COMUNE DI POLLICA

# Condensed Dark Matter
# at
# Galactic Scales

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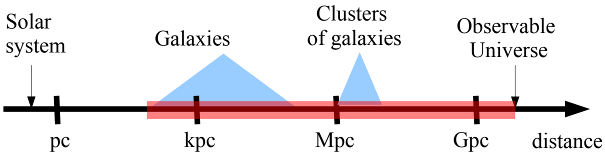
# Matter

## Dark Matter and visible matter in the Universe



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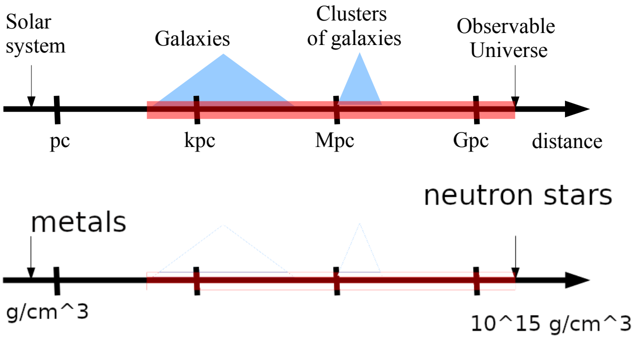

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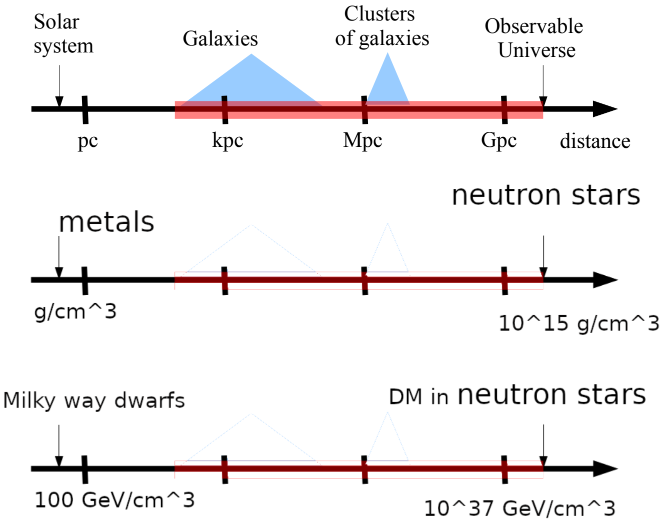


# What is new here?

## Outline

- Fermion asymmetric DM with yukawa interaction at finite density. Going beyond non-interacting scenario Domcke & Urbano '14, Randall et al. '16, Gresham &Zurek '18, and Bosonic DM with effective theory for phonons Berezhiani & Khoury '15
- Delimiting possible phases of the Yukawa theory
- In-medium effects and generalized 'gap equations' and equation of state for arbitrary mediator masses.
- Equation of state and some applications.

# Phases in the Yukawa theory

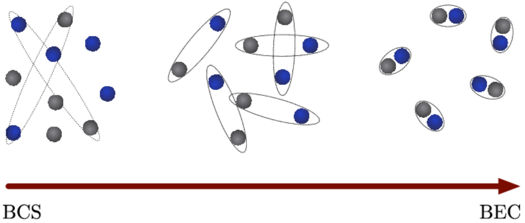
## The model

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + \mu\bar{\psi}\gamma^0\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - g\,\bar{\psi}\psi\phi\ .$$

- 4 free parameters: $m, m_\phi, g$ and the density $\mu$
- Dark particles singlets under SM. The fermion $\psi$ charged under $U(1)_{\mathrm{dark}}$ global
- Fermi energy $E_F = \mu \equiv \sqrt{m^2 + k_F^2}$, number density $n = N/V \equiv \langle\bar{\psi}\gamma_0\psi\rangle$

# Phases in the Yukawa theory

## Scattering in the Yukawa theory




- The scattering length effectively captures the short distance properties of a potential

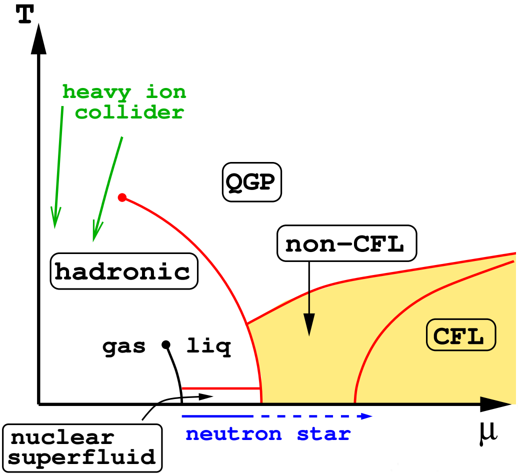
$$\lim_{k\to 0} k \cot \delta_0(k) = -\frac{1}{a} \ .$$

Computable for dilute gases in the non-relativistic limit.

- Anologous to contact interactions in low temperature physics, phases delimitted by dimenionless $k_F a$.

