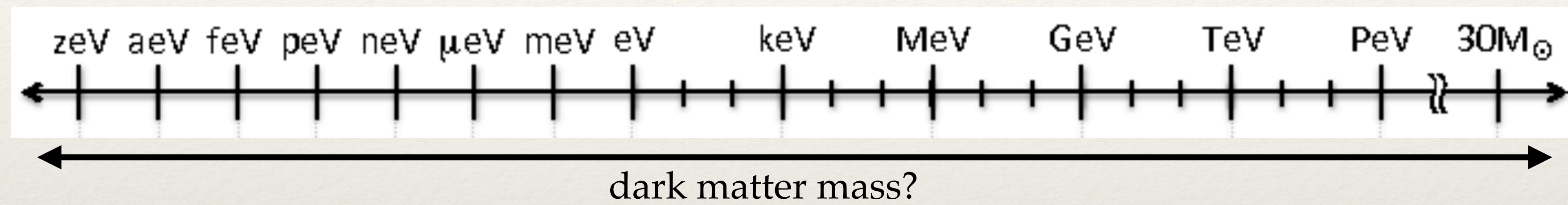


Pollica SIDM Workshop, June 20th 2023

Astrophysical Plasma Instabilities induced by Long-Range Interacting Dark Matter

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University of Washington Physics
in collaboration with
Matthew McQuinn

Dark matter's microscopic properties are poorly constrained



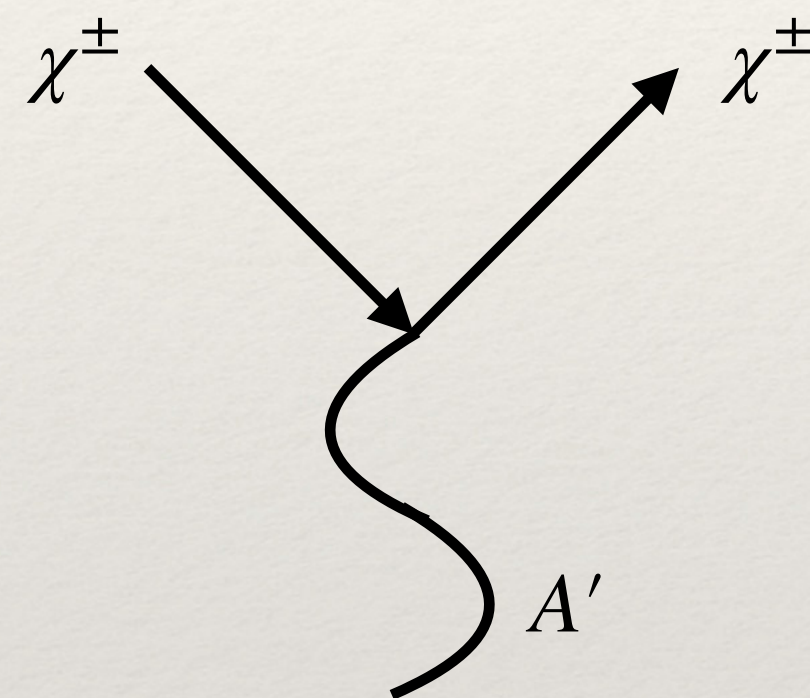
Does dark matter interact beyond gravity?

1. Does it have self-interactions?
2. Does it interact with the Standard Model of particle physics?

What could non-gravitational dark matter interactions look like?

In Cruz & McQuinn [2202.12464] we considered **two** charged dark matter (DM) Models

DM with dark charge

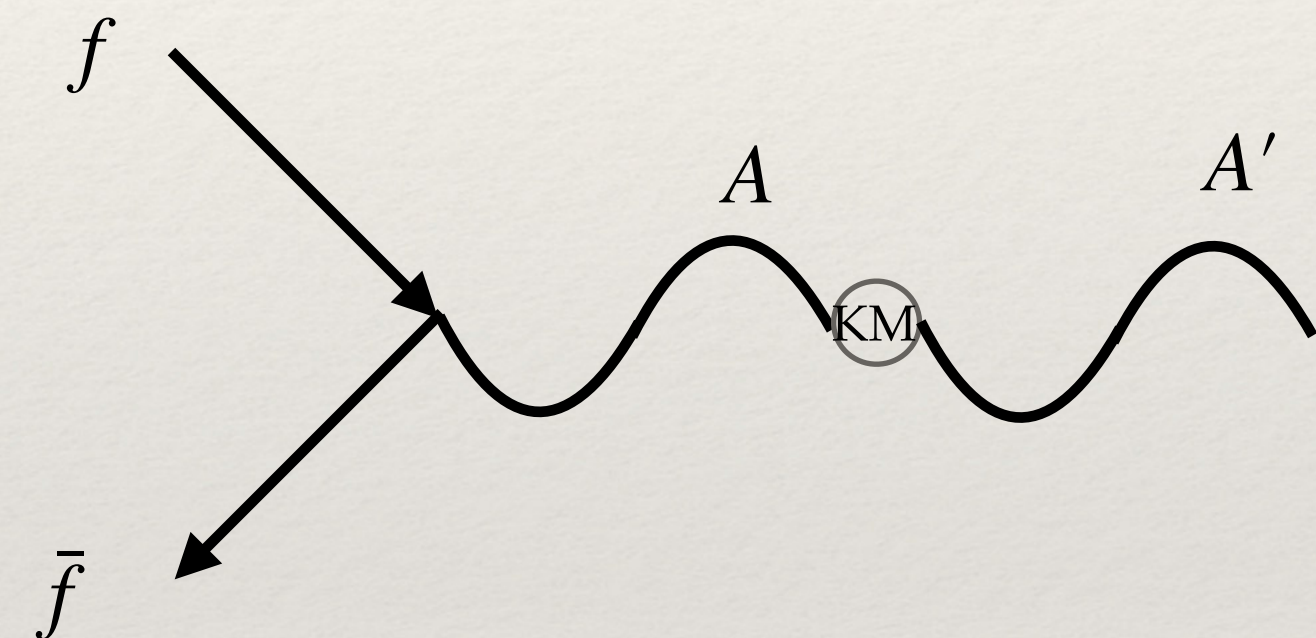


two parameters which we will work to constrain

$$m_\chi, q_\chi$$

DM-DM interactions only

"Milli-charged DM"



two parameters which we will work to constrain

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DM-SM and DM-DM interactions allowed

Two-particle hard scattering constraints

Ackerman et al. 2008 used hard scattering interactions between DM particles χ in the Milky Way to constrain dark charged DM

constraint on $q_\chi - m_\chi$ plane

$$\tau = \frac{1}{\langle n\sigma v \rangle} \quad \text{where} \quad v \simeq \sqrt{\frac{GM_{Gal}}{R}} \simeq \sqrt{\frac{GNm_\chi}{R}}$$

$$\tau_{\text{dyn}} = 2\pi R/v \approx 2 \times 10^8 \text{ yrs for the Milky Way}$$

the average time for a DM hard scatter is greater than the age of the universe if

$$\frac{\tau}{\tau_{\text{dyn}}} = \frac{2R^2}{3N\sigma} \gtrsim 50$$

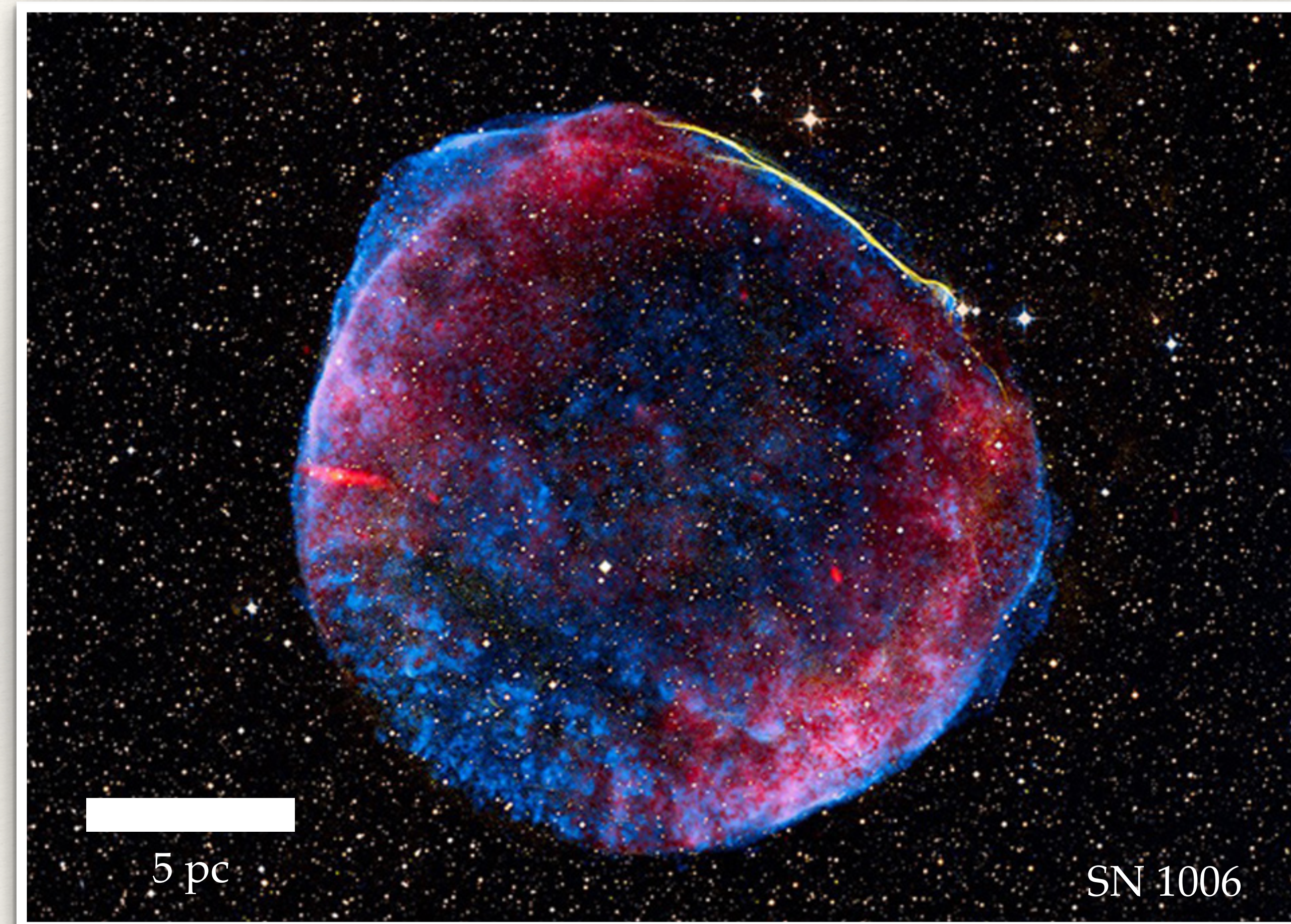
hard scattering

$$b_{\text{hard}} = \frac{2\alpha}{v^2 m_\chi} \quad \sigma_{\text{hard}} = b_{\text{hard}}^2$$

