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POLLICA PHYSICS CENTRE

Self-Interacting Dark Matter:

Bound State within SIMPs

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(2307.XXXXX)

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I. from SIDM to SIMP

We only see dark matter from the sky.

Almost everything fits at large-scales, while several puzzles at **small-scale** remain.

1. **DM cores** preferred by observations in many (sub-)halos [Moore 1994; Burkert 1995, Newman et al 2013, …]

   (core/cusp problem)

2. **Non-observation of massive sub-halos** which should host brightest dwarfs [M.Boylan-Kolchin et al. 2011, 2012, Ferrero et al. 2011 ….]

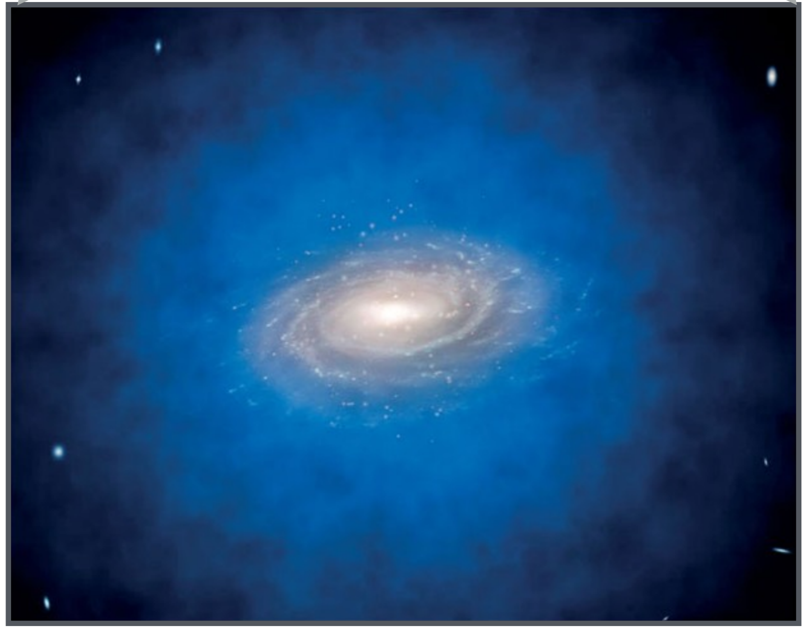
   (too-big-to-fail problem)

3. Some **globular/star clusters** expected to be destroyed, or sink to halo centre for **cuspy** profile [J. Binney & S.Tremaine 2008, F. Contenta et al. 2017, P. Boldrini et al. 2018, …]

   (GC timing problem)

Mass deficit in small halos?

kpc - Mpc



4. Diversity of galaxies, ….

Possible explanations:

Systematic uncertainties?

Baryonic effects (by bursty star formation)?

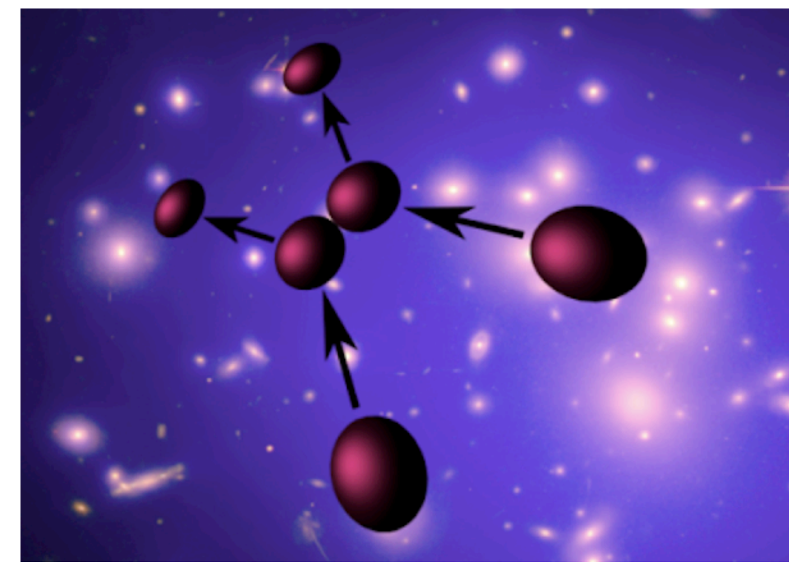
$10^{53} - 10^{55}$ erg

Self-interacting dark matter (SIDM)**?**

Observational evidence for self-interacting cold dark matter

D.N. Spergel and P J. Steinhardt [astro-ph/9909386]

Infalling dark matter is scattered before reaching the center of the galaxy so that the orbit distribution is isotropic rather than radial. These collisions increase the entropy of the dark matter phase space distribution and lead to a dark matter halo profile with a shallower density profile.



Strong DM self-scattering at dwarfs ➡ inner halo **DM self-thermalization**

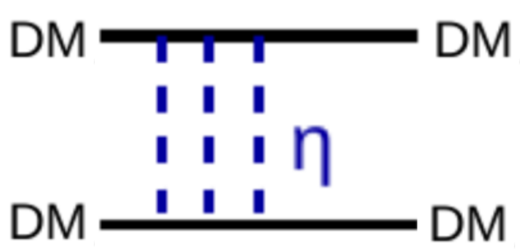
(heating up the halo center)

$$\frac{\sigma_{\rm SI}}{m_{\rm DM}} = 0.2 - 20\,{\rm cm}^2/{\rm g}$$

O(1) scatters per DM particle

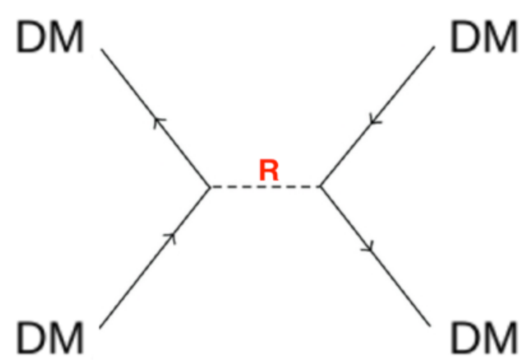
Popular models of SIDM:

1. SIDM via a light mediator *[D.N. Spergel & P. J. Steinhardt 1999, J. Feng, M. Kaplinghat & H.-B. Yu 2009, …]*



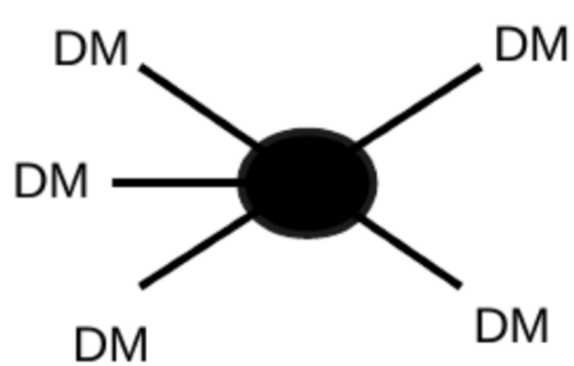
t-channel elastic scattering
&
light mediator enhances cross sections

2. SIDM via Breit-Wigner resonance *[M. Ibe & H.-B. Yu 2009, E. Braaten & E.W.Hammer 2013, **XC**, C. Garcia-Cely, H. Murayama 2018, …]*



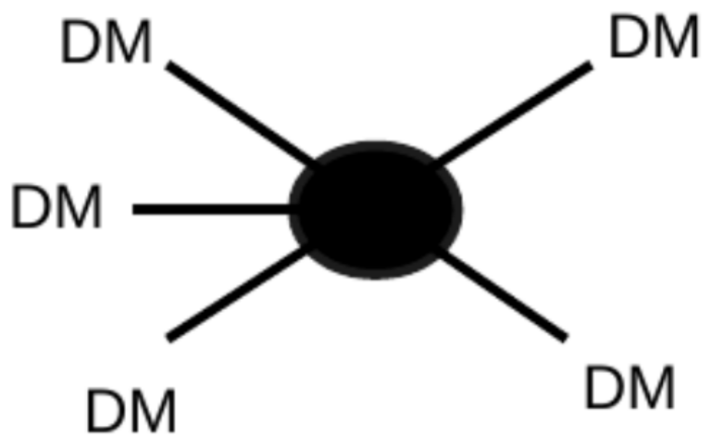
s-channel enhancement
&
velocity-dependence

3. N-to-2 strongly-interacting-massive-particles (SIMP) *[E. Carlson, M. Machacek & L. Hall, 1992, A. de Laix, R.Scherrer & R.Schaefer 1995, Hochberg, Kuflik, Volansky & Wacker 2014, …]*



kinetic equilibrium with radiation
&
relic abundance requires strong interaction.

Zoom-in strongly-interacting-massive-particles (**SIMP**)



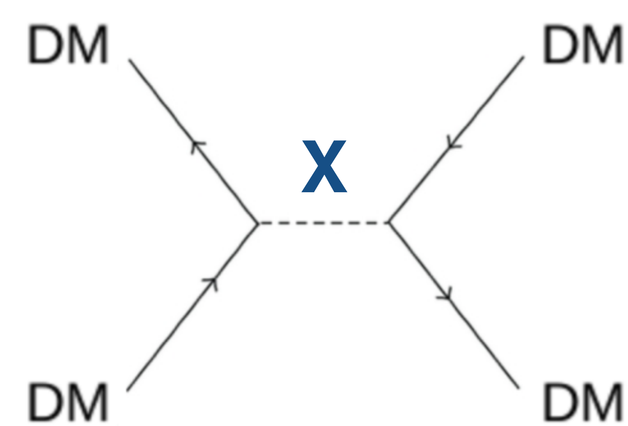
1. **Very-constrained** sub-GeV mass range;
2. Only **constant** self-scattering.

$$\frac{\sigma_{\rm SI}}{m_{\rm DM}} \simeq \frac{1}{m_{\rm DM}^3} \simeq \frac{1}{(60\,{\rm MeV})^3}$$

Meanwhile, **order-one interactions** indicate potentially **on-shell bound state formation**;

Off-shell resonance *in 3→2 SIMP*

[e.g. S. Choi&H.Lee 2016, S.Choi, Y.Kang&H.Lee 2016, A.Berlin, N.Blinov, S.Gori, P.Schuster&N.Toro 2018, U.Dey, T.Maity&T.Ray 2018, P.Braata&M.Postma 2023, …]