# Phases of QCD

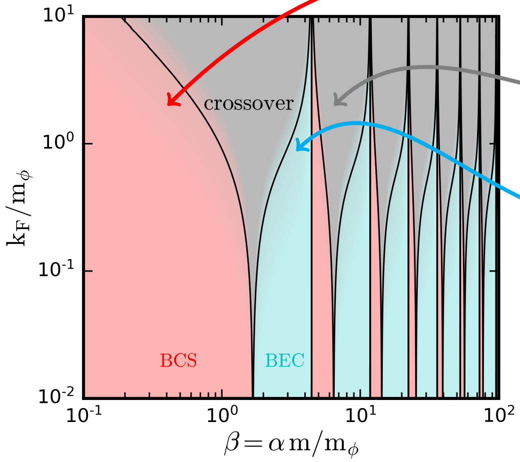
Alford, Schmitt, Rajagopal , Schäfer '08



# Phases in the Yukawa theory at $T = 0$

## Phases for dilute fermi-gas with Yukawa

RG, M.H.G Tytgat and J. Vandecasteele '22



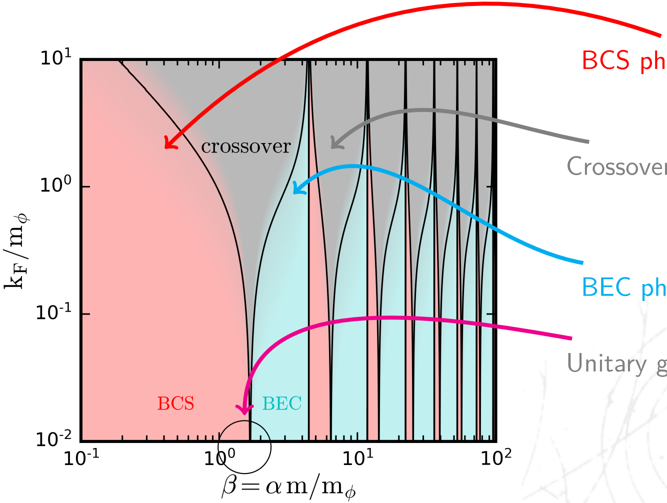

BCS phase: $(k_F\, a)^{-1} <$ -1

Crossover: -1 $\leq (k_F\, a)^{-1} \leq 1$

BEC phase: $(k_F\, a)^{-1} > 1$

# Phases in the Yukawa theory at $T = 0$

Phases for dilute fermi-gas with Yukawa RG, M.H.G Tytgat and J. Vandecasteele '22

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Unitary gas: $(k_F\, a)^{-1} \to 0$

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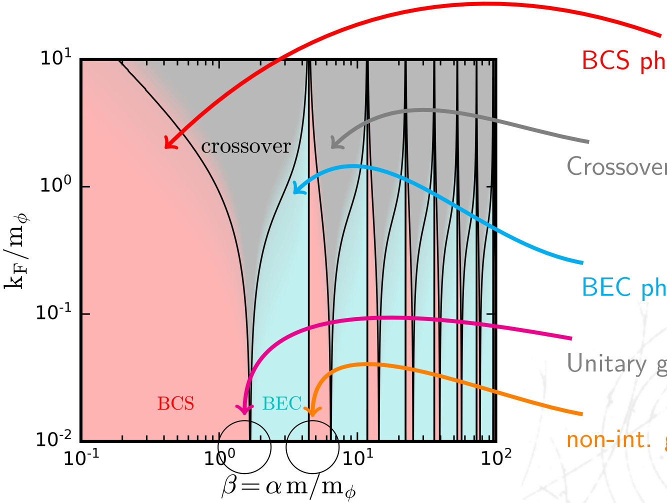


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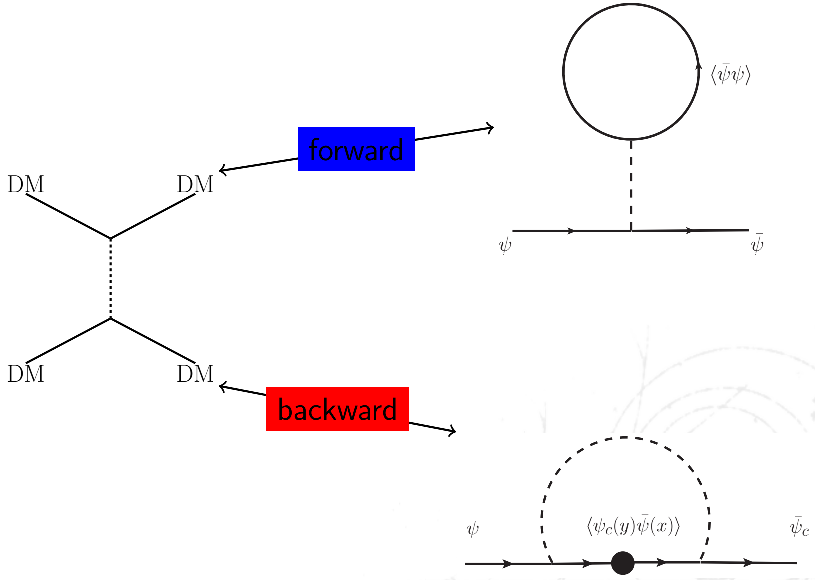
BEC phase: $(k_F\, a)^{-1} > 1$

