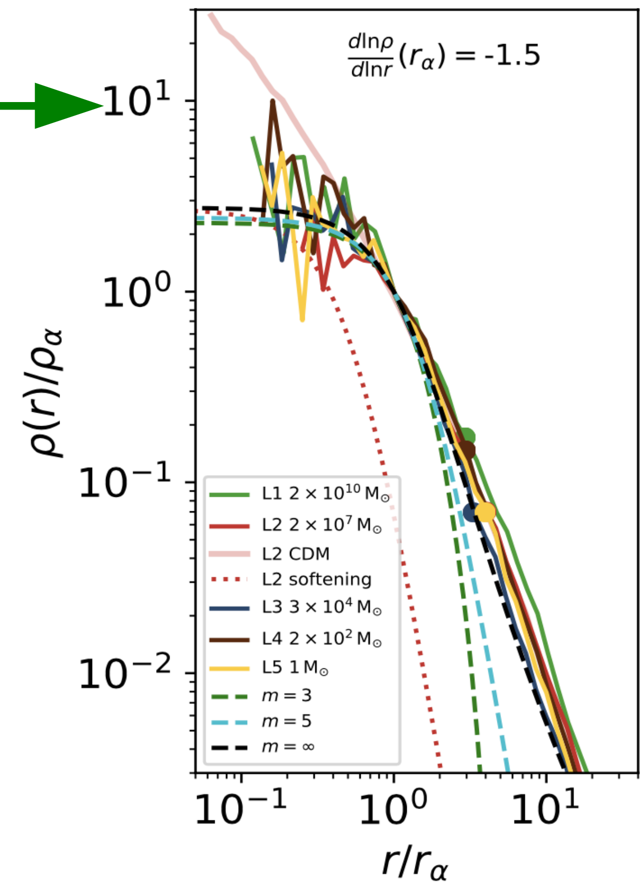
Polytropes are characteristic of **thermodynamic equilibrium**, but **CDM particles do not collide.** Solution? Add collisions to the DM particles …. the simples **more obvious solution** is **SIDM** particles.

Polytropes represent the thermodynamic equilibrium solution of self gravitating N-body systems in numerical simulations



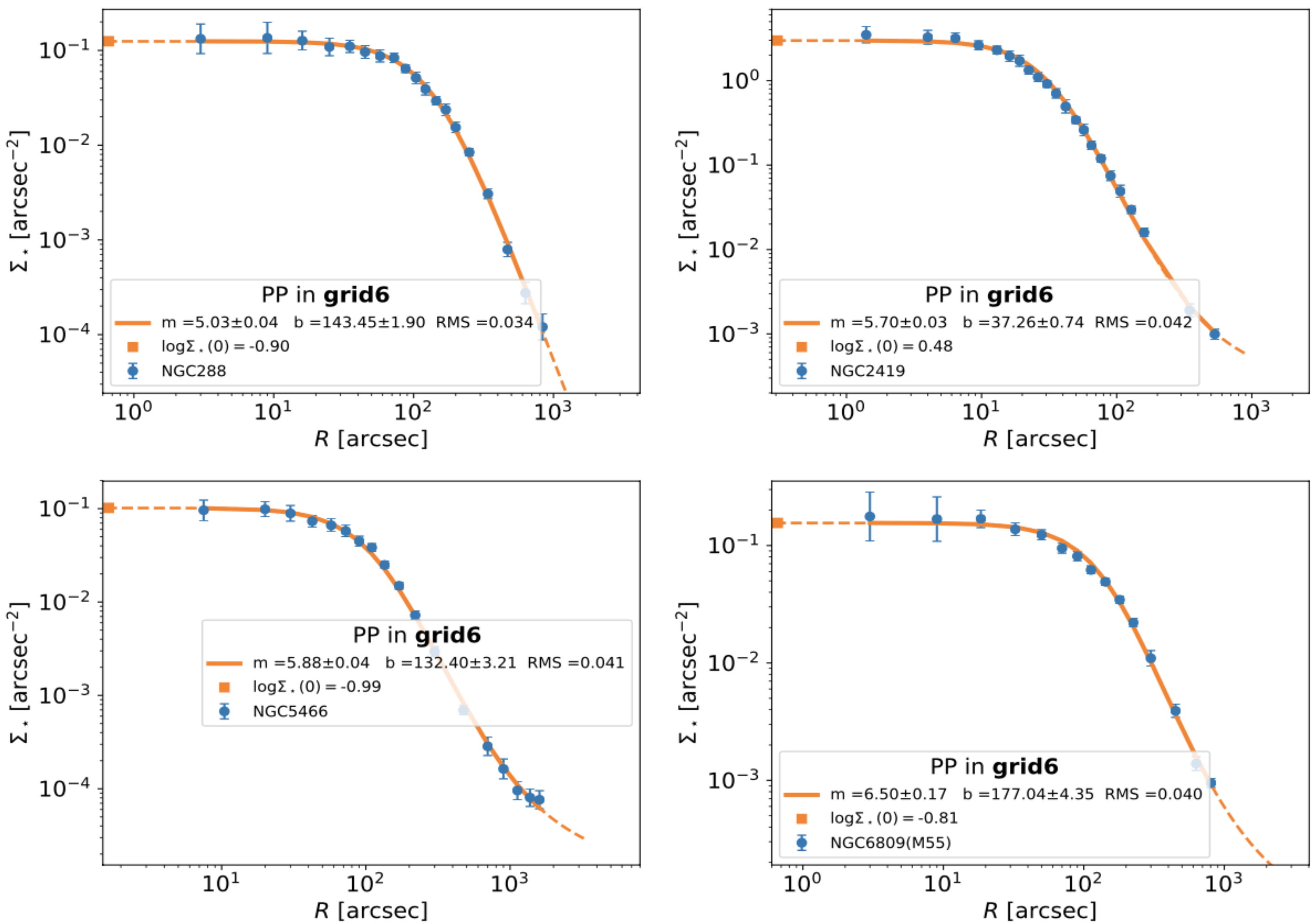
SIDM simulations from various sources (Elbert+15 E+; Bullock & Boylan-Kolchin 17 BB; Robles+17 R+, and Brinckmann+18 B+).

Even in CDM, the artificial cores appearing within the convergence radius are polytropes (Wang+20).



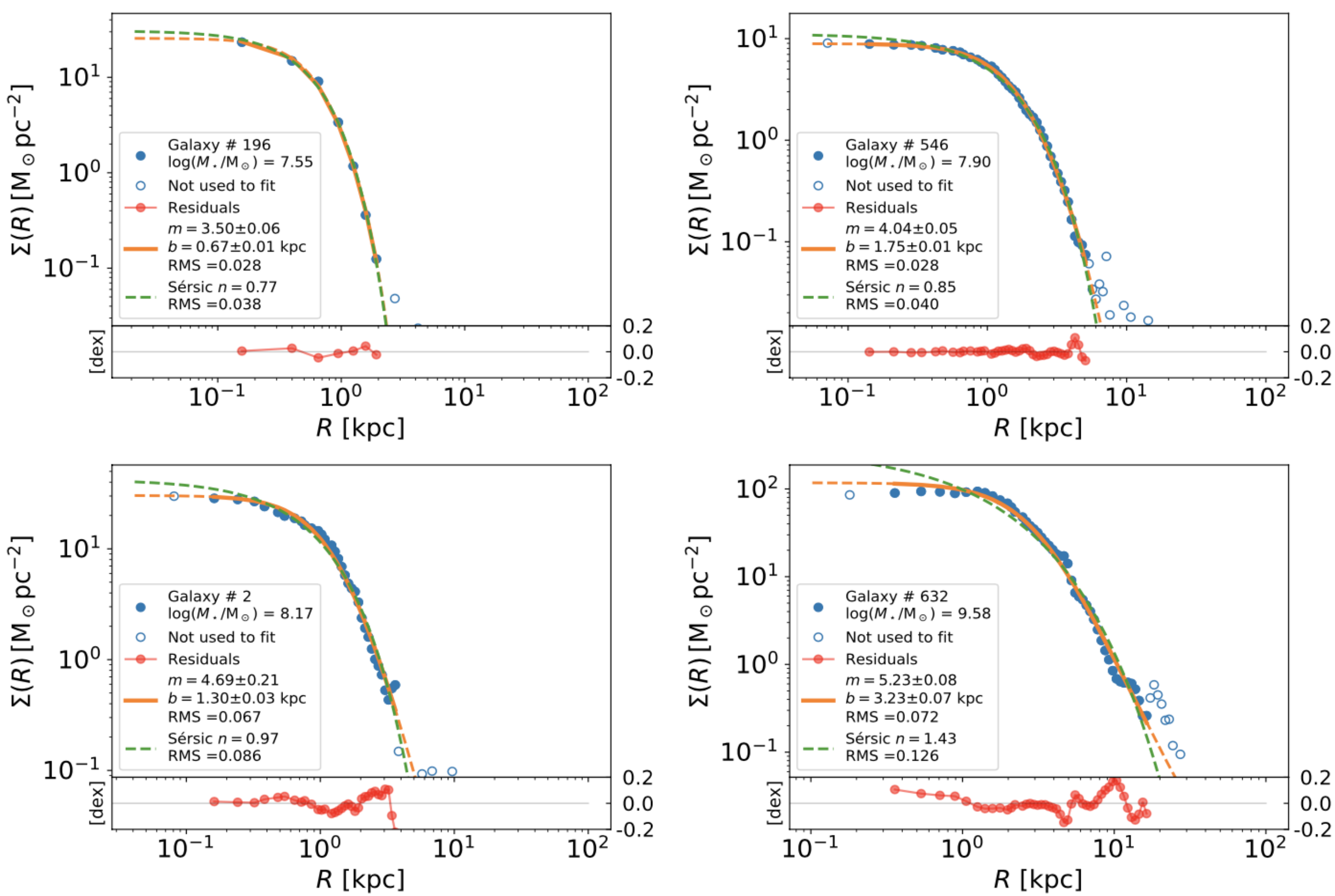
SA & Trujillo, 2021, MNRAS, 504, 2832

Projected Polytropes provide good fits to globular clusters



Trujillo&SA, in prep

Projected polytropes provide good fits to the stellar mass distribution of low mass galaxies