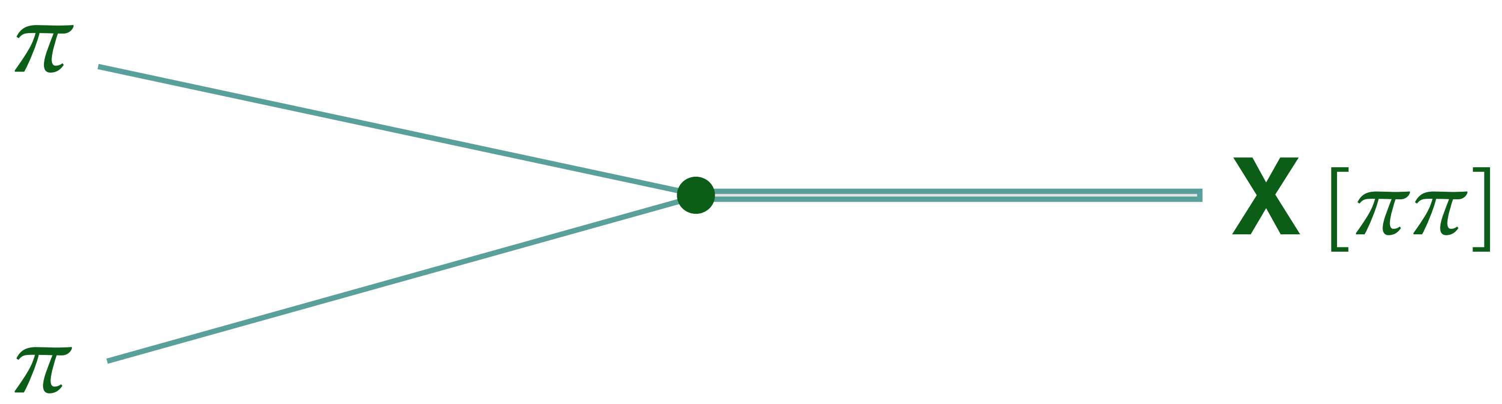


1. Larger mass range for DM;
2. Velocity-dependent self-scattering.

☑ **The framework**

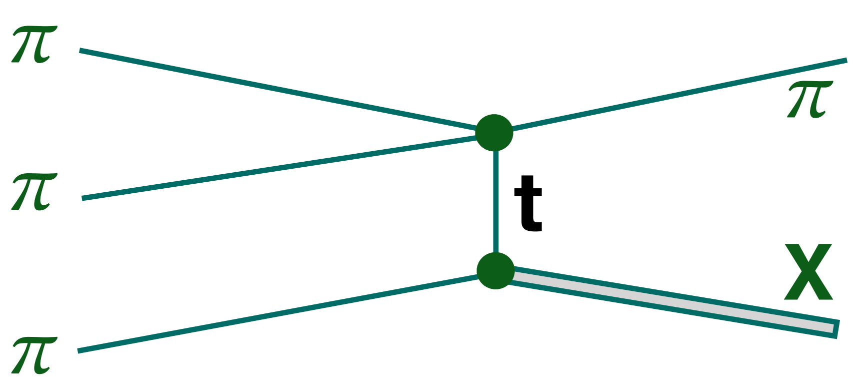
☐ **Discussion of model building**

II. The framework: SIMP + X



<u>step-1</u>: **formation** of bound state, X

- Use Bethe-Sapleter equation at non-relativistic limit *[e.g. K.Petraki, M.Postma&J.de Vries 2016, …]* :



$$M(p_1, p_2, p_3 \to k, Q)_{3\pi\to\pi X}$$

$$\simeq \frac{1}{\sqrt{\mu_{\rm re.}}} \int \frac{d^3q}{(2\pi)^3} \tilde{\psi}^\star_X(\vec{q}) \int \frac{dq_0}{2\pi} \frac{S(q;Q)}{S_0(\vec{q};\vec{Q})} \times M^{\rm free}_{(p_1,p_2,p_3\to k,Q/2+q,Q/2-q)}$$

$$\simeq \frac{\sqrt{2m_X}}{2m_\pi} \int \frac{d^3q}{(2\pi)^3} \int d^3r\, \psi^\star_X(\vec{r})\, e^{-i\vec{q}\vec{r}} \times M^{\rm free}_{(p_1,p_2,p_3\to k,Q/2+q,Q/2-q)}$$

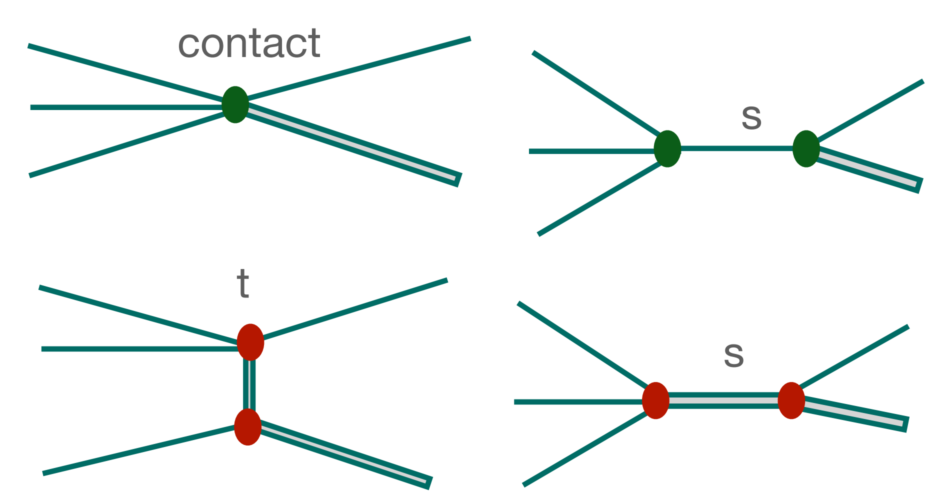
agreeing with Peskin's notation with $\int \frac{d^3p}{(2\pi)^3} \tilde{\psi}^\star_n(\boldsymbol{p})\tilde{\psi}_n(\boldsymbol{p}) = \int d^3r \psi^\star_n(\boldsymbol{r})\psi_n(\boldsymbol{r}) = 1$

[bound state wave-function: **dim-3/2**]

- Set all initial states to be at rest, one obtains a t-channel **enhancement**:

$$\frac{s}{t - m_\pi^2} \propto \frac{m_\pi^2}{m_X^2 - 4m_\pi^2} \propto \frac{m_\pi}{E_B} \gg 1$$

*Subleading (with **odd-number** vertices):*



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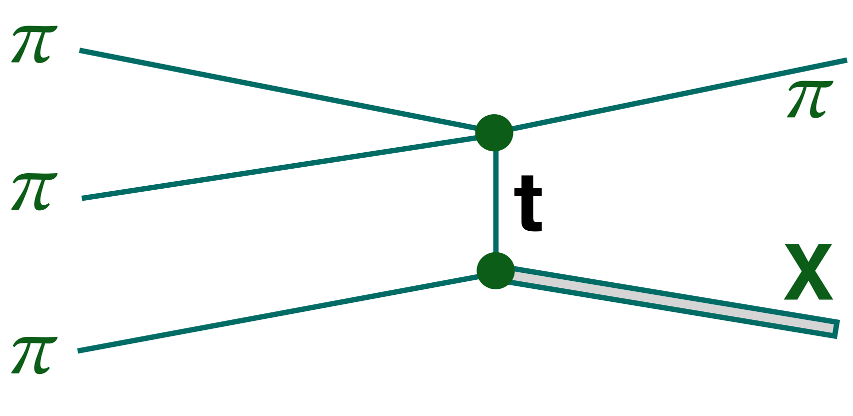

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$$\simeq \frac{\sqrt{2m_X}}{2m_\pi} \int \frac{d^3 q}{(2\pi)^3} \int d^3 r\, \psi^\star_X(\vec{r})\, e^{-i\vec{q}\vec{r}} \times M^{\rm free}_{(p_1,p_2,p_3\to k,Q/2+q,Q/2-q)}$$

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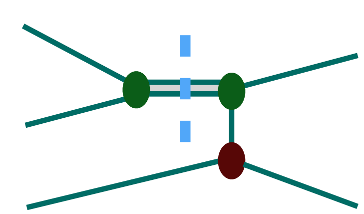
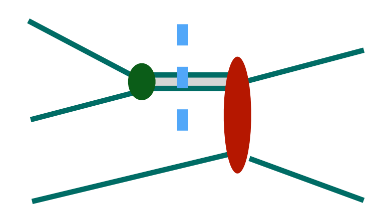
besides, s-channel enhancement is achieved via **on-shellness** of X:



Yukawa-like
symmetric indices



WZW-like
anti-symmetric indices

- Therefore:

1. Only **even-number** vertices involved, so far;
2. The rate of $3\pi\to\pi X$ can be much **larger** than the (v-suppressed) rate of $3\pi\to 2\pi$.

step-2: **mass reduction** with bound state, X

- Processes with mass reduction:

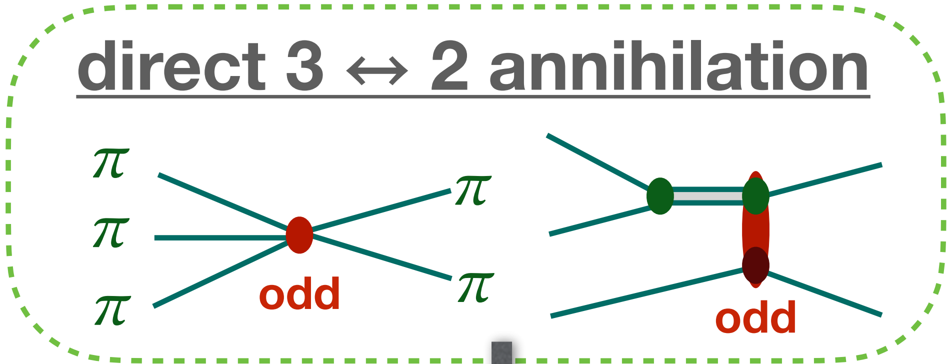
  (total DM number, free or bound, reduces)