Unitary gas: $(k_F\, a)^{-1} \to 0$

non-int. gas: $(k_F\, a)^{-1} \to \infty$

# The BCS phase

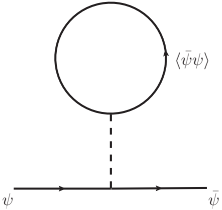
## Cooper pairing and in-medium effects



# Full forward scattering

## Scalar density condensate

- Tadpole $\neq 0$ when $\mu \neq 0$
- The scalar operator $\bar{\psi}\psi$ has a non-zero mean, $n_s = \langle \bar{\psi}\psi \rangle > 0$ Waleck '74, Gresham et al. '18. $\implies$ $n_s$ sources the scalar field due to its Yukawa interactions with the fermions

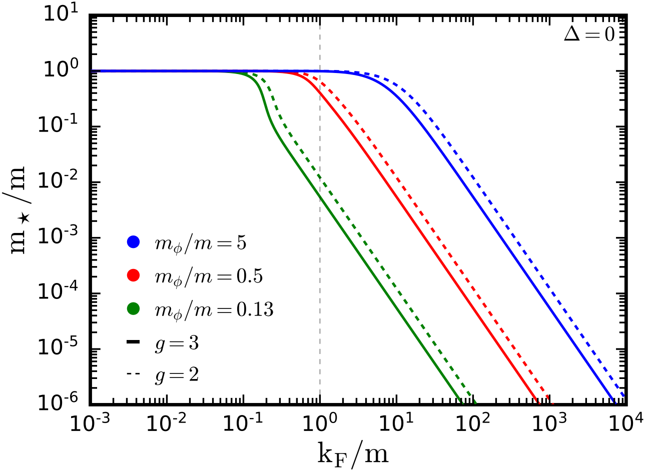



  - $\frac{\delta \mathcal{L}}{\delta \phi} = 0 \rightarrow m_\phi^2 \langle \phi \rangle + g \langle \bar{\psi}\psi \rangle = 0$
  - $m_* = m + g\langle \phi \rangle \rightarrow m_* = m - \frac{g^2}{m_\phi^2} n_s(m_*)$

$\implies$ the fermion mass is reduced in the medium! (similar to NJL model of chiral symmetry breaking)

# Full forward scattering

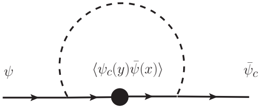
## Results for scalar density condensate RG, M.H.G Tytgat and J. Vandecasteele '22



# Full backward scattering

## Cooper pairing and superfluidity: BCS argument

- Free enrgy for N particles $\Omega_N = E - \mu N$
- Add a particle $\implies$ $\Omega_{N+1} = E_{+1} - \mu\,(N+1)$
- If attractive interactions $\Omega_{N+1} < \Omega_N$
- Formation of many bosonic Cooper pairs which condensate $\sim \langle\psi\psi\rangle$ (Leon Cooper '57). Pairing in $^1S$ channel.
    - Object that gets a vev $\sim \left\langle \psi_C(y)\bar{\psi}(x) \right\rangle$, a $4 \times 4$ quantity



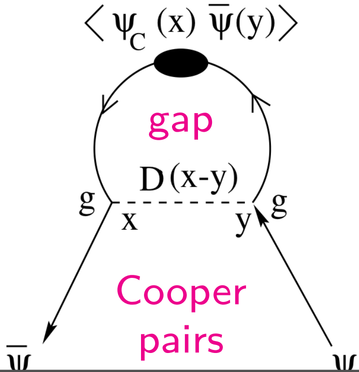
# Full backward scattering

## Qualitative physics

Yukawa theory when $m_\phi \gg m$: $4-$fermion interaction Schmitt '14

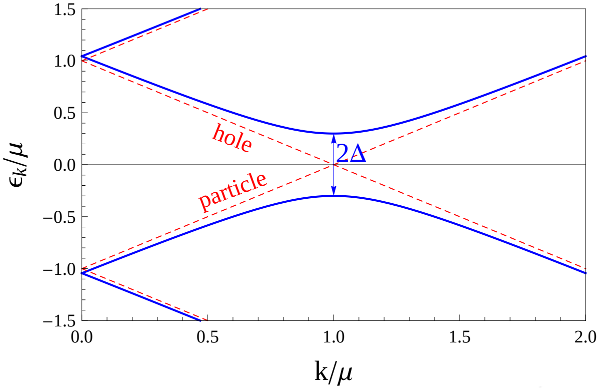
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + \gamma^0\mu - m)\psi + G_\phi \bar{\psi}\bar{\psi}\psi\psi$$

$$\approx \begin{pmatrix} \bar{\psi} & \bar{\psi}_C \end{pmatrix} \begin{pmatrix} k\!\!\!/ + \mu\gamma^0 - m & \langle \psi\bar{\psi}_C \rangle \times G_\phi \\ \langle \psi_C\bar{\psi} \rangle \times G_\phi & k\!\!\!/ - \mu\gamma^0 - m \end{pmatrix} \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$$



