GRAPPA

GRavitation AstroParticle Physics Amsterdam

"Self-Interacting Dark Matter: Models, Simulations and Signals"
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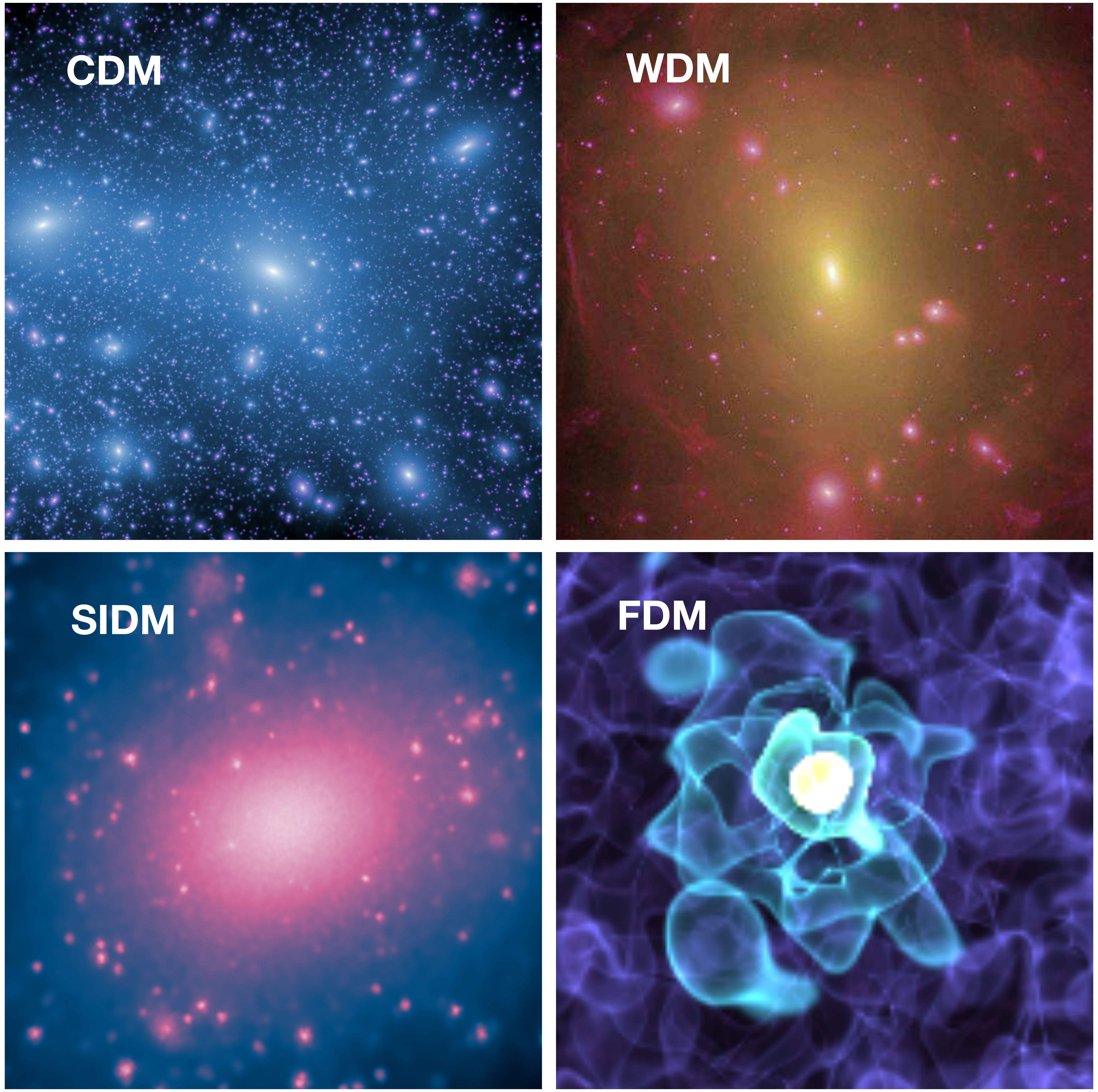
Towards building semi-analytical models of SIDM

Shin'ichiro Ando

University of Amsterdam

Small-scale structure



Scientific goals: develop models of small-scale structure formation, and apply them to various dark matter candidates

- What dark matter particles are determines small-scale distribution
  - Key to identifying particle nature
- Develop **semi-analytical, models**, calibrate with **numerical simulations**, and **establish reliable models** free from shot noise and numerical resolution

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- Semi-analytical models of CDM


Nagisa Hiroshima


Tomoaki Ishiyama

- Extending semi-analytical models to SIDM and calibration with (isolated) N-body simulations


Masato Shirasaki


Takashi Okamoto


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- TangoSIDM: Numerical simulations of SIDM


Camila Correa

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Semi-analytical models of subhalos

- Complementary to numerical simulations
- Light, flexible, and versatile
- Can cover large range for halo masses (**micro-halos to clusters**) and redshifts (***z*** **~ 10 to 0**) based on physics modeling
- **Accuracy:** Reliable if it is **calibrated with simulations** at resolved scales

Semi-analytical modeling

Structures start to form → **Initial condition: Primordial power spectrum**

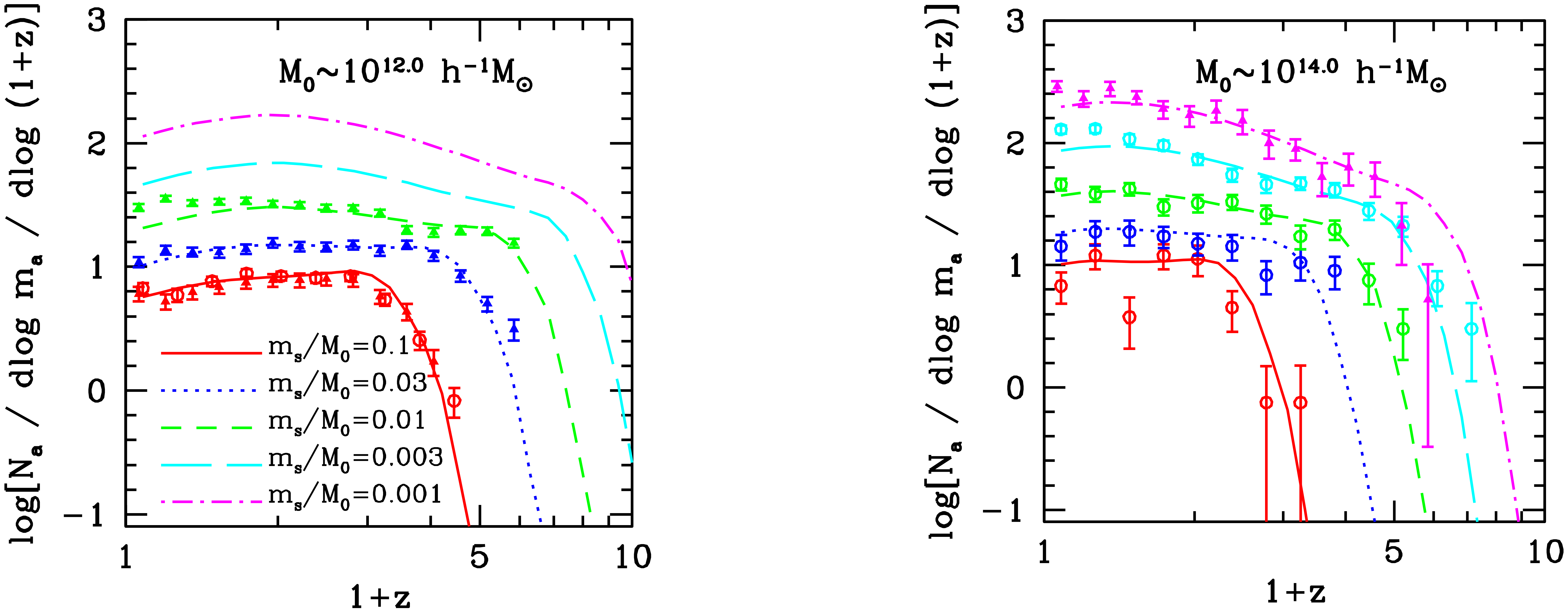
↓

Smaller halos merge and accrete to form larger ones → **Extended Press-Schechter formalism**

↓

Subhalos experience mass loss → **Modeling for tidal stripping and mass-loss rate**

Subhalo accretion



Infall distribution of subhalos:

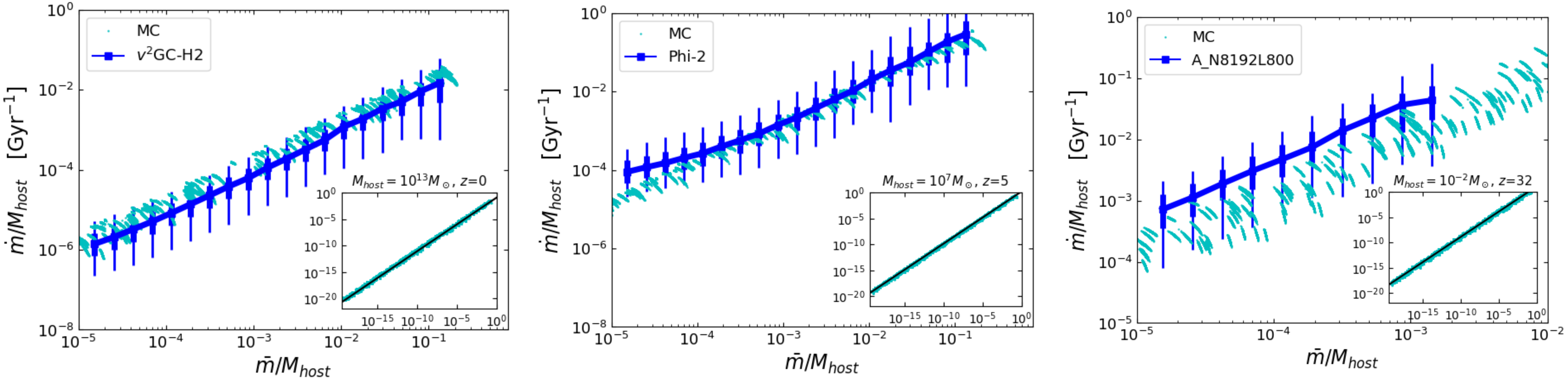
Extended Press-Schechter (EPS) formalism

Yang et al., *Astrophys. J.* **741**, 13, (2011)

$$\frac{d^2N_{\rm sh}}{dm_{\rm acc}dz_{\rm acc}} \propto \frac{1}{\sqrt{2\pi}} \frac{\delta(z_{\rm acc}) - \delta_M}{(\sigma^2(m_{\rm acc}) - \sigma_M^2)^{3/2}} \exp\left[-\frac{(\delta(z_{\rm acc}) - \delta_M)^2}{2(\sigma^2(m_{\rm acc}) - \sigma_M^2)}\right]$$

Subhalo evolution

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)



$$\dot{m} = -A \frac{m(z)}{\tau_{\rm dyn}(z)} \left[\frac{m(z)}{M(z)} \right]^{\zeta}$$

Cf., Jiang, van den Bosch, *Mon. Not. R. Astron. Soc.* **458**, 2848 (2016)

- Monte Carlo approach
  - Determine orbital energy and angular momentum
  - Assume the subhalo loses all the masses outside of its tidal radius instantaneously at its peri-center passage
- Internal structure changes follow Penarrubia et al. (2010)

Semi-analytical modeling

Structures start to form

↓

Smaller halos merge and accrete to form larger ones

↓

Subhalos experience mass loss

✓ **Initial condition: Primordial power spectrum**

✓ **Extended Press-Schechter formalism**

✓ **Modeling for tidal stripping and mass-loss rate**

Distribution of subhalo quantities

Subhalo accretion

$$\frac{dN_{\rm sh}}{d\theta} = \int dm_a \int dz_a \frac{d^2N_{\rm sh}}{dm_a dz_a} \delta(\theta - \theta(m_a, z_a))$$

Subhalo evolution

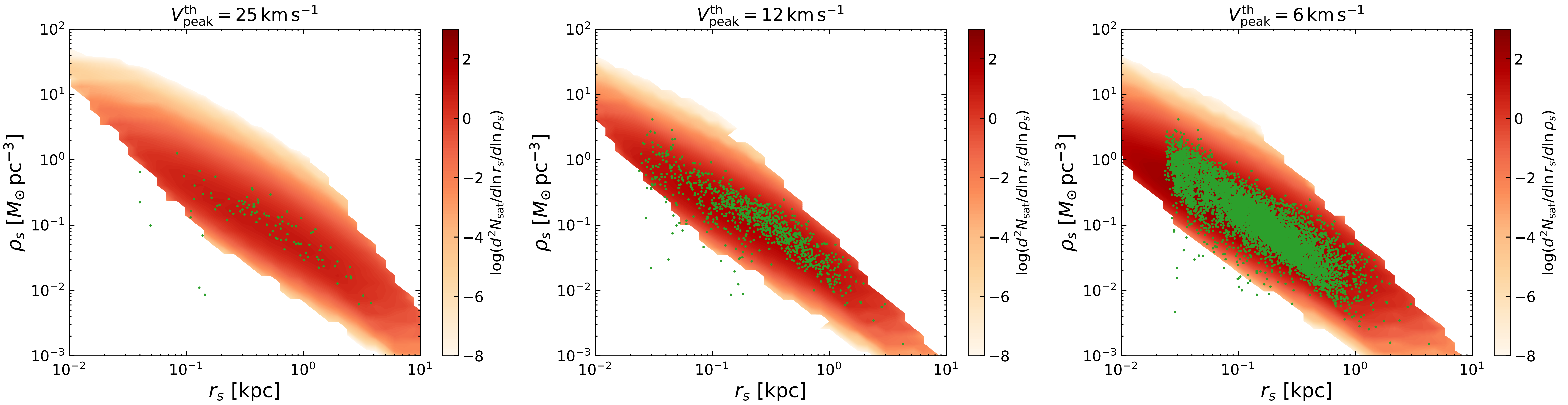
$$\theta = \{m, r_s, \rho_s, \ldots\}$$

Subhalo mass function

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)

