Systematizing the Effective Theory of SIDM

Aditya Parikh
YITP, Stony Brook University

In collaboration with P. Agrawal and M. Reece
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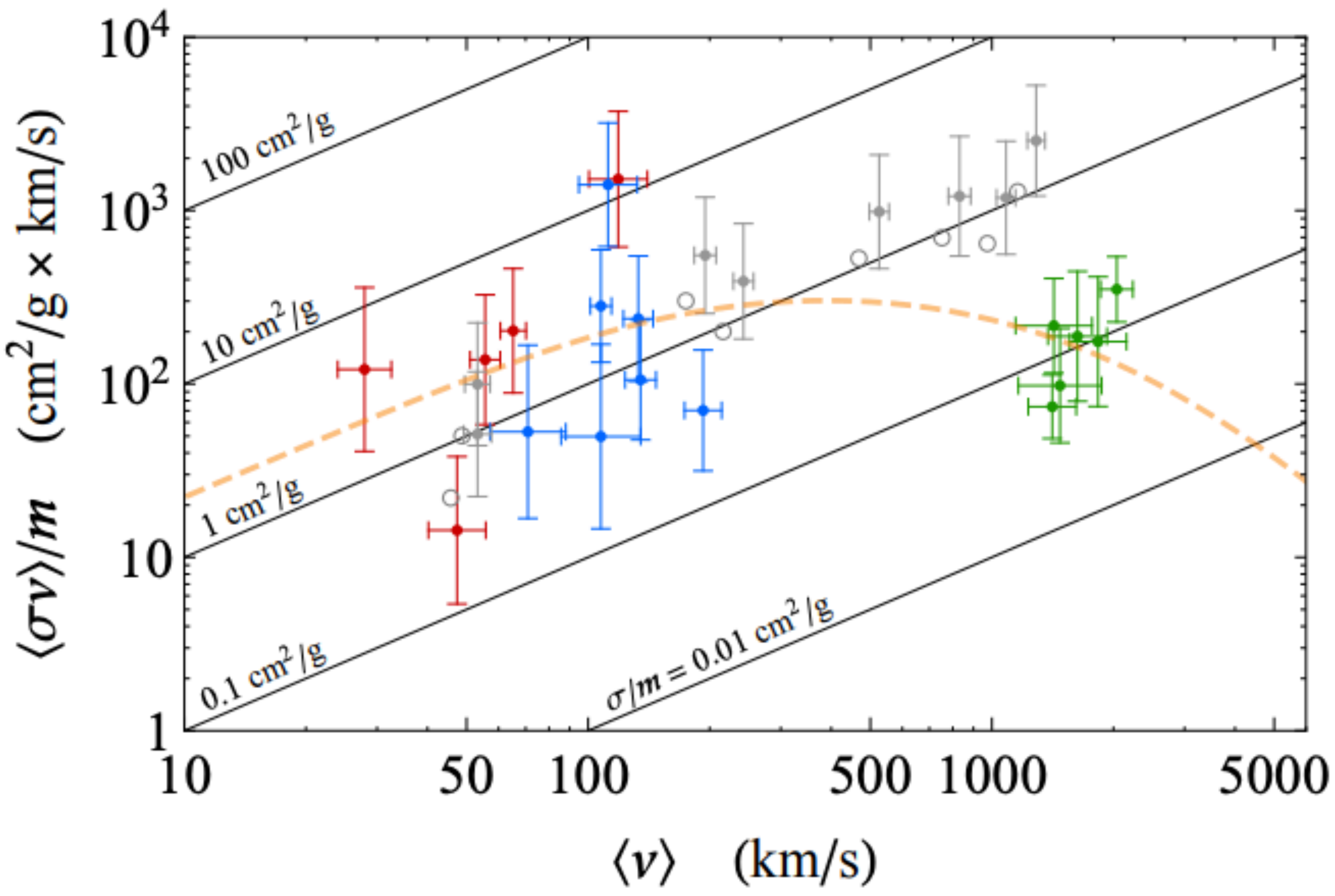
Stony Brook University

(Not So) Boring Dark Sectors

- Different systems map out the velocity dependence of the DM self interaction rate

- What underlying DM microphysics leads to this?

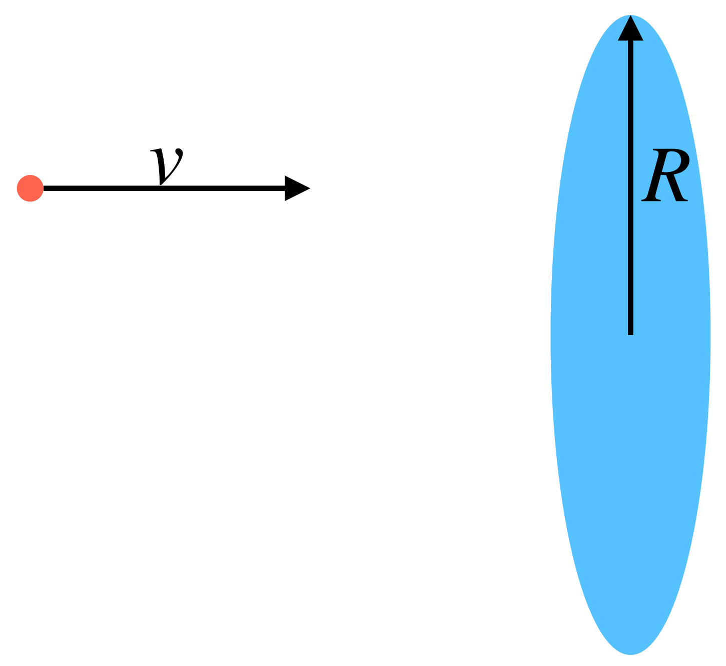
- Possible Sommerfeld enhancement since the DM is non-relativistic



Dwarf, LSB, and Cluster data fit to a Yukawa potential with a light mediator [1508.03339].