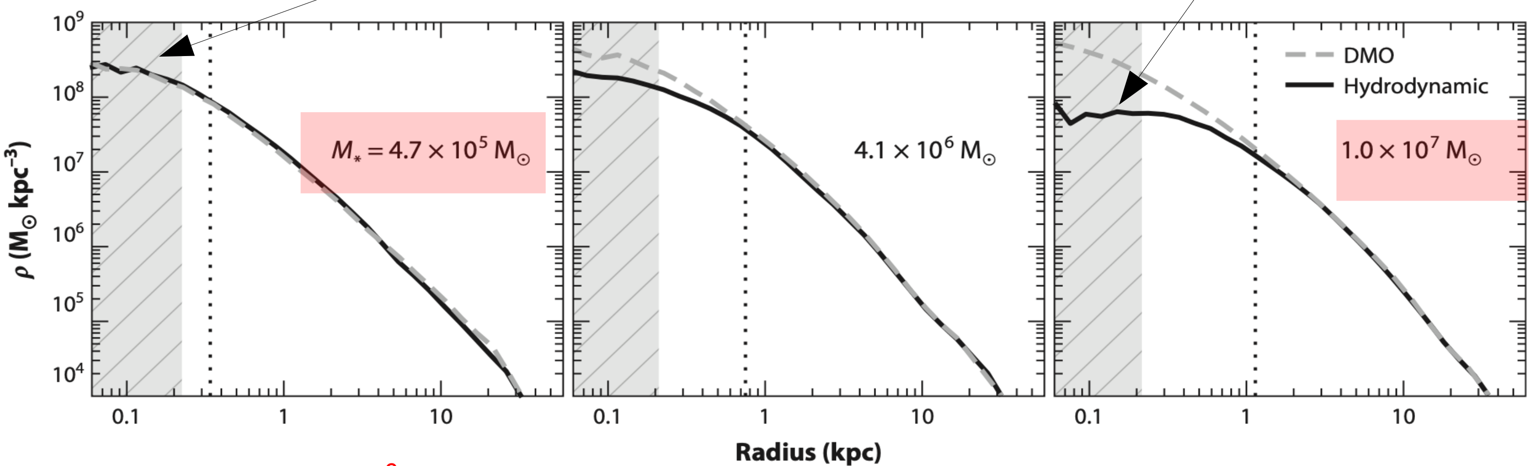


SA+ApJ 2021, 921, 125,

Introduction: motivation

Cusp

Core

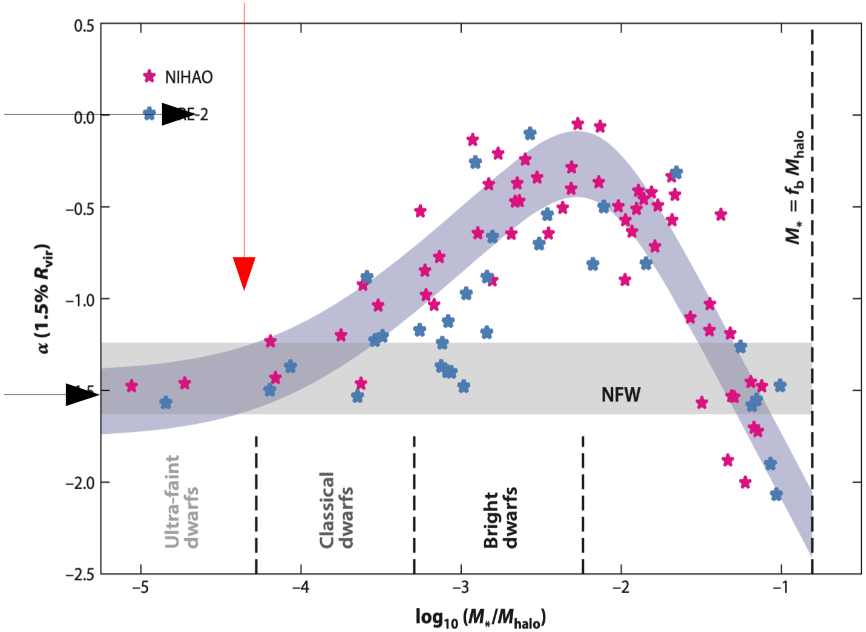


Bullock+17

$M_* < \sim 10^6\ M_\odot$

Core

Cusp



Di Cintio+14

Predictions from CDM: DM cores should go away when $M_* < \sim 10^6\ M_\odot$ (e.g., Peñarubia+12) This limit is model dependent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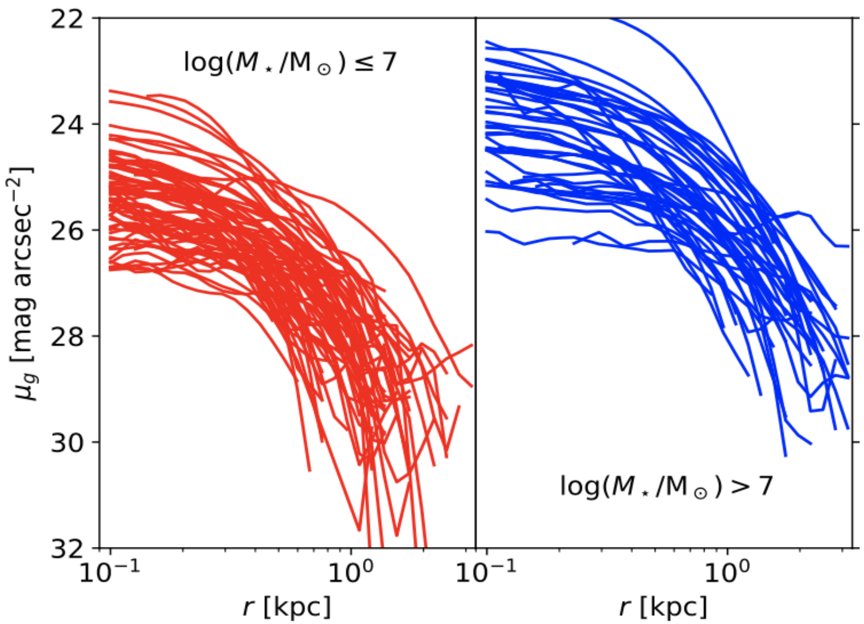
Kinematic measurements at these masses is technically very challenging (if not impossible).

However, photometry is doable. We can measure the mass profile from photometry.

Moreover, the stellar light profiles tend to show inner cores.



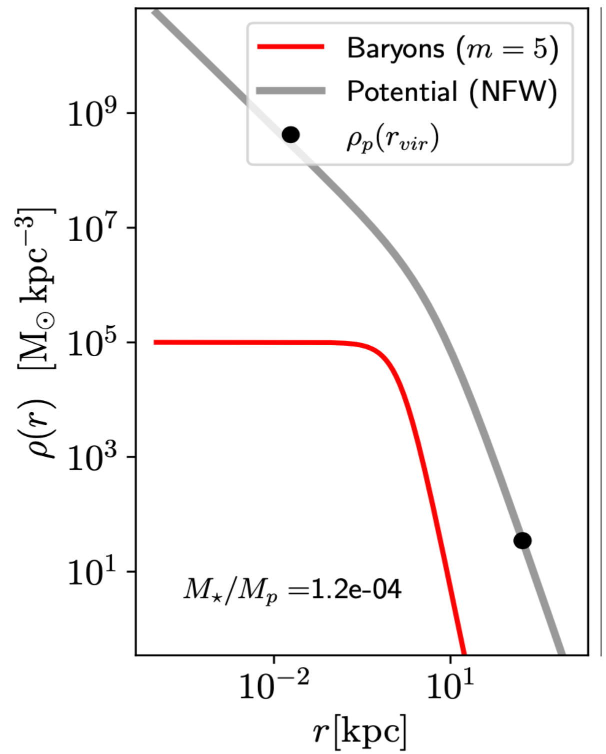
(Carlsten+21,22; MW-like satellites)

Can we constrain the inner slope in the DM distribution only from the stellar distribution?

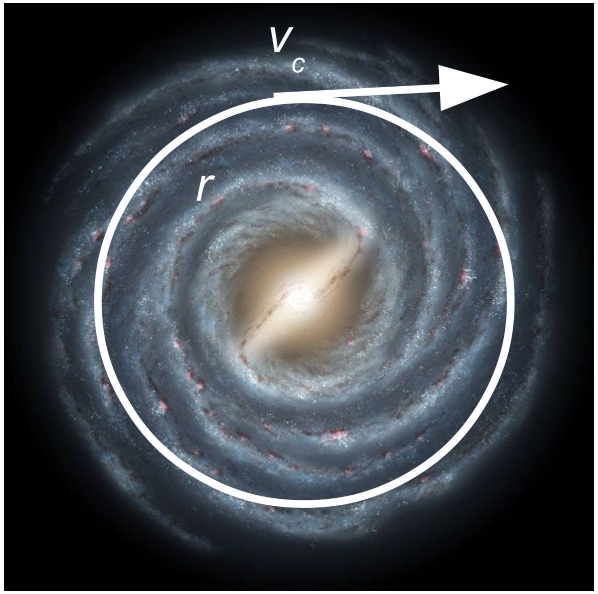
If so, this is doable

$M_* <\sim 10^6\ M_\odot$ galaxies are DM dominated ($\log[M_*/M_{DM}] \sim -4$) so it becomes very natural thinking that stars trace DM and baryon cores trace DM cores.

The question in the title arises:
"**Can CDM halos hold cored stellar mass distributions?**"



However, the answer is not trivial since with circular orbits one can reproduce any density profile (ρ[r]) immersed in any potential.



$$\rho(r) = 2\pi \iint f_c\, v_t\, dv_t\, dv_r,$$

$$v_c^2(r) = \frac{G\, M_p(<r)}{r}.$$

$$F(r) = \frac{\rho(r)}{2\pi\, v_c(r)},$$

$$f_c(r, v_t, v_r) = F(r)\, \delta(v_t - v_c)\, \delta(v_r),$$

The Eddington inversion method comes to help

For spherically symmetric systems of particles with isotropic velocity distribution, the phase-space DF $f(\varepsilon)$ depends only on the particle energy ε.

$$\rho(r) = 4\pi\sqrt{2}\int_0^{\Psi(r)} f(\epsilon)\sqrt{\Psi(r)-\epsilon}\,d\epsilon.$$

$\epsilon = \Psi - \frac{1}{2}v^2$ is the relative energy

$\Psi(r) = \Phi_0 - \Phi(r)$ is the relative potential

$$f(\epsilon) = \frac{1}{\sqrt{2}\pi^2}\int_0^{\epsilon} \frac{d^3\rho}{d\Psi^3}\sqrt{\epsilon-\Psi}\,d\Psi.$$

Give a stellar mass density profile, $\rho(r)$, and a potential, $\Psi(r)$, the Eddington Inversion Method provides the distribution function consistent with both, $f(\varepsilon)$.

- There is no guarantee that two arbitrary $\rho(r)$ and $\Psi(r)$ are physically consistent with each other.

- The absolutely minimum requirement for consistency is $f(\varepsilon) \geq 0$

- Pairs $\rho(r)$ - $\Psi(r)$ leading to $f(\varepsilon) < 0$ can be discarded, thus, given an observed stellar $\rho(r)$ we can constraint the DM $\Psi(r)$.

- Is a cored estellar $\rho(r)$ consistent with a cuspy CDM $\Psi(r)$?