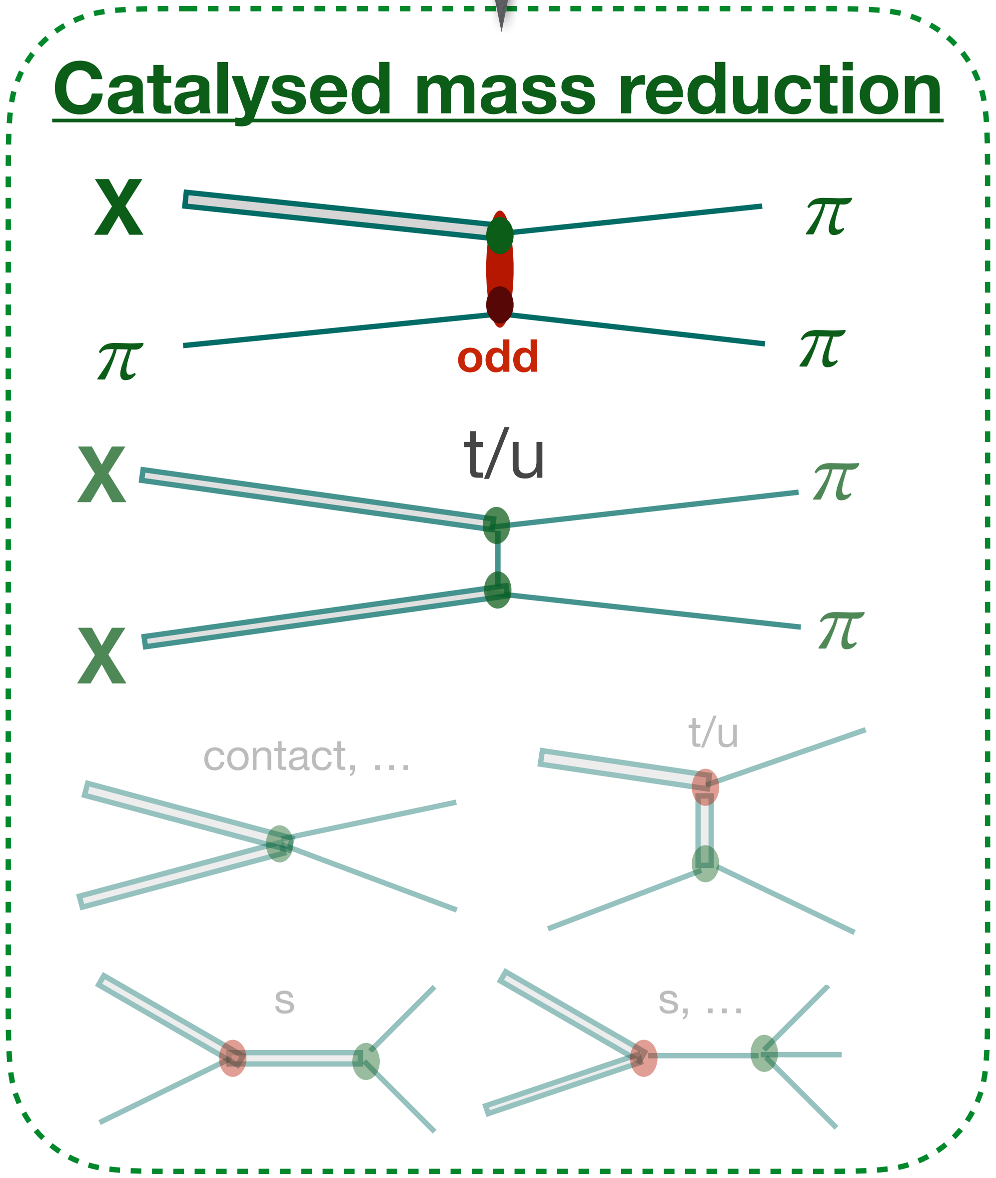
2-body processes generally **sufficient** before BBN (even CMB):

$$\langle \sigma_{\rm anni.} v \rangle \gtrsim \frac{H}{s\, Y_{\rm obs}} \simeq \frac{1}{4\times 10^{-10}\,{\rm GeV}} \frac{m_\pi/T}{m_{\rm pl}} \gtrsim \frac{10^{-9}\,(m_\pi/T)}{{\rm GeV}^2}$$

also from $\frac{\langle \sigma_{\rm anni.} v\rangle}{m_\pi} \gtrsim \frac{T}{\rm MeV} \times 10^{-9}\,{\rm cm}^2/{\rm gram}$

So exact rates of 2-body processes may not be important *[not always true, due to anti-symmetric property of the WZW interaction, or very small abundance of X, etc.]*





step-2: **mass reduction** with bound state, X

- Processes with mass reduction:

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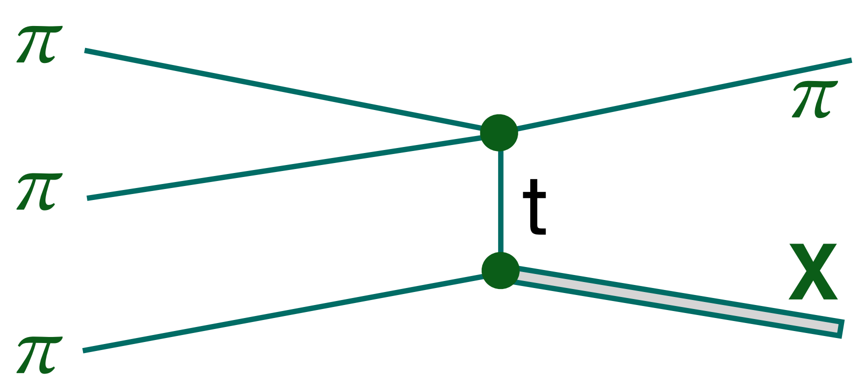
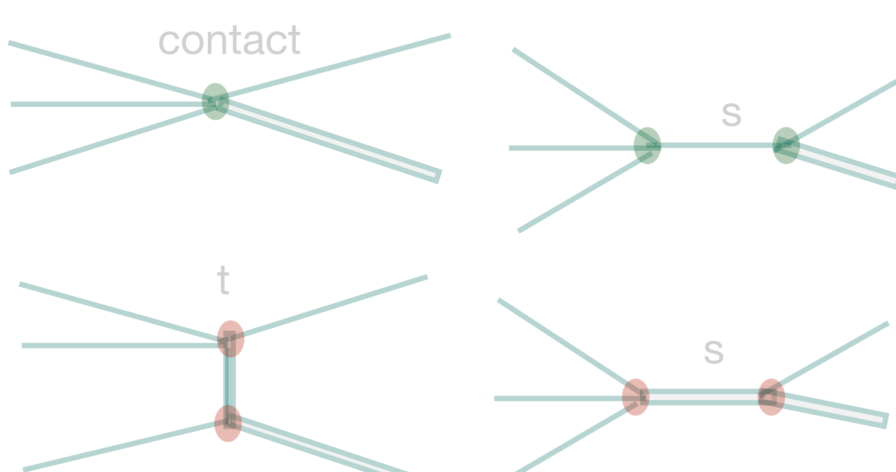
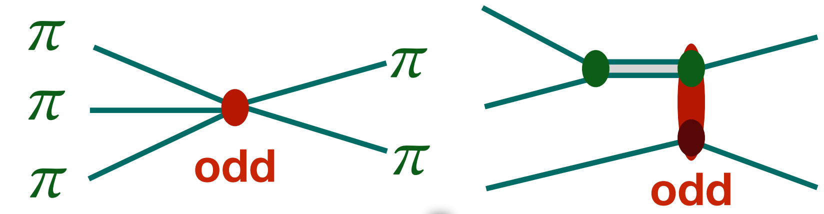
2-body processes generally **sufficient** before BBN (even CMB). That is, the **bottleneck for DM freeze-out** is