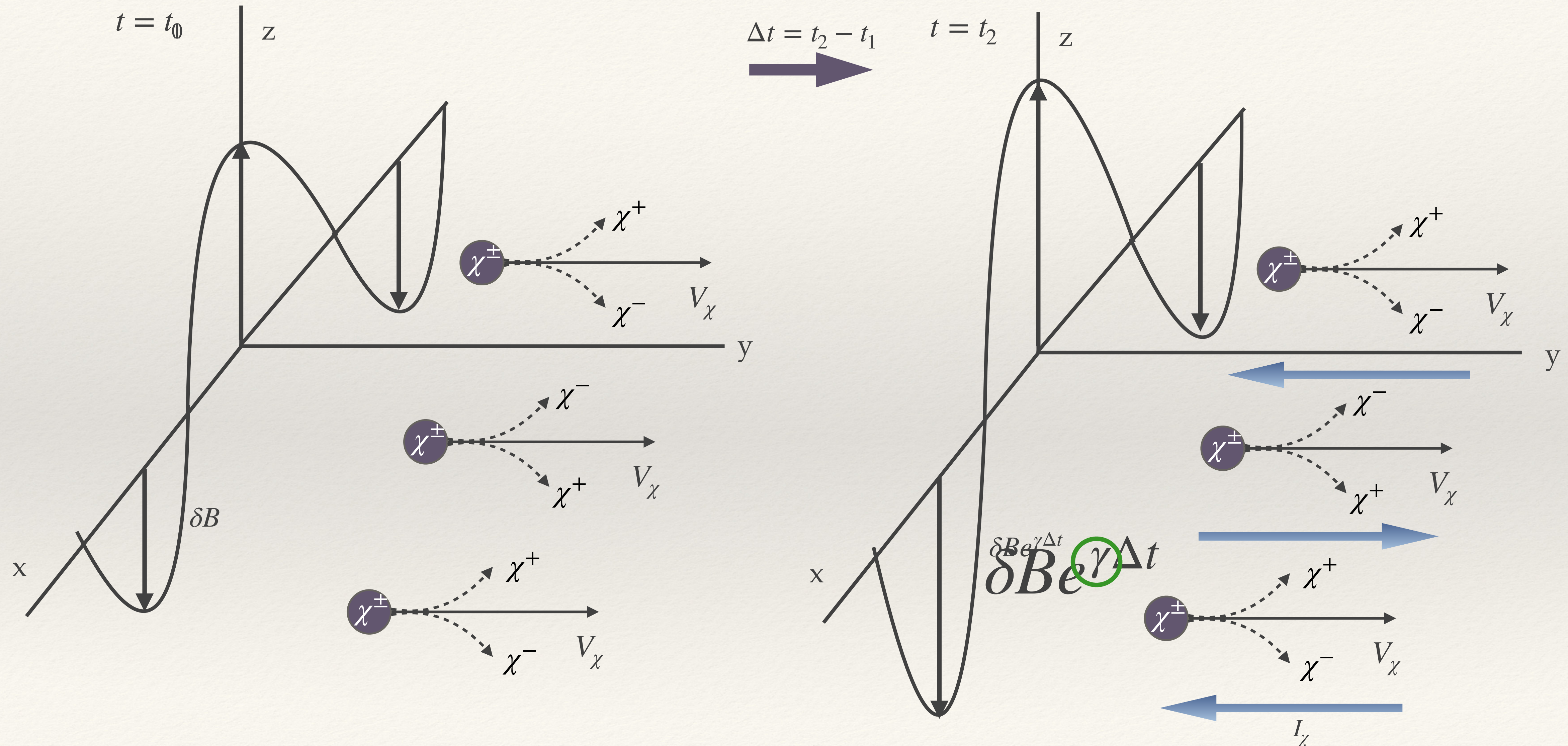
$$\frac{\tau_{\text{hard}}}{\tau_{\text{dyn}}} = \frac{G^2 m_\chi^4 N}{6\alpha^2} \gtrsim 50 \rightarrow \alpha \lesssim \sqrt{\frac{1}{300}} \left(\frac{m_\chi}{\text{TeV}} \right)^{3/2} \rightarrow \sim 2 \times 10^{-2} q_p/m_p$$

Inspiration: astrophysical shocks mediated by collective field interactions

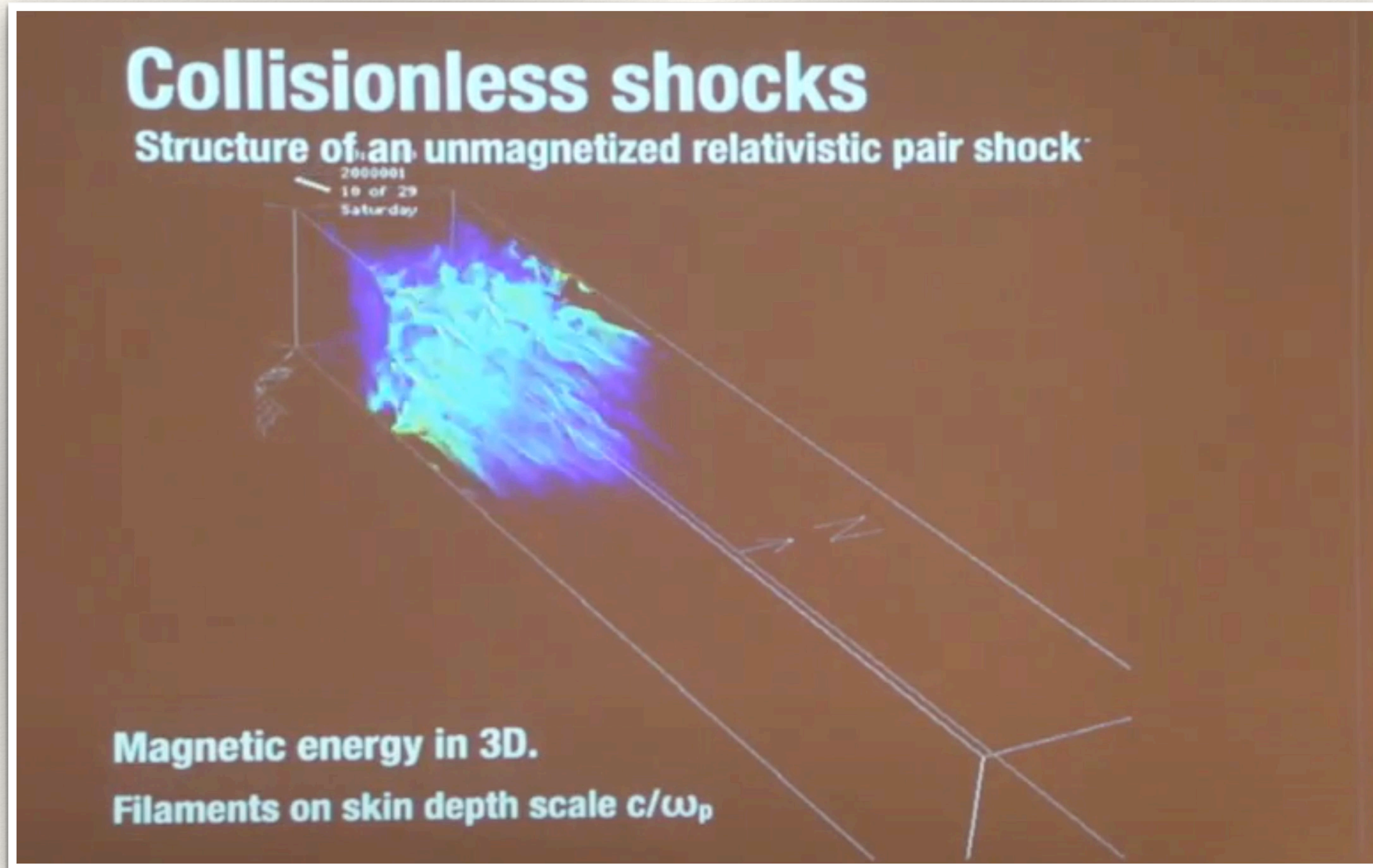
- Mean free paths of Coulomb collisions are huge: 100 pcs for supernova remnants, ~Mpc for galaxy clusters
- Shocks must be mediated without Coulomb collisions, and instead through interactions with collective fields
- In low initial magnetic fields, particles are deflected by self-generated magnetic fields
 - → Filamentation / Weibel instabilities



Weibel instability results from charged counter-streaming matter



3D Weibel instability simulation



Derive instability timescales using Weibel dispersion limits

We examine the linear dispersion^{1,2}

$$0 = D^{\pm} = c^2 k^2 - \omega^2 - \sum_{j=i^+, e^-} \omega_{pj}^2 \left(\frac{\omega}{k v_{T,j}} \right) Z(\xi_j) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{k v_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{v_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$

by expanding the plasma dispersion function $Z(\xi_j)$

where $\xi_j = \frac{\omega + \Omega_{\chi}}{k v_{T,j}}$ indicates whether the instability grows faster or slower than the thermal motion of

particles across the instability scale using

$$Z(\xi_j) = i\sqrt{\pi} \exp(-\xi_j^2) - 2\xi_j + \frac{4}{3}\xi_j^3 - \frac{8}{15}\xi_j^5 + \dots \quad \text{when } |\xi_j| < 1 \quad (\text{warm})$$

and

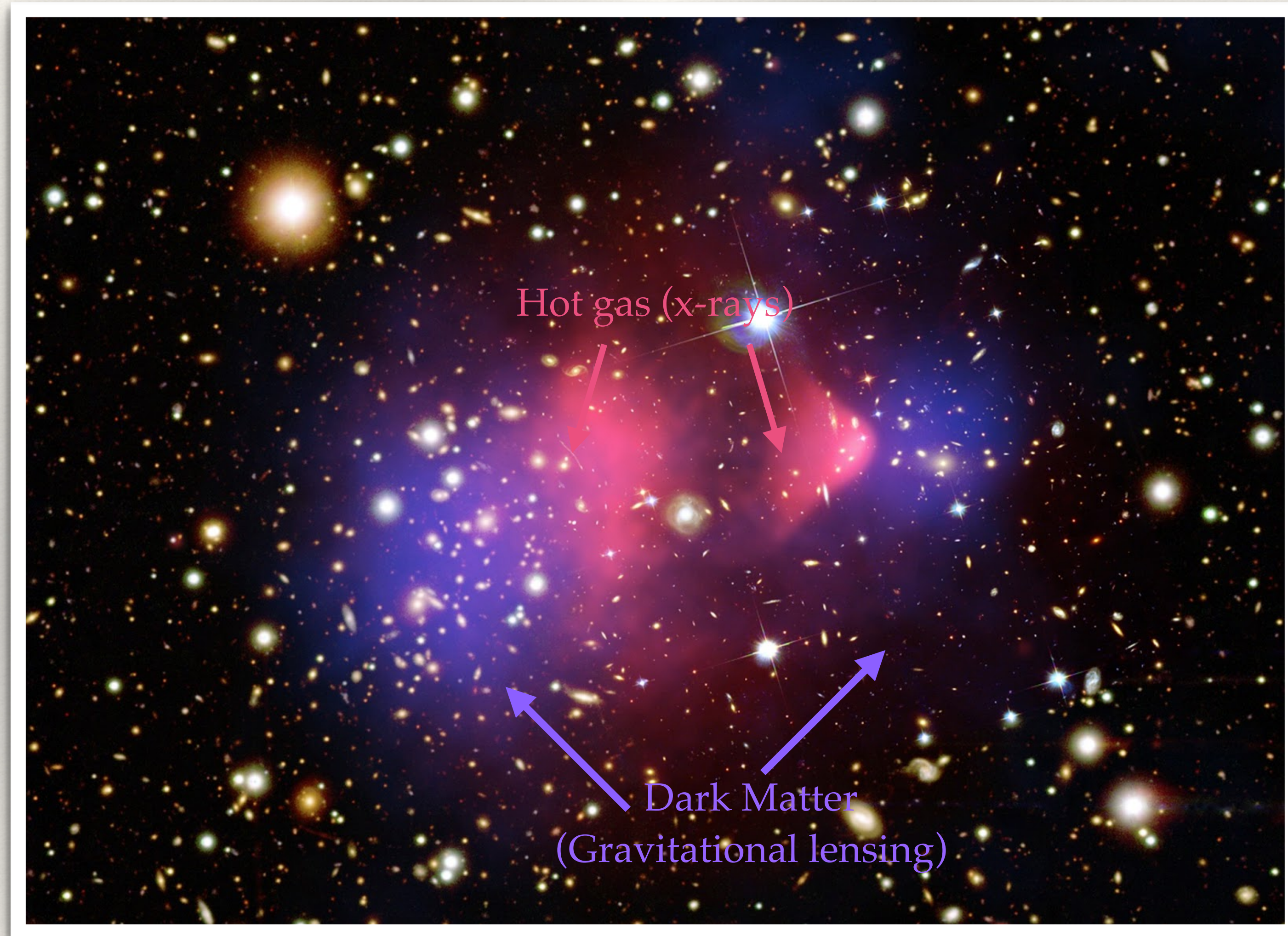
$$Z(\xi_j) = i\sqrt{\pi} \exp(-\xi_j^2) - \frac{1}{\xi_j} - \frac{1}{2\xi_j^3} - \frac{3}{4\xi_j^5} + \dots \quad \text{when } |\xi_j| > 1 \quad (\text{cold})$$

to first order and reduce the dispersion to polynomials and then solve $D^{\pm} = 0$ numerically to confirm analytic solutions