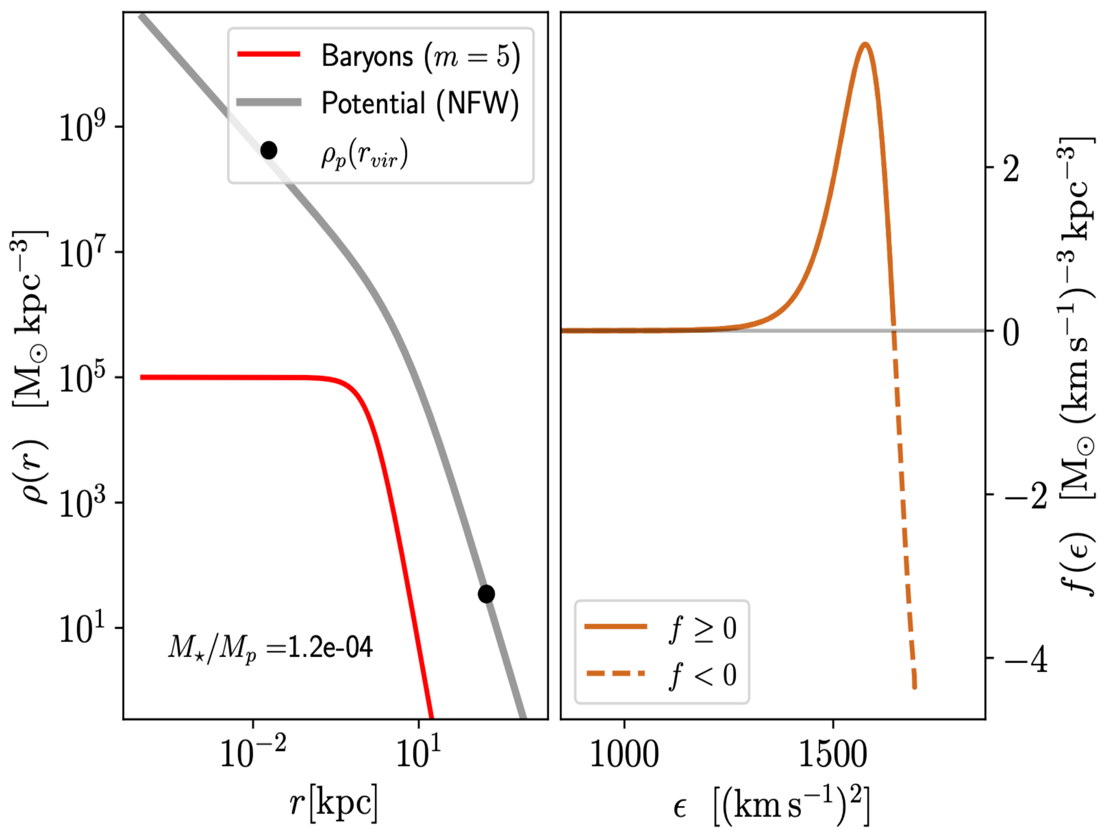
- Is a cored stellar ρ(r) consistent with a cuspy CDM Ψ(r)?

$$\frac{d\rho}{d\Psi} = 2\pi\sqrt{2}\int_0^{\Psi}\frac{f(\epsilon)}{\sqrt{\Psi-\epsilon}}\,d\epsilon.$$ =0 implies f(ε) <0

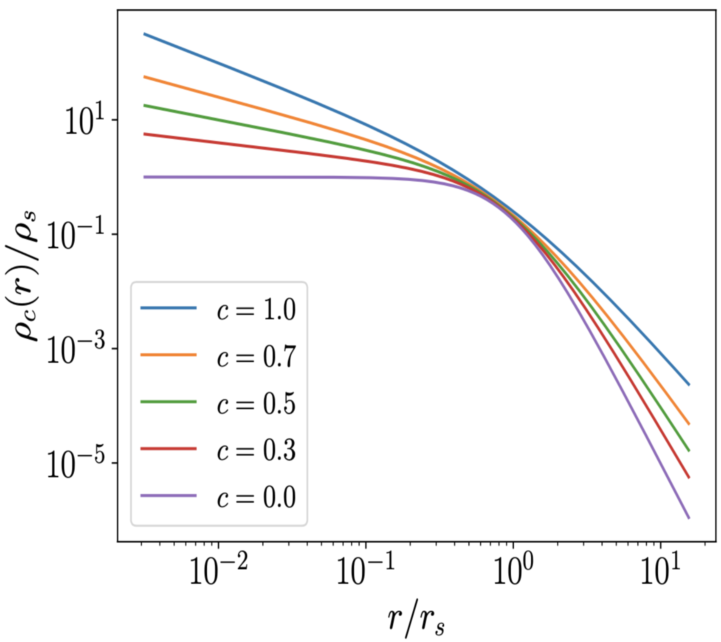
$$\frac{d\rho}{d\Psi} = \frac{d\rho/dr}{d\Psi/dr},$$

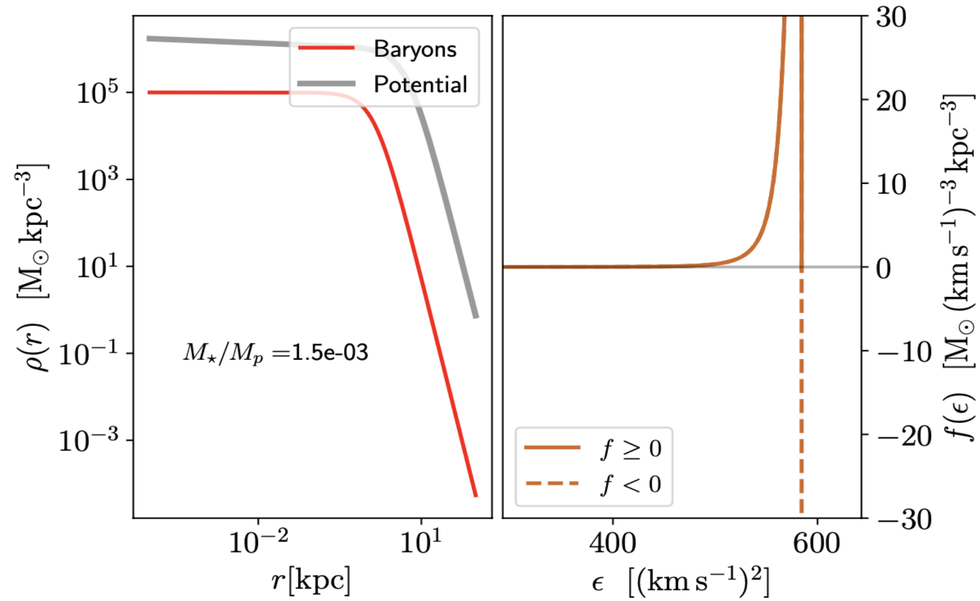
$$\lim_{r\to 0}\frac{d\rho}{dr} = 0,$$

$$\lim_{r\to 0}\frac{d\Psi}{dr} = -\frac{V_c}{2\,r_s^2} \neq 0;$$



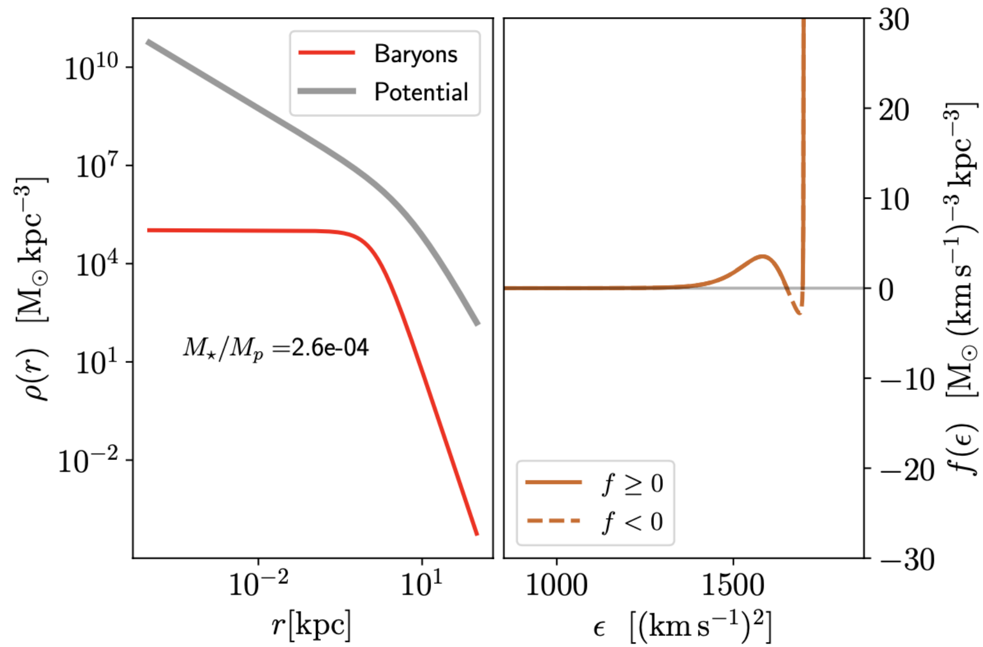
The answer is NO for spherical systems with isotropic velocities (SA+23a)