Sommerfeld Enhancement

- A Classical Analogy
  - w/o gravity $\sigma_0 = \pi R^2$
  - w/ gravity $\sigma = \pi b_{max}^2 = \sigma_0 \left(1 + \frac{v_{esc}^2}{v^2}\right)$



- Non-perturbative effect that can be treated quantum mechanically
  - Match a field theory calculation onto a quantum mechanical potential and solve the corresponding Schrödinger equation

Case Studies: Scalar & Pseudoscalar Exchange

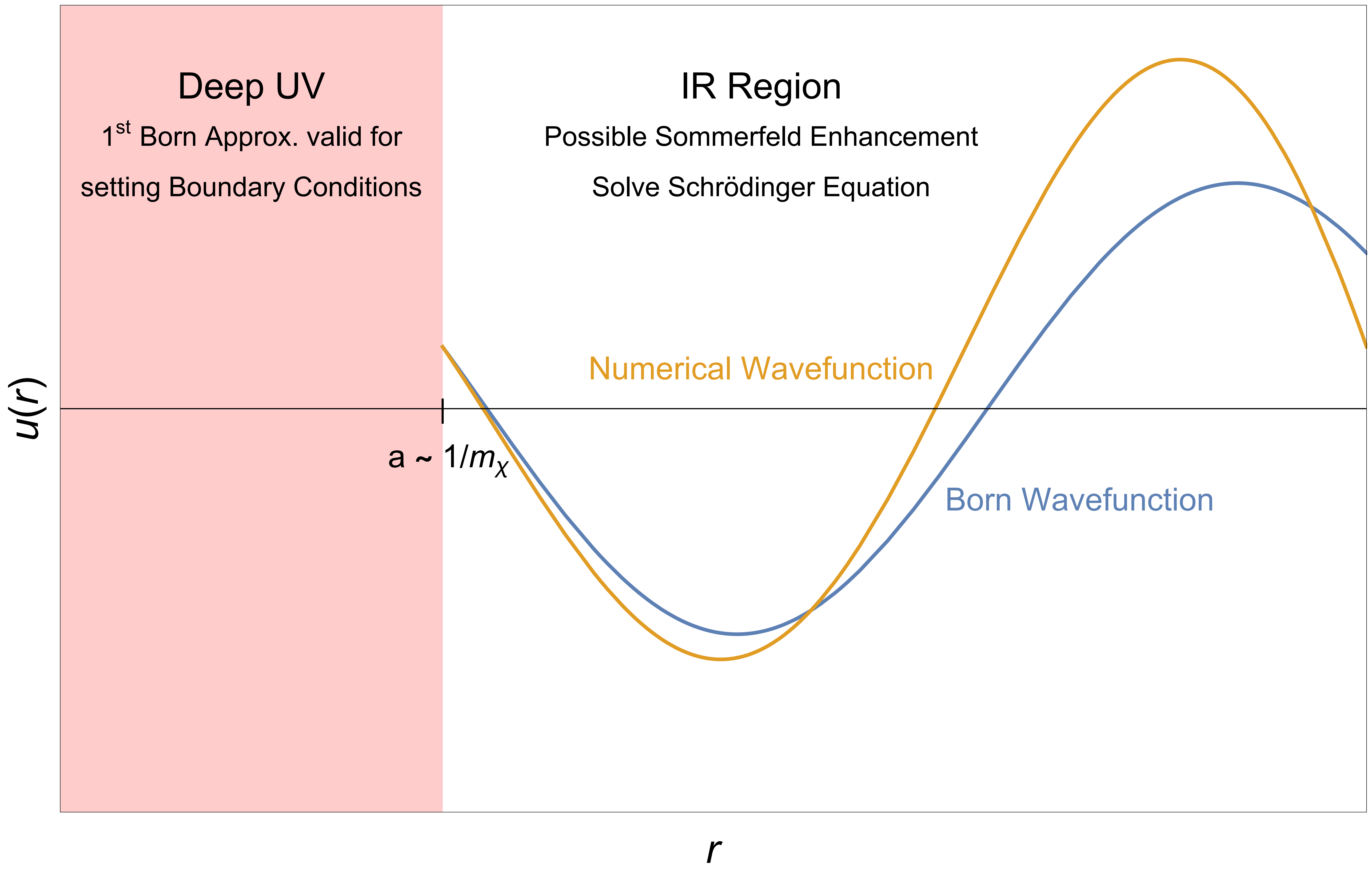
$$V_{\text{scalar}}(r) = -\frac{\lambda^2}{4\pi r}\mathrm{e}^{-m_\phi r}$$

$$V_{\text{pseudoscalar}}(r) = \frac{\lambda^2}{4\pi}\left(\frac{4\pi\delta^3(\vec{r})}{4m_\chi^2 - m_\phi^2}\left(\frac{1}{2} - 2S_1\cdot S_2\right) - \frac{4\pi\delta^3(\vec{r})}{3m_\chi^2}\mathrm{e}^{-m_\phi r}S_1\cdot S_2\right.$$
$$\left. + \frac{\mathrm{e}^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r}S_1\cdot S_2 + \frac{3(S_1\cdot\hat{r})(S_2\cdot\hat{r}) - S_1\cdot S_2}{r^3}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\right]\right)$$

How do we set well-defined boundary conditions for r^{-3} potentials?