bound state formation

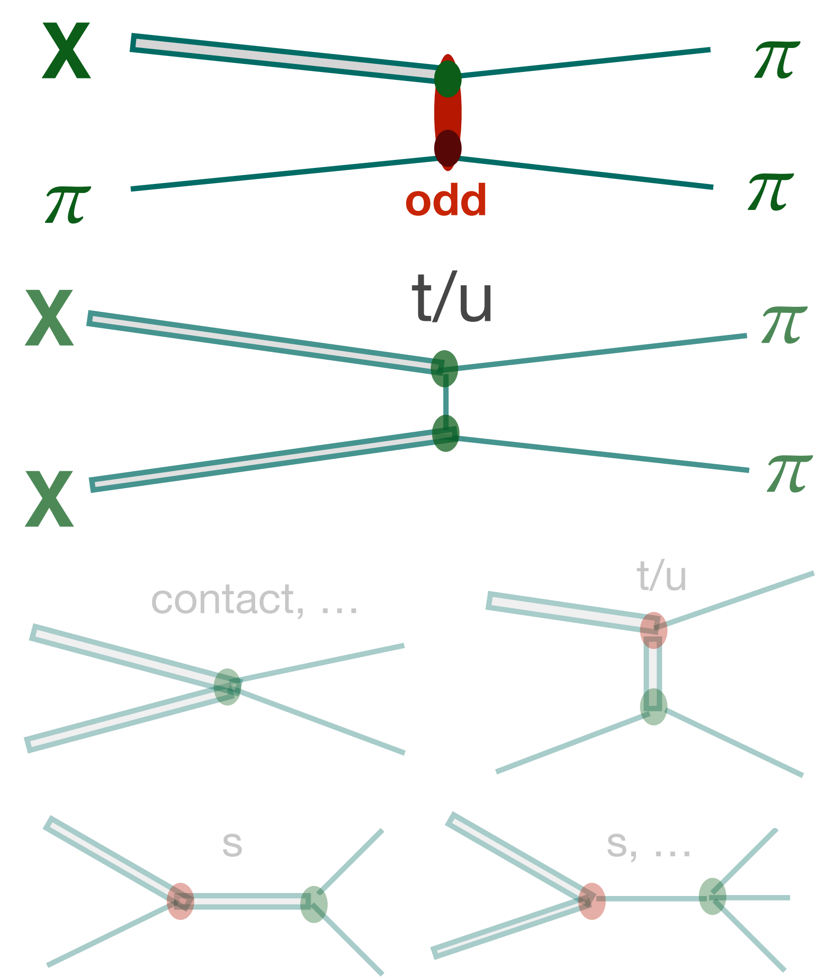




direct 3 ↔ 2 annihilation

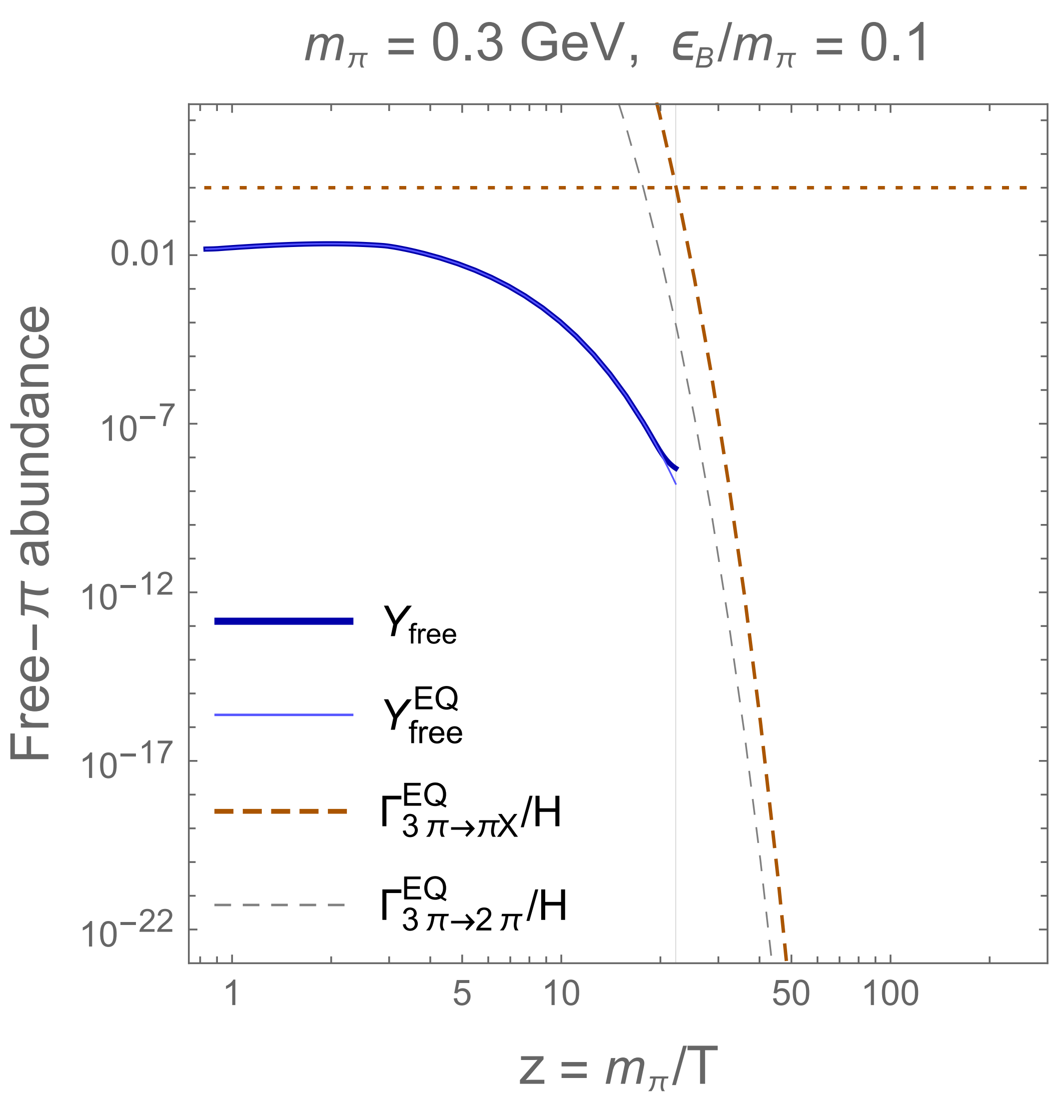


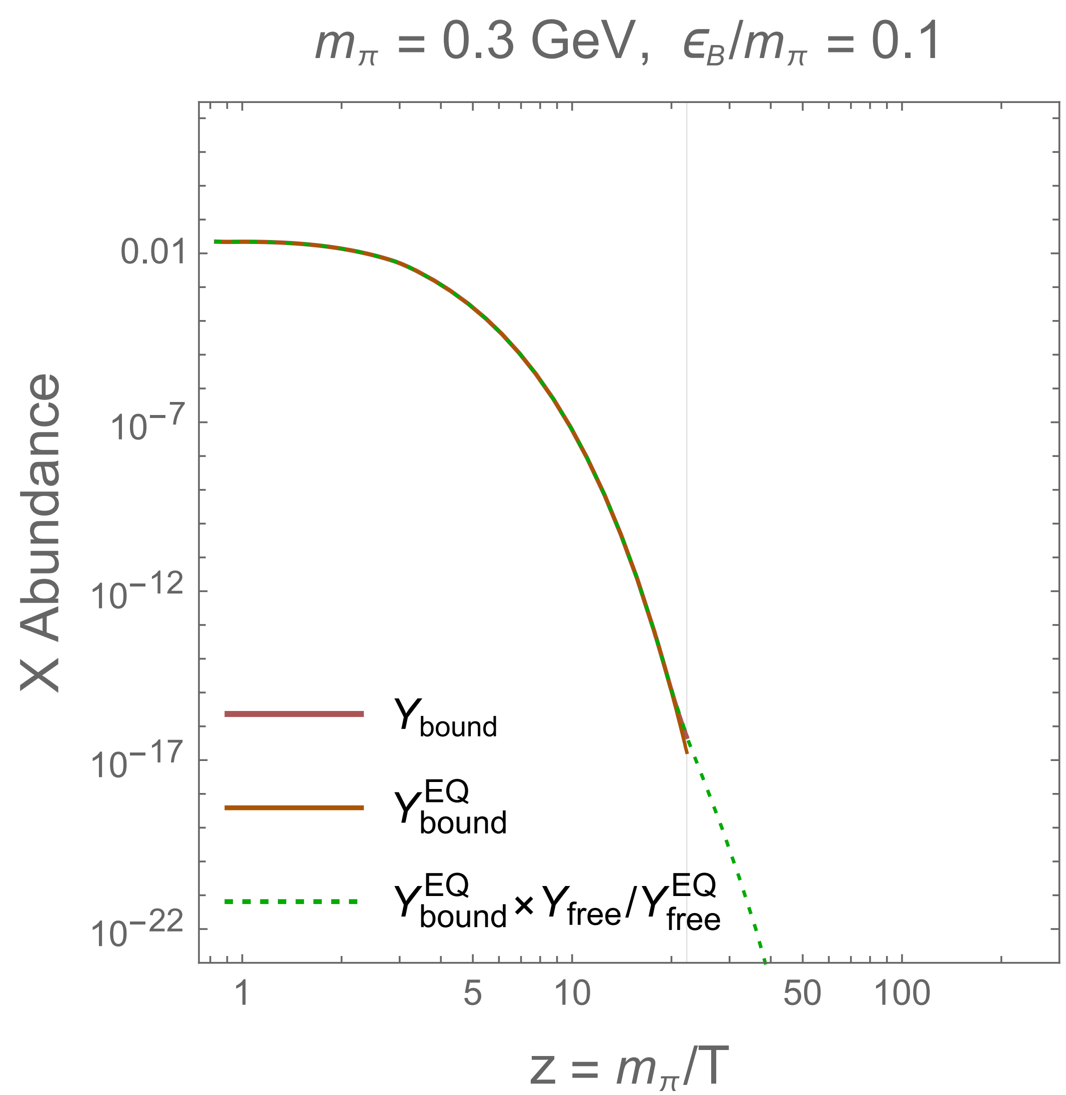
Catalysed mass reduction



One-step freeze-out: numerical solutions

If consider the Wess-Zumino-Witten (WZW) term with simple parametrization:





1. When the $3\pi\to\pi X$ is larger than H, chemical equilibrium lasts;

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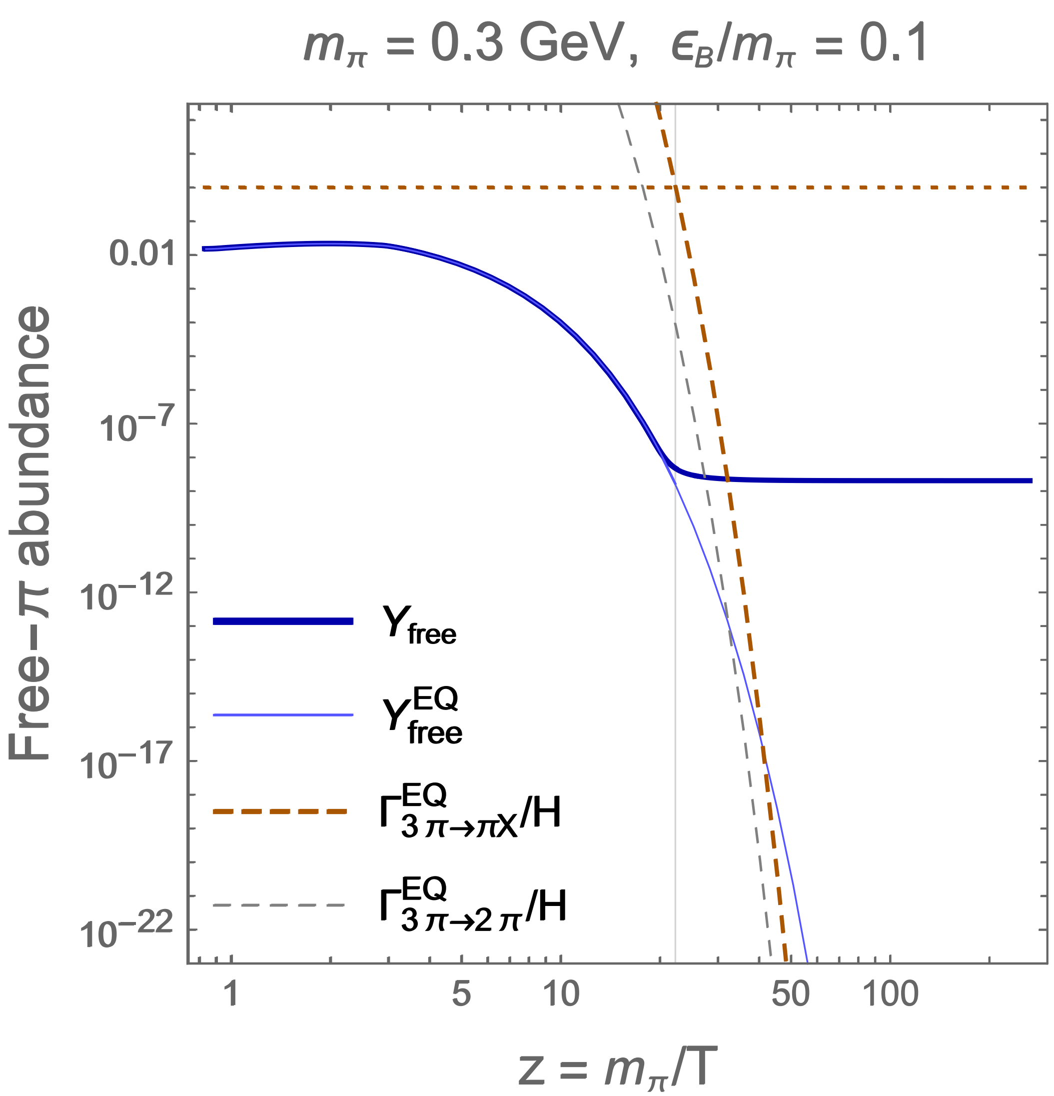
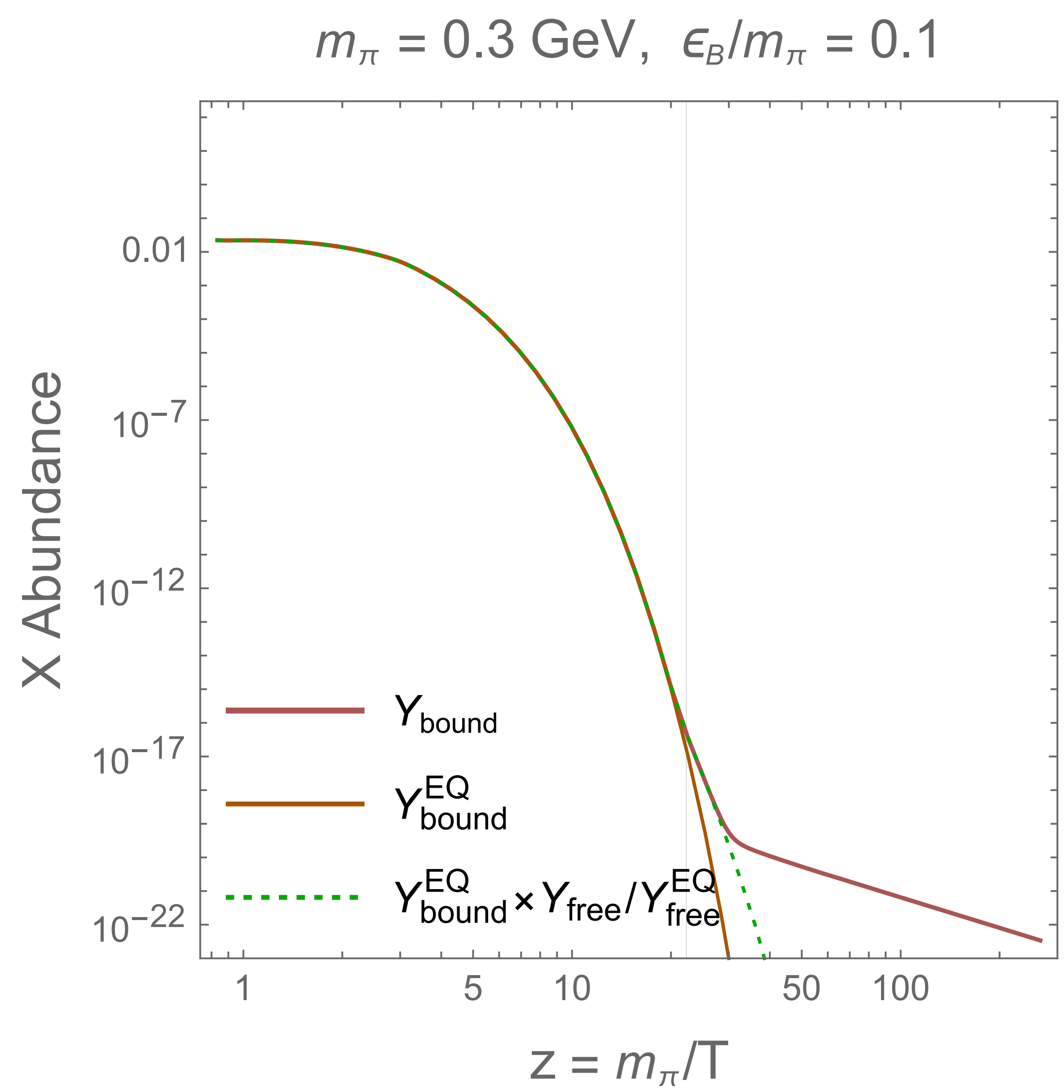