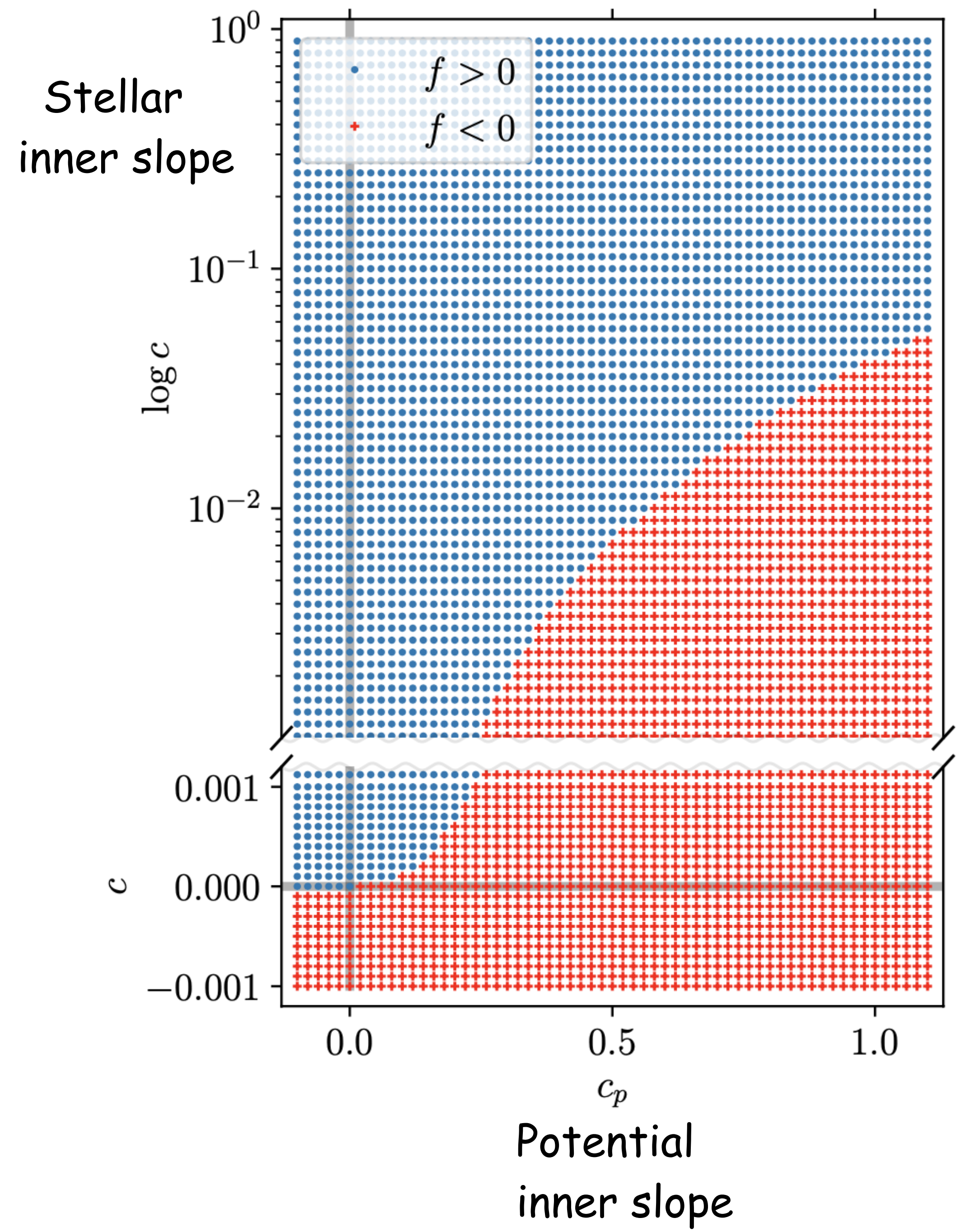


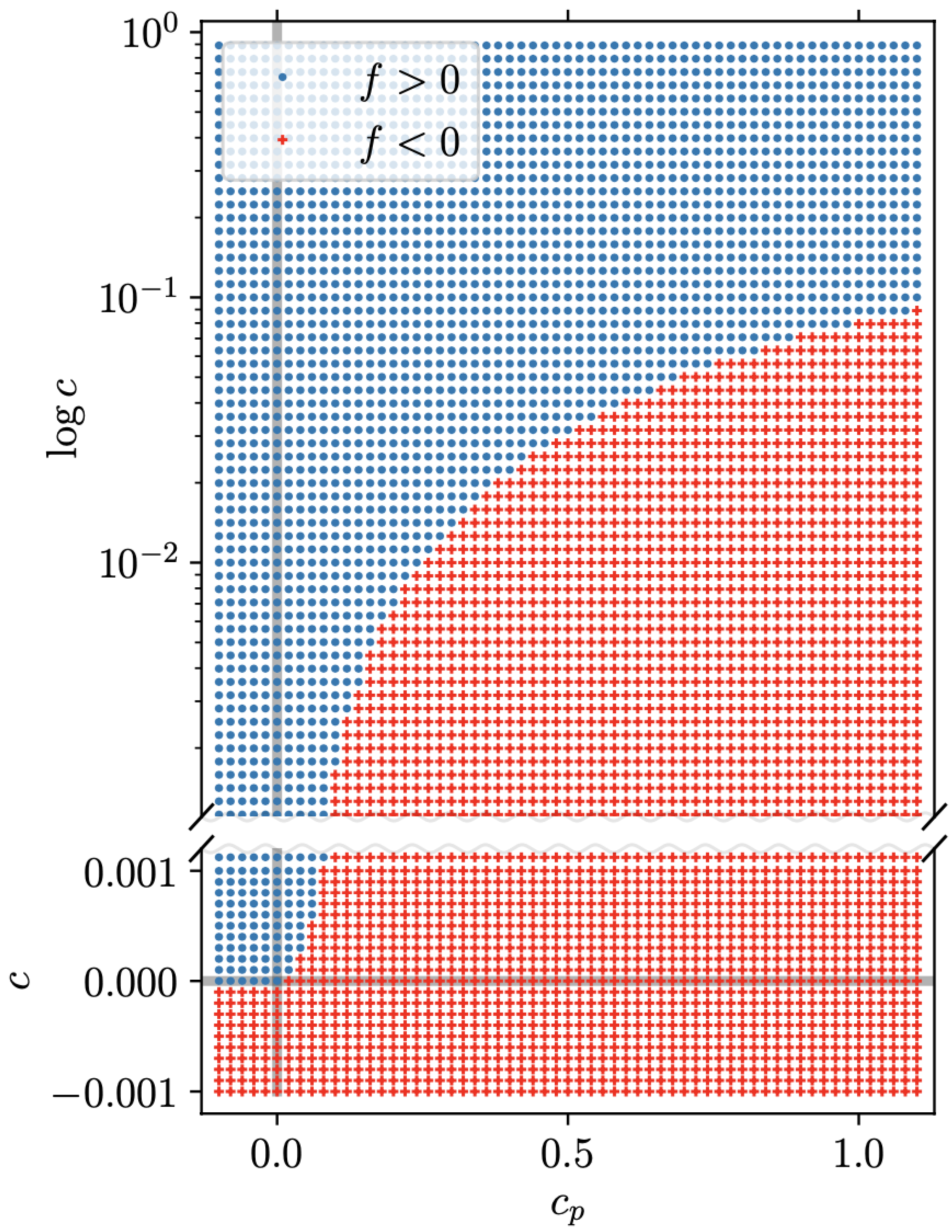


$$\rho_{abc}(r) = \frac{\rho_s}{x^c(1+x^a)^{(b-c)/a}},$$

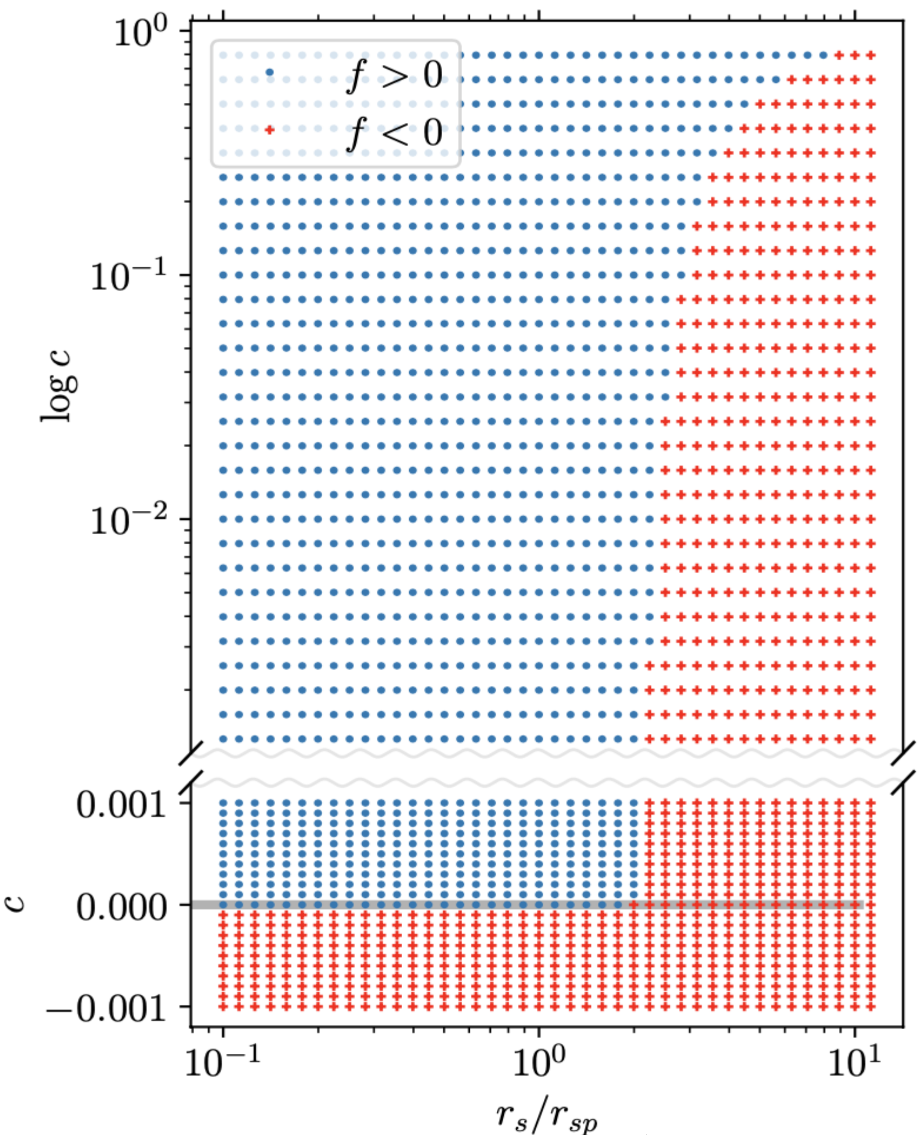
$$\lim_{r\to 0} \frac{d\log\rho_{abc}}{d\log r} = -c.$$







SA+23a, ApJ, submitted



Potential implications for the core-collapse diagnosis: $r_s < 2\, r_{sp}$

Stellar core r_s sets a lower limit for the halo core r_{sp}

SA+23a, ApJ, submitted

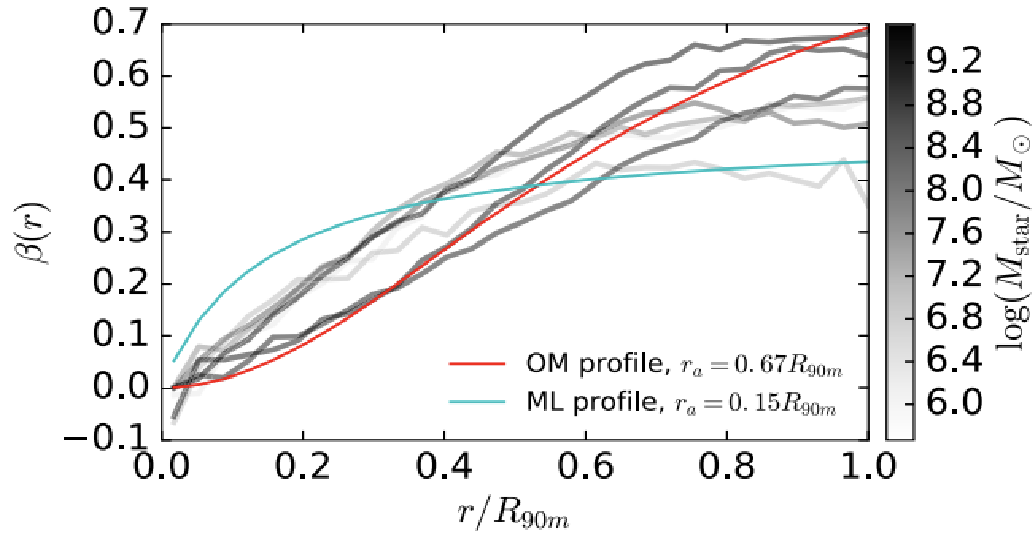
Table 1. Summary of the compatibility between baryon density profile (ρ) and potential

Baryons & Potential, Velocity	Consistency	Comments	Section
(1)	(2)	(3)	(4)
Core[†] & NFW[‡], isotropic	✗	Eqs. (23) and (24). $\beta = 0$[*]. Fig. 1	Sect. 3
Power law[§] & Power law, isotropic	✍	$\alpha > 0$[§] ✓ $\alpha < 0$ ✗. Eq. (25). $\beta = 0$	Sects. 3, 4.2
Core & Soft-core[#], isotropic	✗	$\beta = 0$. Fig. 4. Fig. 5	Sect. 4.2, App. E
Core & Core, isotropic	✍	$\beta = 0$. $a \leq 2$ ✓ $a > 2$ ✗. Fig. 2. Fig. 7	Sects. 4.1, 4.2, App. E
Soft-core & NFW, isotropic	✍	$\beta = 0$. Figs. 5, 6. $c \gtrsim 0.1$ ✓ $c \lesssim 0.1$ ✗.	Sects. 3, 4.2
Soft-core & Soft-core, isotropic	✍	$\beta = 0$. Figs. 5, 6	Sects. 3, 4.2
		$r_s \gtrsim 2\, r_{sp}$ ✗, $c > c_p$ ✓	Sect. 4.2
Core & NFW, O-M model	✗	$\beta(\neq 0)$ in Eq. (12)	Sect. 3
Core & NFW, radially biased	✗	Constant β. $\beta > 0$	Sect. 3, App. D
Core & Any, radially biased	✗	Constant β. $\beta > 0$	Sect. 3, App. D
Power-law & Any, anisotropic	✍	Constant β. $\alpha > 2\beta$	Sect. 3, App. D
Core & NFW, circular	✓	$\beta = -\infty$	App. C
Any & Any, circular	✓	$\beta = -\infty$	App. C
Any & Any, tangentially biased	✍	$\beta < 0$. Eq. (18). ✗ $f_i < 0$	Sects. 2.3, 3

NOTE—
[†] Core $\equiv d\log\rho/d\log r \to 0$ when $r \to 0$.
[‡] Navarro, Frenk, and White potential (Eq. [A6]) produced by a NFW profile (Eq. [A5]).
[*] Velocity anisotropy parameter β defined in Eq. (11).
[§] $\rho \propto r^{-\alpha}$.
[#] Soft-cores defined in Eqs. (30) and (32), and illustrated in Fig. 3. Power laws [§] are a particular type of those.
(1) Description of the baryon density, the gravitational potential, and the velocity distribution.
(2) The symbols ✓, ✗, and ✍ stand for *compatible*, *incompatible*, and *may or may not*, respectively.
(3) Additional comments and keywords.
(4) Section of the text where the combination described in (1) is discussed.