Matching Prescription

- Short distances correspond to semi-relativistic momenta
- QFT is a better description than the effective QM potential
- Sommerfeld enhancement is important at large distances



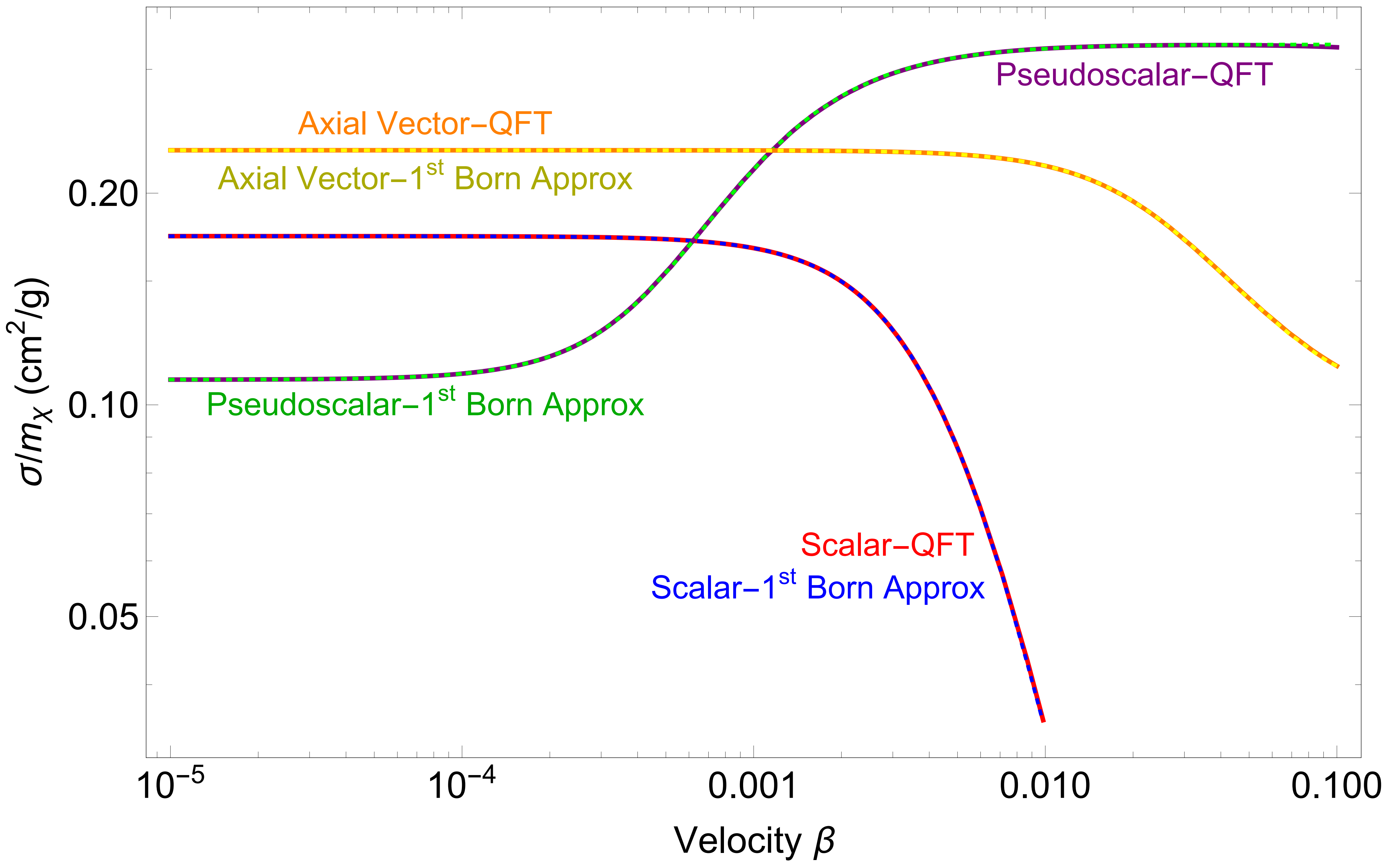
Setting Boundary Conditions

- How do we access the QFT information?
  - The first Born approximation in quantum mechanics faithfully reproduces tree-level QFT

$$K^a_{\ell s,\ell' s'} = -\frac{2\mu}{k}\int_0^a dr s_{\ell'}(kr)V_{\ell s,\ell' s'}(r)s_\ell(kr)$$