1. When the $3\pi\to\pi X$ is larger than H, chemical equilibrium lasts;
2. At $\Gamma_{3\pi\to\pi X}$ = H: **free-π abundance** freezes,

   X abundance keeps decreasing *(EQ, then quasi-EQ)*.

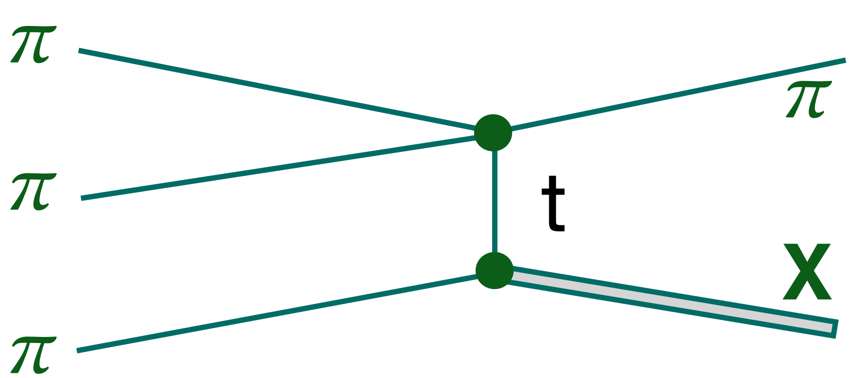
Two-step freeze-out: with only even-number vertex

2-body processes **generally** sufficient before BBN/CMB. **But not in this case with even-number vertex, when**

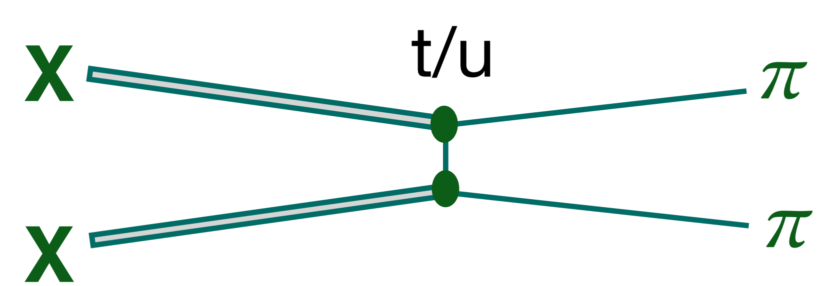
$$Y_X \ll Y_\pi$$

here the bottleneck for DM freeze-out is more complicated:

bound state formation



Catalysed mass reduction

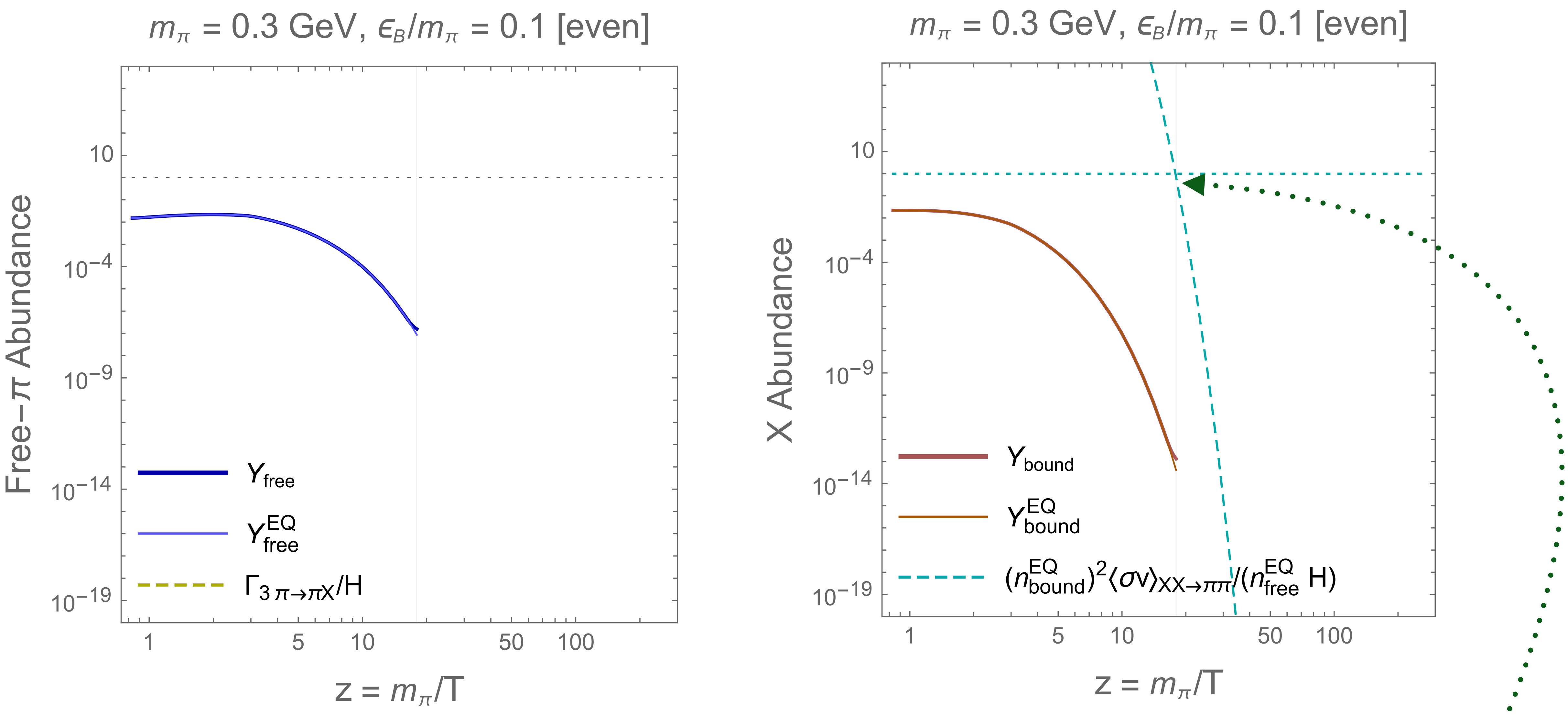


Nevertheless, it is appealing: **odd-number vertex is not necessary for SIMP**.

It thus applies to **fermionic DM**, allowing **much larger masses** than direct 4→2 SIMP *[N.Bernal & **XC**, 2015]*.

Two-step freeze-out: with only even-number vertex

If only consider **even-vertex interactions** (neither Yukawa nor WZW):



1. First **decoupling** happens **when XX→$\pi\pi$ can not change free-π abundance**;

Two-step freeze-out: with only even-number vertex

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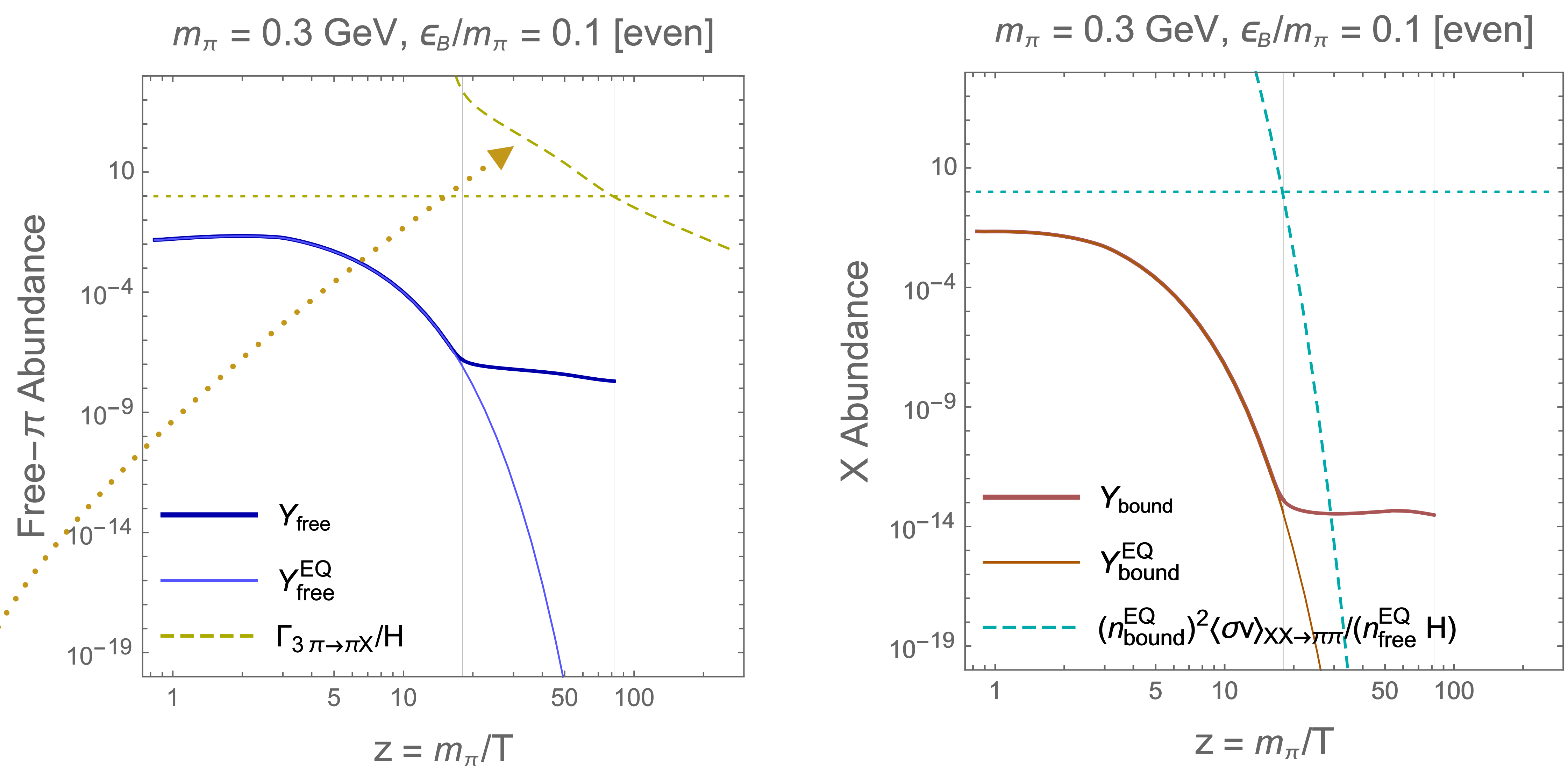