SA+23a, ApJ, submitted

How good or bad are these assumptions? Isotropic velocities and the like

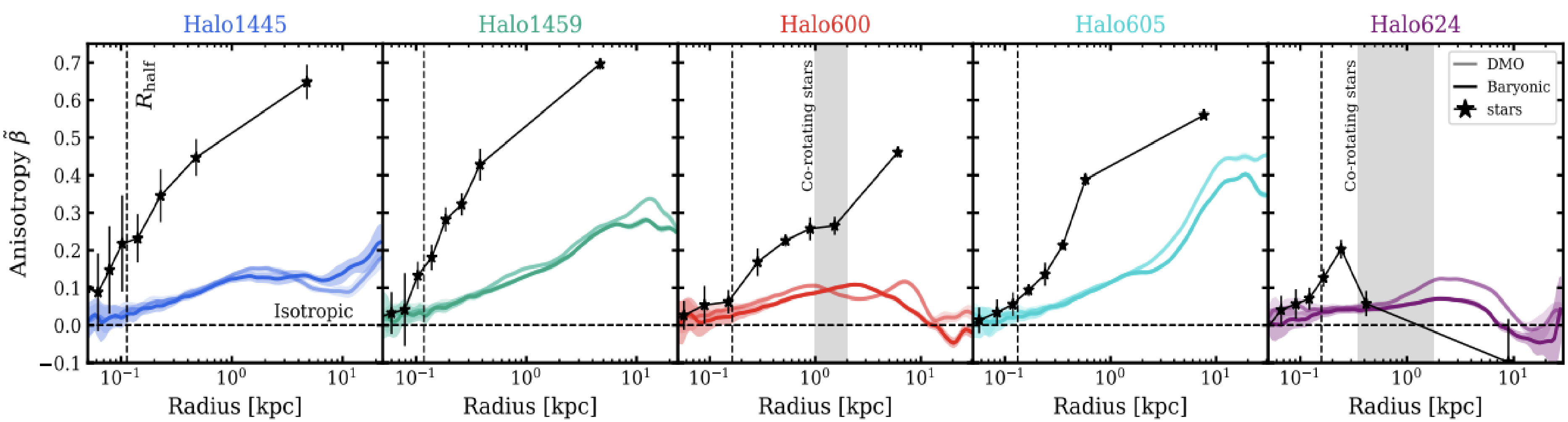


$$\beta(r) = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2},$$

β = 0 Isotropic orbits

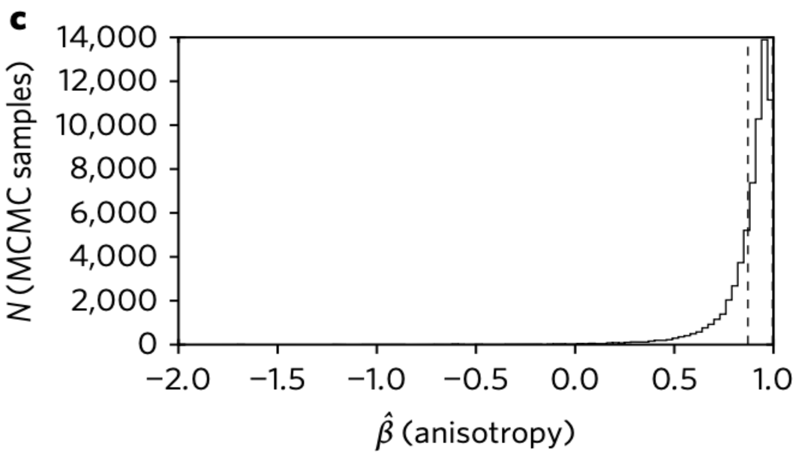
β > 0 Radially biased orbits

FIRE numerical simulation (El-Badry+17, ApJ)

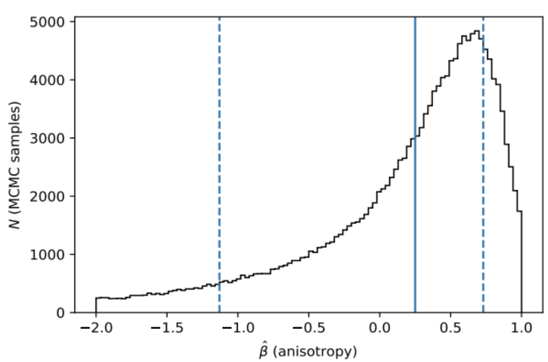


EDGE numerical simulations (Orkey+23, MNRAS)

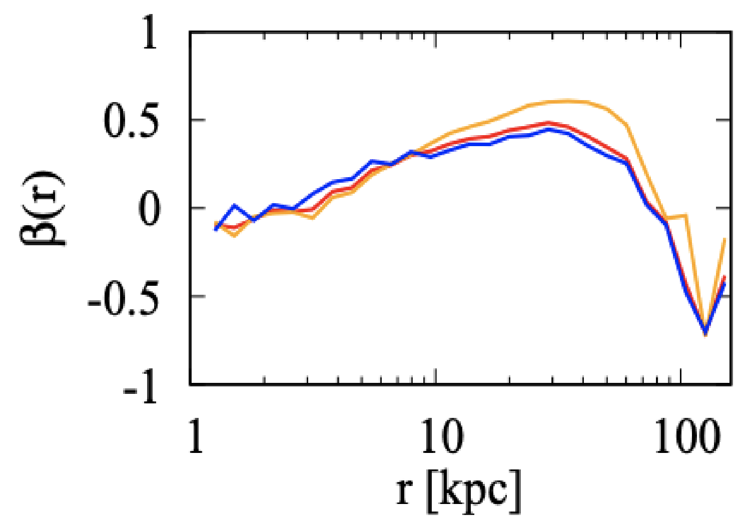
How good or bad are these assumptions? Isotropic velocities and the like

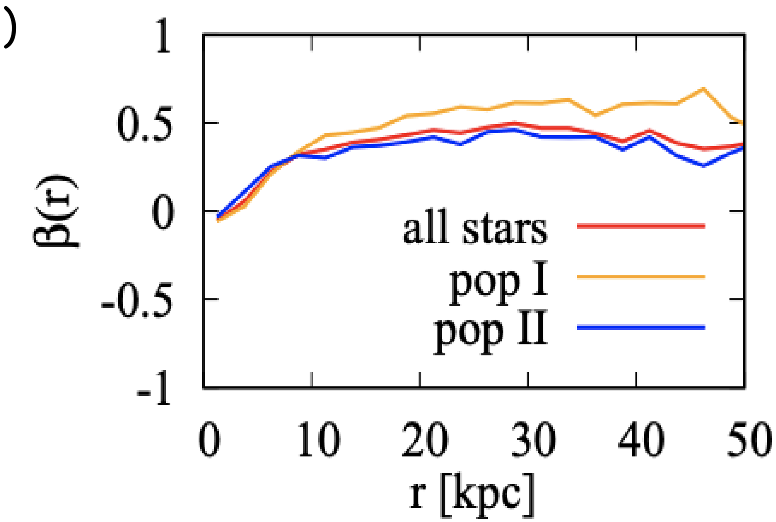


Sculptor, local group dwarfs (Massari+17, NatAstron.)

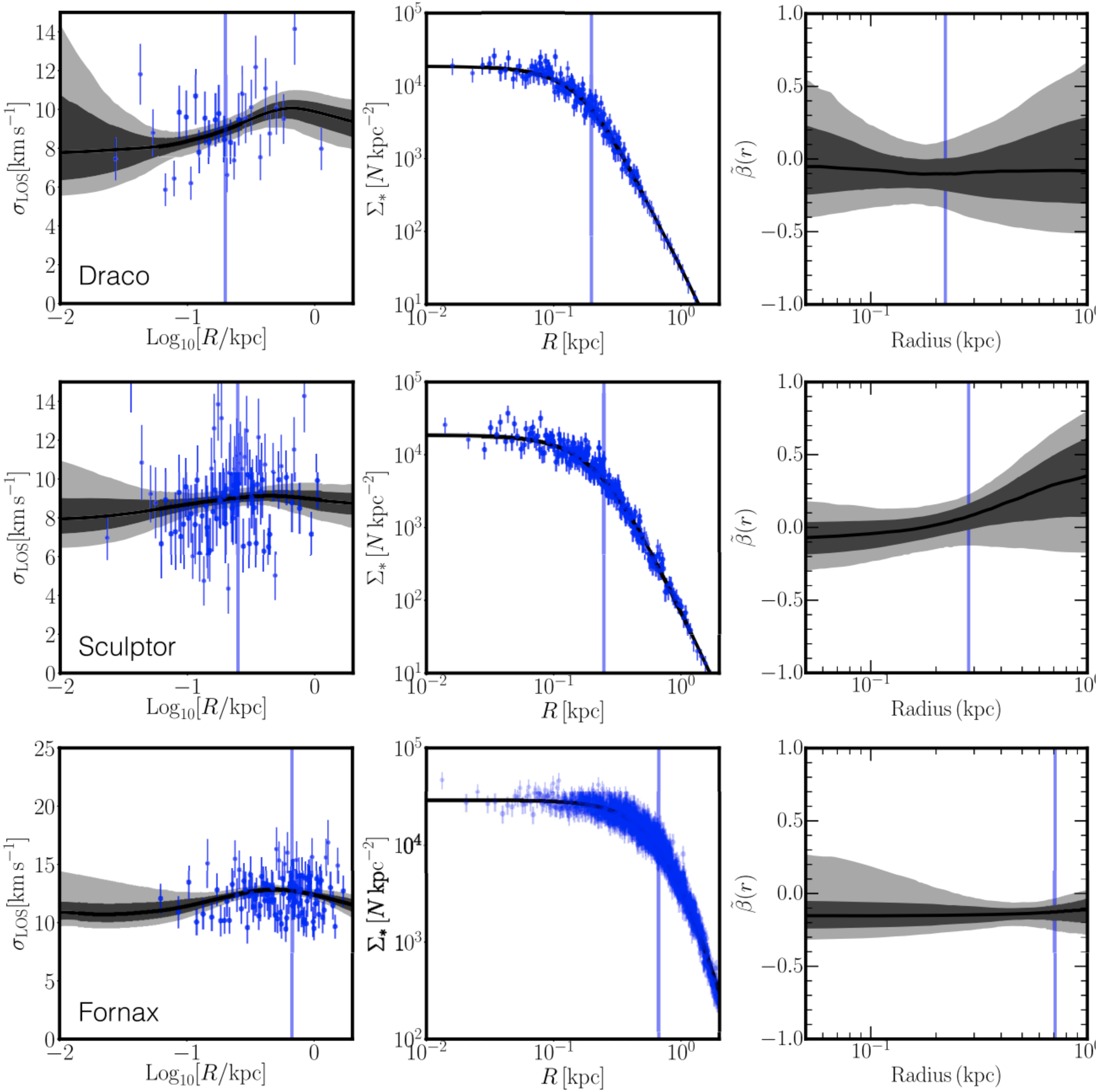


Sculptor, local group dwarfs (Massari+20, A&A)