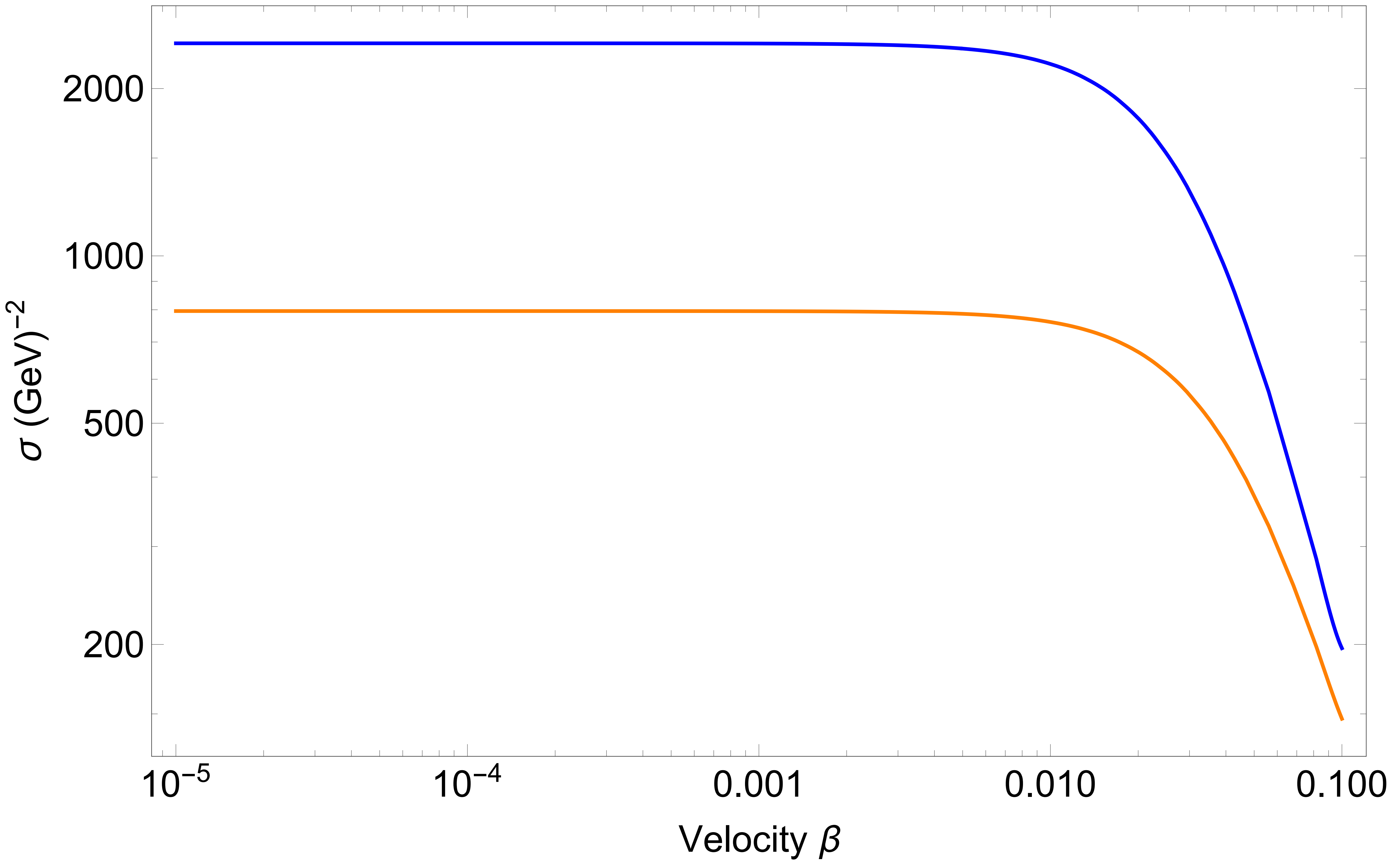
  - This alters the wavefunction and its derivative

$$u_{\ell s,\ell' s'}(a) \sim \delta_{\ell s,\ell' s'}s_\ell(ka) + K^a_{\ell s,\ell' s'}c_\ell(ka)$$



Numerical Results: Scalar Exchange

- Numerical cross section vs. QFT tree-level cross section
- Sommerfeld enhancement at low velocities
- Numerical results agree with and without our matching procedure!



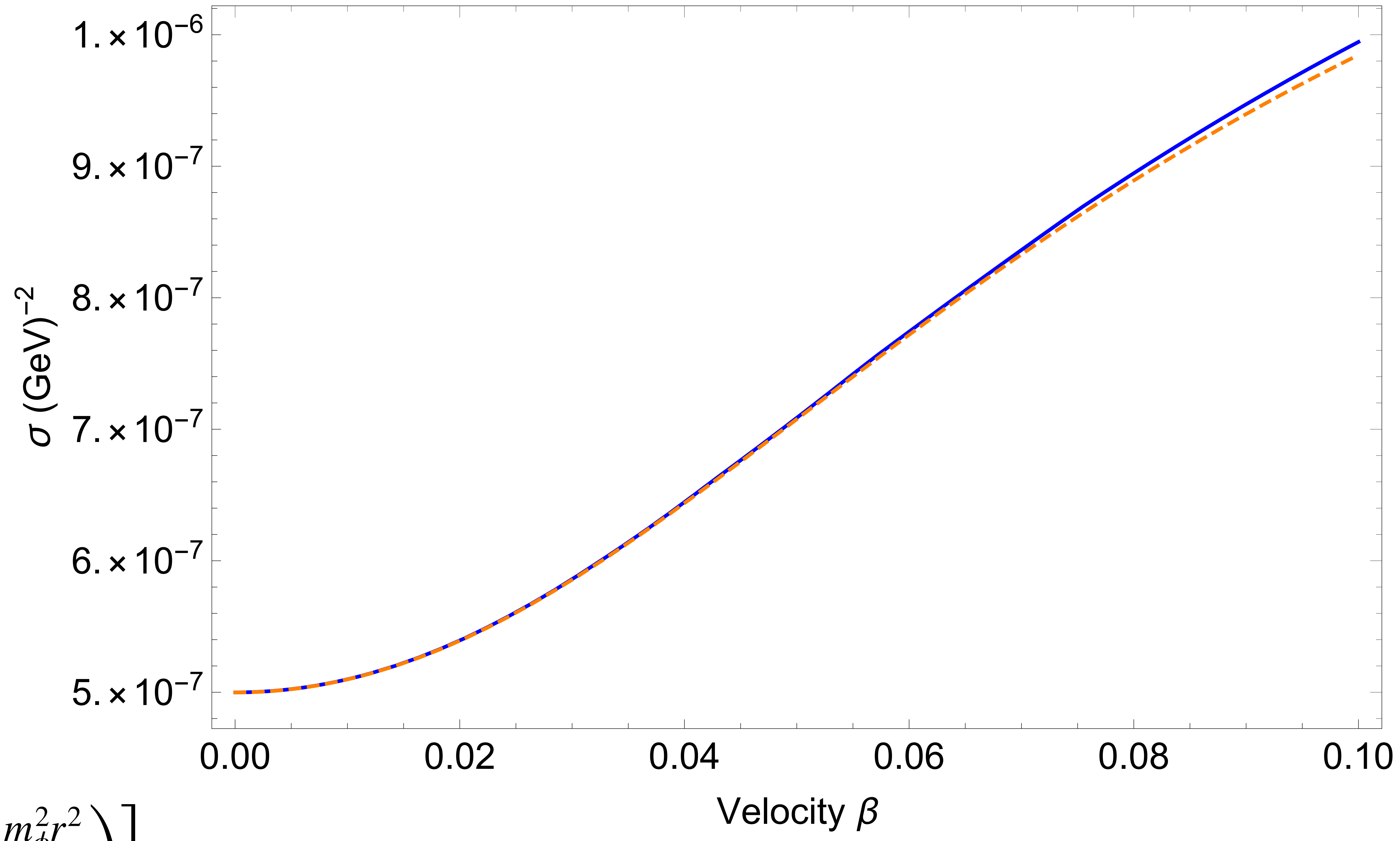
Mediator Mass = 0.1 GeV; Dark Matter Mass = 1 GeV; Coupling = 1