1: Gary 1993

2: Li & Lin [2002.04625]

Strongest dark matter interaction constraints come from the Bullet Cluster

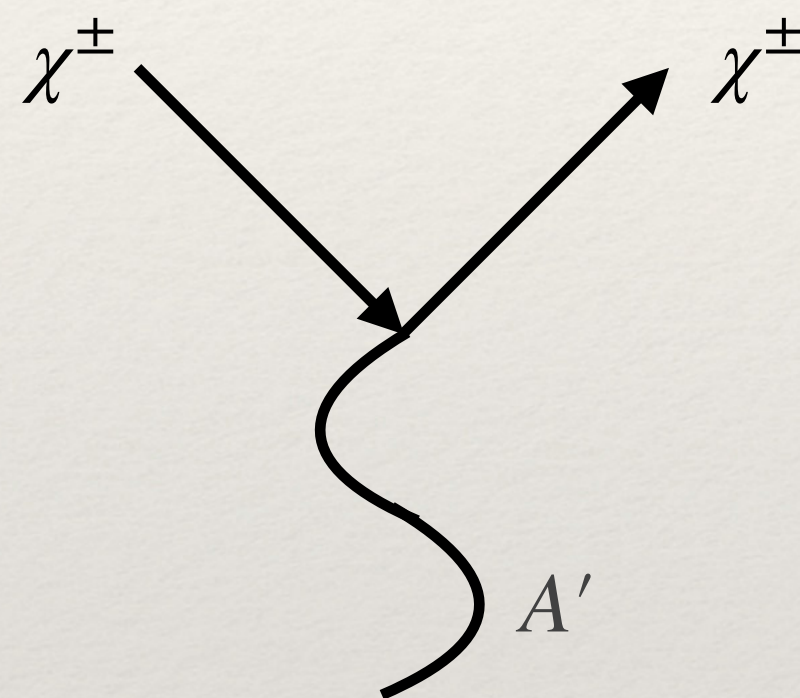


Xray: NASA/CXC/CfA/M.Markevitch, Optical and Lensing: NASA/STSci,
Magellan/U.Arizona/D.Clowe, Lensing map: ESP WFI

Dark matter with a dark charge

In Cruz & McQuinn 2023 we considered **two** charged DM Models

DM with dark charge

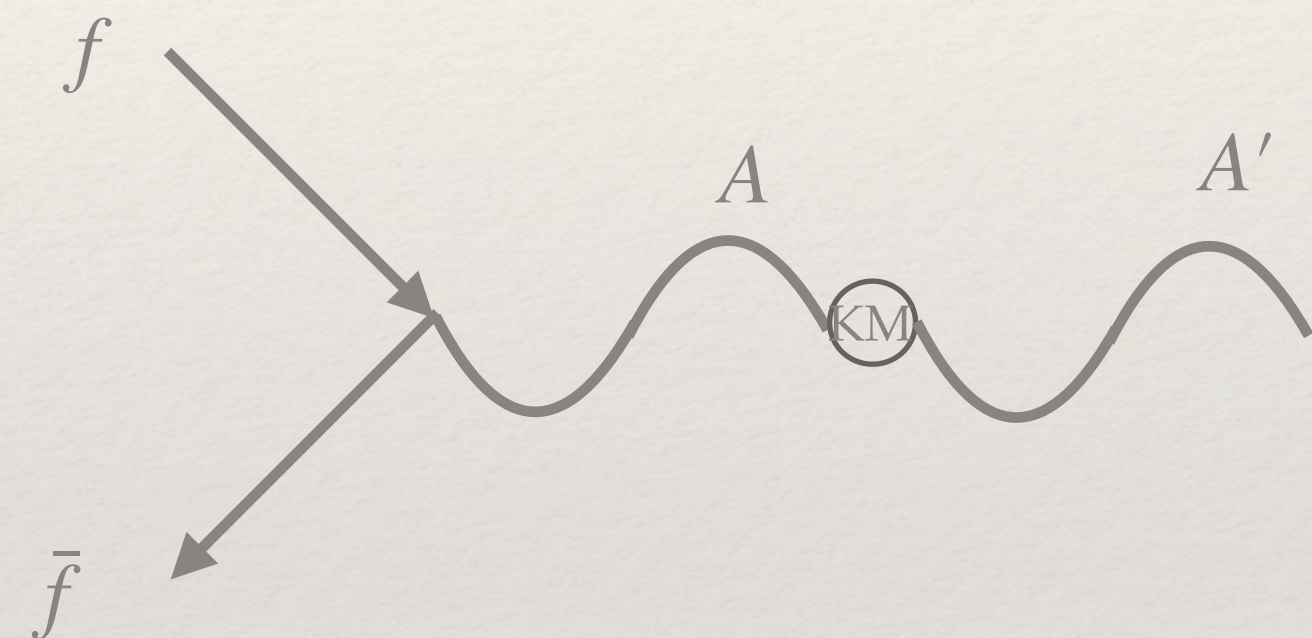


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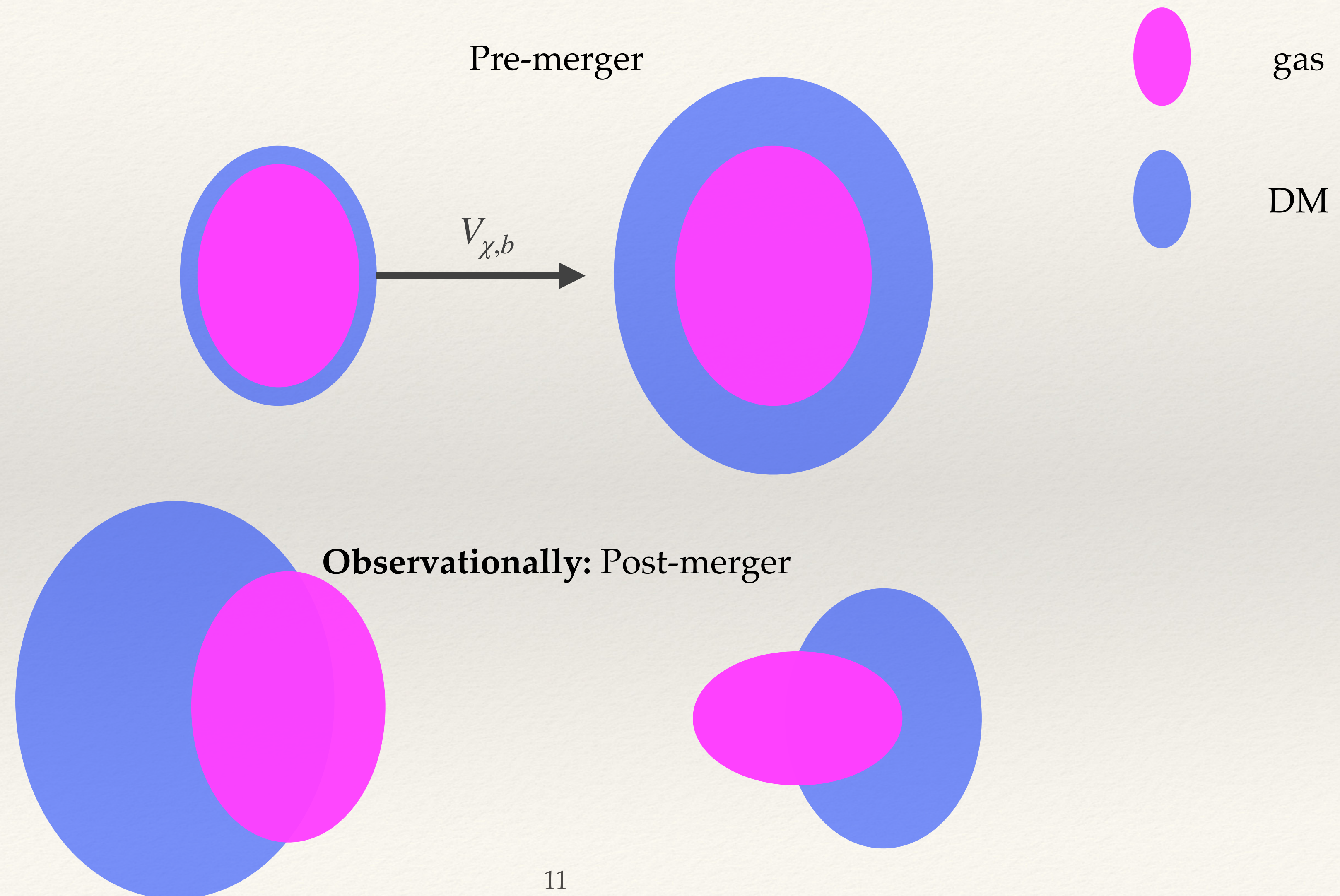


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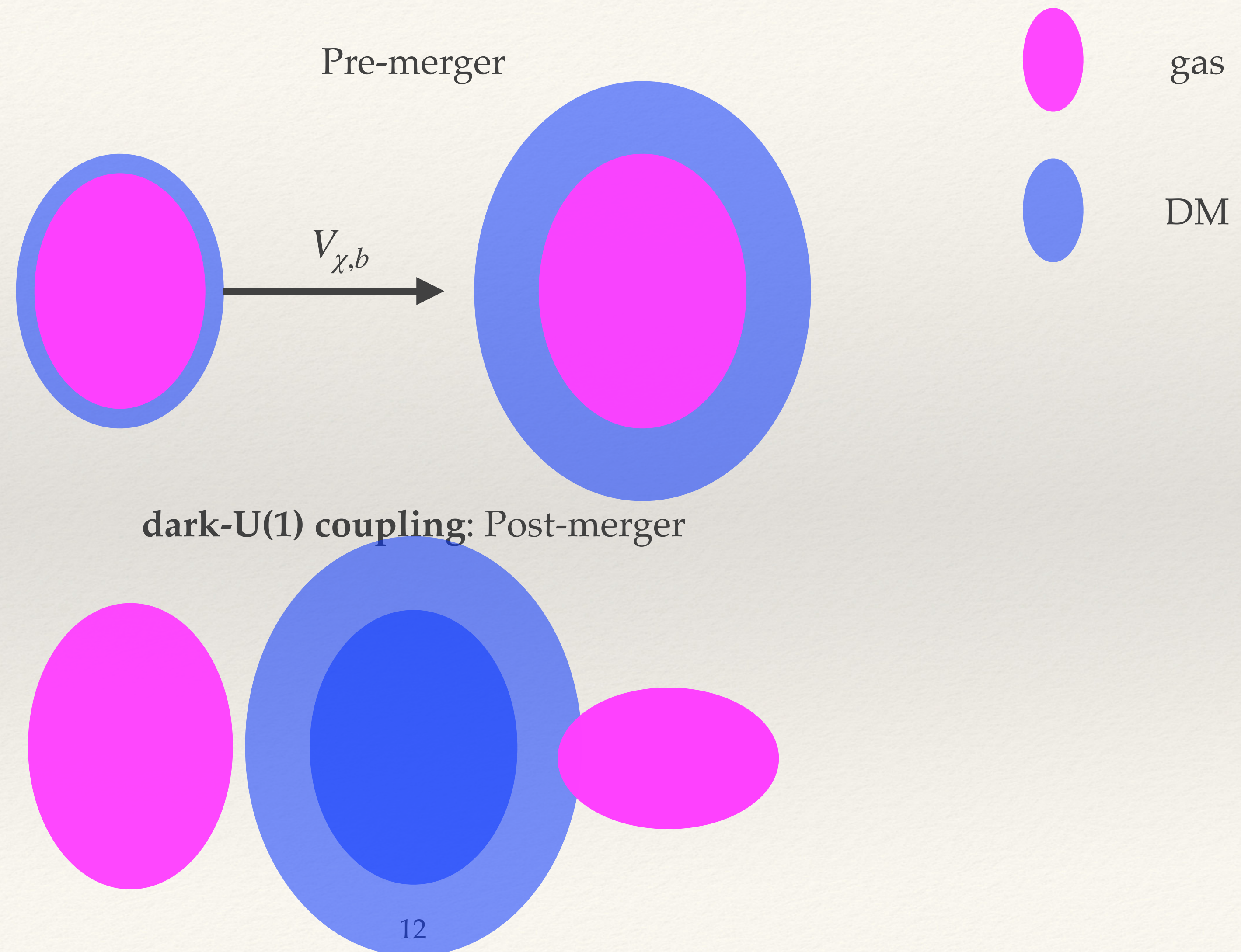
$$m_\chi, q_\chi$$