Fornax, local group dwarfs
(Kowalczyk & Łokas+22, A&A)

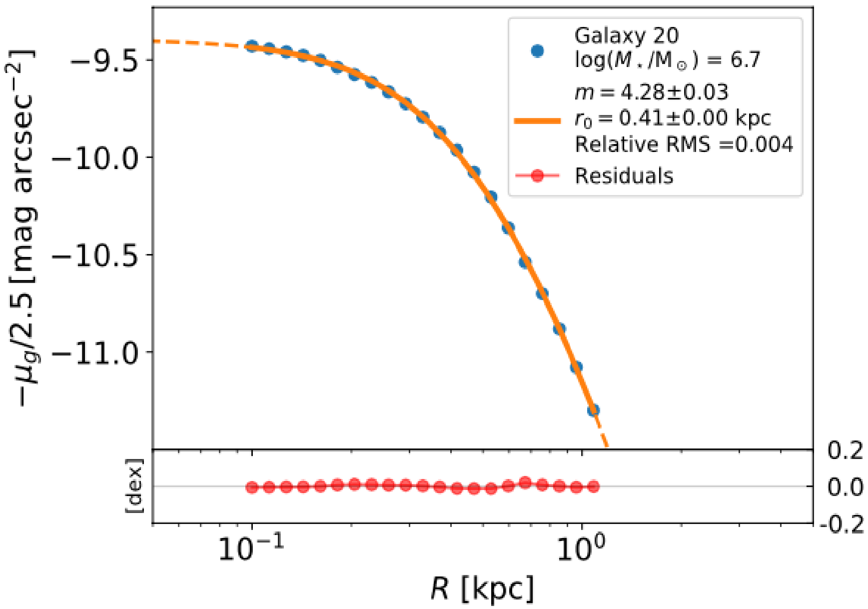


Local group dwarfs
(Read+20, MNRAS)

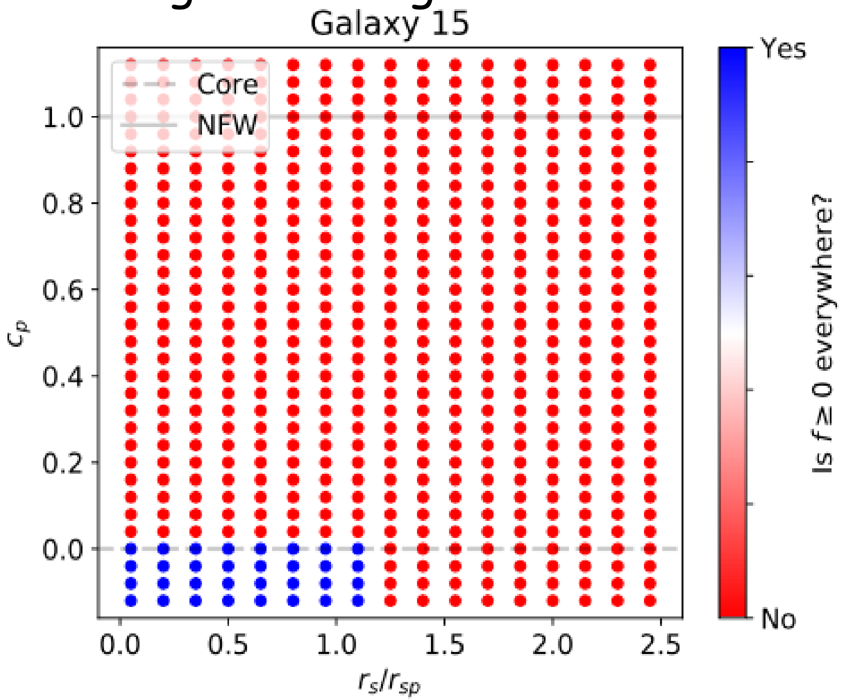
Application to real galaxies

For the galaxies in Carlsten+21,22 (massive satellites) and under the assumption of spherical symmetry and isotropic velocities (SA+23b, in prep.)

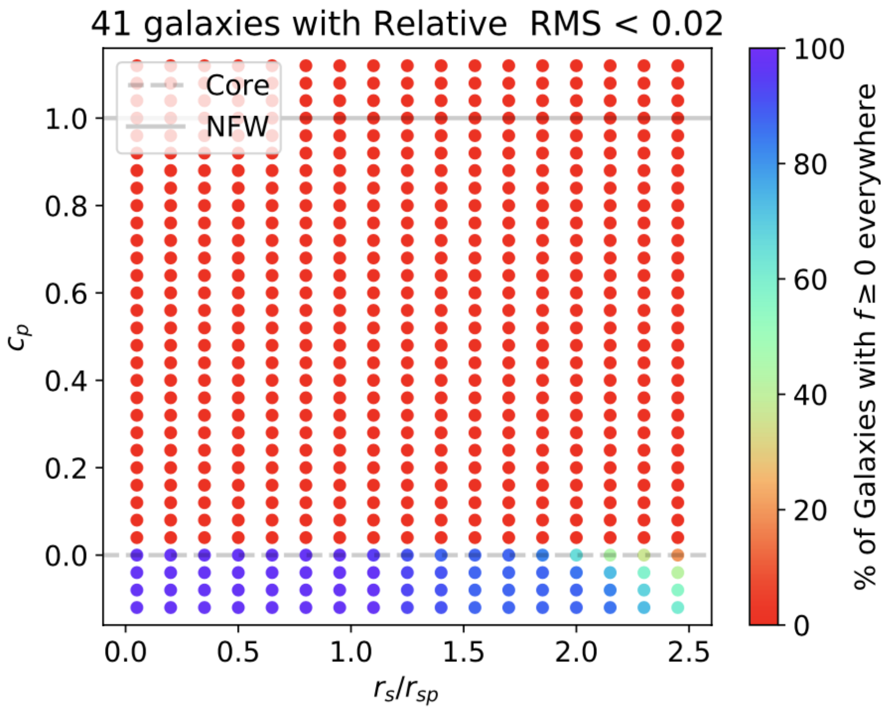
Observation

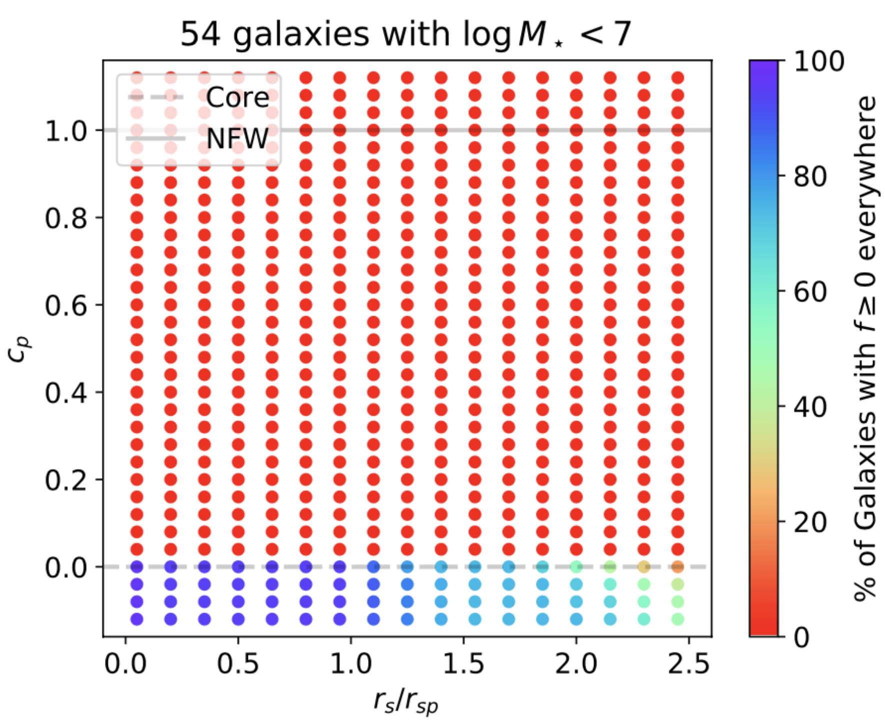


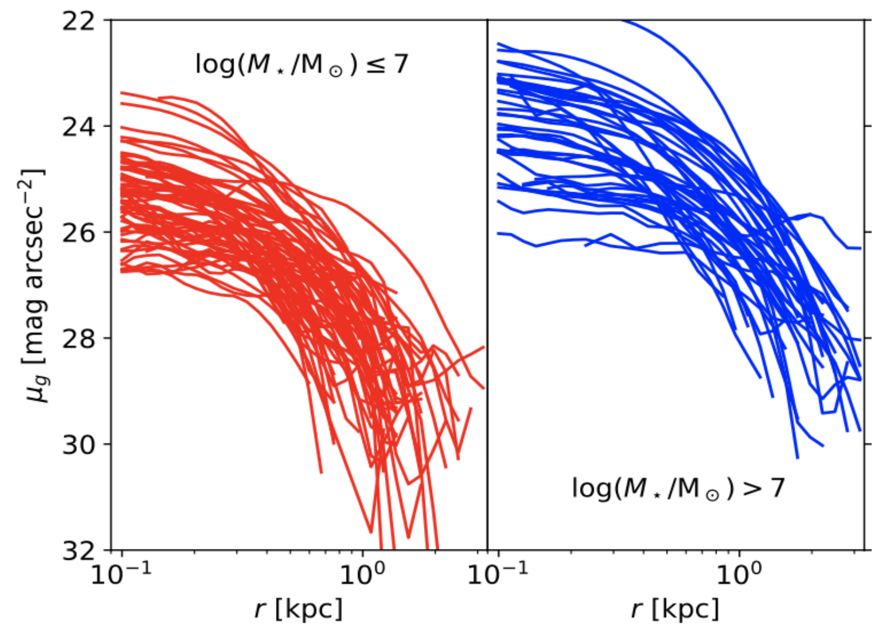
Diagnostic diagram



Is f ≥ 0 everywhere? Yes /No



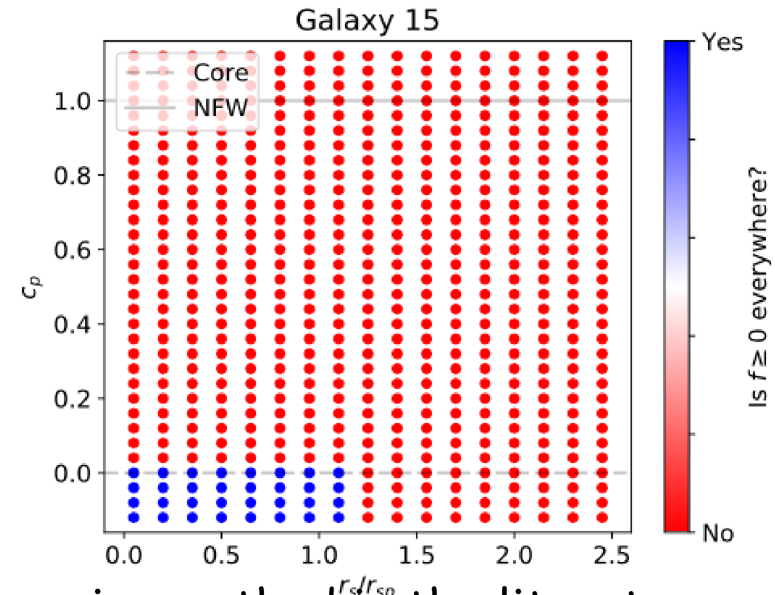




(Carlsten+21,22;
MW-like satellites)