Numerical Results: Pseudoscalar Exchange

- Numerical cross section vs. QFT tree-level cross section

- No Sommerfeld enhancement at low velocities

- Effectively a short-range potential

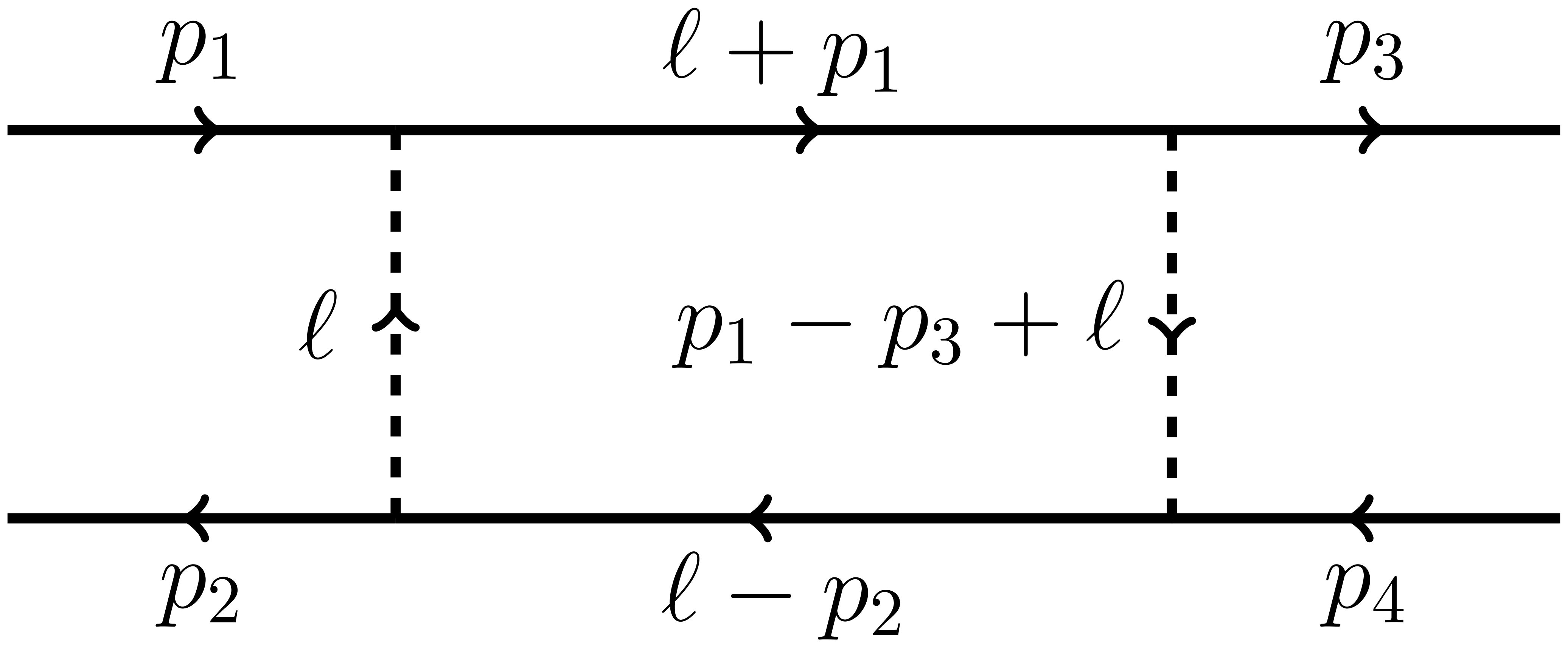


$$V \supset \frac{\lambda^2}{4\pi}\frac{e^{-m_\phi r}}{m_\chi^2}\left[\frac{m_\phi^2}{3r}S_1\cdot S_2 + \frac{3(S_1\cdot\hat{r})(S_2\cdot\hat{r}) - S_1\cdot S_2}{r^3}\left(1 + m_\phi r + \frac{m_\phi^2 r^2}{3}\right)\right]$$