Distribution of r_s and ρ_s

$$\rho(r) = \frac{\rho_s}{(r/r_s)(r/r_s + 1)^2}$$

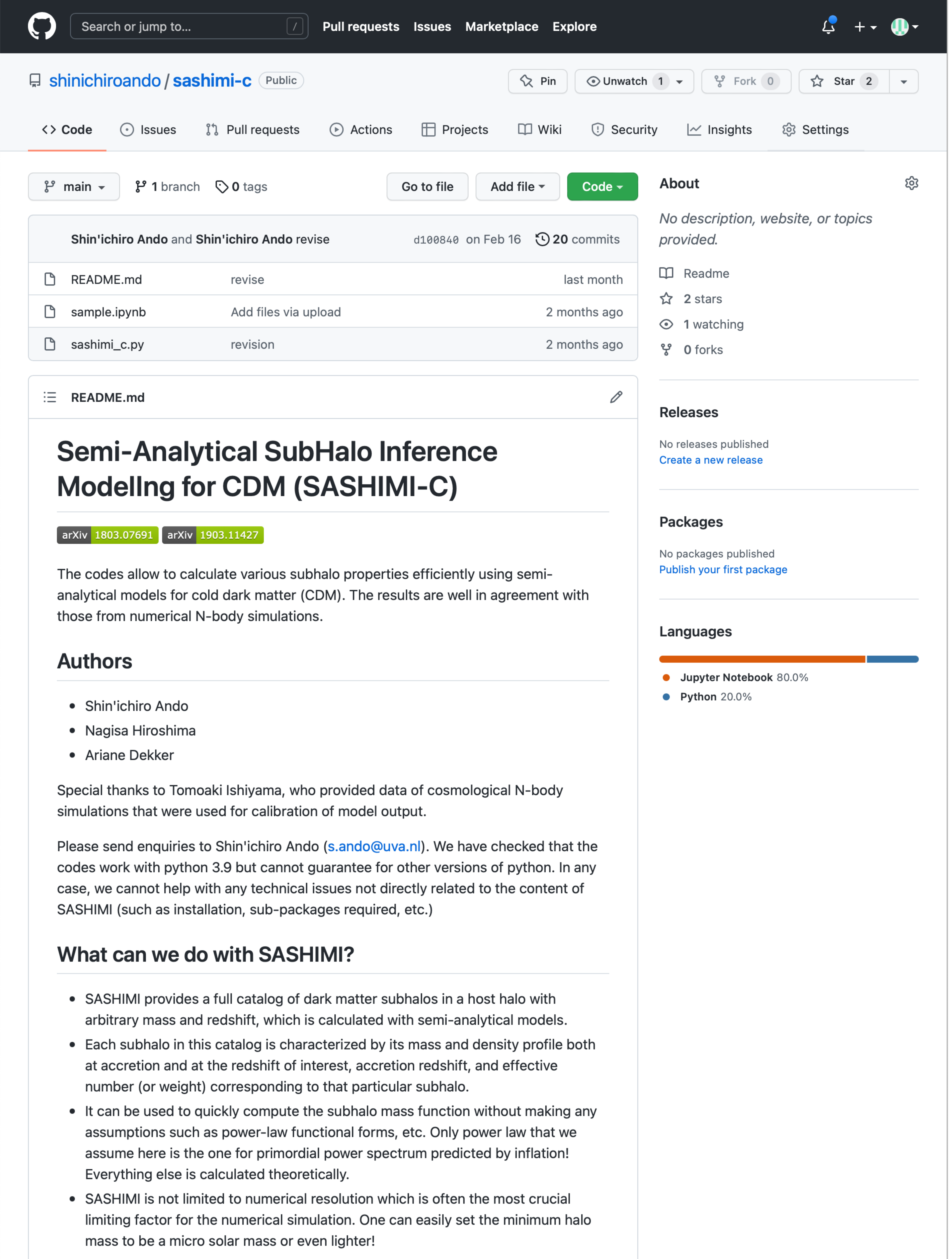


Ando, Geringer-Sameth, Hiroshima, Hoof, Trotta, Walker, *Phys. Rev. D* **102**, 061302 (2020)

Good agreement with simulation results (Vea Lactea II)

Summary: Semi-analytical modeling

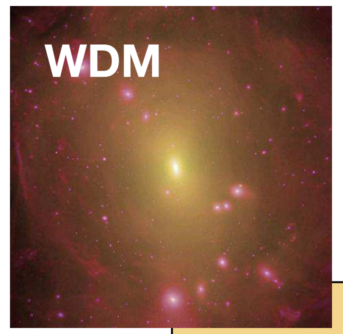
- Benchmark models for CDM
  - Free from resolution (useful for small mass ranges)
  - Free from shot noise (useful for large mass ranges)
  - Well tested against numerical simulations of halos with various masses at various redshifts
  - Quick implementation, which is crucial to survey through parameter spaces for different dark matter models



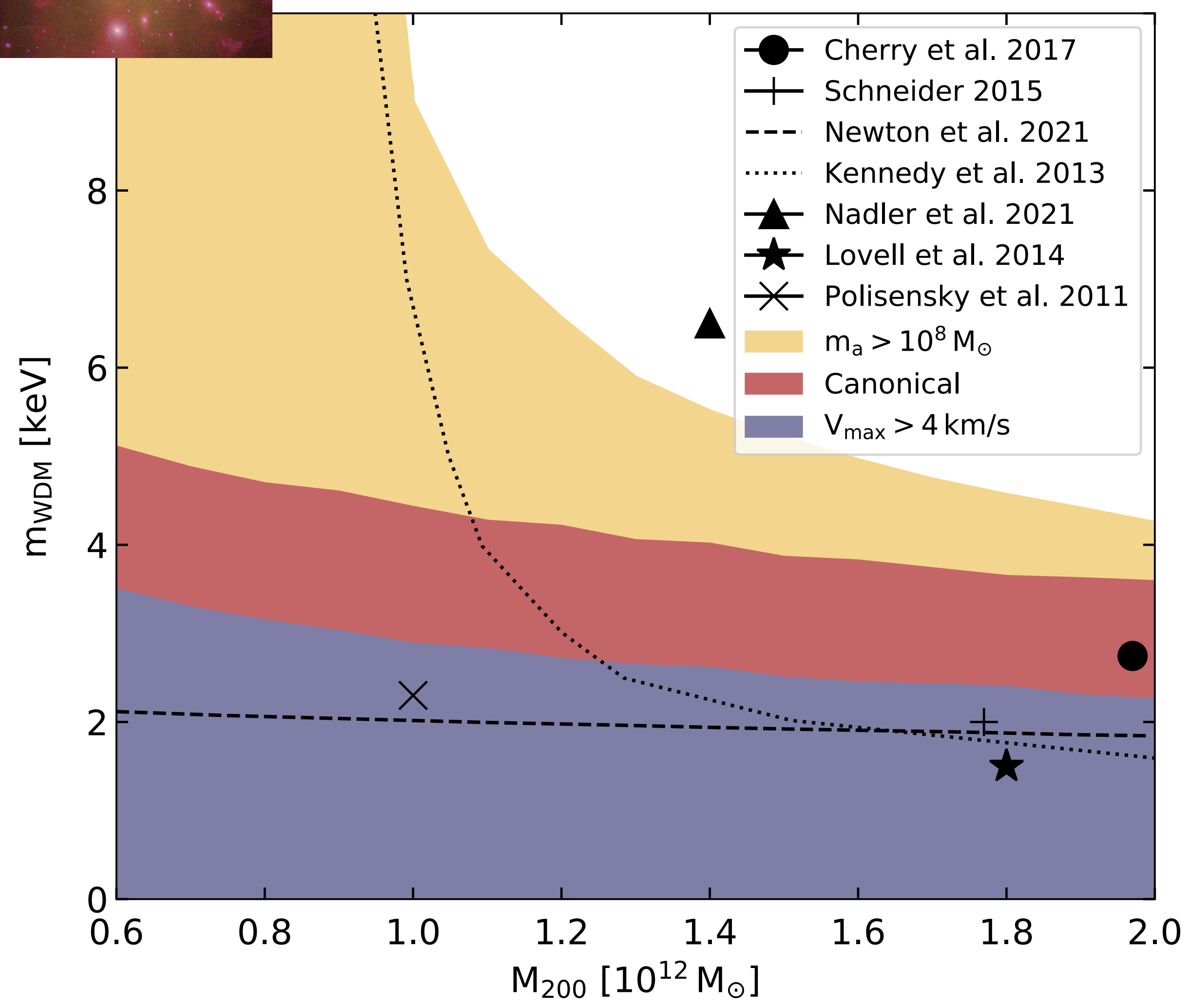
Public code: Semi-analytical subhalo models (CDM)

- **S**emi-**A**nalytical **S**ub**H**alo **I**nference **M**odel**I**ng
- "Cold" SASHIMI: *github.com/shinichiroando/sashimi-c*
- Only 760 lines of simple python codes, which enable to calculate (nearly) everything we did in Hiroshima et al. (2018)
  - Subhalo mass function, substructure boost of dark matter annihilation, etc.
- Well documented and useful sample codes provided