Future pathways - Our roadmap

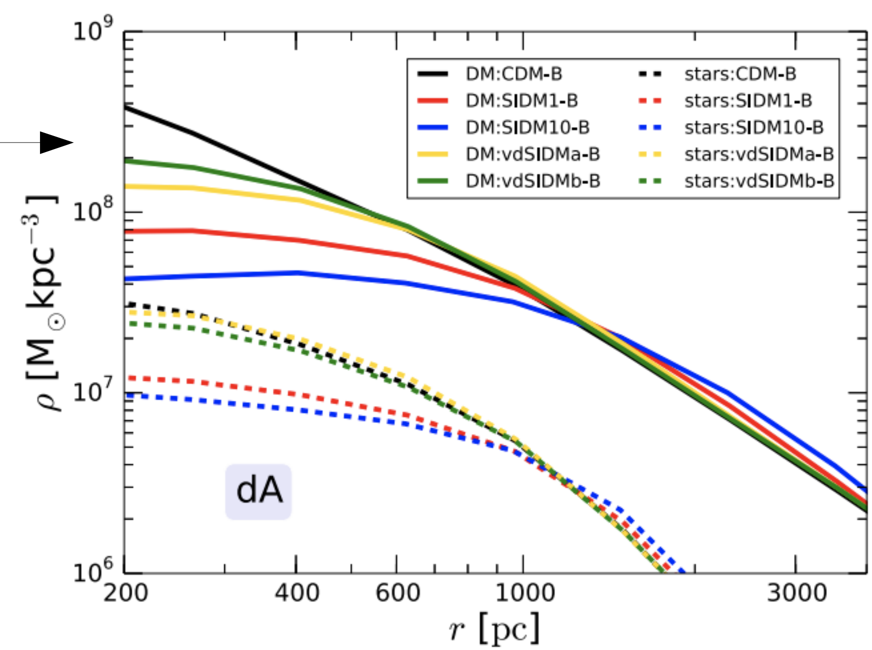
- Consistency test with local group dwarfs having kinematical data, for which we know the potential



- Core collapse: robust diagnostic tool since the stars cannot be more spread-out than the DM core: $r_s < 2\, r_{sp}$

- There are several extensions of the Eddington Inversion method in the literature which allow treating more complicated DFs. In particular extensions that allow to treat axi-symmetrical systems (Lynden-Bell 62). Use them to see how far this idea can be pushed forward.

- Use all possible numerical simulations to check that stelar cores often trace DM cores in numerical simulations.



(Vogelsberger+14, MNRAS)

- Find-define the golden sample of isolated low-mass galaxies ($M_* \ll 10^6\ M_\odot$) to discard (or not) CDM

Conclusions:

- To the question in the title "Can CDM halos hold cored stellar mass distributions?" the answer is … not impossible but improbable

- If we systematically find stellar cores in galaxies with $M_* < \sim 10^6\ M_\odot$, then the DM has a core too and therefore DM must be collisional (SIDM, fuzzy, warm, Black Holes, or else)

- $r_s < 2\ r_{sp}$; the core-collapsed DM haloes should be identifiable as compact stellar systems,

- Observations to provide enough statistics of tiny galaxies are coming up soon (e.g., Trujillo+21, Batagglia 21, Carlsten+21).

- Extensions of the Eddington inversion method to axi-symmetric systems are possible and have to be analyzed (Plastino+23, in prep.)

- This conjecture that stellar cores trace DM cores in low mass systems must be tested using numerical simulations (current and forthcoming and of any kind since it does not depend on the nature of DM).

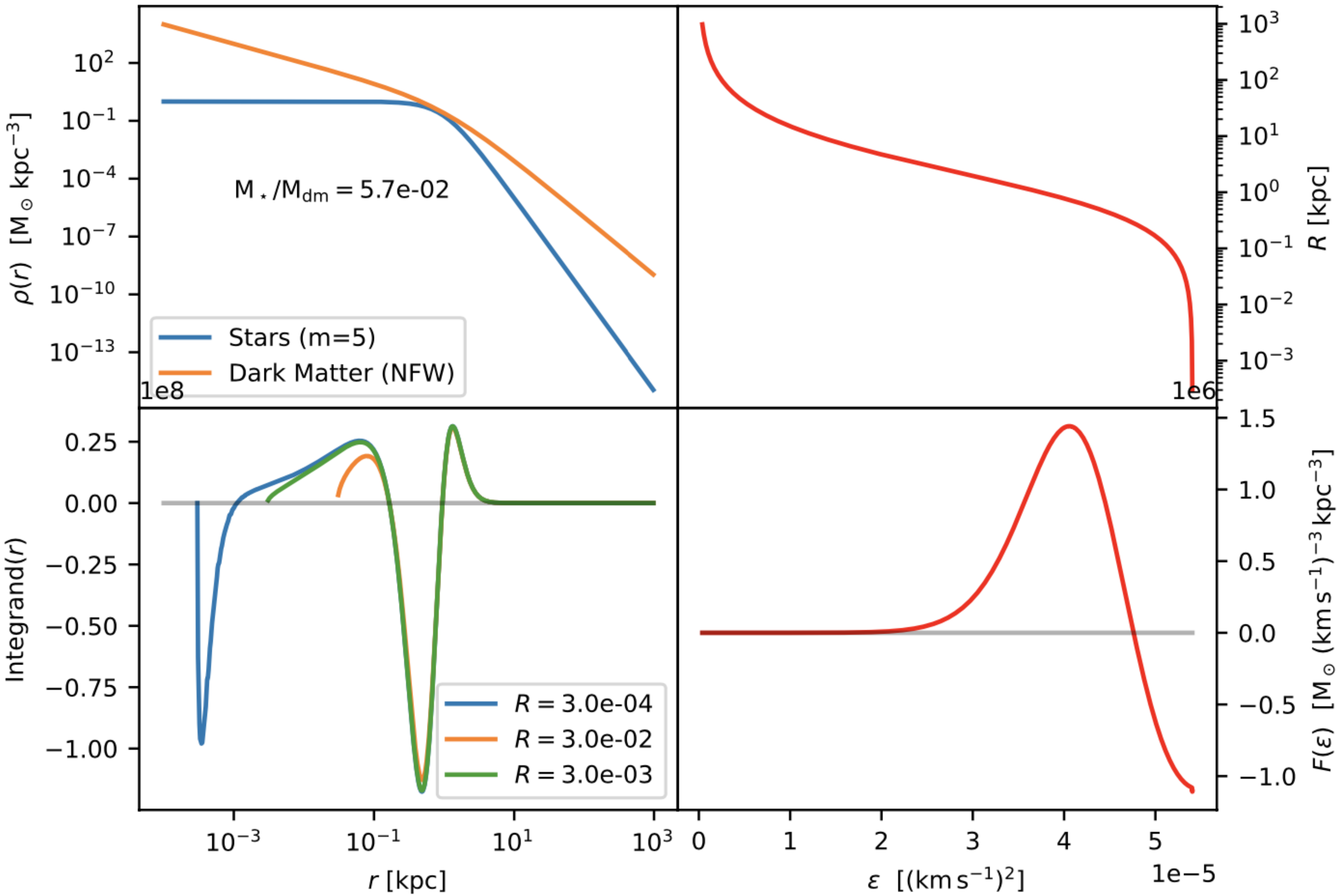
Super Conclusions:

Even if what I presented happen to be a too-naive-to-be-useful approach to the use of photometry to constraint the properties of the dark matter halo, the fact that photometry alone could be used to constraint the DM properties in low stellar mass range is something to consider seriously. Thus,

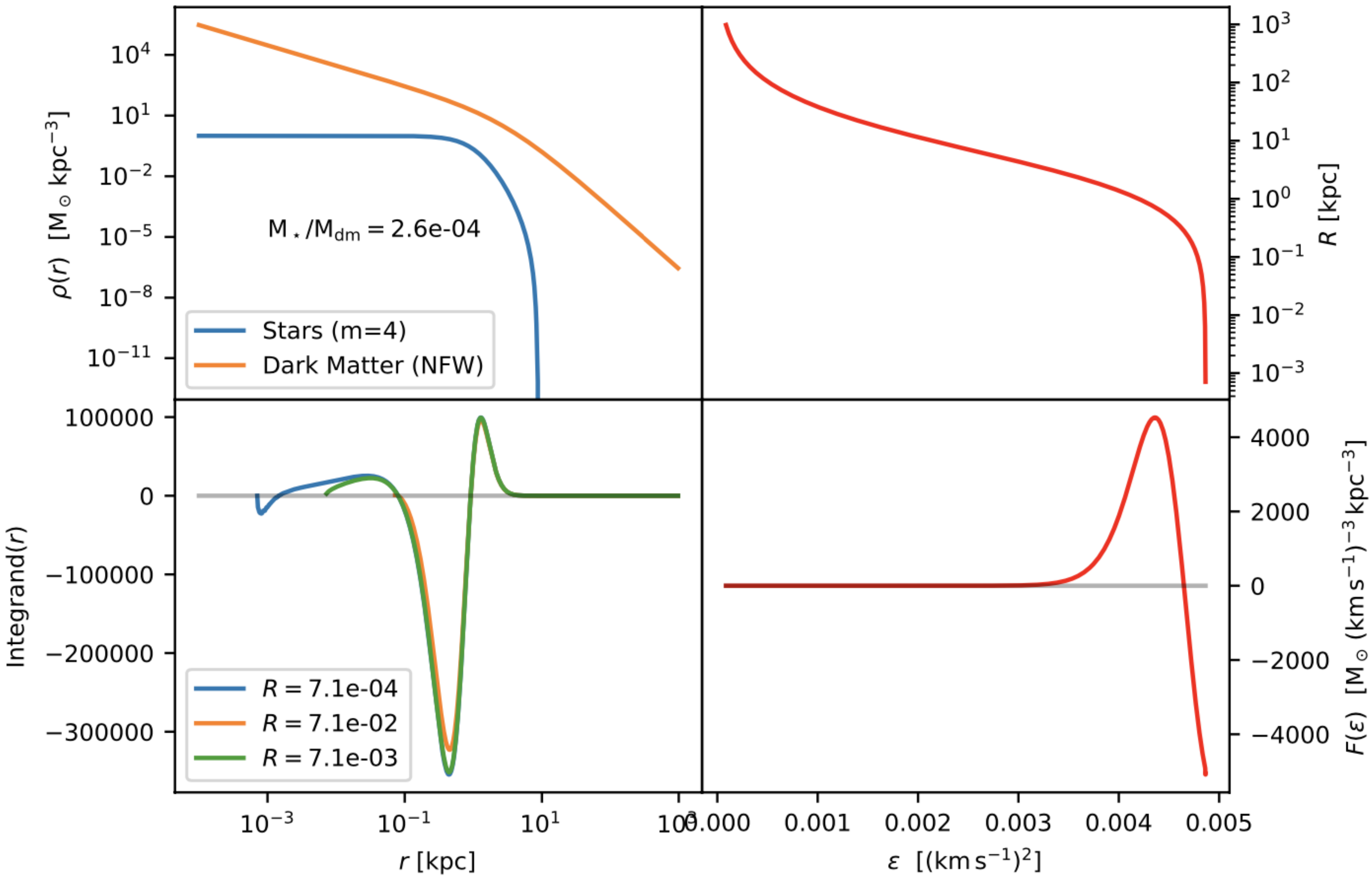
1.- The stellar density profile of the dwarfest galaxies should be characterized observationally.