1. First **decoupling** happens **when XX→ππ can not change free-π abundance**;
2. Fast 3π↔πX leads to **non-zero** chemical potential;

Boltzmann-suppression only from **binding energy:** $Y_\pi^2 = Y_X \frac{(Y_\pi^{\rm EQ})^2}{Y_X^{\rm EQ}} \sim Y_X\, e^{-|E_B|/T}\left(\frac{m_\pi^2}{m_X T}\right)^{3/2}$

Two-step freeze-out: with only even-number vertex

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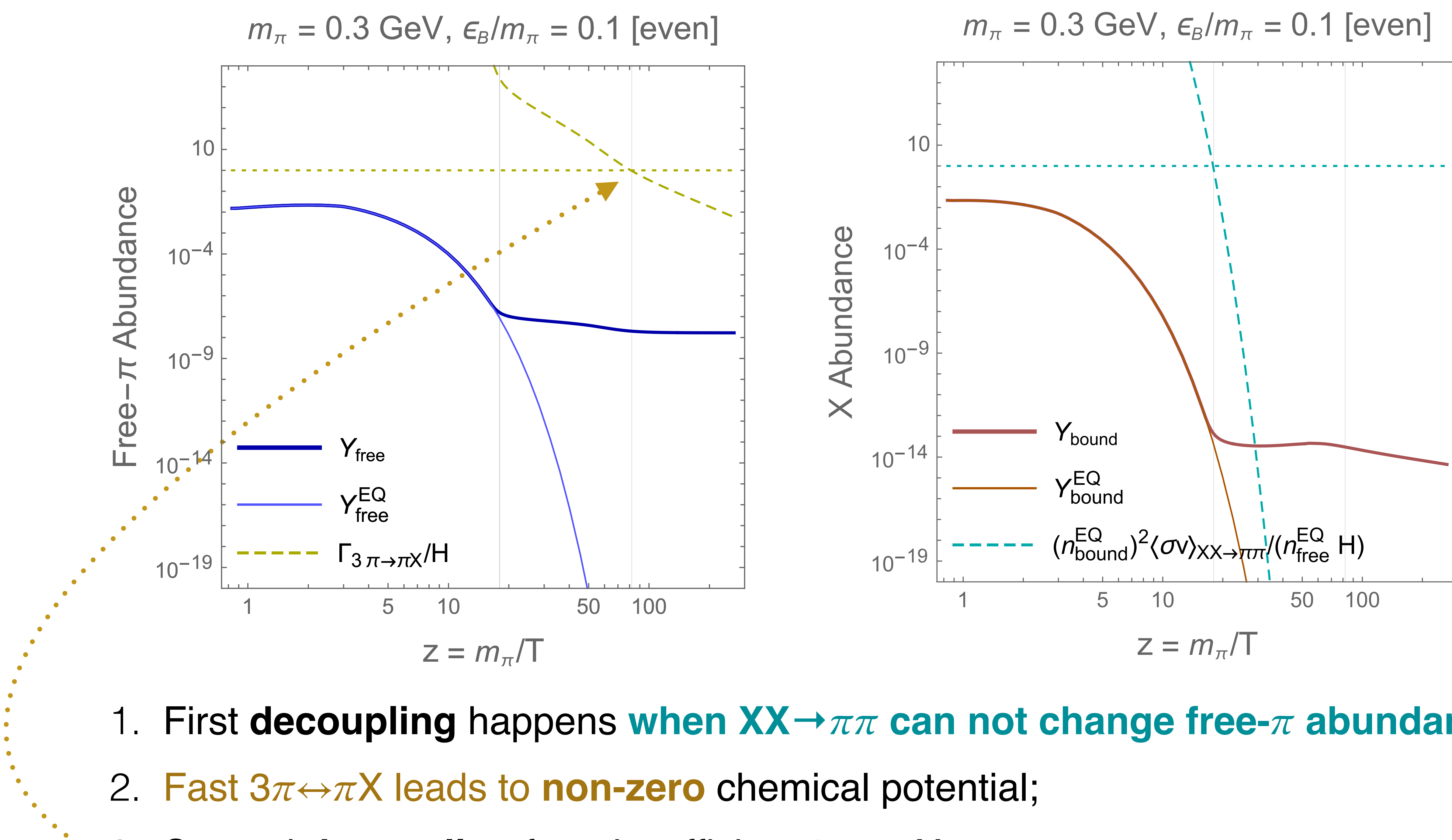
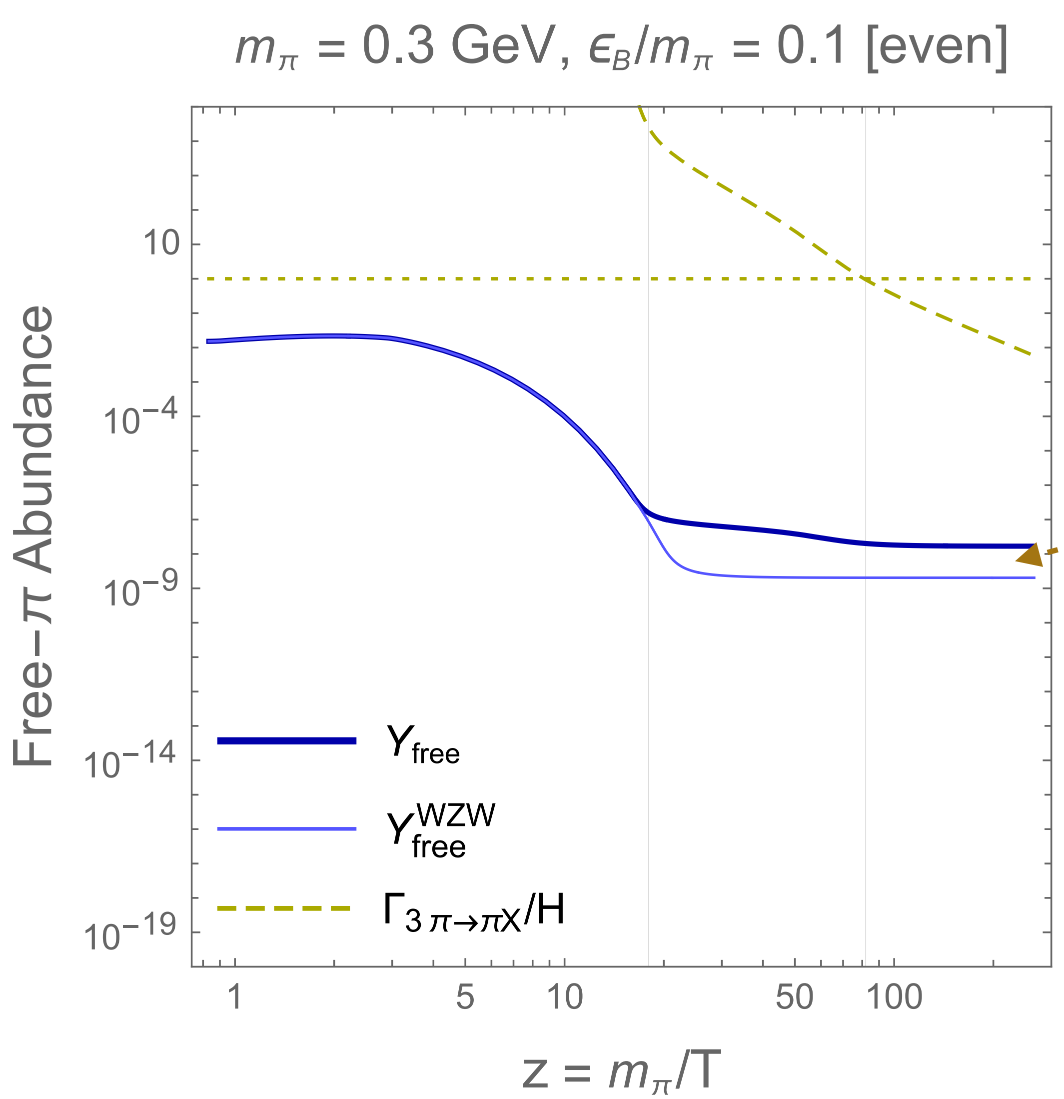


1. First **decoupling** happens **when XX→ππ can not change free-π abundance**;
2. Fast 3π↔πX leads to **non-zero** chemical potential;
3. Second **decoupling** from insufficient 3π↔πX (while X re-starts to decrease via XX→ππ).