Applications

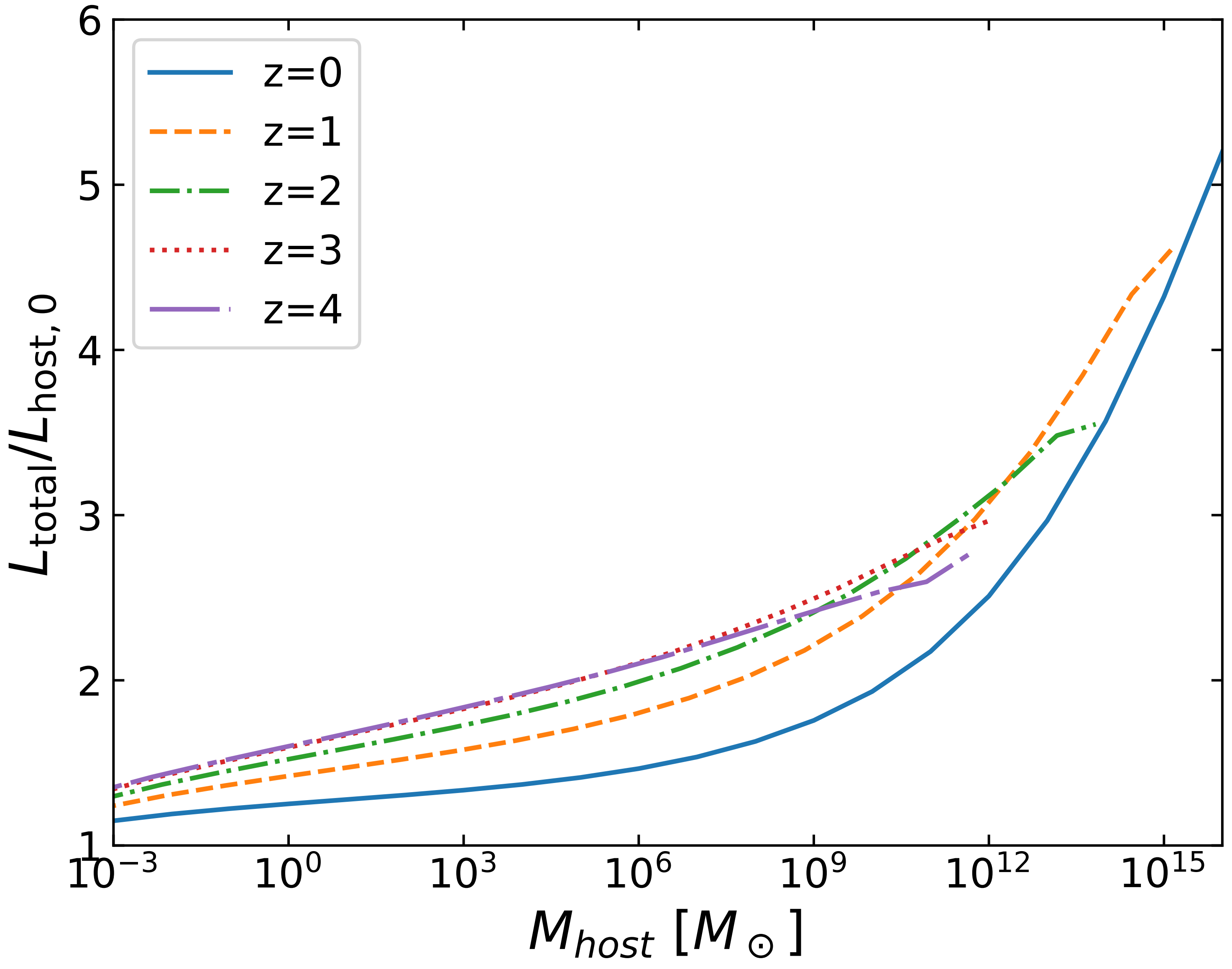


Warm dark matter



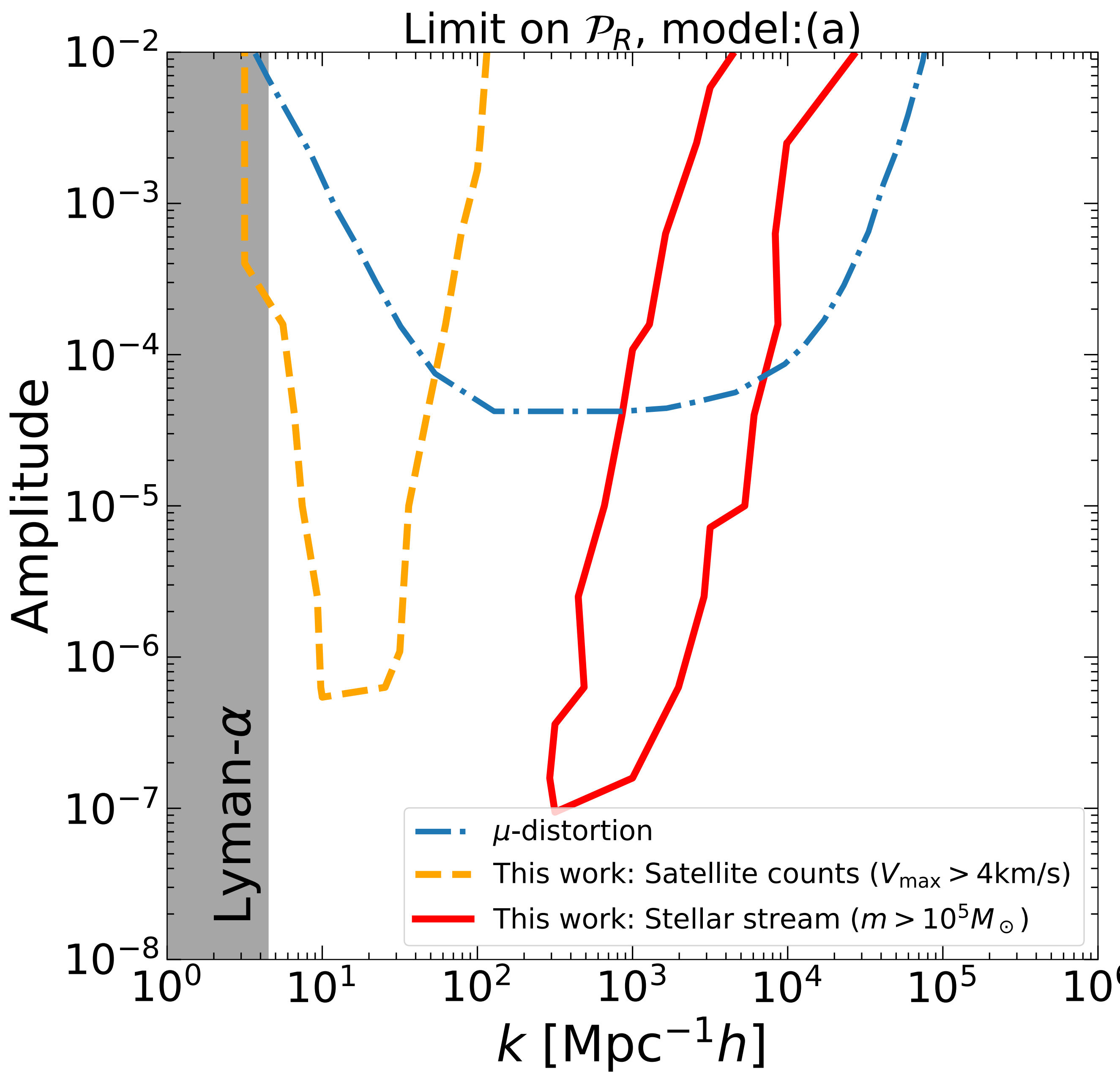
Dekker et al., *Phys. Rev. D* **106**, 123026 (2022)

WIMP annihilation



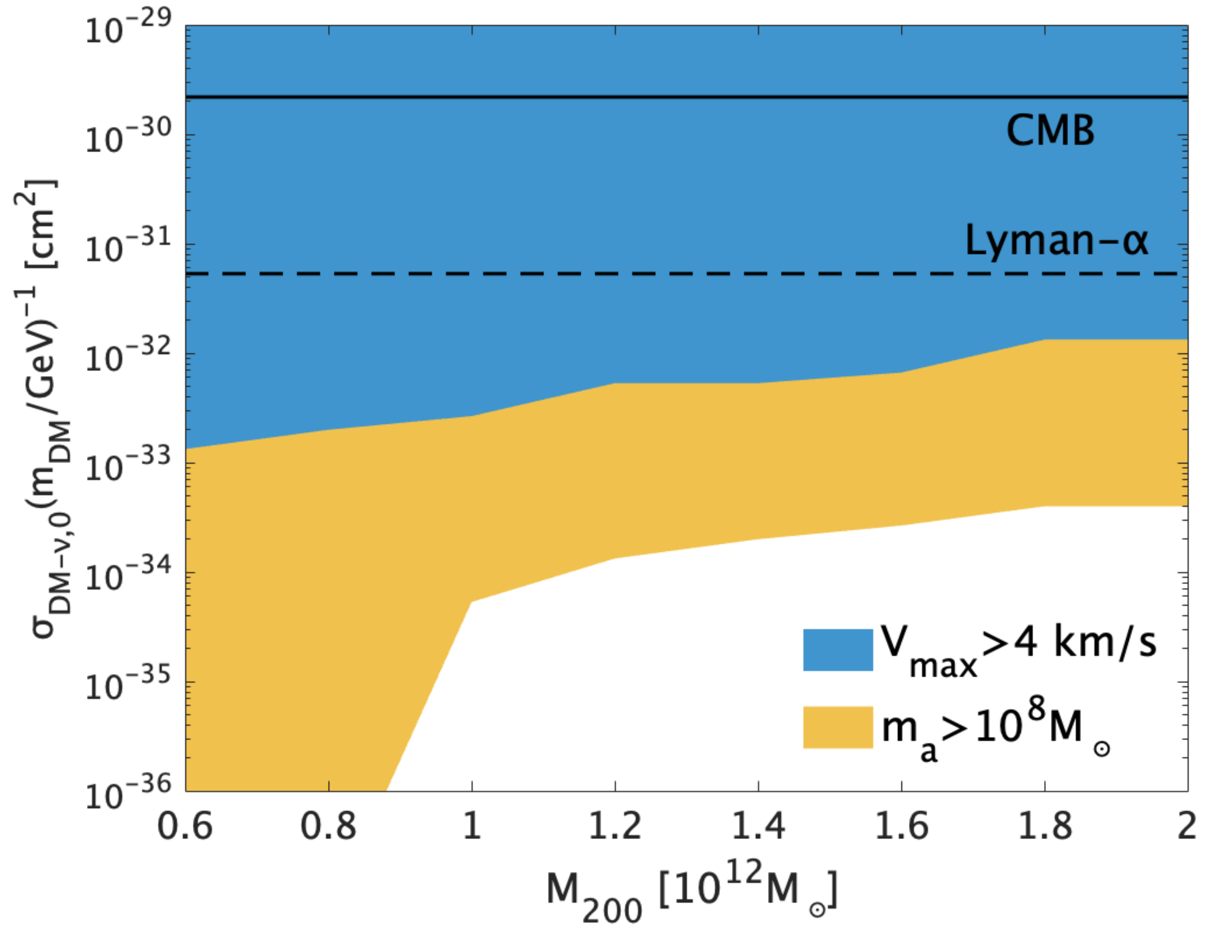
Hiroshima et al., *Phys. Rev. D* **97**, 123002 (2018)

Primordial power spectrum



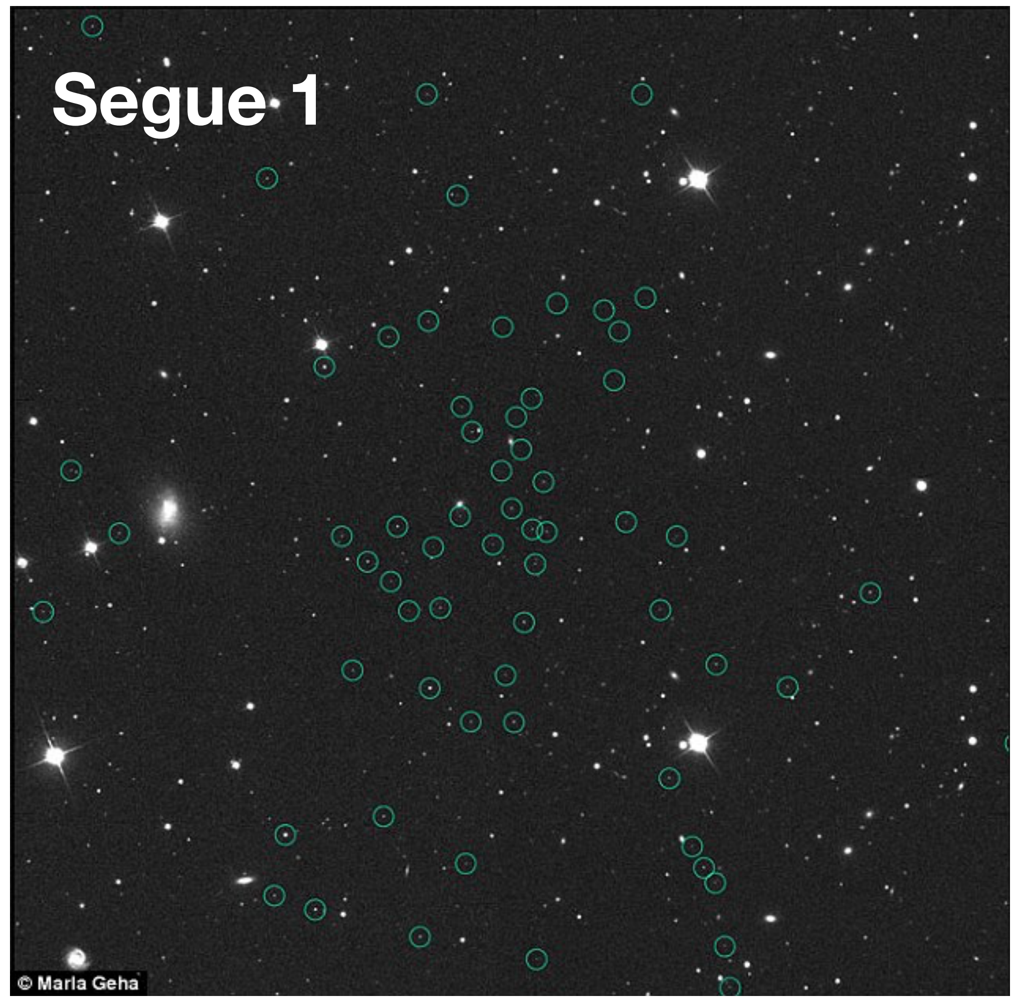
Ando et al., *Phys. Rev. D* **106**, 103014 (2022)

DM-ν interaction



Akita, Ando, arXiv:2305.01913 [astro-ph.CO]