Mediator Mass = 0.1 GeV; Dark Matter Mass = 1 GeV; Coupling = 0.1

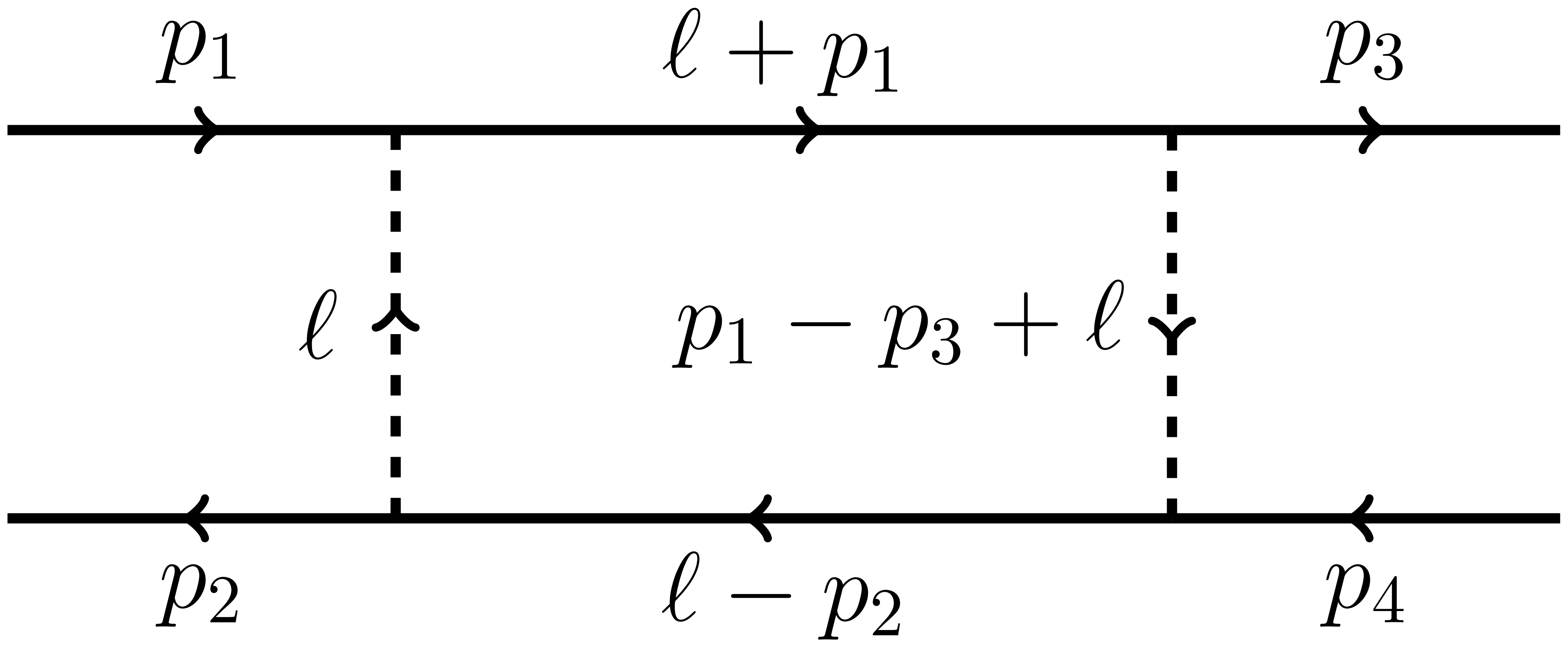
Sommerfeld Enhancement from Feynman Diagrams: Pseudoscalar

- Tree-level has s- and t-channel diagrams
- Box diagram for the 1-loop process
- Pseudoscalar case
  - t-channel velocity suppressed in the NR limit
  - $\frac{M_{1-loop}}{M_{tree}} \sim \frac{\lambda^2}{32\pi^2} \log \frac{m_\chi^2}{m_\phi^2}$



Sommerfeld Enhancement from Feynman Diagrams: Scalar

- Tree-level has s- and t-channel diagrams
- Box diagram for the 1-loop process
- Scalar case
  - t-channel dominant in the NR limit
  - $\dfrac{M_{1-loop}}{M_{tree}} \sim \dfrac{\lambda^2 m_\chi}{4\pi m_\phi} \quad \rightarrow \quad m_\phi \lesssim \dfrac{\lambda^2 m_\chi}{4\pi}$



Singular or Not?

- Is the pseudoscalar potential singular?

- Do singular potentials have phenomenological implications?

    - Extensive reviews in the literature exploring singular potentials. Two primary classification schemes exist. The first classifies potentials as singular if they diverge faster than r^{-2} at the origin. The second classifies potentials as singular if they arise from non-renormalizable operators in QFT.

    - More recently, it has been claimed that this has implications for SIDM and Sommerfeld enhancement for pseudoscalars. In particular, they introduce square-well regulators, but treat the depth of the square well and the coupling strength as free parameters, instead of matching to a perturbative QFT.

Future Directions?

- Our analysis so far focused only on scattering, but we can analyze annihilations as well. [with R. Sato and T. Slatyer]
    - Annihilation is inherently short-range, so it modifies the short distance boundary conditions.
    - Annihilation is absorptive, so the modification contains an imaginary part.
- Can we recast the effective range formalism in this language?
- How does the story change if we consider
    - Loops/Strong coupling? Relativistic corrections?