DM-SM and DM-DM interactions allowed

In Bullet Cluster, gas and DM stream through each other



Dark charge can prohibit DM-DM streaming



Weibel instability in the dark-charged Bullet Cluster

$$0 = D^\pm = c^2 k^2 - \omega^2 - \sum_{b=i^+, e^-} \omega_{pb}^2 \left(\frac{\omega}{k \sigma_{T,b}} \right) Z(\xi_b) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{k \sigma_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{\sigma_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$

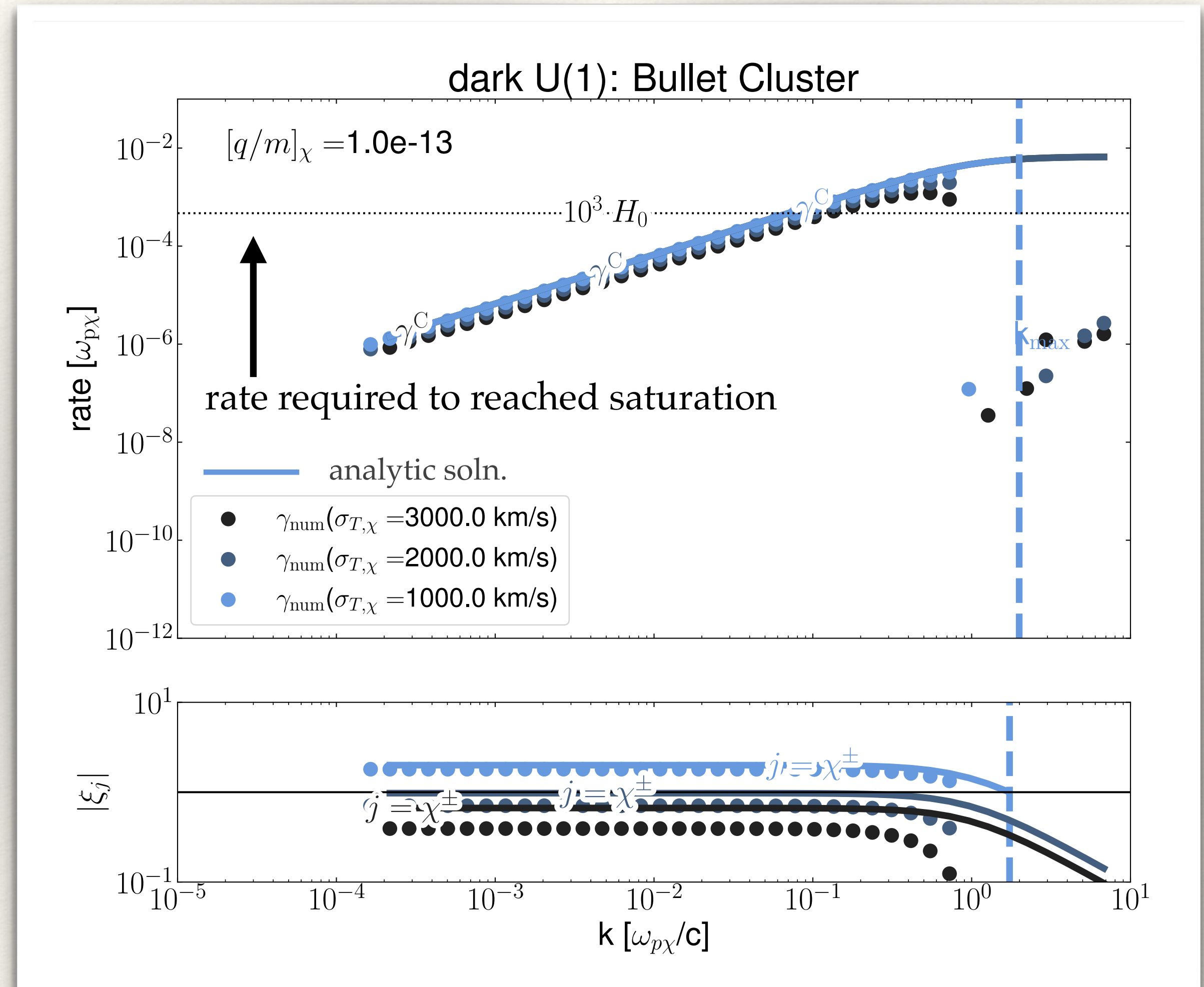
$$\text{where } \xi_j = \frac{\omega + \Omega_j}{k \sigma_{T,j}}$$

purely dark plasma in the cold limit

$$\text{when } |\xi_\chi| > 1 \quad Z(\xi_j) \approx 1/\xi_j + \mathcal{O}(1/\xi_j)$$

$$0 = c^2 k^2 - \omega^2 + \omega_{pp}^2 + \omega_{pe}^2 + \omega_{p\chi}^2 + f_\chi \omega_{p\chi}^2 \left(\frac{V_{b\chi} k}{\omega} \right)^2$$

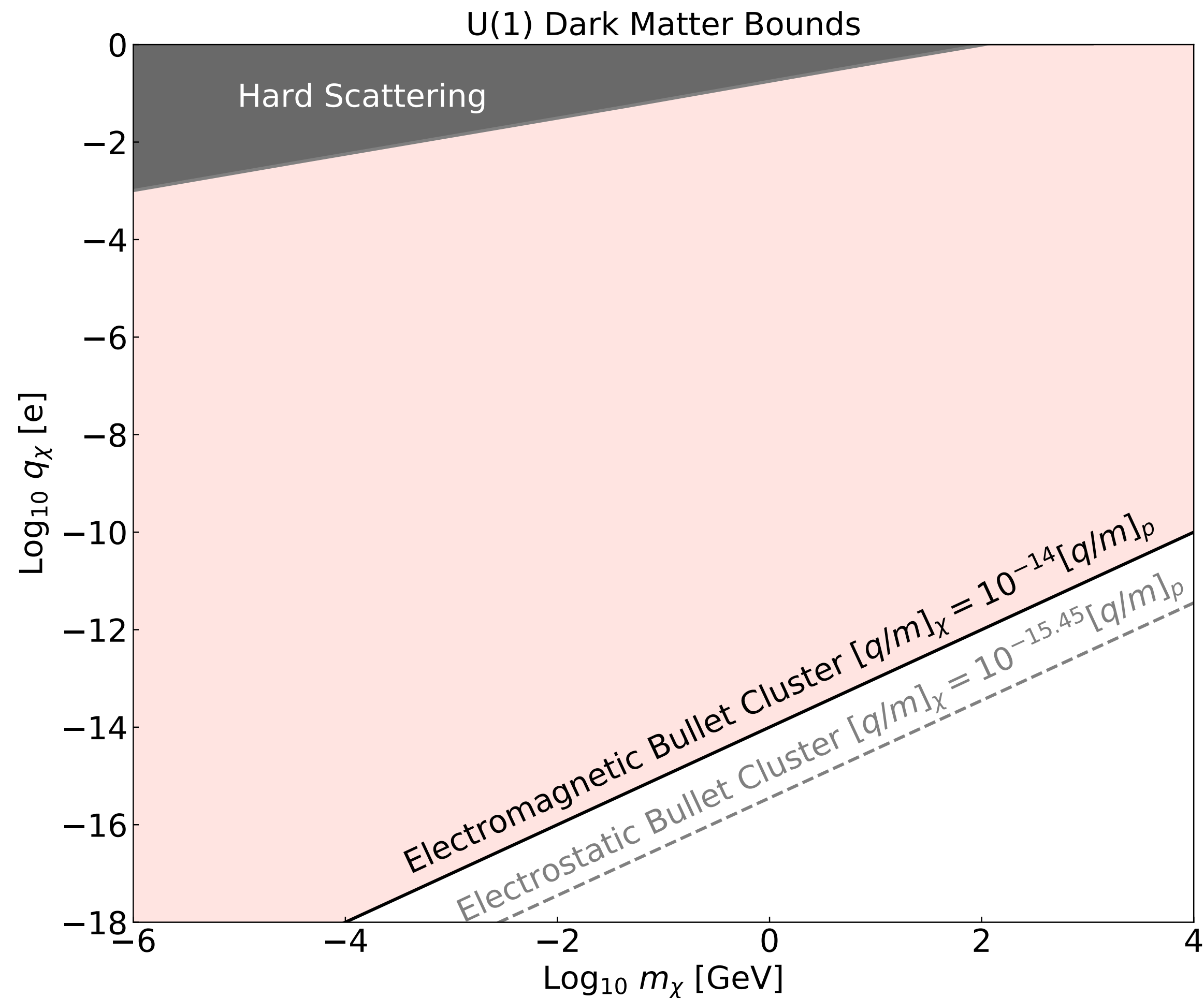
$$\boxed{\gamma \approx \frac{f_\chi^{1/2} \omega_{p\chi} V_{b\chi} k}{(\omega_{pj}^2 + c^2 k^2)^{1/2}}} \quad \text{analytic soln.}$$



purely imaginary root is the classical Weibel solution (Weibel 1959)

Cruz & McQuinn [2202.12464]

Dark-charged DM constraints are stronger than two-body considerations



Cruz & McQuinn [2202.12464]

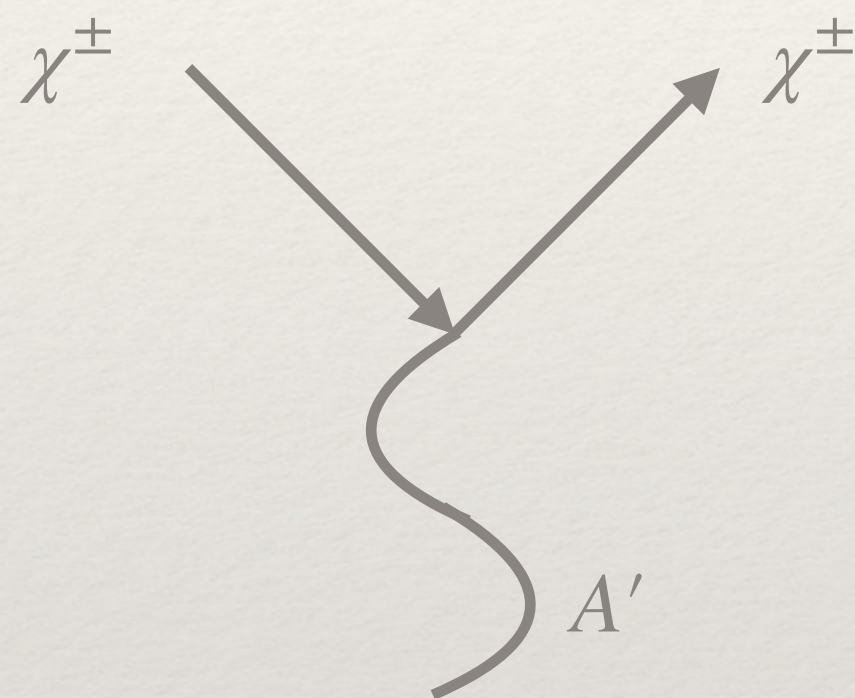
The electrostatic instability growth rate is faster by $\sim c / V_{str}$

However, electrostatic instabilities do not grow once $\sigma_{T,\chi} > 0.4 V_{str}$ — which is problematic in the Bullet Cluster case