Estimating density profiles of dSphs



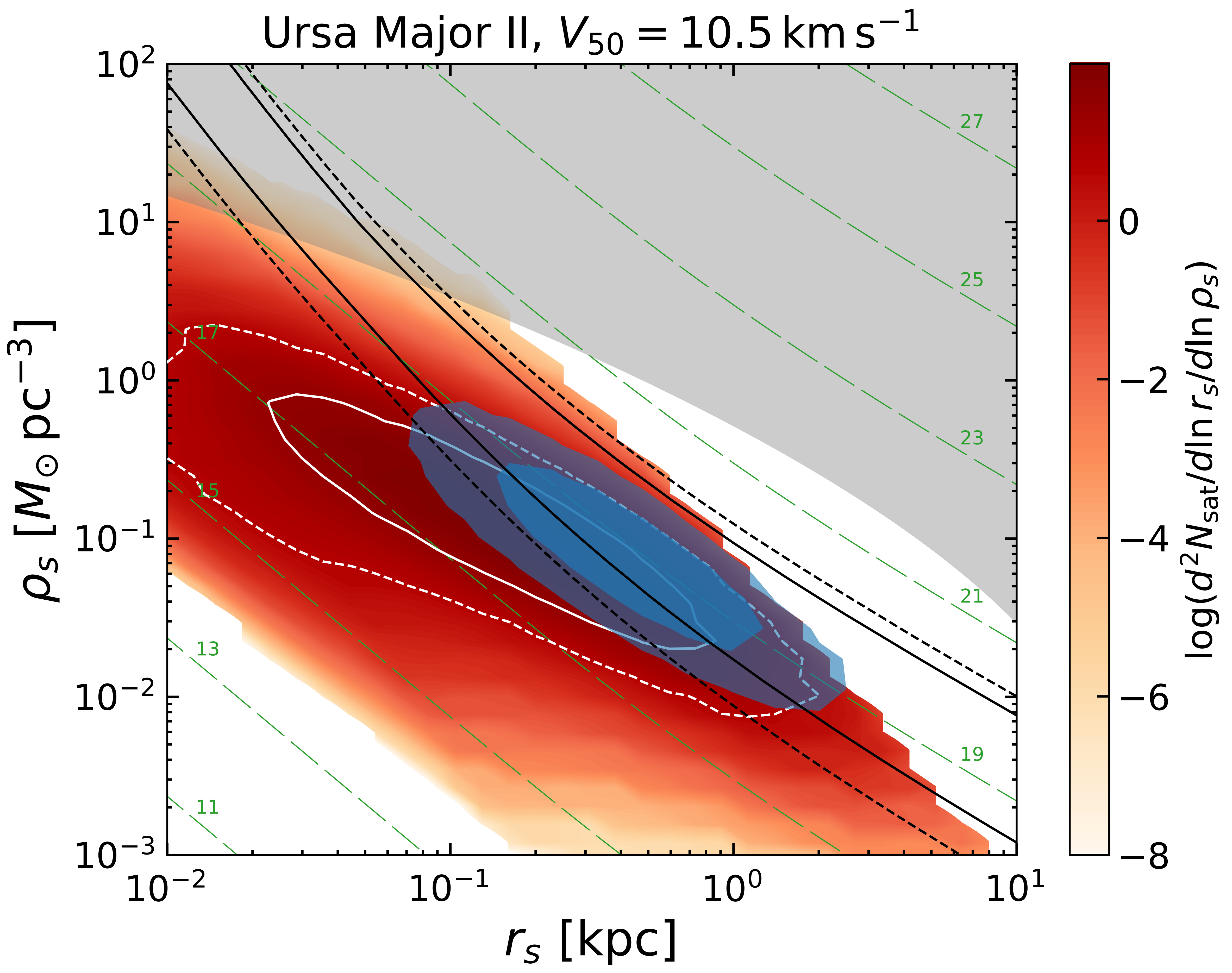
- Estimates of r_s and ρ_s usually rely on Bayesian statistics:

$$P(r_s, \rho_s \,|\, \mathbf{d}) \propto P(r_s, \rho_s)\mathscr{L}(\mathbf{d} \,|\, r_s, \rho_s)$$

- If data are not constraining, **the posterior depends on prior choices**
- Usually **log-uniform priors** are chosen for both r_s and ρ_s
- Doing frequentist way is very challenging, which is done only for *classical* dwarfs (Chiappo et al. 2016, 2018)

Application to ultrafaint dSphs

Ando et al., *Phys. Rev. D* **102**, 061302 (2020)



- **Black: Likelihood contours**
- **Green: log [J/(GeV²/cm⁵)]**
- **Red: Prior density**
- **Blue: Posterior density**

- Having small data only does not break the degeneracy between r_s and ρ_s
- Cosmological arguments have been adopted to chop off upper regions of the parameter space (e.g., Geringer-Sameth et al. 2015)
- Satellite prior does this job naturally as well as breaks the degeneracy
- This is hard to achieve with simulations as they are limited by statistics of finding dwarf candidates

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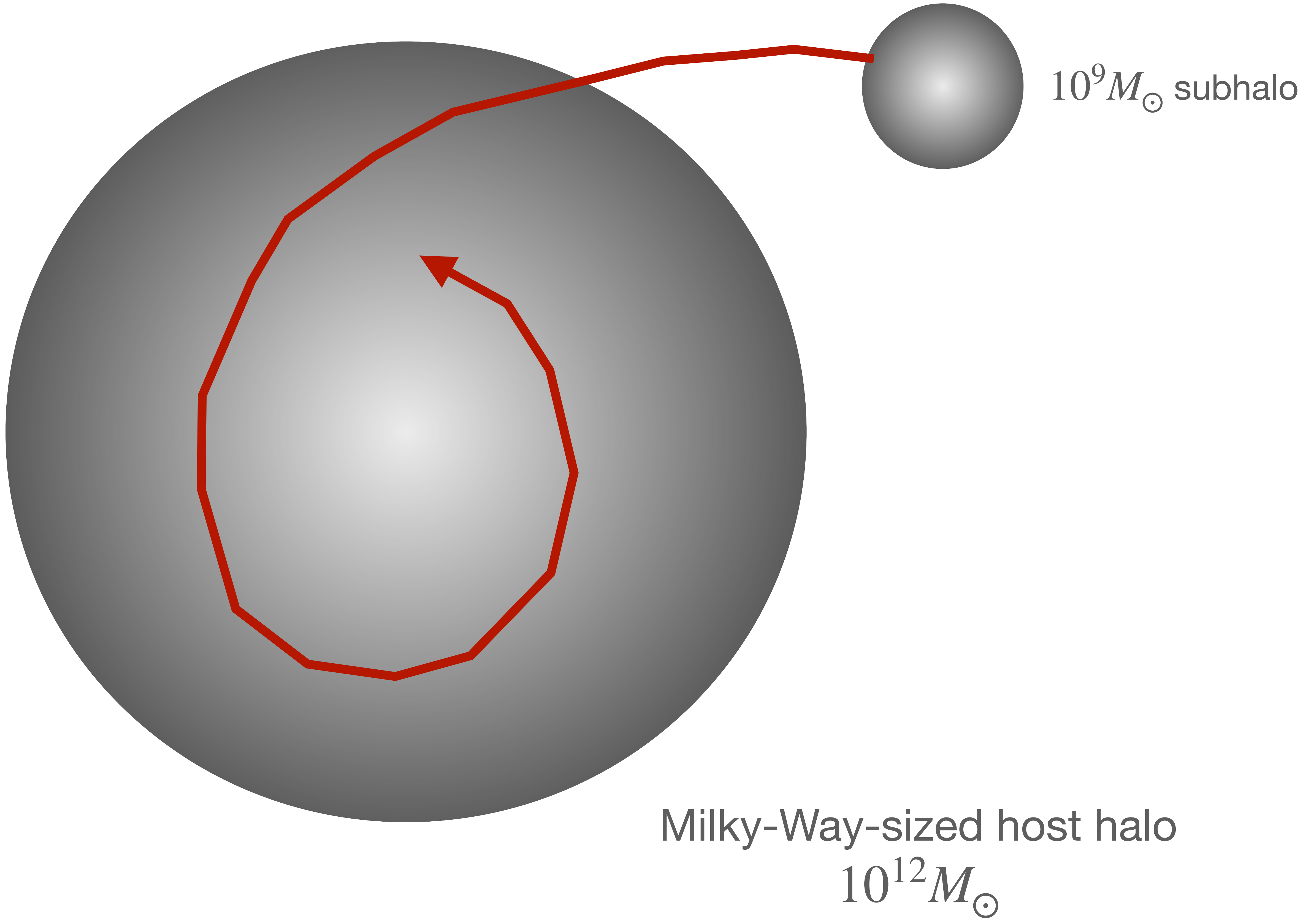
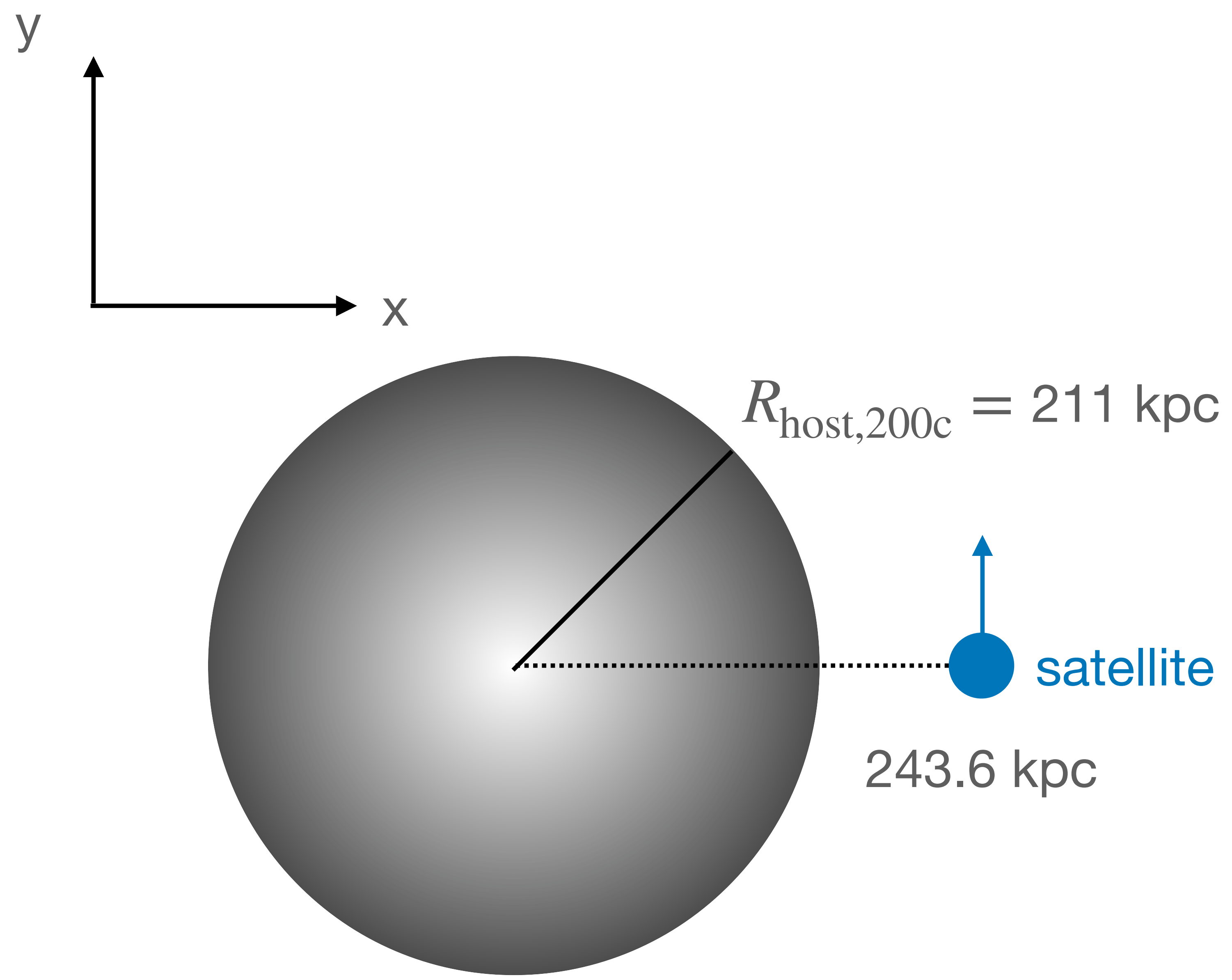
N-body simulations of isolated system

- Testing self-interactions of DM particles would require a precise modeling of
  - Thermalization of SIDM halo and subhalo
  - Tidal stripping / Ram pressure
- Develop a semi-analytic model of infalling subhalos to a MW-sized halo and calibrate it with (isolated) N-body sims