Two-step freeze-out: with only even-number vertex



- That is, with only even-vertex the final **freeze-out happens later**, at

$$Y_\pi^2 s^2 \langle \sigma_{3\pi\to\pi X} v^2 \rangle \sim H$$

fixed cross sections

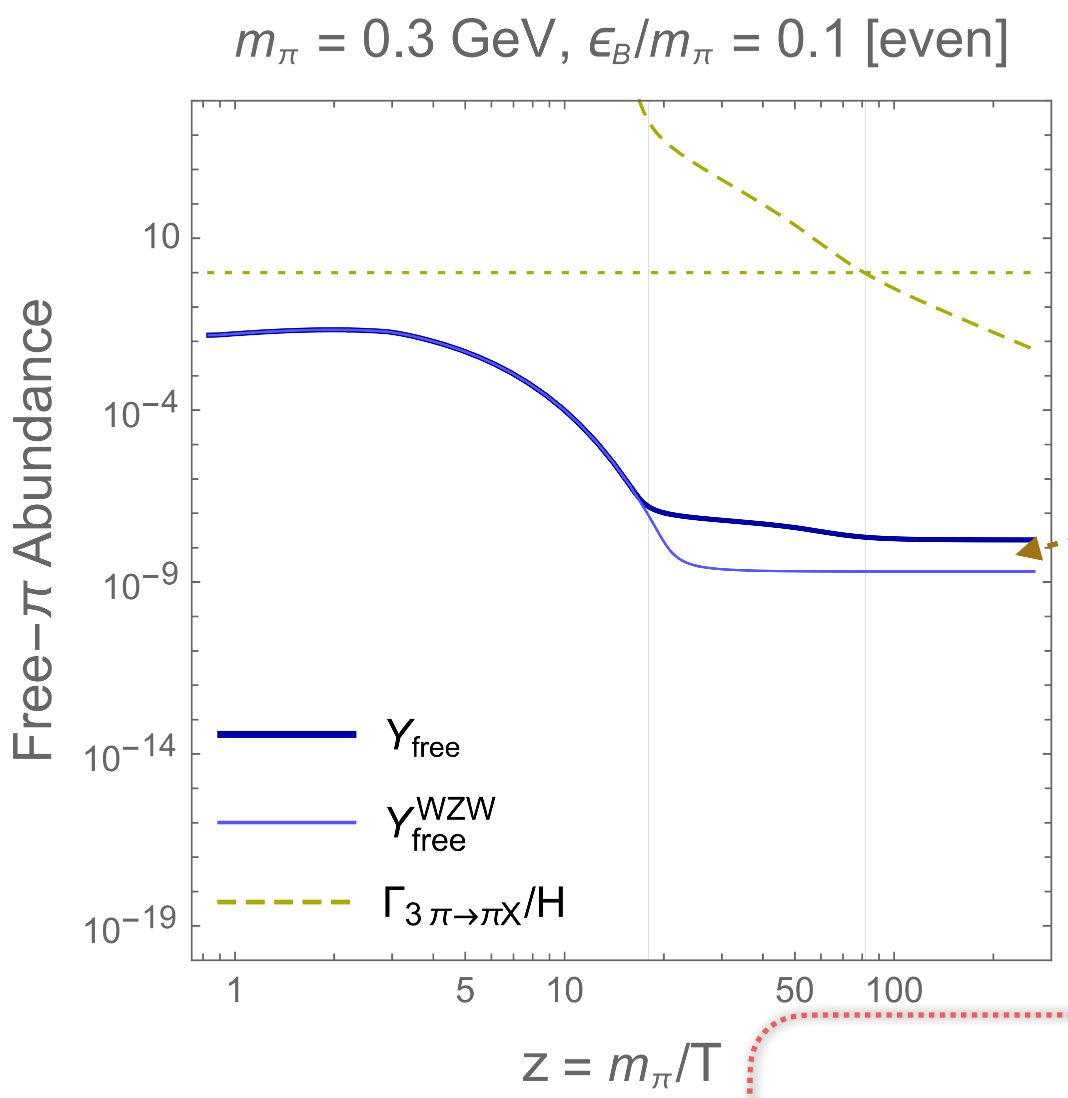
$$Y_\pi \propto \sqrt{H}/s \propto 1/T^2$$

Later freeze-out yields **larger abundance**.

1. Successful **freeze-out for GeV DM**, instead of sub-MeV *[4→2 in N.Bernal & **XC**, 2015]*.

2. Potential **warm DM** if kinetic decoupling happens before final freeze-out.

Two-step freeze-out: with only even-number vertex



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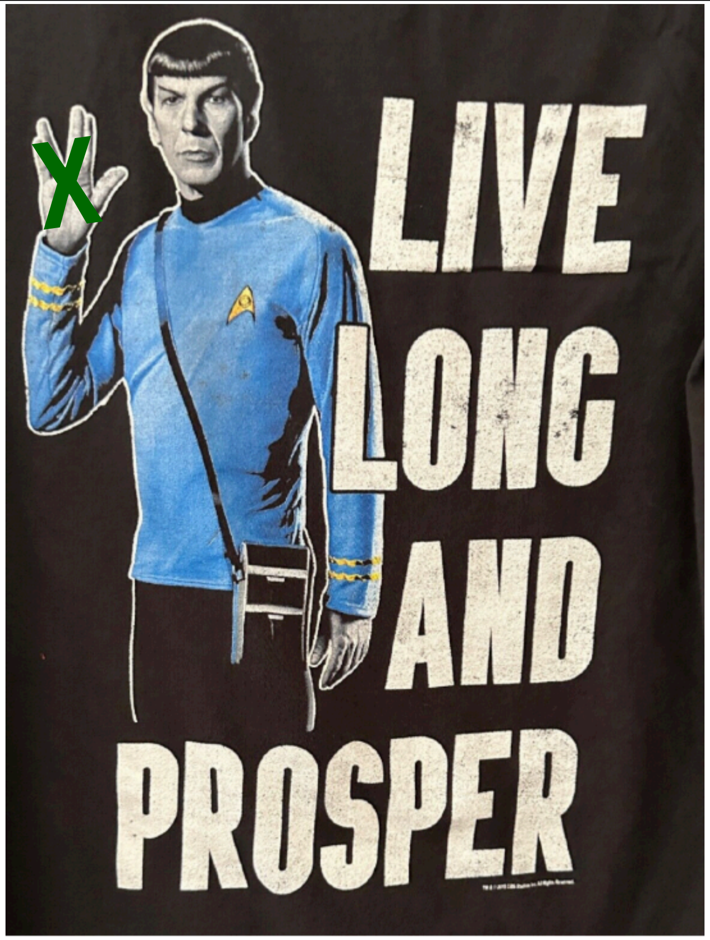
☐ **The framework**

☑ **Discussion of model building**

II. Realization of SIMP + **X**

Requirements on model realization



- **X** needs to be **long-lived**

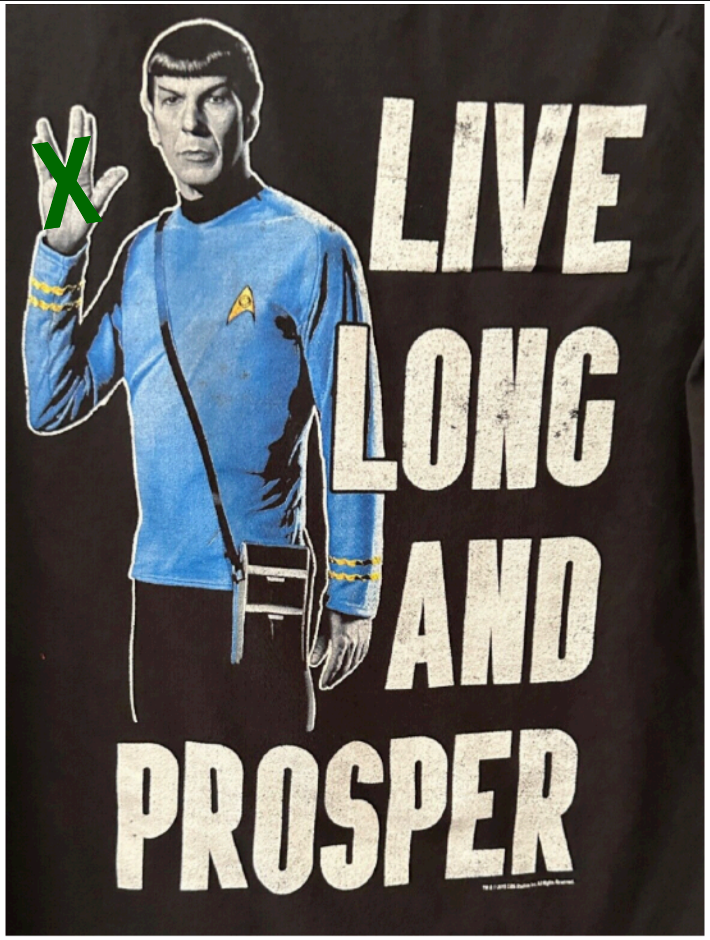
1. suppress **X** → $\pi\pi$: bound states appear in QCD, but they are usually unstable *[e.g. glueball-onium, tetraquark]*.

Easy to achieve with $m_X \leq 2m_\pi$, **induced by dark-QCD with heavy quarks**, or other short-range forces *[e.g. G.Kribs&E.Neil 2016, Y.Tsai, R.McGehee&H.Murayama 2020, R. Mahbubania, M.Redic&A.Tesi 2020, ….]*.

2. suppress **X** → radiation: **number-changing processes** generally leave **X**-stability symmetry-unprotected. Thus, **kinetic equilibrium** of DM and (dark) radiation may force X **quickly decay to radiation.**

Not easy to achieve *[$\mathbb{Z}_4$* DM *is safe, where DM-R* scattering *is tricky].*

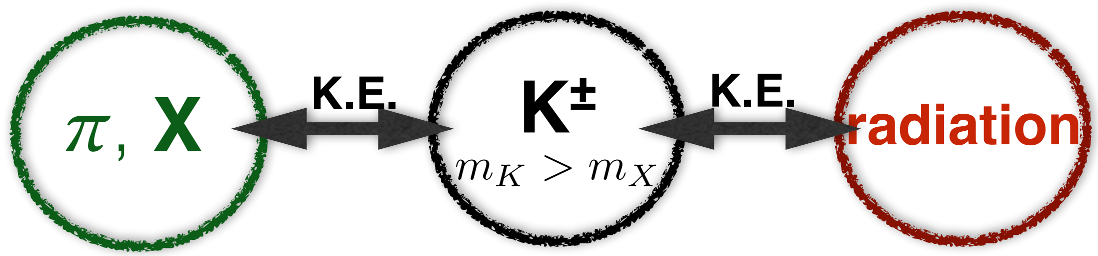
Requirements on model realization



- **X** needs to be **long-lived**

But, there are several ways out:

a) introduce heavier intermediate sates, **$K^{\pm}$**



Extra mass of K may come from heavier dark-quark, additional-charge corrections, or K is X …