# Full backward scattering

## Physical meaning



# The consistent set of gap equation

## Deriving the gap equations with variational methods

- Ansatz for the Dirac structure of gap $\Delta$. In BCS $\Delta = \Delta_1 \gamma^5$.
- Starting from the action,
  $S = \int_{x,y} \left[\bar{\psi}(x) G_0^{-1}(x,y) \psi(y) - \frac{1}{2}\phi(x) D^{-1}(x,y) \phi(y)\right] - g \int_x \bar{\psi}(x) \psi(x) \phi(x)$
  do Hubbard-Stratonovich transformation to introduce gaps as auxilliary fields
  $1 \propto \int \mathcal{D}\bar{\Delta}\mathcal{D}\Delta \exp\{-\frac{1}{2}(\Delta - \psi_c\bar{\psi})\frac{D}{2}(\bar{\Delta} - \psi\bar{\psi}) - \frac{1}{2}(\bar{\Delta} - \psi\bar{\psi})\frac{D}{2}(\Delta - \psi_c\bar{\psi})\}$

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  do Hubbard-Strantanovich transformation to introduce gaps as auxilliary fields
- $S = \int_{x,y} \bar{\psi}(x)\, G_0^{-1}(x,y)\, \psi(y) + \bar{\Delta} D \Delta + \psi \bar{\Delta} \psi + \bar{\psi} \Delta \bar{\psi}$
  $Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\bar{\Delta}\mathcal{D}\Delta\, e^{-S}$
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  $\Omega = -\frac{T}{V} \log Z$

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- Differentiate $\Omega$ w.r.t gaps, set to 0 :
  $\frac{\partial \Omega}{\partial \Delta} = 0$
- Solve by iterative numerical methods. **Very general setup!**

# The consistent set of gap equation

## Advantages of variational methods

- Introduce auxilliary fields to bring fermion bilinears to quadratic form Stratonovich '57, Hubbard '59
- Captures multi-channel condensation Kleinert '11
- Applicable in all regimes without spurious cut-off Alford '01, Alford et al. '07, Kleinert '11

# The consistent set of gap equation

Gap structure and dispersion RG, M.H.G Tytgat and J. Vandecasteele '22

$\Delta$ has fermionic indices, respects Fermi statistics

$$\Delta_{\alpha\beta} \equiv \left\langle \psi_{C,\alpha}(x)\, \bar{\psi}_{\beta}(y) \right\rangle$$

Ansatz for the Yukawa theory Pisarski and Rischke '99, Bailin and Love '84

$$\Delta = \Delta_1 \gamma_5 + \Delta_2 \boldsymbol{\gamma} \cdot \hat{\boldsymbol{k}} \gamma_0 \gamma_5 + \Delta_3 \gamma_0 \gamma_5$$

$$\mathcal{L} = \begin{pmatrix} \bar{\psi} & \bar{\psi}_C \end{pmatrix} \underbrace{\begin{pmatrix} \not{k} + \mu\gamma^0 - m & \Delta(k) \\ \Delta(k) & \not{k} - \mu\gamma^0 - m \end{pmatrix}}_{\text{inverse propagator}} \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}$$

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In BCS, $\Delta \ll \mu$

$$\epsilon_\pm^2 \approx (\omega \pm \mu)^2 + \left(\Delta_1 \pm \left(\frac{k}{\omega}\Delta_2 + \frac{m}{\omega}\Delta_3\right)\right)^2$$

Like standard BCS theory but non-trivial momentum dependence

# The consistent set of gap equation

RG, M.H.G Tytgat and J. Vandecasteele '22

$$\Sigma(0) = \frac{-g^2}{m_\phi^2}\sum_\eta \int \frac{d^3k}{(2\pi)^3}\left\{\frac{m_*}{\omega_k}\left(\frac{\omega_k+\eta\mu}{\epsilon_\eta(k)}-1\right)-\eta\frac{k}{\omega_k}\frac{\tilde{\kappa}(k)}{\omega_k}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}\right\},$$

$$\tilde{\Delta}_\pm(p) = \frac{g^2}{32\pi^2}\sum_\eta \int_0^\infty dk\,\frac{k}{p}\left\{\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\mp\eta\right.$$

$$\frac{kp}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)$$

$$\left.\pm\eta\frac{m_*^2}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)},$$