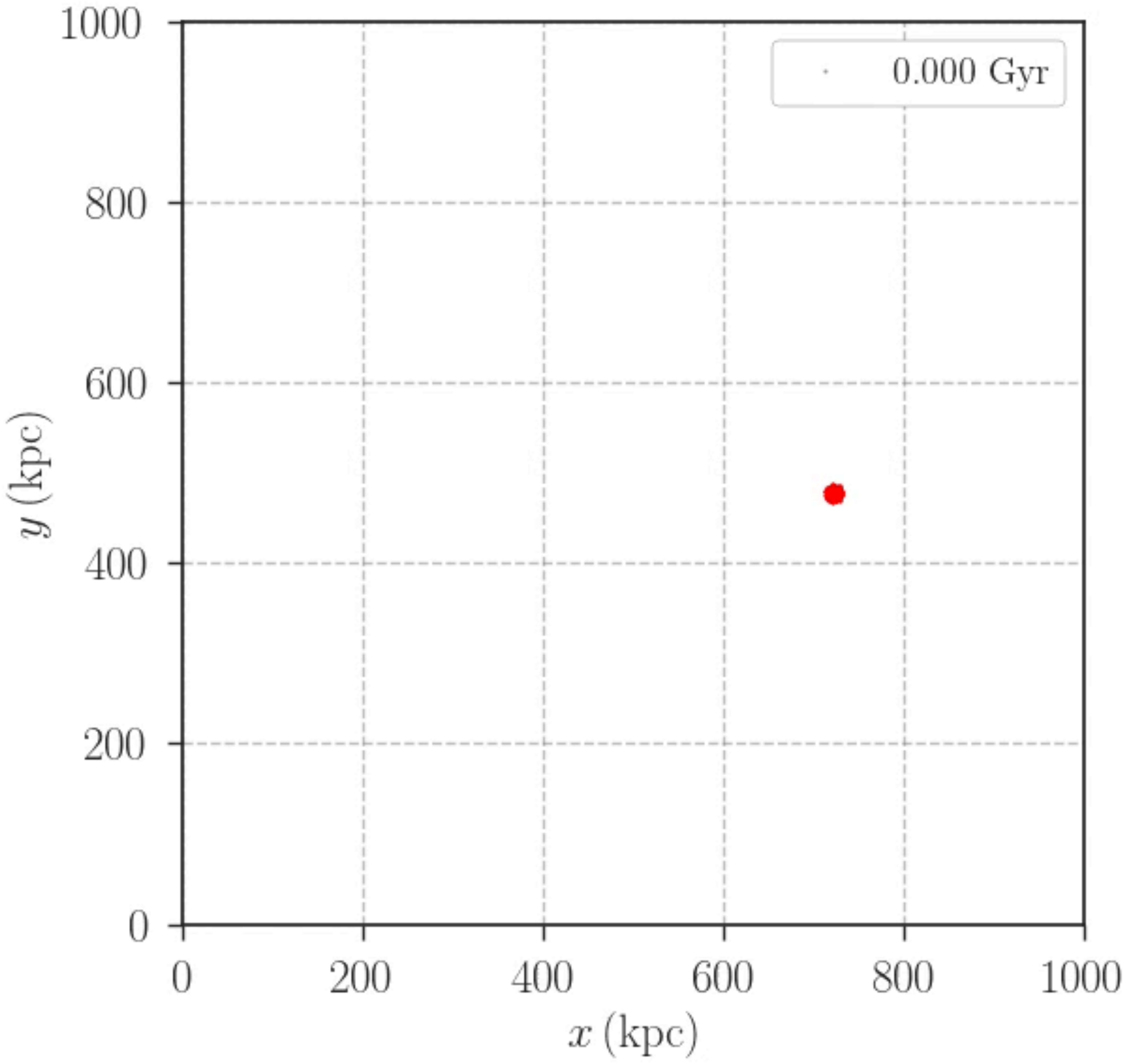


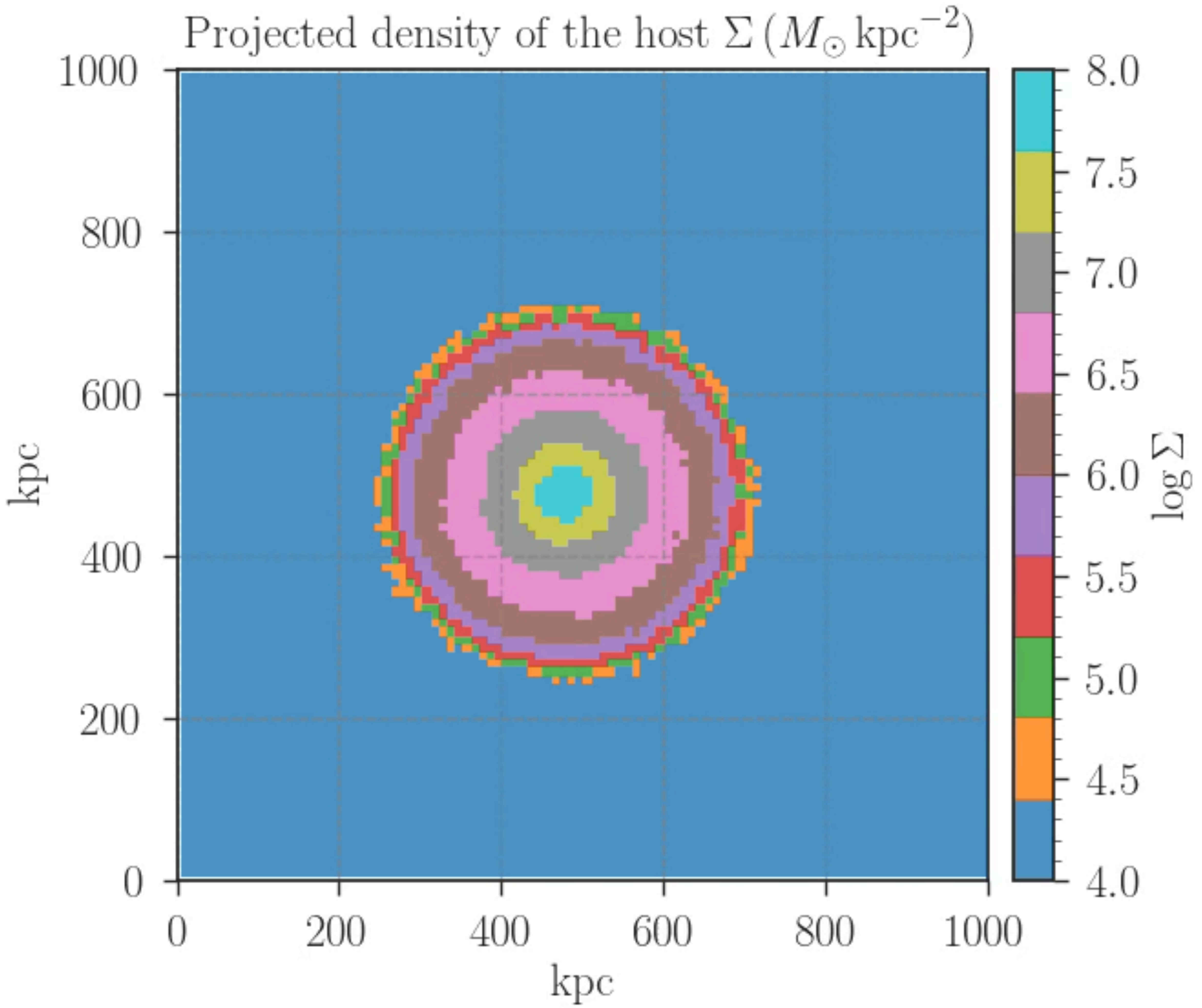
Isolated N-body halos: Initial conditions

- Host: NFW halo ($M_{200c} = 10^{12}\,M_{\odot}, c = 10, r_s = 21.1\,\mathrm{kpc}$)
- Satellite: NFW halo ($M_{200c} = 10^{9}\,M_{\odot}, c = 6, r_s = 1.68\,\mathrm{kpc}$)
- MAGI: Generator of spherical N-body halos in dynamical equilibrium
  - https://bitbucket.org/ymiki/magi/src/master/
  - No disc components for now
- Set the initial condition of the satellite with its energy E and angular momentum L
  - $E = \frac{1}{2}V_c^2 + \Phi_{\mathrm{NFW-host}}(R_c), L = \eta R_c V_c$ where $V_c = (GM_{\mathrm{host}}/R_c)^{1/2}$
  - $x_c = R_c/R_{\mathrm{host,200c}} = 0.5, \eta = 0.6$ → apocenter = 243.6 kpc



Isolated N-body halos ($N_{sat} = 1000$)





Gravothermal fluid model

E.g., Balberg et al. 2002

Mass conservation
$$\mathrm{d}M/\mathrm{d}r = 4\pi r^2 \rho_\mathrm{h}(r)$$

Hydrostatic equilibrium
$$\mathrm{d}(\rho\sigma_v^2)/\mathrm{d}r = -GM\rho/r^2$$

$\text{Heat flux} = -\kappa\,(m/k_B)\,\partial\sigma_v^2/\partial r$ re-arranges ρ_h and σ_v

Host halo density
$\rho_\mathrm{h}(r,t)$ → $\rho_\mathrm{h}(r, t+\Delta t)$

Subhalo

Density $\rho_\mathrm{sub}(r,t)$ → **Density** $\rho_\mathrm{sub}(r, t+\Delta t)$

Gravothermal fluid model (e.g. Balberg+2002)

CDM-like tidal evolution proposed in Green & van den Bosch (2019)

Bound Mass $M_\mathrm{sub}(t)$ → **Bound Mass** $M_\mathrm{sub}(t+\Delta t)$

$\dot{M}_\mathrm{sub} = (\text{Tidal stripping}) + (\text{Ram pressure evaporation})$

Analytic model in Chandrasekhar 1943

Analytic model in Kummer+2018

Position & Velocity $\mathbf{x}_\mathrm{sub}(t), \mathbf{v}_\mathrm{sub}(t)$ → **Position & Velocity** $\mathbf{x}_\mathrm{sub}(t+\Delta t), \mathbf{v}_\mathrm{sub}(t+\Delta t)$

$\ddot{\mathbf{x}}_\mathrm{sub} = -\nabla\Phi_\mathrm{h} + (\text{Dynamical Friction}) + (\text{Ram Pressure Deceleration})$

Test 1: CDM-like tidal stripping model can work or not

- Green and van den Bosch (2019) have found that the tidal stripping effect in CDM subhaloes can be expressed as

$$\rho_{\rm sat}(r,t) = H_{\rm GB19}(r, f_b(t))\,\rho_{\rm NFW}(r)$$

Mass fraction of subhalos at t

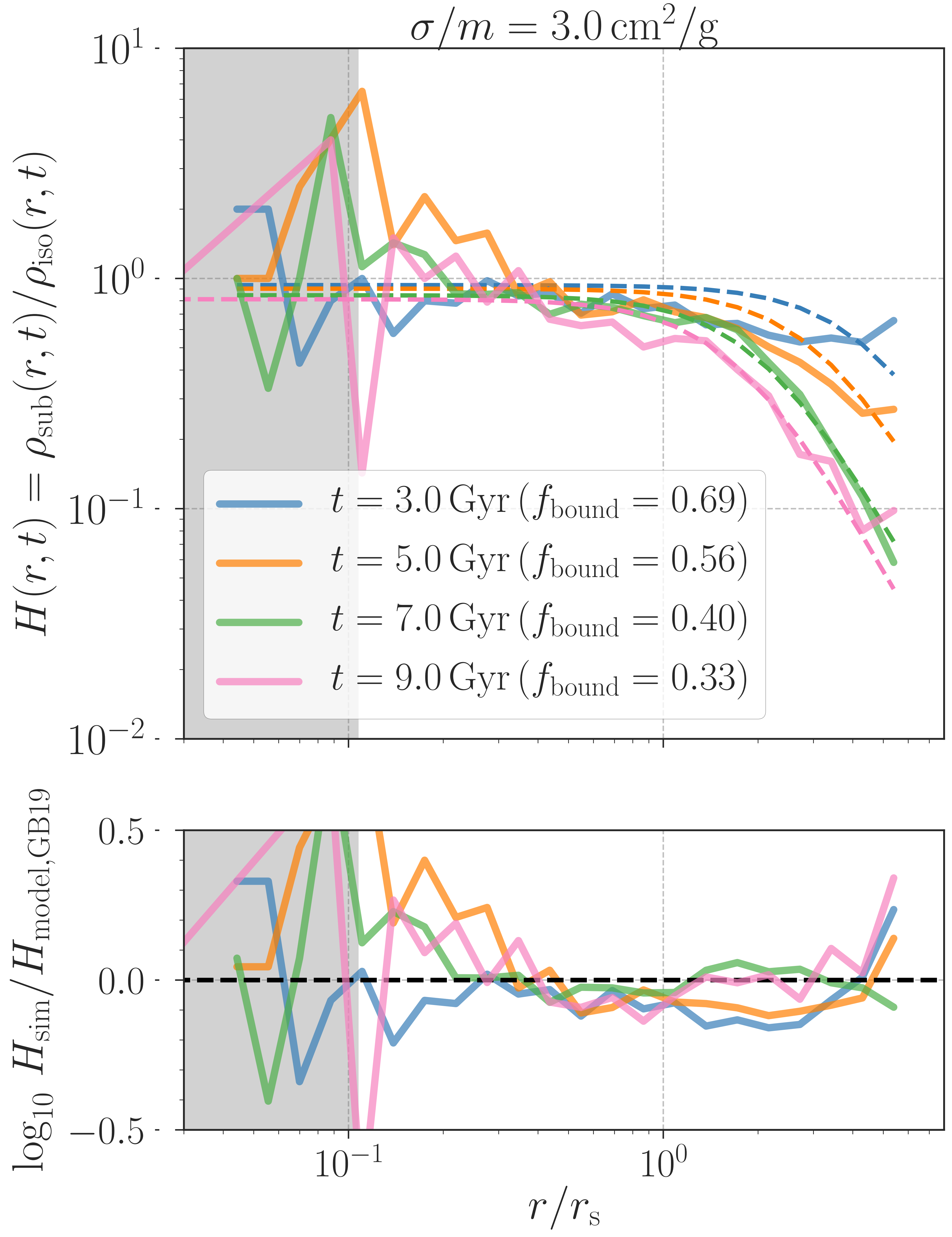
- In the SIDM case, we naively expect that

$$\rho_{\rm SIDM,sat}(r,t) = H_{\rm GB19}(r, f_b(t))\,\rho_{\rm SIDM,iso}(r,t)$$

- We evolved isolated haloes with the same mass as the subhalo at initial states and then compute

$$H_{\rm sim}(r,t) = \frac{\rho_{\rm sat}(r,t)}{\rho_{\rm iso}(r,t)}$$

- Confirmed $H_{\rm sim} \simeq H_{\rm GB19}$ in our simulations



Shirasaki, Okamoto, Ando, *Mon. Not. R. Astron. Soc.* **516**, 4594 (2022)