Milli-charged dark matter

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DM with dark charge

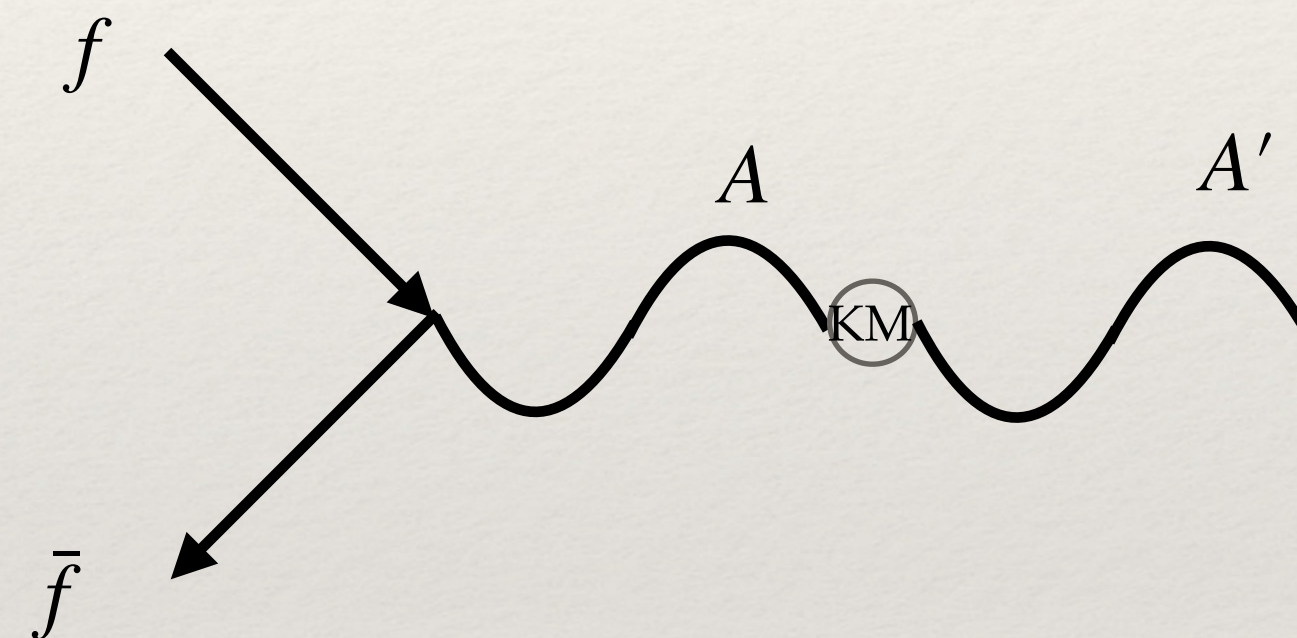


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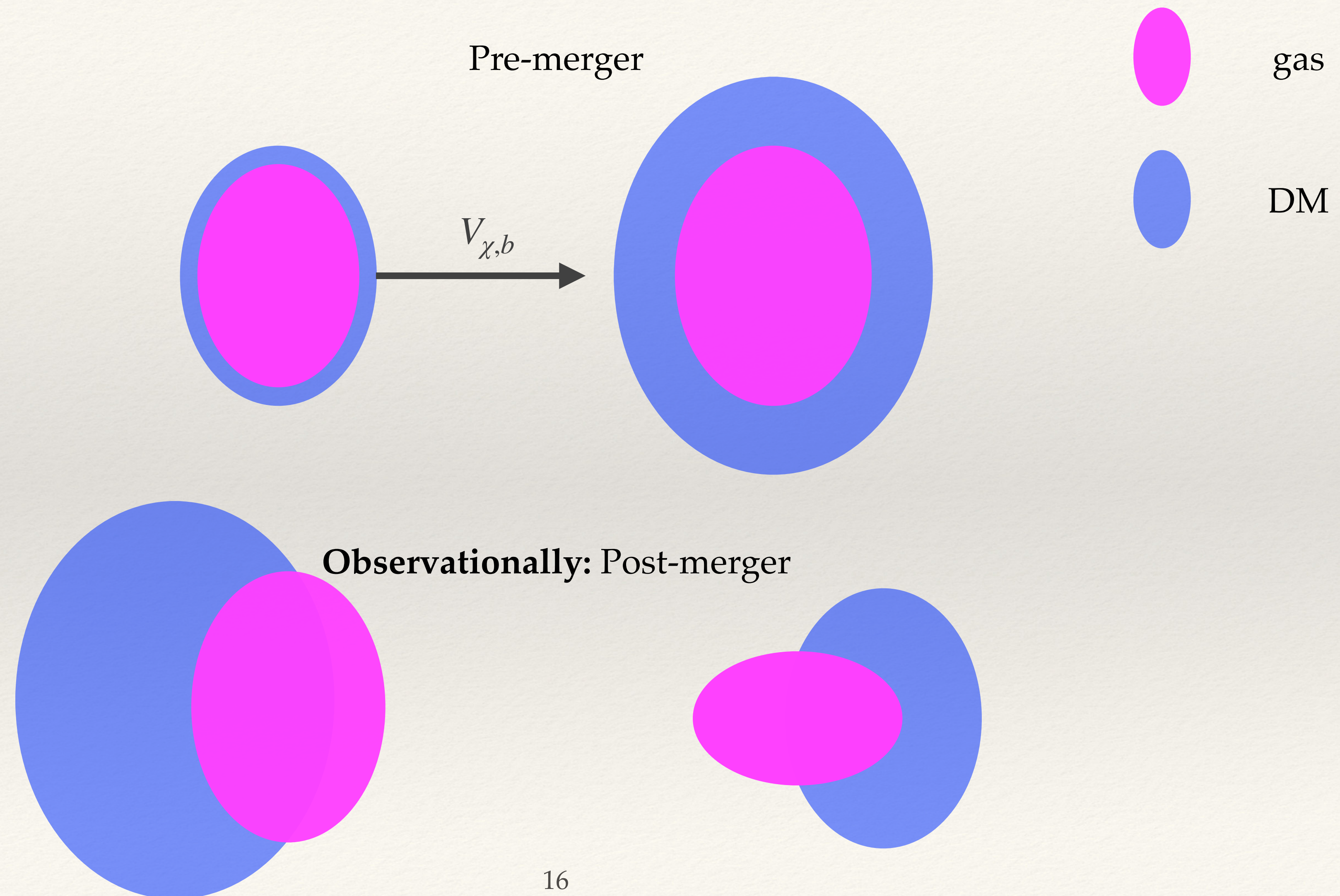


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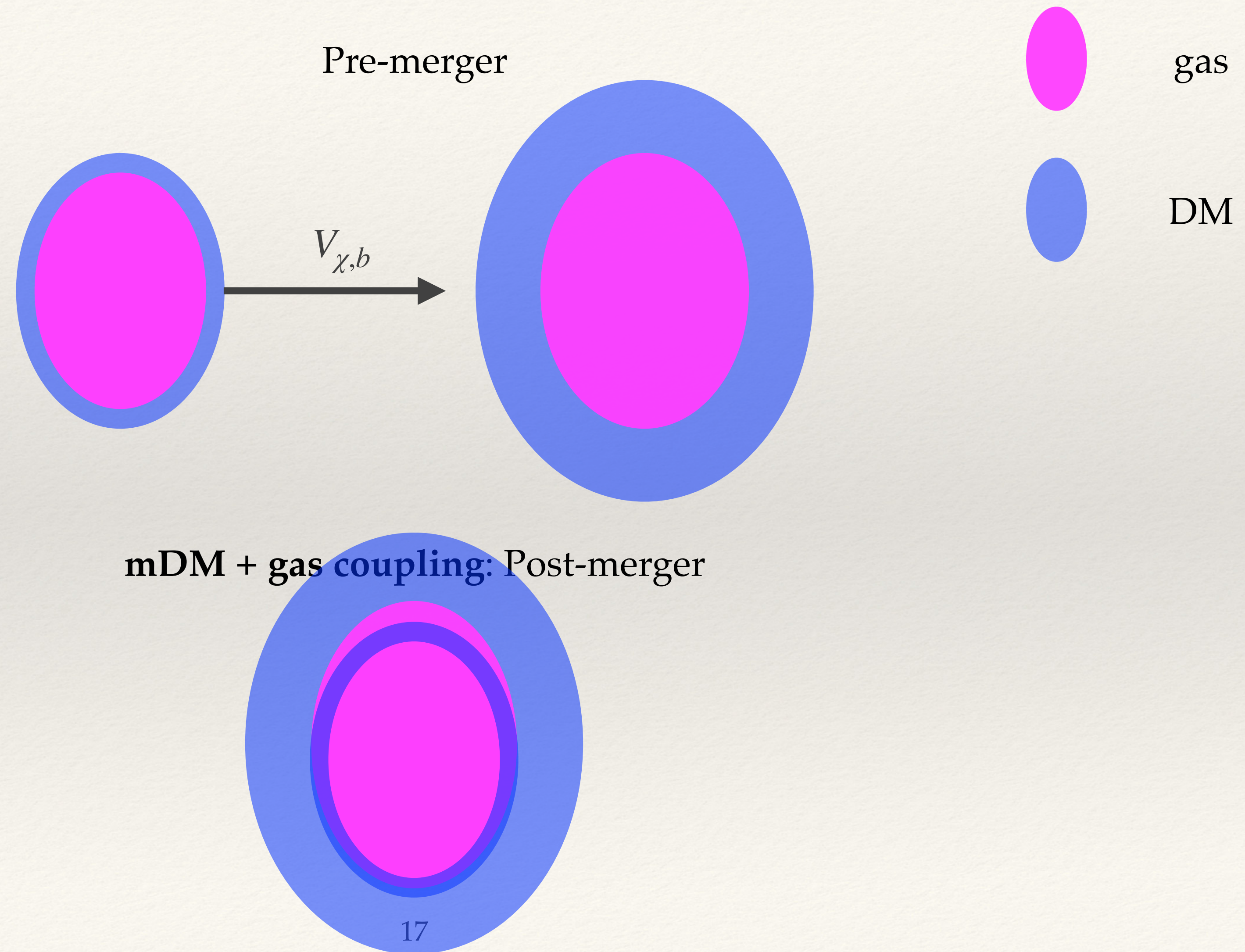
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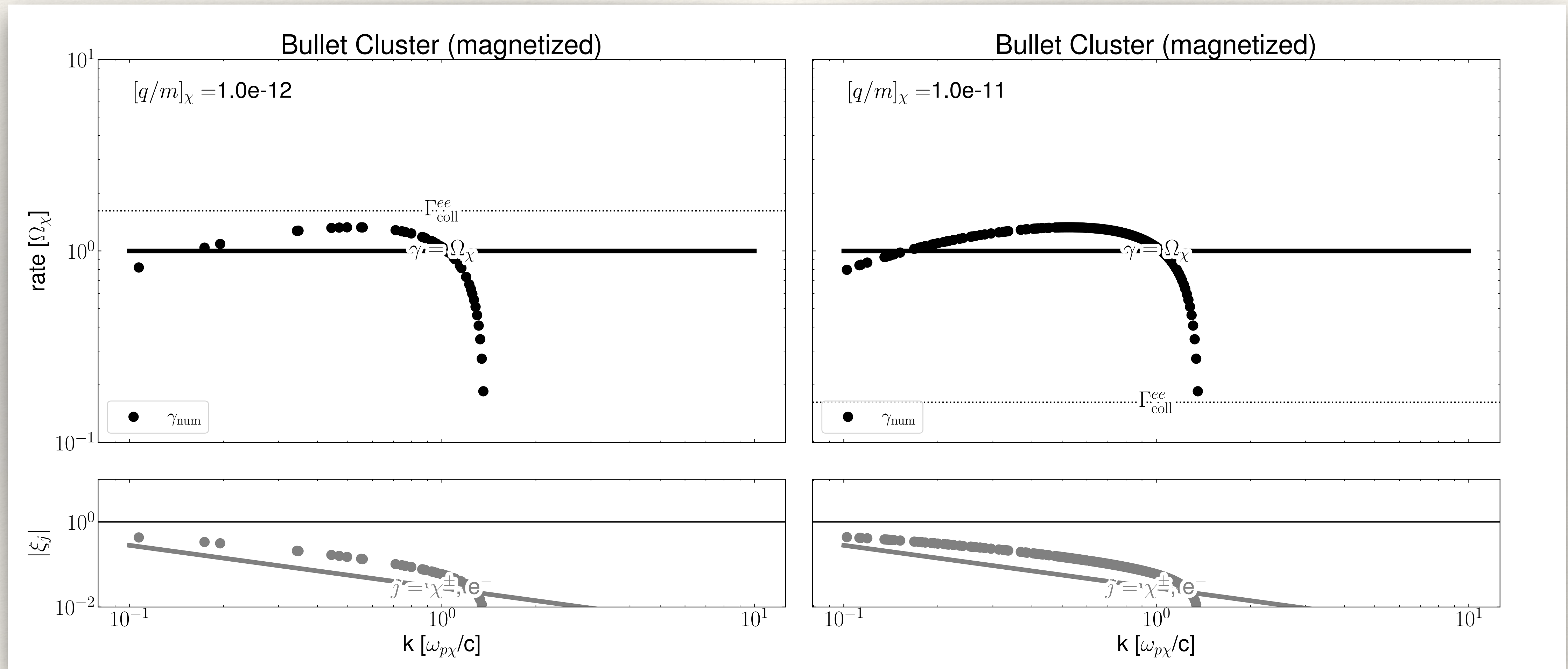