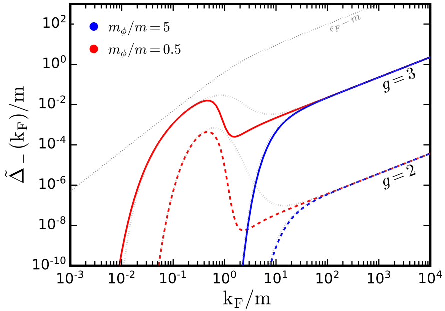
$$\tilde{\kappa}(p) = \frac{g^2}{32\pi^2}\sum_\eta \int_0^\infty dk\,\frac{k}{p}\left\{-\eta\frac{m_*k}{\omega_p\omega_k}\left(-2+\frac{m_\phi^2+k^2+p^2}{2kp}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right)\right.$$

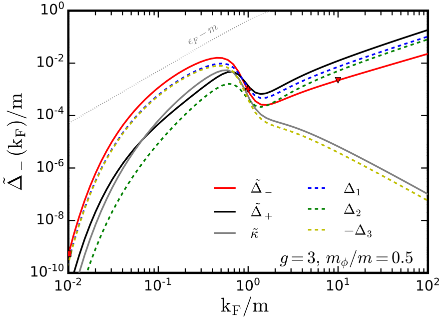
$$\left.-\eta\frac{m_*p}{\omega_p\omega_k}\log\frac{m_\phi^2+(p+k)^2}{m_\phi^2+(p-k)^2}\right\}\frac{\tilde{\Delta}_\eta(k)}{\epsilon_\eta(k)}.$$

# The consistent set of gap equation

## Solution to gap equations: moderately heavy mediators RG, M.H.G Tytgat and J. Vandecasteele '22

$$\Delta = \Delta_1\gamma_5 + \Delta_2\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}\gamma_0\gamma_5 + \Delta_3\gamma_0\gamma_5$$



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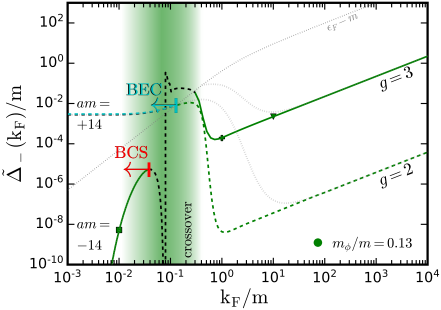
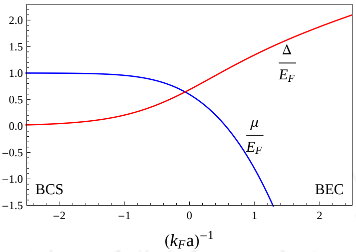


# Applications

# Bullet cluster constraints

## Effective range formalism

- s-wave scatterings are best understood in terms of scattering length $a$, effective range $r_e$ H. Bethe '49, Chu et al. '20

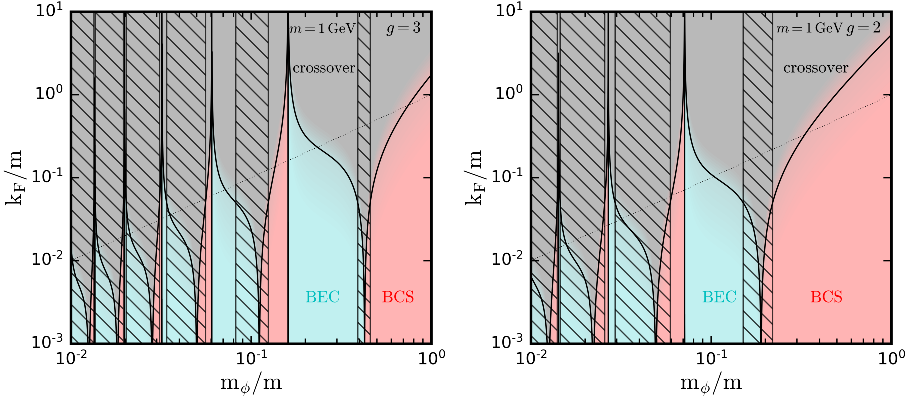
$$\mathcal{M}^{l=0} = \frac{1}{k\left(\cot\delta - i\right)}\,,\;\; k\cot\delta\big|_{\text{s-wave}} \approx \frac{-1}{a} + \frac{r_e}{2}k^2$$

$$\sigma_0 \approx \frac{4\pi a^2}{1 + k^2\left(a^2 - a r_e\right) + \frac{1}{4}a^2 r_e^2 k^4}$$

- Phase shift $\leftrightarrow$ Schrödinger equation(dilute gases, non-relativistiic)
- also gap equation $\leftrightarrow$ like Schrödinger equation but at large density + relativistic effects

# Bullet cluster constraints

$\sigma/m \approx$ barn/GeV



# Equation of state

## Thermodynamic considerations

- Consider first the case when $\Delta = 0$ but $m_\star \ll m$. Free energy Kapusta and Gale '11 also Gresham and Zurek '18

$$\Omega = -2T \int \frac{d^3p}{(2\pi)^3} \left[\log\left(1 + e^{-\beta(\omega_* - \mu)}\right) + \log\left(1 + e^{-\beta(\omega_* + \mu)}\right)\right] + \frac{1}{2} m_\phi^2 \phi_0^2$$