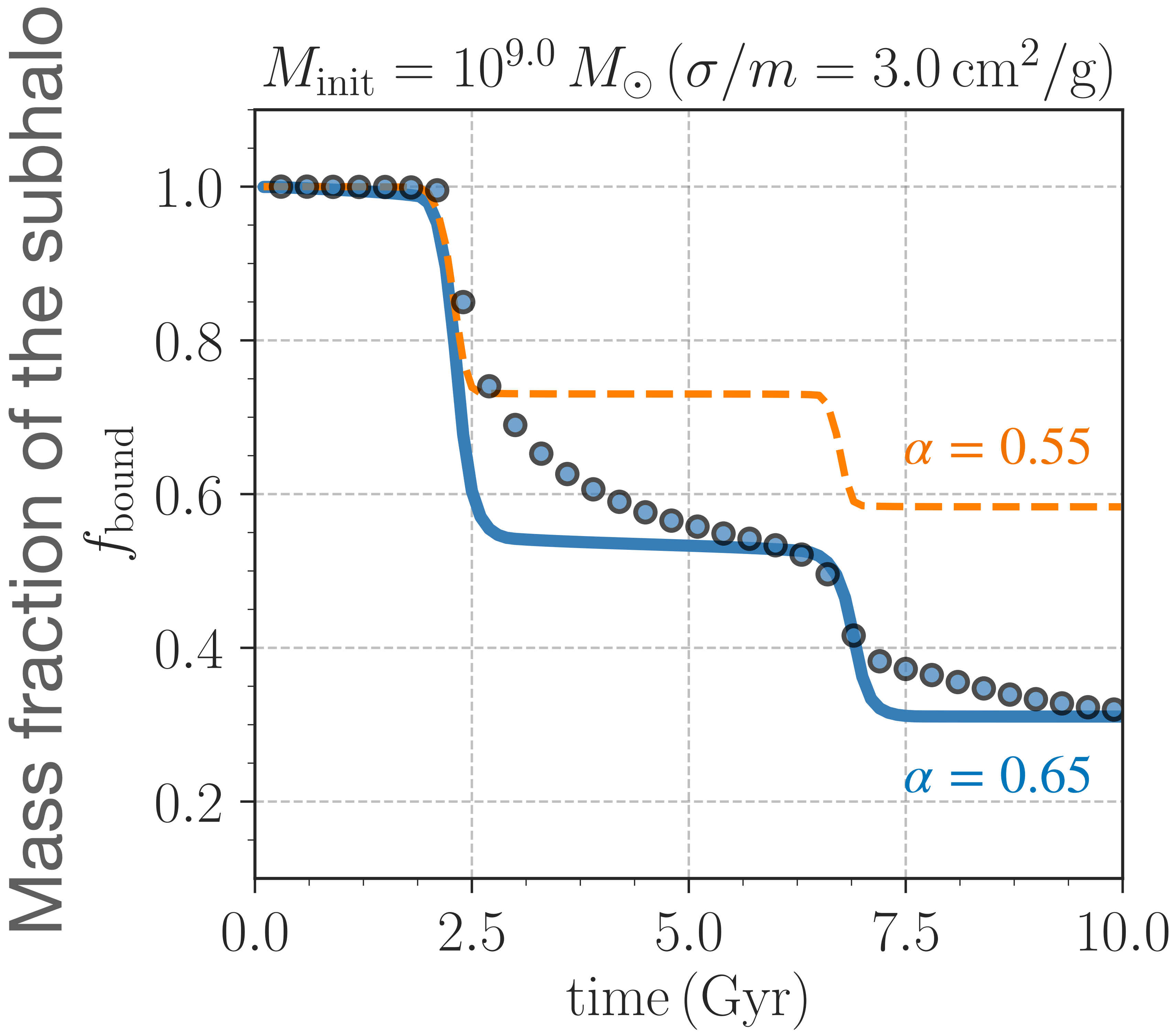
Test 2: Subhalo mass loss rate in SIDM

Shirasaki, Okamoto, Ando, *Mon. Not. R. Astron. Soc.* **516**, 4594 (2022)

- The mass loss rate by tidal stripping effects is commonly modeled as

$$\frac{\mathrm{d}M_{\mathrm{sub}}}{\mathrm{d}t} = -A\frac{M_{\mathrm{sub}}(r > r_t, t)}{\tau_{\mathrm{dyn}}}$$

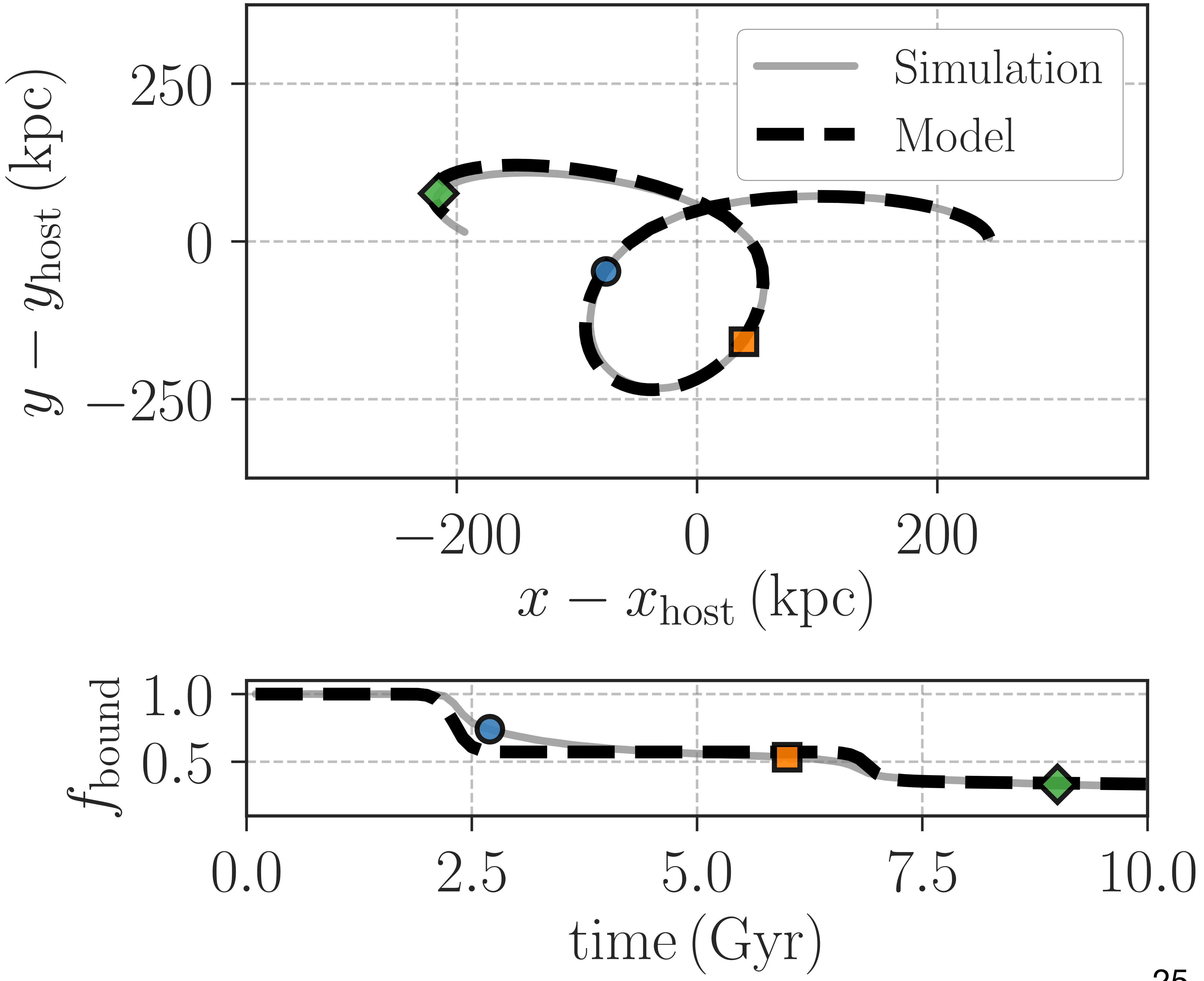
- $A = 0.55$ can explain the CDM simulation results
- Our simulations indicate that A depends on σ/m
- We find $A \simeq 0.65$ provides a better fit to the simulation results with $\sigma/m = 3\,\mathrm{cm^2/g}$

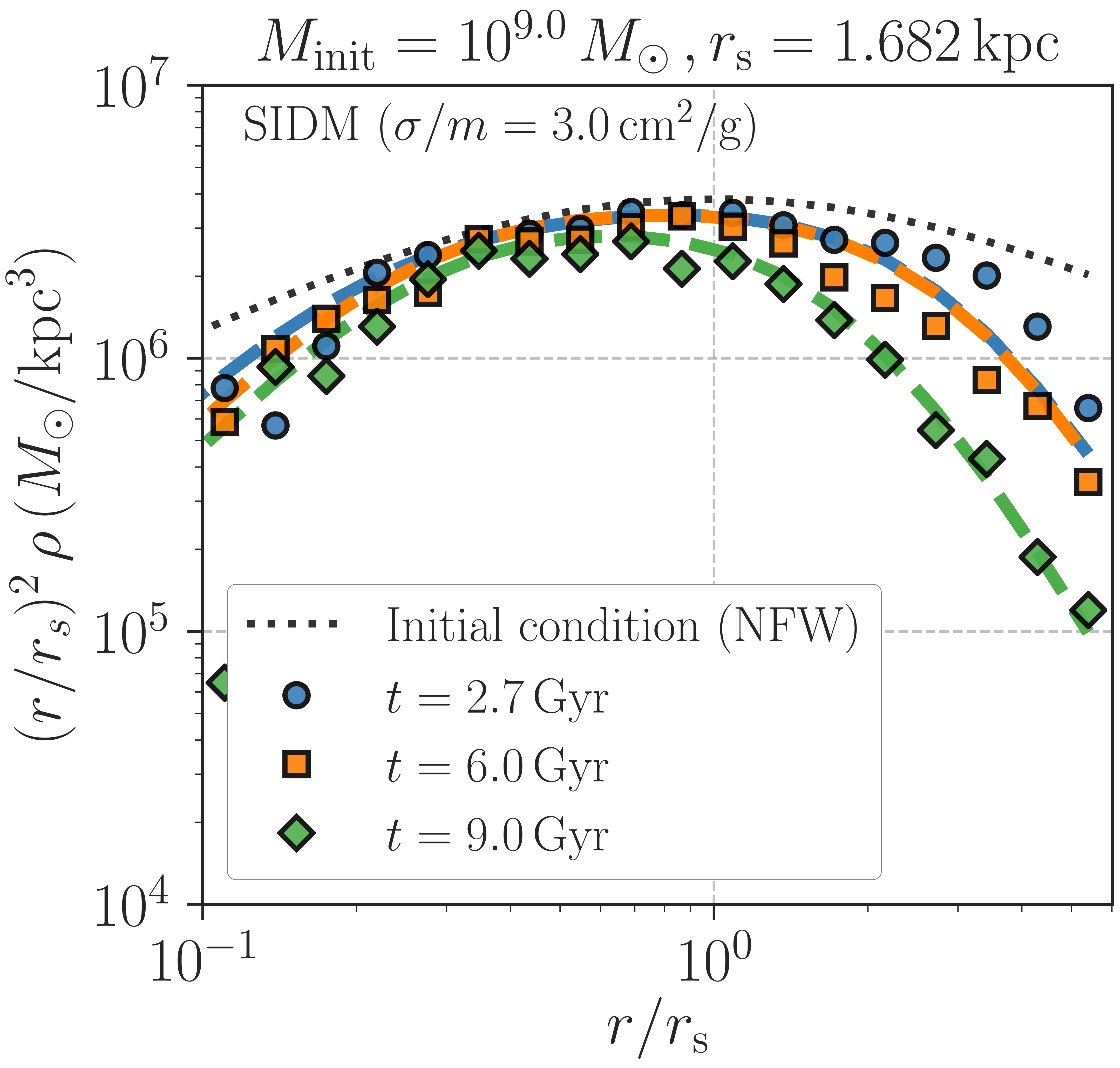


Comparison with our model and simulations

Shirasaki, Okamoto, Ando, *Mon. Not. R. Astron. Soc.* **516**, 4594 (2022)