mDM can prohibit cluster streaming

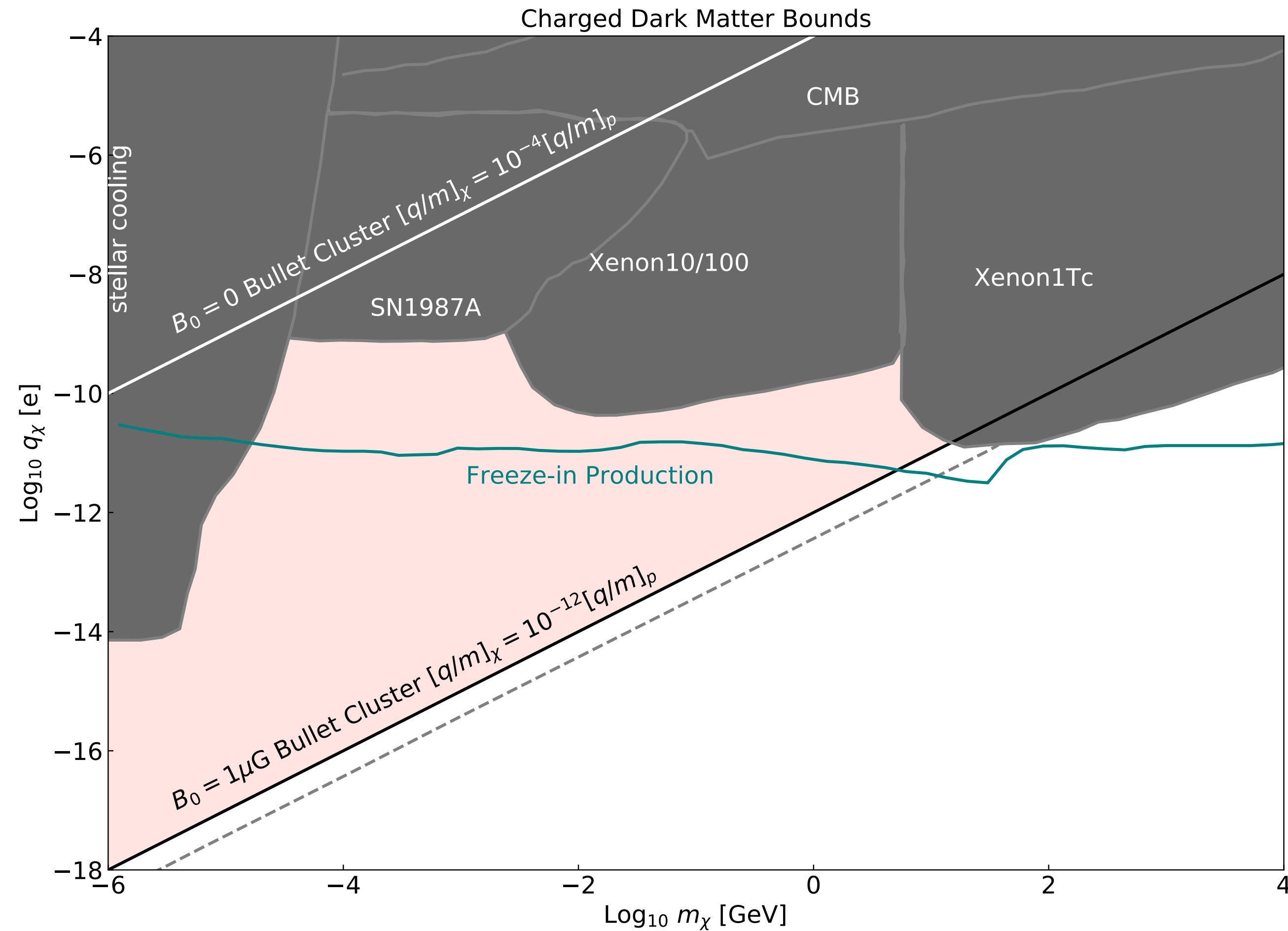


Magnetized mDM Bullet Cluster solutions

Solving $D^\pm = 0$ numerically with starting guess $\omega_0 = \Omega_\chi$



mDM leads to strict constraints below GeV mass



Cruz & McQuinn [2202.12464]

Darkly charged dark matter result summary

1. If dark matter is milli-charged or darkly charged, collective plasma processes may dominate momentum exchange over direct, short range particle collisions
2. When a magnetic field is added consistent with cluster observations, Weibel instabilities result in the mDM constraint $[q/m]_\chi \gtrsim 10^{-12} [q/m]_p$, **~ 10 orders of magnitude lower than short range particle collision constraints**
3. The constraints are even stronger in the case of a dark-charge, ruling out $[q/m]_\chi \gtrsim 10^{-14} [q/m]_p$ in the Bullet Cluster system, **~ 12 orders of magnitude lower than short range particle collision constraints¹**

Thank you!

Questions?

Weibel Instability: Underlying Physics

unperturbed distribution function $f_{s,0}(\mathbf{v}) = \frac{n_s}{(2\pi)^{3/2}v_{T,s}^3} \exp\left[-\frac{(\mathbf{v} - \mathbf{u}_s)^2}{v_{T,s}^2}\right]$

Boltzmann $\frac{\partial f_{s,1}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{s,1}}{\partial \mathbf{r}} + \frac{q_s}{m_s} \left(\frac{\mathbf{v}}{c} \times \mathbf{B}_0 \right) \cdot \frac{\partial f_{s,1}}{\partial \mathbf{v}} = -\frac{q_s}{m_s} \left(\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1 \right) \cdot \frac{\partial f_{s,0}}{\partial \mathbf{v}}$

Maxwell $\nabla \cdot \mathbf{E} = \sum_j 4\pi \int f_j(\mathbf{x}, \mathbf{v}, t) d^3\mathbf{v}$

Non-trivial solution if the linear dispersion is zero

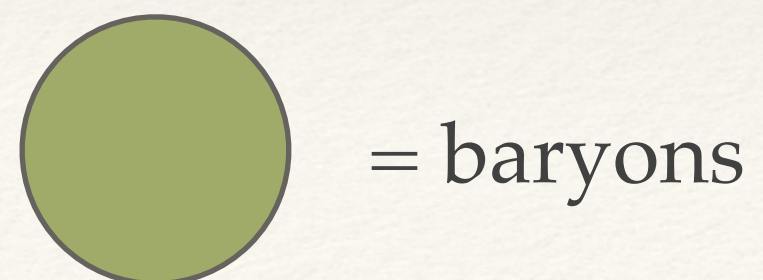
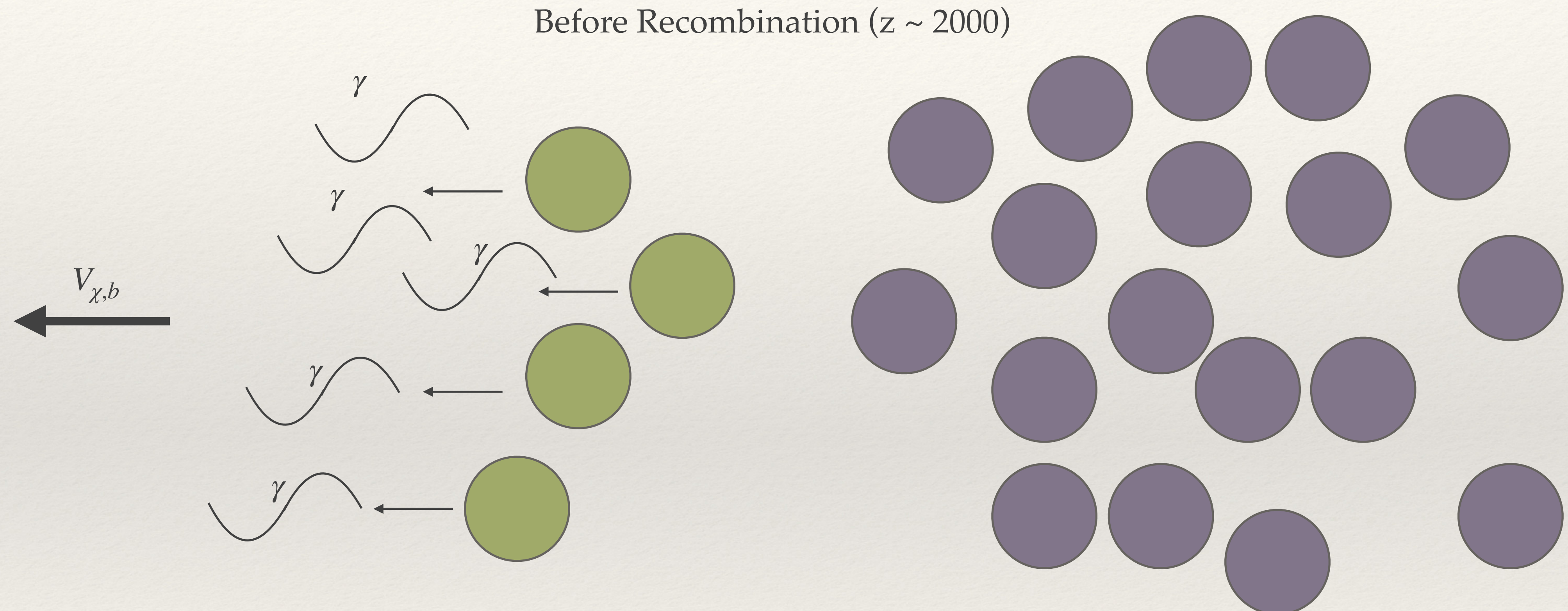
$$0 = D(k, \omega, \xi_{j,s}, V_{b,\chi}, v_{T,j,s}, \omega_{p,j,s}) = c^2 k^2 - \omega^2 - \sum_{j=i^+, e^-} \omega_{pj}^2 \left(\frac{\omega}{kv_{T,j}} \right) Z(\xi_j) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{kv_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{v_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$

and $\omega = i\gamma$, i.e. is purely imaginary

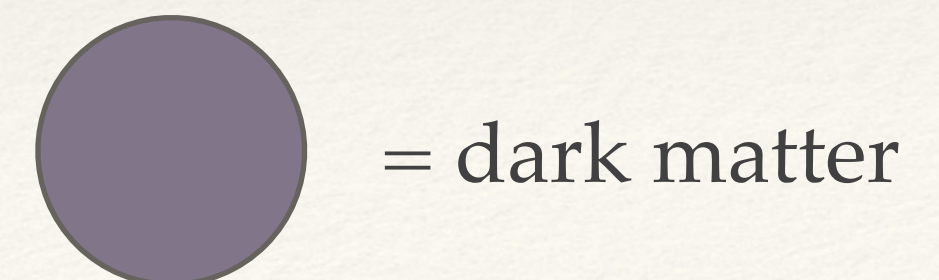
(Gary 1993, Li&Lin 2020)