Conclusions

- Using the QFT to set the boundary conditions, we analyzed a variety of potentials. We were able to reproduce the known results for the Yukawa potential.
- Pseudoscalar potentials don't generate Sommerfeld enhancement. Tree-level perturbative QFT is a good approximation to scattering mediated by pseudoscalars.
- QFT seems to produce highly non-generic quantum mechanical potentials. In some cases, we have very non-trivial cancellations occurring so as to preserve the non-singular behavior of these potentials.
- The naive classifications of singular potentials based on the scaling with r near the origin or the dimensionality of the QFT operator from which it arises are insufficient.

aditya.parikh@stonybrook.edu

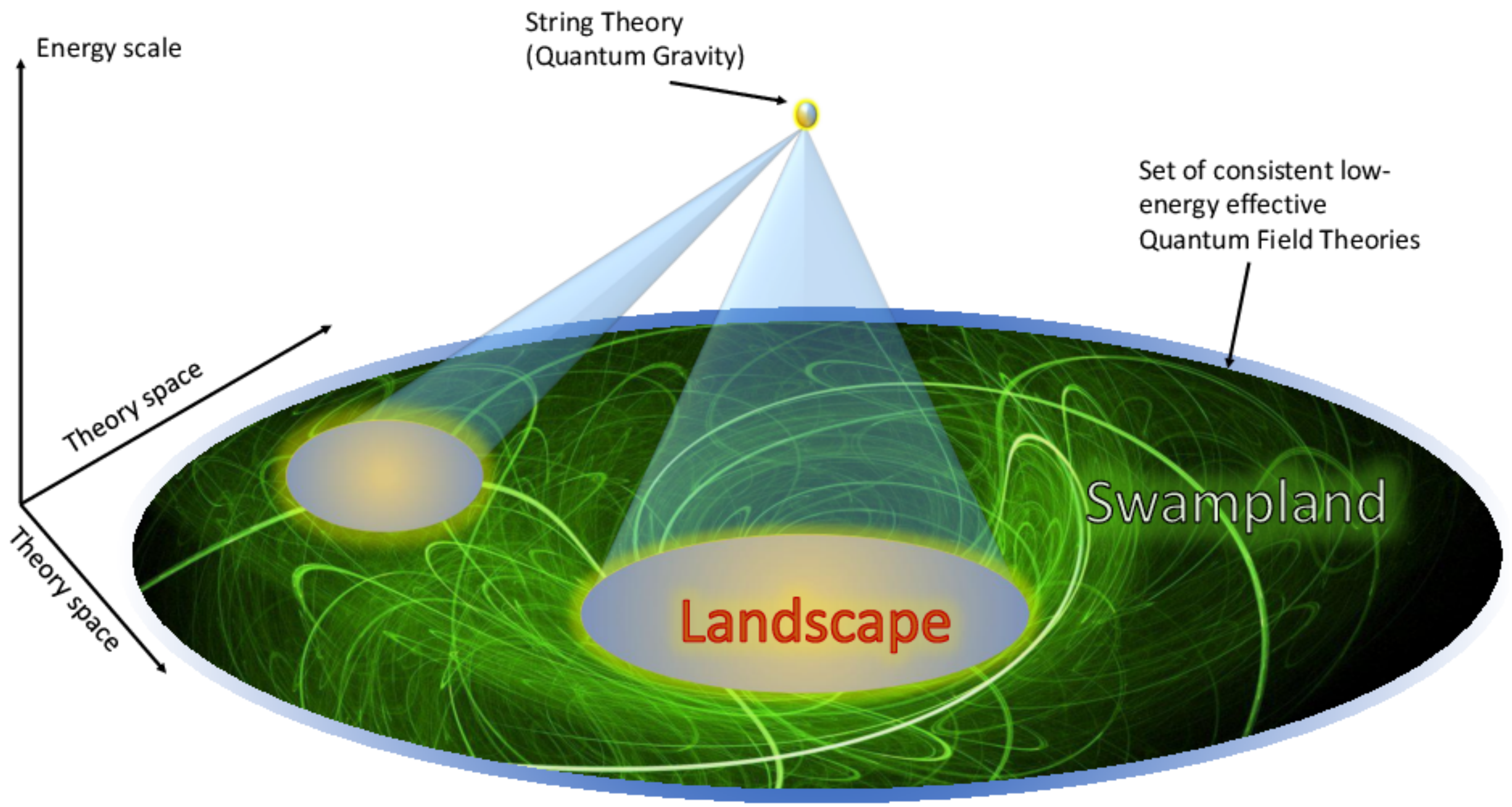
Backup

An Aside: Swampland Overview

- Consistency is the key to the Swampland program

- Frame conjectures in terms of criteria that low energy QFTs must satisfy to reside in the Landscape

- Apply this philosophy to the study of QM potentials



Palti [1903.06239]

The Quantum Mechanics Swampland

- The Landscape consists of all quantum mechanical potentials that can be derived from well-defined tree-level QFTs. A potential resides in the Swampland if it is singular.

- **Diagnostic:** If the first Born approximation diverges for *any* combination of incoming and outgoing states, then the potential is singular.

$$K_{\ell s,\ell' s'} \propto \int_0^a dr s_{\ell'}(kr) V_{\ell s,\ell' s'}(r) s_\ell(kr) \approx \int_0^a dr (kr)^{\ell'+1} V_{\ell s,\ell' s'}(r) (kr)^{\ell+1}$$

- As an example, we'll evaluate the pseudoscalar potential and the four-fermion version of it.