- parameterized by $C_\phi^2 = \frac{4}{3\pi}\alpha\frac{m^2}{m_\phi^2}$, and $m_*/m = 1 - \varphi$, $n = k_F^3/3\pi^2$

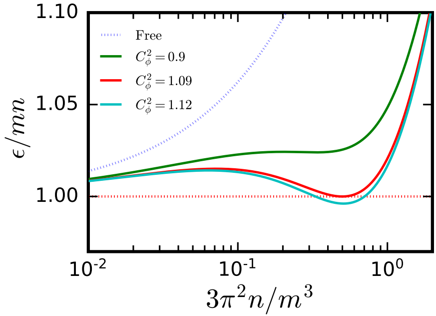
$$P = -\Omega = \frac{m^4}{3\pi^2}\left(-\frac{\varphi^2}{2C_\phi^2} + \int_0^{k_F/m} \frac{x^4}{\sqrt{x^2 + (1-\varphi)^2}} dx\right)$$

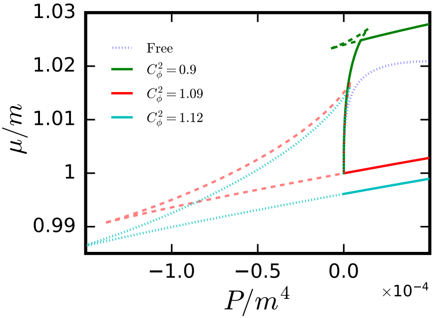
$$\epsilon = \mu n - P = \frac{m^4}{3\pi^2}\left(\frac{\varphi^2}{2C_\phi^2} + 3\int_0^{k_F/m} x^2 \sqrt{x^2 + (1-\varphi)^2} dx\right) ,$$

# Equation of state

## With Maxwell construction

$$C_\phi^2 = \frac{4}{3\pi}\alpha\frac{m^2}{m_\phi^2},\; n = k_F^3/3\pi^2$$

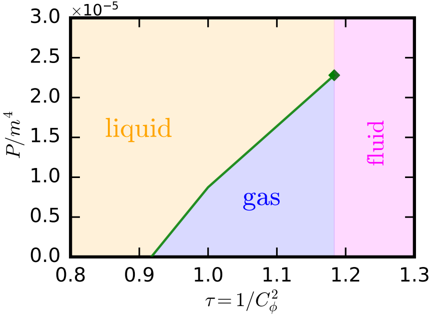


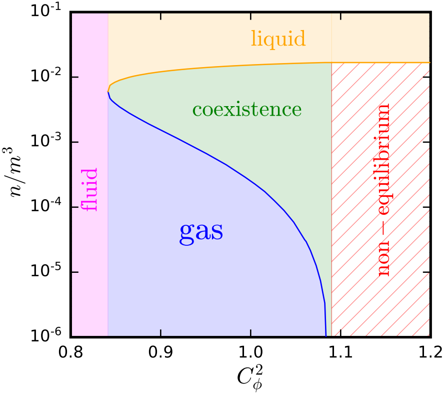



# Equation of state

## as a function of coupling

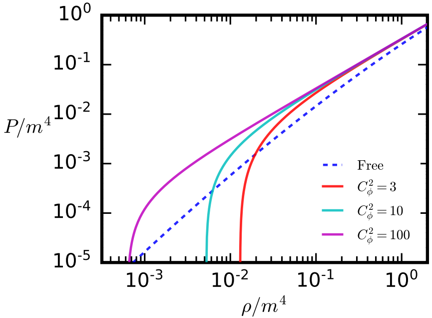
$$C_\phi^2 = \frac{4}{3\pi}\alpha\frac{m^2}{m_\phi^2},\ n = k_F^3/3\pi^2$$

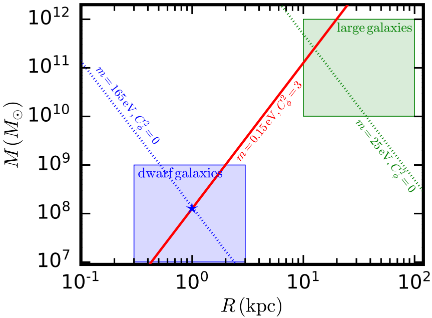


# Equation of state

## Application to Halos



# Applications and consequences

## Collapse to Black hole: classical

Minimize total energy per particle

- For non-interacting bosons

$$E_{\rm tot} \sim -\frac{GNm_\chi^2}{R} + \frac{1}{R} \implies N_{\rm ch}^{\rm bosons} \simeq 10^{34}\left(\frac{100{\rm GeV}}{m_\chi}\right)^2$$

- For non-interacting fermions

$$E_{\rm tot} \sim -\frac{GNm_\chi^2}{R} + \left(\frac{N}{g_f}\right)^{1/3}\frac{1}{R} \implies N^{\rm ch}_{\rm fermions} \simeq 10^{51} g_f^{-1/2}\left(\frac{100\,{\rm GeV}}{m_{DM}}\right)^3$$