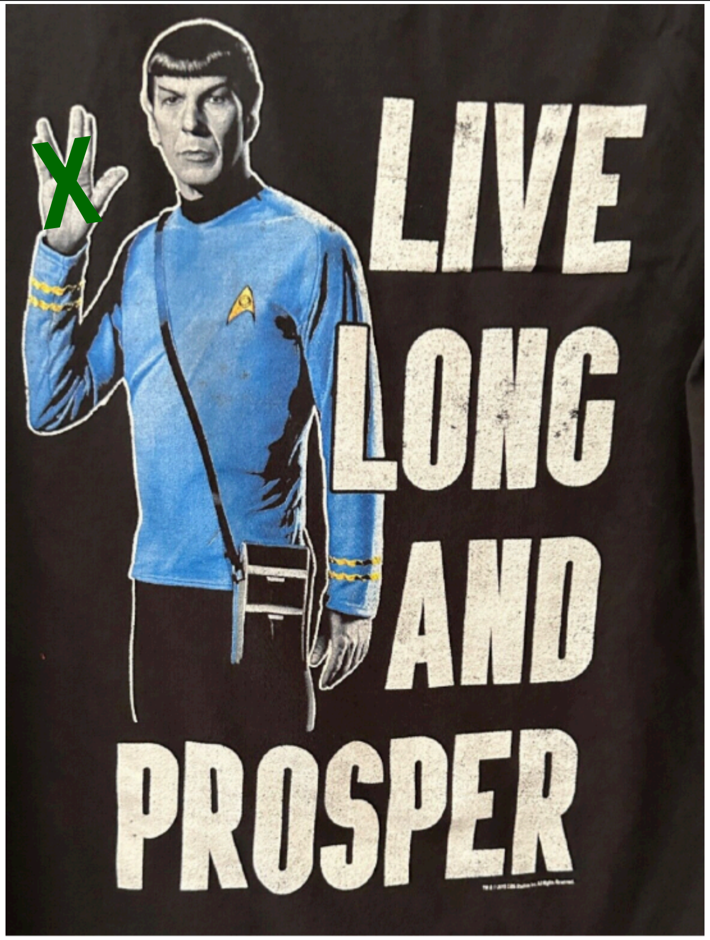
b) impose quantum number for X (e.g. dark pion case)

$$X_{ij} = [\pi_i \pi_j]$$

Energy leak via $[\pi_i \pi_i]$ could last:

improve by enhancing off-diagonal potential?

Requirements on model realization

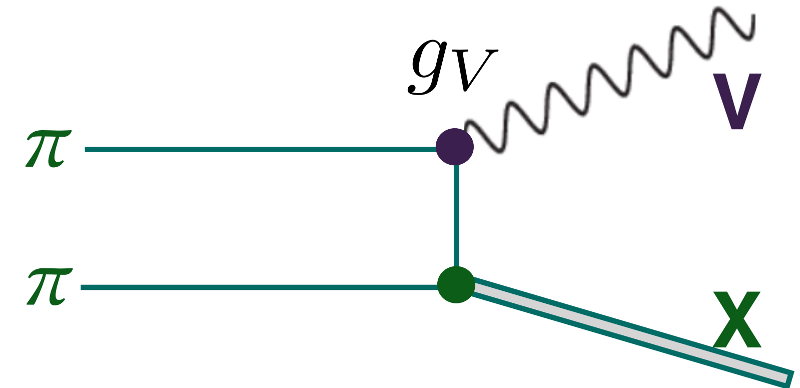


✳ **Boost the formation** of **X**

$3\pi \to \pi X$ needs **three initial states:**

a) what if **gauged**, to

use **two** initial states?

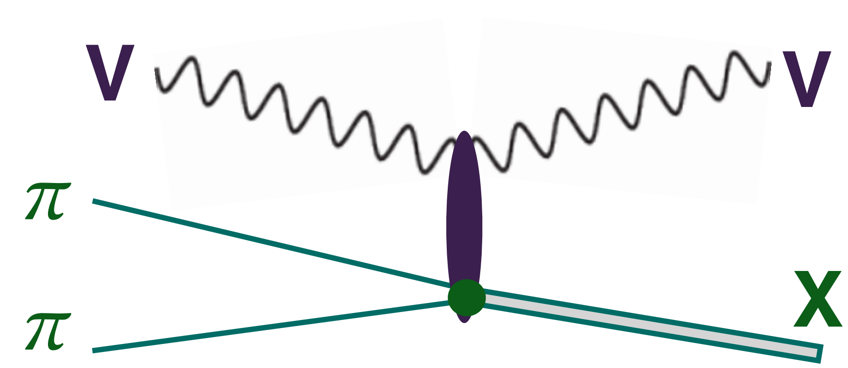


X is bound by **strong** force, as gV is feeble.

1. Introducing **nearly-massless gauge boson** strong enough to make DM in **kinetic equilibrium** with (dark) radiation *[e.g. N.Bernal, C.Garcia-Cely&R.Rosenfeld 2015, …];*
2. **Additional channel $\pi\pi \to$ VV** and/or **long-liveness of X**: $g_V \sim 10^{-7} - 10^{-5}$

b) **Radiation-assisted** processes?

$$n_V \gg n_\pi$$



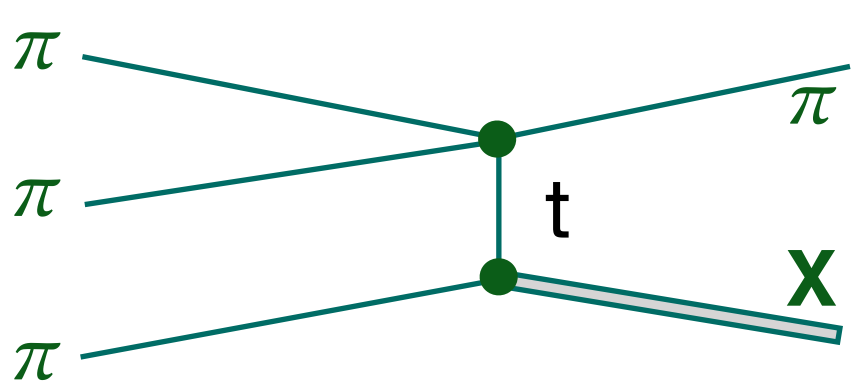
After taking a short-range potential:

In practice, all information is embedded in the **wave-functions**, or for ground state, the effective **volume of the potential**:

$$\psi_X^\star(\vec{r}=0) \sim \frac{1}{\sqrt{V_{\rm pot.}}}$$

Relevant Processes with bound states:

1. As mentioned already one takes Peskin's notation (or Bethe-Sapleter equation):



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$$\simeq \frac{\sqrt{2m_X}}{2m_\pi}\int\frac{d^3q}{(2\pi)^3}\int d^3r\,\psi_X^\star(\vec{r})\,e^{-i\vec{q}\vec{r}}\times M^{\rm free}_{(p_1,p_2,p_3\to k,Q/2+q,Q/2-q)}$$

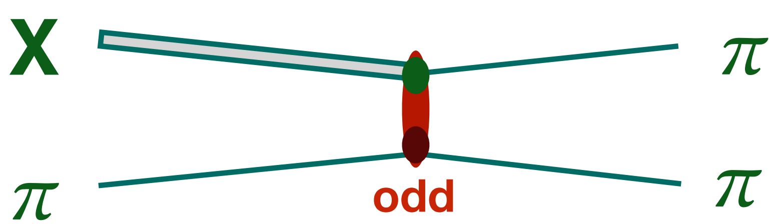
- For constant *M*: $\psi_X^\star(\vec{r}=0) \sim \frac{1}{\sqrt{V_{\rm pot.}}}$
  non-vanish for **ground-state wave-function** w.r.t. a finite-range spherical potential.

- For $M \propto p_i p_j$: $\nabla_i\nabla_j\psi_X^\star(\vec{r}=0)$ divergent?
  to be **regularized** to ~ $|m_\pi E_B|\psi(0)$ *[e.g. N.Brambilla, D.Eiras et al, 2002, S. Biondinia&V.Shtabovenko 2020, ..]* .

After taking a short-range potential:

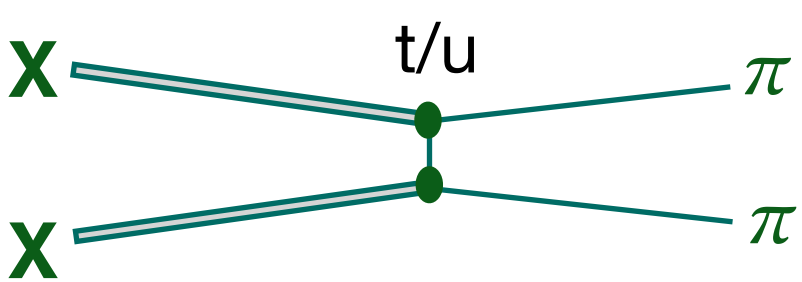
Relevant Processes with bound states:

2. At leading-order, $X\pi\to\pi\pi$ may need $\ell>0$ bound states:



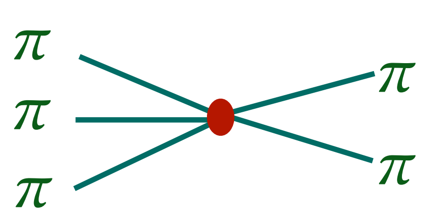
$$
\begin{aligned}
&M(P,p\to k_1,k_2)_{\pi X\to 2\pi} \\
&\simeq \frac{\sqrt{2m_X}}{2m_\pi}\int\frac{d^3q}{(2\pi)^3}\int d^3r\,\psi_X(\vec{r})\,e^{i\vec{q}\vec{r}}\times M^{\rm WZW}_{(P/2+q,P/2-q,p\to k_1,k_2)} \\
&\simeq \frac{\sqrt{2m_X}}{2m_\pi}\frac{N_c}{2\sqrt{2}\pi^2 f_\pi^5}\iint\frac{d^3r d^3q}{(2\pi)^3}\,\psi_X(\vec{r})\,e^{i\vec{q}\vec{r}}\times q^\nu(\varepsilon_{\mu\nu\rho\sigma}P^\mu p^\rho k_1^\sigma)
\end{aligned}
$$

and/or XX→$\pi\pi$ can be considered as co-decay:



$$
M(P_1,P_2\to k_1,k_2)\propto\frac{\psi_{X,1}(\vec{r}_1=0)\psi_{X,2}(\vec{r}_2=0)}{m_\pi^3(1-2m_\pi^2/m_X^2)}
$$

3. All free-states processes, e.g. WZW-like interaction



Free DM particles via WZW interactions:

$$
(\sigma v^2)_{3\to 2}\overset{N.R}{=}\frac{\sqrt{5}}{3}\frac{|\mathcal{M}|^2_{3\to 2}}{64\pi m_\pi^3}\overset{x\gg 1}{=}\frac{25N_c^2 m_\pi^5}{16\pi^5\,64\sqrt{5}f_\pi^{10}x^2}+O(1/x^3)
$$

IV. Conclusions

Conclusions

- Halo mass deficit may be **hint of DM self-interaction,** such as **SIMP**
- **SIMP** involves N-body processes, where **bound state X** could play a role
    - automatically induce velocity-dependence;
    - modify N→2 freeze-out significantly;
- **Freeze-out** enhanced by **on-shell X** and **t-channel** resonance
    - w/o odd-number vertex: **one-step/two-step**;
    - bottleneck is **three initial-states**: better with extensions;
- **Concrete model** has to satisfy several conditions, leading to constraints
- X can be a **fundamental** particle, sitting near 2π-resonance by accident

Thanks!