where $Z(\xi_j) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x - \xi_j} dx$ and $\xi_j = \frac{\omega + \Omega_j}{kv_{T,j}}; \Omega_j = \frac{q_j B_0}{m_j c}; \xi_s = \frac{\omega + \Omega_s}{kv_{T,\chi}}; \Omega_s = \frac{q_s B_0}{m_s c} \equiv \frac{RB_0}{c}$

mDM Streaming in the Early Universe



Tseliakhovich & Hirata 2010



Early Universe

warm/cold limit

$$|\xi_e| < 1 \quad \text{and} \quad |\xi_\chi| > 1$$

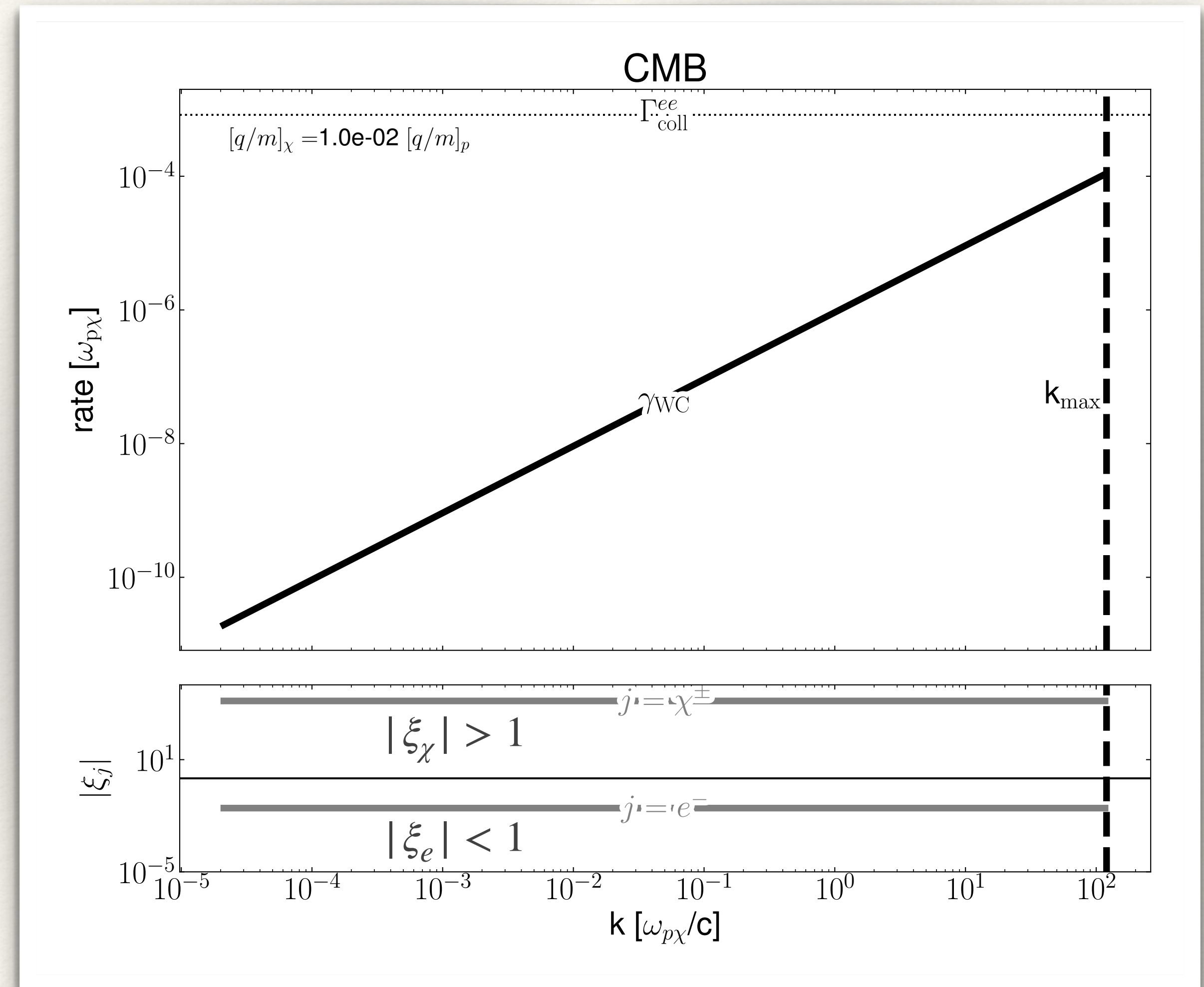
$$0 = D^\pm = c^2 k^2 - \omega^2 - \sum_{j=i^+, e^-} \omega_{pj}^2 \left(\frac{\omega}{k v_{T,j}} \right) Z(\xi_j) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{k v_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{v_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$



$$c^2 k^2 - \omega^2 - i\sqrt{\pi} \omega_{pj}^2 \left(\frac{\omega}{k \sigma_{T,j}} \right) + f_\chi \omega_{p\chi}^2 \left(\frac{V_{b\chi} k}{\omega} \right)^2 = 0$$



$$\gamma^{\text{WC}} \approx \left(\frac{f_\chi}{\sqrt{\pi}} \frac{\omega_{p\chi}^2}{\omega_{pj}^2} (k \sigma_{T,j}) (k V_{b\chi})^2 \right)^{1/3} \quad \text{analytic soln.}$$



Unmagnetized mDM Bullet Cluster solutions

warm limit

when $|\xi_e| < 1$ and $|\xi_\chi| < 1$ $Z(\xi_j) = i\sqrt{\pi} + \mathcal{O}(\xi_j)$

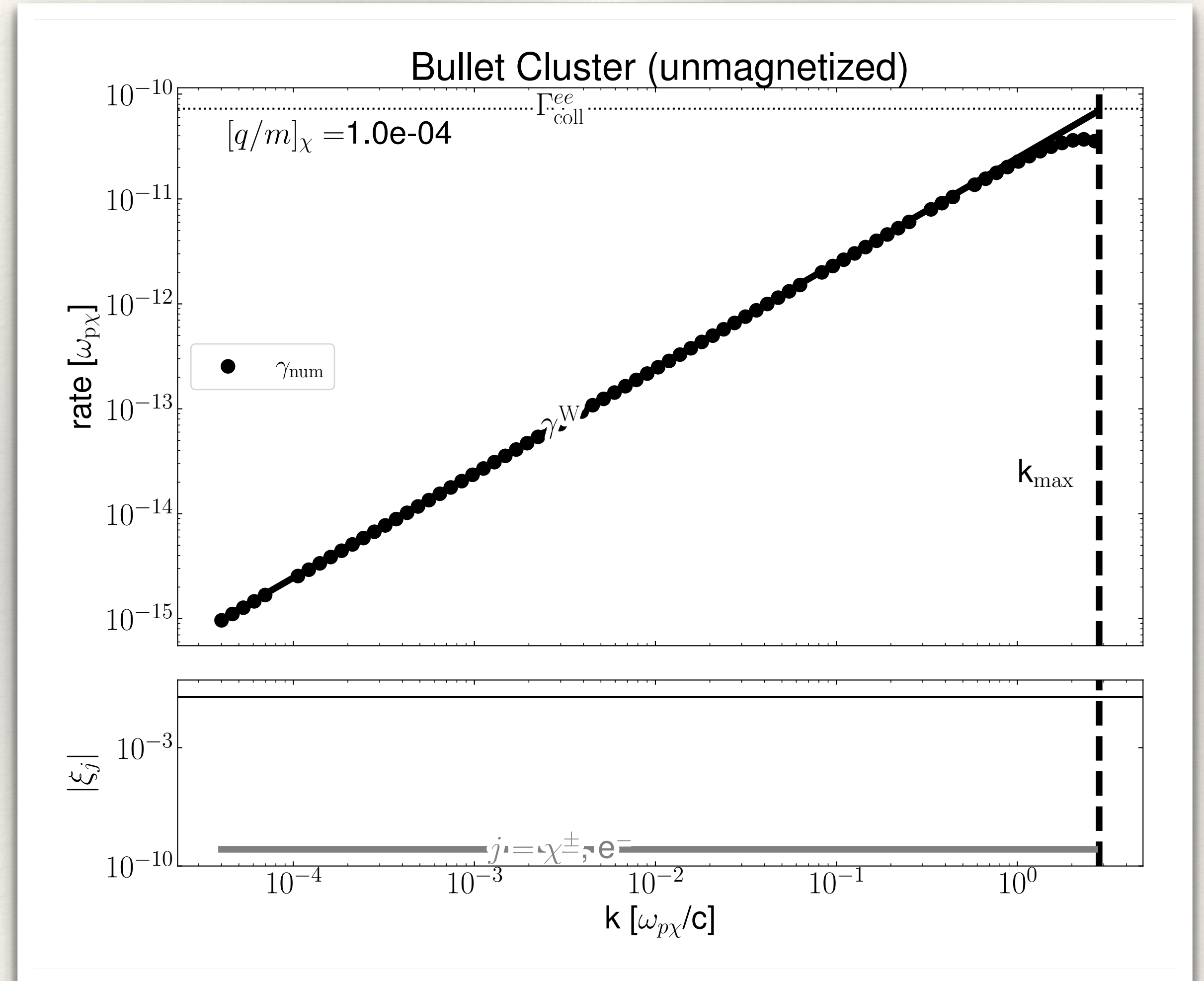
$$0 = D^\pm = c^2 k^2 - \omega^2 - \sum_{j=i^+, e^-} \omega_{pj}^2 \left(\frac{\omega}{k v_{T,j}} \right) Z(\xi_j) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{k v_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{v_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$

$$0 = c^2 k^2 - \omega^2 - i\sqrt{\pi} \omega_{pj}^2 \left(\frac{\omega}{k \sigma_{T,j}} \right) - f_\chi \omega_{p\chi}^2 \left(\frac{V_{b\chi}}{\sigma_{T,\chi}} \right)^2$$

$$\omega = -\frac{i\sqrt{\pi}}{2} \left(\frac{\omega_{pj}^2}{k \sigma_{T,j}} \right) \pm \frac{1}{2} \sqrt{-\pi \left(\frac{\omega_{pj}^2}{k \sigma_{T,j}} \right)^2 - 4 \left(\omega_{p\chi}^2 f_\chi \left(\frac{V_{b\chi}}{\sigma_{T,\chi}} \right)^2 - c^2 k^2 \right)}$$

keeping Weibel mode ($\text{Re}[w] \sim 0$ and $\text{Im}[w] > 0$)

$$\gamma^W = \frac{1}{\sqrt{\pi}} \left(\frac{k \sigma_{T,j}}{\omega_{pj}^2} \right) \omega_{p\chi}^2 f_\chi \left(\frac{V_{b\chi}}{\sigma_{T,\chi}} \right)^2 \quad \text{analytic soln.}$$



Dark U(1) Bounds

From the cold limit, we derive the maximum growth rate possible by setting $\xi = 1$

$$\frac{\gamma_{\max}^{\text{C}}(k = k_{\max})}{k_{\max} \sigma_{T,j}} = 1 \quad \longrightarrow \quad ck_{\max} \approx \left(\left(\frac{f_{\chi}^{1/2} \omega_{\text{p}\chi} V_{b\chi}}{\sigma_{T,j}} \right)^2 - \omega_{\text{p},j}^2 \right)^{1/2}$$



$$\gamma_{\max}^{\text{C}} \approx \omega_{\text{p}\chi} f_{\chi}^{1/2} \frac{V_{b\chi}}{c}$$