- For fermions with Yukawa interaction: $E_{\rm int} = \pm\sum_{i\neq j}\alpha\frac{e^{-m_\phi r_{ij}}}{r_{ij}}$ Kouvaris et al '15, '18

$$E_{\rm tot} \sim -\frac{GNm_\chi^2}{R} + \left(\frac{N}{g_f}\right)^{1/3}\frac{1}{R} + E_{\rm int} \implies N_{\rm ch} \approx \left(\frac{m_\phi}{m_\chi\sqrt{\alpha}}\right)^3\left(\frac{M_{\rm pl}}{m_\chi}\right)^3$$

# Applications and consequences

## Collapse to Black hole: reality RG, Tytgat and Vandecasteele '22 and also Gresham and Zurek '18

- Classical calculation fails as the system becomes more relativistic!
- In-medium corrections very important
- Yukawa theory: attractive interaction leads to non-trivial phases
- Collapse to blackhole impeded as interactions $\implies$ additional pressure
- True condition for collapse even in the presence of attractive interactions $\implies N_{\rm ch} \approx \left(\frac{M_{\rm pl}}{m_\chi}\right)^3$. The usual Chandrasekhar limit holds always!
- Impact of superfluid gaps parameterically very small.

# Conclusions and Outlook

- Emergent phenomena can be realized in dark sectors due to DM$-$DM interactions.
- Most important effects: finite density correction to mass and paramteric change to the dispersion relation.
- We have delimitted phases of Yukawa theory!
- Very general framework to describe superfluidity, motivated by DM phenomenology. For arbitrary mediator masses all the way from non-relativistic limit to relativistic limit.
- We are at the crossroad of many areas in physics.
- Construct EoS that corectly interpolates between condensate dominated high density regions and low density Maxwellian regimes $\rightarrow$ realistic description of DM halos at dwarf galaxy scales?

# Additional information

# Scattering length in the non-relativistic limit

- Wave-function in CM frame $\psi = e^{ikz} + f(\theta)\frac{e^{ikr}}{r}$
- The s-wave amplitude $f_0 = \frac{e^{2i\delta_0} - 1}{2ik}$
- At $k \to 0$ the wave function is $\psi = 1 - \frac{a}{r}$
- Scattering length $\lim_{k\to 0} k \cot \delta_0(k) = -\frac{1}{a}$
- Deutron (n-p spin triplet) $a = +7$ fm, whereas n-p spin singlet $a = -20$ fm
- In co-ordinate space Yukawa potential is $\pm\alpha\frac{e^{-m_\phi r}}{r}$
- Solve

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{l,k}}{dr}\right) + \left(k^2 - \frac{l(l+1)}{r^2} - m\,V(r)\right)R_{l,k} = 0\ ,$$

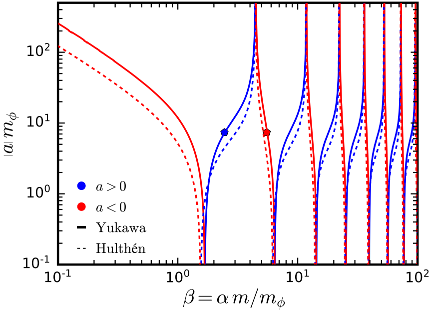
with boundary condition $r R_{l,k} = 0$ at $r = 0$

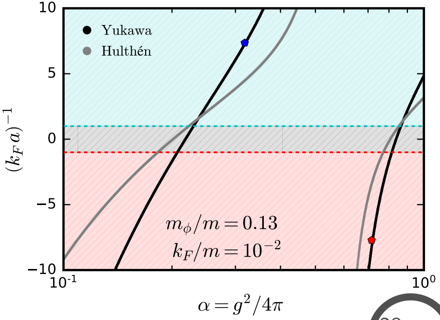
# Scattering length in the non-relativistic limit continued

- With further massaging (see Chu, Garcia-Cely, Murayama '19)

$$\frac{d\delta_{l,k}(r)}{dr} = -k\,m\,r^2\,V(r)\,\mathrm{Re}\left[e^{i\delta_{l,k}(r)}h_l^{(1)}(kr)\right]^2 \ ,$$

which is to be solved with boundary conditions $\delta_{l,k}(0) = 0$ and $\delta_{l,k} \to \delta_l$ at $r \to \infty$.