$\sigma/m = 3\,\mathrm{cm}^2/\mathrm{g}$

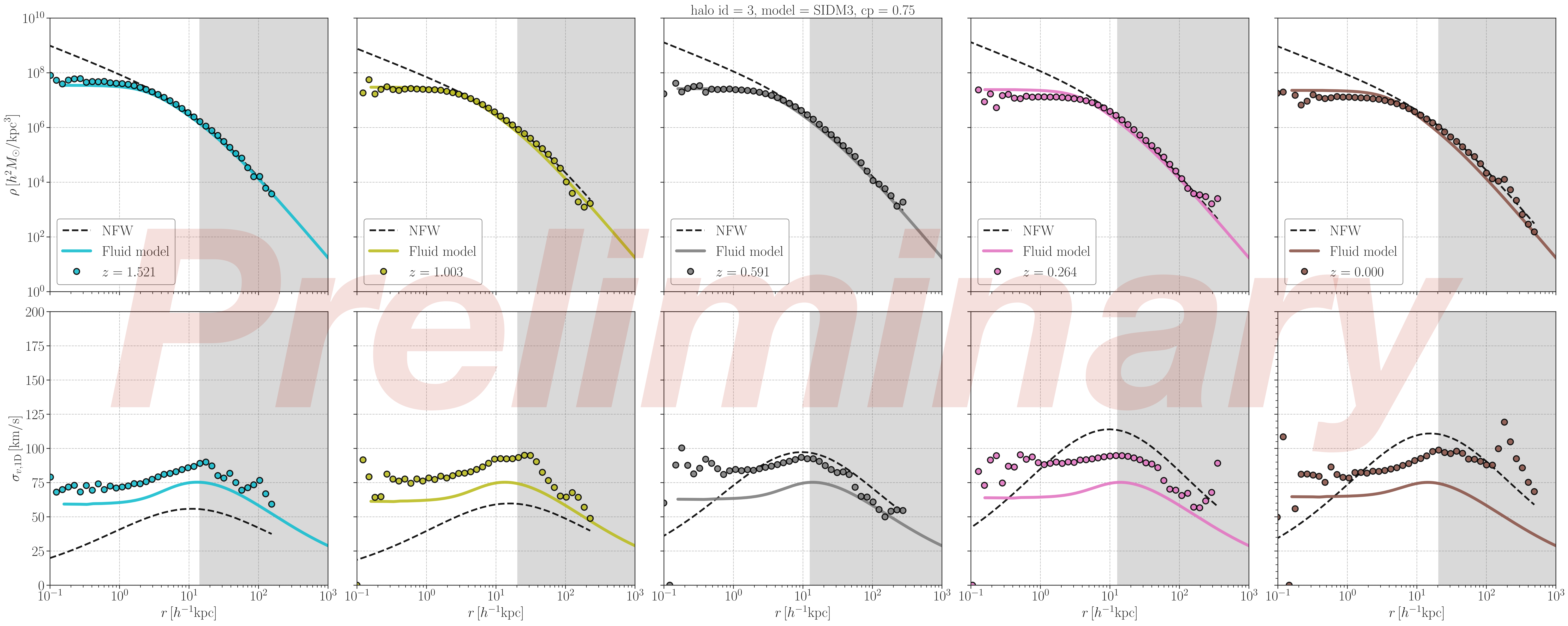




Caveats and outlook

- We developed a semi-analytic model of SIDM subhaloes in a MW-sized host with ideal N-body sims
    - We found a non-trivial effect in the subhalo mass loss rate for the SIDM scenario
    - We tested our models with simulations by varying subhalo orbits, SIDM cross sections, initial subhalo profiles
- To do:
    - (1) Comparisons of our model with **cosmological simulations**; (2) Include the **baryonic disc** in a host halo; (3) **Velocity dependence** of cross section; (4) **Gravothermal collapse**

Comparison with cosmological simulations



Ongoing work with Masato Shirasaki, Shunichi Horigome et al., with simulation data from Ebisu, Ishiyama, Hayashi, *Phys. Rev. D* **105**, 023016 (2022)

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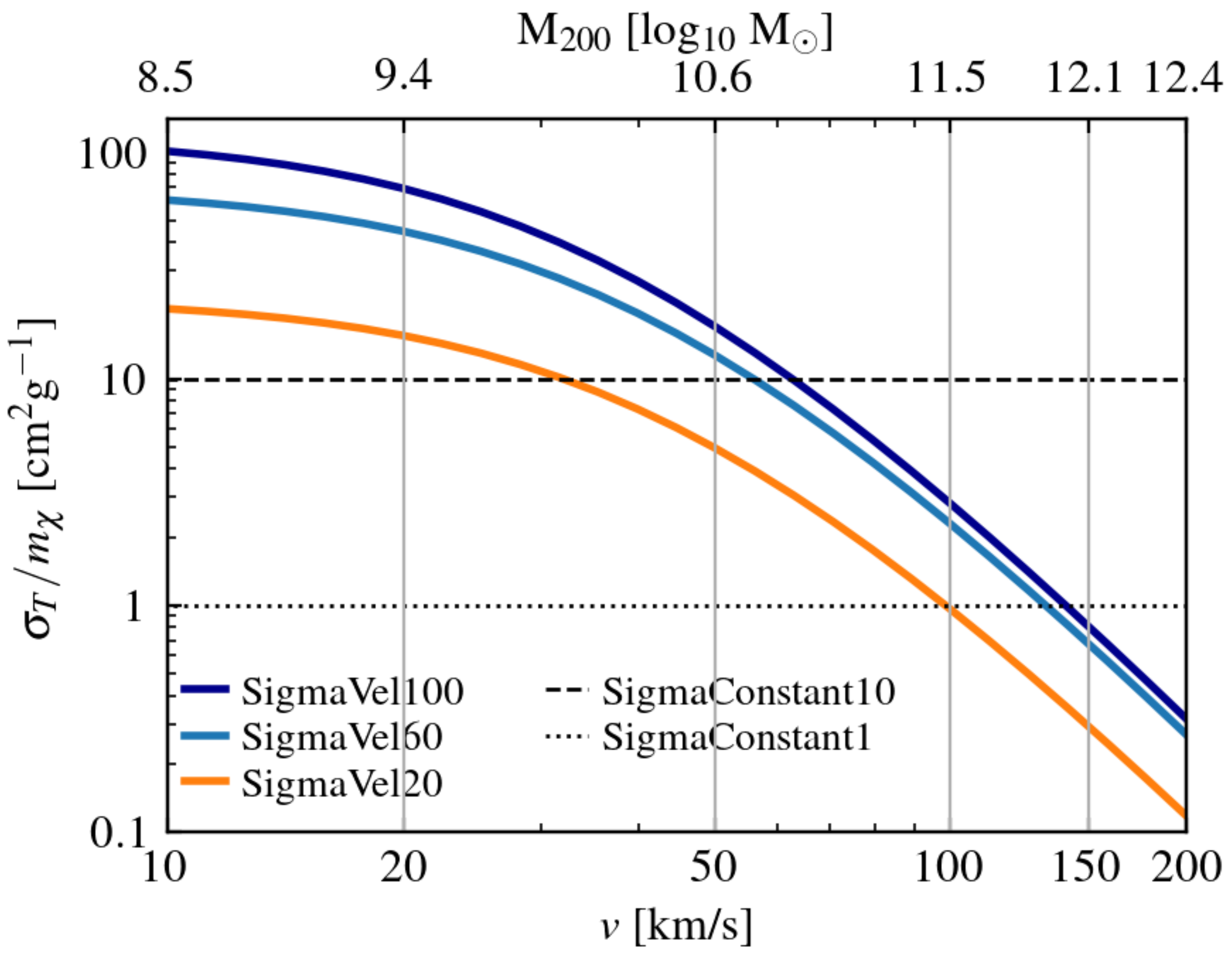
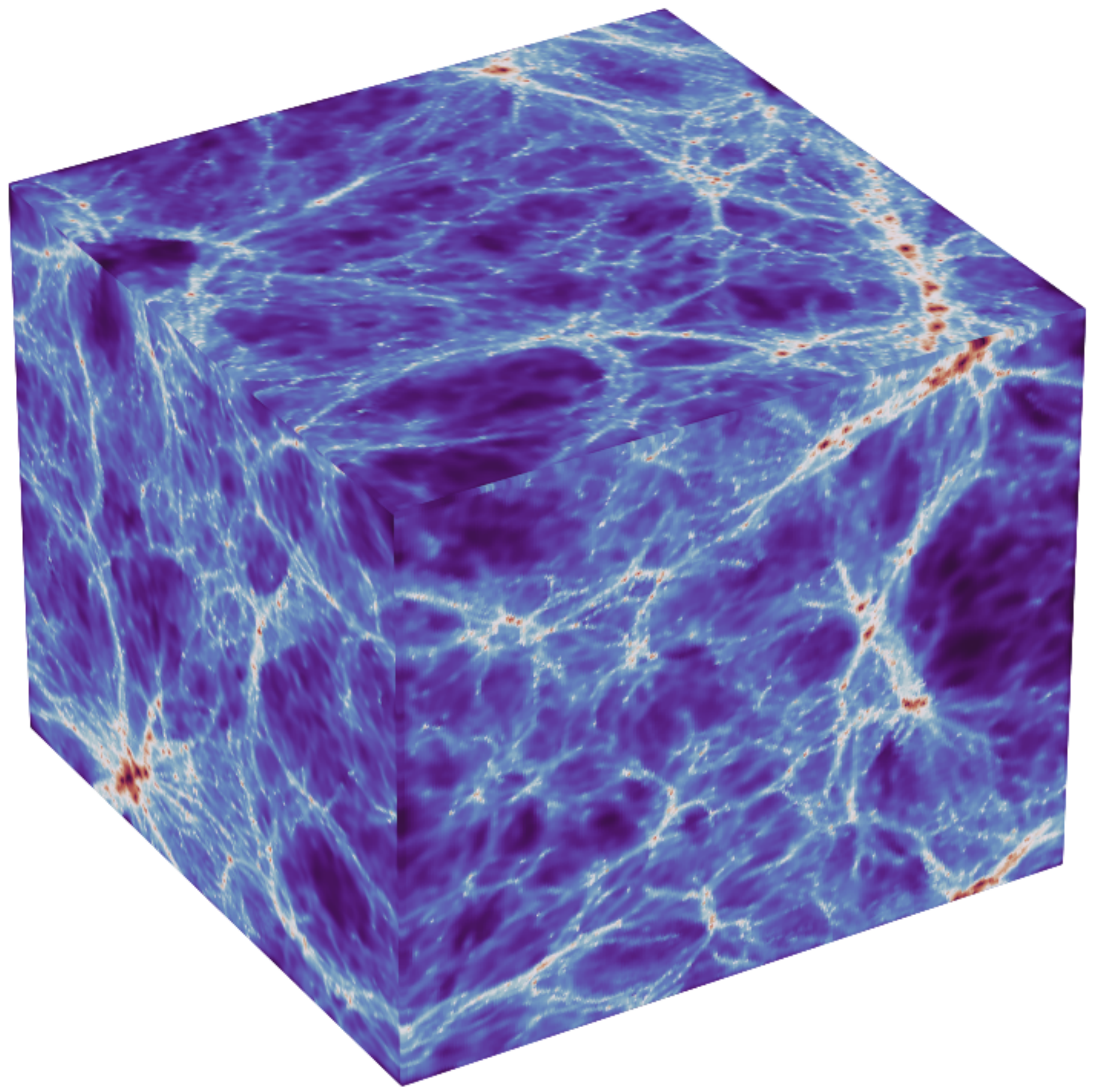
TangoSIDM: Tantalising models of SIDM



We implemented SIDM on the gravity and hydrodynamics solver code: **SWIFT**

TangoSIDM consists on a set of DM-only and hydrodynamical cosmological simulations of (25 Mpc)^3

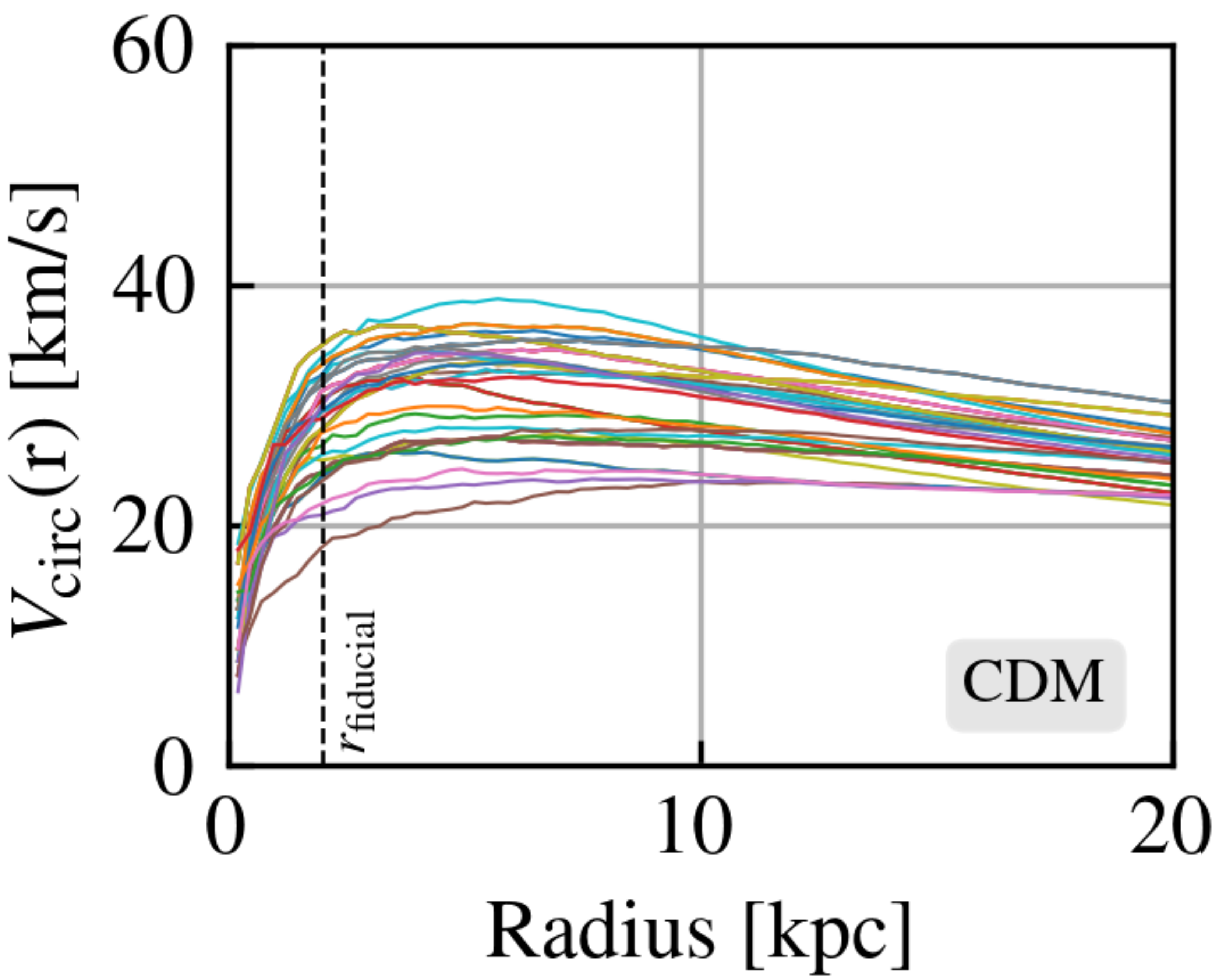


Correa, Schaller, Ploeckinger, Anau Montel, Weniger, Ando, *Mon. Not. R. Astron. Soc.* **517**, 3045 (2022)