2.- The impact of the assumptions on the DM nature on the shapes of the dwarfest galaxies should be characterized using numerical simulations (with baryon physics included).

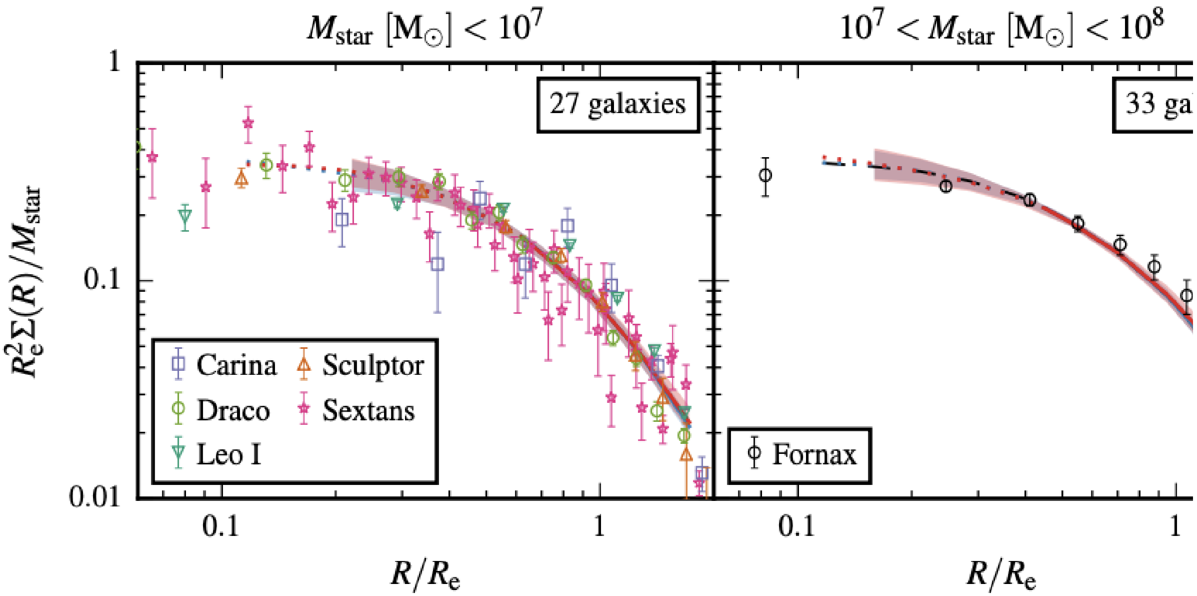
(Fornax Dwarf - ESO)



$\rho(r)$ [$M_\odot$ kpc^{-3}]
10^{2}
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
1e8
$M_\star/M_{dm}$ = 5.7e-02
Stars (m=5)
Dark Matter (NFW)
R [kpc]
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
1e6
Integrand(r)
0.25
0.00
−0.25
−0.50
−0.75
−1.00
R = 3.0e-04
R = 3.0e-02
R = 3.0e-03
10^{-3}
10^{-1}
10^{1}
10^{3}
r [kpc]
$F(\varepsilon)$ [$M_\odot$ (km s^{-1})$^{-3}$ kpc^{-3}]
1.5
1.0
0.5
0.0
−0.5
−1.0
0
1
2
3
4
5
1e−5
ε [(km s^{-1})^2]



$M_\star/M_{dm} = 2.6\text{e-}04$
Stars (m=4)
Dark Matter (NFW)
$\rho(r)$ [$M_\odot$ kpc^{-3}]
R [kpc]
Integrand(r)
$R = 7.1\text{e-}04$
$R = 7.1\text{e-}02$
$R = 7.1\text{e-}03$
r [kpc]
ε [(km s^{-1})^2]
$F(\varepsilon)$ [$M_\odot$ (km s^{-1})$^{-3}$ kpc^{-3}]



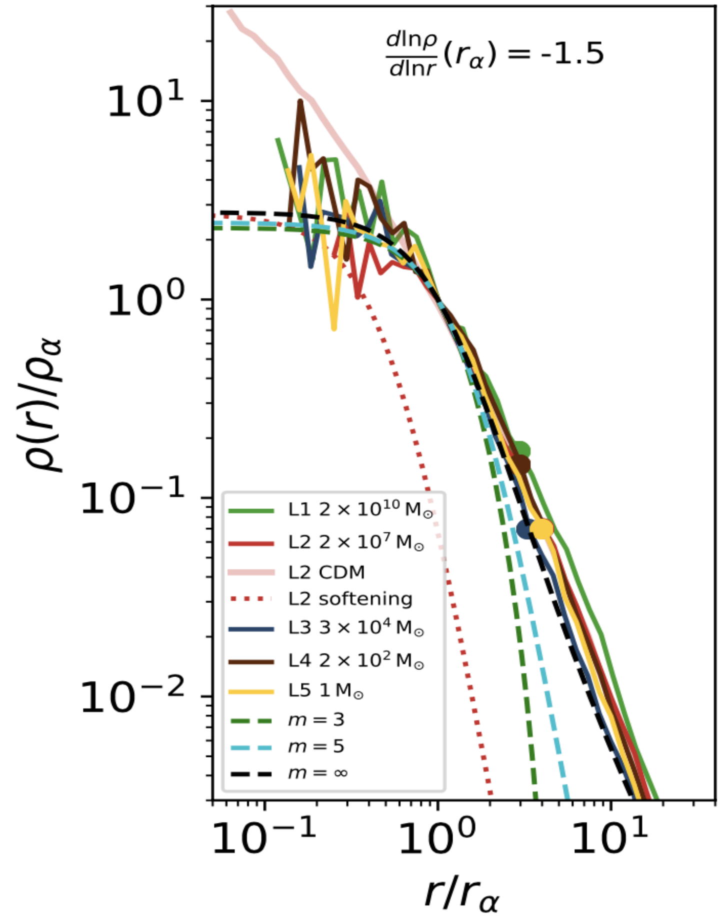
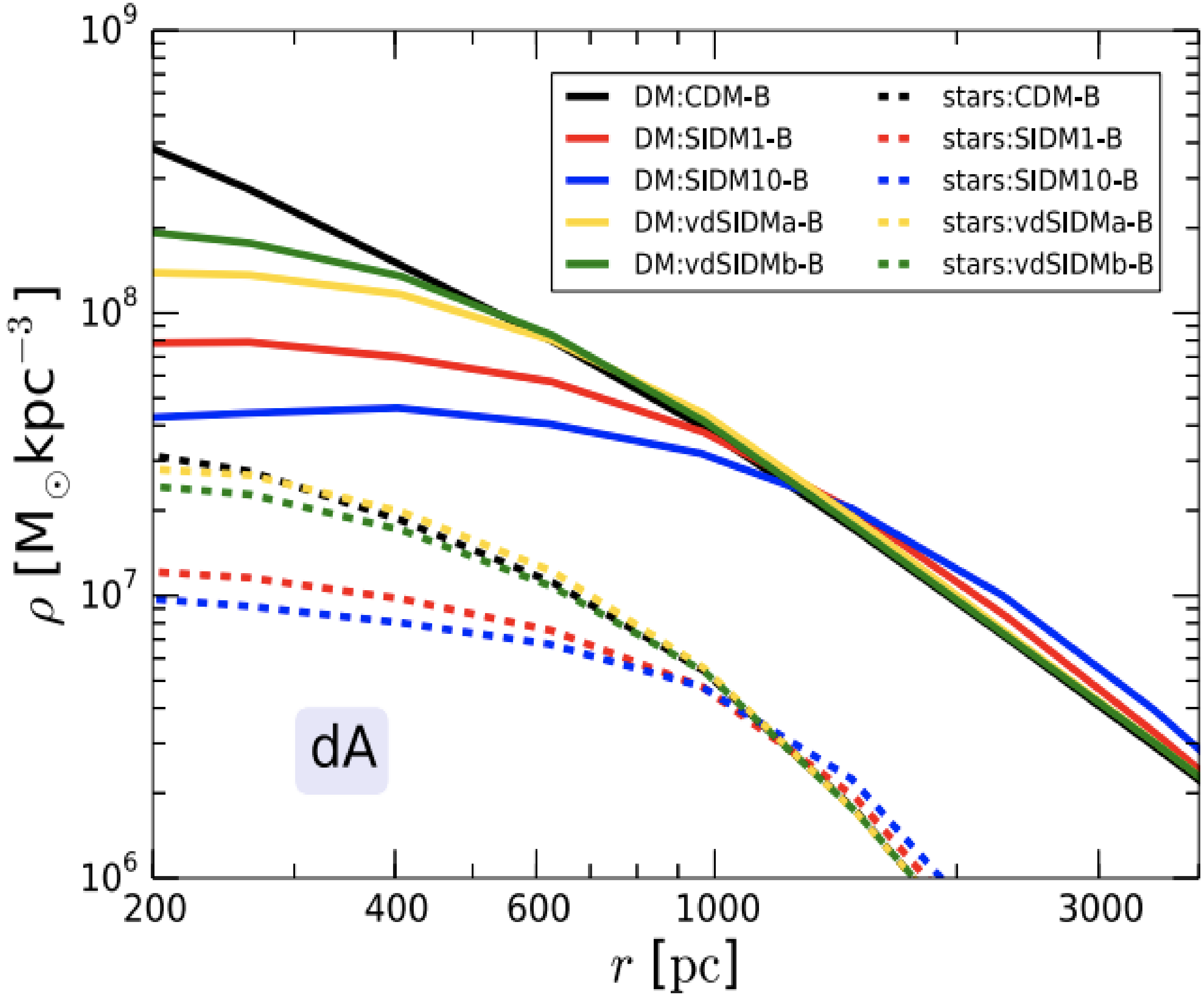


Figure 3. Projected stellar density profiles, $\Sigma(R)$, for dispersion-dominated ($\kappa_{\rm rot} < 0.5$) field and satellite galaxies in the HR simulations. The solid lines are the median projected profiles obtained from 1536 evenly distributed projections for all galaxies of a given type (different colours; legend in right panel), around which the shaded regions of the same colour indicate the $16^{\rm th} - 84^{\rm th}$ percentile spread. Each individual profile is scaled, prior to stacking, by the projected stellar half-mass radius, $R_{\rm e}$, for that line of sight, and the stellar mass, $M_{\rm star}$. The shaded regions are not shown below the gravitational softening (see Table 1; in units of the median $R_{\rm e}$ in each panel), and the median profiles are dotted below 2.8 times the softening. The panels show different stellar mass ranges, and are labelled with the number of simulated galaxies they contain. As a visual aid, the median profile for all galaxies in the lowest mass bin is repeated as a dashed black line in the higher mass bins. The symbols with error bars show data for dSph satellites of the MW, where the data for each dSph are plotted in the relevant panel for its stellar mass following McConnachie (2012). The data shown are for Carina (Muñoz et al. 2006), Draco (Odenkirchen et al. 2001), Fornax (Coleman et al. 2005), Leo I (Smolčić et al. 2007), Sculptor (Battaglia et al. 2008), and Sextans (Irwin & Hatzidimitriou 1995). The measurements are scaled in the same way as the simulation predictions, assuming either a Gaussian (Leo I) or Plummer (all others) density profile fit.

Campbell+17, MNRAS
Wang+20, Nat.



(Vogelsberger+14, MNRAS)