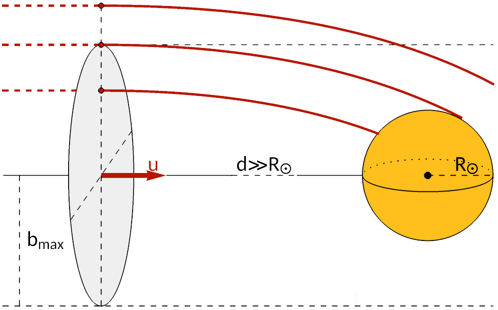


# Applications and consequences

## Black hole formation from DM collapse

- All particles that intersect the celestial body is captured if $\frac{\Delta E}{E} \gtrsim \frac{u^2}{u^2+v_e^2}$




$$\mathcal{C}_{\max} \approx \pi R_\star^2 \left(1 + \frac{v_e^2}{v_d^2}\right) \left(\frac{\rho_\chi}{m_\chi}\right) v_\infty$$

$$\mathcal{N}_{\max} = t_{\rm age} \mathcal{C}_{\max}$$

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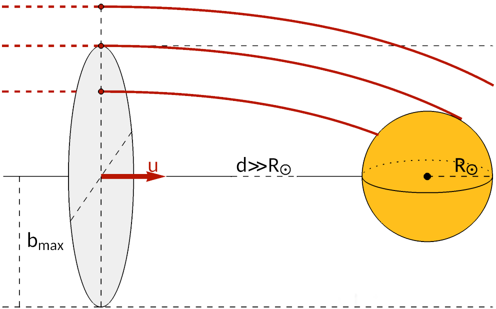
- Estimate simple when $\sigma n_\star R_\star \gtrsim 1$. In this limit the rate is independent of $\sigma$

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# Applications and consequences

## Comparison

- Sufficiently weak, $\sigma n_\star R_\star \sim 1$
- Geometric cross section, $\sigma_\star = \pi R_\star^2/N$
- The maximal capture rate

$$\mathcal{C}_{\rm max} \approx \pi R_\star^2 \left(1 + \frac{v_e^2}{v_d^2}\right)\left(\frac{\rho_\chi}{m_\chi}\right) v_\infty$$

| | $\sigma_\star$ [cm$^2$] | $\sim M_{\rm max}$/Gyr |
|---|---|---|
| Sun | $10^{-35}$ | $10^{-11}\mathrm{M}_\odot$ |
| White Dwarf | $10^{-39}$ | $10^{-19}\mathrm{M}_\odot$ |
| Neutron Star | $10^{-45}$ | $10^{-15}\mathrm{M}_\odot$ |

# A cosmological dark-QCD model

## The model

- Can dark matter (DM) be a baryon/pion of new confining dark sectors? $\implies$ composite DM Bai, Hill '10 + Boddy et.al. '14 +Gresham, Lou, Zurek '17 + Bai, Long, Lu '18 + many more

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- Here we focus on $SU(3)$

$$\int d^4x\,\sqrt{-g}\left[\mathcal{L}_{\rm SM}-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}+\bar{\psi}_i\left(\not{D}-m_i\right)\psi_i+\sum\frac{\mathcal{O}_{\rm SM}\mathcal{O}_{\rm dark}}{M_{\rm pl}^{\#}}\right].$$

with possibility of both dark-pion and -baryon DM!

# A cosmological dark-QCD model

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- Global chiral symmetry SU($N_F$)$\times$ SU($N_F$) $\rightarrow$ SU($N_F$)

$$\mathcal{L}_\pi = \frac{f^2}{4}\mathrm{Tr}(\partial_\mu U)^2 + b\mathrm{Tr}[MU + h.c.] + \mathrm{WZW}, \qquad U = \exp[i\pi/f]$$

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- Dark pions RG, Michele Redi, Andrea Tesi: Similar to SM we choose $M_\pi < 5f$
- Stability: not absolute. Violated by

$$\frac{1}{\Lambda_5}\bar{\Psi}^i\gamma^5\Psi^j|H|^2 + \frac{1}{\Lambda_6^2}\bar{\Psi}^i\gamma^\mu\gamma^5\Psi^j\bar{f}\sigma^\mu f\,.$$

$\langle 0|\bar{\Psi}\gamma^5\Psi|\pi\rangle = c\,4\pi f^2 \implies$ mixing with higgs $\frac{4\pi f^2}{\Lambda_5}|H^2|\pi$

# Is emergent phenomenon realized in dark-QCD models?

Yes

- Much richer structure than the Yukawa theory

$$g\bar{\psi}\psi\phi \to g\bar{\psi}_\alpha \gamma^\mu T_a^{\alpha\beta}\psi_\beta A_\mu^a$$

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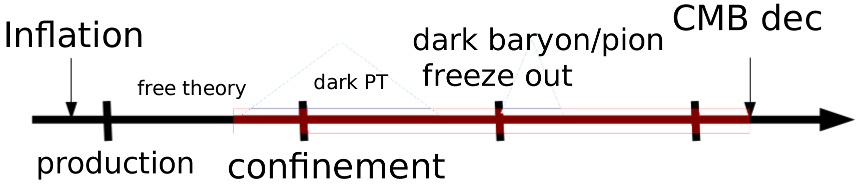
- Pairing is most attractive in $^1S$ : $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$
- Wavefunction has to be overall antisymmetric
- Need to be also antisymmetric in flavour ✓

# Dark-QCD model

## Cosmology



# Dark-QCD model

## Results RG, M. Redi, A. Tesi '21