Pseudoscalar Potentials

- Mediated by renormalizable operators, we get
  - $\mathscr{L}_{int} = i\lambda\phi\overline{\psi}\gamma^5\psi \quad \to \quad V \supset \boxed{\frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^3}\bigg(1}+ m_\phi r + \frac{m_\phi^2 r^2}{3}\bigg)\frac{e^{-m_\phi r}}{m_\chi^2}$
- Diagnostic check - the only singular term has a vanishing matrix element!
  - $K_{\ell s,\ell' s'} \supset \int_0^a dr s_{\ell'}(kr)\frac{N_{\ell,\ell'}}{r^3}s_\ell(kr) \approx N_{\ell,\ell'}\int_0^a dr r^{\ell+\ell'-1}$
  - $N_{0,0} = \langle \ell' = 0 | 3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2 | \ell = 0 \rangle = 0$

Pseudoscalar Potentials

- Mediated by non-renormalizable operators, we get
  - $\mathscr{L}_{int} = \frac{\lambda}{\Lambda^2}\overline{\psi}_1\gamma^5\psi_1\overline{\psi}_2\gamma^5\psi_2 \quad \rightarrow \quad V \supset \frac{\lambda}{m_1 m_2 \Lambda^2}(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla})\delta^3(\vec{r})$
- Diagnostic check
  - $\int_0^a dr s_{\ell'}(kr)(\vec{S}_1 \cdot \vec{\nabla})(\vec{S}_2 \cdot \vec{\nabla})\delta^3(\vec{r})s_\ell(kr) \approx S_1^i S_2^j \int_0^a dr \nabla_i \nabla_j \frac{\delta(r)}{r^2}(kr)^{\ell+1}(kr)^{\ell'+1}$
  - $S_1^i S_2^j \nabla_i \nabla_j \delta(r) r^{\ell+\ell'} = \delta(r) r^{\ell+\ell'-2}\left[(\ell+\ell'-1)\delta_{ij} + (3 + (\ell+\ell')(\ell+\ell'-4))\hat{r}_i\hat{r}_j\right] S_1^i S_2^j$
- Case I: $\ell = \ell' = 0 \rightarrow \frac{3(S_1 \cdot \hat{r})(S_2 \cdot \hat{r}) - S_1 \cdot S_2}{r^2}\delta(r)$ Case II: $\ell + \ell' = 1 \rightarrow 0$

Extensions to Higher Dimensions

- Coulomb potentials in d spatial dimensions
  - No operator structure! Problematic for d > 4?

$$V(r) = \frac{\alpha}{r^{d-2}}$$

- To compute the diagnostic, we need the free particle solutions in d spatial dimensions. Let's turn to solving the free Schrödinger equation in d dimensions, which will give us these solutions.

Solving the Schrödinger Equation in Higher Dimensions

- Consider the free particle Schrödinger equation in d spatial dimensions

$$-\frac{1}{2\mu}\nabla_d^2\Psi(r) = E\Psi(r) \qquad \nabla_d^2 = \partial_r^2 + \frac{d-1}{r}\partial_r + \frac{1}{r^2}\Omega^2$$

- The wavefunction is a product of a radial function and Gegenbauer polynomials. They are the higher dimensional generalization of the spherical harmonics.

$$\partial_r^2 R + \frac{d-1}{r}\partial_r R - \frac{\ell(\ell+d-2)}{r^2}R = -k^2 R$$

- Change variables to cancel the first derivative: $u(r) = r^{(d-1)/2}R(r)$

$$\partial_r^2 u + \left[k^2 - \frac{j(j+1)}{r^2}\right]u = 0 \qquad j = \ell + \frac{d-3}{2}$$

Higher Dimensional Coulomb Potentials

- Coulomb potentials in d spatial dimensions
  - No operator structure! Problematic for d > 4? $V(r) = \dfrac{\alpha}{r^{d-2}}$
- Free particle solutions in d spatial dimensions

$$s_j(kr) = krj_j(kr) \qquad c_j = -\,kry_j(kr) \qquad j = \ell + \frac{d-3}{2}$$

- Diagnostic check

$$K_{js,j's'} = \frac{-2\mu}{k}\int_0^a dr s_{j'}(kr) V_{js,j's'}(r) s_j(kr) \approx \frac{-2\alpha\mu}{k}\int_0^a dr r^{j'+1} r^{2-d} r^{j+1} \approx \frac{-2\alpha\mu}{k}\int_0^a dr r^{\ell'+\ell+1}$$

Scalar-Scalar Potentials

- Scalars don't possess any intrinsic spin. This allows us to uniquely fix the non-relativistic limit of the amplitude.

$$\tilde{V}(\vec{q}) = \frac{f(q^2)}{q^2 + m^2} = \sum_{n=0}^{\infty} \frac{a_n q^{2n}}{q^2 + m^2} = \frac{\tilde{a}_{-1}}{q^2 + m^2} + \sum_{n=0}^{\infty} \tilde{a}_n q^{2n}$$

- Every factor of q gives us another derivative, so the potential is the sum of a Yukawa term and even derivatives of delta functions.

- Diagnostic check

$$\nabla^{2n} \delta(r) r^{\ell + \ell'} = (\ell + \ell')(\ell + \ell' - 1) \cdots (\ell + \ell' + 1 - 2n) \delta(r) r^{\ell + \ell' - 2n}$$

Conclusions

- QFT seems to produce highly non-generic quantum mechanical potentials. In some cases, we have very non-trivial cancellations occurring so as to preserve the non-singular behavior of these potentials.

- The naive classifications of singular potentials based on the scaling with r near the origin or the dimensionality of the QFT operator from which it arises are insufficient.

- These results are not generic and extend to scalars as well. Preliminary results also show that these results hold in higher dimensions.