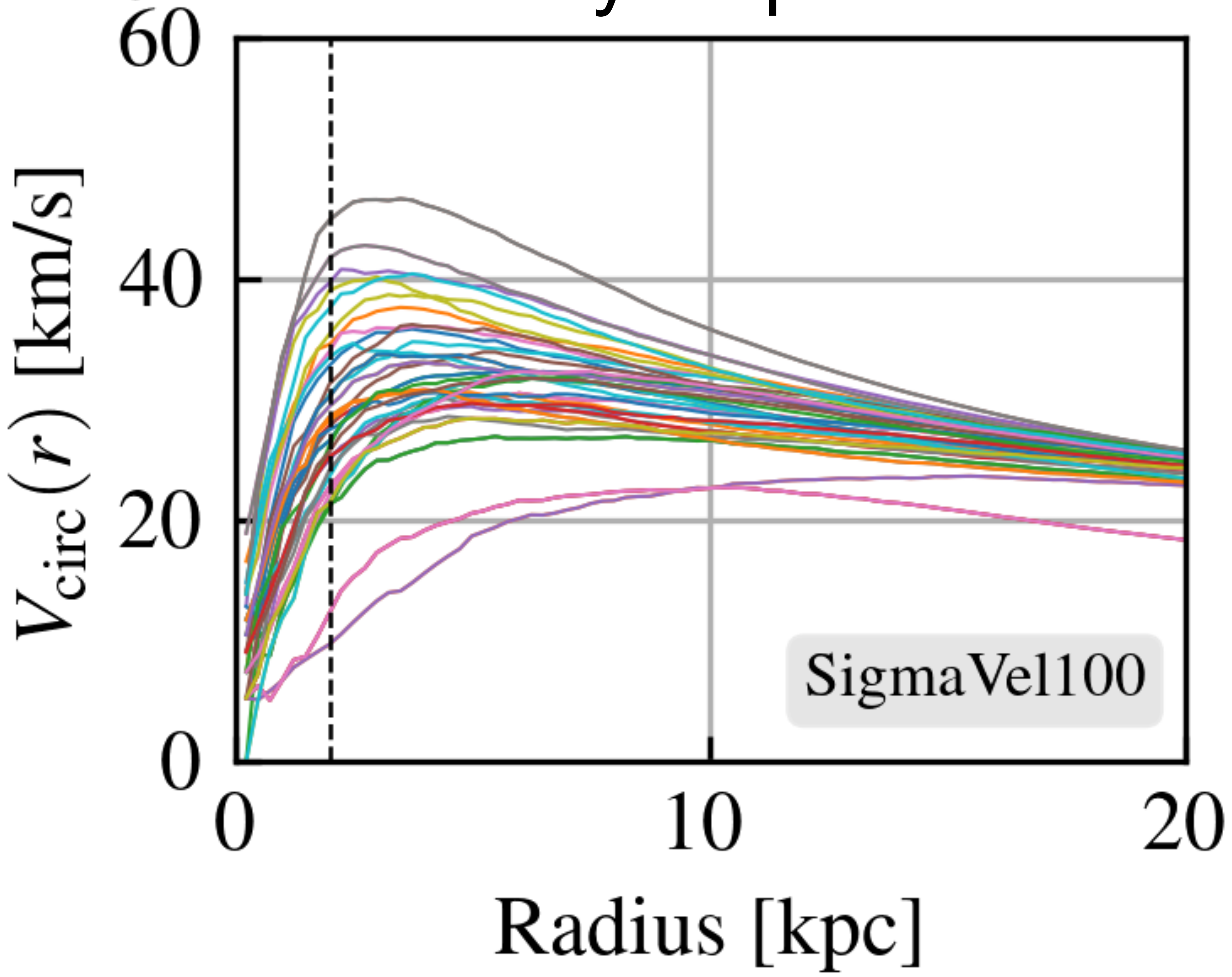
TangoSIDM: Rotation curves

Example of circular velocities from $10^{9.5} M_{\odot}$ satellite haloes

CDM model



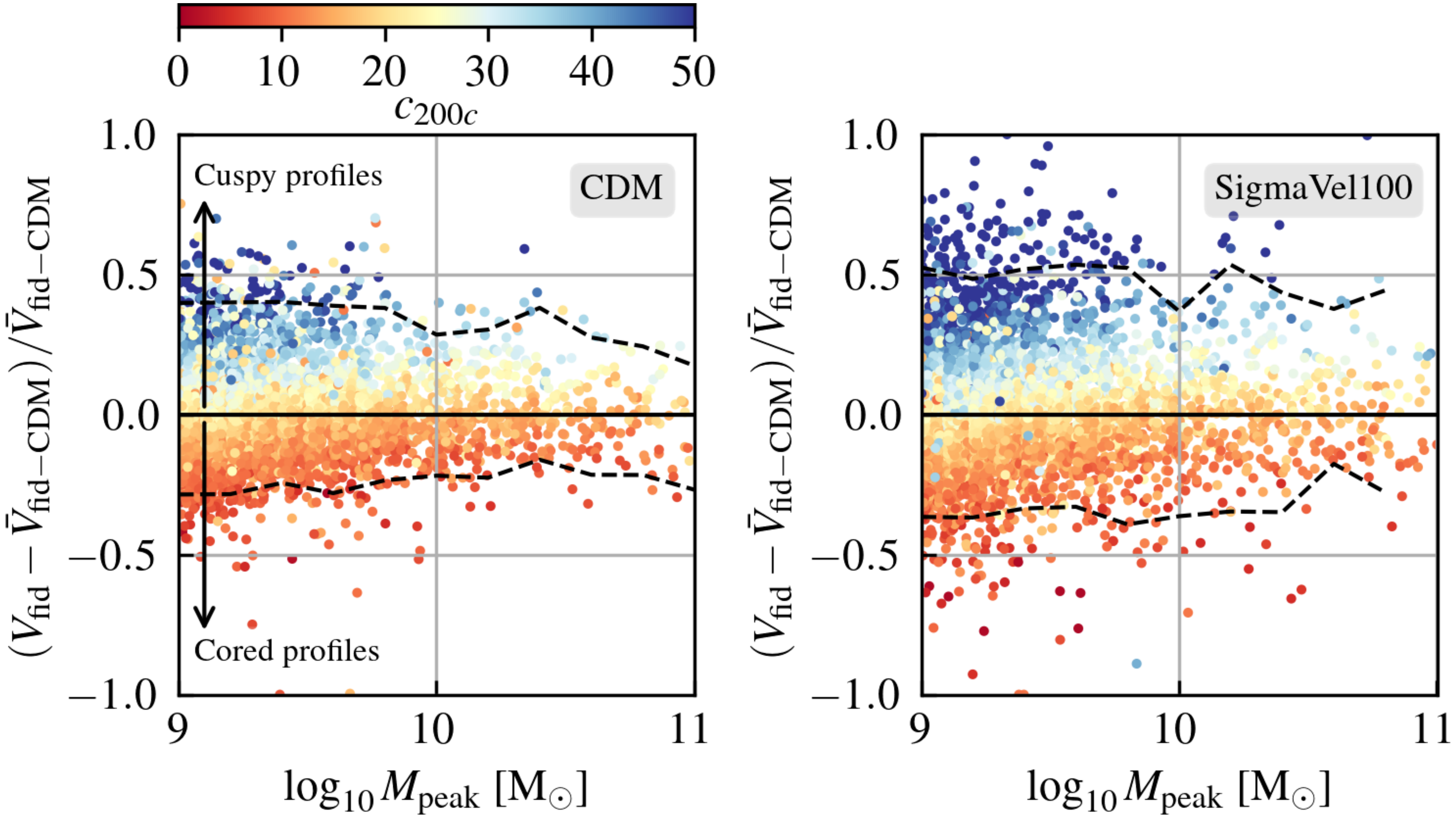
SIDM velocity-dependent model



Velocity-dependent SIDM models in cosmological DM-only simulations are able to produce a “diversity” in the rotation curves of low-mass halos

Correa et al., *Mon. Not. R. Astron. Soc.* **517**, 3045 (2022)

TangoSIDM: Rotation curves



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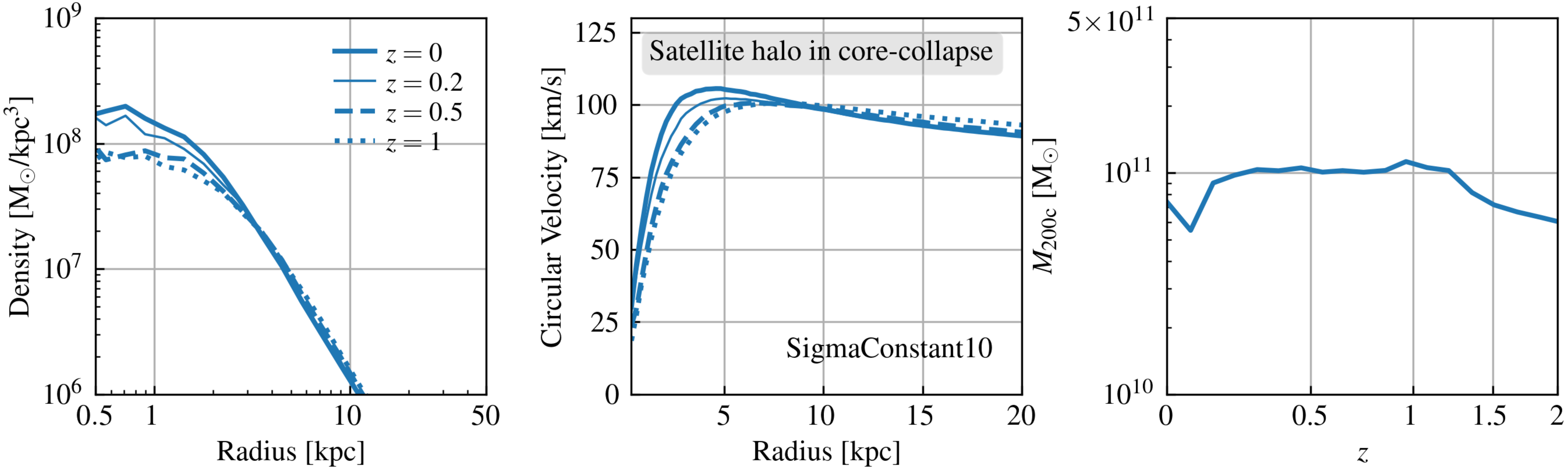
Correa et al., *Mon. Not. R. Astron. Soc.* **517**, 3045 (2022)

TangoSIDM: Gravothermal collapse



Collapse time-scale: $t_c \propto (\sigma/m_\chi)^{-1} M_{200}^{-1/3} c_{200}^{-7/2}$

Correa et al., *Mon. Not. R. Astron. Soc.* **517**, 3045 (2022)

Conclusions and prospects

- **Small-scale distribution** of dark matter is essential in discriminating different particle dark matter candidates
- We base our theoretical studies on **benchmark subhalo models for CDM/WIMP**
- We theoretically model the evolution of **SIDM subhalos** using **gravothermal fluid** model and calibrate the model parameters against idealized N-body simulations of minor merger
- Goal: refine this calibration procedure, incorporate these SIDM mass-loss models with EPS theory, and make SASHIMI-I