Electromagnetic

Lasenby 2021 showed

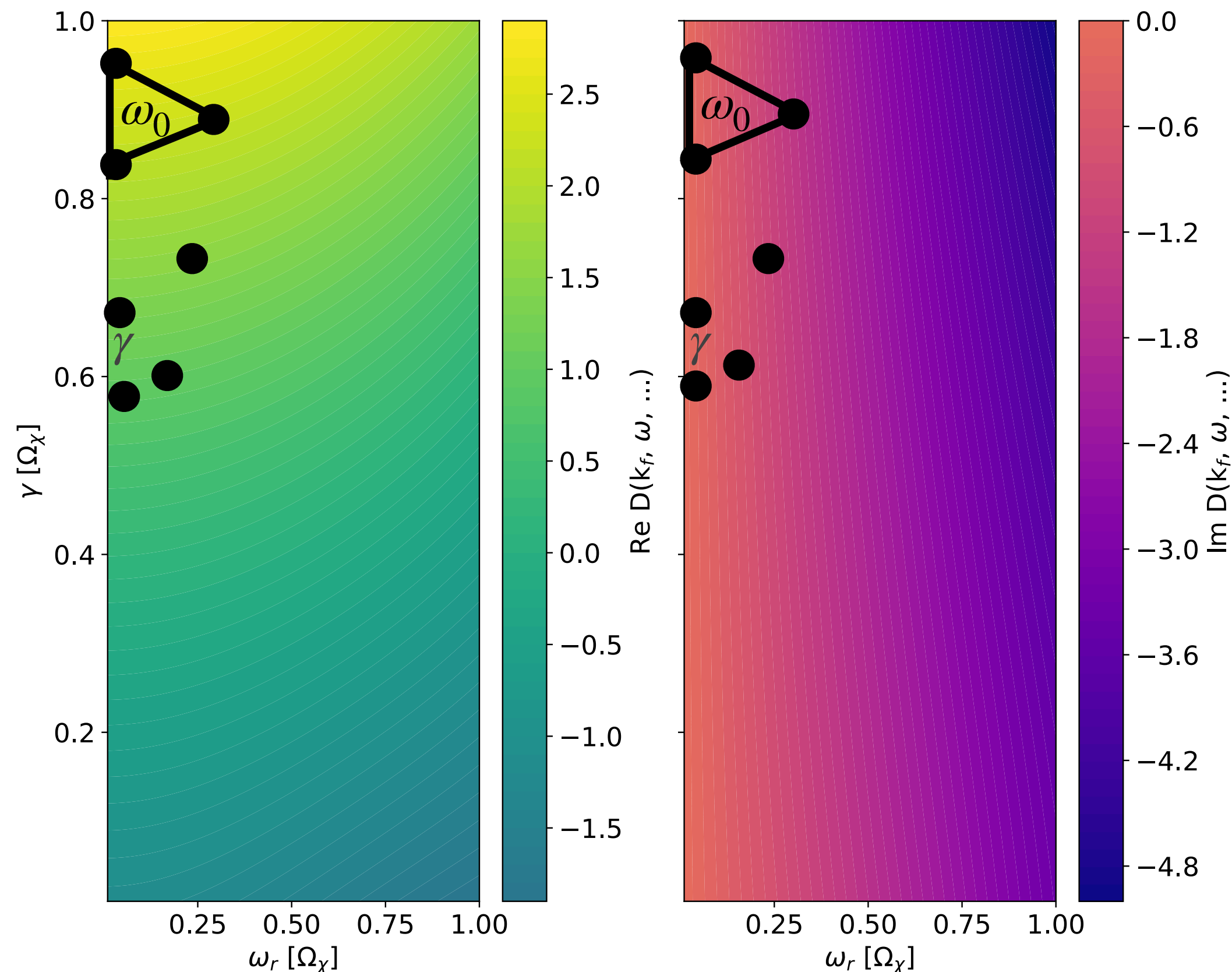
$$\gamma_{\max} \sim \omega_{\text{p}\chi}$$

Electrostatic

which grows faster by a factor of 10^2 in the Bullet Cluster case

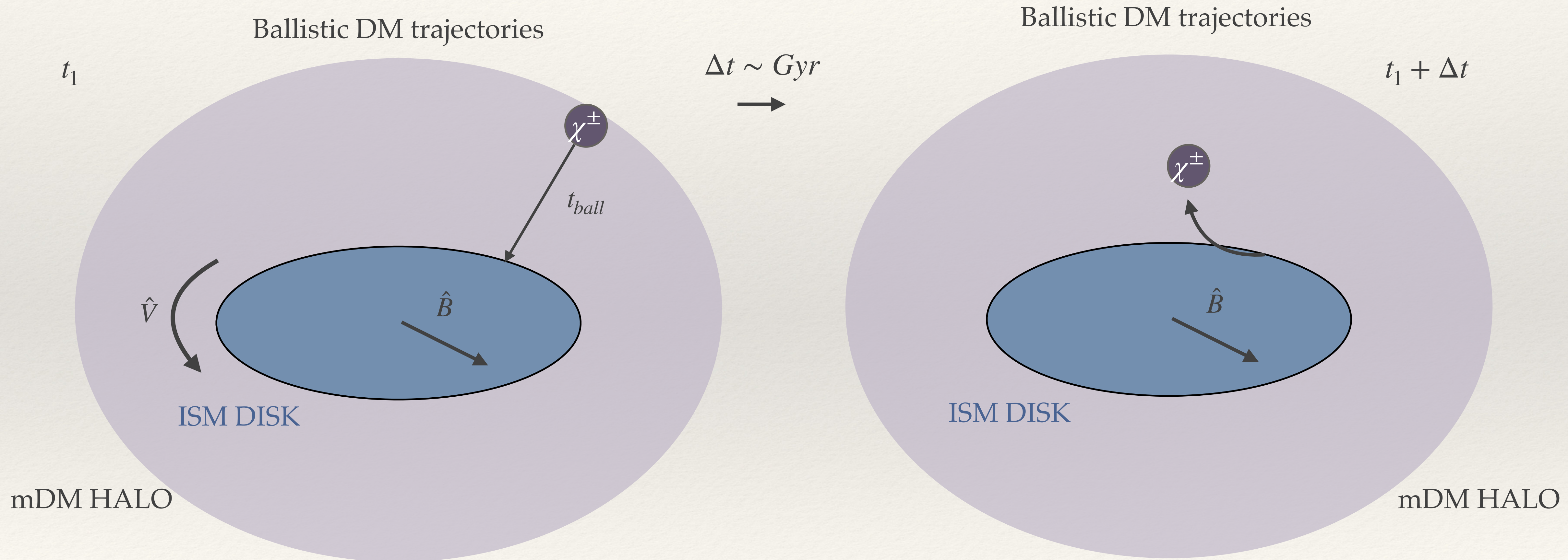
Nelder-Mead Numerical Method

$$0 = D(k, \omega, \xi_{j,s}, V_{b,\chi}, v_{T,j,s}, \omega_{p,j,s}) = c^2 k^2 - \omega^2 - \sum_{j=i^+, e^-} \omega_{pj}^2 \left(\frac{\omega}{k v_{T,j}} \right) Z(\xi_j) - \sum_{s=\chi^+, \chi^-} \omega_{ps}^2 \left[\left(\frac{\omega}{k v_{T,s}} \right) Z(\xi_s) + \left(\frac{V_{b\chi}}{v_{T,\chi}} \right)^2 (1 + \xi_s Z(\xi_s)) \right]$$



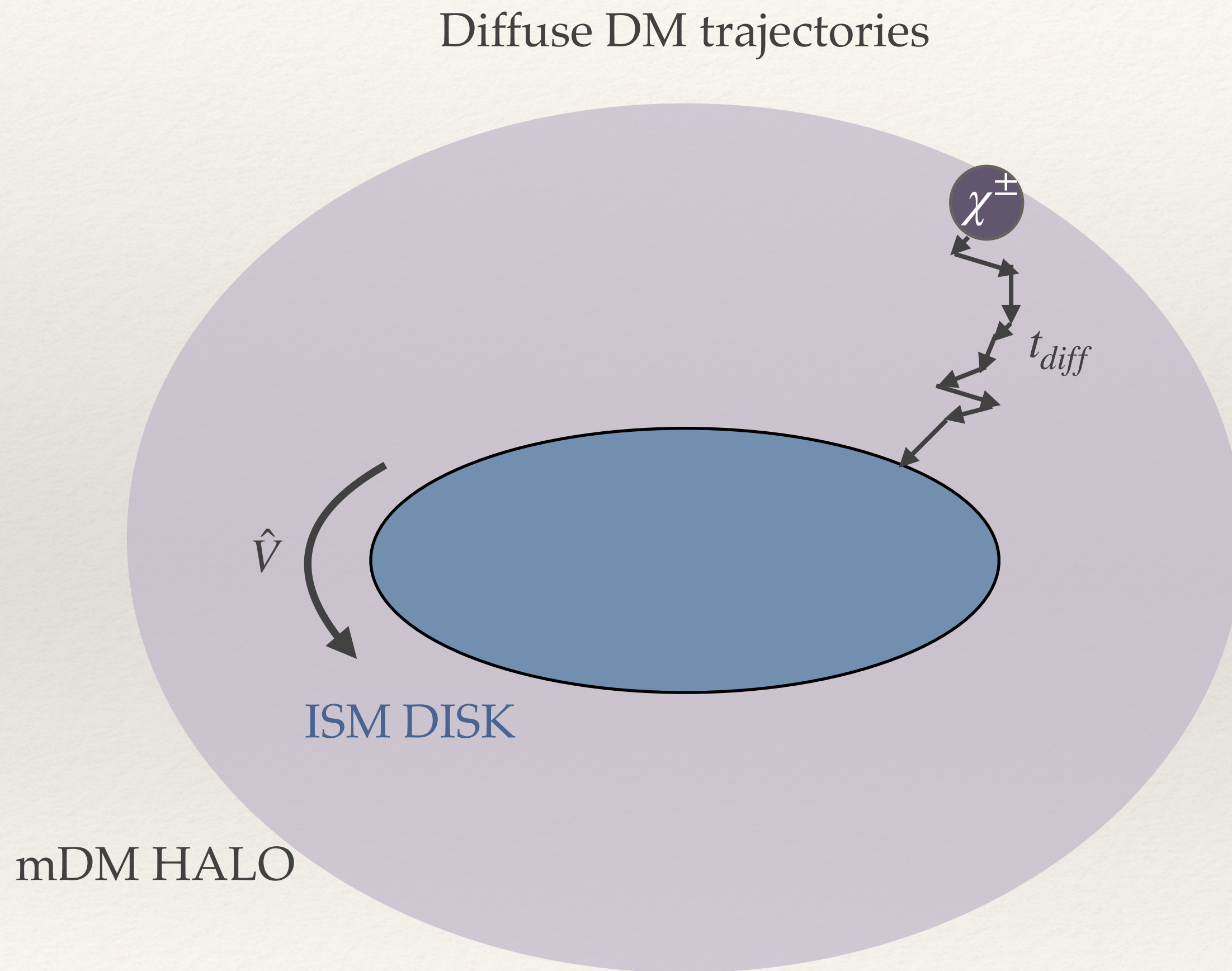
- Since the linear dispersion is a complex function, we look to find purely imaginary frequency solutions of the square modulus of the linear dispersion
- We use the Nelder-Mead algorithm, a simplex search algorithm for multidimensional unconstrained optimization of a given non-linear function (in our case $|D|^2 : \mathbb{R}^2 \rightarrow \mathbb{R}$)
- A simplex in \mathbb{R}^2 is defined as the smallest subset of \mathbb{R}^2 that contains each whole line segment joining any two points of 3 vertices $\in \mathbb{R}^2$ (i.e a simplex is a triangle in \mathbb{R}^2)
- In our application, the simplex is initialized around an initial frequency ω_0 motivated by our analytic analysis
- Then at each iteration of the algorithm, the vertices of the simplex are transformed
- This is repeated until the simplex is sufficiently small or the number of iterations surpasses some set maximum value

Revisiting ISM Spin Down Constraints



mDM is deflected, and angular momentum is exchanged causing the ISM disk to spin down

Revisiting ISM Spin Down Constraints



- The Weibel instabilities excites small scale magnetic inhomogeneities, below the Larmor radius of dark matter
- The later nonlinear stages of the instability lead to turbulence and a very inhomogeneous magnetic field
- The dark matter then DIFFUSES through the resulting magnetic inhomogeneities, rather than traveling ballistically as is assumed in Stebbins 2019.
- This diffusion alters the DM ability to reach the ISM disk and thus to exchange momentum and cause spin down within the lifetime (~ 10 Gyrs) of Milky Way